Probing the odderon through η_c production in diffractive collisions at the EIC

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Benić, Horvatić, Kaushik, Vivoda, 2306.10626

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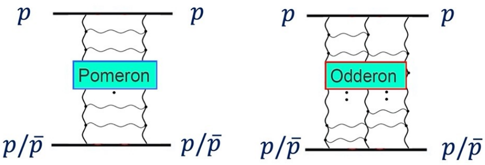
Odderon

C-odd counterpart of the C-even pomeron: a colour-neutral t-channel strong exchange.

- First suggested as mechanism to explain differences in pp vs $p\bar{p}$ elastic cross-sections

  Łukaszuk, Nicolescu, LNC 8 (1973) 40

- At lowest order in QCD, exchange of three gluons in a colour singlet state: $d^{abc}A^a_\mu A^b_\nu A^c_\lambda$



https://blog.hip.fi/the-discovery-of-the-odderon/

- Recent results by TOTEM and D0 indicate a non-zero odderon

  TOTEM, D0, PRL 127 (2021) 6, 062003

How do we understand it in a perturbative QCD framework?

Exclusive η_c production: $ep \to e + p + \eta_c$

Production of C-even mesons in ep collisions offers a clean environment to probe the odderon

- In particular, η_c (1S, $J^{PC} = 0^{-+}$) has been suggested as a golden probe: η_c has $C = +1$, photon has $C = -1$, therefore strong exchange should have $C = -1$.
- Charm quark production ensures sensitivity to gluon content of proton.
- So far η_c production not observed at HERA or JLab. Could be measured at the Electron-Ion Collider.

  Null result from HERA for π^0 as well.

  H1, PLB 544 (2002) 35-43

Exclusive η_c production: $ep \to e + p + \eta_c$

Lots of work done on this probe:

Czyzewski, Kwiecinski, Motyka, Sadzikowski, PLB 398 (1997) 400 [Erratum PLB 411 (1997) 402]
Engel, Ivanov, Kirschner, and Szymanowski, EPJC 4 (1998) 93
Bartels, Braun, Colferai, Vacca, EPJC 20 (2001) 323
Dumitru, Stebel, PRD 99 (2019) 094038
...

- Studies so far focused on dilute regime, moderate-x, gluon density not too large
- Newer calculations sugggest far smaller differential cross-sections than older calculations: $d\sigma/d|t| \sim O(\text{fb}/\text{GeV}^2)$ vs $O(\text{pb}/\text{GeV}^2)$

In this work:

- We focused on the dense regime, small-x, where gluon density is larger and saturation effects may be relevant
- We considered nuclear targets as well, which can be studied at the EIC and which again offer a dense gluon environment

Odderon in a CGC framework

Small-x regime, dense target $\Longrightarrow$ Colour-Glass condensate framework

- Gluon distributions are given through correlators of Wilson lines

$$V(\mathbf{z}_\perp) = \mathcal{P}\exp\left\{ig\int dz^- A^+(z^-,\mathbf{z}_\perp)\right\}$$

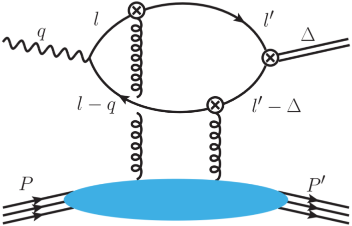
- Odderon is the imaginary parton of the dipole distribution,

$$\mathcal{O}(\mathbf{x}_\perp,\mathbf{y}_\perp) \equiv -\frac{1}{2iN_c}\mathrm{tr}\langle V(\mathbf{x}_\perp)V^\dagger(\mathbf{y}_\perp) - V(\mathbf{y}_\perp)V^\dagger(\mathbf{x}_\perp)\rangle$$

Kovchegov, Szymanowski, Wallon, PLB 586, 267 (2004)
Hatta, Iancu, Itakura, McLerran, Nucl.Phys.A 760 (2005) 172-207

- Energy evolution given by JIMWLK equations, reduces to coupled BK equations for the odderon and the pomeron in the large N_c limit

Calculating η_c production in a CGC framework



$$\mathcal{S}_\lambda = (eq_c)\int_{ll'} \mathrm{Tr}\left[S(l)\not\epsilon(\lambda,q)S(l-q)\tau(l-q,l'-\Delta)S(l'-\Delta)(i\gamma_5)S(l')\tau(l',l)\right]$$

