Discussion: Spin & TMDs at small x

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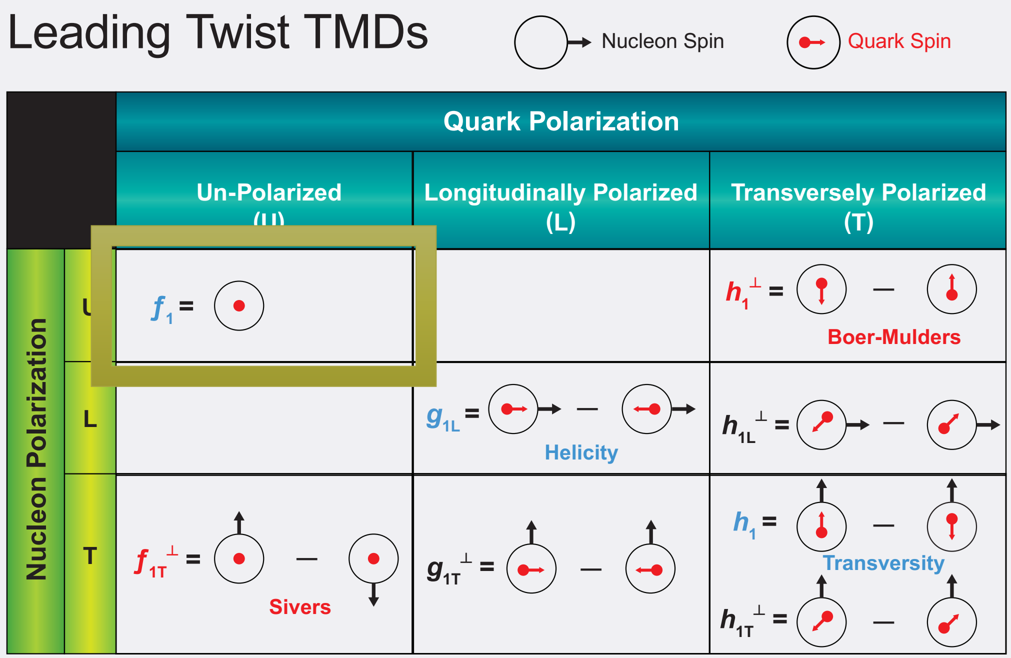
Topics

- Small-x asymptotics of various leading-twist TMDs. (Linear regime or the saturation region.)
- Are any of the features of small-x TMDs useful/relevant at large x?
- Bridging small-x and large-x formalisms.
- Today's topics:
  - Sudakov FF at small x
  - Higher twists (genuine and kinematic)
  - Soft factor at small x : initial conditions? part of evolution?
  - DGLAP evolution for diffractive PDFs

Unpolarized TMDs

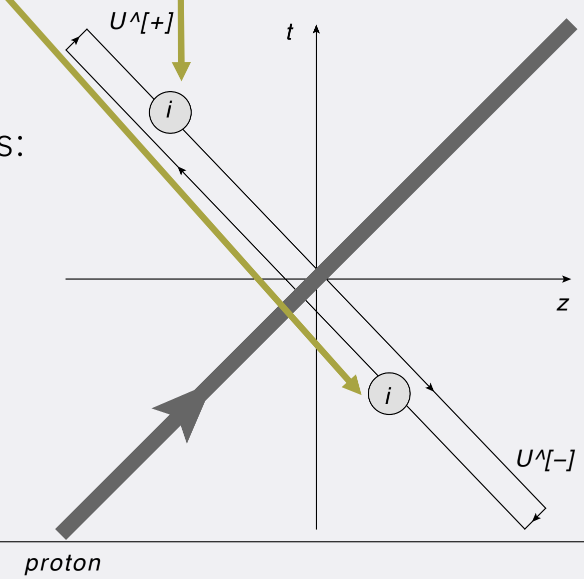


We will concentrate on the region where x is small, but not small enough to include the nonlinear saturation effects.

Dipole Gluon TMD

$$f_1^{G\,dip}(x, k_T^2) = \frac{2}{x\,P^+} \int \frac{d\xi^-\, d^2\xi}{(2\pi)^3}\, e^{ixP^+\,\xi^- - i\underline{k}\cdot\underline{\xi}}\, \langle P| \mathrm{tr}\left[F^{+i}(0)\,\mathcal{U}^{[+]}[0,\xi]\, F^{+i}(\xi)\,\mathcal{U}^{[-]}[\xi,0]\right]|P\rangle_{\xi^+=0}$$

- Here $U^{[+]}$ and $U^{[-]}$ are future and past-pointing fundamental Wilson line staples
- Dipole gluon TMD enters a number of cross sections: DIS, DY, SIDIS, hadronproduction in pA.
- Dominguez, Marquet, Xiao, Yuan '11; M. Braun '00; YK, Tuchin '01, Kharzeev, YK, Tuchin '03.



Dipole Gluon TMD

- One can show that the gluon dipole TMD at small *x* is indeed related to the dipole amplitude N= 1-S (Dominguez et al, '11; M. Braun '00; YK, Tuchin '01, Kharzeev, YK, Tuchin '03):

$$f_1^{G\,dip}(x, k_T^2) = \frac{k_T^2\, N_c}{(2\pi)^3\, \pi\, \alpha_s\, x} \int d^2b\, d^2r\, e^{-i\vec{k}_\perp \cdot \vec{r}_\perp}\, S(\vec{r}_\perp, \vec{b}_\perp, Y = \ln(1/x))$$
$$= - \frac{k_T^2\, N_c}{(2\pi)^3\, \pi\, \alpha_s\, x} \int d^2b\, d^2r\, e^{-i\vec{k}_\perp \cdot \vec{r}_\perp}\, N(\vec{r}_\perp, \vec{b}_\perp, Y = \ln(1/x))$$

- The resulting small-*x* asymptotics is given by the BFKL evolution,

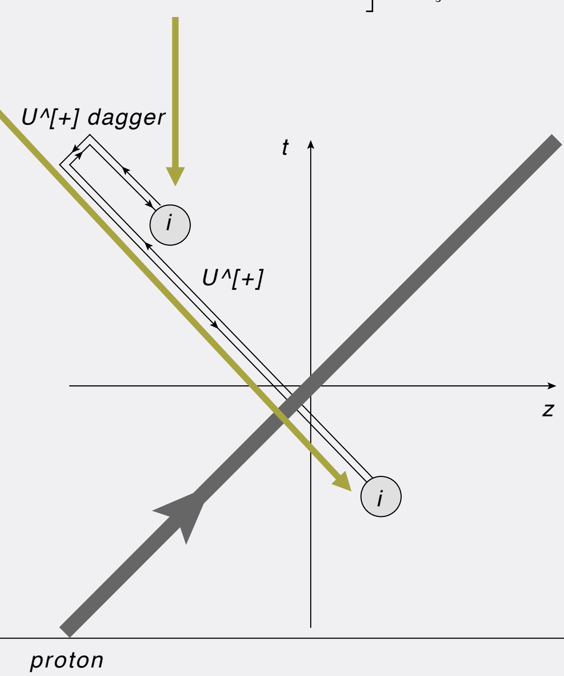
$$f_1^{G\,dip}(x, k_T^2) \sim \frac{1}{x}\, N(\vec{r}_\perp, \vec{b}_\perp, Y = \ln(1/x)) \sim \left(\frac{1}{x}\right)^{1+\frac{4\,\alpha_s\,N_c}{\pi}\,\ln 2 + \mathcal{O}(\alpha_s^2)}$$

- The k_T dependence is also determined by the small-x evolution.

