Exclusive J/psi photoproduction: connecting PDFs and GPDs

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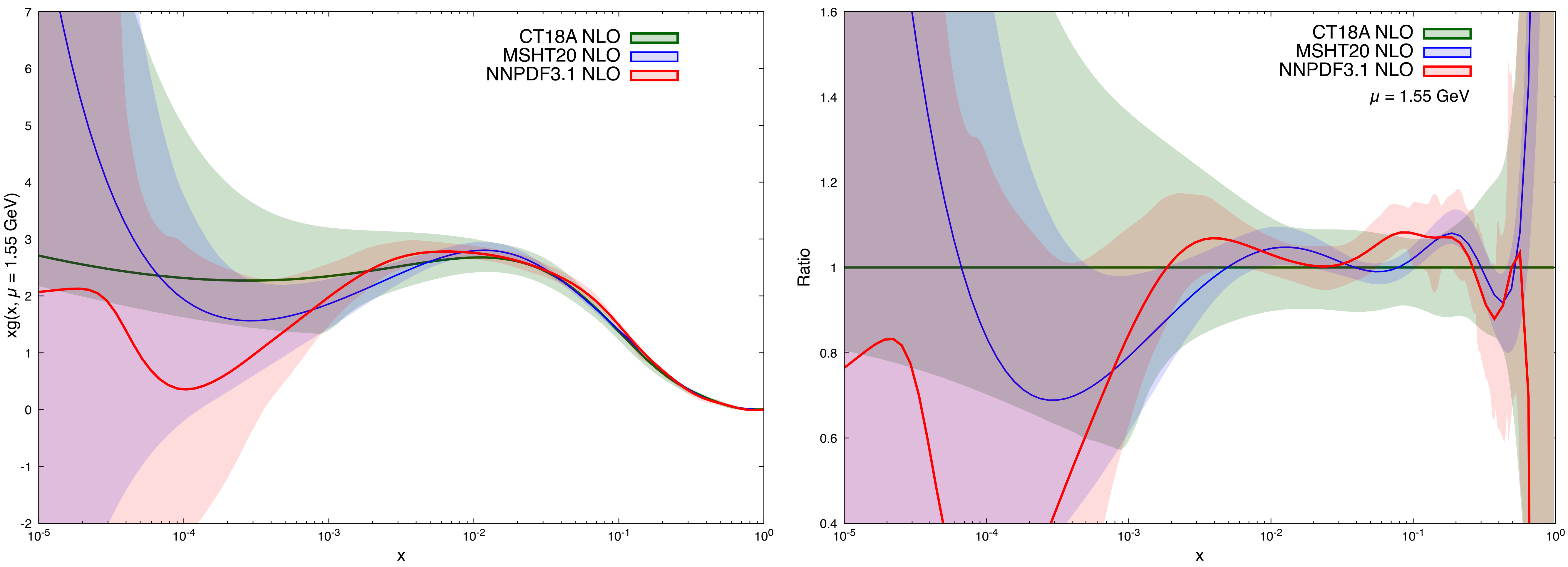
Introduction

- Inclusive processes do not well constrain small x/Regge limit domain of PDFs
- Exclusive processes offer sensitive probe of this domain but as of yet not included in global analyses PDF determination - why?
    1. Off forward kinematics imply sensitivity to *GPD* over conventional PDFs
    2. Scale dependence and stability of theoretical predictions



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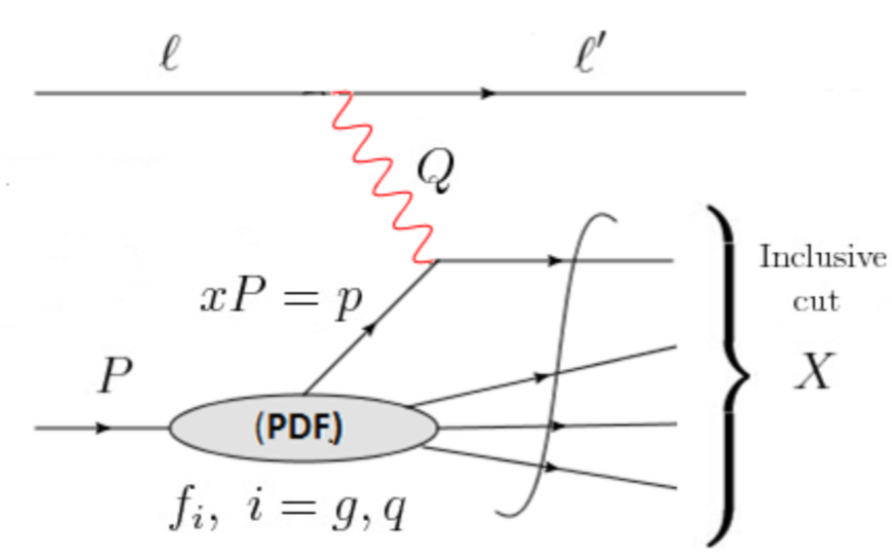
- As higher CM energies are realised at LHC, pushed towards small x domain, W ~ 1/x

Ryskin 1993

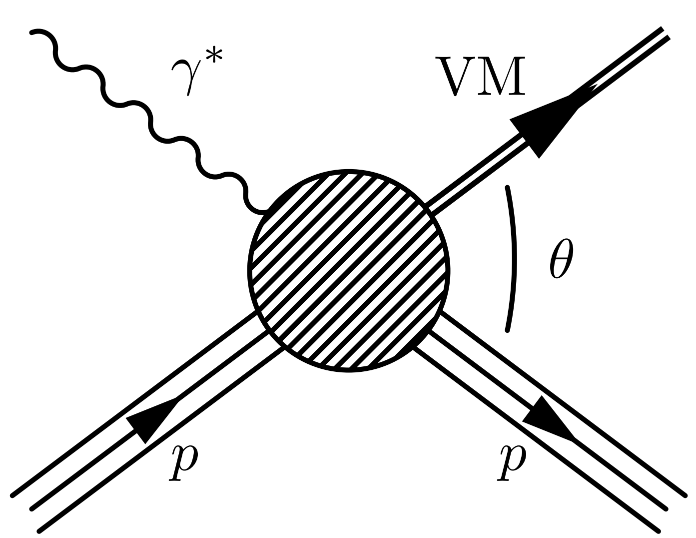
LLx exclusive J/psi production:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}(\gamma^* p \to J/\psi p)\Big|_{t=0} = \frac{\Gamma_{ee}^{J/\psi} M_{J/\psi}^3 \pi^3}{48\alpha_{\rm em}} \left[\frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4} \quad xg(x,\bar{Q}^2)\right]^2 \left(1 + \frac{Q^2}{M_{J/\psi}^2}\right)$$

Inclusive - e.g. DIS included in global parton analyses



Exclusive - can we use the data?



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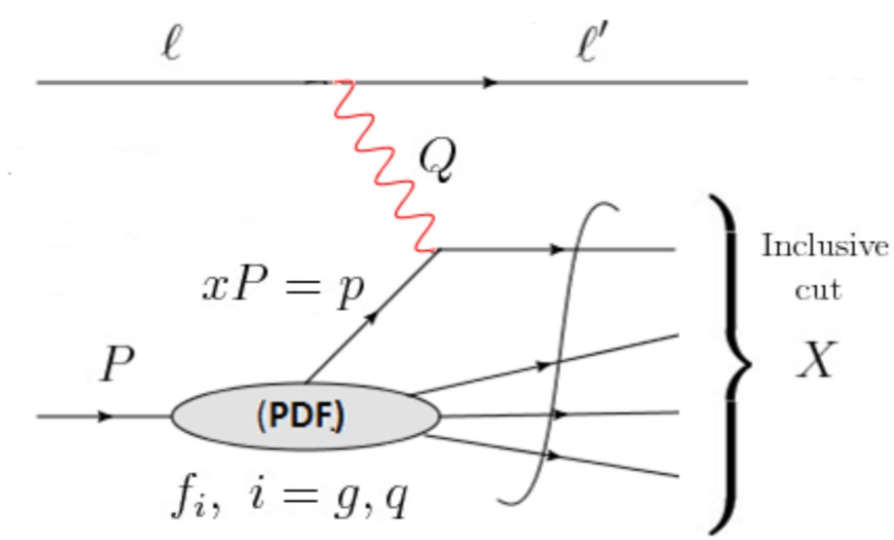

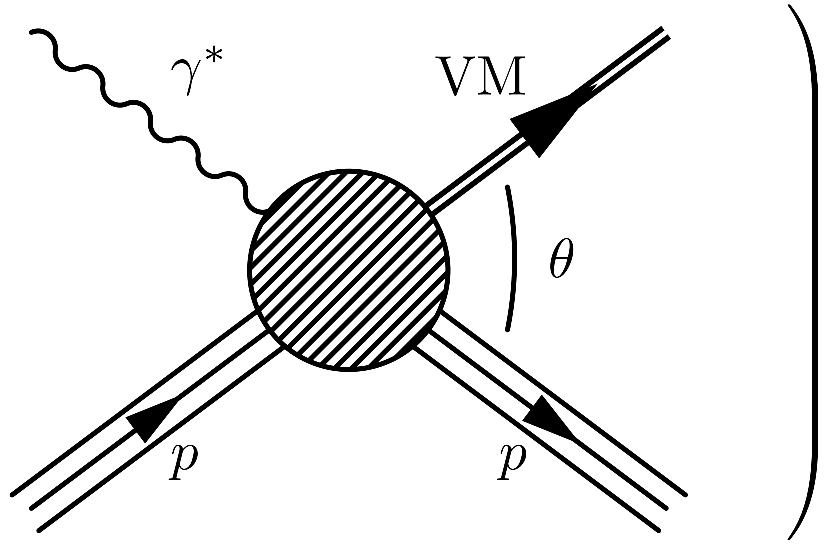

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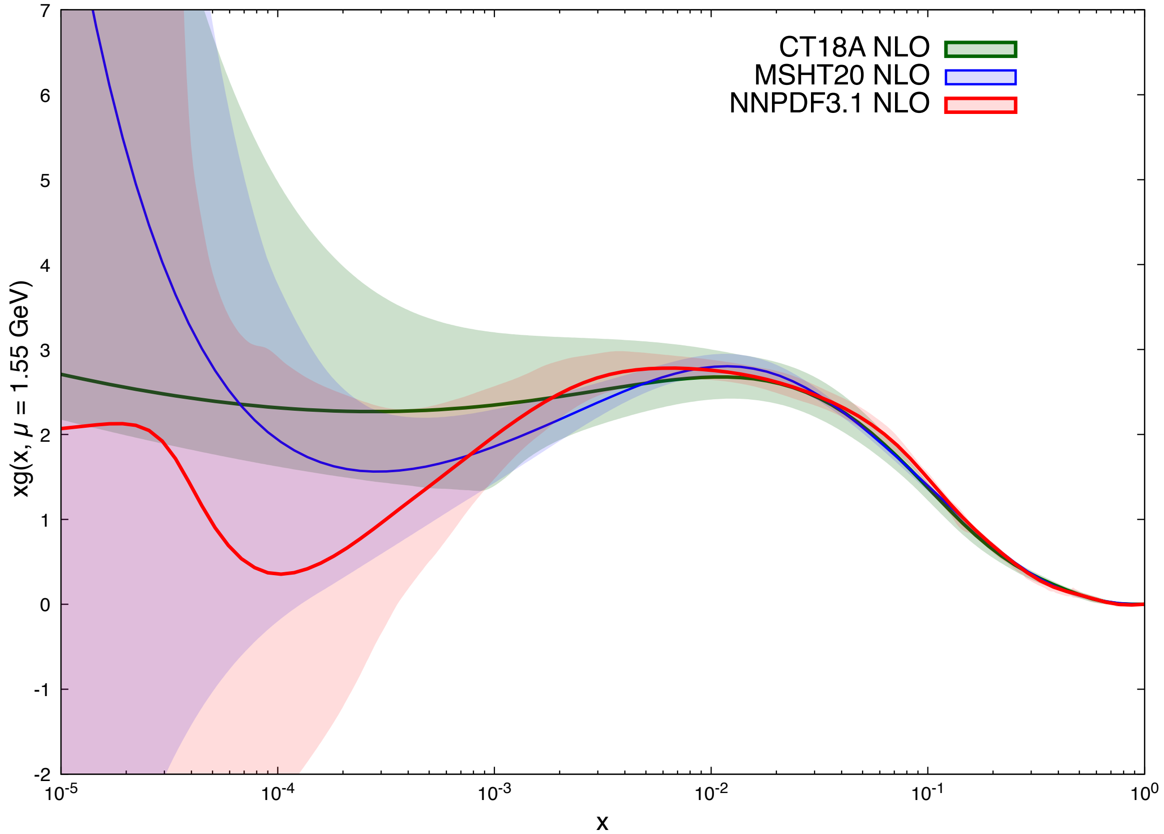
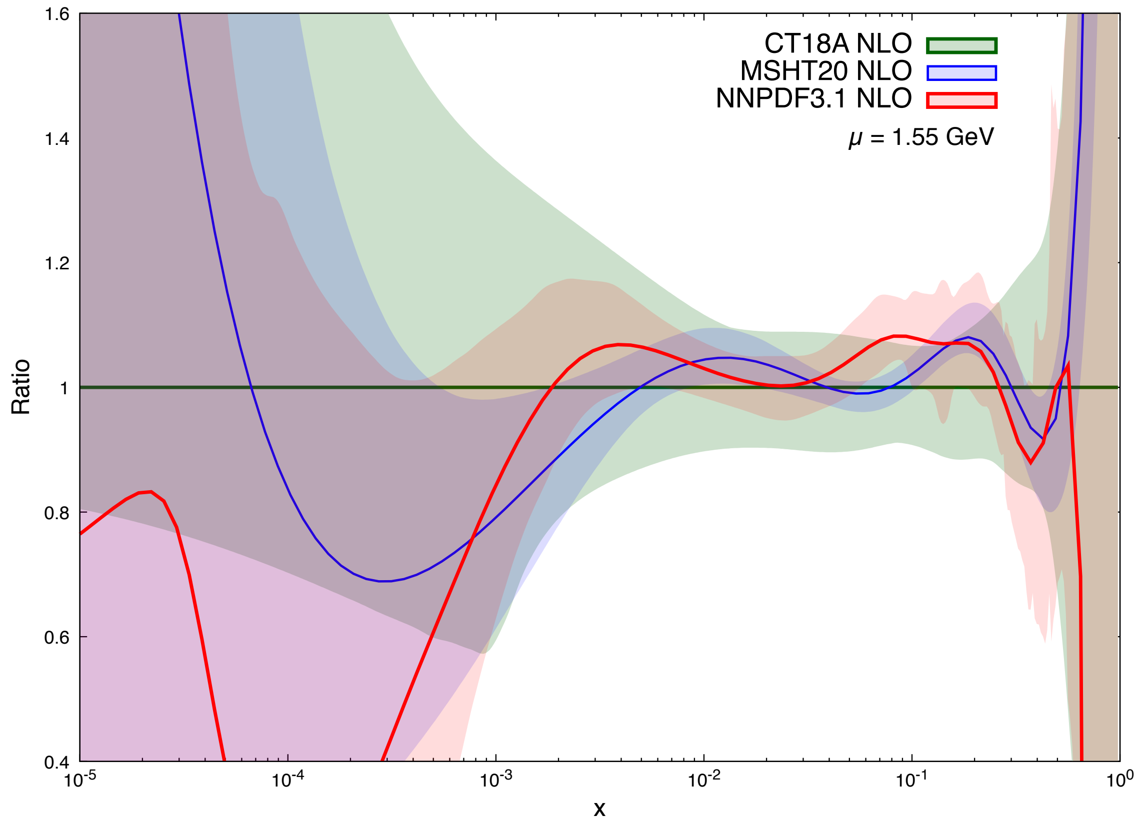
This talk: how to counteract these problems and so allow exclusive J/psi data to probe gluon PDF down to

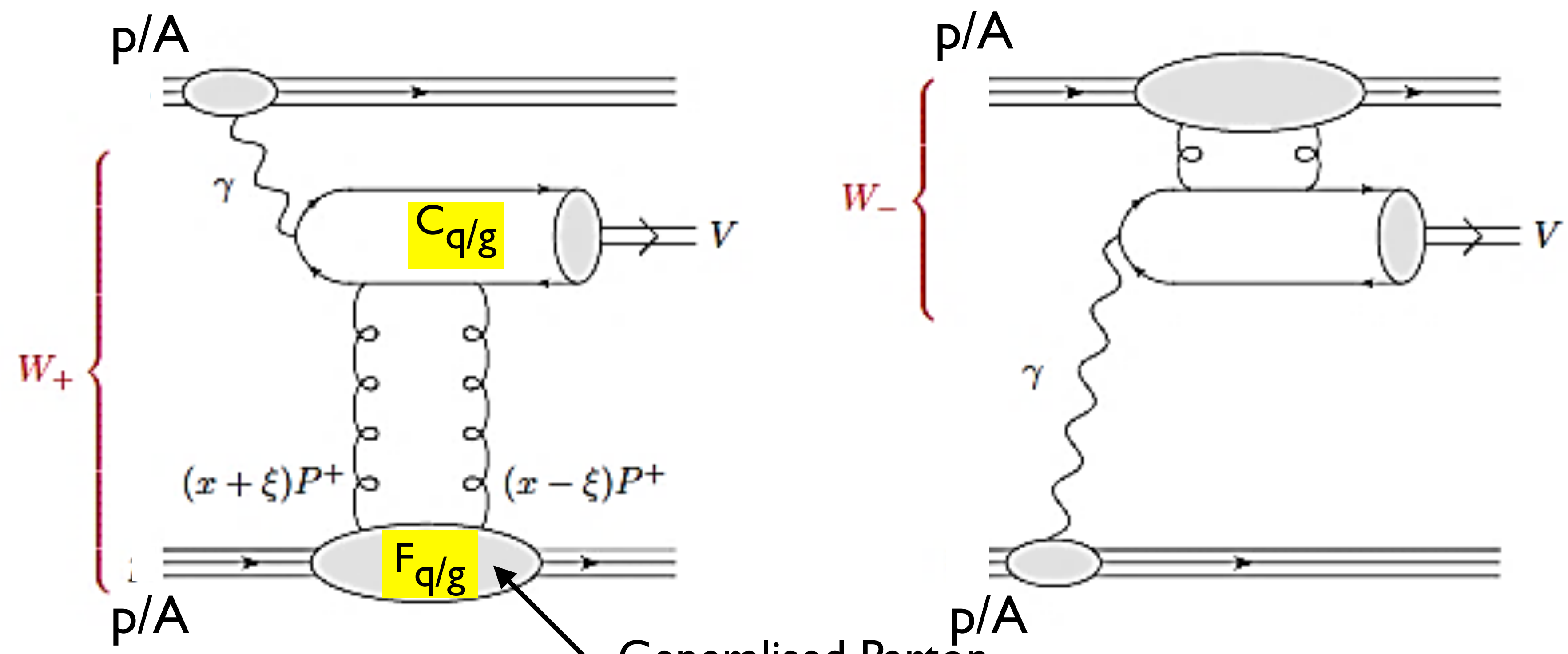
$$x \sim 3 \times 10^{-6} \quad \& \quad \mu = O(M_{J/\psi}/2)$$





General Set up and Framework

Exclusive J/psi photoproduction in p+p (A+A) UPC collisions in collinear factorisation



Setup for $\gamma p \to J/\psi p$ follows:
Ivanov, Schäfer, Szymanowski, Krasnikov, 04

$C_{q/g}$

Photoproduction:

- hep-ph/0401131

Ivanov, Schäfer, Szymanowski, Krasnikov, 04

Electroproduction:

