Accessing GPDs with nucleon LFWFs

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In collaboration with:
M. Riberdy, J. M. Morgado Chavez,
and many more !

- Generalised Parton Distributions (GPDs):

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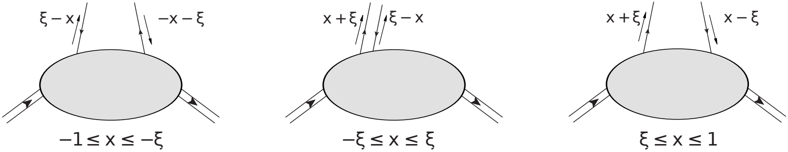
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,



  - ★ x: average momentum fraction carried by the active parton
  - ★ ξ: skewness parameter $\xi \simeq \frac{x_B}{2-x_B}$
  - ★ t: the Mandelstam variable

- Generalised Parton Distributions (GPDs):
  - "hadron-parton" amplitudes which depend on three variables (x, ξ, t) and a scale μ,
  - are defined in terms of a non-local matrix element,

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}\gamma^+u + E^q(x,\xi,t)\bar{u}\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u\right].$$

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\gamma_5\psi^q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0}$$
$$= \frac{1}{2P^+}\left[\tilde{H}^q(x,\xi,t)\bar{u}\gamma^+\gamma_5 u + \tilde{E}^q(x,\xi,t)\bar{u}\frac{\gamma_5\Delta^+}{2M}u\right].$$

D. Müller *et al.*, Fortsch. Phy. 42 101 (1994)
X. Ji, Phys. Rev. Lett. **78**, 610 (1997)
A. Radyushkin, Phys. Lett. **B380**, 417 (1996)

4 GPDs without helicity transfer + 4 helicity flip GPDs

Generalised Parton Distributions

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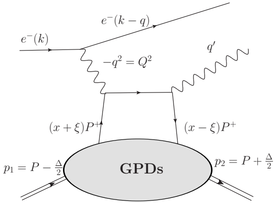
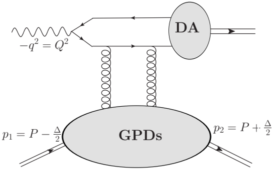
  - are related to PDF in the forward limit $H(x, \xi = 0, t = 0; \mu) = q(x; \mu)$
  - are universal, *i.e.* are related to the Compton Form Factors (CFFs) of various exclusive processes through convolutions

$$\mathcal{H}(\xi, t) = \int \mathrm{d}x \; C(x, \xi) H(x, \xi, t)$$





Properties

- Polynomiality Property:

$$\int_{-1}^{1} \mathrm{d}x\, x^m H^q(x,\xi,t;\mu) = \sum_{j=0}^{\left[\frac{m}{2}\right]} \xi^{2j} C^q_{2j}(t;\mu) + mod(m,2)\xi^{m+1} C^q_{m+1}(t;\mu)$$

X. Ji, J.Phys.G 24 (1998) 1181-1205
A. Radyushkin, Phys.Lett.B 449 (1999) 81-88

Special case :

$$\int_{-1}^{1} \mathrm{d}x\ H^q(x,\xi,t;\mu) = F^q_1(t)$$

Lorentz Covariance

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

$$\left| H^q(x,\xi,t) - \frac{\xi^2}{1-\xi^2} E^q(x,\xi,t) \right| \leq \sqrt{\frac{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}{1-\xi^2}}$$

A. Radysuhkin, Phys. Rev. **D59**, 014030 (1999)
B. Pire *et al.*, Eur. Phys. J. **C8**, 103 (1999)
M. Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)
P.V. Pobilitsa, Phys. Rev. **D65**, 114015 (2002)

Positivity of Hilbert space norm

Properties

- Polynomiality Property:

Lorentz Covariance

- Positivity property:

Positivity of Hilbert space norm

- Support property:

$$x \in [-1; 1]$$

M. Diehl and T. Gousset, Phys. Lett. **B428**, 359 (1998)

Relativistic quantum mechanics

Properties

- Polynomiality Property:

  Lorentz Covariance

- Positivity property:

  Positivity of Hilbert space norm

- Support property:

  Relativistic quantum mechanics

- Continuity at the crossover lines
  $\rightarrow$ GPDs are continuous albeit non analytical at $x = \pm\xi$

  J. Collins and A. Freund, PRD 59 074009 (1999)

  Factorisation theorem

Properties

- Polynomiality Property:

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- Positivity property:

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- Scale evolution property
  $\rightarrow$ generalization of DGLAP and ERBL evolution equations

  D. Müller *et al.*, Fortschr. Phys. 42, 101 (1994)

  Renormalization

Properties

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  Relativistic quantum mechanics

- Continuity at the crossover lines

  Factorisation theorem

- Scale evolution property

  Renormalization

Problem

- There is hardly any model fulfilling *a priori* all these constraints.
- Lattice QCD computations remain very challenging.