WW Gluon TMD

$$f_1^{G\,WW}(x,k_T^2) = \frac{2}{x\,P^+}\int \frac{d\xi^-\,d^2\xi}{(2\pi)^3}\,e^{ixP^+\xi^- - i\underline{k}\cdot\underline{\xi}}\,\langle P|\mathrm{tr}\left[F^{+i}(0)\,\mathcal{U}^{[+]}[0,\xi]\,F^{+i}(\xi)\,\mathcal{U}^{[+]\,\dagger}[\xi,0]\right]|P\rangle_{\xi^+=0}$$

- Here $U^{[+]}$ is the future-pointing fundamental Wilson line staple (can use past-pointing too)
- Jalilian-Marian, Kovner, McLerran, Weigert '97; Dominguez, Marquet, Xiao, Yuan '11.
- WW gluon TMD can be measured in dijet production in DIS and in pA



WW Gluon TMD

- At small *x* the WW gluon TMD is proportional to a different object, now made out of 4 Wilson lines, the quadrupole amplitude Q:

$$Q(x_1, x_2, x_3, x_4) = \frac{1}{N_c} \langle \mathrm{tr}[V_1\, V_2^\dagger\, V_3\, V_4^\dagger] \rangle$$

- Small-*x* evolution for the quadrupole amplitude Q is given by an evolution equation different from BK. (Jalilian-Marian, YK '04; Dominguez, Mueller, Munier, Xiao '11.)
- In the linear regime the dipole amplitude Q obeys BFKL equation, such that the small-*x* asymptotics of the WW gluon TMD is the same as for the dipole gluon TMD:

$$f_1^{G\ WW}(x, k_T^2) \sim \frac{1}{x}\, Q \sim \left(\frac{1}{x}\right)^{1+\frac{4\,\alpha_s\,N_c}{\pi}\ \ln 2+\mathcal{O}(\alpha_s^2)}$$

- The difference between the two TMDs is inside the saturation region.

Linearly Polarized Gluon TMD

- If we keep the indices of the two F^{+i} different, we get access to the linearly polarized (WW) gluon TMD $h_1^{\perp}$ (Metz, Zhou, '11):

$$\frac{1}{P^+}\int \frac{d\xi^- \, d^2\xi}{(2\pi)^3} \, e^{ixP^+\xi^- - i\underline{k}\cdot\underline{\xi}} \, \langle P|\mathrm{tr}\left[F^{+i}(0)\,\mathcal{U}^{[+]}[0,\xi]\,F^{+j}(\xi)\,\mathcal{U}^{[+]\,\dagger}[\xi,0]\right]|P\rangle_{\xi^+=0}$$
$$= \frac{1}{2}\,\delta^{ij}\,x\,f_1^{G\ WW}(x,k_T^2) + \frac{2k^i k^j - k_T^2\,\delta^{ij}}{4\,k_T^2}\,x\,h^{\perp}_{1,\ WW}(x,k_T^2)$$

- The linearly polarized WW gluon TMD is thus also related to the color-quadrupole amplitude Q.
- In the linear (BFKL) regime the small-x asymptotics is the same,

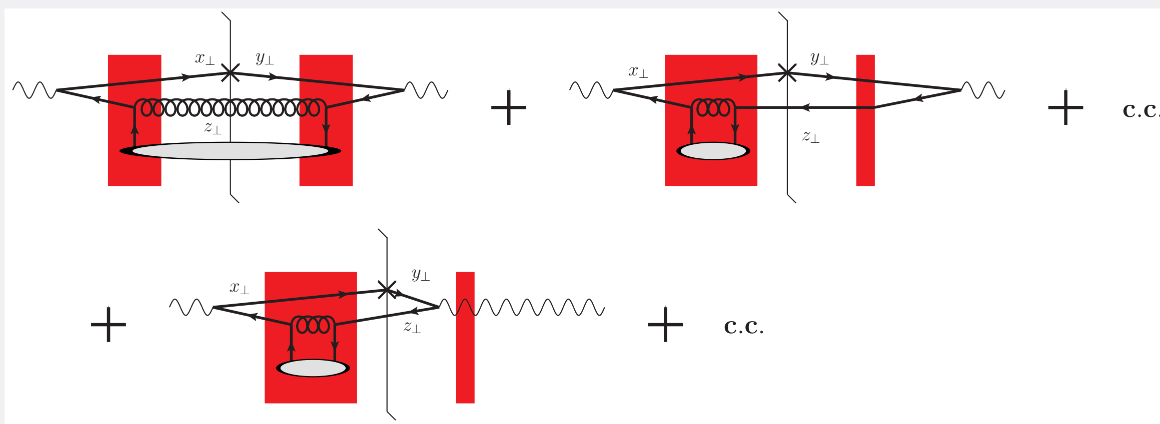
$$h^{\perp}_{1,\ WW}(x,k_T^2) \sim \frac{1}{x}\,Q \sim \left(\frac{1}{x}\right)^{1+\frac{4\,\alpha_s\,N_c}{\pi}\ \ln 2 + \mathcal{O}(\alpha_s^2)}$$

- For more on small-x evolution of the linear gluon polarization see the work by Dumitru, Skokov '17.

Unpolarized Quark TMD

- The small-*x* asymptotics of the unpolarized quark TMD is (Mueller, 2003)

$$f_1^q(x, k_T^2) \sim \frac{1}{x} N \sim \left(\frac{1}{x}\right)^{1+\frac{4\,\alpha_s\,N_c}{\pi}\,\ln 2+\mathcal{O}(\alpha_s^2)}$$



