The Gluon GPD from Small to Moderate x using Partial Twist Expansion

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In collaboration with R. Boussarie (in preparation)
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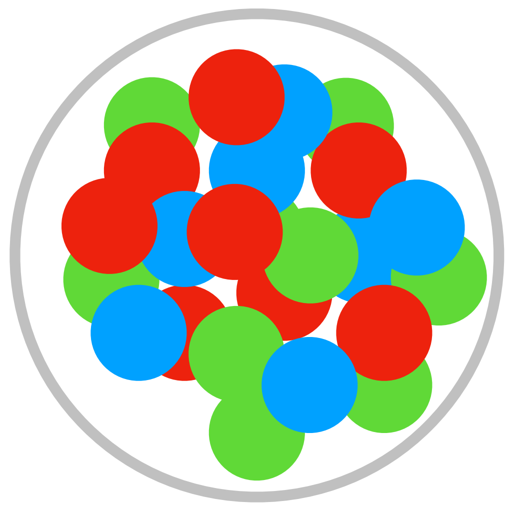
Outline

- Motivation
- Philosophy: partial resummation of higher twists
- Applications: inclusive DIS, exclusive Compton scattering

Motivation

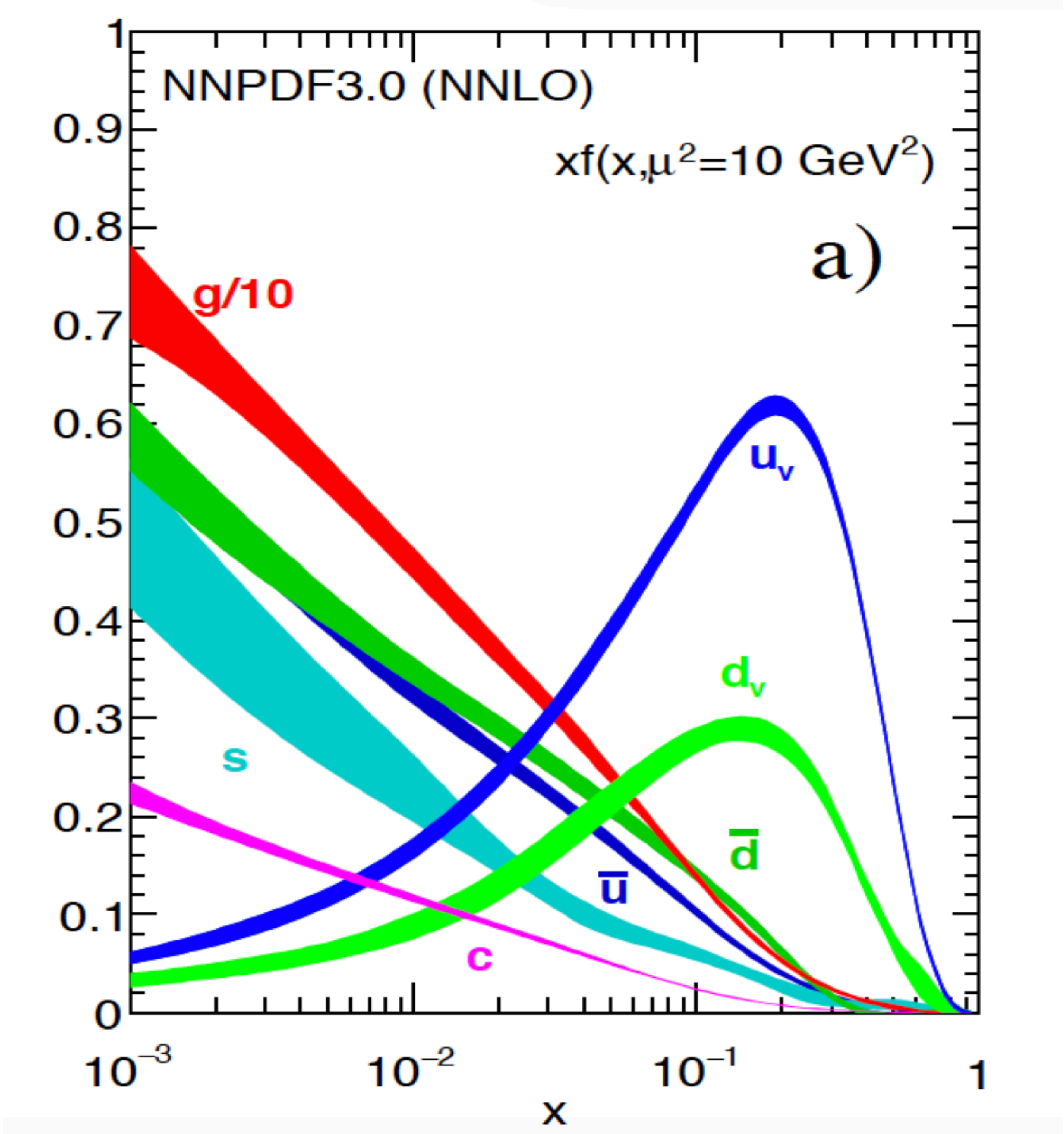
Regge limit (small x)

$s \gg Q^2$



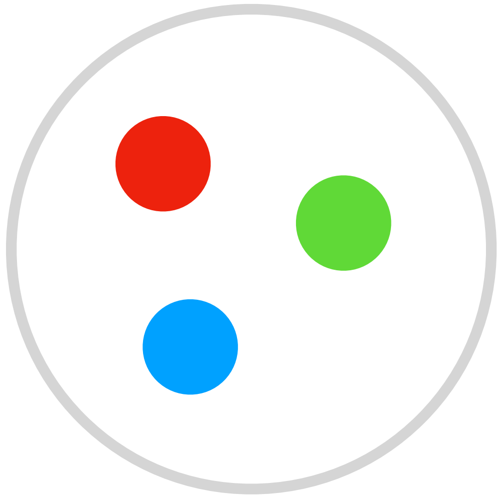
Strong fields

$A^\mu \sim 1/g$



Bjorken limit (moderate x)

