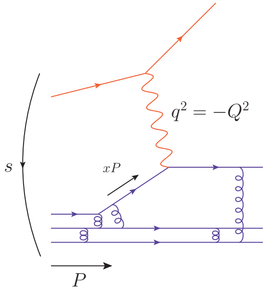
Generalized TMD distributions

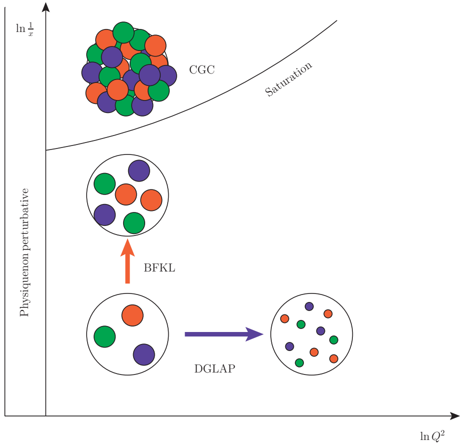
Renaud Boussarie

QCD Evolution 2023

Accessing the partonic content of hadrons with an electromagnetic probe

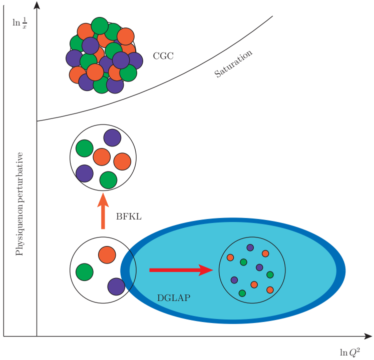


Electron-proton collision (parton model)



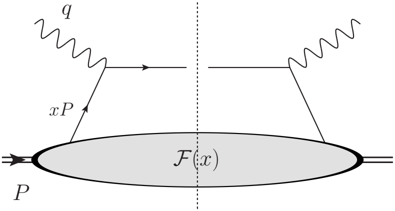
QCD at moderate $x_{\rm Bj} \sim Q^2/s$

Bjorken limit: $Q^2 \sim s$



QCD factorization

processes with a hard scale $Q \gg \Lambda_{QCD}$



$$\sigma = \mathcal{F}(x, \mu) \otimes \mathcal{H}(x, \mu)$$

At a scale μ, the process is factorized into:

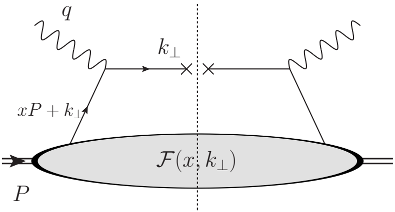
- A hard scattering subamplitude $\mathcal{H}(x, \mu)$
- A Parton Distribution Function (PDF) $\mathcal{F}(x, \mu)$

μ independence: DGLAP renormalization equation for $\mathcal{F}$

Transverse Momentum Dependent (TMD) factorization:

semi-inclusive processes with one hard and one semihard scale

$$Q \sim \sqrt{s} \gg k_\perp$$



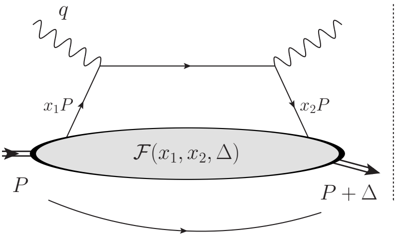
$$\sigma = \mathcal{F}(x, k_\perp, \zeta, \mu) \otimes \mathcal{H}(\mu) \otimes \hat{\mathcal{F}}(\hat{x}, \hat{k}_\perp, \hat{\zeta}, \mu)$$

At a scale μ, the process is factorized into:

- A hard scattering subamplitude $\mathcal{H}(\mu)$
- A TMD PDF $\mathcal{F}(x, k_\perp, \zeta, \mu)$
- A TMD FF $\hat{\mathcal{F}}(\hat{x}, \hat{k}_\perp, \hat{\zeta}, \mu)$

$\mu, \zeta, \hat{\zeta}$ independence: TMD evolution for $\mathcal{F}, \hat{\mathcal{F}}$