Mini-Summary

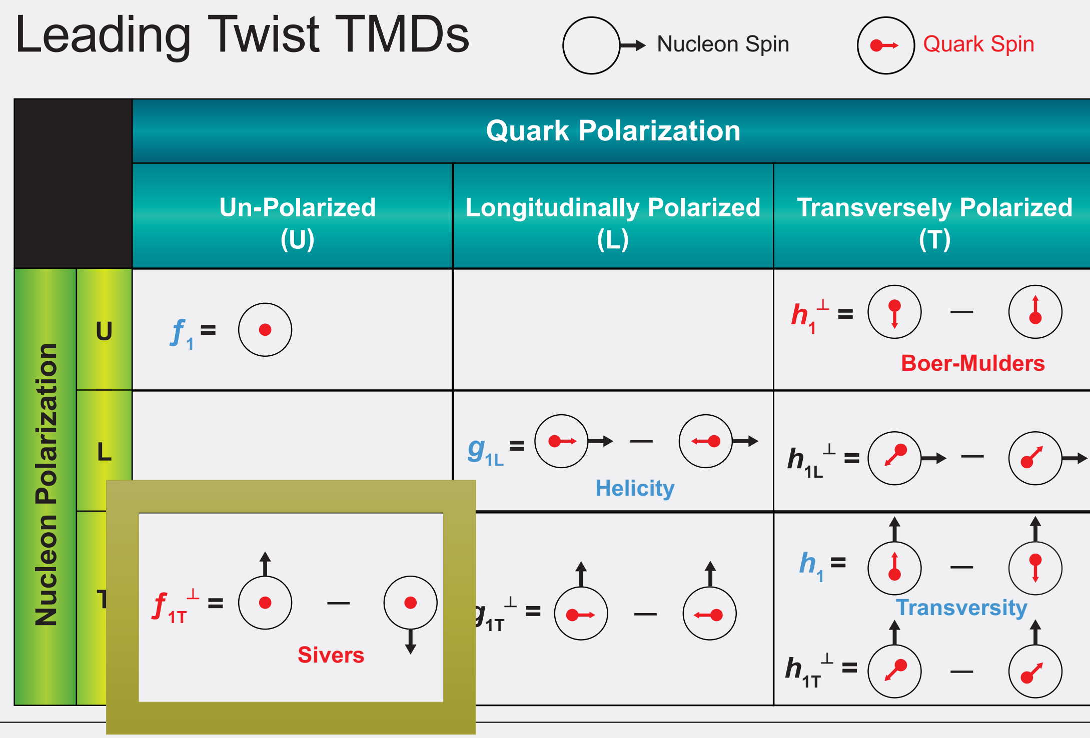
- (Almost) all the unpolarized quark and gluon TMDs for an unpolarized nucleon have the same *x*-dependence at small *x*,

$$\mathrm{TMD}^{q,G}_{unpolarized}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{1+\frac{4\,\alpha_s\,N_c}{\pi}\,\ln 2+\mathcal{O}(\alpha_s^2)}$$

  in the linear region (outside the saturation region).

- Inside the saturation region the behavior is different for different TMDs.
- For many more unpolarized TMDs see Bury, Kotko, Kutak '18.

Sivers function



Sivers function

- Another TMD receiving an eikonal contribution is the Sivers function (quark or gluon one).
- Consider the quark Sivers function:

$$f_1^q(x, k_T^2) - \frac{\underline{k} \times \underline{S}_P}{M_P} f_{1\,T}^{\perp\,q}(x, k_T^2) = \int \frac{dr^- d^2 r_\perp}{2\,(2\pi)^3} e^{ik\cdot r} \langle P, S | \bar{\psi}(0) \mathcal{U}[0, r] \frac{\gamma^+}{2} \psi(r) | P, S \rangle$$

- The small-x asymptotics of the Sivers function is given by the odderon exchange (Boer, Echevarria, Mulders and Zhou '15 (gluon); Dong, Zheng, Zhou '18; YK, Santiago '21 (quark))

$$-\frac{\underline{k} \times \underline{S}_P}{M_P} f_{1\,T}^{\perp\,q}(x, k_T^2)\Big|_{\text{eikonal}} = \frac{4i\,N_c\,p_1^+}{(2\pi)^3} \int d^2\zeta_\perp d^2 w_\perp \frac{d^2 k_{1\perp} dk_1^-}{(2\pi)^3} e^{i(\underline{k}_1 + \underline{k})\cdot(\underline{w} - \underline{\zeta})} \theta(k_1^-)$$
$$\times \left[\frac{2\,\underline{k}\cdot\underline{k}_1}{(xp_1^+ k_1^- + \underline{k}_1^2)(xp_1^+ k_1^- + \underline{k}^2)} + \frac{\underline{k}_1^2}{(xp_1^+ k_1^- + \underline{k}_1^2)^2} \right] \mathcal{O}_{\underline{\zeta}\underline{w}}$$

Small-x asymptotics of the Sivers function

The leading (eikonal) small-x asymptotics is given by the spin-dependent odderon (Boer, Echevarria, Mulders and Zhou '15 (gluon); Dong, Zheng, Zhou '18 (quark)):

$$f_{1\,T}^{\perp\,G}(x, k_T^2) \sim \frac{1}{x} \qquad\qquad f_{1\,T}^{\perp\,q}(x, k_T^2) \sim \frac{1}{x}$$

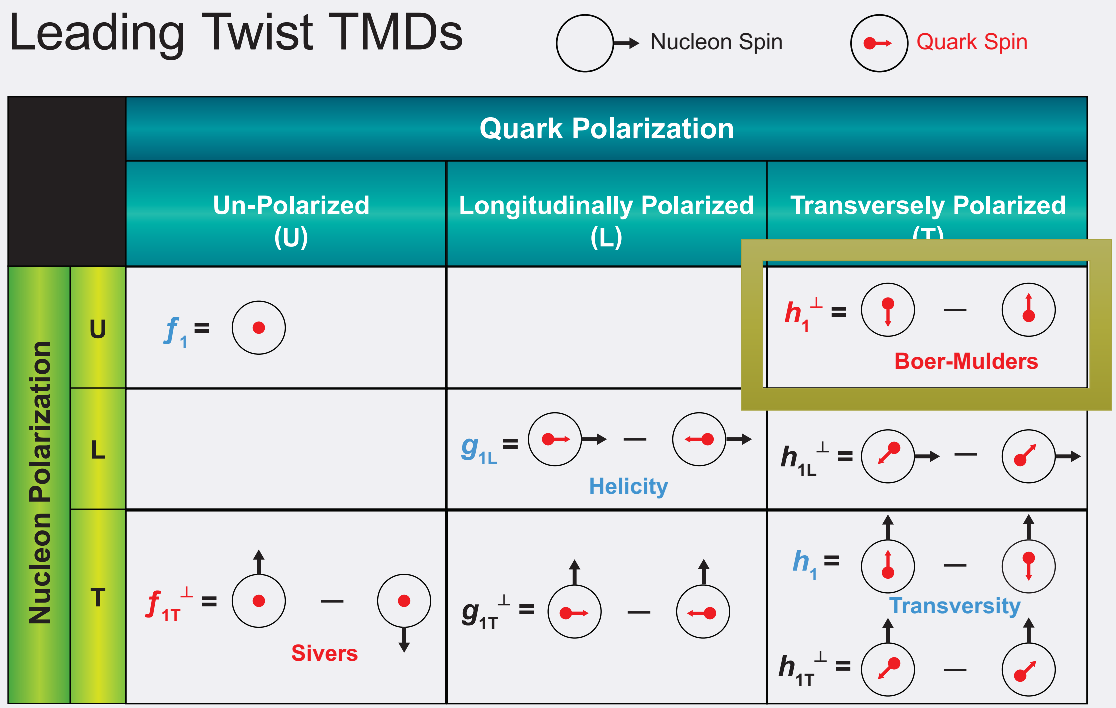
Sub-eikonal (suppressed by x) correction to the quark Sivers function has been calculated recently (YK, Santiago '22). The power of 1/x (the intercept) is also close to 1…

$$f_{1\,T}^{\perp\,NS}(x, k_T^2) = C_O(x, k_T^2)\,\frac{1}{x} + C_1(x, k_T^2)\,\left(\frac{1}{x}\right)^{3.4\sqrt{\frac{\alpha_s N_c}{4\pi}}} + \ldots$$