- GPDs are related to Double Distributions (DDs) through:

$$H(x,\xi,t) = \int_{\Omega} \mathrm{d}\beta \mathrm{d}\alpha \left(F(\beta,\alpha,t) + \xi G(\beta,\alpha,t)\right) \delta\left(x - \beta - \xi\alpha\right)$$

The Dirac δ insures that the polynomiality is fulfilled, independently of our choice of F and G

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- DDs have been widely used for phenomenological purposes (VGG, GK...)
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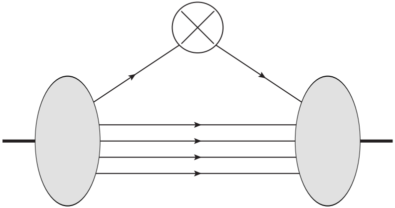
Positivity property is not guaranteed, and may be violated.

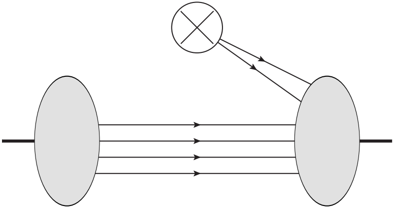
- On the light front, hadronic states can be expanded on a Fock basis

DGLAP: $|x| > |\xi|$



- Same N LFWFs
- No ambiguity

ERBL: $|x| < |\xi|$



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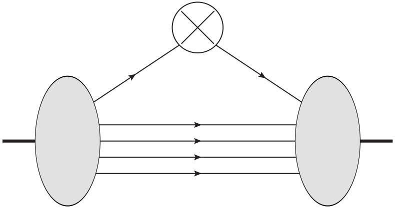

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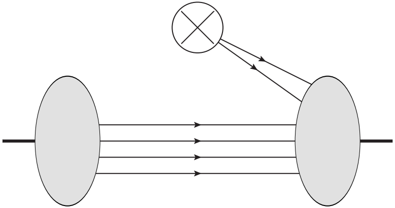

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LFWFs formalism has the positivity property inbuilt but polynomiality is lost by truncating both in DGLAP and ERBL sectors.

A new modelling procedure

Chouika et al., EPJC 77, 906

Lightfront
Wave
Functions

A new modelling procedure

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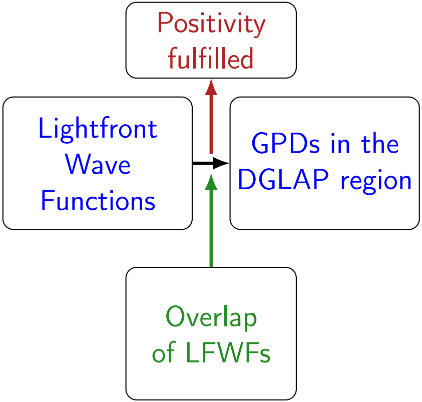

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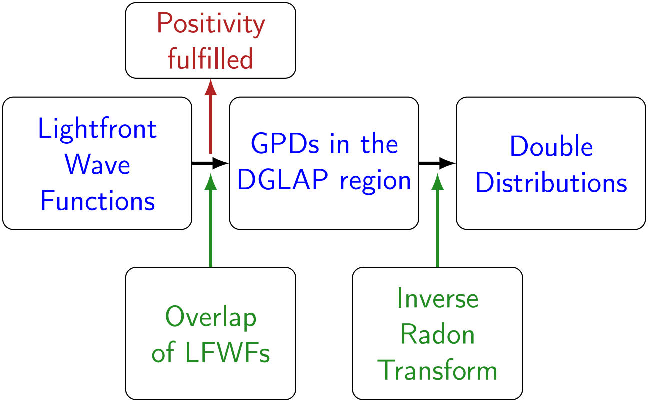

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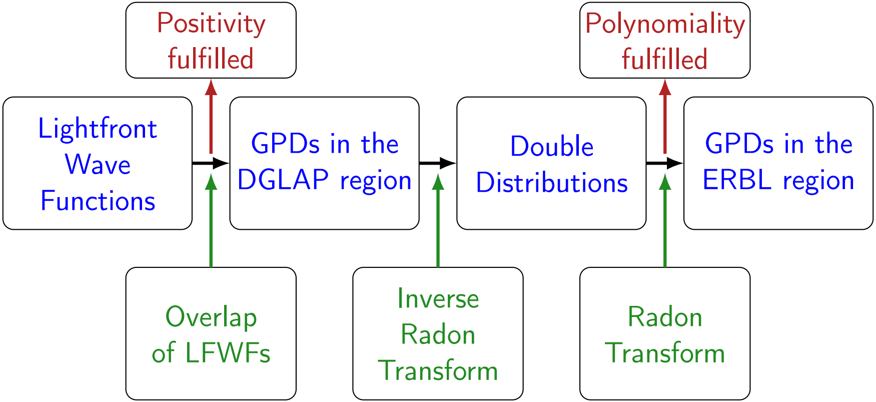