$s \sim Q^2$



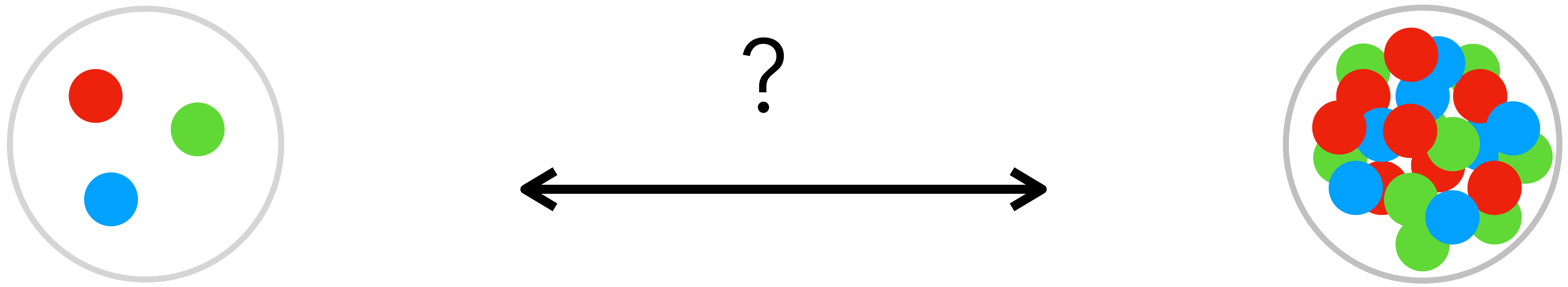
Partons

- Parton picture expected to break down at small $x \sim Q^2/s$
- Gluon Saturation $Q < Q_s(x) \sim x^{-\lambda}$: Relevant d.o.f.'s are strong classical fields (or Wilson lines)

Motivation

Bjorken limit (moderate x) | Regge limit (small x)

$s \sim Q^2$ | $s \gg Q^2$



Partons | Strong fields

How to connect the two regimes of QCD from first principles ?

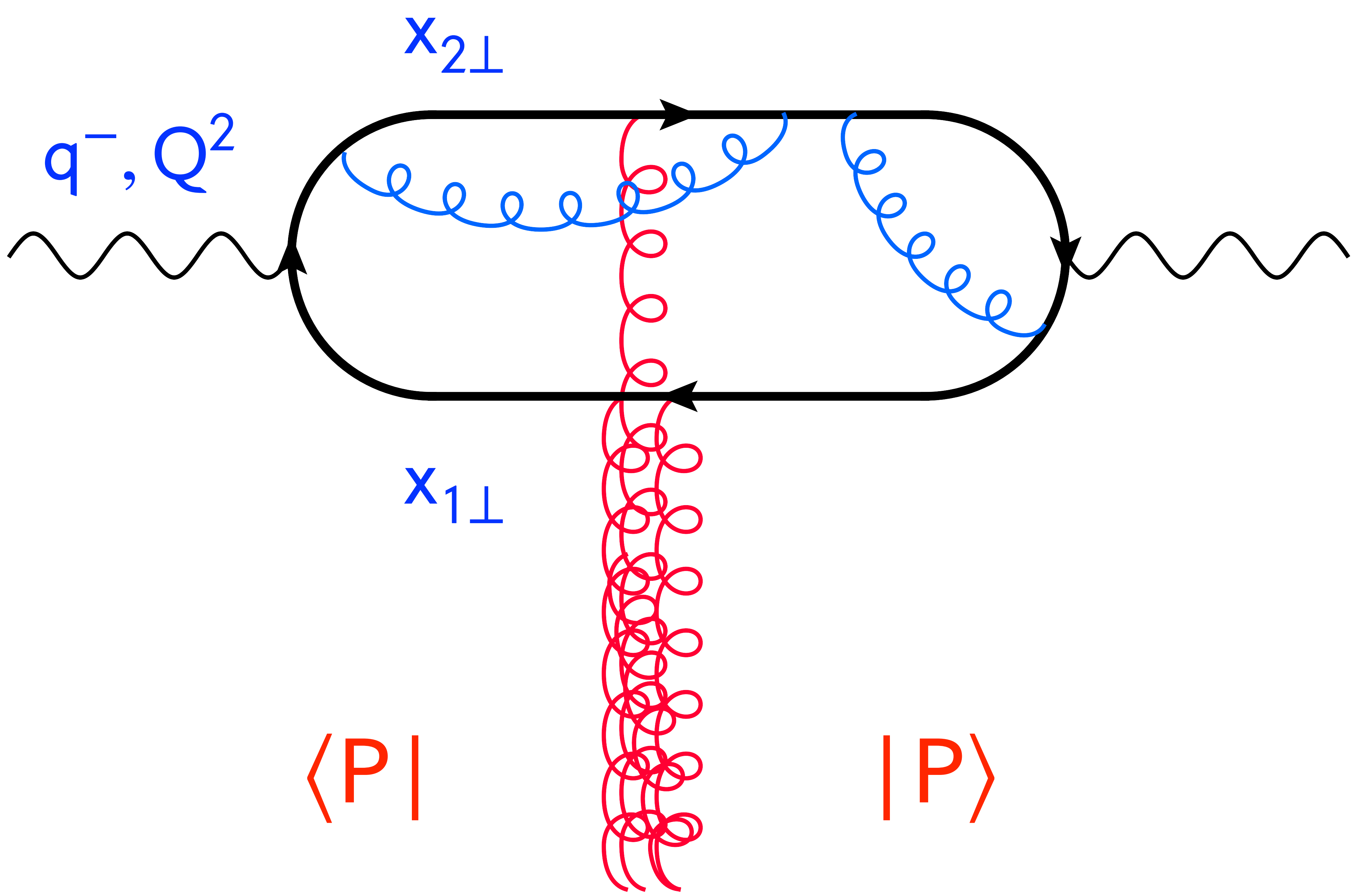
Previous attempts in the dilute regime and leading twist: CCFM, unified DGLAP/BFKL [Kwiecinski, Martin, Stasto] Duality [Altarelli, Ball, Forte]

The dipole picture at small x

- High energy factorization: $O(1/s)$, all powers in $x_1 - x_2 \sim 1/Q$

Long lived
$q\bar{q}$ fluctuation

$$S = \int dx_{1\perp} dx_{2\perp} \; H(x_1, x_2) \; \langle P' | \, [\mathrm{Tr}(U_{x_{1\perp}} U^\dagger_{x_{2\perp}}) - N_c] \, | P \rangle_Y$$



- Building block: infinite Wilson lines

$$U_{r_\perp} \equiv [+\infty, -\infty]_{r_\perp} = P\exp\left[ig \int_{-\infty}^{+\infty} du^- A^+(u^-, r_\perp) \right]$$