Spin-dependent odderon

Sub-eikonal correction

Boer–Mulders function



Boer-Mulders Function

- Similar procedure yields, for the Boer-Mulders function,

$$h_1^{\perp NS}(x \ll 1, k_T^2) \sim \left(\frac{1}{x}\right)^{-1}$$

YK, M. G. Santiago,
2209.03538 [hep-ph]

- Boer-Mulders function is sub-eikonal at small *x*. Moreover, the DLA correction to the sub-eikonal power of *x* is zero.

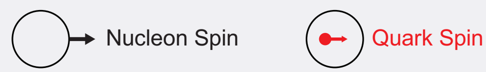
- We observe that the intercepts of the leading small-x asymptotics terms for the T-odd leading-twist quark TMDs, the Sivers function and Boer-Mulders function, appear to receive no perturbative corrections (no α_s corrections). Could this be an exact statement in QCD?

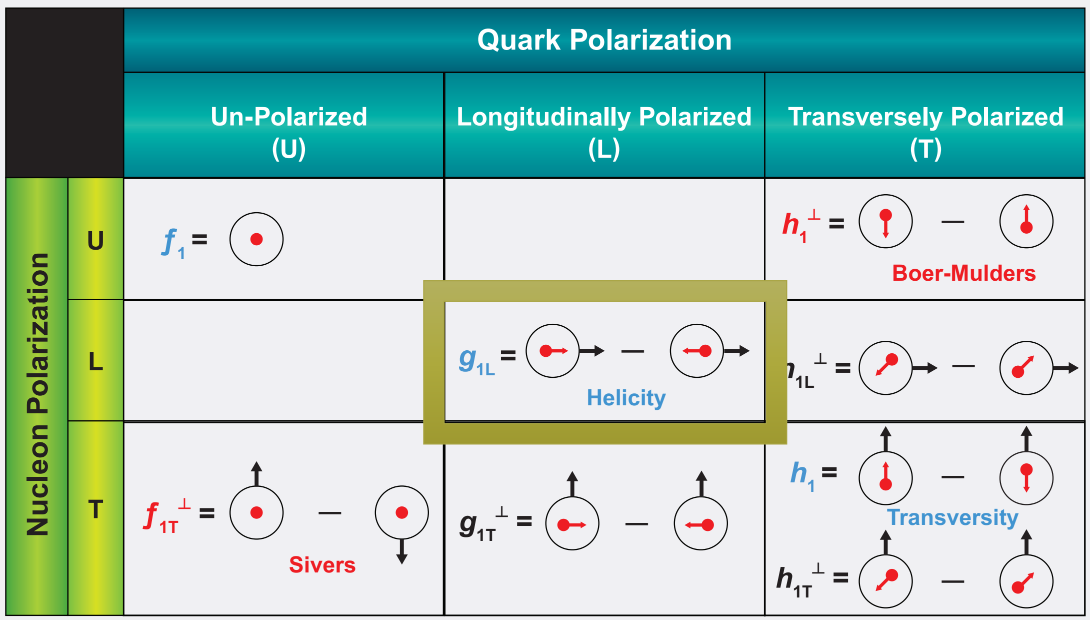
Eikonality

- One can classify various TMDs by their small-*x* asymptotics.
- Eikonal behavior corresponds to (up to ~α_S corrections in the power) $f(x, k_T^2) \sim \frac{1}{x}$
  Examples: unpolarized TMDs, Sivers function.
- Sub-eikonal behavior corresponds to $g(x, k_T^2) \sim \left(\frac{1}{x}\right)^0 = \mathrm{const}$
  Example: helicity TMDs.
- Sub-sub-eikonal behavior is $h(x, k_T^2) \sim x$
  Examples: transversity, Boer-Mulder function.
- We've been calling the leading power of x "eikonality".

Helicity

Leading Twist TMDs





Small-x Asymptotics for Helicity Distributions

- Let's take a closer look at the anomalous dimension:

$$\Delta G(x, Q^2) = \int \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} \left(\frac{Q^2}{\Lambda^2}\right)^{\Delta\gamma_{GG}(\omega)} \Delta G_\omega(\Lambda^2)$$

- In the pure-glue case, Bartels, Ermolaev and Ryskin's (BER, '96) anomalous dimension can be found analytically. It reads (KPS '16)

$$\Delta\gamma_{GG}^{BER}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \frac{1 - \frac{3\,\bar{\alpha}_s}{\omega^2}}{1 - \frac{\bar{\alpha}_s}{\omega^2}}}\right] \qquad \bar{\alpha}_s = \frac{\alpha_s\, N_c}{2\pi}$$

- Our (KPS-CTT) evolution's anomalous dimension can be found analytically at large-N_c (J. Borden, YK, 2304.06161 [hep-ph]):

$$\Delta\gamma_{GG}^{us}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \sqrt{1 - \frac{4\,\bar{\alpha}_s}{\omega^2}}}\right]$$

A Tale of Two Anomalous Dimensions

- The two anomalous dimensions look similar enough but are not the same function.

$$\Delta\gamma_{GG}^{BER}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \frac{1 - \frac{3\,\bar{\alpha}_s}{\omega^2}}{1 - \frac{\bar{\alpha}_s}{\omega^2}}}\right] \qquad \Delta\gamma_{GG}^{us}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \sqrt{1 - \frac{4\,\bar{\alpha}_s}{\omega^2}}}\right]$$

- Their expansions in α_S start out the same, then differ at four (!) loops (the first 3 terms agree with the existing finite-order calculations, the four-loop result is unknown):

$$\Delta\gamma_{GG}^{BER}(\omega) = \frac{4\,\bar{\alpha}_s}{\omega} + \frac{8\,\bar{\alpha}_s^2}{\omega^3} + \frac{56\,\bar{\alpha}_s^3}{\omega^5} + \frac{504\,\bar{\alpha}_s^4}{\omega^7} + \ldots$$

$$\Delta\gamma_{GG}^{us}(\omega) = \frac{4\,\bar{\alpha}_s}{\omega} + \frac{8\,\bar{\alpha}_s^2}{\omega^3} + \frac{56\,\bar{\alpha}_s^3}{\omega^5} + \frac{496\,\bar{\alpha}_s^4}{\omega^7} + \ldots$$

A Tale of Two Intercepts

$$\Delta G(x, Q^2) = \int \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} \left(\frac{Q^2}{\Lambda^2}\right)^{\Delta\gamma_{GG}(\omega)} \Delta G_\omega(\Lambda^2)$$

$$\Delta\gamma_{GG}^{BER}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \frac{1 - \frac{3\,\bar{\alpha}_s}{\omega^2}}{1 - \frac{\bar{\alpha}_s}{\omega^2}}}\right] \qquad \Delta\gamma_{GG}^{us}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \sqrt{1 - \frac{4\,\bar{\alpha}_s}{\omega^2}}}\right]$$

- The intercept (largest power Re[ω]) is given by the right-most singularity (branch point) of the anomalous dimension.