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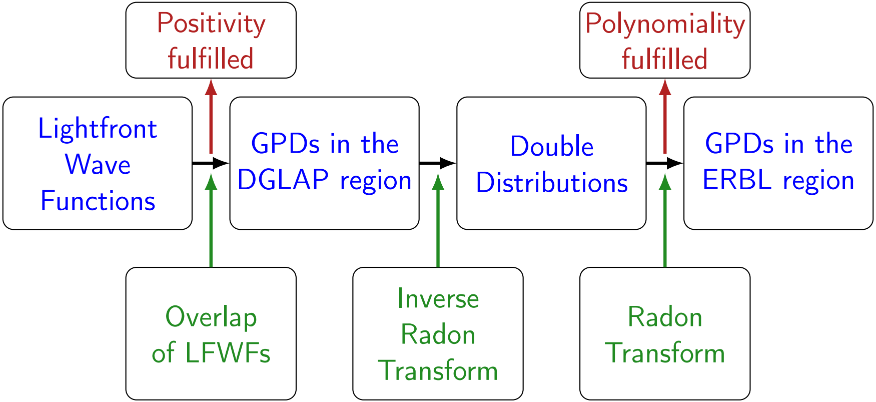
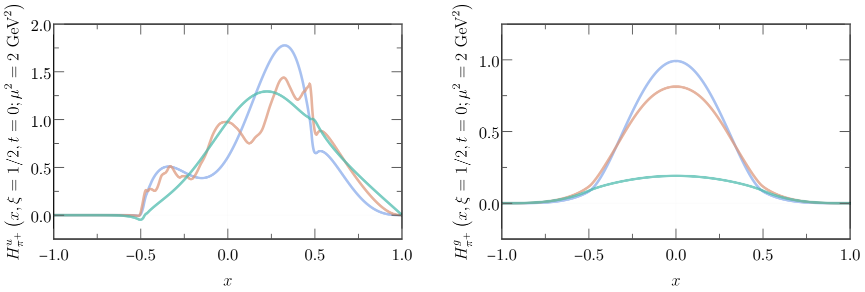


Not necessary to start from LFWFs
→ Fulfilling the positivity and forward limit properties is enough

An example on the pion

Morgado *et al.*, Phys.Rev.D 105 (2022) 9, 094012



- Blue: GPD based on algebraic PDFs model
- Orange: GPD based on refine numerical PDF model
- Green: GPD based on standard Ansatz (RDDA)

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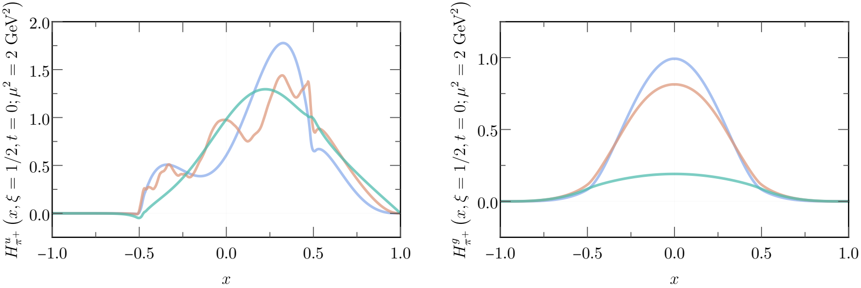

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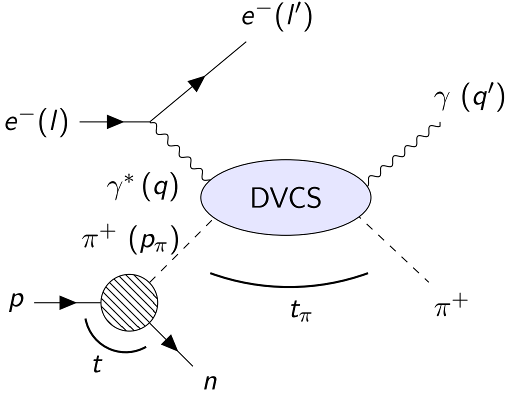
All theoretical constraints are fulfilled by construction !

Can we measure DVCS on a virtual pion ?

D. Amrath *et al.*, EPJC 58 (2008) 179-192

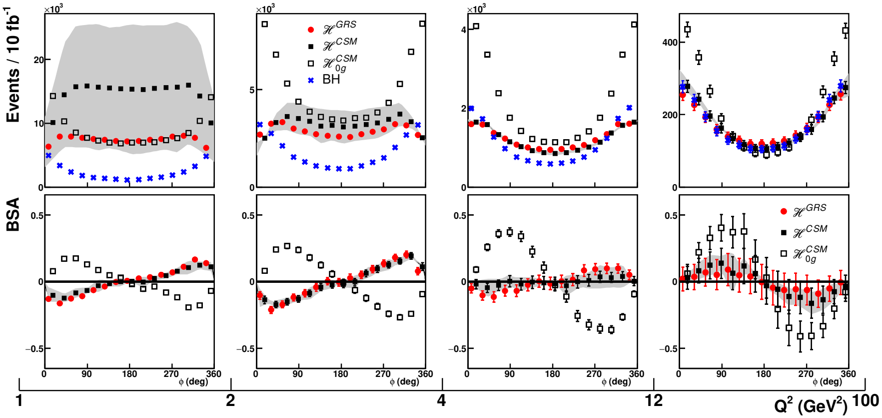


- $e^- p \to e^- \gamma \pi^+ n$
- kinematical cuts to avoid N^* resonances
- Already used to extract pion EFF at JLab
- Considered for pion structure function at EIC and EicC