- $Y \equiv \ln k^-$ dependence via BK/JIMWLK: resums $\ln s$

$\Delta x_\gamma^- \gg \Delta x^- \sim 1/P^+$

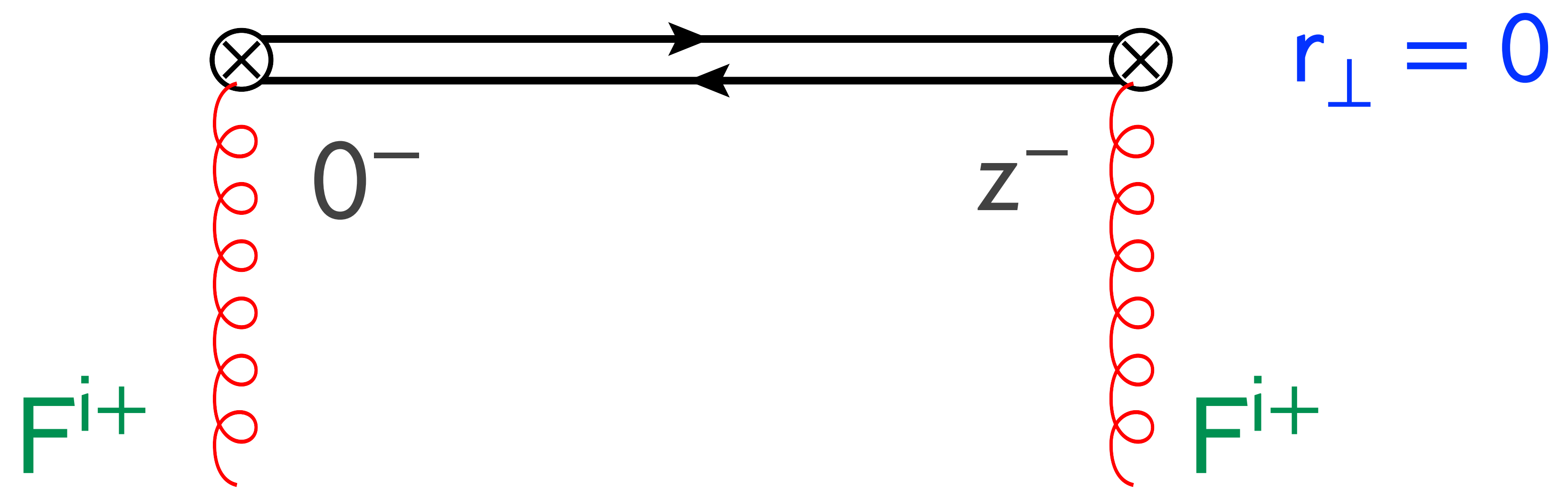
[Balitsky, Kovchegov, Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

A tale of two distributions

$r_\perp = 0 \quad \rightarrow \quad$ Gluon PDF

$$\int_x \langle P | \, F^{i+}(z^-) \, W \, F^{i+}(0^-) \, W^\dagger \, | P \rangle \, e^{iz^- x P^+}$$

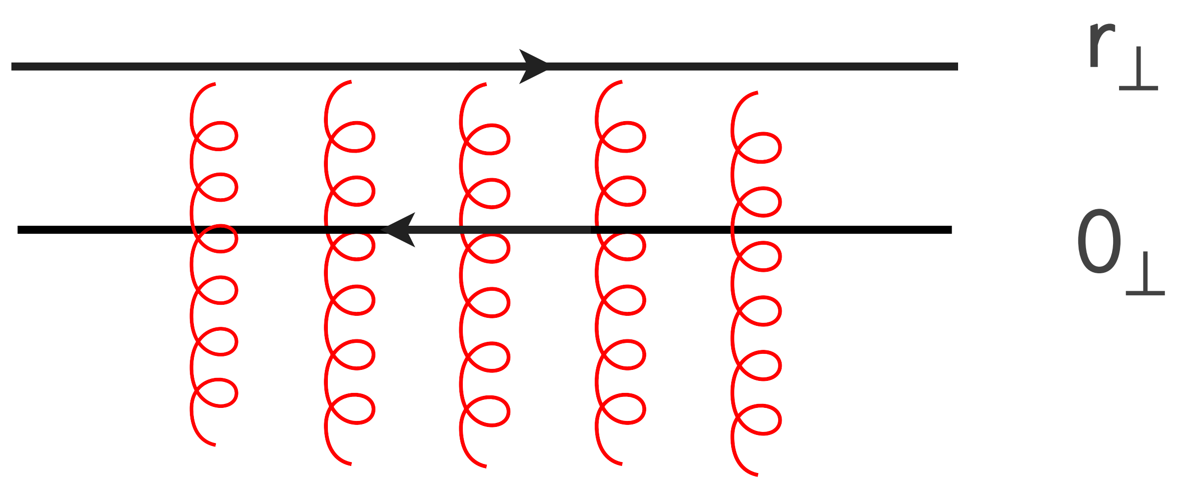
$$W \equiv [0^-, z^-]_{r_\perp = 0} = P \exp\left[ig \int_0^{z^-} du^- A^+(u^-, r_\perp = 0) \right]$$



$x = 0 \quad \rightarrow \quad$ Dipole gluon distribution

$$\int_k \langle P | \, \mathrm{Tr} \, U_r \, U_0^\dagger \, | P \rangle \; e^{-i r_\perp \cdot k_\perp}$$

$$U_{r_\perp} \equiv [+\infty, -\infty]_{r_\perp} = P \exp\left[ig \int_{-\infty}^{+\infty} du^- A^+(u^-, r_\perp) \right]$$



