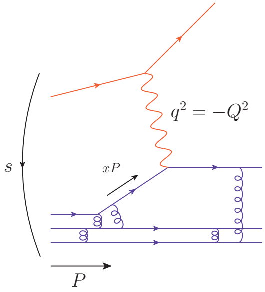
# Probing the gluon orbital angular momentum with double spin asymmetries

Renaud Boussarie

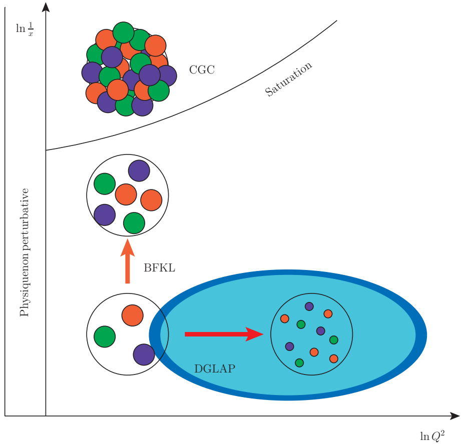
REVESTRUCTURE workshop

[Bhattacharya, RB, Hatta, PRL 128 (2022) 18, 182002]

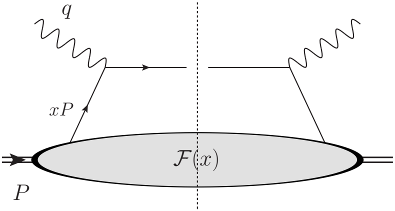
# Accessing the partonic content of hadrons with an electromagnetic probe




Electron-proton collision (parton model)



## QCD factorization
processes with a hard scale $Q \gg \Lambda_{QCD}$




$$\sigma = \mathcal{F}(x, \mu) \otimes \mathcal{H}(x, \mu)$$

At a scale $\mu$, the process is factorized into:

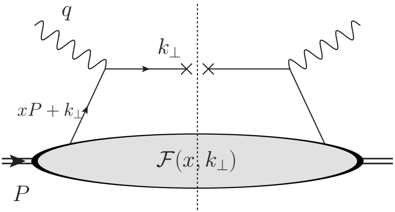
- A hard scattering subamplitude $\mathcal{H}(x, \mu)$
- A Parton Distribution Function (PDF) $\mathcal{F}(x, \mu)$

$\mu$ independence: DGLAP renormalization equation for $\mathcal{F}$

# Transverse Momentum Dependent (TMD) factorization:

## semi-inclusive processes with one hard and one semihard scale

$$Q \sim \sqrt{s} \gg k_\perp$$



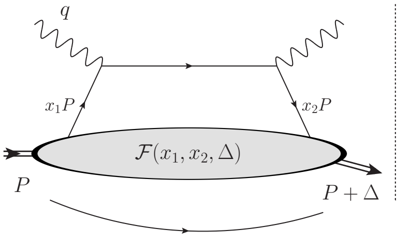

$$\sigma = \mathcal{F}(x, k_\perp, \zeta, \mu) \otimes \mathcal{H}(\mu) \otimes \hat{\mathcal{F}}(\hat{x}, \hat{k}_\perp, \hat{\zeta}, \mu)$$

At a scale $\mu$, the process is factorized into:

- A hard scattering subamplitude $\mathcal{H}(\mu)$
- A TMD PDF $\mathcal{F}(x, k_\perp, \zeta, \mu)$
- A TMD FF $\hat{\mathcal{F}}(\hat{x}, \hat{k}_\perp, \hat{\zeta}, \mu)$

$\mu, \zeta, \hat{\zeta}$ independence: TMD evolution for $\mathcal{F}, \hat{\mathcal{F}}$

# Factorization with Generalized Parton Distributions (GPD): exclusive processes with one hard scale $Q \sim \sqrt{s}$




$$\sigma = \mathcal{F}(x_1, x_2, |\Delta_\perp|, \mu) \otimes \mathcal{H}(x_1, x_2, \mu)$$

At a scale $\mu$, the process is factorized into:

- A hard scattering subamplitude $\mathcal{H}(x_1, x_2, \mu)$
- A Generalized Parton Distribution (GPD) $\mathcal{F}(x_1, x_2, |\Delta_\perp|, \mu)$