EIC Yellow report, arXiv:2103.05419
EicC white paper, arXiv:2102.09222

An example on the pion

Morgado *et al.*, Phys.Rev.Lett. 128 (2022) 20, 202501



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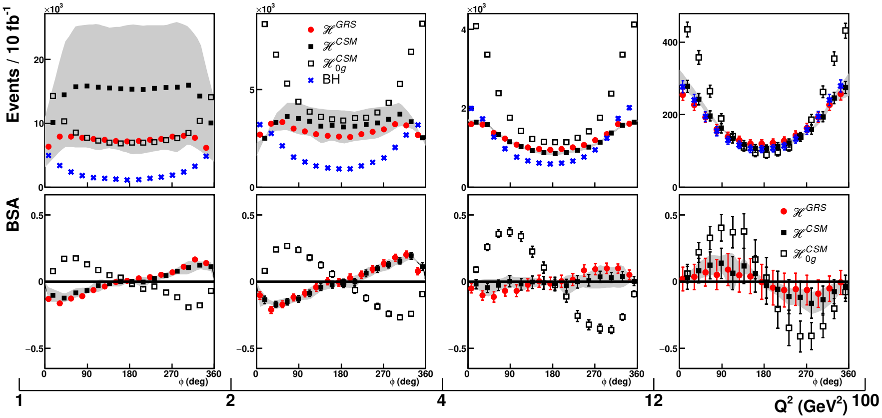

DVCS off virtual pion measurable at EIC and EicC

Can we do the same program for the nucleon ?

Definitions and Classification of LFWFs

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

$$|P,N\rangle \propto \sum_\beta \Psi_\beta^{qqq}|qqq\rangle + \sum_\beta \Psi_\beta^{qqq,q\bar{q}}|qqq,q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

LFWFs: definitions

$$\langle 0|O^{\alpha,\cdots}(\{z_1^-, z_{\perp 1}\}, \ldots, \{z_n^-, z_{\perp n}\})|P,\lambda\rangle\Big|_{z_i^+=0}$$

- Lightfront operator O of given number of quark and gluon fields

LFWFs: definitions

$$\langle 0| O^{\alpha,\cdots}(\{z_1^-, z_{\perp 1}\}, \ldots, \{z_n^-, z_{\perp n}\})|P,\lambda\rangle\big|_{z_i^+=0} = \sum_j^n \tau_j^{\alpha,\cdots} N(P,\lambda) F_j(z_i)$$

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Both mesons and baryons can (in principle) have multiple independent leading-twist LFWFs

Nucleon LFWFs classification

- In the nucleon case, the procedure applies with three quarks at leading Fock state:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P,\uparrow\rangle$$

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X. Ji, *et al.*, Nucl Phys B652 383 (2003)

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X. Ji, *et al.*, Nucl Phys B652 383 (2003)

- The LFWFs carry different amount of OAM projections:

states	$\langle\downarrow\downarrow\downarrow\|P,\uparrow\rangle$	$\langle\downarrow\downarrow\uparrow\|P,\uparrow\rangle$	$\langle\uparrow\downarrow\uparrow\|P,\uparrow\rangle$	$\langle\uparrow\uparrow\uparrow\|P,\uparrow\rangle$
OAM	2	1	0	-1
LFWFs	ψ^6	ψ^3, ψ^4	ψ^1, ψ^2	ψ^5

- Since the Faddeev wave function χ is given as:

$$\langle 0| T\left\{q(z_1)q(z_2)q(z_3)\right\}|P,\lambda\rangle = \frac{1}{4} f_N N_\sigma(P,\lambda)$$
$$\times \int \prod_{j=1}^{3} \mathrm{d}^{(4)}k_j e^{-ik_j z_j} \delta^{(4)}(P - \sum_j k_j)\chi_\sigma(k_1,k_2,k_3),$$

Relation with the Faddeev Wave function

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$$\psi^i = \int \prod_{j=1}^{3}[\mathrm{d}k_j^-]\mathcal{P}_i\chi$$

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Important

The FWF allows a **consistent** derivation of the 6 leading-fock states LFWFs of the nucleon

Modelling the Faddeev wave Function

Baryon and Diquarks

- The Faddeev equation provides a covariant framework to describe the nucleon as a bound state of three dressed quarks.

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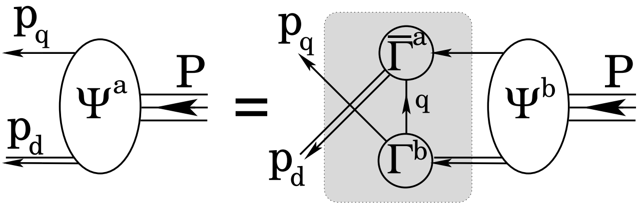
- It predicts the existence of strong diquarks correlations inside the nucleon.



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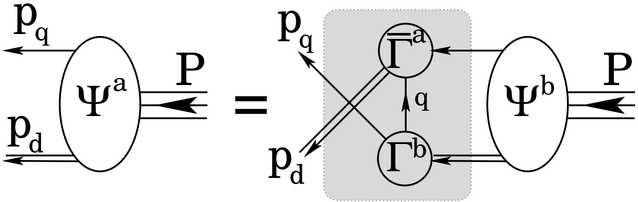

- Mostly two types of diquark are dynamically generated by the Faddeev equation:
  - Scalar diquarks,
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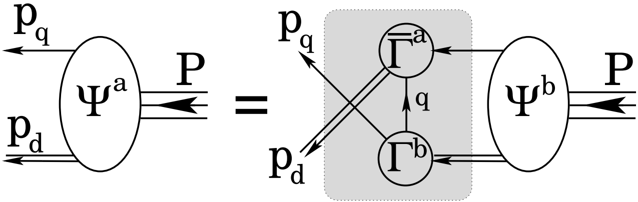