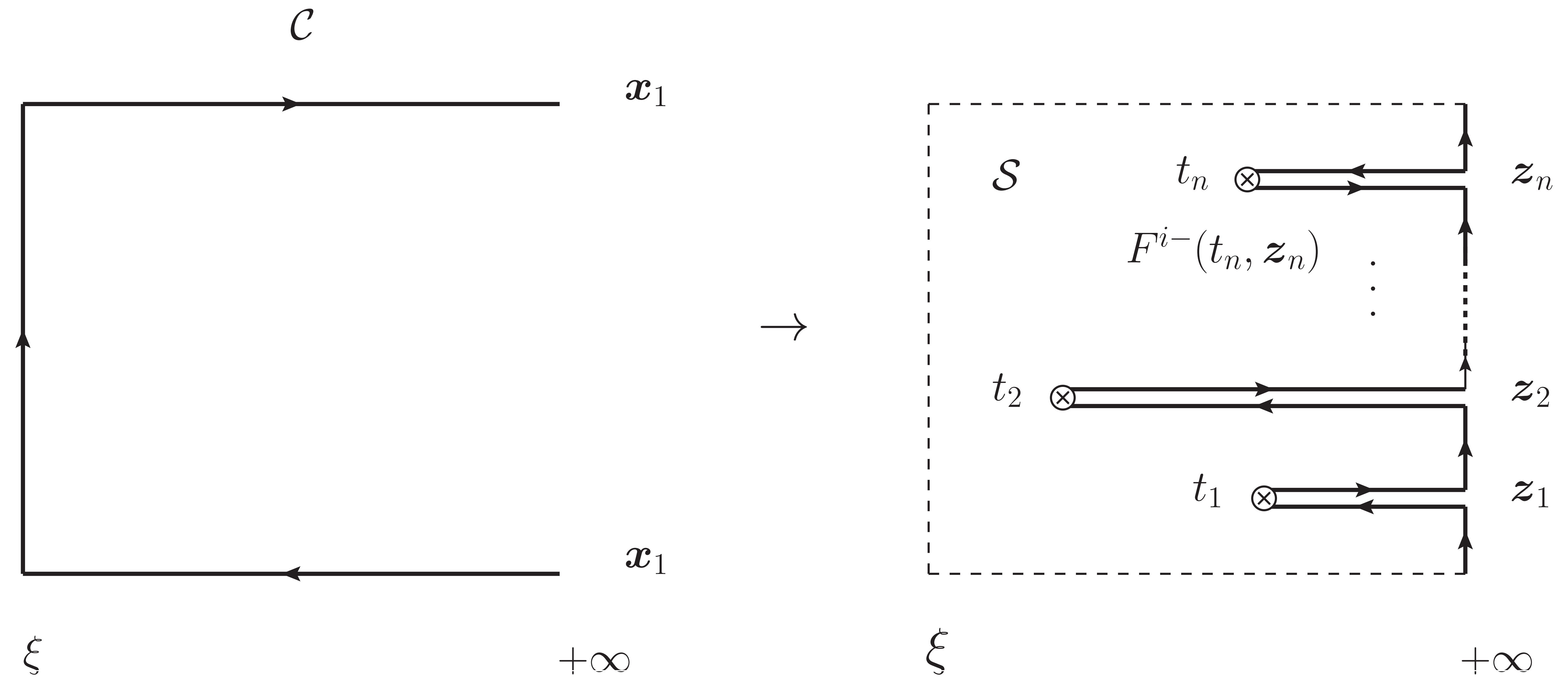
Beyond shock wave

- Subeikonal expansion about the shock wave: NL power in $1/s$, all powers in $1/Q^2$ [Agostini, Altinoluk, Armesto, Beuf, Martinez, Moscoso, Salgado, Czajka, Tymowska] [Kovchegov, Sievert, Pitonyak, Tarasov] [Chirilli]
- Similar to higher twist corrections: increasing number of operators in power suppressed corrections
- All power corrections are not created equal!
    - Keeping Q^2 fixed and decreasing s: a subset of power corrections become important when s tends to Q^2
    - Conversely, keeping s fixed and decreasing Q^2 : a tower of higher twist corrections dominate when $Q^2 \ll s$

Dipole ↔ parton distribution equivalence

YMT, Boussarie (2020)

- non-Abelian Stokes' theorem: the dipole operator can be written as a path ordered tower of "twisted" field strength tensor (i.e. dressed with future pointing Wilson lines)



$$U_{\boldsymbol{x}_2} U^{\dagger}_{\boldsymbol{x}_1} \equiv \mathrm{P}\exp\left[\oint_C dx_\mu A^\mu(x)\right] = \mathrm{P}\exp\left[-ig\int_S dx^- dz^i_\perp \, [+\infty, x^-]_{x_\perp} \, F^{i+}(x^-, x_\perp) \, [x^-, +\infty]_{x_\perp}\right]$$

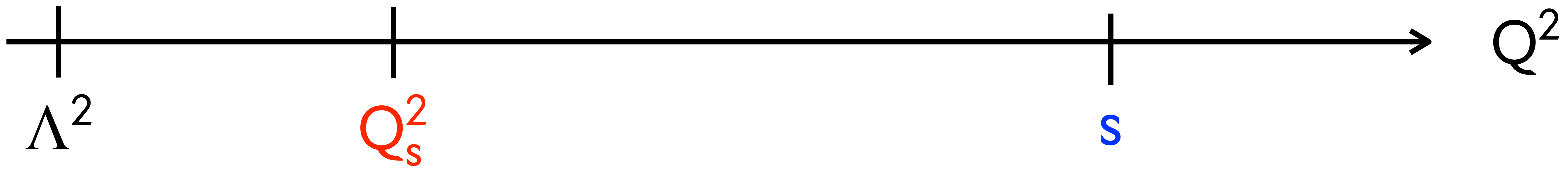
Partial Twist Expansion (PTE)

- Resum subleading powers of s necessary to bridge the Bjorken and Regge limits

Regge limit $(s \gg Q^2)$	Bjorken limit $(s \sim Q^2)$
leading in $\frac{\Lambda^2}{s}, \frac{Q^2}{s}$ all powers of $\frac{Q_s^2}{Q^2}$	leading in $\frac{\Lambda^2}{s}, \frac{Q_s^2}{Q^2}$ all powers of $x = \frac{Q^2}{s}$

$\xrightarrow{\text{PTE}}$ leading in $\frac{\Lambda^2}{s}$ all powers of $\underbrace{\frac{Q_s^2}{Q^2}, \frac{Q^2}{s}}$

cannot be large simultaneously since $s \gg Q_s^2$



High energy factorization revisited

- Resum to all orders the subleading powers of s necessary to bridge the Bjorken and Regge limits