$\mu$ independence: DGLAP/ERBL renormalization equation for $\mathcal{F}$

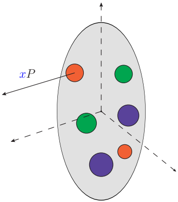
# Operator definition for parton distributions

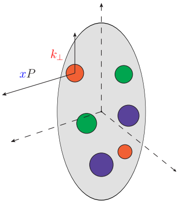
## Parton distribution function

$$\mathcal{F}(x) \propto \int \mathrm{d}z^- \mathrm{e}^{ixP^+z^-} \left\langle P \left| F^{+i}(z^-) \left[z^-, 0^-\right] F^{+i}(0) \left[0^-, z^-\right] \right| P \right\rangle$$

## Transverse Momentum Dependent distribution

$$\mathcal{F}(x, k_\perp) \propto \int \mathrm{d}^4 z \delta(z^+) \mathrm{e}^{ixP^+z^- + i(k_\perp \cdot z_\perp)} \left\langle P \left| F^{+i}(z) \mathcal{U}_{z,0} F^{+i}(0) \mathcal{U}_{0,z} \right| P \right\rangle$$



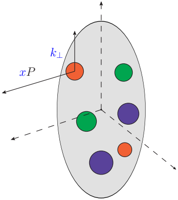
# Operator definition for parton distributions

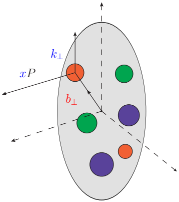
## TMD distribution

$$\mathcal{F}(x, k_\perp) \propto \int \mathrm{d}^4 z \delta(z^+) \mathrm{e}^{ixP^+ z^- + i(k_\perp \cdot z_\perp)} \left\langle P \left| F^{+i}(z) \mathcal{U}_{z,0} F^{+i}(0) \mathcal{U}_{0,z} \right| P \right\rangle$$

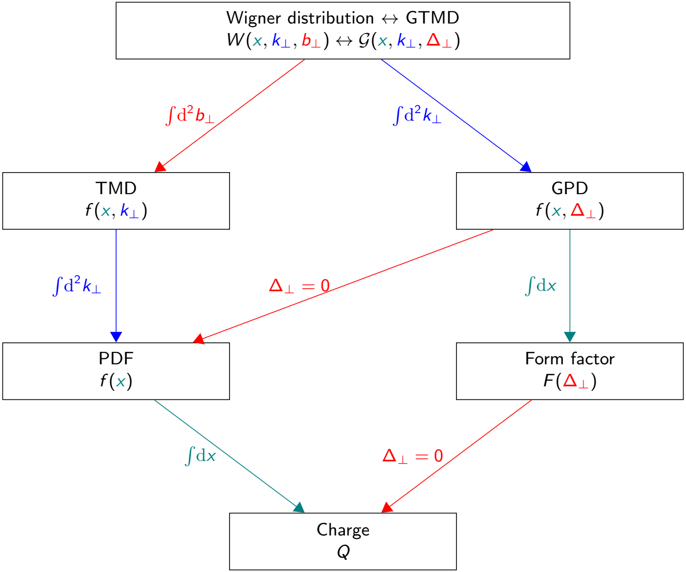
## Generalized TMD distribution

$$\mathcal{F}(x, k_\perp, \Delta) \propto \int \mathrm{d}^4 z \delta(z^+) \mathrm{e}^{ixP^+ z^- + i(k_\perp \cdot z_\perp)} \left\langle P + \Delta \left| F^{+i}(z) \mathcal{U}_{z,0} F^{+i}(0) \mathcal{U}_{0,z} \right| P \right\rangle$$



# The family tree of parton distributions



# Wigner distributions in NRQM

## Wigner distributions in Quantum Mechanics [Wigner, 1932]

Defined via wavefunctions

$$W(x,k)=\int\frac{\mathrm{d}x'}{2\pi}\mathrm{e}^{-i(k\cdot x')}\psi\left(x+\frac{x'}{2}\right)\psi^*\left(x-\frac{x'}{2}\right)$$