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- In the following we build a model inspired by numerical solutions of the Faddeev equations

Example on the Nucleon Distribution Amplitude

- DA is obtained by integrating the transverse momentum degrees of feedom

$$\langle 0|\epsilon^{ijk}\left(u_{\uparrow}^{i}(z_1^-,0_\perp)C\not{n}u_{\downarrow}^{j}(z_2^-,0_\perp)\right)\not{n}d_{\uparrow}^{k}(z_3^-,0_\perp)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

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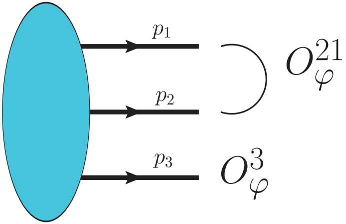
- We can apply it on the wave function:



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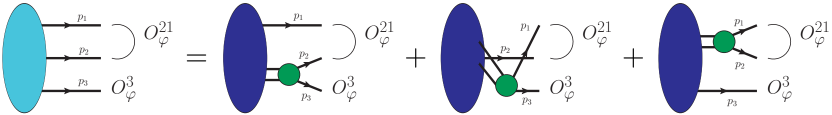

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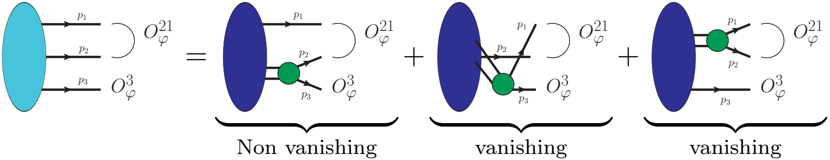

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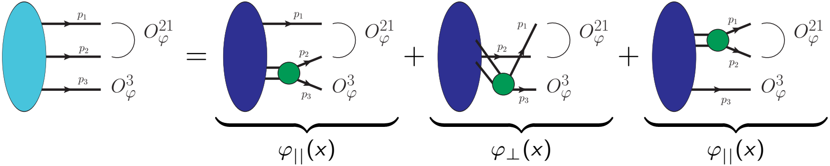

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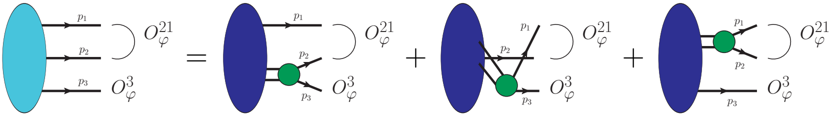
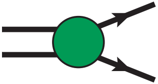


- The operator then selects the relevant component of the wave function.
- Our ingredients are:
  - Nakanishi based diquark Bethe-Salpeter-like amplitude (green disks)
  - Nakanishi based quark-diquark amplitude (dark blue ellipses)
  - Caveat : how to treat the gauge links remains an open question

Scalar Diquark part of the nucleon

Modelling the Scalar Diquark DA

- We need to obtain the structure of the scalar diquark itself



$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{(\Lambda_q^2 + (q + \frac{z}{2}K)^2)}$$

  - q is the relative momentum between the quarks and K the total diquark momentum
  - Λ_q is a free parameter to be fit on DSE computations
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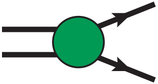

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- We couple this with a simple massive fermion propagator:

$$S(p) = \frac{-ip \cdot \gamma + M}{p^2 + M^2}$$

Adjusting the parameters

- Mass of the quarks: $M = 2/5 M_N$
  - Sum of the frozen mass bigger than the nucleon mass for stability (binding energy)
  - Avoid singularities in the complex plane

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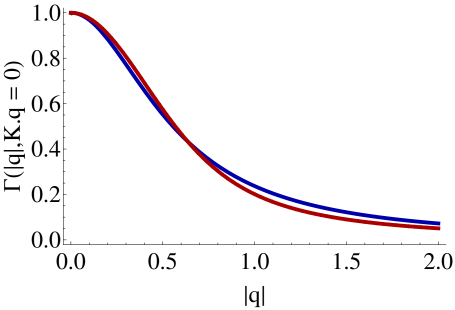
- Width of the diquark BSA $\Lambda_q = 3/5 M_N$ fitted on previous computations:



red curve from Segovia *et al.*,Few Body Syst. 55 (2014) 1185-1222

Scalar Diquark DA

- From that we can compute the scalar diquark DA as:

$$\phi(x) \propto \int d^4q\delta\left(q\cdot n - xK\cdot n\right)\mathrm{Tr}\left[S\Gamma^{0T}S^{T}L^{\downarrow}C^{\dagger}n\cdot\gamma L^{\uparrow}\right]$$

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- We compute Mellin moments $\rightarrow$ avoid difficulties with lightcone in euclidean space
- Nakanishi representation $\rightarrow$ analytic treatments of singularities and analytic reconstruction of the function from the moment

$$\phi(x) = \int_x^1 \mathrm{d}u \int_0^x \mathrm{d}v \frac{F(u,v,x)}{M_{eff}^2(u,v,x,M^2,\Lambda^2) + K^2}$$

F and M_{eff} are computed analytically

Analytic results

- In the specific case $M^2 = \Lambda_q^2$, the PDA can be analytically obtained:

$$\phi(x) \propto \frac{M^2}{K^2}\left[1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}\right]$$