- For BER this gives $\alpha_h = \sqrt{\frac{17 + \sqrt{97}}{2}} \sqrt{\frac{\alpha_s\,N_c}{2\pi}} \approx 3.664 \sqrt{\frac{\alpha_s\,N_c}{2\pi}}$

- For us $\alpha_h = \frac{4}{3^{1/3}} \sqrt{\mathrm{Re}\left[(-9 + i\,\sqrt{111})^{1/3}\right]} \sqrt{\frac{\alpha_s\,N_c}{2\pi}} \approx 3.661 \sqrt{\frac{\alpha_s\,N_c}{2\pi}}$

A Tale of Two Intercepts

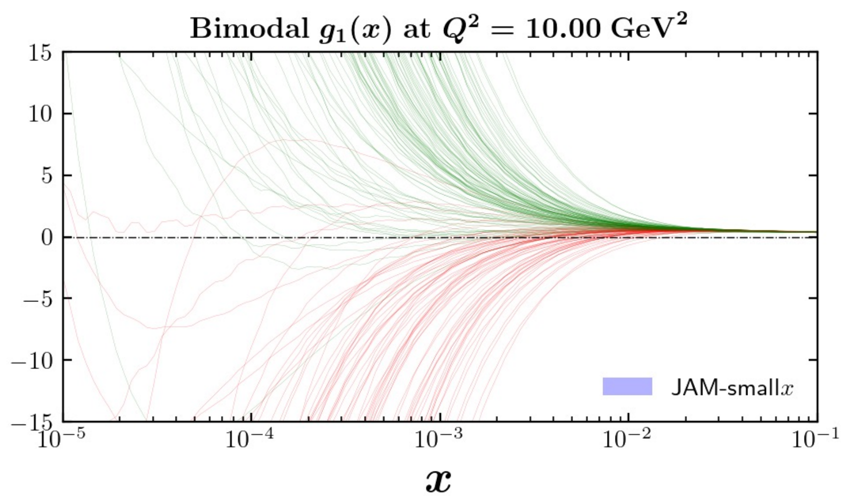
$$\Delta\Sigma(x, Q^2)\Big|_{x \ll 1} \sim \Delta G(x, Q^2)\Big|_{x \ll 1} \sim \left(\frac{1}{x}\right)^{\alpha_h}$$

- BER: $$\alpha_h = \sqrt{\frac{17 + \sqrt{97}}{2}}\sqrt{\frac{\alpha_s\, N_c}{2\pi}} \approx 3.664\sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$
- Us: $$\alpha_h = \frac{4}{3^{1/3}}\sqrt{\mathrm{Re}\left[(-9 + i\,\sqrt{111})^{1/3}\right]}\sqrt{\frac{\alpha_s\, N_c}{2\pi}} \approx 3.661\sqrt{\frac{\alpha_s\, N_c}{2\pi}}$$
- Our 2022 numerical solution also gave the intercept of 3.660 or 3.661, but we believed we had larger error bars.
- We (still) disagree with BER. Albeit in the 3rd decimal point…

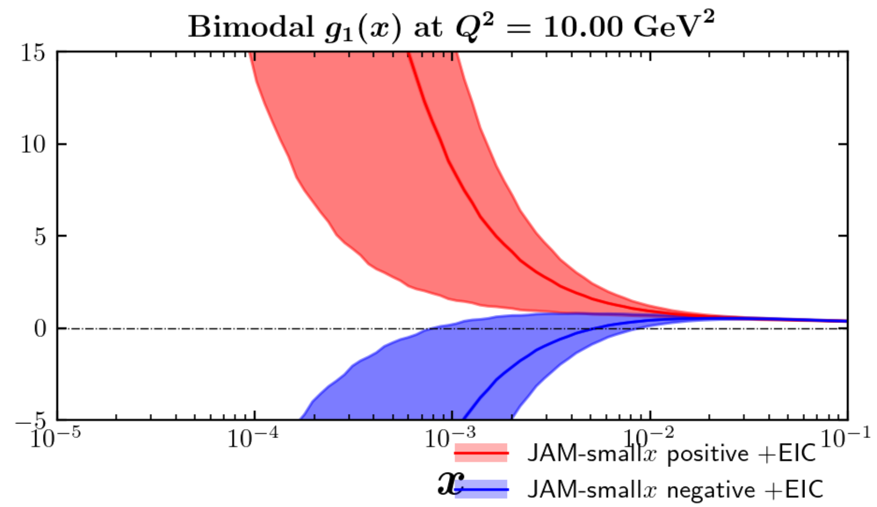
J. Borden, YK, 2304.06161 [hep-ph]

Proton g_1 structure function

JAM-smallx preliminary (D. Adamiak, N. Baldonado, YK, W. Melnitchouk, D. Pitonyak, N. Sato, M. Sievert, A. Tarasov, Y. Tawabutr)



Replicas



Error bands

- JAM is based on a Bayesian Monte-Carlo: it uses replicas.
- Due to the lack of constraints, everything is bi-modal!
- Thin lines in the middle – PDF extractions using EIC pseudo-data, it would resolve the bi-modality.