- arXiv:1903.00171
- arXiv:2105.07657

Chen, Qiao, 19
CAF, Gracey, Jones, Teubner, 21

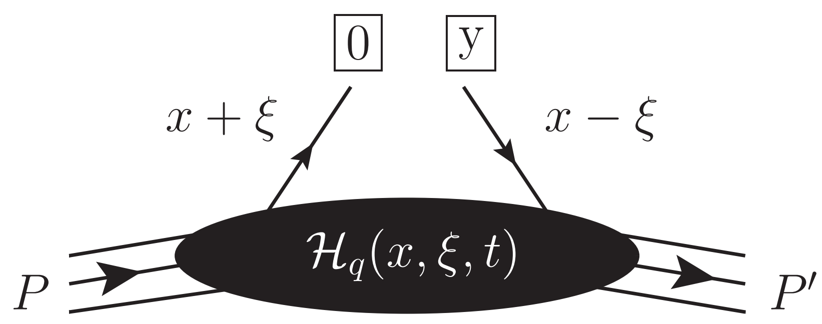
- Factorisation: $F_{q/g} \otimes C_{q/g} \otimes \phi^V_{Q\bar{Q}}$
- Leading zeroth order term in rel. velocity (NRQCD)
- Colour singlet exchange between hard and soft sectors

$$A \propto \int_{-1}^{1} \mathrm{d}x \left[C_g(x,\xi) F_g(x,\xi) + \sum_{q=u,d,s} C_q(x,\xi) F_q(x,\xi) \right]$$

GPDs and the Shuvaev transform

GPDs generalise PDFs: outgoing/incoming partons carry different momentum fractions

Müller 94; Radyushkin 97; Ji 97



$\langle P'|\,\overline{\psi}_q(y)\mathcal{P}\{\}\psi_q(0)\,|P\rangle$

Shuvaev: Relates GPDs to PDFs at small x under physically motivated assumptions c.f analyticity

Shuvaev 99   Martin et al. 09

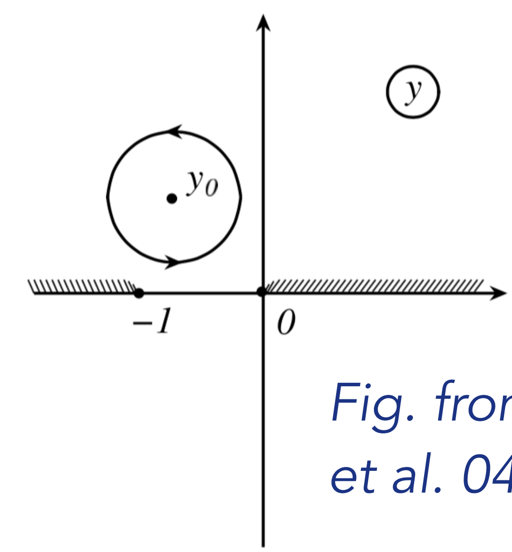


Fig. from Ivanov et al. 04

Idea: Conformal moments of GPDs = Mellin moments of PDFs
(up to corrections of O(xi^2) @ LO and O(xi) @ NLO)

- Construct GPD grids in multidimensional parameter space x, xi/x, qsq with forward PDFs from LHAPDF
- Costly computationally due to slowly converging double integral transform
- Regge theory considerations => Shuvaev transform valid in space-like (DGLAP) region only. In time-like (ERBL) region imaginary part of coefficient function is zero

Shuvaev transform

Full Transform:

$$\mathcal{H}_q(x,\xi) = \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi}\mathrm{Im}\int_0^1 \frac{\mathrm{d}s}{y(s)\sqrt{1-y(s)x'}}\right] \frac{\mathrm{d}}{\mathrm{d}x'}\left(\frac{q(x')}{|x'|}\right),$$

$$\mathcal{H}_g(x,\xi) = \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi}\mathrm{Im}\int_0^1 \frac{\mathrm{d}s(x+\xi(1-2s))}{y(s)\sqrt{1-y(s)x'}}\right] \frac{\mathrm{d}}{\mathrm{d}x'}\left(\frac{g(x')}{|x'|}\right),$$

$$y(s) = \frac{4s(1-s)}{x+\xi(1-2s)}.$$

[Shuvaev et. al 1999]

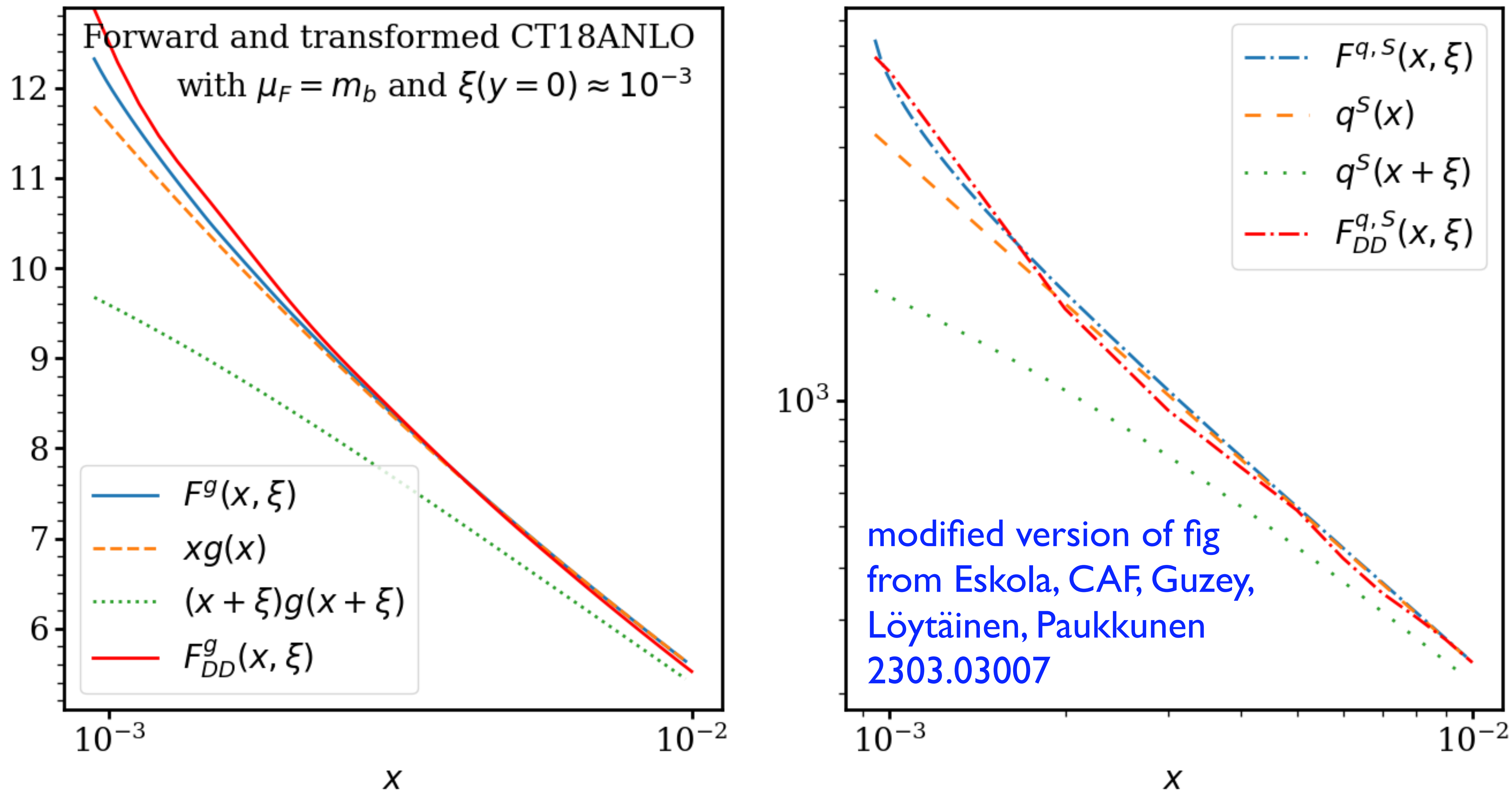
Shuvaev transform vs. DD model

DD model:

$$H^{g,q}(x,\xi,t=0,Q_0) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x-\beta-\alpha\xi) h(\beta,\alpha) H^{g,q}(x,\xi=0,t=0,Q_0)$$

Radyushkin hep-ph/9704207

$$h(\beta,\alpha) = \frac{\Gamma(b+3/2)}{\sqrt{\pi}\Gamma(b+1)} \frac{[(1-|\beta|)^2-\alpha^2]^b}{(1-|\beta|)^{2b+1}}$$



results in qualitative agreement with the LO evolution of GPDs in DGLAP region

 Dutrieux, Winn, Bertone 2302.07861

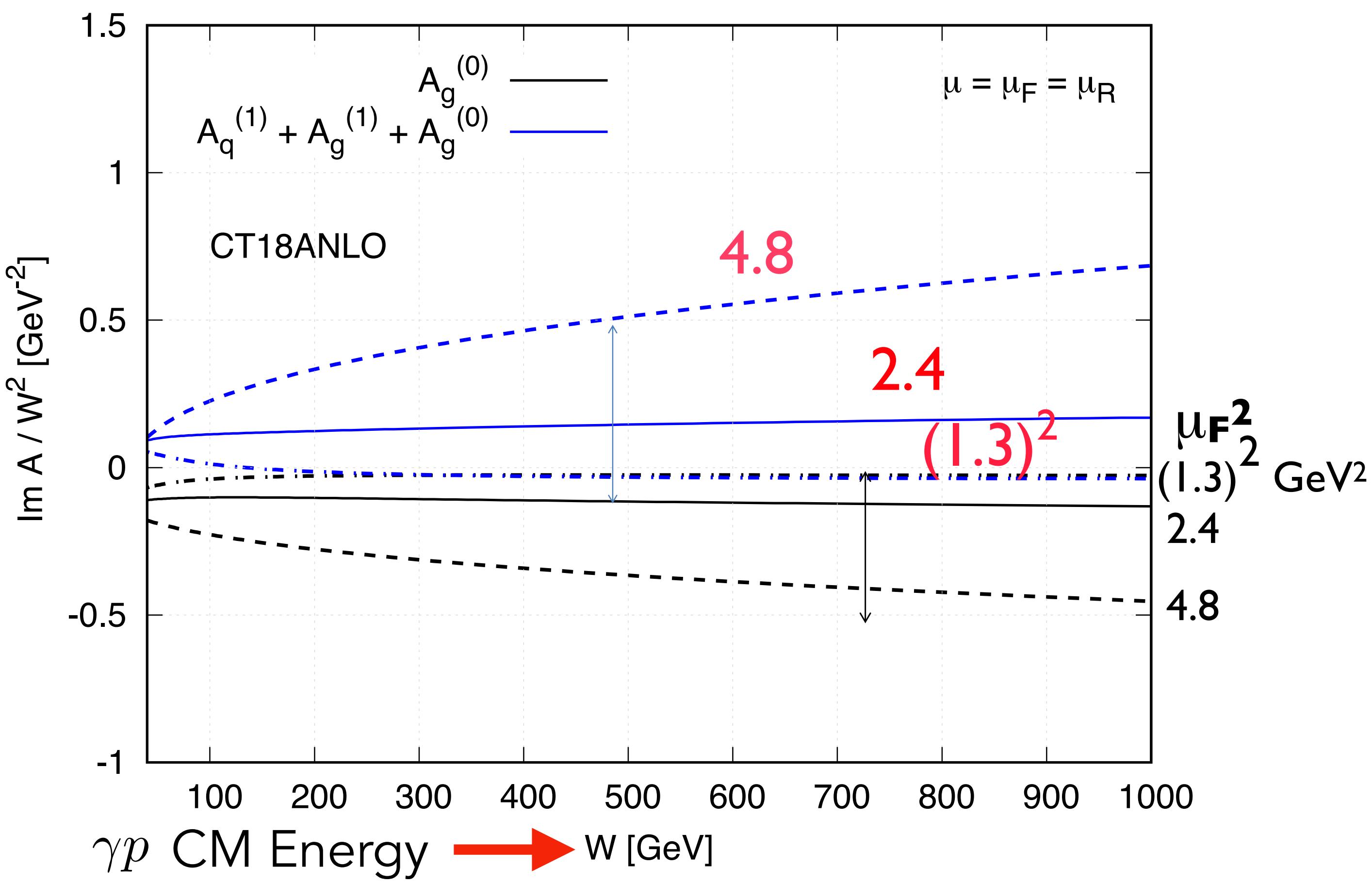
Stability of NLO prediction I+II

NLO in MSbar scheme

hep-ph/0401131

A. Bad perturbative convergence $|NLO_{correctn.}| > |LO|$ and opp. sign

B. Strong dependence on scale μ_F

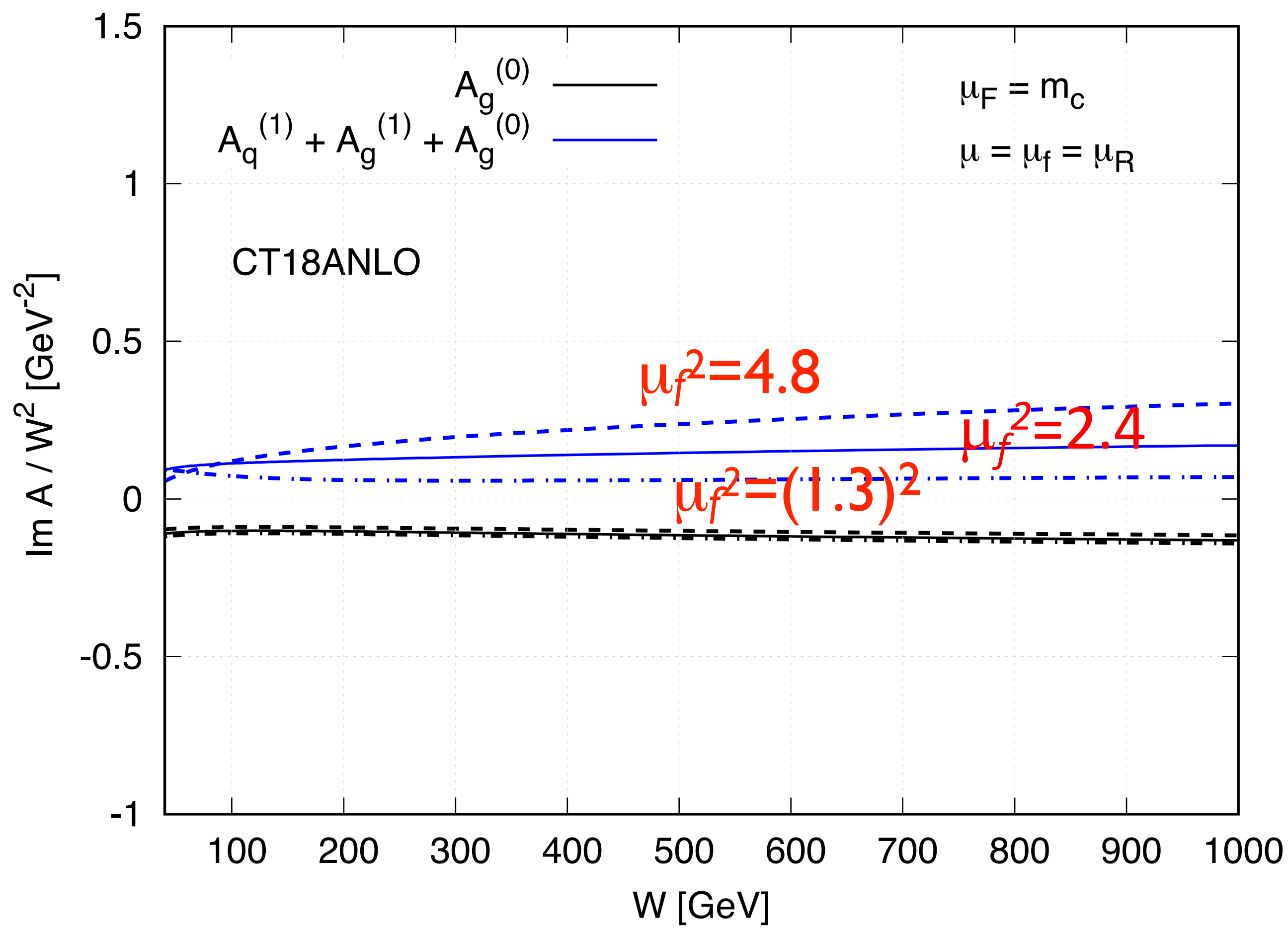


'Effective' small-x resummation

Jones et al., 1507.06942

Resummation of

$(\alpha_S \ln(1/\xi) \ln(\mu_{F/m}))^n$



There exists another numerically sizeable correction that can reduce variations further -> implementation of a **`Q0'** cut