C. Mezrag *et al.*, Springer Proc.Phys. 238 (2020) 773-781

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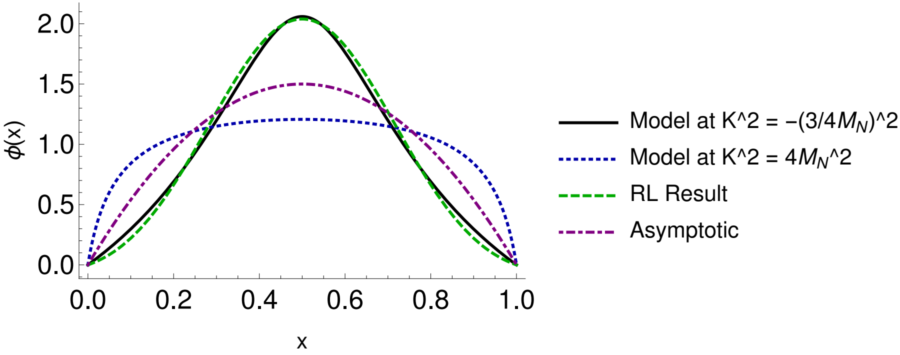
- Note that expanding the log, one get:

$$\phi(x) \propto \frac{1}{2}x(1-x) - \frac{1}{3}K^2/M^2x^2(1-x)^2 + \dots$$

so that:

- at the end point the DA remains linearly decreasing (important impact on observable)
- at vanishing diquark virtuality, one recovers the asymptotic DA

Comparison with DSE results



RL results from Y. Lu *et al.*, Eur.Phys.J.A 57 (2021) 4, 115

- Complex plane singularities for large timelike virtualities

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  - Need of spectral representation with running mass to bypass this?

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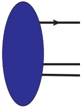
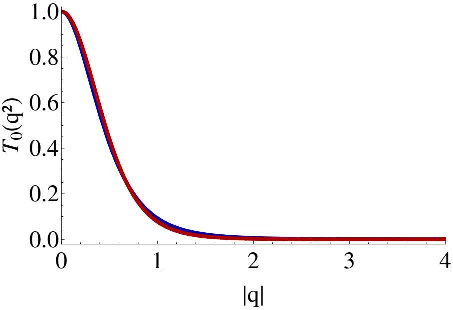
But overall, we expect to gain insights from this simple model

Quark-diquark amplitude



$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}(z) = \prod_j (z - a_j)(z - \bar{a}_j)$$

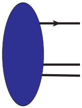
Fits of the parameters through comparison to Chebychev moments:



red curve from Segovia *et al.*,

Nucleon Quark-Diquark Amplitude

Scalar diquark case



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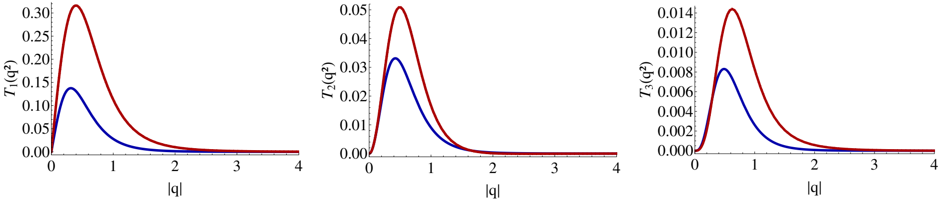

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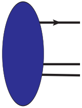

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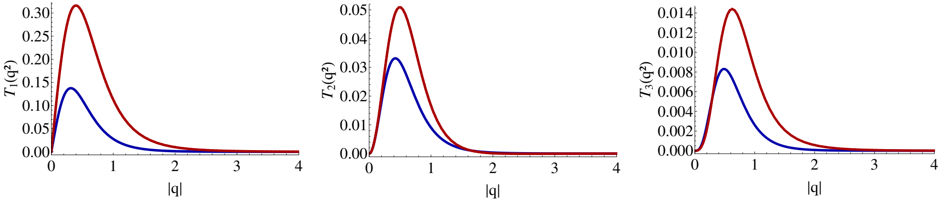


red curves from Segovia *et al.*,

Modification of the $\tilde{\rho}$ Ansatz ? $\tilde{\rho}(z) \to \tilde{\rho}(\gamma, z)$?

Mellin Moments

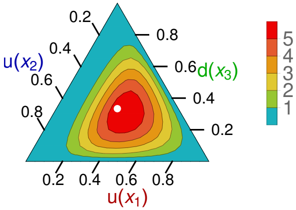
- We do not compute the PDA directly but Mellin moments of it:

$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \; x_1^m x_2^n \varphi(x_1, x_2, 1 - x_1 - x_2)$$

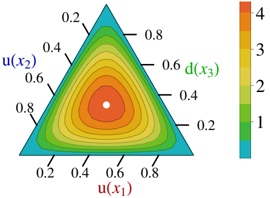
- For a general moment $\langle x_1^m x_2^n \rangle$, we change the variable in such a way to write down our moments as:

$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta \; \alpha^m \beta^n f(\alpha, \beta)$$

- f is a complicated function involving the integration on 6 parameters
- Uniqueness of the Mellin moments of continuous functions allows us to identify f and φ