- CGC vertex:
  $\tau(p,p') = (2\pi)\delta(p^- - p'^-)\gamma^-\mathrm{sgn}(p^-)\int_{\mathbf{z}_\perp} e^{-i(\mathbf{p}_\perp - \mathbf{p}'_\perp)\cdot\mathbf{z}_\perp} V^{\mathrm{sgn}(p^-)}(\mathbf{z}_\perp)$

Calculating η_c production in a CGC framework

After some algebra,

$$\langle \mathcal{S}_\lambda \rangle = -\langle \mathcal{M}_\lambda \rangle (2\pi)\delta(q^- - \Delta^-)$$

$$\langle \mathcal{M}_\lambda \rangle = (eq_c) \int_{\mathbf{r}_\perp} \int_{ll'} (2\pi)\delta(l^- - l'^-)\theta(l^-)\theta(q^- - l^-) \mathrm{e}^{-\mathrm{i}(\boldsymbol{l}'_\perp - \boldsymbol{l}_\perp - \frac{1}{2}\boldsymbol{\Delta}_\perp)\cdot \mathbf{r}_\perp}$$
$$\times (-\mathrm{i}N_c)\mathcal{O}(\mathbf{r}_\perp, \boldsymbol{\Delta}_\perp) \mathrm{tr}\left[S(l)\not{\epsilon}(\lambda, q)S(l-q)\gamma^- S(l'-\Delta)(\mathrm{i}\gamma_5)S(l')\gamma^-\right] ,$$

$$\mathbf{r}_\perp = \mathbf{x}_\perp - \mathbf{y}_\perp, \qquad \boldsymbol{b}_\perp = \frac{\mathbf{x}_\perp + \mathbf{y}_\perp}{2}$$

- “Boosted Gaussian” for nonperturbative scalar part of η_c wavefunction:

$$\psi^{\eta_c}(r_\perp, z) \propto \frac{\bar{u}(r_\perp, z)}{\sqrt{z}}(\mathrm{i}\gamma^5)\frac{v(r_\perp, z)}{\sqrt{1-z}}\phi^{\mathcal{P}}(r_\perp, z)$$

$$\phi^{\mathcal{P}}(r_\perp, z) = \mathcal{N}_P z\bar{z} \exp\left(-\frac{m_c^2 \mathcal{R}_\mathcal{P}^2}{8z\bar{z}} - \frac{2z\bar{z}r_\perp^2}{\mathcal{R}_\mathcal{P}^2} + \frac{1}{2}m_c^2\mathcal{R}_\mathcal{P}^2\right)$$

Dumitru, Stebel, PRD 99 (2019) 9, 094038

Calculating η_c production in a CGC framework

General features of amplitude:

- Longitudinal polarisation $\lambda = 0$ decouples, only transverse photon $\lambda = \pm 1$ contributes

$$\langle \mathcal{M}_\lambda \rangle = q^- \lambda e^{i\lambda\phi_\Delta} \lambda \langle \mathcal{M} \rangle$$

Polarisation independent part of amplitude:

$$\langle \mathcal{M} \rangle = 8\pi i e q_c N_c \sum_{k=0}^{\infty} (-1)^k \int_z \int_0^\infty r_\perp dr_\perp \mathcal{O}_{2k+1}(r_\perp, \Delta_\perp) \times \mathcal{A}(r_\perp) \left[J_{2k}(r_\perp \delta_\perp) - \frac{2k+1}{r_\perp \delta_\perp} J_{2k+1}(r_\perp \delta_\perp) \right] .$$

$$\mathcal{O}(\mathbf{r}_\perp, \mathbf{\Delta}_\perp) = 2 \sum_{k=0}^{\infty} \mathcal{O}_{2k+1}(r_\perp, \Delta_\perp) \cos((2k+1)\phi_{r\Delta})$$

- Amplitude proportional to m_c:

$$\mathcal{A}(r_\perp) = -\frac{\sqrt{2} m_c}{2\pi} \frac{1}{z\bar{z}} \left[K_0(\epsilon r_\perp) \partial_{r_\perp} \phi_{\mathcal{P}}(z, r_\perp) - \epsilon K_1(\epsilon r_\perp) \phi_{\mathcal{P}}(z, r_\perp) \right] .$$

γ splits into a spin 1 $q\bar{q}$ state which transitions to an spin 0 meson $\rightarrow$ spin flip provided by m_c