Stability of NLO prediction II+III

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Resummation of

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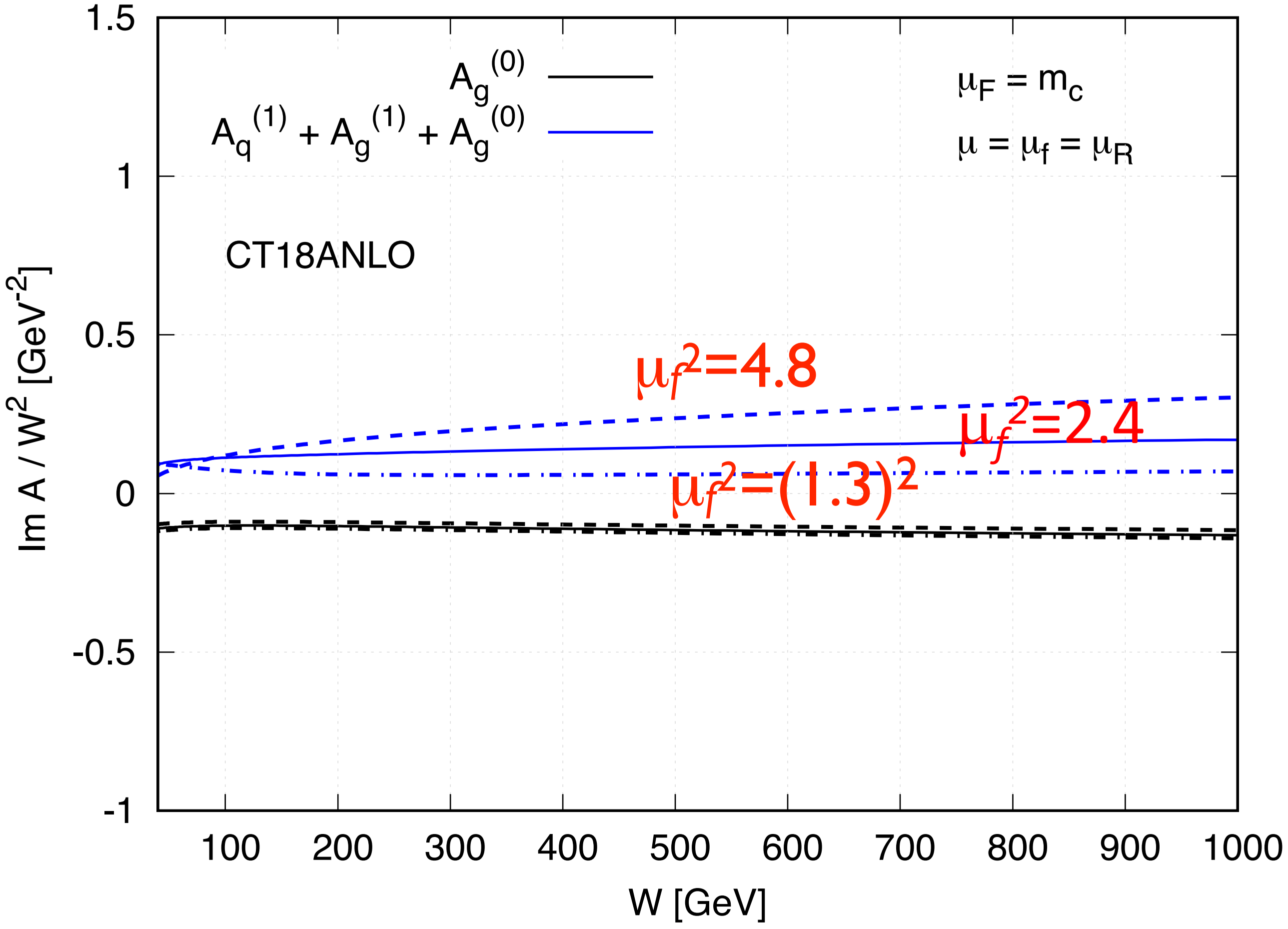
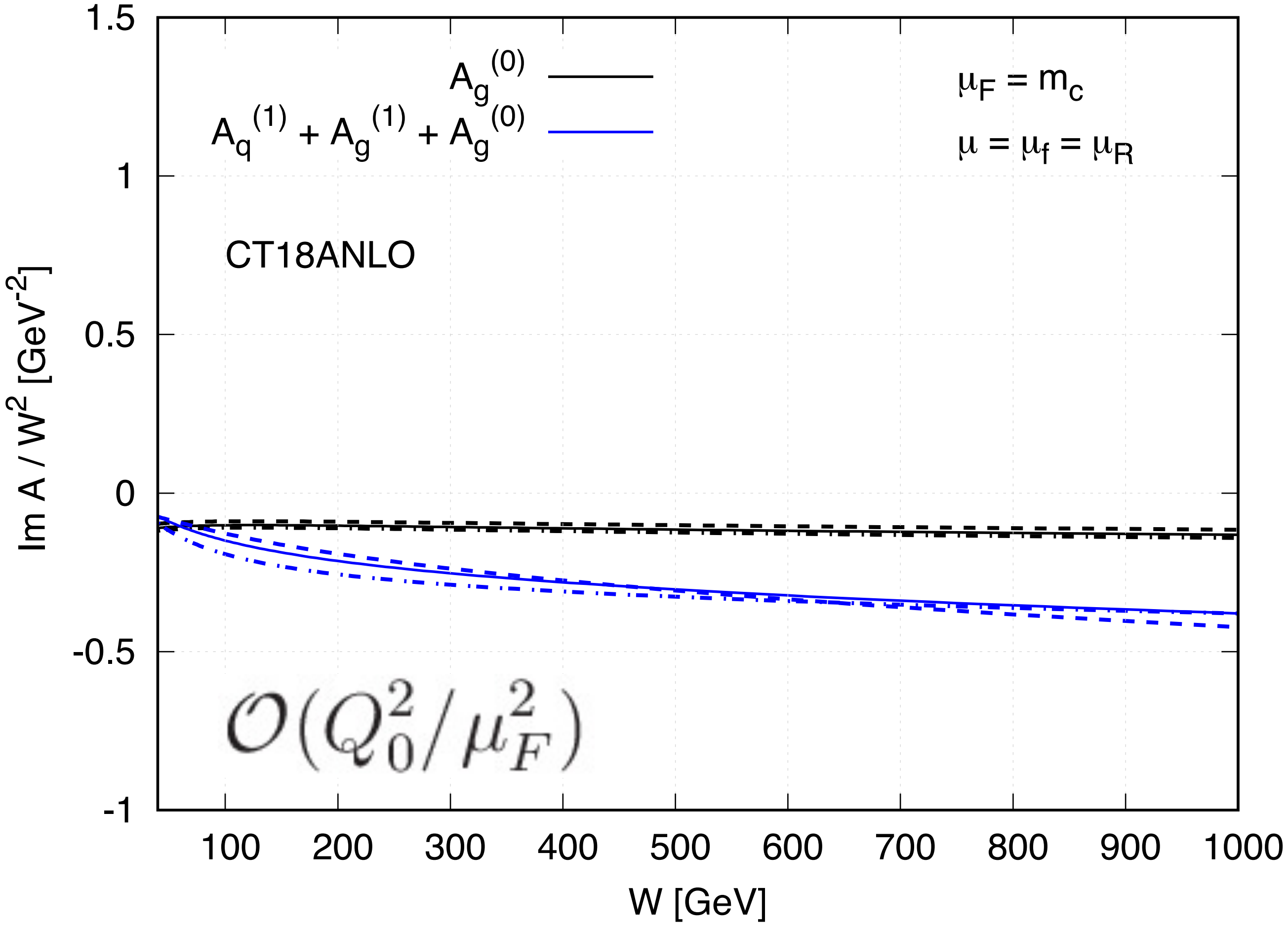


Low $l_t < Q_0$ subtraction

Jones et al., 1610.02272

Subtract DGLAP contribution NLO ($|\ell^2| < Q_0^2$) from known NLO MSbar coefficient function to avoid a double counting with input GPD at Q_0.



Predictions based on three global PDF analyses differ dramatically in large energy LHC region but are compatible in lower energy HERA region*

*See backup slides for details/plots

CAF, Jones, Martin, Ryskin, Teubner, 1908.08398

Extraction of low x gluon PDF via exclusive J/psi

Error budgets: errors due to parameter variations in global fits >> experimental uncertainty and scale variations in the theoretical result

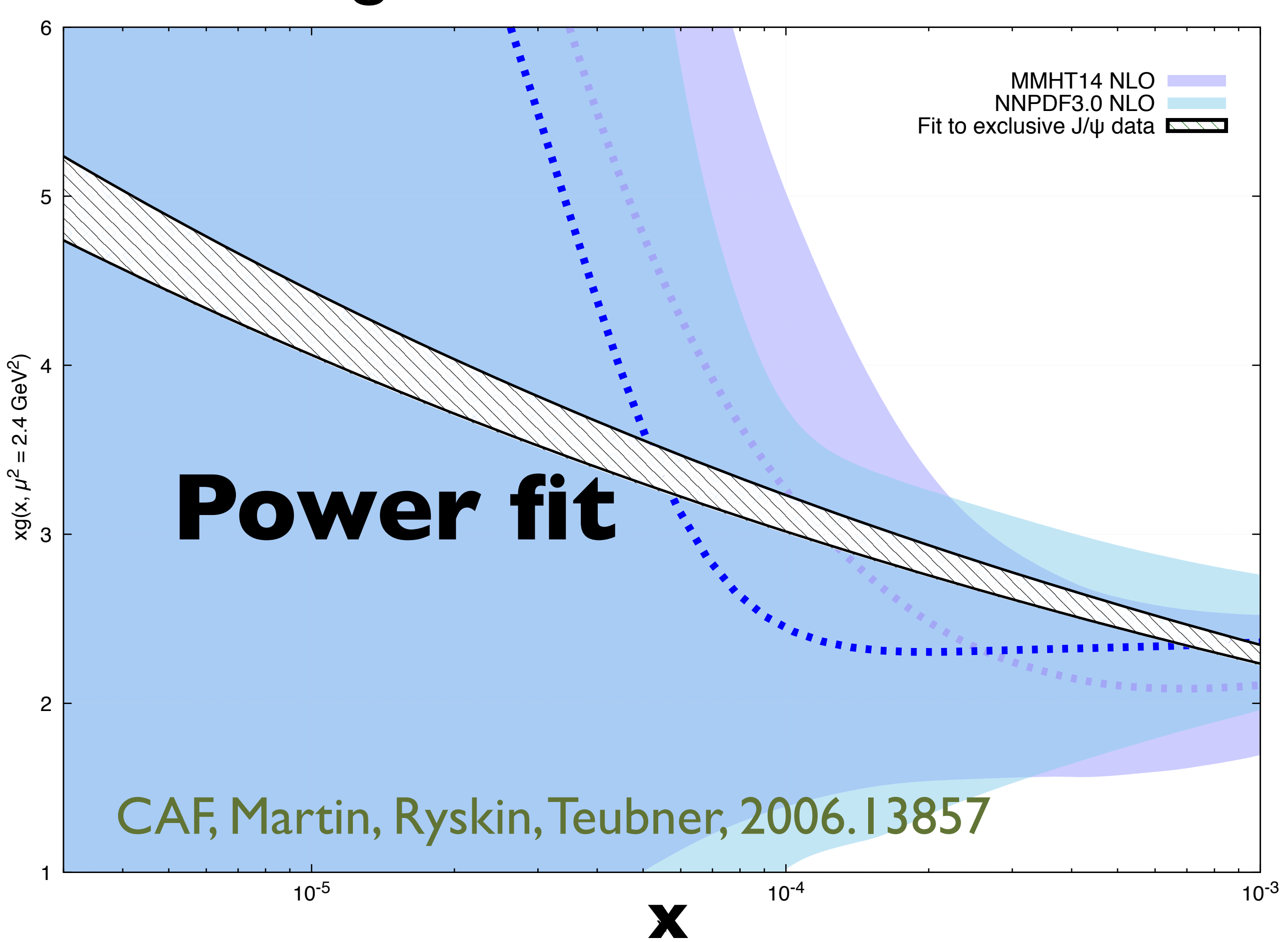
	λ	n	$\chi^2_{\rm min}$	$\chi^2_{\rm min}/{\rm d.o.f}$
NNPDF3.0	0.136	0.966	44.51	1.04
MMHT14	0.136	1.082	47.00	1.09
CT14	0.132	0.946	48.25	1.12

$$xg^{\rm new}(x,\mu_0^2) \;=\; nN_0\;(1-x)\;x^{-\lambda}$$

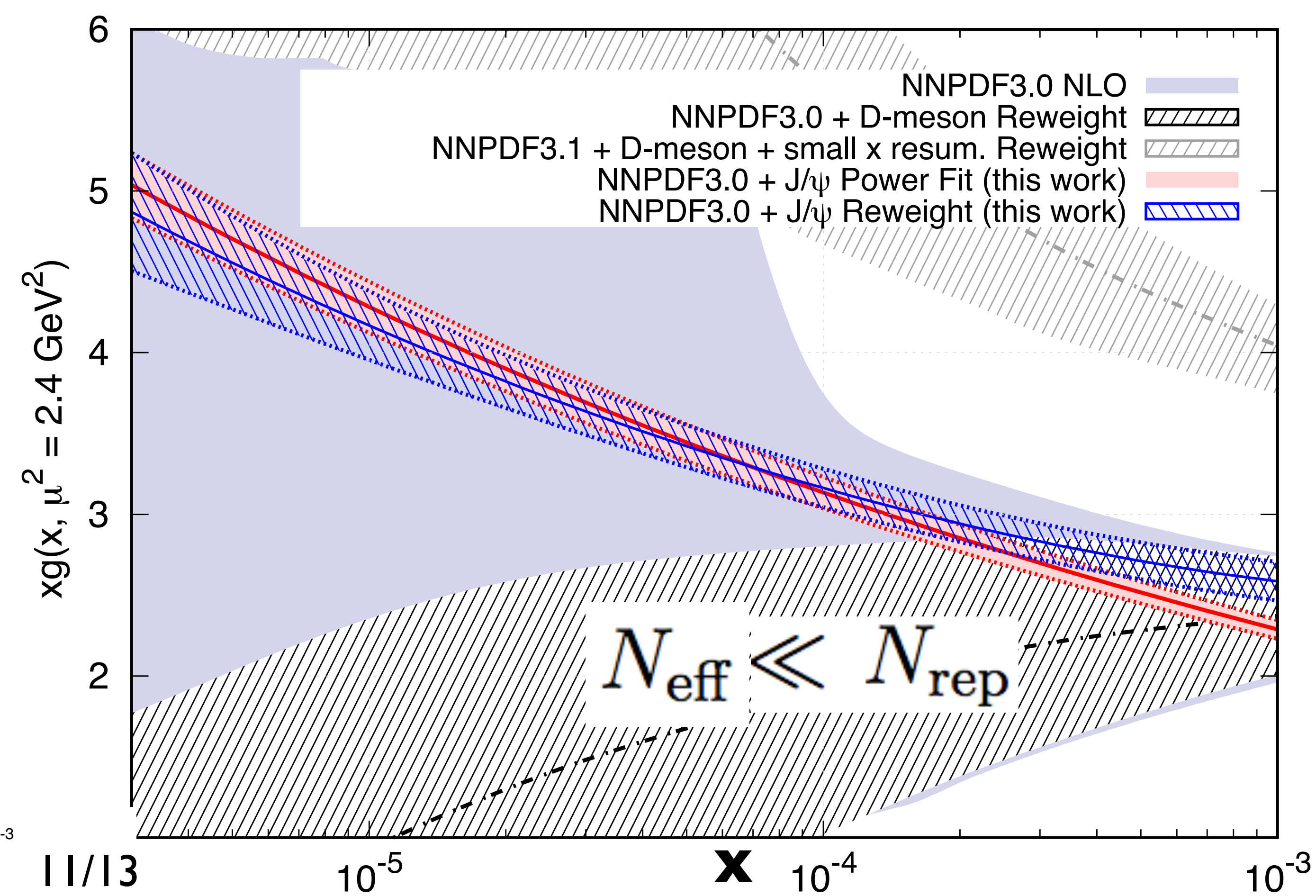
lambda = 0.136 +/- 0.006

n = 0.966 +/- 0.025

Fit a low x gluon PDF ansatz to the data



Bayesian profile current global PDF analyses



Profiling in xFitter

NNPDF30_nlo_as_0118

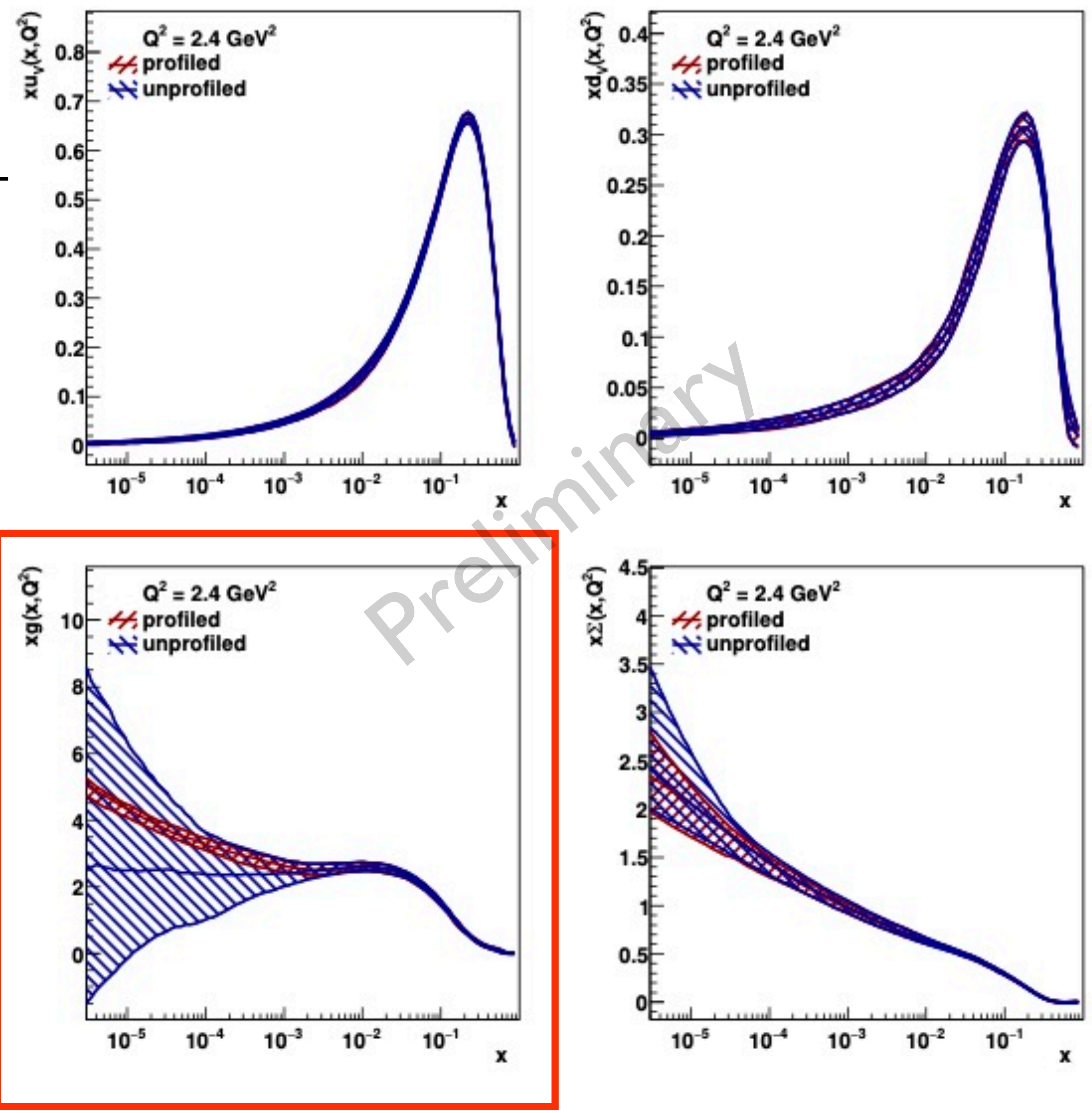
$N_{rep} = 1000$

profiled with
LHCb 13 TeV excl. J/psi
data 1806.04079

$N_{eff} = 63 << N_{rep}$

Condition $N_{eff} << N_{rep}$
expected here

Precursor to full fit



Profiling in xFitter

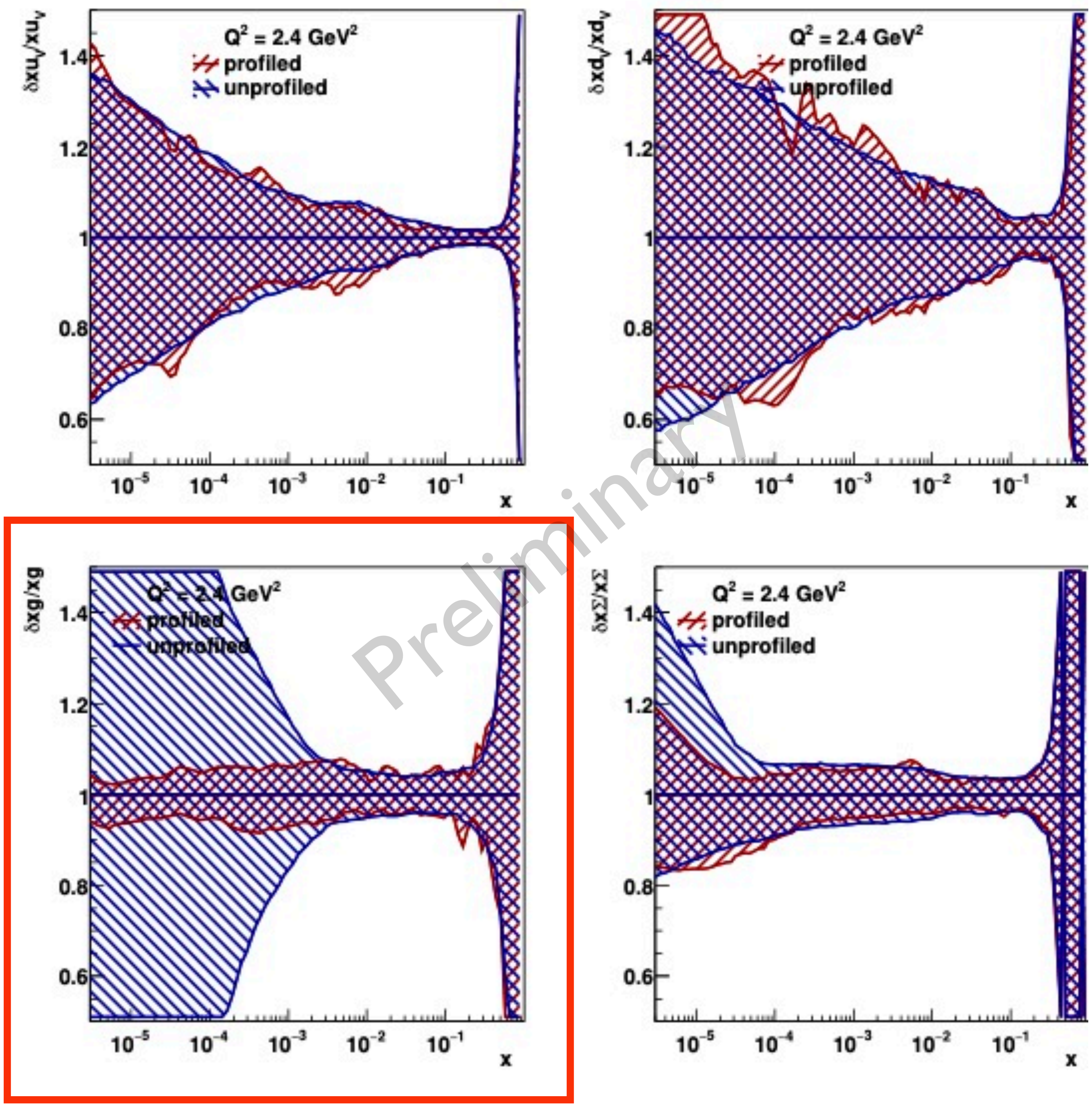
NB:

The condition $N_{eff} << N_{rep}$ implies the data adds a lot of new information which can lead to overestimation of PDF errors in the Hessian profiling procedure.