Connection with probability densities:

$$|\psi(x)|^2=\int\mathrm{d}k\,W(x,k)$$

and

$$|\psi(k)|^2=\int\mathrm{d}x\,W(x,k)$$

Connection with observables:

$$\langle O\rangle=\int\mathrm{d}x\int\mathrm{d}k\,O(x,k)\,W(x,k)$$

# Wigner distributions in QCD

## QCD Wigner distributions

Defined as the Fourier transform of a GTMD

$$\mathcal{W}^g(x, k_\perp, b_\perp) = \int \mathrm{d}^2\Delta_\perp \mathrm{e}^{i(b_\perp \cdot \Delta_\perp)} \times \int \mathrm{d}^4 z \delta(z^+) \mathrm{e}^{ixP^+ z^- + i(k_\perp \cdot z_\perp)} \left\langle P + \Delta_\perp \left| F^{+i}(z) \mathcal{U}_{z,0} F^{+i}(0) \mathcal{U}_{0,z} \right| P \right\rangle$$

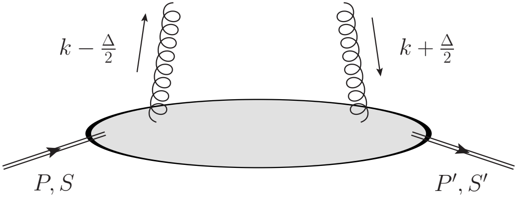
Connection with observables: e.g. orbital angular momentum of gluons inside a proton

$$\langle L_z^g \rangle = \int \mathrm{d}x \int \mathrm{d}^2 k_\perp \int \mathrm{d}^2 b_\perp (b_\perp \times k_\perp)_z W^g(x, k_\perp, b_\perp)$$

# GTMD

## Parametrization and coupling to the target hadron

[Meissner, Metz, Schlegel, 2009], [Lorcé, Pasquini, 2013]




$$\int \mathrm{d}^4 v \delta(v^+) e^{ix\bar{P}^+ v^- - i(\boldsymbol{k}\cdot\boldsymbol{v})} \langle P'S'|F^{i+}(-\frac{v}{2})\mathcal{U}^{[+]}_{\frac{v}{2},-\frac{v}{2}} F^{i+}(\frac{v}{2})|PS\rangle$$

$$= (2\pi)^3 \frac{\bar{P}^+}{2M} \bar{u}_{P'S'} \left[ F^g_{1,1} + i\frac{\sigma^{i+}}{\bar{P}^+}(\boldsymbol{k}^i F^g_{1,2} + \Delta^i F^g_{1,3}) + i\frac{\sigma^{ij}\boldsymbol{k}^i\Delta^j}{M^2} F^g_{1,4} \right] u_{PS}$$

# Leading twist GTMDs

## Leading twist GTMDs in a spin 1/2 hadron

[Meissner, Metz, Schlegel, 2009], [Lorcé, Pasquini, 2013]

- Unpolarized parton pairs: $F_{1,1}$, $F_{1,2}$, $F_{1,3}$ and $F_{1,4}$
- Polarized parton pairs: $G_{1,1}$, $G_{1,2}$, $G_{1,3}$ and $G_{1,4}$
- Transversity distributions: $H_{1,1}$, $H_{1,2}$, $H_{1,3}$, $H_{1,4}$, $H_{1,5}$, $H_{1,6}$, $H_{1,7}$ and $H_{1,8}$

16 distributions with real and imaginary values at leading twist

# Leading twist GTMDs vs GPDs

## GTMDs span GPDs

- Unpolarized parton pairs:
  $\mathrm{Re}(F_{1,1})$, $\mathrm{Re}(F_{1,2})$, $\mathrm{Re}(F_{1,3}) \to H, E$
- Polarized parton pairs:
  $\mathrm{Re}(G_{1,2})$, $\mathrm{Re}(G_{1,3})$, $\mathrm{Re}(G_{1,4}) \to \tilde{H}, \tilde{E}$
- Transversity distributions:
  $\mathrm{Re}(H_{1,3})$, $\mathrm{Re}(H_{1,4})$, $\mathrm{Re}(H_{1,5})$, $\mathrm{Re}(H_{1,6})$, $\mathrm{Re}(H_{1,7})$, $\mathrm{Re}(H_{1,8}) \to H_T, E_T$