Scalar diquark Only

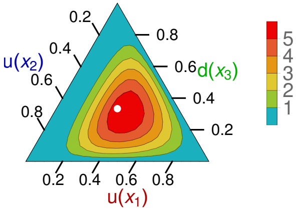


Asymptotic DA

- Typical symmetry in the pure scalar case

C. Mezrag *et al.*, Phys.Lett. B783 (2018)

Preliminary Results



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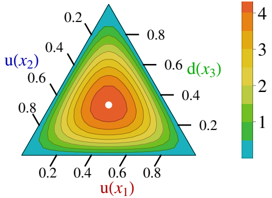

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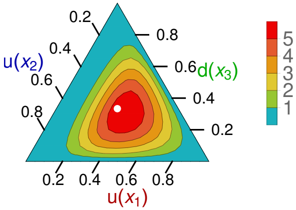

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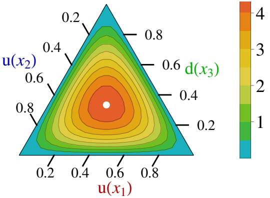

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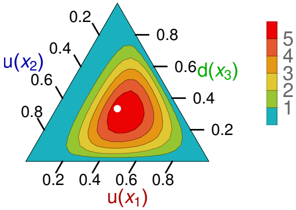

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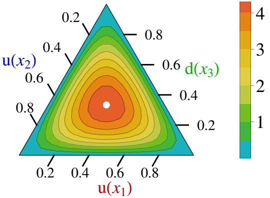

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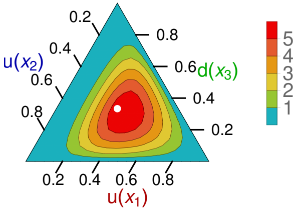

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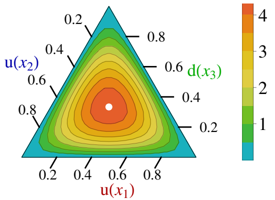

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  - Impact of the virtuality dependence of the diquark WF
- These properties are consequences of our quark-diquark picture
- Improvement in the modelling with respect to our previous work

C. Mezrag *et al.*, Phys.Lett. B783 (2018)

LFWFs and images of the nucleon with GPDs

- In the limit $\xi \to 0$ the connection between GPDs and LFWFs can be computed:

$$\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi} \langle P + \frac{\Delta}{2}|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|P - \frac{\Delta}{2}\rangle \mathrm{d}z^- \Big|_{\substack{z^+=0, \mathbf{z}=0 \\ \xi=0}}$$

$$\underset{N=3}{=} \sum_{\beta,\beta'} \int \left[\prod_{i=1}^{3} \mathrm{d}x_i \mathrm{d}k_\perp^i\right] K_{\beta,\beta'}(x,\xi,t,k_\perp^i)\psi^*_{\beta'}\psi_\beta$$

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- The impact of LFWFs with definite OAM projection can be followed up to the GPD expressions for $|x|\geq|\xi|$:

$$H(x,\xi,t) = F_H(x,\xi,t,\psi_1,\ldots,\psi_6)$$
$$E(x,\xi,t) = F_E(x,\xi,t,\psi_5,\psi_6)$$

M. Riberdy *et al.*, in preparation

Consequences I: OAM deformation of the nucleon

- OAM projection dependence on the 3D probability density:

$$F_H(x,\xi,t,\psi_1,\ldots,\psi_6) \to \rho(x,b_\perp,\psi_1,\ldots,\psi_6)$$

Visualisation of the impact of OAM

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Visualisation of the impact of OAM

- NR charged proton radius

$$F_1(t) = \int \mathrm{d}x\ H(x,0,t,\psi_1,\ldots,\psi_6)$$

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- However, no input on pressure or energy distributions (no access to D-term)

- In principle we could assess the impact of OAM pojection on GPD-sensitive observables

Consequences II: Experimental sensibility to OAM

- In principle we could assess the impact of OAM pojection on GPD-sensitive observables
- but two main difficulties:
  - Evolution mixes OAM projections → one can draw conclusion only for OAM at the original scale
  - amplitude convolution filters the information accessible experimentally

Consequences II: Experimental sensibility to OAM

- In principle we could assess the impact of OAM pojection on GPD-sensitive observables
- but two main difficulties:
  - Evolution mixes OAM projections $\rightarrow$ one can draw conclusion only for OAM at the original scale
  - amplitude convolution filters the information accessible experimentally
- Observables sensitive to GPD E remain the best ones able to tell us something on OAM projection within the nucleon, but they are hard to measure (requires polarised proton targets or the ability to measure the polarisation of the recoil proton)

O. Bessidskaia Bylund *et al.*, Phys.Rev.D 107 (2023) 1, 014020

Summary

Summary and conclusion

Achievements

- **DSE compatible** framework for Nucleon LFWFs computations
- Based on the Nakanishi representation
- Improved models from the first exploratory work
- Relation between LFWFs and GPDs has been worked out