Accounting for small-x effects: BK equation

The Balitsky-Kovchegov equation describes the small-x evolution of the dipole distribution:

$$\frac{\partial \mathcal{D}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2} \int_{\boldsymbol{r}_{1\perp}} \frac{\mathbf{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2} \left[\mathcal{D}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_{1\perp}) \mathcal{D}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_{2\perp}) - \mathcal{D}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) \right]$$

$$\boldsymbol{r}_{2\perp} = \mathbf{r}_\perp - \boldsymbol{r}_{1\perp}$$

$$\mathcal{D}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) \equiv \frac{1}{N_c} \mathrm{tr} \left\langle V\left(\boldsymbol{b}_\perp + \frac{\mathbf{r}_\perp}{2}\right) V^\dagger\left(\boldsymbol{b}_\perp - \frac{\mathbf{r}_\perp}{2}\right) \right\rangle = 1 - \mathcal{N}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) + i\mathcal{O}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)$$

BK nonlocal in $\boldsymbol{b}_\perp$: $\boldsymbol{b}_{1\perp} = \boldsymbol{b}_\perp + (\mathbf{r}_\perp - \boldsymbol{r}_{1\perp})/2\,,\ \boldsymbol{b}_{2\perp} = \boldsymbol{b}_\perp - \boldsymbol{r}_{1\perp}/2$
and Odderon explicitly depends on $\boldsymbol{b}_\perp$

- In principle, we need to solve the fully impact parameter dependent BK
- In practice, we treat impact parameter $\boldsymbol{b}_\perp$ as an external parameter
  Lappi, Mäntysaari, PRD 88 (2013) 114020

$$\boldsymbol{r}_{1\perp},\ \boldsymbol{r}_{2\perp} << \boldsymbol{b}_\perp$$

BK equation

$$\frac{\partial \mathcal{N}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2} \int_{\boldsymbol{r}_{1\perp}} \frac{\mathbf{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2} \left[\mathcal{N}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp) + \mathcal{N}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{N}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) + \mathcal{N}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp)\mathcal{N}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{O}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp)\right],$$

$$\frac{\partial \mathcal{O}(\mathbf{r}_\perp, \boldsymbol{b}_\perp)}{\partial Y} = \frac{\alpha_S N_c}{2\pi^2} \int_{\boldsymbol{r}_{1\perp}} \frac{\mathbf{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2} \left[\mathcal{O}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp) + \mathcal{O}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{O}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) - \mathcal{N}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp) - \mathcal{O}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_\perp)\mathcal{N}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_\perp)\right].$$

Kovchegov, Szymanowski, Wallon, PLB 586, 267 (2004)
Hatta, Iancu, Itakura, McLerran, NPA 760 (2005) 172-207
Lappi, Ramnath, Rummukainen, Weigert, PRD 94, 054014 (2016)
Yao, Hagiwara, Hatta, PLB 790 (2019) 361-366

- Odderon and pomeron evolution coupled by nonlinear terms

Small $r_\perp$ limit: system decouples, odderon exponentially suppressed

$$\mathcal{O} \sim \exp(-cY)$$

Large $r_\perp$ limit: $\mathcal{N}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) \to 1$, nonlinear terms result in exponential suppression

$$\mathcal{O} \sim \exp(-cY)$$

In numerical computations we replace $\frac{\alpha_S N_c}{2\pi^2} \frac{\mathbf{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2}$ by Balitsky's prescription for the running coupling kernel.

Initial conditions

For pomeron, we use a fit to HERA data,

$$\mathcal{N}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) = 1 - \exp\left[-\frac{1}{4}\mathbf{r}_\perp^2 A T_A(\boldsymbol{b}_\perp)\frac{\sigma_0}{2} Q_{S,0}^2 \log\left(\frac{1}{r_\perp \Lambda_{\mathrm{QCD}}} + e_c \mathrm{e}\right)\right]$$

Lappi, Mäntysaari, PRD 88 (2013) 114020

Woods-Saxon transverse profile:

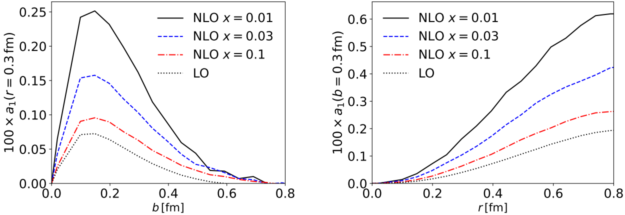
$$T_A(\boldsymbol{b}_\perp) = \int_{-\infty}^{\infty} \mathrm{d}z \frac{n_A}{1 + \exp\left[\frac{\sqrt{\boldsymbol{b}_\perp^2 + z^2} - R_A}{d}\right]}$$