Factorization with Generalized Parton Distributions (GPD): exclusive processes with one hard scale $Q \sim \sqrt{s}$



$$\sigma = \mathcal{F}(x_1, x_2, |\Delta_\perp|, \mu) \otimes \mathcal{H}\,(x_1, x_2, \mu)$$

At a scale μ, the process is factorized into:

- A hard scattering subamplitude $\mathcal{H}(x_1, x_2, \mu)$
- A Generalized Parton Distribution (GPD) $\mathcal{F}(x_1, x_2, |\Delta_\perp|, \mu)$

μ independence: DGLAP/ERBL renormalization equation for $\mathcal{F}$

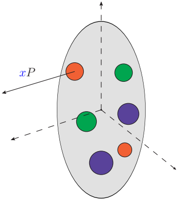
Operator definition for parton distributions

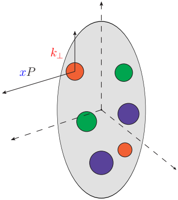
Parton distribution function

$$\mathcal{F}(x) \propto \int dz^+ e^{ixP^- z^+} \left\langle P \left| F^{-i}(z^+) \left[z^+, 0^+\right] F^{-i}(0) \left[0^+, z^+\right] \right| P \right\rangle$$

Transverse Momentum Dependent distribution

$$\mathcal{F}(x, k_\perp) \propto \int d^4 z \delta(z^-) e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P \left| F^{-i}(z) \mathcal{U}_{z,0} F^{-i}(0) \mathcal{U}_{0,z} \right| P \right\rangle$$





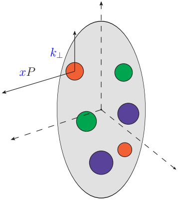
Operator definition for parton distributions

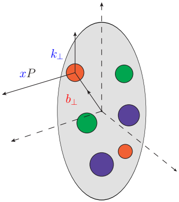
TMD distribution

$$\mathcal{F}(x, k_\perp) \propto \int d^4z\delta(z^-)e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P \middle| F^{-i}(z)\mathcal{U}_{z,0}F^{-i}(0)\mathcal{U}_{0,z} \middle| P \right\rangle$$

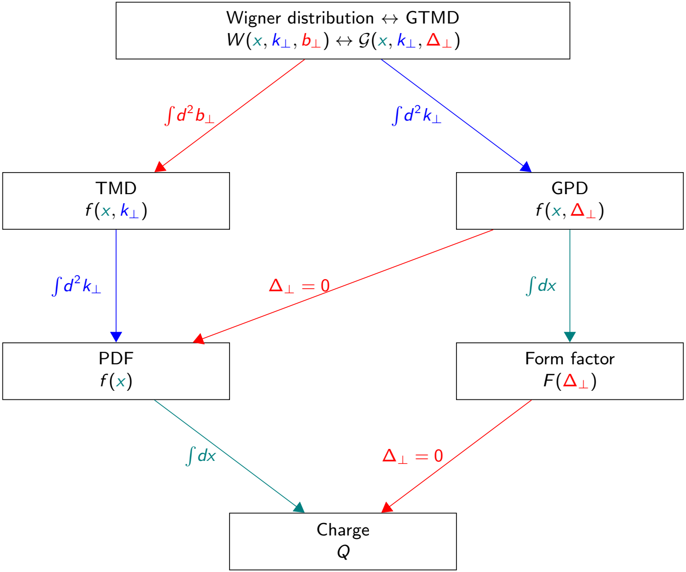
Generalized TMD distribution

$$\mathcal{F}(x, k_\perp, \Delta) \propto \int d^4z\delta(z^-)e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P+\Delta \middle| F^{-i}(z)\mathcal{U}_{z,0}F^{-i}(0)\mathcal{U}_{0,z} \middle| P \right\rangle$$





The family tree of parton distributions



Wigner distributions in NRQM

Wigner distributions in Quantum Mechanics [Wigner, 1932]