Most distributions have a null GPD limit

# GTMDs and Orbital Angular Momentum

# Spin sum rule

Jaffe Manohar spin sum rule

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

- Quark helicity $\Delta\Sigma$
- Gluon helicity $\Delta G$
- Quark OAM $L^q$
- Gluon OAM $L^g$

## Gluon OAM

### Gluon orbital angular momentum

'Intuitive' (canonical) definition for the OAM

$$L_z^g = \int \mathrm{d}^2 k_\perp (\Delta_\perp \times k_\perp)_z \, x f^g(x, \xi, k_\perp, \Delta_\perp)$$

with $xf^g(x, \xi, k_\perp, \Delta_\perp)$ the gluon GTMD correlator

# Gluon OAM

## Gluon orbital angular momentum

$$xf_g(x,\xi,\boldsymbol{k},\Delta)$$
$$= \frac{1}{2M}\bar{u}(p',h_{p'})\left[F_{1,1} + i\frac{\sigma^{j+}}{P^+}(\boldsymbol{k}^j F_{1,2} + \Delta^j F_{1,3}) + i\frac{\sigma^{ij}\boldsymbol{k}^i\Delta^j}{M^2}F_{1,4}\right]u(p,h_p)$$

Symmetry properties:

$$F^*_{1,(1,3,4)}(\xi,\boldsymbol{k}\cdot\Delta,\boldsymbol{k}^2,\Delta^2) = F_{1,(1,3,4)}(-\xi,-\boldsymbol{k}\cdot\Delta,\boldsymbol{k}^2,\Delta^2)$$
$$F^*_{1,2}(\xi,\boldsymbol{k}\cdot\Delta,\boldsymbol{k}^2,\Delta^2) = -F_{1,2}(-\xi,-\boldsymbol{k}\cdot\Delta,\boldsymbol{k}^2,\Delta^2)$$

## Gluon OAM

### Gluon orbital angular momentum

Symmetry properties:

$$F^*_{1,(1,3,4)}(\xi, \boldsymbol{k}\cdot\Delta, \boldsymbol{k}^2, \Delta^2) = F_{1,(1,3,4)}(-\xi, -\boldsymbol{k}\cdot\Delta, \boldsymbol{k}^2, \Delta^2)$$

$$F^*_{1,2}(\xi, \boldsymbol{k}\cdot\Delta, \boldsymbol{k}^2, \Delta^2) = -F_{1,2}(-\xi, -\boldsymbol{k}\cdot\Delta, \boldsymbol{k}^2, \Delta^2)$$

so we can write

$$F_{1,(1,3,4)} = \left[R^{(1)}_{1,(1,3,4)} + \xi(\boldsymbol{k}\cdot\Delta)R^{(2)}_{1,(1,3,4)}\right] + i\left[\xi I^{(1)}_{1,(1,3,4)} + (\boldsymbol{k}\cdot\Delta)I^{(2)}_{1,(1,3,4)}\right]$$

$$F_{1,2} = \left[\xi R^{(1)}_{1,2} + (\boldsymbol{k}\cdot\Delta)R^{(2)}_{1,2}\right] + i\left[I^{(1)}_{1,2} + \xi(\boldsymbol{k}\cdot\Delta)I^{(2)}_{1,2}\right]$$

# Gluon OAM

## Gluon orbital angular momentum

$$xf_g(x,\xi,\boldsymbol{k},\Delta)$$
$$= \frac{1}{2M}\bar{u}(p',h_{p'})\left[F_{1,1} + i\frac{\sigma^{j+}}{P^+}(\boldsymbol{k}^j F_{1,2} + \Delta^j F_{1,3}) + i\frac{\sigma^{ij}\boldsymbol{k}^i\Delta^j}{M^2}F_{1,4}\right]u(p,h_p)$$

with

$$F_{1,(1,3,4)} = \left[R^{(1)}_{1,(1,3,4)} + \xi(\boldsymbol{k}\cdot\Delta)R^{(2)}_{1,(1,3,4)}\right] + i\left[\xi I^{(1)}_{1,(1,3,4)} + (\boldsymbol{k}\cdot\Delta)I^{(2)}_{1,(1,3,4)}\right]$$

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Identify the terms which survive the convolution with $(\boldsymbol{k}\times\Delta)$