→ interpretation of these results to be taken with care

Compare shape of the gluon PDF favored by the exclusive J/psi data to that from e.g. inclusive open charm production or eta_c hadroproduction

Results support doing full fit in this framework (in progress)



Summary

- Bottlenecks of exclusive J/psi photoproduction in global PDF analyses

  - Sensitivity to GPD rather than PDF
  - Conventional MSbar NLO pQCD result exhibits large factorisation scale dep.

- Use Shuvaev's integral transform as reliable means at small xi to relate PDF and GPD

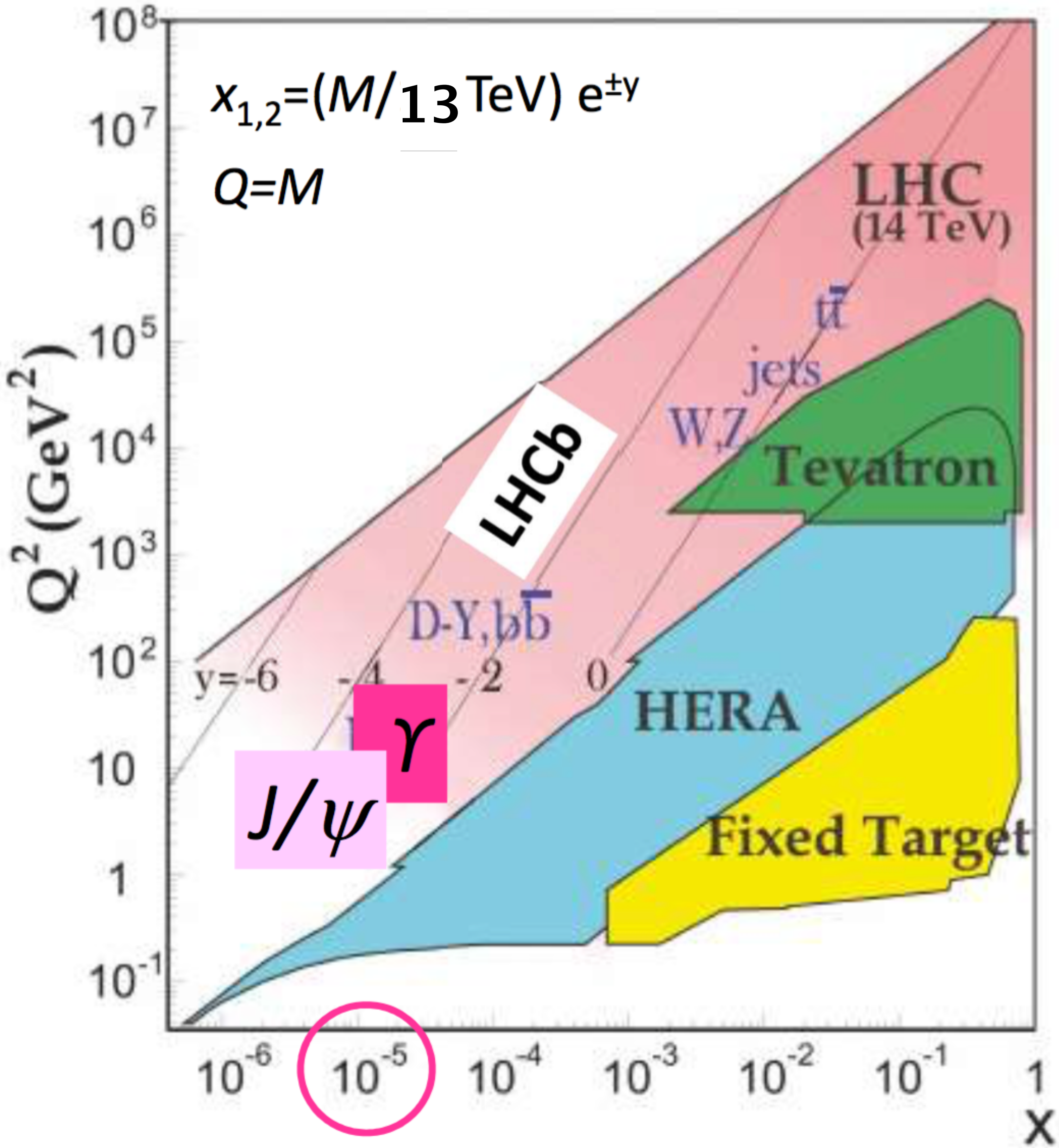
- Systematic taming via implementation of low ‘Q0’ subtraction and effective small-x resummation of large logarithmic contributions collectively reduce wild scale variations at NLO

- Large difference between cross section predictions based on global PDFs in LHCb regime while compatible at HERA energies -> motivates extraction of low x and low scale gluon PDF. Profiling and fitting exercises performed with exclusive data.

- Upshot: In a position to finally use exclusive J/psi data in a global fitter framework. Interfaced code to public PDF fitting tool xFitter. Profiling and fitting exercises in progress…

Thank you

Kinematic coverage

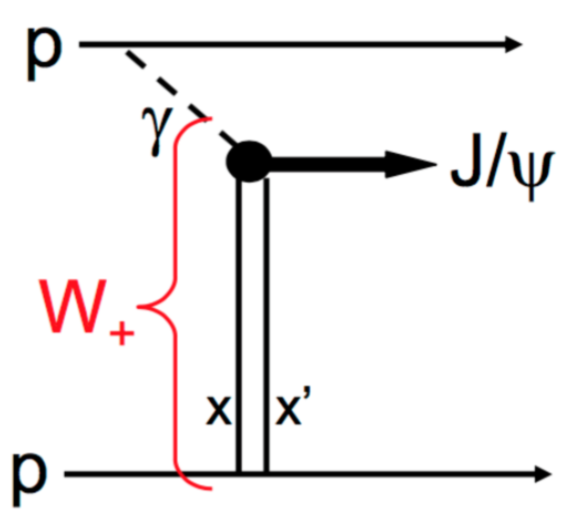


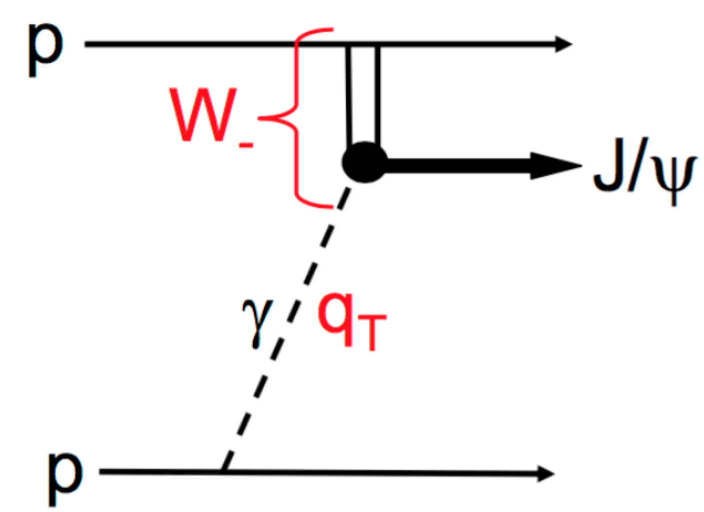
LHCb with $2 < y < 4.5$ can probe gluon down to $x \sim 10^{-5}$

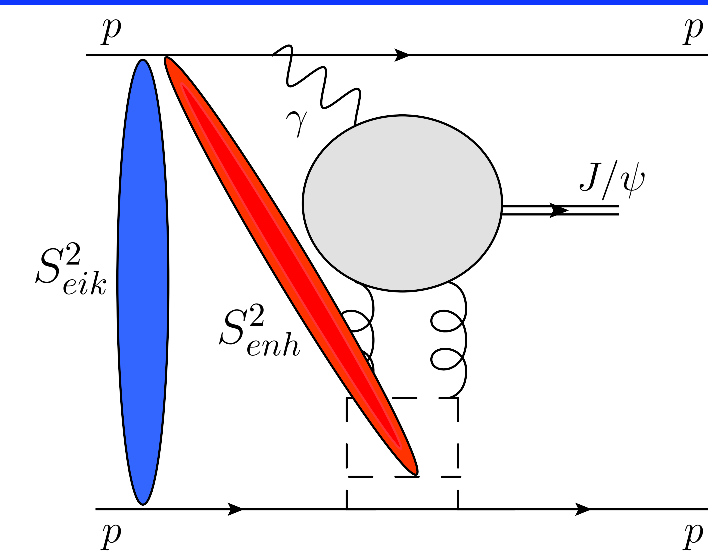
exclusive J/ψ, Υ
[$Q=M_V/2$ (scale)]

Why are these LHCb data not used in global PDF fits ??

General Set up and assumptions







LHCb data

$$\frac{d\sigma(pp)}{dy} = S^2(W_+)\left(k_+\frac{dn}{dk_+}\right)\sigma_+(\gamma p) + S^2(W_-)\left(k_-\frac{dn}{dk_-}\right)\sigma_-(\gamma p)$$

survival probability factors

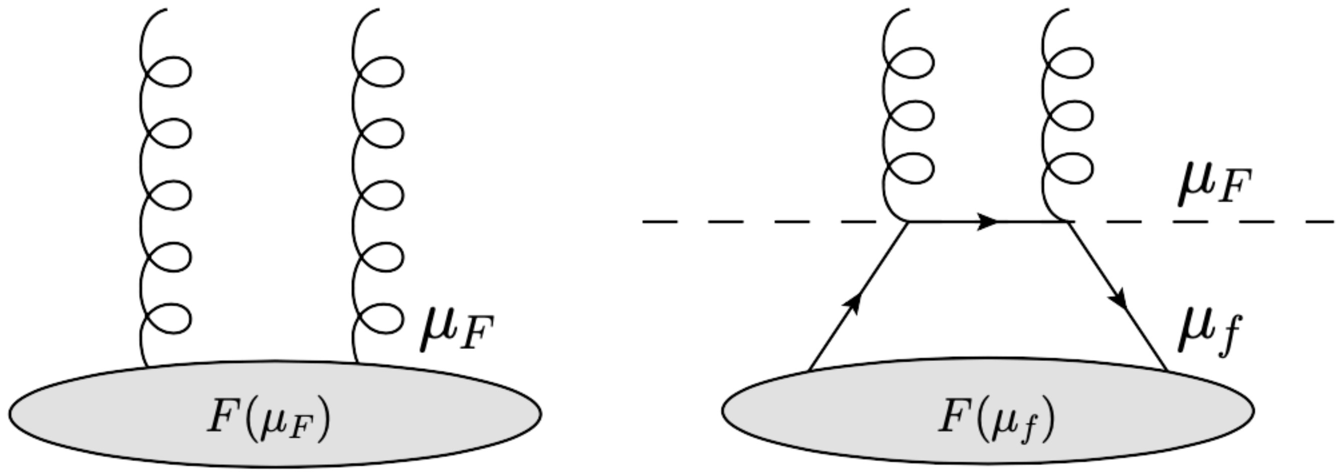
LHCb 'data'

photon flux

HERA gives W-

$$W_\pm^2 = M_{J/\psi}\sqrt{s}e^{\pm|y|} \Rightarrow x_\pm = \begin{cases}10^{-5}\\ 0.02\end{cases} \quad \text{at } y=4,\ \sqrt{s}\text{=13 TeV}$$

Treatment of double logarithmic contribution



Ideology: Use scale shifting to find optimal scale that removes the largest contribution from the NLO correction *

At fact. scale. μ_f , quark contribution is part of NLO hard matrix element

At fact. scale μ_F, absorbed quark contribution into LO result

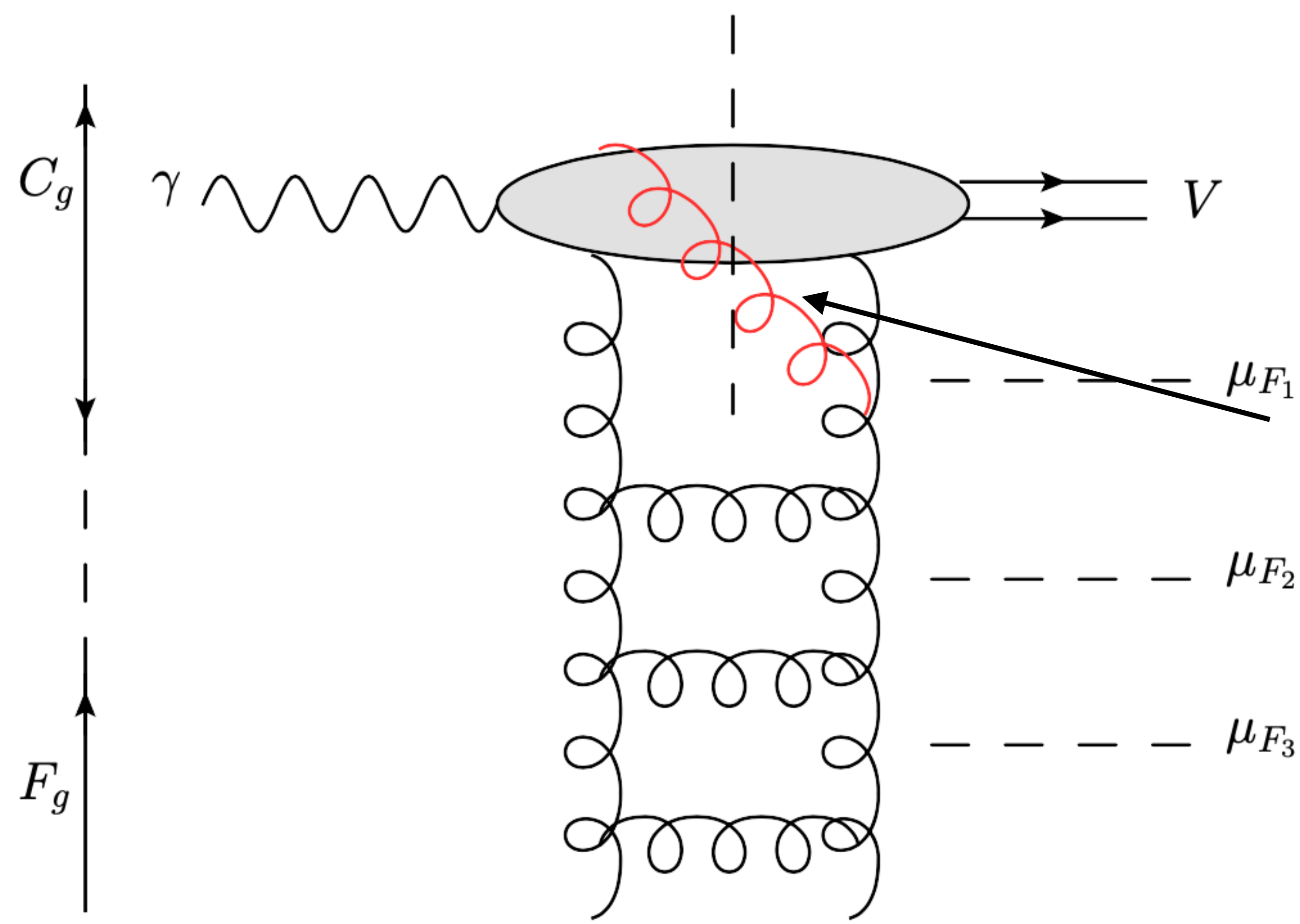
Effect of scale change driven by (generalised, skewed) DGLAP evolution:

$$A^{(0)}(\mu_f) = \left(C^{(0)} + \frac{\alpha_s}{2\pi} \ln\left(\frac{\mu_f^2}{\mu_F^2}\right) C^{(0)} \otimes V \right) \otimes F(\mu_F)$$

---

* At small xi, this is the double logarithmic contribution ~ln(1/xi) ln(muF^2/mc^2)

Treatment of double logarithmic contribution



The red gluon cannot be resummed in this scale shifting approach and so will always be treated as part of the higher order correction

Choice muF = mc ‘resums’ the gluon ladder contributions, enhanced by this double logarithmic contribution. They are intrinsically resummed within the kt factorisation framework* and here by judicious choice of factorisation scale

---

* But kt fact. framework treats only a *subset* of NLO corrections, those belonging to equivalence class of gluon-ladder diagrams

Shuvaev Transform cont.

The conformal moments H_i^N of the GPDs are given by

$$H_i^N \equiv \int_{-1}^{1} \mathrm{d}x R_{N,i}(x_1, x_2) H_i(x, \xi), \qquad i = q, g,$$

Ohrndorf, 82

The conformal moments are polynomials in even powers of ξ,

$$H_i^N = \sum_{k=0}^{\lfloor (N+1)/2 \rfloor} c_{k,i}^N \xi^{2k} = c_{0,i}^N + c_{1,i}^N \xi^2 + c_{2,i}^N \xi^4 + \ldots, \qquad , \; c_{0,i}^N = f_i^N$$

Leading term is Mellin moment of PDF

- Provided inverse exists then can relate GPDs to PDFs with suppression of order xi (i.e. good low x approx)

Shuvaev Transform cont.