Defined via wavefunctions

$$W(x,k) = \int \frac{dx'}{2\pi} e^{-i(k\cdot x')} \psi\left(x + \frac{x'}{2}\right) \psi^*\left(x - \frac{x'}{2}\right)$$

Connection with probability densities:

$$|\psi(x)|^2 = \int dk\, W(x,k)$$

and

$$|\psi(k)|^2 = \int dx\, W(x,k)$$

Connection with observables:

$$\langle O \rangle = \int dx \int dk\, O(x,k)\, W(x,k)$$

Wigner distributions in QCD

QCD Wigner distributions [Belitsky, Ji, Yuan, 2003]

Defined as the Fourier transform of a GTMD

$$\mathcal{W}^g(x, k_\perp, b_\perp) = \int d^2\Delta_\perp e^{i(b_\perp \cdot \Delta_\perp)} \times \int d^4 z \delta(z^-) e^{ixP^- z^+ + i(k_\perp \cdot z_\perp)} \left\langle P+\Delta_\perp \middle| F^{-i}(z)\mathcal{U}_{z,0} F^{-i}(0)\mathcal{U}_{0,z} \middle| P \right\rangle$$

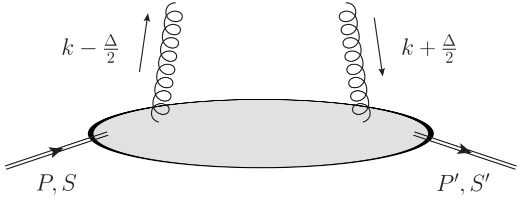
Connection with observables: e.g. orbital angular momentum of gluons inside a proton

$$\langle L_z^g \rangle = \int dx \int d^2 k_\perp \int d^2 b_\perp (b_\perp \times k_\perp)_z W^g(x, k_\perp, b_\perp)$$

GTMD

Parametrization and coupling to the target hadron

[Meissner, Metz, Schlegel, 2009], [Lorcé, Pasquini, 2013]



$$\int d^4v\,\delta(v^-)e^{ix\bar{P}^- v^+ - i(\boldsymbol{k}\cdot\boldsymbol{v})}\langle P'S'|\mathrm{Tr}\left[F^{i-}(-\frac{v}{2})\mathcal{U}^{[+]}_{\frac{v}{2},-\frac{v}{2}}F^{i-}(\frac{v}{2})\mathcal{U}^{[-]}_{-\frac{v}{2},\frac{v}{2}}\right]|PS\rangle$$

$$= (2\pi)^3\frac{\bar{P}^-}{2M}\bar{u}_{P'S'}\left[F^g_{1,1} + i\frac{\sigma^{i-}}{\bar{P}^-}(\boldsymbol{k}^i F^g_{1,2} + \Delta^i F^g_{1,3}) + i\frac{\sigma^{ij}\boldsymbol{k}^i\Delta^j}{M^2}F^g_{1,4}\right]u_{PS}$$

Leading twist gluon TMD distributions

Leading twist gluon TMD distributions

Hadron pol. \ Parton	Unpolarized	Circular	Linear
Unpolarized	f_1^g	$\emptyset$	$h_1^{\perp g}$
Longitudinal	$\emptyset$	g_{1L}^g	$h_{1L}^{\perp g}$
Transverse	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

PDF-spanning

- Unpolarized f_1^g
- Helicity g_{1L}^g

Naive T-even pure TMDs

- Worm-gear $h_{1L}^{\perp g}, g_{1T}^g$
- Pretzelosity $h_{1T}^{\perp g}$
- Transversity h_1^g

Naive T-odd pure TMDs

- Boer-Mulders $h_1^{\perp g}$
- Sivers $f_{1T}^{\perp g}$

Leading twist GPDs

Leading twist GPDs

- Unpolarized parton pairs: H and E
- Polarized parton pairs: $\tilde{H}$ and $\tilde{E}$
- Transversity distributions: H_T, $\tilde{H}_T$, E_T and $\tilde{E}_T$