- Restore finite x dependence: convolution in both x and kt

$$\sigma \sim H(k_\perp, x) \otimes f(k_\perp, x) + O\left(\frac{x_{Bj}}{Q^2}\right)$$

$$\sigma \sim H(k_\perp = 0, x) \otimes \int_k f(k_\perp, x)$$

QCD factorization

$$\sigma \sim \left(\int_x H(k_\perp, x)\right) \otimes f(k_\perp, x = 0)$$

High energy factorization

Partial Twist Expansion

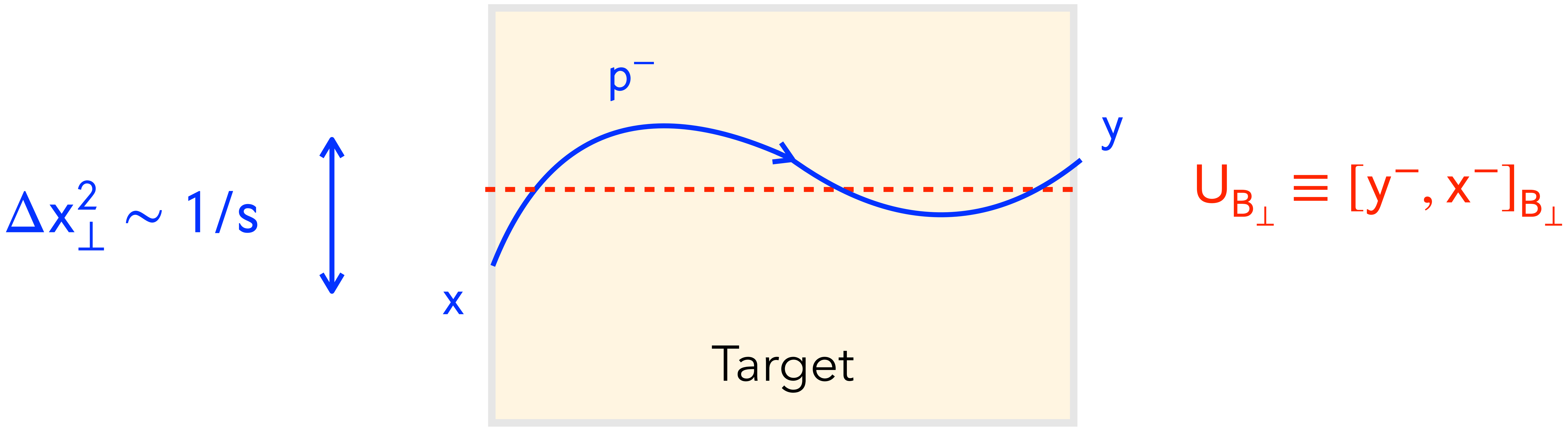
- Background field method (Balitsky, Braun): factorizing gluons in k^- in $A^- = 0$ gauge

$$A^\mu(x) \sim A^+(x^-, x_\perp, x^+ = 0)\, n^\mu$$

- Transverse field at leading twist from gauge invariance (Stokes' theorem)

- Neglect recoil of the high energy parton in the target field

Propagator in the target background field



- Shock wave approximation: projectile $p^- \to \infty$

$$D(x-y) \sim \delta(x_\perp - y_\perp)\; U_x(x^-, y^-)$$

- Partial Twist Expansion: $\Delta x_\perp = x_\perp - y_\perp \ll B_\perp = (ux_\perp + (1-u)y_\perp)/2$

$$D(x-y) \sim \frac{p^-}{2i\pi \Delta x^-}\, e^{i\frac{(x-y)_\perp^2}{\Delta x^-}p^-}\; U_B(x^-, y^-) \;+\; O(|\Delta x_\perp|/|B_\perp|)$$

- N.B.: u dependence power suppressed (reparametrization invariance)

Longitudinal phase

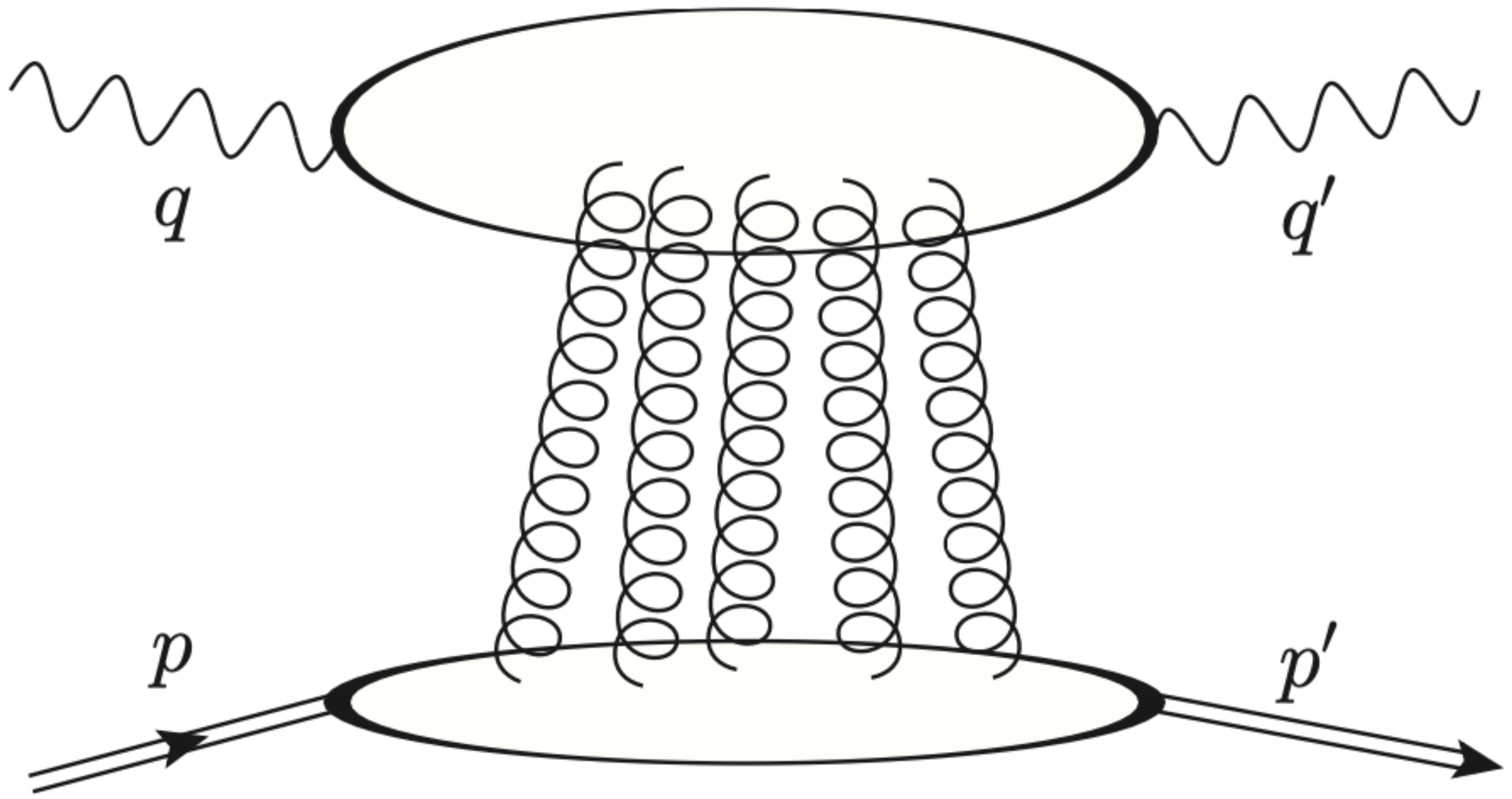
- x dependence encoded in noneikonal phase

$$\frac{p^-}{2i\pi\Delta x^-} e^{i\frac{(x-y)_\perp^2}{\Delta x^-}p^-} \xrightarrow{\text{Fourier T}} e^{ixP^+\Delta x^-}$$