Widely debated, certain conditions needing upheld, e.g lack of singularities in Re N > 1 plane

e.g Diehl, Kugler, 08

Regge theory considerations => condition met Martin, Nockles, Ryskin, Teubner, 09

- Can check in physically motivated ansatz, e.g MSTW2008 global partons input parametrisation

Martin, Stirling,Thorne, Watt, 09

$$xg(x,Q_0^2) = A_g x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\sqrt{x}+\gamma_g x) + A_{g'} x^{\delta_{g'}}(1-x)^{\eta_{g'}}.$$

Expand about x ~ 0

$$xg(x,Q_0^2) = A_g x^{\delta_g} + A_{g'} x^{\delta_{g'}} + \ldots,$$

Mellin transform:

$$\begin{aligned} xg^N(Q_0^2) &= \int_0^1 \mathrm{d}x x^{N-1}(A_g x^{\delta_g} + A_{g'} x^{\delta_{g'}}) + \ldots \\ &= \frac{A_g}{N+\delta_g} + \frac{A_{g'}}{N+\delta_{g'}} + \ldots, \end{aligned}$$

Fits to data (including 1sig. errors) suggest $\delta_g > -1$ and $\delta_{g'} > -1$

- Shuvaev transform describes HVM and GDVCS data well

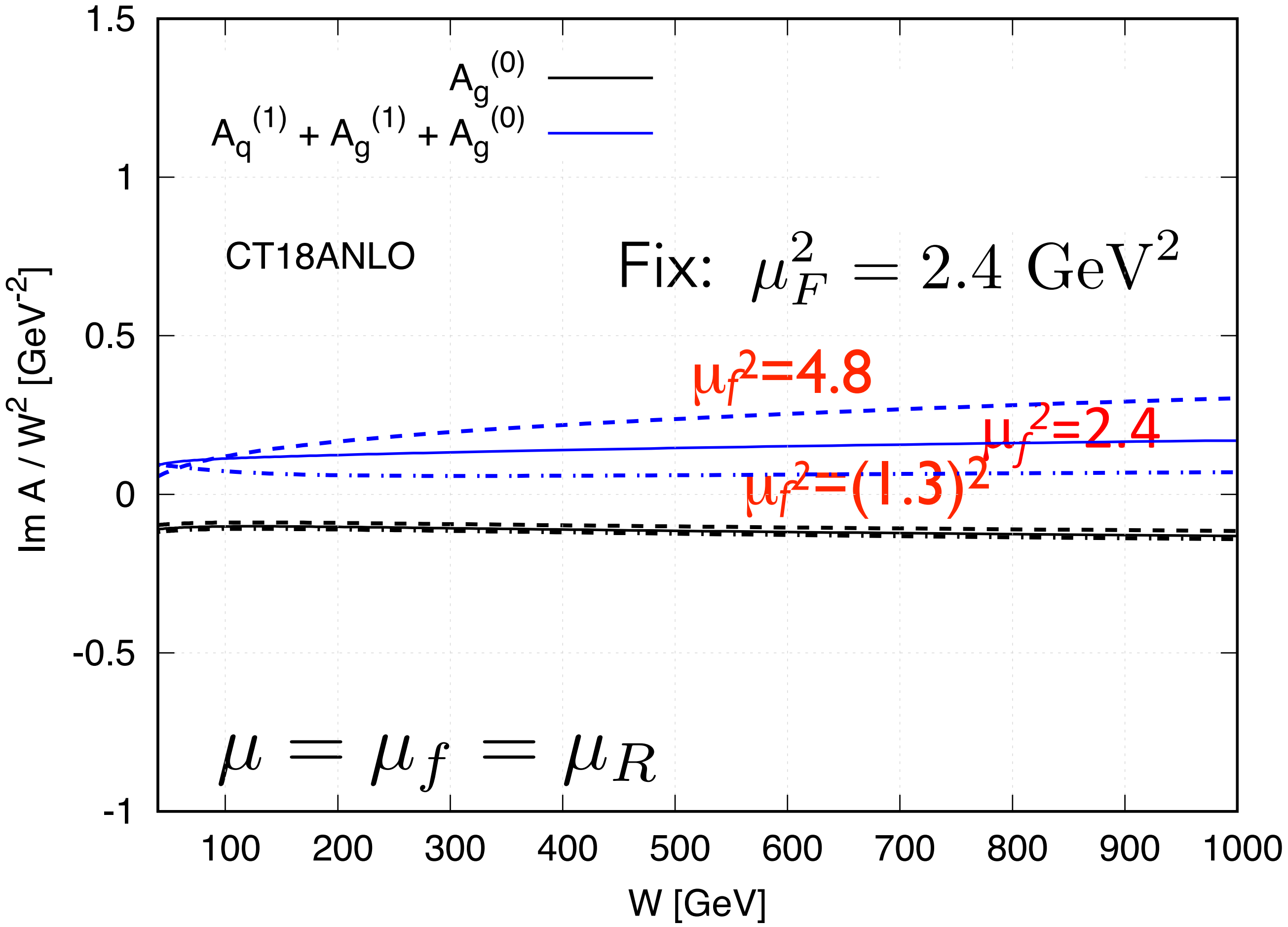
Kumericki, Muller, 10

Stability of prediction II

'Scale Fixing'

`Optimal' factorisation scale $\mu_F = m$ eliminates large logs at NLO

Jones et al., 1507.06942



Resummation of $(\alpha_S \ln(1/\xi) \ln(\mu_F/m))^n$

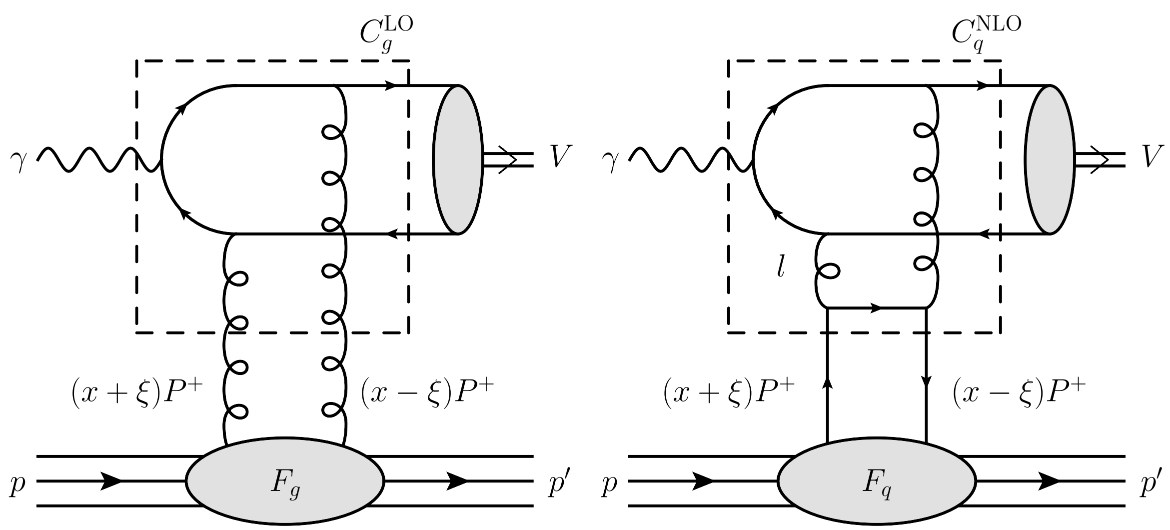
terms into LO PDF, leaving remnant NLO coefficient and residual, μ_f, scale dependence

$$A(\mu_f) = C^{LO} \times GPD(\mu_F) + C^{NLO}(\mu_F) \times GPD(\mu_f)$$

Look for another sizeable correction that can reduce variations further -> implementation of a **`Q0'** cut

Stability of prediction III

`Q0' cut Jones et al., 1610.02272



Fundamentally ubiquitous* and typically power suppressed, but sizeable here

$$\mathcal{O}(Q_0^2/\mu_F^2)$$

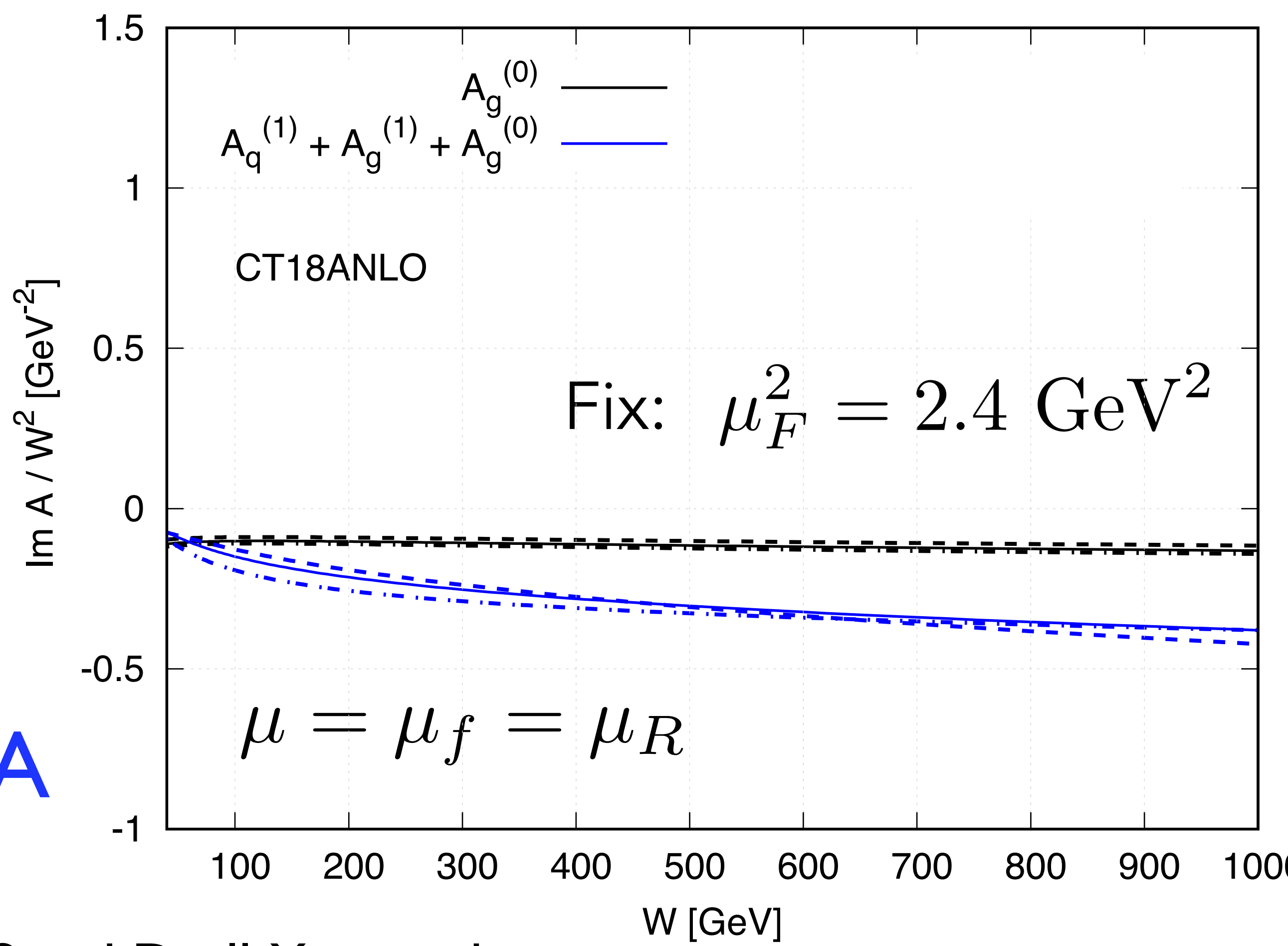
$\longrightarrow$

How do these predictions compare with the data at HERA and LHCb?

Subtract DGLAP contribution

NLO (| ℓ^2 | < Q_0^2)

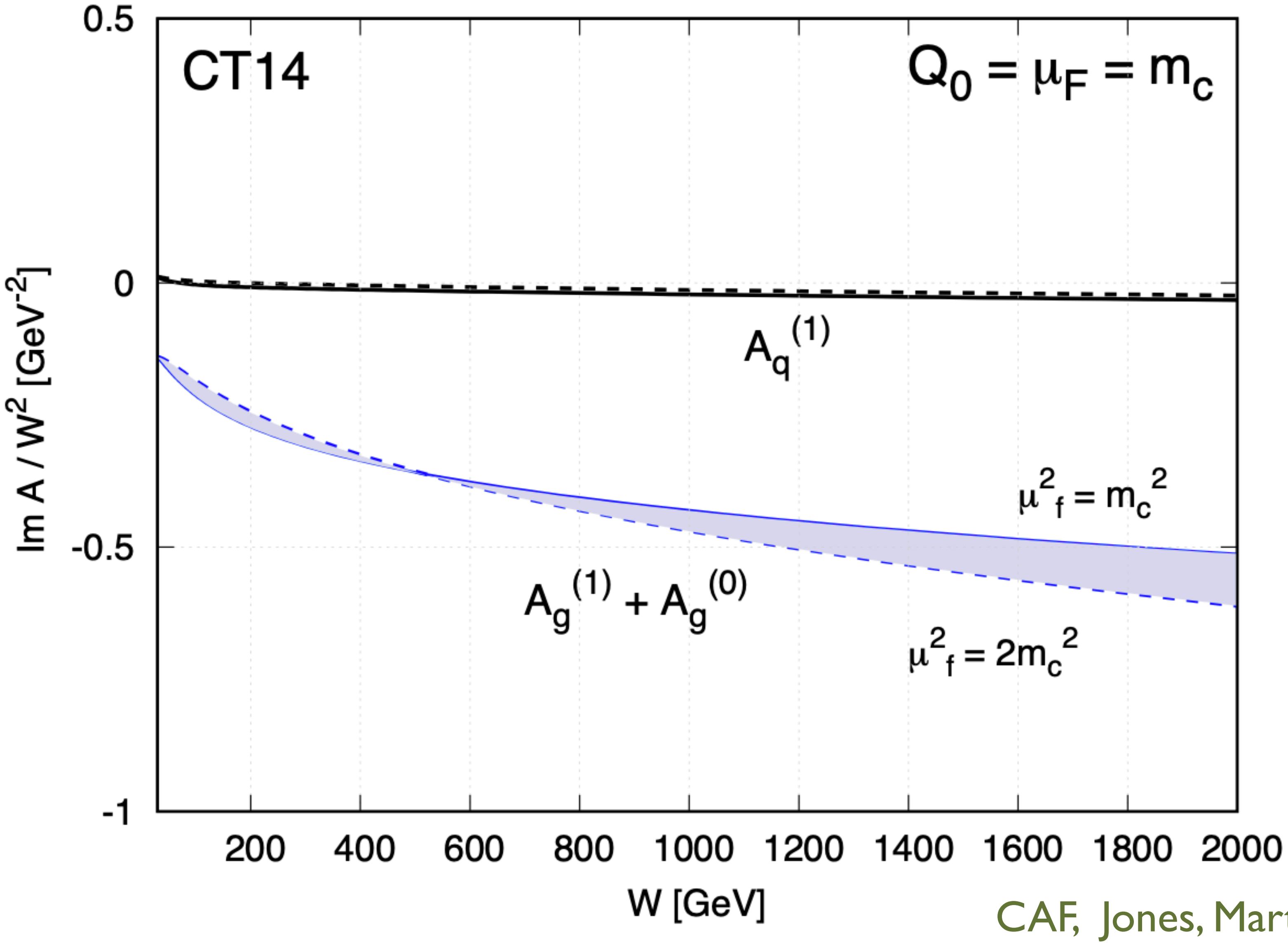
from known NLO MSbar coefficient function to avoid a double count with input GPD at Q_0.

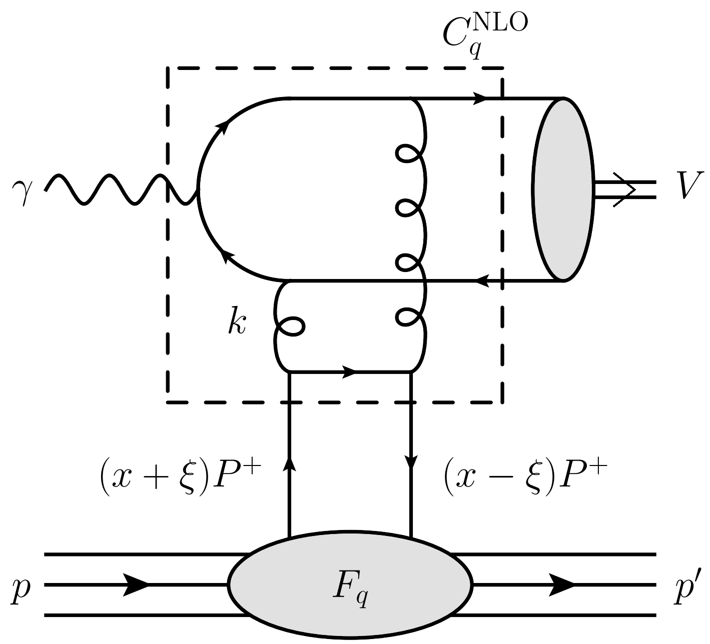


*see 1912.09304 for procedure applied to inclusive DIS and Drell-Yan production

Interplay of quark and gluons at NLO

After Q_0 subtraction:





CAF, Jones, Martin, Ryskin, Teubner, 1908.08398

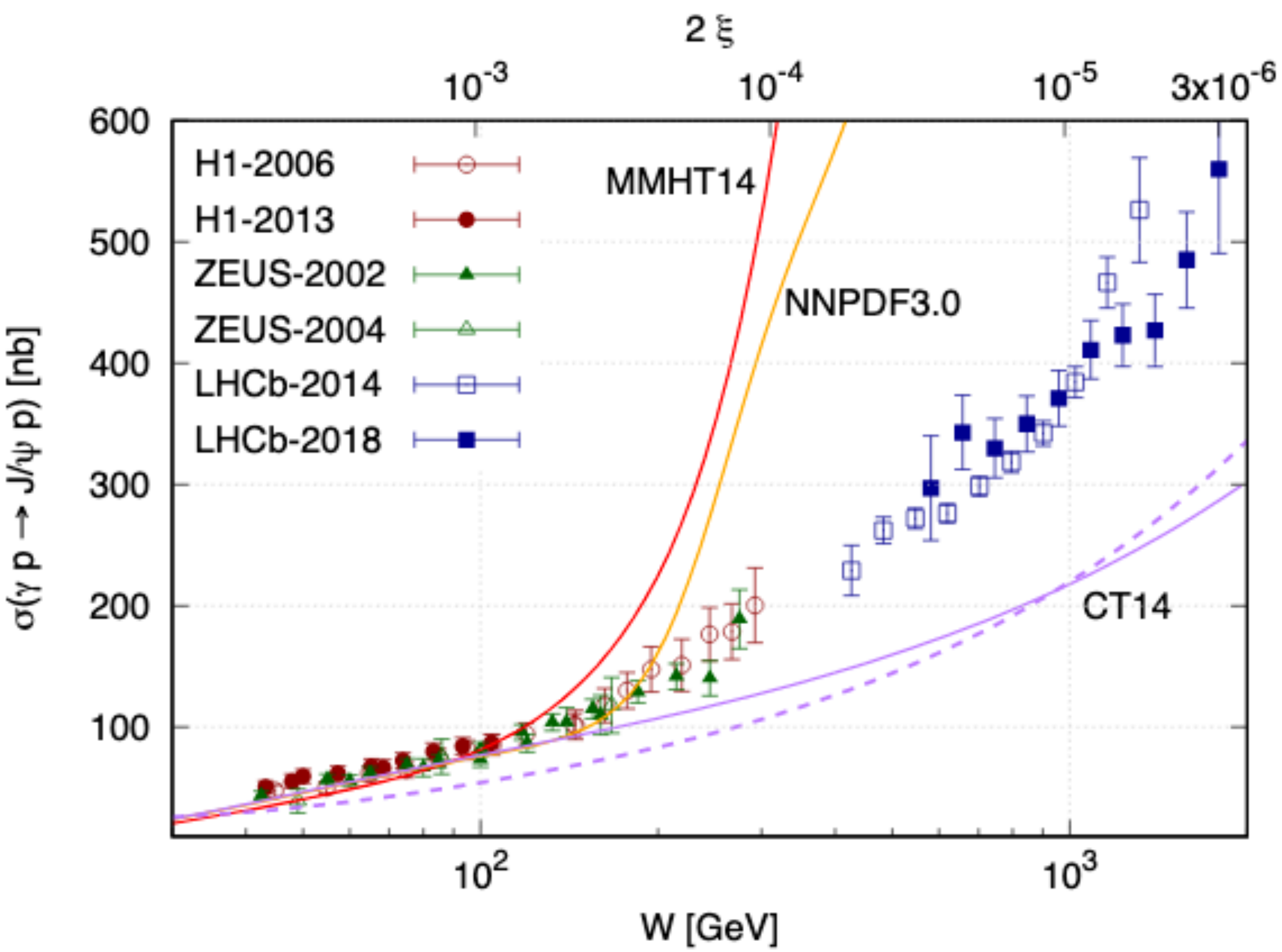
Quark contribution separated from hard scattering by at least *one* step of DGLAP evolution and is therefore removed after imposition of Q_0 subtraction (as reflected in the numerics)