PDF-spanning

Unpolarized $H^q \to q, -\bar{q}$ and $H^g \to xg$

Helicity $\tilde{H}^q \to \Delta q, \Delta\bar{q}$ and $\tilde{H}^g \to x\Delta g$

Quark transversity $H_T^q \to \delta q$

Leading twist GTMDs

Leading twist GTMDPDFs in a spin 1/2 hadron

[Meissner, Metz, Schlegel, 2009], [Lorcé, Pasquini, 2013]

- Unpolarized parton pairs: $F_{1,1}$, $F_{1,2}$, $F_{1,3}$ and $F_{1,4}$
- Polarized parton pairs: $G_{1,1}$, $G_{1,2}$, $G_{1,3}$ and $G_{1,4}$
- Transversity distributions: $H_{1,1}$, $H_{1,2}$, $H_{1,3}$, $H_{1,4}$, $H_{1,5}$, $H_{1,6}$, $H_{1,7}$ and $H_{1,8}$

16 distributions with real and imaginary values at leading twist

Some questions about GTMDs

- Do they actually map onto GPDs and TMDs?
- Can we measure them in observables? Do these observables factorize?
- Are they universal?

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Matching onto GPDs

Matching onto GPDs at the operator level

Integrating the operators w.r.t $k_\perp$ yields GPDs:

$$H(x,\xi,t) = \int \mathrm{d}^2 k_\perp \left[\mathrm{Re}(F_{1,1}) + 2\xi^2 \left(\frac{k_\perp \cdot \Delta_\perp}{\Delta_\perp^2} \mathrm{Im}(F_{1,2}) + \mathrm{Re}(F_{1,3})\right)\right]$$

and similar relations for other GTMDs.

But evolution spoils the matching

GTMD evolution equations complicate the dependence on $k_\perp$
$\Rightarrow$ we need matching functions.

Known at one loop for WW type distributions [Bertone, 2022]

Matching onto GPDs

Matching onto TMDs

The forward limit $\Delta = 0$ yields TMDs:

- $\mathrm{Re}(F_{1,1}) \to f_1$
- $\mathrm{Im}(F_{1,2}) \to -f_{1T}^{\perp}$
- $\mathrm{Re}(G_{1,2}) \to g_{1T}$
- $\mathrm{Re}(G_{1,4}) \to g_{1L}$
- $\mathrm{Im}(H_{1,1}) \to -h_1^{\perp}$
- $\mathrm{Re}(H_{1,3}) \to h_{1T}$
- $\mathrm{Re}(H_{1,4}) \to h_{1T}^{\perp}$
- $\mathrm{Re}(H_{1,7}) \to h_{1L}^{\perp}$

Experimentally, Δ is never exactly 0

Can we really constrain TMDs using GTMD observables?
Similar questions for GPD vs PDF

see e.g. [Dutrieux, Winn, Bertone, 2023]

Some questions about GTMDs

- Do they actually map onto GPDs and TMDs?
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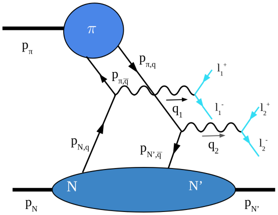
Exclusive double Drell Yan

Quark GTMDs: exclusive double Drell Yan

Suggested by [Bhattacharya, Metz, Zhou, 2017]



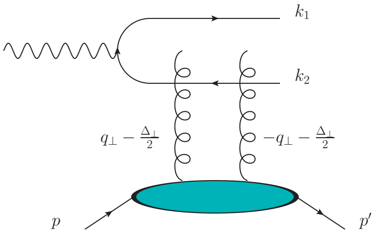
Factorization formula from SCET (and figure stolen from) [Echevarria, Gutierrez Garcia, Scimemi, 2023]

If Glauber gluons cancel, factorization theorem

Exclusive dijet production

Gluon GTMDs: at small x in exclusive dijet production

Suggested by [Hatta, Xiao, Yuan, 2016]
Known at full NLL accuracy
[RB, Grabovsky, Szymanowski, Wallon, 2016]