- PTE resums all powers of $\frac{Q^2}{s}$ in addition to saturation enhanced twists $\frac{Q_s^n}{Q^n}$

Application to the exclusive $\gamma^{(*)}(q)P(p) \to \gamma^{(*)}(q')P(p')$ amplitude

Double, Spacelike, and Timelike exclusive Compton Scattering



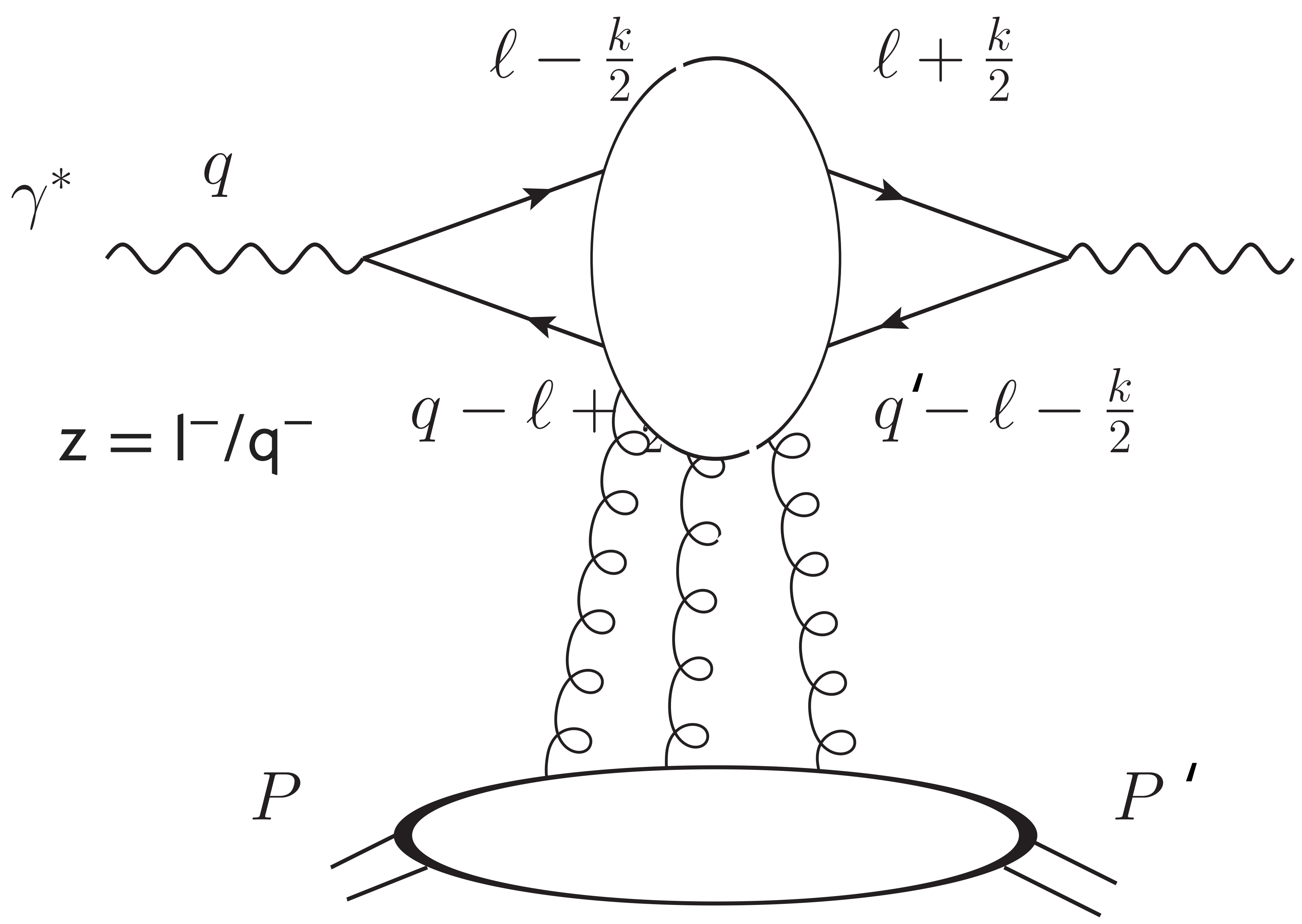
Longitudinal momentum variables:

$$x, \quad \xi \sim \frac{-q^2 + q'^2}{2q\cdot(p+p')}, \quad x_{\mathrm{Bj}} = \frac{-q^2 - q'^2}{2q\cdot(p+p')}$$

Factorization formula for the amplitude

$$A = g^2 \sum_f q_f^2 \int_0^1 \frac{dz}{2\pi} \int \frac{d^d l_\perp}{(2\pi)^d} \int d^d k_\perp \, (\partial^i \Phi)(z, l_\perp - k_\perp/2)(\partial^j \Phi^*)(z, l_\perp + k_\perp/2)$$

$$\times \int dx \, \frac{G^{ij}(x, \xi, k_\perp - \frac{z-\bar{z}}{2}\Delta_\perp, \Delta_\perp)}{x - x_{Bj} - \frac{l_\perp^2}{2z\bar{z}q^- P^+} + i0}$$