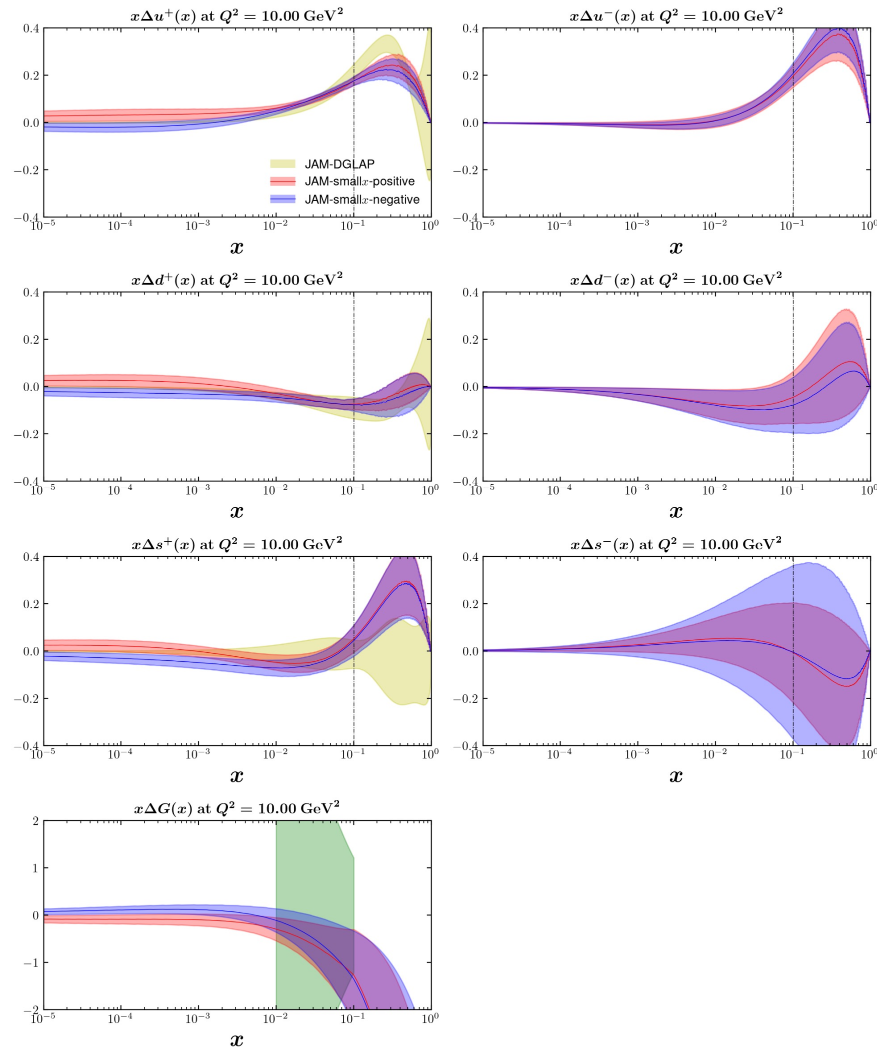
Helicity PDFs:

JAM-smallx preliminary

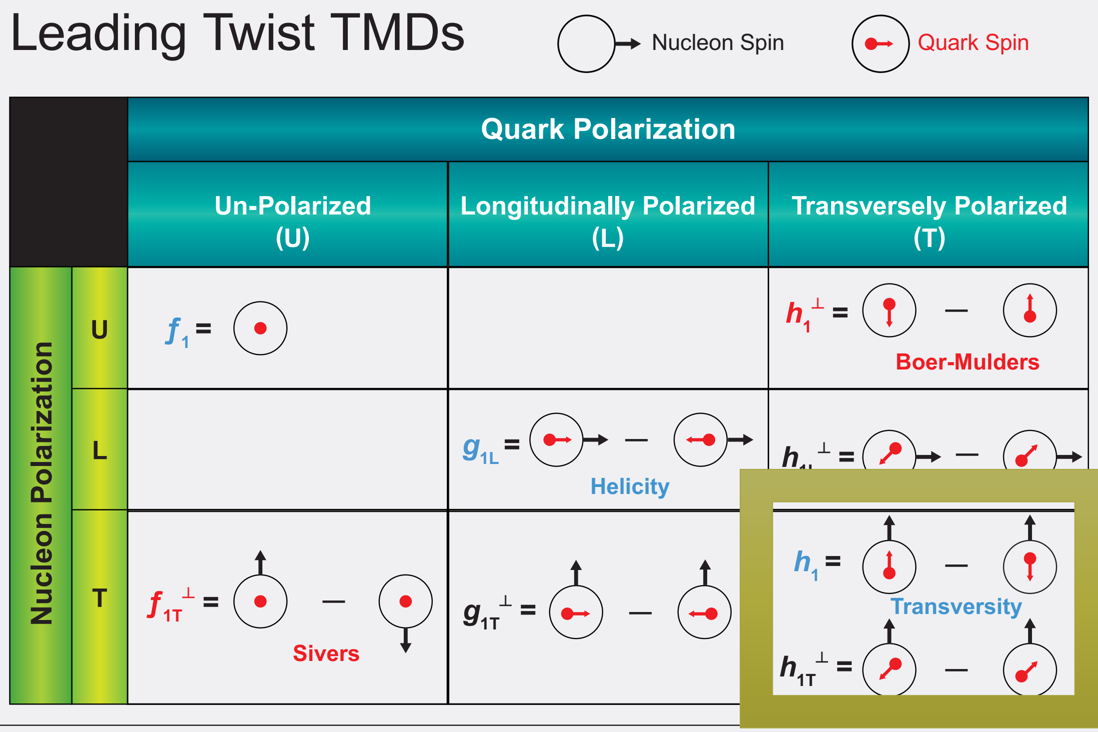
- Here too, everything is bi-modal.
- Again, lines in the middle are the EIC pseudo-data.

$$\Delta q^+ = \Delta q + \Delta \bar{q}$$

$$\Delta q^- = \Delta q - \Delta \bar{q}$$



Transversity



Small-x Asymptotics of Quark Transversity

- The small-x asymptotics of quark transversity is (Kirschner et al, 1996; YK, Sievert 2018)

$$h_{1T}^q(x,k_T^2) \sim h_{1T}^{\perp\,q}(x,k_T^2) \sim \left(\frac{1}{x}\right)^{\alpha_t^q} \quad \text{with} \quad \alpha_t^q = -1 + 2\sqrt{\frac{\alpha_s\,C_F}{\pi}}$$

- Note the suppression by x^2 compared to the unpolarized quark TMDs: transversity is sub-sub-eikonal.
- For α_s = 0.3 we get $$h_{1T}^q(x,k_T^2) \sim h_{1T}^{\perp\,q}(x,k_T^2) \sim x^{0.286}$$
- This certainly satisfies the Soffer bound, but is not likely to produce much tensor charge from small x.

$$\delta q(Q^2) = \int\limits_0^1 dx\, h_1(x,Q^2)$$

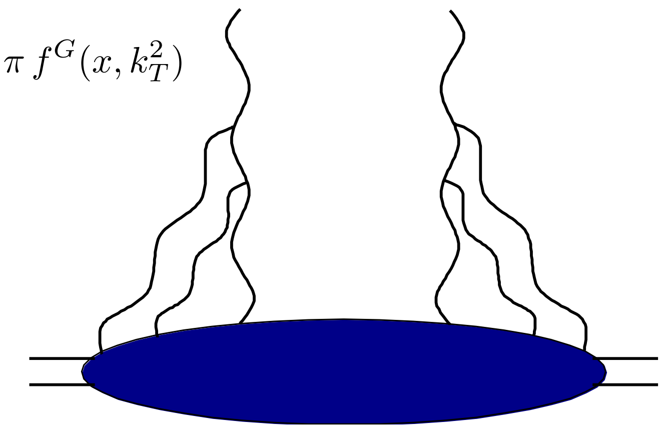
Classical Gluon Field of a Nucleus

Using the obtained classical gluon field one can construct corresponding gluon distribution function (gluon WW TMD):

$$\phi_A(x,k^2) \sim \left\langle \underline{A}(-k) \cdot \underline{A}(k) \right\rangle$$

with the field in the $A^+=0$ gauge

$$\phi(x, k_T^2) = x\,\pi\, f^G(x, k_T^2)$$



$$\phi_A(x, k_T^2) = \frac{C_F}{\alpha_s\,\pi} \int \frac{d^2 x_\perp}{x_\perp^2}\, e^{i\,\underline{k}\cdot\underline{x}} \left[1 - \exp\left(-\frac{x_\perp^2\, Q_s^2}{4}\,\ln\frac{1}{x_\perp\,\Lambda}\right)\right]$$

J. Jalilian-Marian et al, '97; Yu. K. and A. Mueller, '98

⇨ $Q_S=\mu$ is the saturation scale $Q_S^2 \sim A^{1/3}$

⇨ Note that $\phi \sim \langle A_\mu A_\mu \rangle \sim 1/\alpha$ such that $A_\mu \sim 1/g$, which is what one would expect for a classical field.