Would also work at moderate x, if factorizable

Exclusive dijet production

$k_\perp$-moments of gluon GTMDs: in exclusive dijet electroproduction

[Ji, Yuan, Zhao, 2016]: Longitudinal SSA allows to probe the first $k_\perp$-moment of GTMD $F_{1,4}$

$\Rightarrow$ Probe of the gluon orbital angular momentum contribution to proton spin

Rather explicitly breaks factorization

Exclusive dijet production

$k_\perp$-moments of gluon GTMDs: in exclusive dijet electroproduction

[Bhattacharya, RB, Hatta, 2022]: Longitudinal DSA allows to probe the first $k_\perp$-moment of GTMD $F_{1,4}$

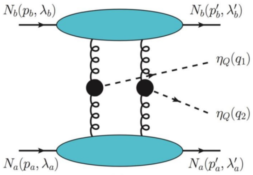
$\Rightarrow$ Probe of the gluon orbital angular momentum contribution to proton spin

Requires very specific kinematics

Exclusive double quarkonium production in pp

Gluon GTMDs: exclusive double quarkonium production in *pp* collisions

Suggested by [Bhattacharya, Metz, Ojha, Tsai, Zhou, 2018]

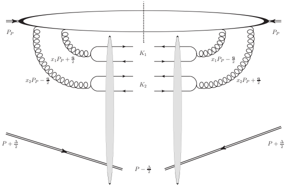


Neglects all gauge links: factorization is uncertain

Exclusive forward double quarkonium production in pp

Gluon GTMDs: at small x in forward diffractive double quarkonium production in pp and pA collisions

Suggested by [RB, Hatta, Xiao, Yuan, 2018]



Relies on the hybrid factorization ansatz

Some questions about GTMDs

- Do they actually map onto GPDs and TMDs?
- Can we measure them in observables? Do these observables factorize?
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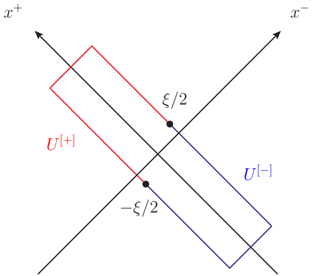
TMD gauge links

"Non-universality" of quark TMD distributions

Gauge links can be future-pointing or past-pointing



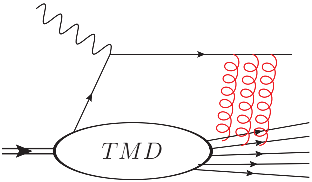
$$q^{[+]}(x, k_\perp) \propto \left\langle P \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[+]}_{\frac{z}{2},-\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P \right\rangle$$

$$q^{[-]}(x, k_\perp) \propto \left\langle P \left| \bar{\psi}\left(\frac{z}{2}\right) \mathcal{U}^{[-]}_{\frac{z}{2},-\frac{z}{2}} \psi\left(-\frac{z}{2}\right) \right| P \right\rangle$$

For naive T-odd distributions, $q^{[+]} = -q^{[-]}$: Sivers effect

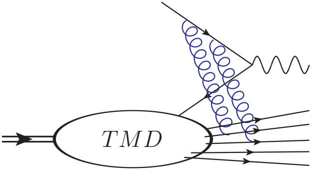
The Sivers effect

SIDIS



Final state interactions: $q^{[+]}$

Drell-Yan

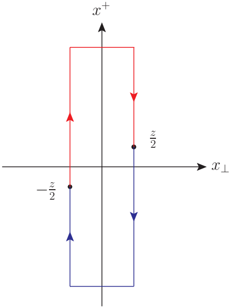


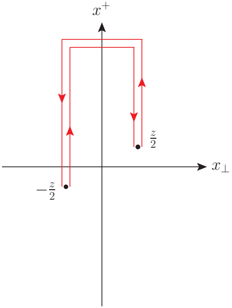
Initial state interactions: $q^{[-]}$

The Sivers distribution comes with a relative $-$ sign between SIDIS and DY: different gauge links for a naive T-odd quantity!