⟶ Gluon GPD driven like at LO

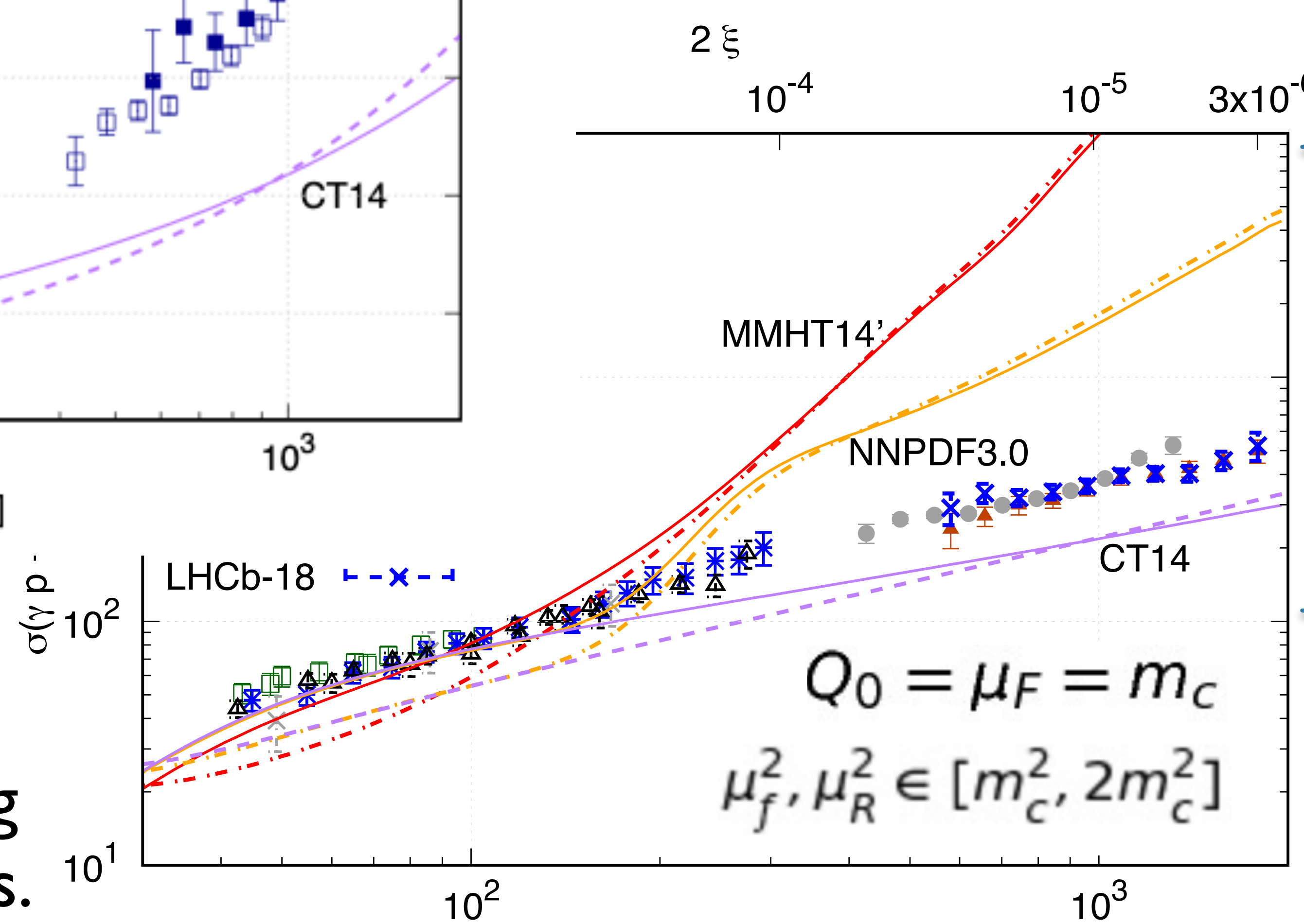
Towards the bigger picture

Plots demonstrates good scale stability of our NLO predictions in LHCb regime

Predictions at optimal scale (solid) agree better with HERA data



CAF, Jones, Martin, Ryskin, Teubner, 1907.06471 & 1908.08398



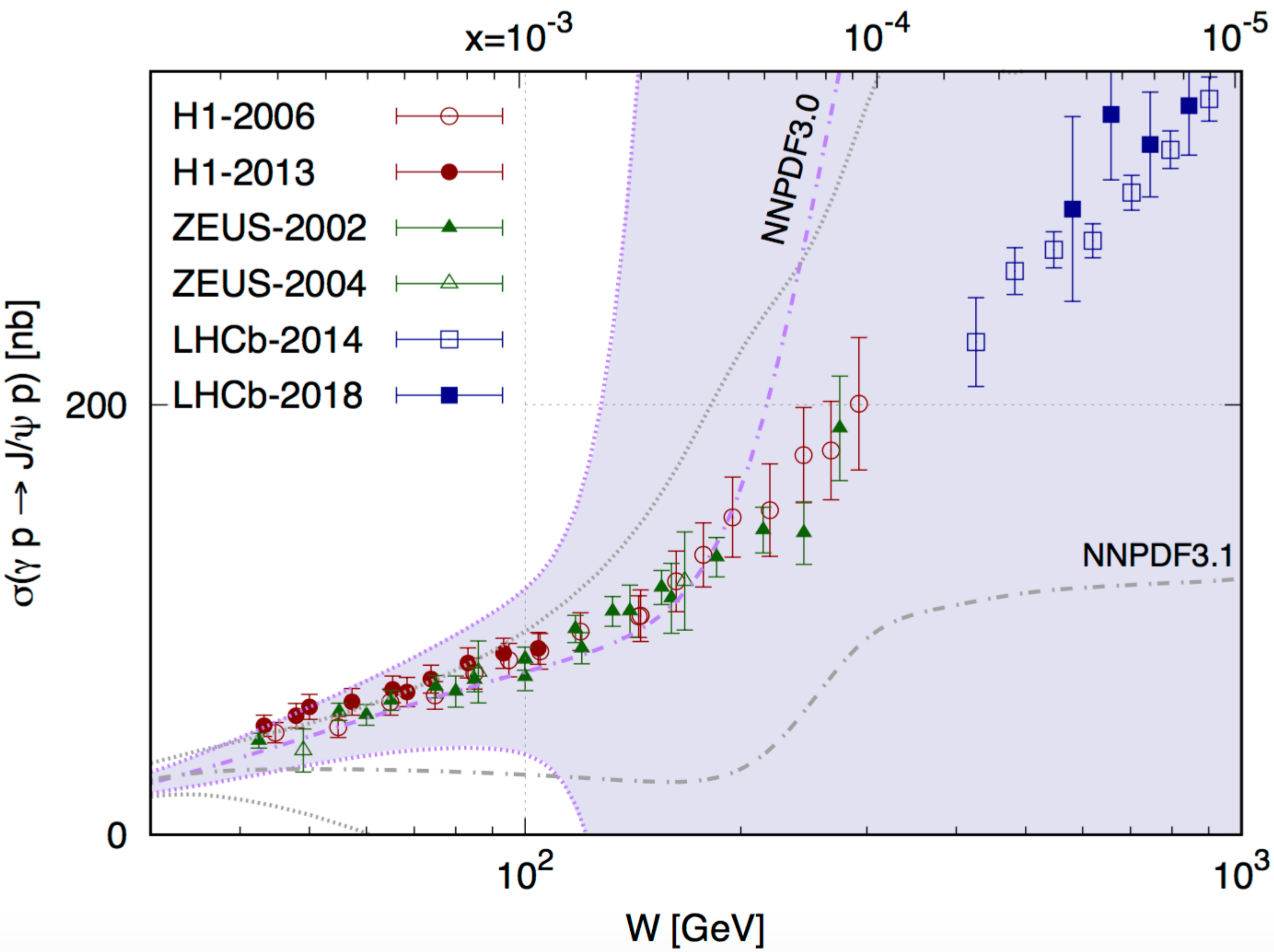
Diversity between predictions based on current global PDFs in unconstrained phase space -> important message

Repeat NB: Convoluting with existing global partons. Here, MMHT14, NNPDF3.0 & CT14

$$\frac{\mathrm{Re}\mathcal{M}}{\mathrm{Im}\mathcal{M}} \sim \frac{\pi}{2}\lambda = \frac{\pi}{2}\frac{\partial \ln \mathrm{Im}\mathcal{M}/W^2}{\partial \ln W^2} \quad \text{with} \quad \mathcal{M} \sim x^{-\lambda}$$

Error budgets: errors due to parameter variations in global fits >> experimental uncertainty and scale variations in the theoretical result

…… exclusive data now in a position to readily improve global analyses

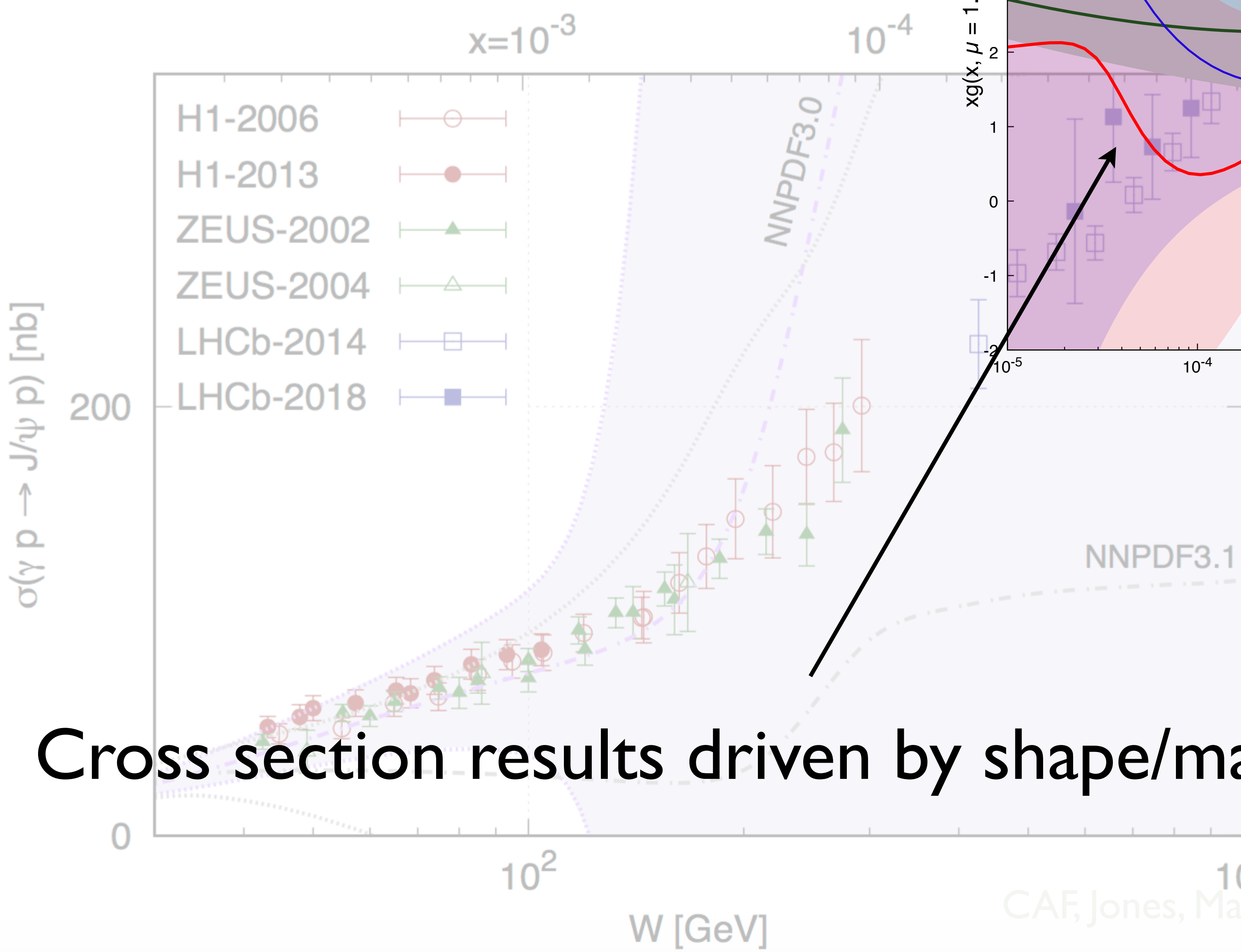


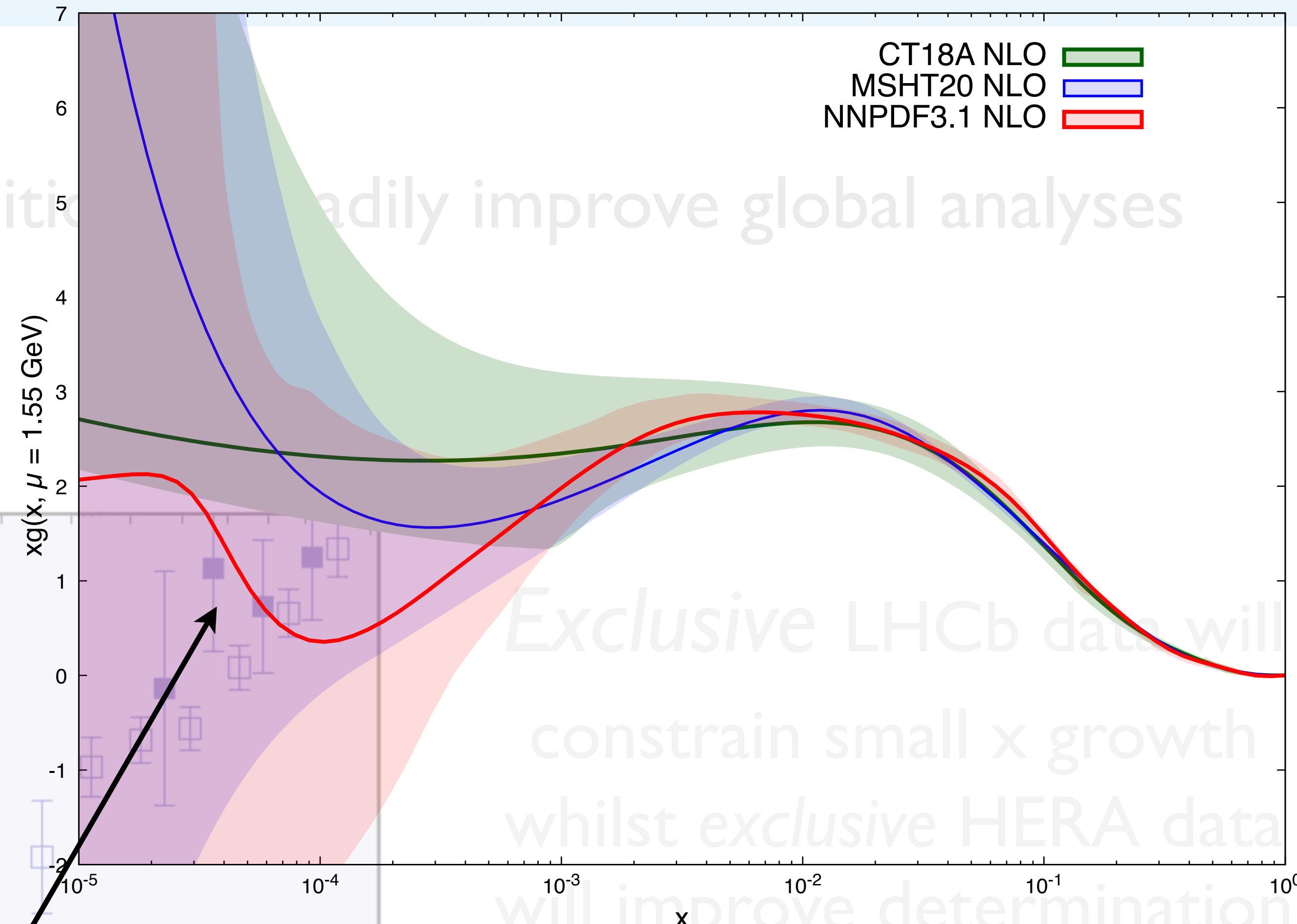
Exclusive LHCb data will constrain small x growth whilst *exclusive* HERA data will improve determination of partons in regime with data constraints already from diffractive DIS HERA data

CAF, Jones, Martin, Ryskin, Teubner, 1907.06471, 1908.08398

Error budgets: errors due to parameter variations in global fits >> experimental uncertainty and scale variations in the theoretical result

…… exclusive data now in a positi adily improve global analyses





Exclusive LHCb data will constrain small x growth whilst *exclusive* HERA data will improve determination of partons in regime with data constraints already from diffractive DIS HERA data

Cross section results driven by shape/magnitude of gluon PDF

CAF, Jones, Martin, Ryskin, Teubner, 1907.06471, 1908.08398

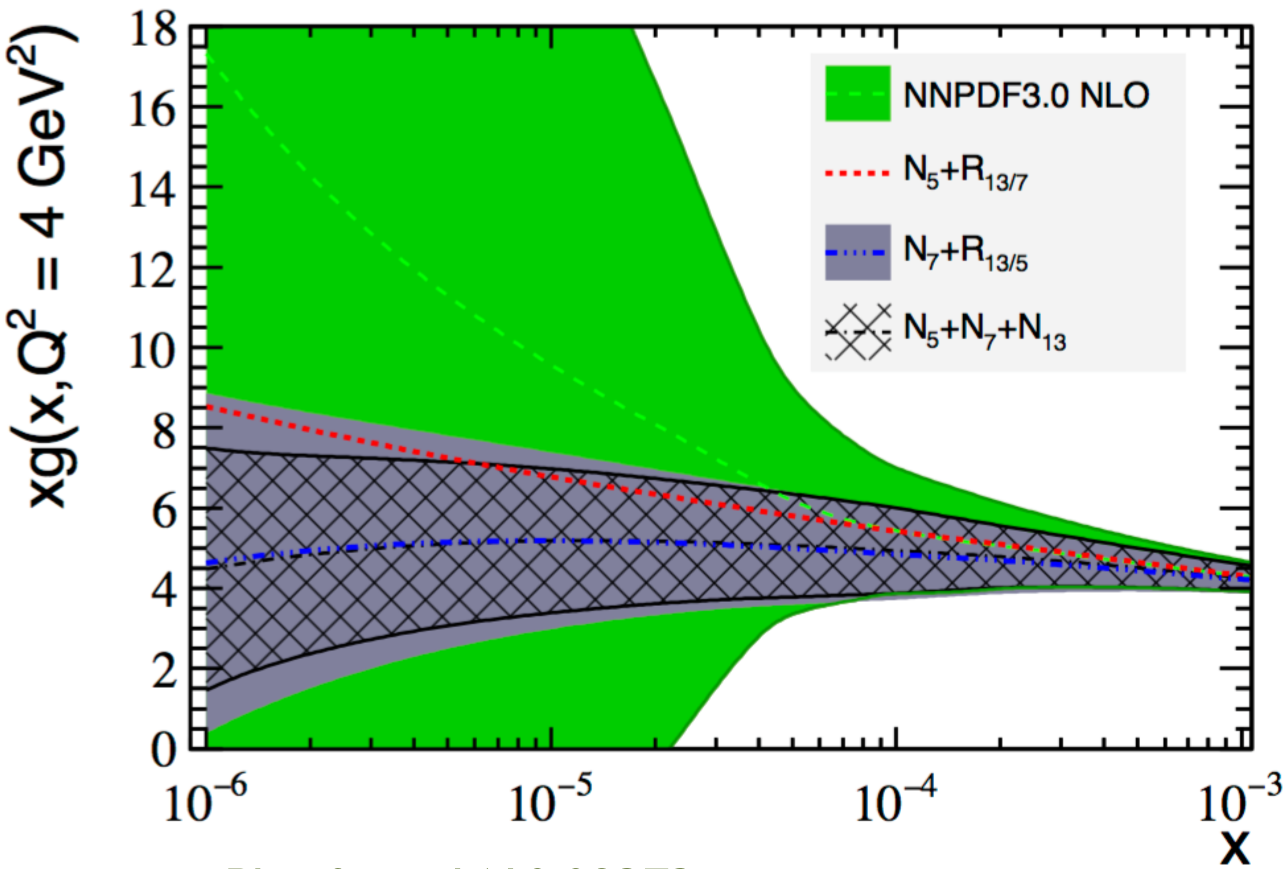
Constraints from inclusive D meson production data