# Gluon OAM

## Gluon orbital angular momentum

$$\int \mathrm{d}^2\boldsymbol{k}(\boldsymbol{k}\times\Delta)xf_g(x,\xi,\boldsymbol{k},\Delta)$$
$$= \int \mathrm{d}^2\boldsymbol{k}\,\frac{i(\boldsymbol{k}\times\Delta)}{2M}\times\bar{u}(p',h_{p'})\left[\xi\frac{\boldsymbol{k}^i\sigma^{i+}}{P+}R^{(1)}_{1,2}+\frac{\sigma^{ij}\boldsymbol{k}^i\Delta^j}{M^2}F_{1,4}\right]u(p,h_p)$$

At small $\xi$, only $F_{1,4}$ contributes

# Gluon OAM

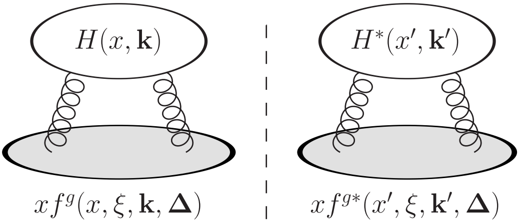
## Gluon orbital angular momentum as a GTMD

$$\int \mathrm{d}^2 \boldsymbol{k} \frac{\boldsymbol{k}^2}{M^2} F_{1,4}^{g}(x, 0, \boldsymbol{k}, 0) = -x L_g(x, 0)$$

# Where to constrain the OAM GTMD?

## How to constrain the OAM GTMD

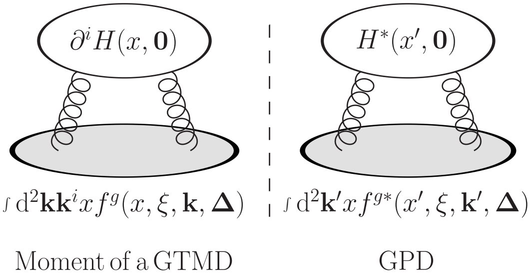
Exclusive process with general(-ish) gluon kinematics




Squared amplitude of a process involving a gluon GTMD

# How to constrain the OAM GTMD

Power expand the cross section




We need to study the GTMD*GPD proton structure

# How to constrain the OAM GTMD

GTMD*GPD correlators, summed over outgoing helicities

$$
\sum_{h_p'} \frac{1}{2P^+} \left[x f_g(x,\xi,\boldsymbol{k},\Delta)\right] \left[x' F_g(x',\xi,\Delta)\right]^*
$$

$$
= H_g F_{1,1} + \frac{\xi^2}{1-\xi^2} E_g F_{1,1}
$$

$$
+ \frac{1}{2}\frac{\boldsymbol{k}\cdot\Delta}{M^2} E_g F_{1,2} + \frac{1}{2}\frac{\Delta^2}{M^2} E_g F_{1,3}
$$

$$
+ \frac{iS^+ \epsilon^{ij} \boldsymbol{k}^i \Delta^j}{(1+\xi)P^+ M^2} \left( H_g F_{1,4}^g - \frac{\xi^2}{1-\xi^2} E_g F_{1,4} - \frac{1}{2} E_g F_{1,2} \right)
$$

Longitudinal spin asymmetries select $F_{1,4}^g$

# How to constrain the OAM GTMD

Target correlator for longitudinal target spin asymmetry

$$\sum_{h'_p} \frac{1}{2P^+} \left[x f_g(x,\xi,\boldsymbol{k},\Delta)\right] \left[x' F_g(x',\xi,\Delta)\right]^* \Bigg|_{S^+}$$

$$= \frac{iS^+ \epsilon^{ij} \boldsymbol{k}^i \Delta^j}{(1+\xi)P^+ M^2} \left( H_g F^g_{1,4} - \frac{1}{2} E_g F_{1,2} - \frac{\xi^2}{1-\xi^2} E_g F_{1,4} \right)$$

- Phenomenologically, $E_g \ll H_g$
- The moment of $F^g_{1,2}$ is a genuine higher twist while that of $F^g_{1,4}$ has a kinematic part