TMD gauge links

"Non-universality" of gluon TMD distributions





$$\mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[-]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right] \qquad \mathrm{Tr}\left[F^{i-}\left(\frac{z}{2}\right)\mathcal{U}^{[+]\dagger}F^{i-}\left(-\frac{z}{2}\right)\mathcal{U}^{[+]}\right]$$

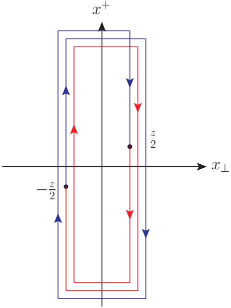
Only the WW-type TMD is easy to factorize

GTMD soft factors only known for WW

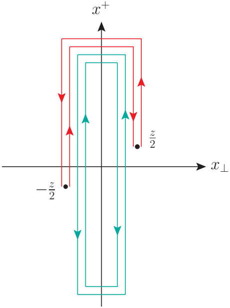
[Echevarria, Idilbi, Kanazawa, Lorcé, Metz, Pasquini, Schlegel, 2016]

TMD gauge links

"Non-universality" of gluon TMD distributions



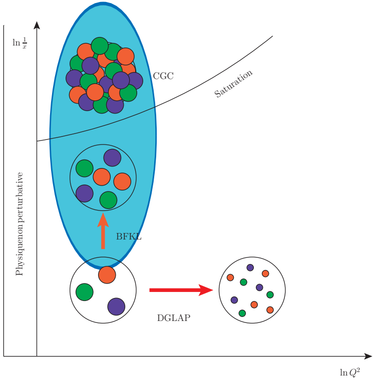
$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}F^{i-}\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]$$



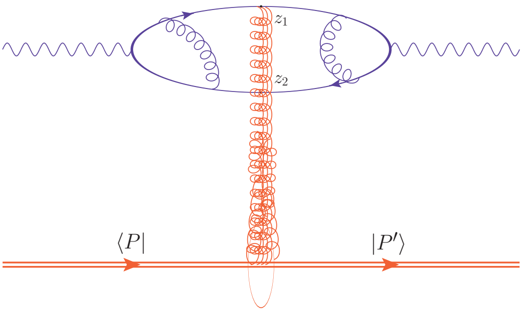
$$\mathrm{Tr}\left[F^{i-}\mathcal{U}^{[+]\dagger}F^{i-}\mathcal{U}^{[+]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]$$

QCD at small $x_{\mathrm{Bj}} \sim Q^2/s$

Regge limit: $Q^2 \ll s$



Factorized picture



Factorized amplitude

$$\mathcal{S} = \int \mathrm{d}\boldsymbol{x}_1 \mathrm{d}\boldsymbol{x}_2 \, \Phi^Y(\boldsymbol{x}_1, \boldsymbol{x}_2) \, \langle P' | [\mathrm{Tr}(U^Y_{\boldsymbol{x}_1} U^{Y\dagger}_{\boldsymbol{x}_2}) - N_c] | P \rangle$$

Written similarly for any number of Wilson lines in any color representation!

Y independence: B-JIMWLK, BK equations. Resums logarithms of s

The seemingly incompatible nature of the distributions

Two different kinds of gluon distributions

Moderate x distributions

TMD, PDF...

$$\langle P|F^{-i}WF^{-j}W|P\rangle$$

Low x distributions

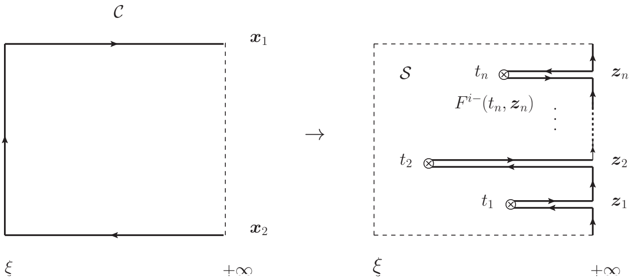
Dipole scattering amplitude

$$\langle P|\mathrm{tr}(U_1 U_2^\dagger)|P\rangle$$

The Wilson line ↔ parton distribution equivalence

Most general equivalence: use the Non-Abelian Stokes theorem

[RB, Mehtar-Tani, 2020]



$$\mathcal{P}\exp\left[\oint_{\mathcal{C}} dx_\mu A^\mu(x)\right] = \mathcal{P}\exp\left[\int_{\mathcal{S}} d\sigma_{\mu\nu}\, W F^{\mu\nu} W^\dagger\right]$$