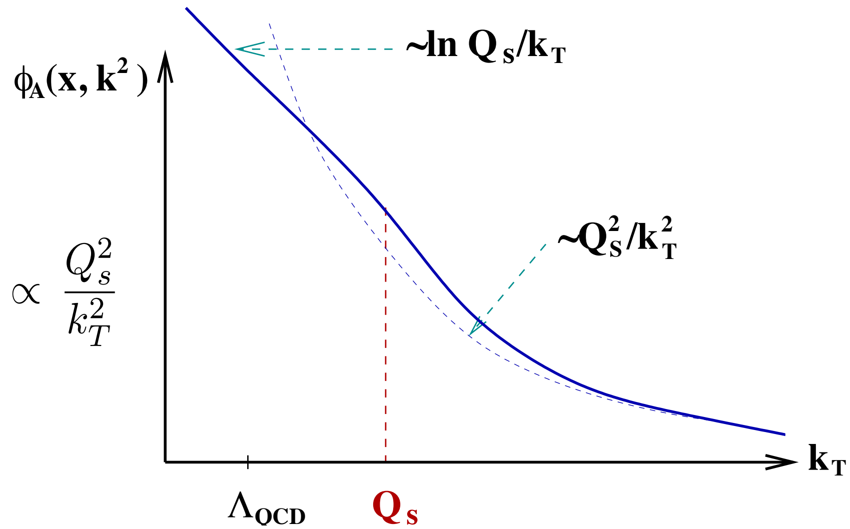
$$\boxed{\phi_A(x,k_T^2) = \frac{C_F}{\alpha_s\,\pi}\int \frac{d^2x_\perp}{x_\perp^2}\,e^{i\,\underline{k}\cdot\underline{x}}\left[1-\exp\left(-\frac{x_\perp^2\,Q_s^2}{4}\,\ln\frac{1}{x_\perp\,\Lambda}\right)\right]}$$

⇨ In the UV limit of k→∞,

x_T is small and one obtains

$$\phi_A(x,k_T^2) \sim \int d^2x_\perp\,e^{i\,\underline{k}\cdot\underline{x}}\,Q_s^2\,\ln\frac{1}{x_\perp\,\Lambda} \propto \frac{Q_s^2}{k_T^2}$$

which is the usual LO result.



⇨ In the IR limit of small k_T,

x_T is large and we get

$$\phi_A(x,k_T^2) \approx \frac{C_F}{\alpha_s\,\pi}\int_{1/Q_s}\frac{d^2x_\perp}{x_\perp^2}\,e^{i\,\underline{k}\cdot\underline{x}} \propto \ln\frac{Q_s}{k_T}$$

SATURATION !

Divergence is regularized.

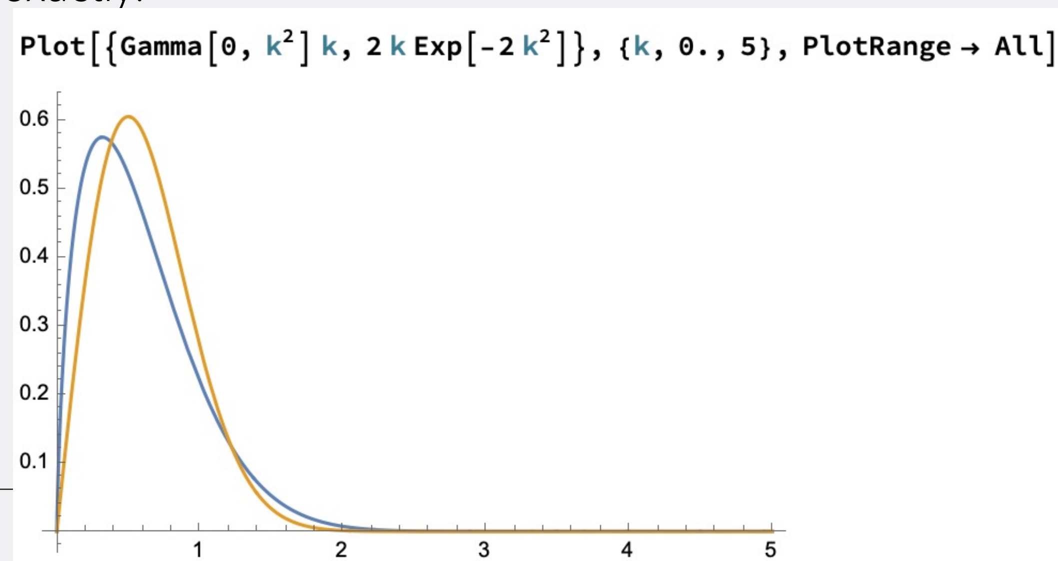
Features of small-x WW gluon TMD

$$\phi_A(x, k_T^2) = \frac{C_F}{\alpha_s \, \pi} \int \frac{d^2 x_\perp}{x_\perp^2} \, e^{i \, \underline{k} \cdot \underline{x}} \left[1 - \exp\left(- \frac{x_\perp^2 \, Q_s^2}{4} \, \ln \frac{1}{x_\perp \, \Lambda} \right) \right]$$

- One can discard the log and integrate this exactly:

$$\phi_A(x, k_T^2) = \frac{C_F}{\alpha_s} \, \Gamma\left(0, \frac{k_T^2}{Q_s^2}\right)$$

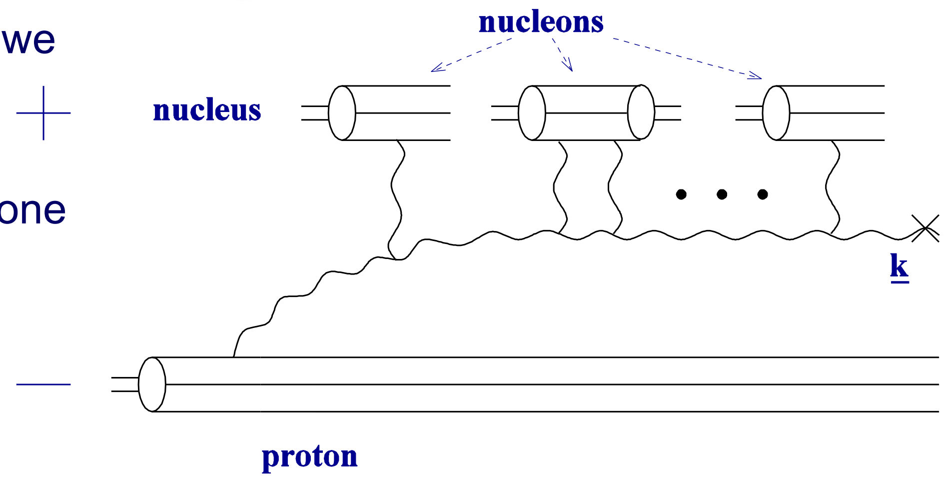
- When multiplied by phase space factor k_T, it looks a lot like a Gaussian. But it is not a Gaussian.