- Hard factor: standard photon wave functions

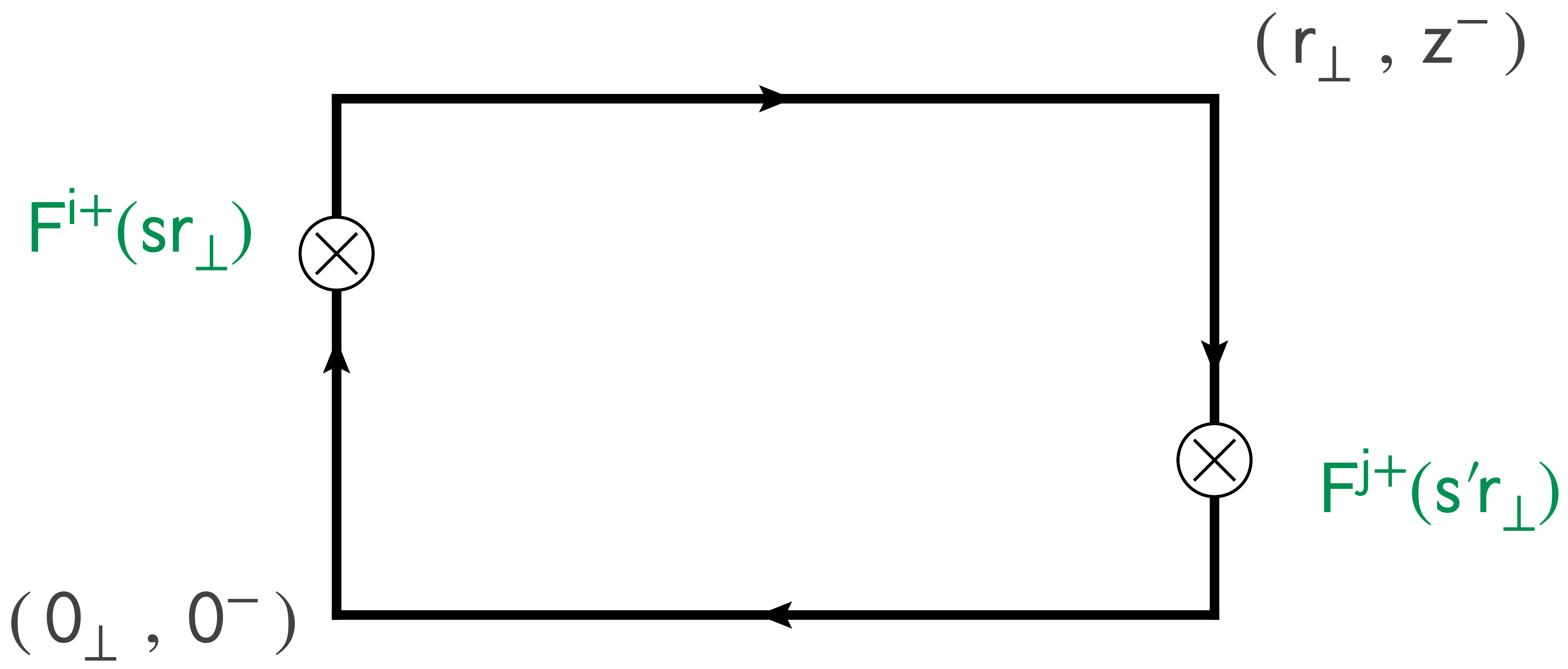
- x-dependent unintegrated gluon GPD $G^{ij}(x, \xi, \boldsymbol{k}, \boldsymbol{\Delta})$

x-dependent unintegrated gluon GPD

$$G^{ij}(x,\xi,k_\perp,\Delta_\perp) \equiv \frac{1}{P^+}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \int \frac{d^d r_\perp}{(2\pi)^d} e^{-ik_\perp\cdot r_\perp} \int_0^1 ds ds' \times \left\langle p' \middle| \mathrm{Tr}\left[z^-,0^-\right]_0 F^{i+}\left(0^-, s r_\perp\right) \left[0^-,z^-\right]_{r_\perp} F^{j+}\left(z^-, s' r_\perp\right) \middle| p \right\rangle$$

$$P^+ \equiv \frac{p^+ + p'^+}{2}$$

$$\Delta \equiv p' - p$$



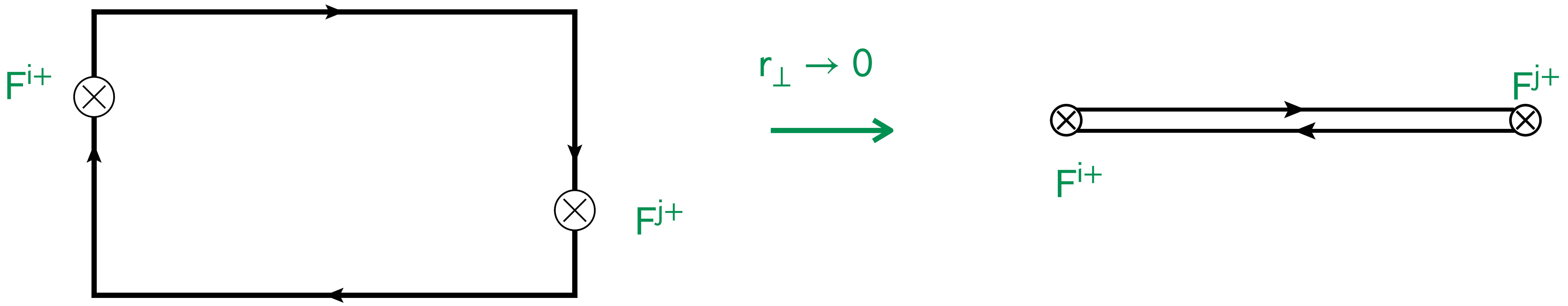
We find a nonlocal gluon operator in longitudinal and transverse directions