Work in progress/future work

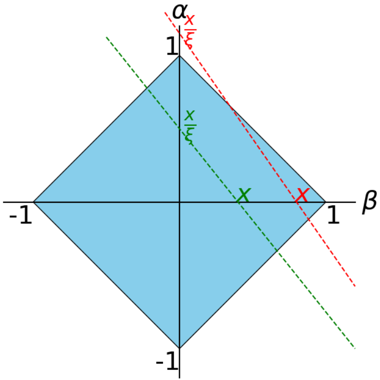
- Finishing the computation for the 6 LFWFs
- Tackling the AV-diquark contributions
- Improvement of the Nakanishi Ansätze
- Computations of GPDs
- Going beyond $\xi = 0$ with the covariant extension

Thank you for your attention

Back up slides

Intuitive picture

$$H(x,\xi) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x-\beta-\alpha\xi)\left[F(\beta,\alpha)+\xi G(\beta,\alpha)\right]$$



- DGLAP (red) and ERBL (green) lines cut $\beta = 0$ outside or inside the square
- Every point $(\beta \neq 0, \alpha)$ contributes **both** to DGLAP and ERBL regions
- For every point $(\beta \neq 0, \alpha)$ we can draw an infinite number of DGLAP lines.

Intuitive picture

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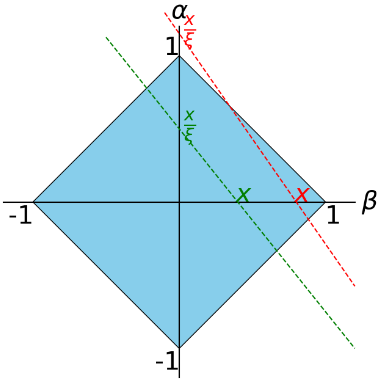

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Is it possible to recover the DDs from the DGLAP region only?

GPD properties from Radon Transform

- Double Distribution representation:

$$H(x,\xi) = D(x/\xi) + \int_{\Omega} \mathrm{d}\beta \mathrm{d}\alpha \delta(x - \beta - \alpha\xi) F_D(\beta, \alpha)$$

GPD properties from Radon Transform

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$$H(x,\xi) = D(x/\xi) + \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x - \beta - \alpha\xi) F_D(\beta,\alpha)$$

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- Since DD are compactly supported, we can use the **Boman and Todd-Quinto theorem** which tells us

$$H(x,\xi) = 0 \quad \text{for} \quad (x,\xi) \in \text{DGLAP} \Rightarrow F_D(\beta,\alpha) = 0 \quad \text{for all} \quad (\beta \neq 0, \alpha) \in \Omega$$

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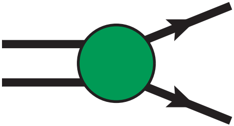
New modeling strategy

- Compute the DGLAP region through overlap of LFWFs
  $\Rightarrow$ **fulfilment of the positivity property**
- Extension to the ERBL region using the Radon inverse transform
  $\Rightarrow$ **fulfilment of the polynomiality property**

Nakanishi Representation



At all order of perturbation theory, one can write (Euclidean space):

$$\Gamma(k,P) = \mathcal{N}\int_0^\infty \mathrm{d}\gamma \int_{-1}^{1} \mathrm{d}z \frac{\rho_n(\gamma,z)}{(\gamma + (k+\frac{z}{2}P)^2)^n}$$

We use a "simpler" version of the latter as follow:

$$\tilde{\Gamma}(q,P) = \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{\rho_n(z)}{(\Lambda^2 + (q+\frac{z}{2}P)^2)^n}$$

We introduce to lightlike vector p and n such that:

$$P^\mu = p^\mu + n^\mu \frac{M^2}{2p\cdot n} \quad \text{and} \quad P^+ = p^+$$

see for instance V. Braun *et al.*, Nucl Phys B589 381 (2000)

From these four-vectors we define the projectors:

$$N(p) = \underbrace{\frac{p\cdot\gamma n\cdot\gamma}{2p\cdot n} N(P)}_{\substack{\text{Dominant contribution} \\ \text{when } P^+\to\infty}} + \frac{n\cdot\gamma p\cdot\gamma}{2p\cdot n} N(p) = N^+(P) + N^-(P)$$

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The same procedure is applied to all quark fields (and a similar one to gluon fields), selecting the leading twist contributions

$$\langle 0|\tilde{O}^{\alpha,\cdots}(\{z_1^-, z_{\perp 1}\}, \ldots, \{z_n^-, z_{\perp n}\})|P,\lambda\rangle\Big|_{z_i^+=0} = \sum_{\substack{j \\ \mathrm{LT}}}^{n} \tilde{\tau}_j^{\alpha,\cdots} N^+(P,\lambda)\tilde{F}_j(z_i)$$

- With the previous procedure we can select the leading-twist combinations scalar functions $\tilde{F}$

Sorting LFWFs

$$\langle 0|\tilde{O}^{\alpha,\cdots}(\{z_1^-, z_{\perp 1}\}, \ldots, \{z_n^-, z_{\perp n}\})|P,\lambda\rangle\Big|_{z_i^+=0} \underset{\text{LT}}{=} \sum_j^n \tilde{\tau}_j^{\alpha,\cdots} N^+(P,\lambda)\tilde{F}_j(z_i)$$

- With the previous procedure we can select the leading-twist combinations scalar functions $\tilde{F}$
- But an additional classification can be performed, by selecting the helicity projection of the quark fields involved through:

$$\psi = \frac{1+\gamma_5}{2}\psi + \frac{1-\gamma_5}{2}\psi = \psi^\uparrow + \psi^\downarrow$$