Gluon Production in pA: McLerran-Venugopalan model

Classical gluon production: we need to resum only the multiple rescatterings of the gluon on nucleons. Here's one of the graphs considered.

Yu. K., A.H. Mueller, hep-ph/9802440



Resulting inclusive gluon production cross section is given by

$$\frac{d\sigma}{d^2k\,dy} = \frac{1}{(2\pi)^2}\int d^2b\,d^2x\,d^2y\,e^{i\underline{k}\cdot(\underline{x}-\underline{y})}\underbrace{\frac{\alpha C_F}{\pi^2}\frac{\underline{x}\cdot\underline{y}}{\underline{x}^2\underline{y}^2}}_{\text{proton's wave function}}\left[N_G(\underline{x})+N_G(\underline{y})-N_G(\underline{x}-\underline{y})\right]$$

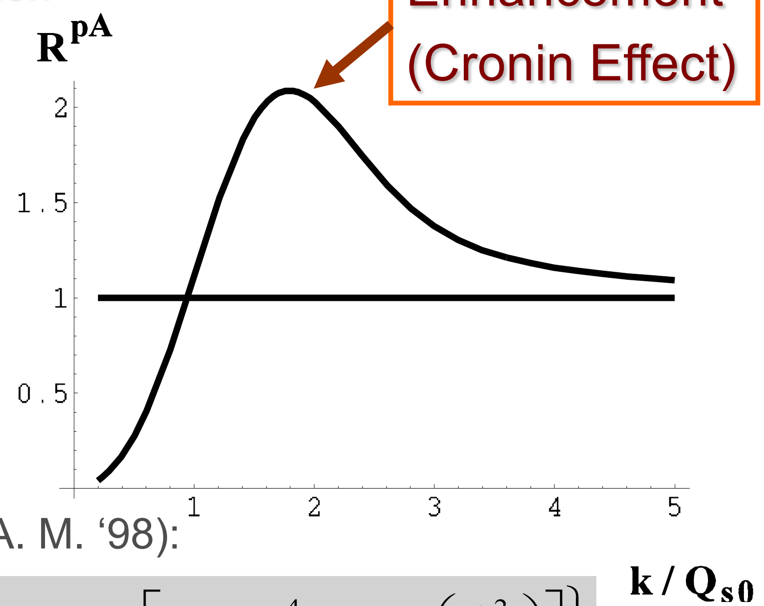
With the gluon-gluon dipole-nucleus forward scattering amplitude

$$N_G(x, Y=0) = 1 - e^{-x^2 Q_s^2/4}$$

McLerran-Venugopalan model: Cronin Effect

To understand how the gluon production in pA is different from independent superposition of *A* proton-proton (pp) collisions one constructs the quantity

$$R^{pA} = \frac{\frac{d\sigma^{pA}}{d^2k\,dy}}{A\,\frac{d\sigma^{pp}}{d^2k\,dy}}$$



We can plot it for the quasi-classical cross section calculated before (Y.K., A. M. '98):

$$R^{pA}(k_T) = \frac{k^4}{Q_S^4}\left\{-\frac{1}{k^2} + \frac{2}{k^2}e^{-k^2/Q_S^2} + \frac{1}{Q_S^2}e^{-k^2/Q_S^2}\left[\ln\frac{Q_S^4}{4\Lambda^2 k^2} + Ei\left(\frac{k^2}{Q_S^2}\right)\right]\right\}$$

Classical gluon production leads to Cronin effect!
Nucleus pushes gluons to higher transverse momentum!

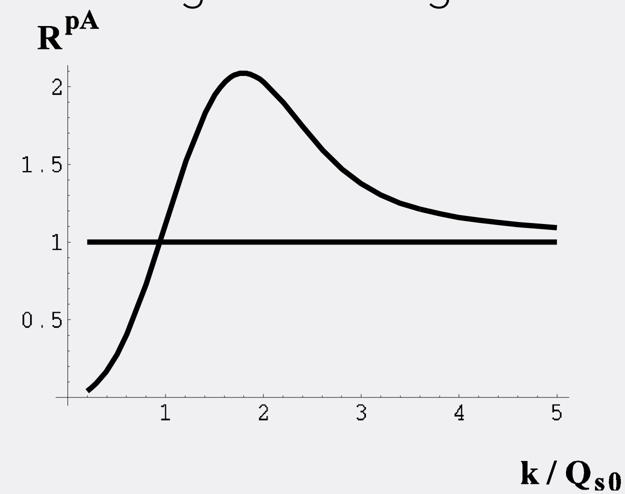
Kharzeev
Yu. K.
Tuchin '03

(see also Kopeliovich et al, '02; Baier et al, '03; Accardi and Gyulassy, '03)

Nuclear Modification Factor

- Nuclear modification factor is like a ratio of the all-twist expression to the leading-twist expression.
- At large k_T one can expand it in a series, which is an asymptotic non-sign-alternating series:

$$R^{pA}(k_T) = 1 + 2\,\frac{Q_{s0}^2}{\underline{k}^2} + 6\,\frac{Q_{s0}^4}{\underline{k}^4} + 24\,\frac{Q_{s0}^6}{\underline{k}^6} + \ldots = \sum_{n=0}^{\infty} n! \left(\frac{Q_{s0}^2}{\underline{k}^2}\right)^n$$



- Explains the approach to 1 from above at large k_T. At low k_T, the divergent series represents a well-behaved function.

Bridging small-x and large-x physics

- At small *x* we have the expansion in powers of $x \approx Q^2/s$ (eikonal, sub-eikonal, etc.). Logarithms of *x* are kept and resummed. Logarithms of Q^2 are kept, but not resummed (at a given power in *x* and given order in the coupling α_s). Some (A-enhanced) powers of Λ^2/Q^2 are also resummed.
- At large *x* we have the twist expansion, i.e., an expansion in the powers of Λ^2/Q^2 . Logarithms of Q^2 are kept and resummed. Logarithms and powers of *x* are kept but not re-summed (at a given power in Q^2 and a given order in α_s). ← Collinear formalism.
- Can we unify the two approaches? Thoughts?