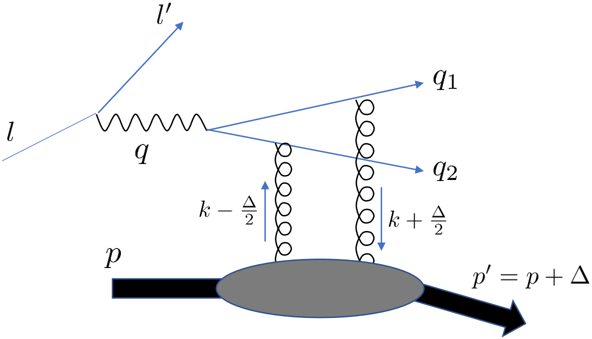
Longitudinal spin asymmetry selects the OAM GTMD $F^g_{1,4}$

# How to constrain the gluon OAM

A process will contrain the gluon OAM if:

- It involves gluons
- It is exclusive enough
- It involves a longitudinal target spin asymmetry
- Its hard part involves $\boldsymbol{k} \times \Delta$

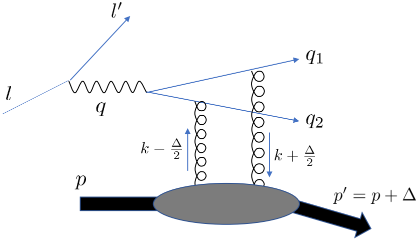
# Exclusive dijet electroproduction and gluon OAM




[Ji, Yuan, Zhao], [Bhattacharya, RB, Hatta]

# Kinematics

## Dijet kinematics




Longitudinal fractions

$$z = p \cdot q_1 / p \cdot q$$
$$\bar{z} = p \cdot q_2 / p \cdot q$$

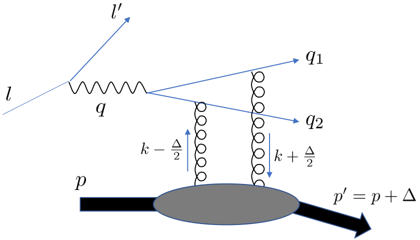
Transverse momenta

$$q_{1\perp} = q_\perp - z\Delta_\perp$$
$$q_{2\perp} = -q_\perp - \bar{z}\Delta_\perp$$

Consider longitudinal target spin asymmetry, try to select $(k \times \Delta)$

# SSA kinematics

## Dijet kinematics

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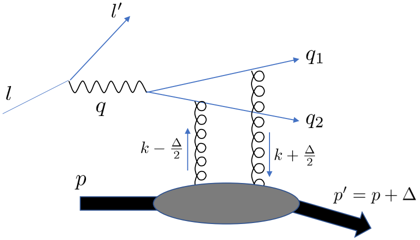
Single (target) spin asymmetry [Ji, Yuan, Zhao]

$$\mathrm{d}\sigma^{h_p} \sim h_p(z - \bar{z}) \sin(\phi_{q_\perp} - \phi_{\Delta_\perp}) \mathrm{Re}(A_2 A_3^*)$$

## SSA kinematics

### Dijet kinematics

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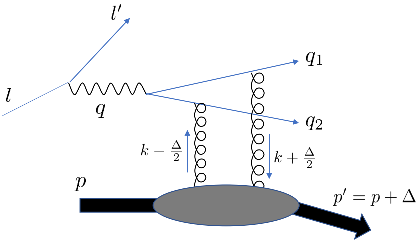
BUT $A_3$ has end point singularities (higher poles in $x \pm \xi$)

# DSA kinematics

## Dijet kinematics




Longitudinal fractions

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Transverse momenta

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$$q_{2\perp} = -q_\perp - \bar{z}\Delta_\perp$$

Double spin asymmetry [Bhattachary, RB, Hatta]

$$\mathrm{d}\sigma^{h_p h_l} \sim h_p h_l \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})\mathrm{Re}(A_2' A_3'^*)$$

$A_3'$ has end point singularities BUT they cancel at $z = 1/2$

# DSA

## Best gluon OAM observable so far: DSA

- Consider exclusive DIS dijet production
- Make sure gluons are involved (quark-induced jets, $c\bar{c}$ pair...)
- Measure double spin asymmetries
- Focus on $\cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$ at $z = 1/2$

# DSA

## Best gluon OAM observable so far: DSA

- Consider exclusive DIS dijet production
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- Measure double spin asymmetries
- Focus on $\cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$ at $z = 1/2$