Idea: Construct ratios of observables in y and p_t bins to combat various uncertainties

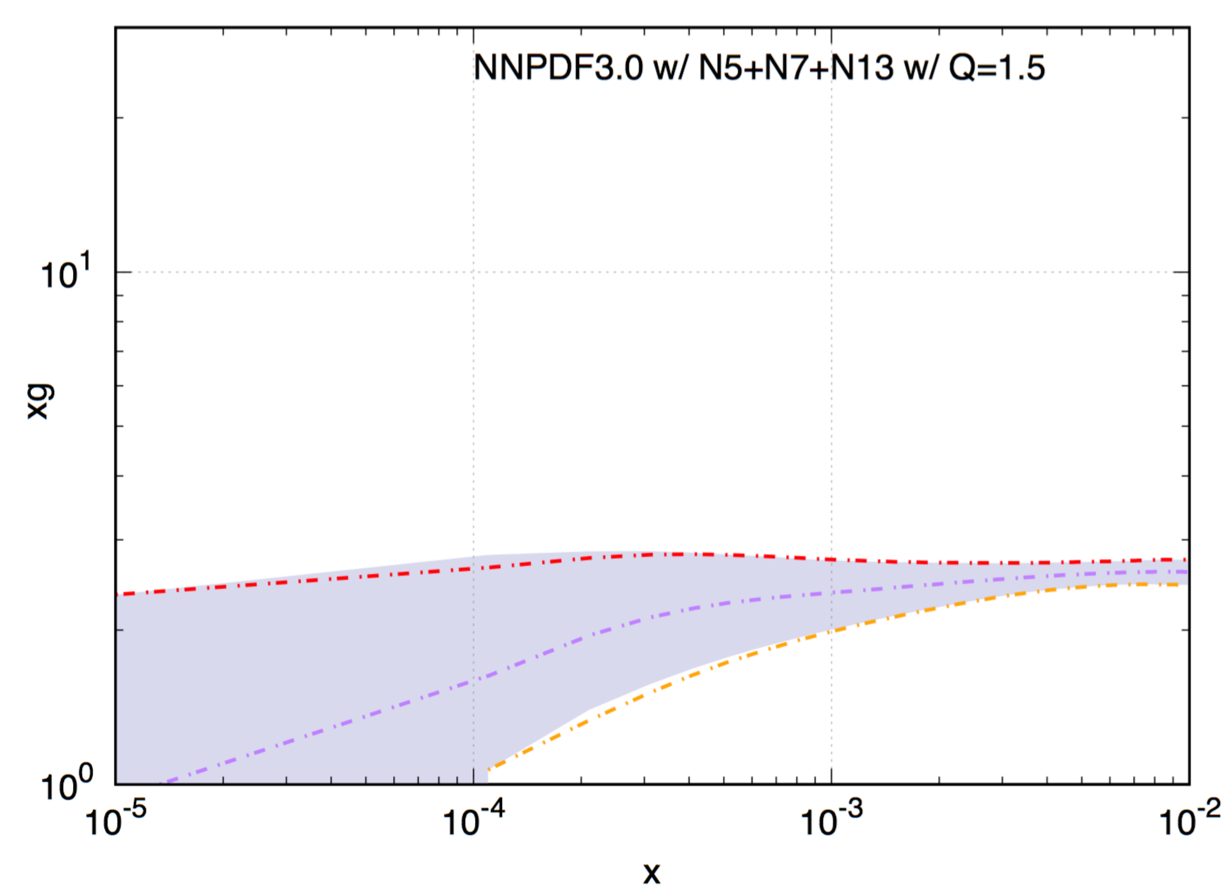
$$N_X^{ij} = \frac{d^2\sigma(\text{X TeV})}{dy_i^D d(p_T^D)_j} \Big/ \frac{d^2\sigma(\text{X TeV})}{dy_{\text{ref}}^D d(p_T^D)_j}$$

$$R_{13/X}^{ij} = \frac{d^2\sigma(13\text{ TeV})}{dy_i^D d(p_T^D)_j} \Big/ \frac{d^2\sigma(\text{X TeV})}{dy_\text{i}^D d(p_T^D)_j}$$

$\longrightarrow$ find decreasing gluon at the lowest x they may probe



Plot from 1610.09373

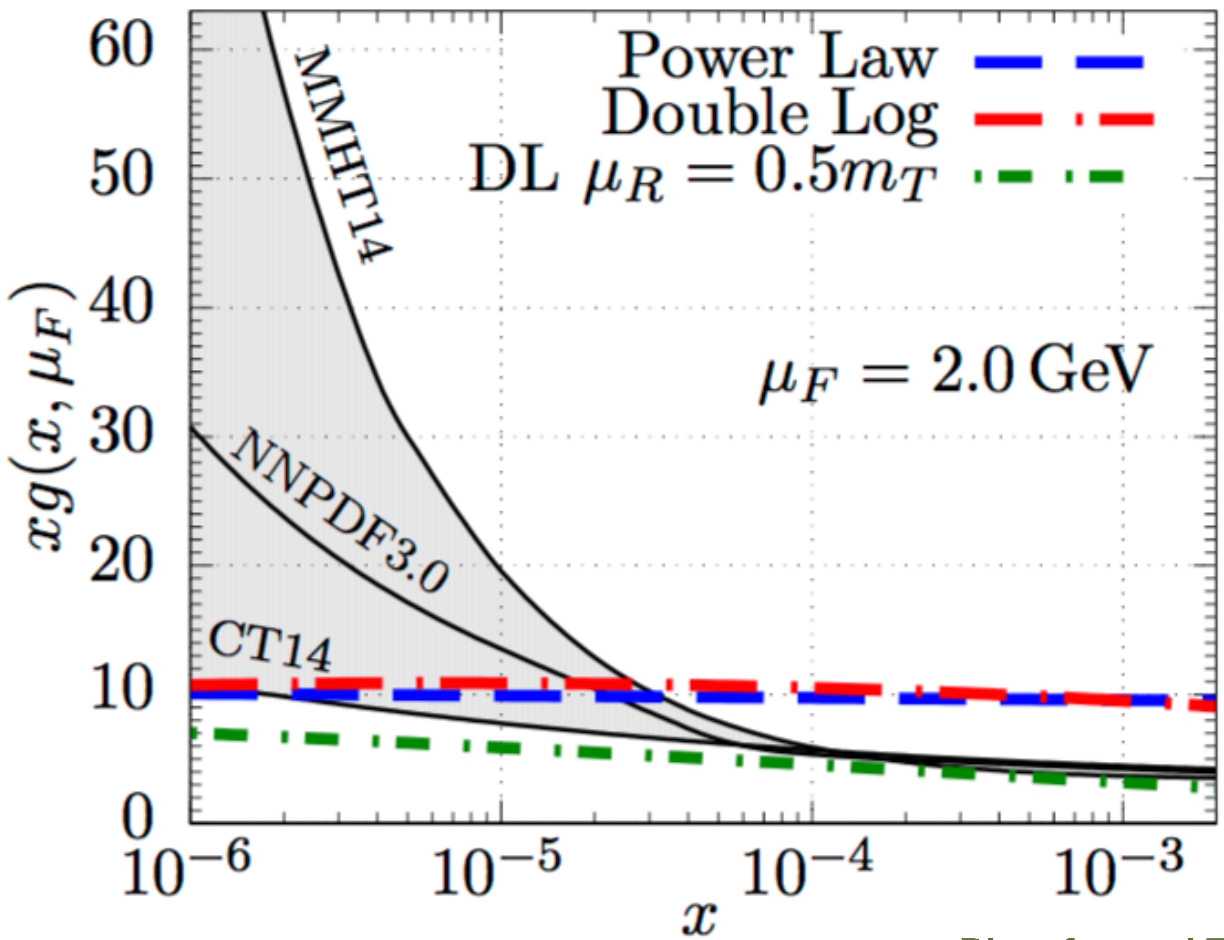


Tension with the J/psi data

We need a much harder gluon at low x to describe the exclusive J/psi LHCb data.

What's the reconciliation?

Indications of **inconsistencies** in the inclusive D experimental measurement (see next slide)

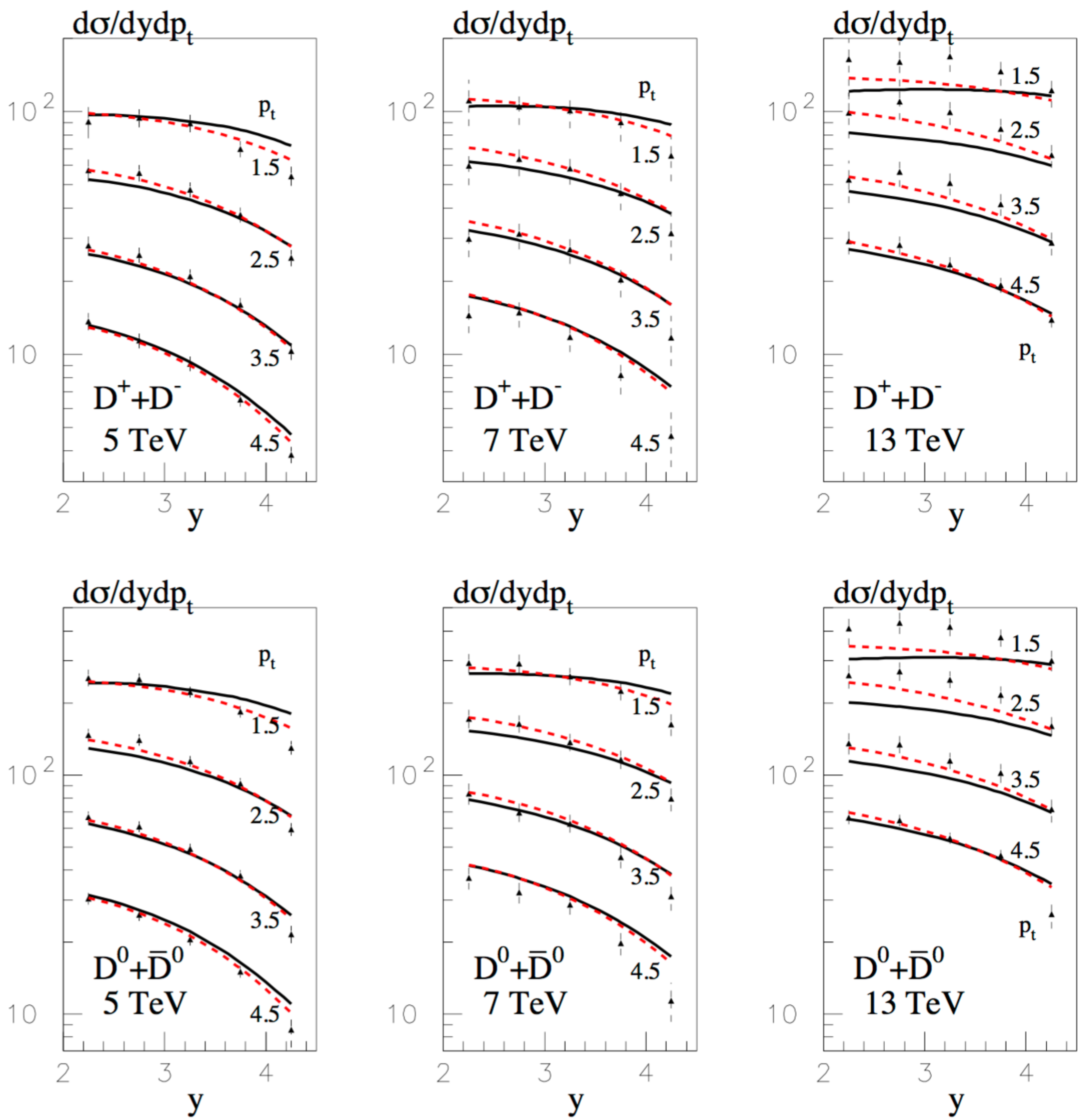


Plot from 1712.06834

$$xg(x) = N \left(\frac{x}{x_0}\right)^{-\lambda}$$

$$xg(x,\mu^2) \;=\; N^{\mathrm{DL}} \left(\frac{x}{x_0}\right)^{-a} \left(\frac{\mu^2}{Q_0^2}\right)^{b} \exp\left[\sqrt{16(N_c/\beta_0)\ln(1/x)\ln(G)}\right]$$

Rapidity and energy dependence of open charm cross section



Plot from 1712.06834

- Need *slower* increasing gluon with decreasing x to describe rapidity dependence
- Need *faster* increasing gluon with decreasing x to describe energy dependence

y ~ ln(1/x) !!

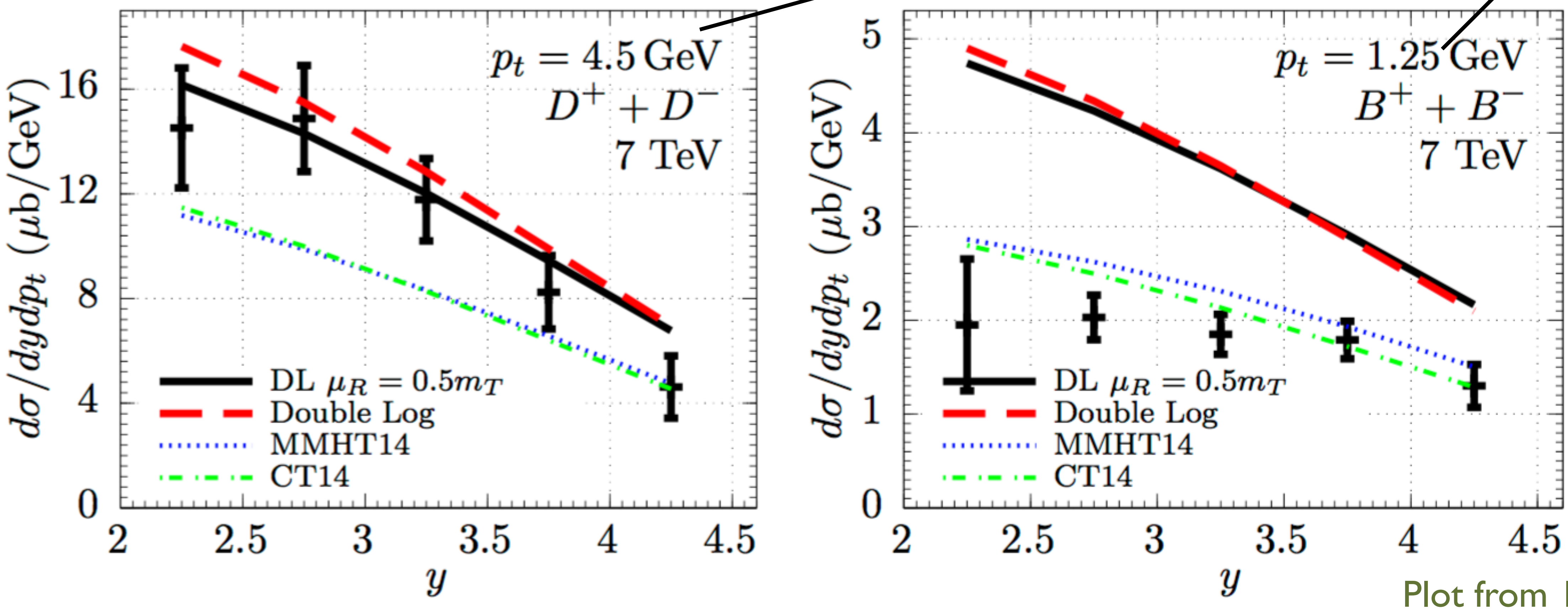
dash Q_0=1 GeV and $\mu_F = \mu_R = 0.85 m_T$

solid $\mu_f = \mu_R = 0.5 m_T$ and Q_0=0.5 GeV

Open beauty results

B sector has something to say…

p_t chosen to sample gluon at same factorisation scale and x



Plot from 1712.06834

Gluon found through fit to D meson data fails to describe the B meson distribution

Should we really trust the decreasing nature of the low -scale and -x gluon PDF obtained via fit to LHCb open charm data?