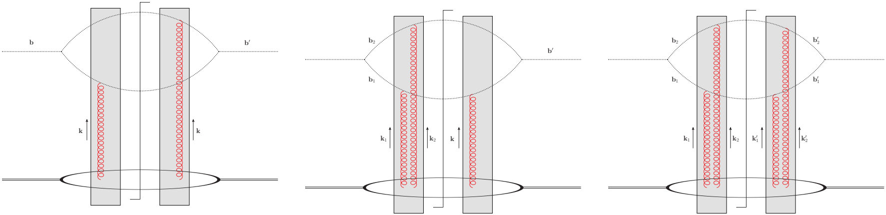
$$U_{x_{1\perp}} U^\dagger_{x_{2\perp}} = [\hat{x}_{1\perp}, \hat{x}_{2\perp}]$$

Inclusive low x cross section

Inclusive low x cross section = TMD cross section

[Altinoluk, RB, Kotko, 2019], [Altinoluk, RB, 2019]

Generalizes [Dominguez, Marquet, Xiao, Yuan, 2011]



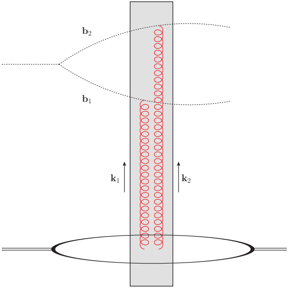
$$\sigma = \mathcal{H}_2^{ij}(\boldsymbol{k}) \otimes f_2^{ij}(x=0, \boldsymbol{k})$$
$$+ \mathcal{H}_3^{ijk}(\boldsymbol{k}, \boldsymbol{k}_1) \otimes f_3^{ijk}(x=0, x_1=0, \boldsymbol{k}, \boldsymbol{k}_1)$$
$$+ \mathcal{H}_4^{ijkl}(\boldsymbol{k}, \boldsymbol{k}_1, \boldsymbol{k}_1') \otimes f_4^{ijkl}(x=0, x_1=0, x_1'=0, \boldsymbol{k}, \boldsymbol{k}_1, \boldsymbol{k}_1')$$

All twists but $x_n = 0$ and $F^{12}=0$

Exclusive low x cross section

Exclusive low x amplitude = GTMD amplitude

[Altinoluk, RB, 2019], [RB, Mehtar-Tani, 2020]



$$\mathcal{H}^{ij}(\boldsymbol{k}_1, \boldsymbol{k}_2) \otimes f^{ij}(x = 0, \xi = 0; \boldsymbol{k}, \Delta)$$

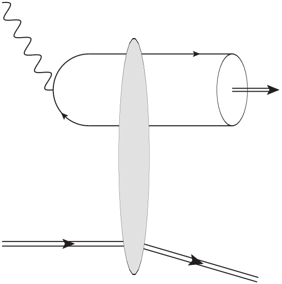
Every exclusive low x process probes a Wigner distribution!

All twists but $x_n = 0$ and $F^{12}=0$

Exclusive low x cross section

Exclusive low x electroproduction of a C-even meson

[RB, Hatta, Szymanowski, Wallon, 2019]



Odderon GTMDs contributing:
$\mathrm{Im}(F^g_{1,1})$, $\mathrm{Im}(F^g_{1,2})$, $\mathrm{Im}(F^g_{1,3})$

Leading term for small t: $\mathrm{Im}(F^g_{1,2})$
i.e. the gluon Sivers function

Proton spin physics without proton spin

Summary

- GTMDs are the Mother Distributions
- They raise similar questions to both TMD and GPD physics
- Significant progress recently on building experimental probes for GTMDs