Bjorken limit

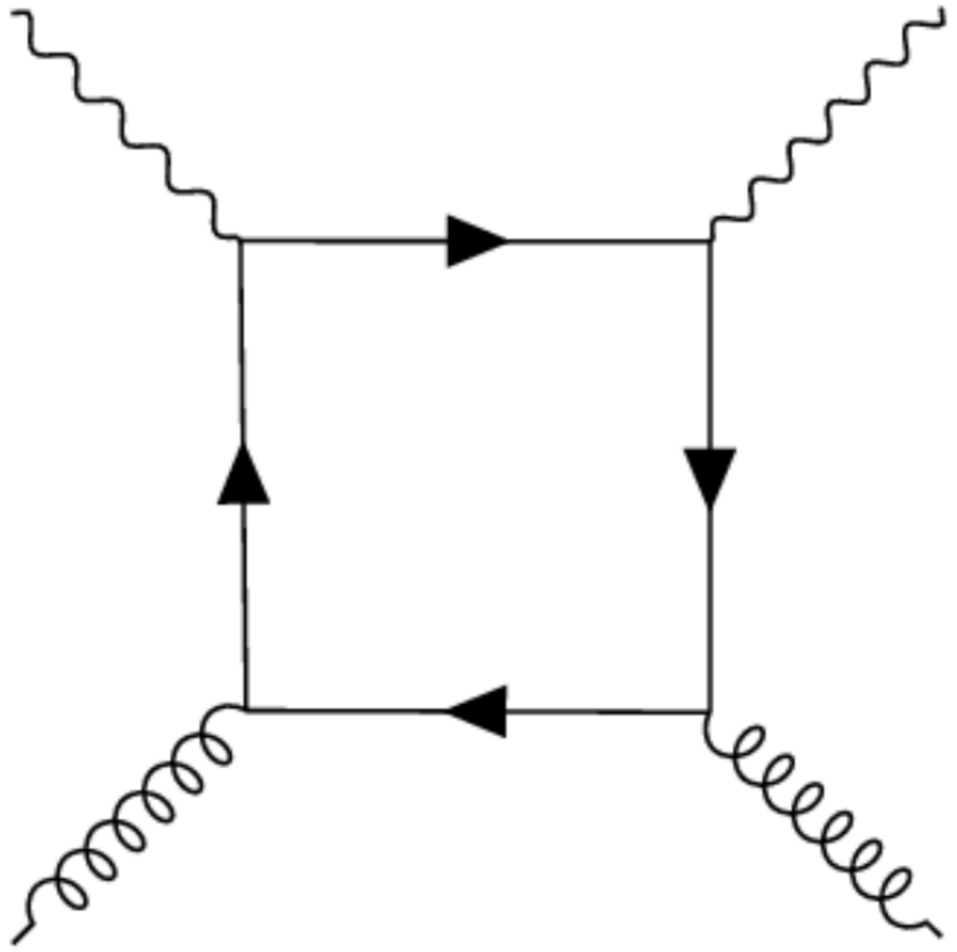
- Neglecting transverse momentum transfer from the target

$$k_\perp \ll l_\perp \sim Q$$

- uGPD integrates into gluon GPD $\longrightarrow$ $\int d^dk_\perp\, G^{ij}(x,k_\perp) = G^{ij}(x,k_\perp)$



Gluon mediated inclusive DIS: one-loop leading twist



$$F_T(x_{Bj}, Q^2 \to \infty) = \frac{\alpha_s}{\pi} \sum_f q_f^2 \int_{x_{Bj}}^1 dy \left[xg(x)\right]_{x=x_{Bj}/y}$$

$$\times \left\{ \frac{1}{\epsilon} \left(\frac{e^{\gamma_E}}{4\pi} \right)^{\epsilon} \left[(1-y)^2 + y^2\right] + \left[(1-y)^2 + y^2\right] \ln \left[\frac{Q^2(1-y)}{\mu^2 y} \right] - 1 + 4y\left(1-y\right) \right\}$$

We fully recover the well-known one-loop exclusive Compton scattering amplitudes

Polarized contribution

$$2\alpha_{em}\alpha_s \sum_f q_f^2 \int dx \frac{\epsilon^{mn} e_h^m e_{h'}^{\prime n*} \widetilde{G}(x,\xi,\mathbf{\Delta})}{(x+\xi-i0x_{Bj})^2(x-\xi+i0x_{Bj})^2} \Bigg\{ \Bigg[2(2x^2+\xi^2)\ln\left(\frac{x_{Bj}+x-i0}{\xi}\right)$$

$$+3x(x_{Bj}+\xi)\ln\left(\frac{x_{Bj}+x-i0}{x_{Bj}+\xi-i0}\right) + 3x(x_{Bj}-\xi)\ln\left(\frac{x_{Bj}+x-i0}{x_{Bj}-\xi-i0}\right) - \frac{1}{2}x(x_{Bj}+\xi)\left[\ln^2\left(\frac{x_{Bj}+x-i0}{\xi}\right) - \ln^2\left(\frac{x_{Bj}+\xi-i0}{\xi}\right)\right]$$

$$-\frac{1}{2}x(x_{Bj}-\xi)\left[\ln^2\left(\frac{x_{Bj}+x-i0}{\xi}\right) - \ln^2\left(\frac{x_{Bj}-\xi-i0}{\xi}\right)\right] -\frac{1}{2}(x^2+\xi^2)\ln^2\left(\frac{x_{Bj}+x-i0}{\xi}\right)\Bigg] - (x \to -x)\Bigg\}$$

Transversity contribution

$$2\alpha_{em}\alpha_s \sum_f q_f^2 \tau^{mn,ij} e_h^m e_{h'}^{\prime n*} \int dx \frac{G_T^{ij}(x,\xi,\mathbf{\Delta})}{(x-\xi+i0x_{Bj})^2(x+\xi-i0x_{Bj})^2}$$

$$\times\left[(x^2-\xi^2) + (x_{Bj}^2-\xi^2)\ln\frac{(x_{Bj}-x-i0)(x_{Bj}+x-i0)}{(x_{Bj}-\xi-i0)(x_{Bj}+\xi-i0)}\right]$$

[Hoodboy, Ji (1998)]

[Pire, Szymanowski, Wagner (2011)]

Regge limit

- Setting $x = 0$ in the 3D gluon operator and using the property

$$\frac{\partial}{\partial x^-}[y^-, x^-]_x[x^-, y^-]_y = ig\int_z [y^-, x^-]_x F^{i+}(x^-, z)\,[x^-, y^-]_y$$

- We recover the dipole operator at small x:

$$r^i r^j\, G^{ij}(x = 0\,, r_\perp) \;\; \rightarrow \;\; \langle P|\,\mathrm{Tr}\; U_{r_\perp}\; U^\dagger_{0_\perp}\,|P\rangle$$

Interpolating scheme for exclusive Compton scattering

Overarching scheme

$$\int \mathrm{d}x \int \mathrm{d}^d k\, G^{ij}(x, \xi, k, \Delta) H^{ij}(x, \xi, k, \Delta)$$

Bjorken limit

$$\int \mathrm{d}x H^{ij}(x, \xi, 0, \Delta)$$
$$\times [\int \mathrm{d}^d k\, G^{ij}(x, \xi, k, \Delta)]$$

Regge limit

$$\lim_{\xi \to 0} \int \mathrm{d}^d k\, G^{ij}(0, \xi, k, \Delta)$$
$$\times \int \mathrm{d}x H^{ij}_{\mathrm{cut}}(x, \xi, k, \Delta)$$

Conclusions

- Minimal correction to the semi-classical approach to small x to restore x dependence using Partial Twist Expansion for DIS, DVCS, TCS, DDVCS:
    - Controlled interpolation between Regge and Bjorken limit ✓
    - Gauge invariance order by order ✓
    - Operator defintion for 3D gluon PDF, GPD ✓
- Outlook:
    - Quantum evolution
    - Application to TMD's
    - Compute 3D gluon PDF on Lattice

Thank you!