Extraction of low x gluon PDF via exclusive J/psi

Left

Approach I: Fit ... luon PDF ansatz to the data

...eight current global PDF analyses

$\ldots) = nN_0\,(1-x)\,x^{-\lambda}$

a = 0.136 +/- 0.006

0.966 +/- 0.025

Reweighted gluon PDF extractions via exclusive J/psi data and inclusive D meson production differ:

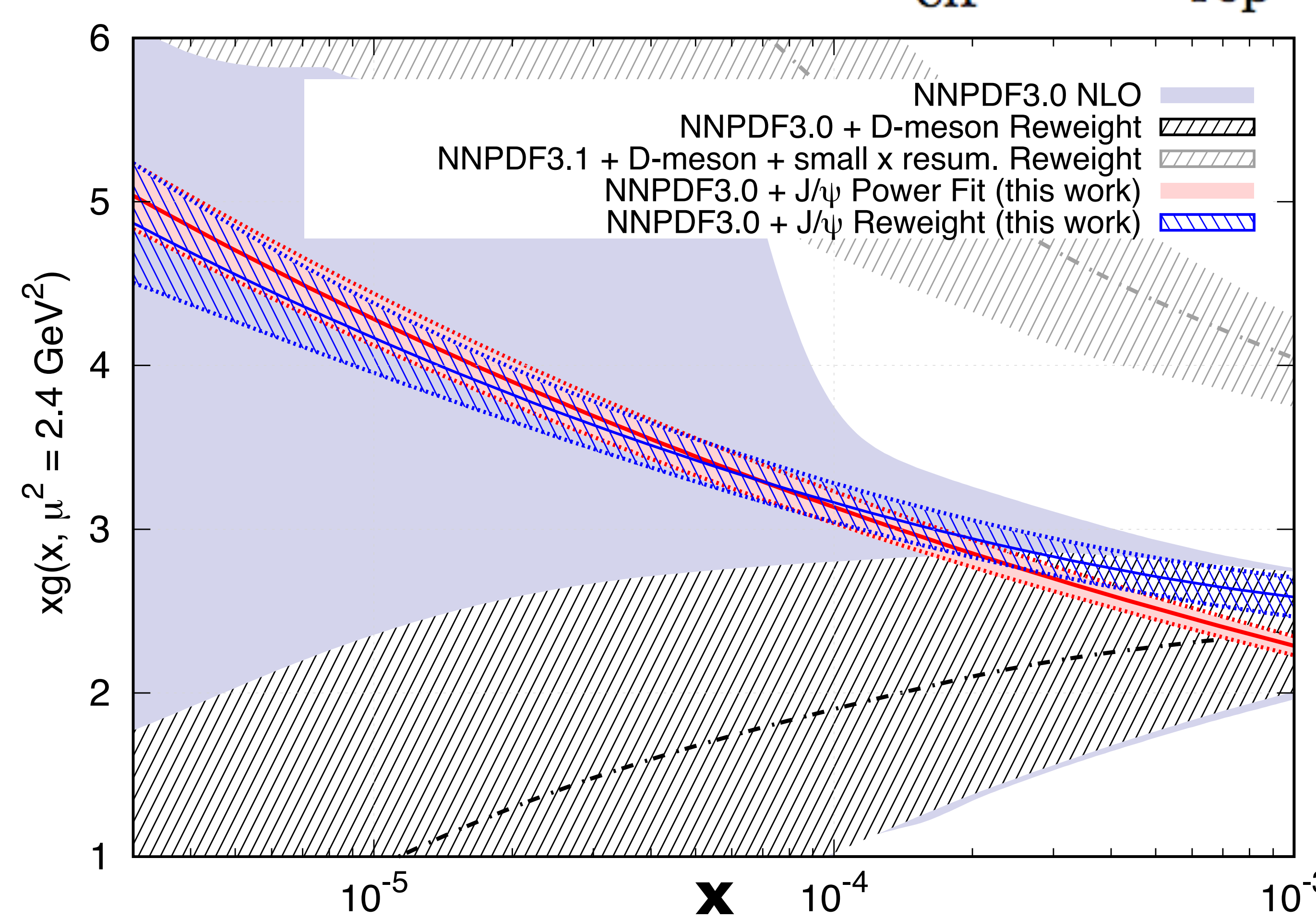
- Experimental inconsistencies in measurement of inclusive D meson production (?) (rapidity detection efficiency and self inconsistency with inclusive B meson detection) ,

  Oliveira, Martin, Ryskin, 1712.06834

- etac hadroproduction (conventional inclusive mode) favours harder gluon than that obtained from inclusive D meson production,

  Lansberg, Ozcelik, 2012.00702

$N_{\text{eff}} \ll N_{\text{rep}}$



General Set up and Framework

ccbar->J/psi:

- Effective field theory for production of heavy quarkonium [Bodwin et al. 1995]

$$\sigma_V = \sigma_{q\bar{q}} \cdot \langle O \rangle_V$$

- Relativistic corrections systematically computed by expanding matrix elements in powers of r:

$$\mathcal{M}[J/\psi] \propto (\mathcal{A}_\rho + \mathcal{B}_{\rho\sigma} r^\sigma + \mathcal{C}_{\rho\sigma\tau} r^\sigma r^\tau + \ldots)\epsilon^\rho_{J/\psi}$$

$$r^\mu = q_1^\mu - q_2^\mu$$

$\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ - matrix elements $\epsilon^\rho_{J/\psi}$ - J/ψ polarization

- We will compute to leading order in relative quark velocity v, for J/ψ:

$$\mathcal{M}[J/\psi] = \left(\frac{\langle O_1 \rangle_{J/\psi}}{2N_c m_C}\right)^{\frac{1}{2}} \mathcal{A}_\rho \epsilon^\rho_{J/\psi}$$

$$\langle O_1 \rangle_{J/\psi} \equiv \langle O_1(^3S_1) \rangle_{J/\psi}$$

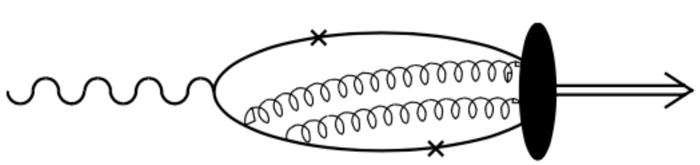
$$\mathcal{O}_1(^3S_1) = \psi^\dagger \boldsymbol{\sigma} \chi \cdot \chi^\dagger \boldsymbol{\sigma} \psi$$

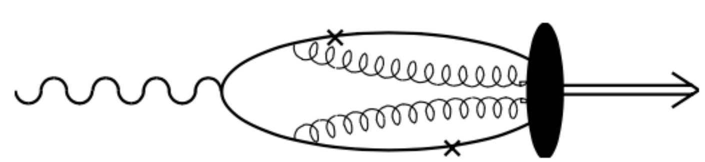
- Compute $\Gamma_{ee} \propto \langle O_1 \rangle_{J/\psi}$
  - Extract $\langle O_1 \rangle_{J/\psi}$ from measurement of Γ_{ee}

$$\langle O_1 \rangle_V = \frac{N_c}{2\pi} |R_S(0)|^2 + \mathcal{O}(v^2)$$

- Leading zeroth order term in rel. velocity (NRQCD)
- First non-vanishing O(v^2) relativistic correction small AFTER additional ccbar+gg Fock state component considered for gauge invariance

Hoodbhoy 97





- O(6%) cross section correction factor proportional to derivative of *square* of J/psi w.f. at origin (and affecting normalisation only and not energy dependence)

Sensitivity to the MSbar gluon PDF

- Remain in MSbar scheme with Q0 subtracted coefficient functions to NLO accuracy
- Subtraction does not affect IR or UV divergence renormalisation procedures
- Soft singularity at l=0 is removed after subtracting off the LO part of the NLO coefficient function before integral over loop momentum from 0 to Q0 is performed

$$\Delta \mathrm{Im}\mathcal{M}^q \;=\; \frac{\alpha_s^2}{2\pi}\int_\xi^1 dx\,(F_q(x,\xi,m_c)-F_q(-x,\xi,m_c))\left(\int_0^{Q_0^2}(M_a^q+M_b^q)\frac{2\pi m_c^4}{\hat{s}^2}dl^2\right)$$

- Precisely this FINITE contribution that is subtracted from full MSbar coefficient functions to avoid double counting inherent within MSbar scheme (subtraction fundamentally ubiquitous but numerically relevant for low scale processes only*)

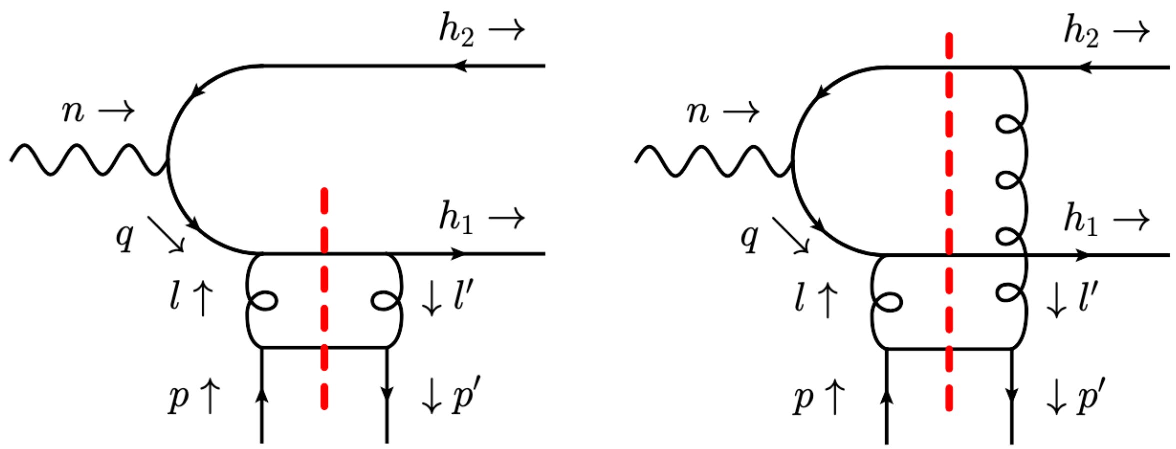
---

*see 1912.09304 for procedure applied to inclusive DIS and Drell-Yan production

Sensitivity to the MSbar gluon PDF

$$\Delta \mathrm{Im}\mathcal{M}^q = \frac{\alpha_s^2}{2\pi}\int_\xi^1 dx\,(F_q(x,\xi,m_c) - F_q(-x,\xi,m_c))\left(\int_0^{Q_0^2}(M_a^q + M_b^q)\frac{2\pi m_c^4}{\hat{s}^2}dl^2\right)$$

- Precisely this FINITE contribution that is subtracted from full MSbar coefficient functions to avoid double counting inherent within MSbar scheme (subtraction fundamentally ubiquitous but numerically relevant for low scale processes only)



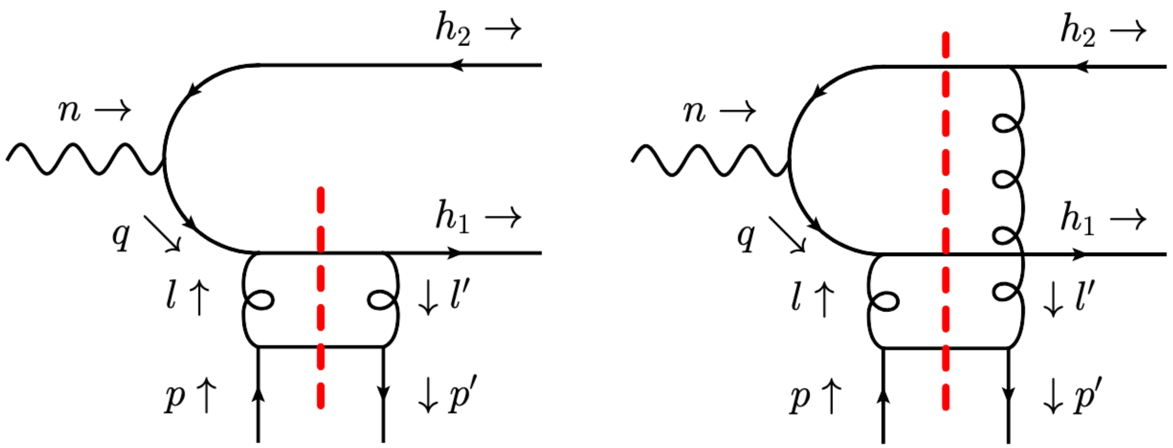
- NLO diagrams for quark and gluon channel considered. Contain both LO and NLO contributions. Subtract off LO contribution (part given by LO (generalised) DGLAP evolution P_LO x C^0, see previous) before integration over l is performed, cancelling soft singularity dl^2/l^2.

Q0 subtraction - further comments

- Factorisation ansatz of the form C(x) X PDF(Q)
- Bare C is computed with the loop momentum from 0 to infinity, with the extremes giving IR and UV divergences dealt with in a consistent renormalisation procedure to the desired order in perturbation theory.
- e.g. in MSbar, and dim reg.., 1/eps collinear and ubiquitous finite term \sim GammaEuler absorbed from bare C
- The PDFs in the factorisation ansatz are parametrised from some PDF input scale Q0 and the low momentum region lt < Q0 is already taken into account in the PDFs at Q0
- So, in convoluting C with PDFs in the factorisation ansatz, there exists a double counting of the lt < Q0 region. We must subtract off this region contributing to C.

Q0 subtraction - further comments

At NLO consider the following (cut) diagrams (+ gluon initiated):



Need to subtract off the lt < Q_0 contribution of these diagrams. This contains IR divergence. Formally cancelled at amplitude level through mass counter term:

$$\Delta_g^{col}(x,\xi) = -\frac{\alpha_S}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\left(\frac{\mu_F^2}{\mu^2}\right)\right)\int_{-1}^{1} dv\, \tilde{T}_g^{(0)}(v,\xi)\, V_{gg}(v,x)$$

$$\Delta_q^{col}(x,\xi) = -\frac{\alpha_S}{2\pi}\left(\frac{1}{\hat{\epsilon}} + \ln\left(\frac{\mu_F^2}{\mu^2}\right)\right)\int_{-1}^{1} dv\, \tilde{T}_g^{(0)}(v,\xi)\, V_{gq}(v,x)$$

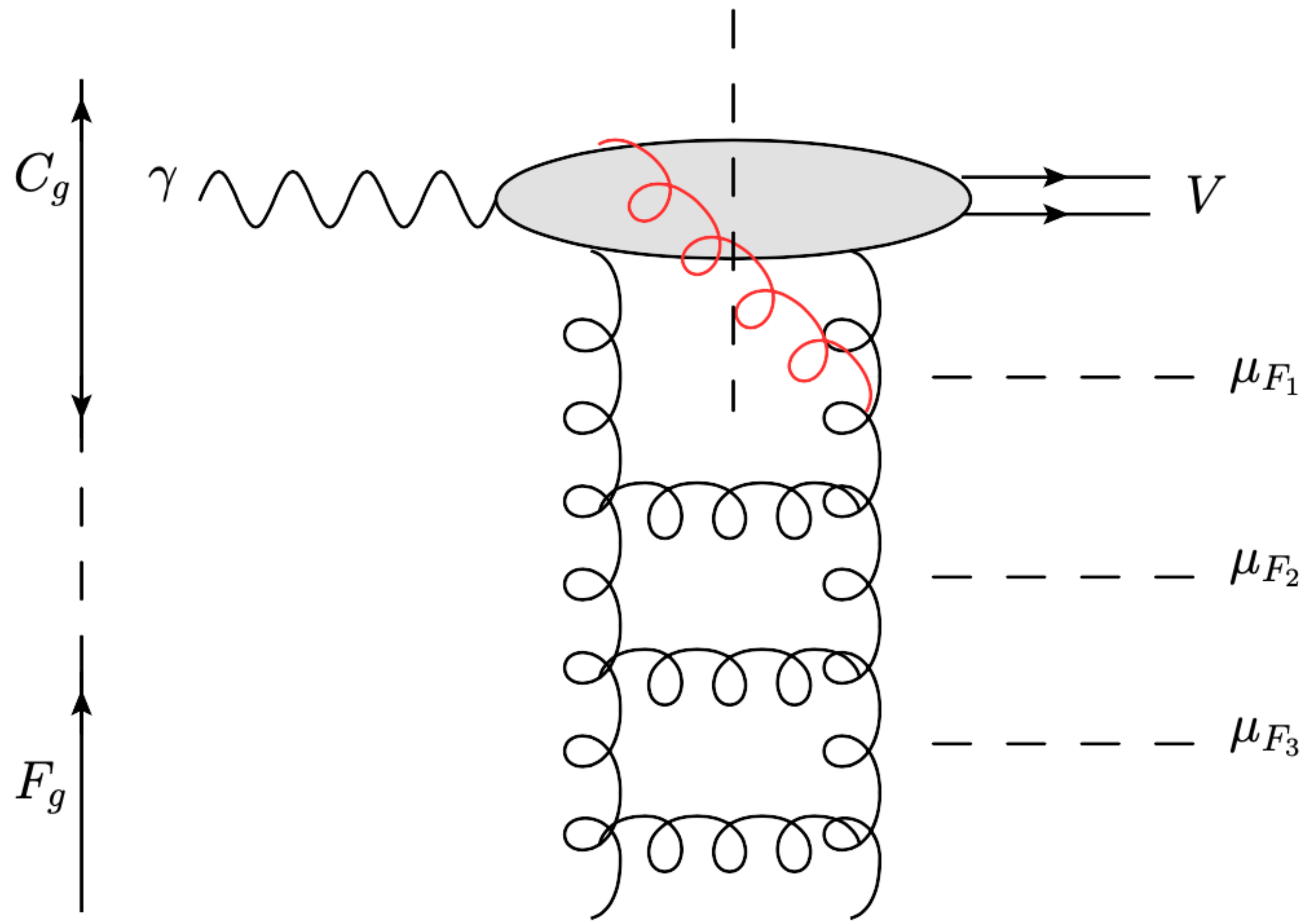
Diagrams where the form of the NLO correction at some factorisation scale can be included as part of evolution of PDFs at another factorisation scale carries a double counting - with \mu_F = mc & small xi this is the equivalence class of gluon ladder diagrams

Q0 subtraction - further comments

Remove at diagram level through the convolution P_LO X C_LO. This removes the dl^2/l^2 divergence of the diagram leaving a finite integration from 0 to Q_0 which can be computed and subtracted off together with this contribution amongst the other diagrams from the renormalised MSbar coefficient function

$$\Delta \mathrm{Im} \mathcal{M}^q \;=\; \frac{\alpha_s^2}{2\pi} \int_\xi^1 dx \left(F_q(x,\xi,m_c) - F_q(-x,\xi,m_c) \right) \left(\int_0^{Q_0^2} (M_a^q + M_b^q) \frac{2\pi m_c^4}{\hat{s}^2} dl^2 \right)$$

DLL effective small x resummation - further comments



Use scale shifting approach to move large corrections from NLO coefficient function to LO contribution. At small xi, these are logarithmically enhanced terms ~ln(1/xi) ln(muF^2/mc^2). Accomplish through scale choice muF = mc

--> LO GPDs at muF = mc include such DLLA logarithmically enhanced terms

$$A(\mu_f) = C^{LO} \times GPD(\mu_F) + C^{NLO}(\mu_F) \times GPD(\mu_f)$$

Higher twist contributions

- Absorptive corrections, which provide the saturation, are described by higher-twist operators and formally not known within the collinear factorisation approach.
- The relative size of the contribution of the next twist absorptive correction is driven by parameter:

$$c = \alpha_s \frac{xg(x)}{R^2 \mu_0^2}$$

- Factor appearing in GLR equation (Phys. Rept. 100 (1983) 1–150) provides non-linear terms through computation of so-called 'fan' diagrams in pQCD that tame (linear) BFKL evolution

- Semi-quantitative estimate based on this scaling gives higher-twist term of O(few percent*). Details in 2006.13857.

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*If one takes into consideration the colour factor calculated assuming that the low x gluon is emitted by the valence quark in the proton, then there is an additional factor of 81/16 which enhances the estimate to ~6.5%. However, the point is that the higher-twist contribution may be relatively small and that, together with the additional factor of alphas from <v2> \sim alphas, all the parametric dependence is included in the GLR factor c.

Alternate small x resummation

- By fixing the scale in the way described previously, we may miss terms containing a large ln(1/xi) not enhanced by a logarithm depending on the factorisation scale, previously considered $(\alpha_S \ln(1/\xi)\ \ln(\mu_{F/m}))^n$
- Can also consider terms $(\alpha_S \ln(1/\xi))^n$:

$$\mathcal{I}m\mathcal{M}^g \sim H^g(\xi,\xi) + \int_\xi^1 \frac{dx}{x} H^g(x,\xi) \sum_{n=1} C_n(L) \frac{\bar{\alpha}_s^n}{(n-1)!} \log^{n-1}\frac{x}{\xi}$$

$$A \sim 1 + z\ln\left(\frac{m^2}{\mu_F^2}\right) + z^2\left[\frac{\pi^2}{6} + \frac{1}{2}\ln^2\left(\frac{m^2}{\mu_F^2}\right)\right] + \ldots, \quad z^n \sim \alpha_s^n \ln^n(1/\xi)$$

1601.07338

$$a) \quad (\mu_F = M_V): \quad 1 - 1.39\,z + 2.61\,z^2 + 0.481\,z^3 - 4.96\,z^4 + \ldots$$
$$b) \quad (\mu_F = M_V/2): \quad 1 + 0.\,z + 1.64\,z^2 + 3.21\,z^3 + 1.08\,z^4 + \ldots .$$

- **To investigate:** Supplement the fixed order NLO code with the resummed coefficients (with and without a Q0 subtraction)