**HOWEVER**

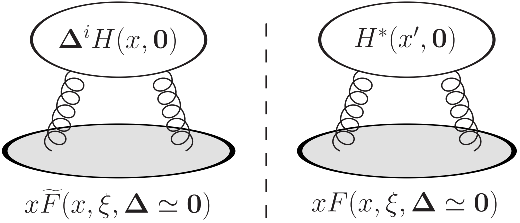
# Helicity contribution

## Another contribution to that observable

Polarized gluon GPDs contribute to

$$\mathrm{d}\sigma^{h_p h_l} \sim h_p h_l \cos(\phi_{l_\perp} - \phi_{\Delta_\perp})\mathrm{Re}(A_2' A_3'^*)$$

at the same order as the OAM term



# Cross section

## Cross section for DSA in exclusive DIS dijet production

Neglecting GPD $E$ wrt GPD $H$

$$\frac{d\sigma}{\mathrm{d}y\mathrm{d}Q^2\mathrm{d}\phi_{l_\perp}\mathrm{d}z\mathrm{d}\boldsymbol{q}^2\mathrm{d}^2\Delta} = \frac{\alpha_s^2\alpha_{\mathrm{em}}^2 e_q^2 y}{2\pi^3 N_c Q^2}\xi\frac{h_l h_p|\boldsymbol{l}||\Delta|\cos(\phi_{l_\perp}-\phi_{\Delta_\perp})}{(W^2+Q^2)(-\boldsymbol{q}^2+W^2/4)(\boldsymbol{q}^2+Q^2/4)^2}$$
$$\times\,\mathrm{Re}\left(\mathcal{H}_g^{(1)*}\widetilde{\mathcal{H}}_g^{(2)}-\mathcal{H}_g^{(1)*}\mathcal{L}_g-\frac{4\boldsymbol{q}^2}{\boldsymbol{q}^2+z\bar{z}Q^2}\mathcal{H}_g^{(2)*}\mathcal{L}_g\right)$$

Compton form factors for helicity ($\widetilde{\mathcal{H}}$) and for OAM ($\mathcal{L}$).

Positive or negative interference depending on $\boldsymbol{q}^2$ vs $Q^2$
$\Rightarrow$ insight on their interplay

## Cross section estimates

### Numerical estimation

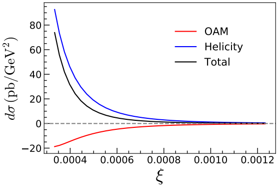
- $H_g(x,\xi)$, $\tilde{H}_g(x,\xi)$: double distribution ansatz $\Leftrightarrow xg(x), x\Delta g(x)$
- $xg(x), x\Delta g(x)$: use known PDF sets, such as JAM
- $L_g(x,\xi)$: double distribution ansatz $L_g(x)$
- $L_g(x)$: known in terms of $xg(x)$ and $x\Delta g(x)$ in the WW approximation

$$L_g(x) \simeq x \int_x^1 \frac{\mathrm{d}x'}{x'^2} \left[x'g(x') - 2\Delta g(x')\right]$$

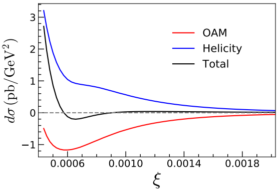
[Hatta, Yoshida]

# Cross section estimates

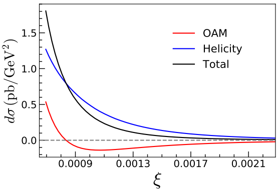
## Numerical estimation




$Q^2 = 2.7\mathrm{GeV}^2$




$Q^2 = 4.8\mathrm{GeV}^2$




$Q^2 = 10\mathrm{GeV}^2$

$$\left.\frac{d\sigma}{\mathrm{d}y\mathrm{d}Q^2\mathrm{d}z\mathrm{d}\xi\mathrm{d}\delta\phi}\right|_{\delta\phi\simeq 0, y=0.7}$$

# Summary

## Conclusions

- GTMDs provide an intuitive definition of the OAM
- One GTMD in particular defines it
- Longitudinal SSA and DSA can constrain this GTMD
- DSA in exclusive DIS dijet probes both OAM and helicity