Initial conditions

For odderon, depending on the target,

1. **DMP**: For proton, we use a recent light-front NLO calculation of the odderon by Dumitru, Mäntysaari and Paatelainen

   Dumitru, Mäntysaari, Paatelainen, PRD 107 (2023) 1, L011501



- Initial $x = 0.01$ (black curve)
- Odderon peak lies well within the proton $\sim\ 0.25 \times R_p$

Initial conitions

2. **JV**: For nuclear targets, we adopt the Jeon-Venugopalan model with impact parameter dependence introduced

$$W[\rho] = \exp\left[-\int_{\mathbf{x}_\perp}\left(\frac{\delta_{ab}\rho^a\rho^b}{2\mu^2} - \frac{d_{abc}\rho^a\rho^b\rho^c}{\kappa}\right)\right]$$

where $\mu^2 = \frac{g^2}{2}\frac{A}{\pi R_A^2}$, $\kappa = g^3 N_c \frac{A^2}{(\pi R_A^2)^2}$.

Jeon, Venugopalan, PRD 71 (2005) 125003

Odderon from the above functional:

$$\mathcal{O}(\mathbf{x}_\perp, \mathbf{y}_\perp) = -g^3 C_{3F}\frac{\mu^6}{\kappa}\Theta(\mathbf{x}_\perp, \mathbf{y}_\perp)\exp\left[-\frac{g^2 C_F \mu^2}{2}\Gamma(\mathbf{x}_\perp, \mathbf{y}_\perp)\right],$$

where

$$C_F = \frac{N_c^2 - 1}{2N_c}, \qquad C_{3F} = \frac{(N_c^2-1)(N_c^2-4)}{4N_c^2},$$

and

$$\Gamma(\mathbf{x}_\perp, \mathbf{y}_\perp) = (\pi R_A^2)\int_{\mathbf{z}_\perp} T_A(\mathbf{z}_\perp)\left[G(\mathbf{x}_\perp - \mathbf{z}_\perp) - G(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^2,$$

$$\Theta(\mathbf{x}_\perp, \mathbf{y}_\perp) = (\pi R_A^2)\int_{\mathbf{z}_\perp} T_A(\mathbf{z}_\perp)\left[G(\mathbf{x}_\perp - \mathbf{z}_\perp) - G(\mathbf{y}_\perp - \mathbf{z}_\perp)\right]^3,$$

$G(\mathbf{x}_\perp - \mathbf{z}_\perp) = \int_{k_\perp} \frac{e^{-ik_\perp\cdot(\mathbf{x}_\perp - \mathbf{y}_\perp)}}{k_\perp^2 + m^2}$ is a 2D Green function.

Initial conditions

$$\mathcal{O}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) = \frac{\lambda}{8}\left[R_A \frac{\mathrm{d}T_A(\boldsymbol{b}_\perp)}{\mathrm{d}b_\perp} A^{2/3}\frac{\sigma_0}{2}\right] A^{1/2}(Q^3_{S,0} r^3_\perp)(\hat{\boldsymbol{r}}_\perp \cdot \hat{\boldsymbol{b}}_\perp) \log\left(\frac{1}{r_\perp \Lambda_{\mathrm{QCD}}} + e_c \mathrm{e}\right)$$
$$\exp\left[-\frac{1}{4}\mathbf{r}^2_\perp A T_A(\boldsymbol{b}_\perp)\frac{\sigma_0}{2} Q^2_{S,0} \log\left(\frac{1}{r_\perp \Lambda_{\mathrm{QCD}}} + e_c \mathrm{e}\right)\right],$$

- $\lambda_{\mathrm{JV}} = -\frac{3}{16}\frac{N_c^2-4}{(N_c^2-1)^2}\frac{Q^3_{S,0}A^{1/2}R_A^3}{\alpha_S^3 A^2}$
- We also explore different strengths. λ_{max} is given by a group theoretic constraint
  Lappi, Ramnath, Rummukainen, Weigert, PRD 94, 054014 (2016)

$$(4 - 3\mathcal{N}(\mathbf{r}_\perp, \boldsymbol{b}_\perp))\,\mathcal{N}^3(\mathbf{r}_\perp, \boldsymbol{b}_\perp) - 6\left(6 - 6\mathcal{N}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) + \mathcal{N}^2(\mathbf{r}_\perp, \boldsymbol{b}_\perp)\right)\mathcal{O}^2(\mathbf{r}_\perp, \boldsymbol{b}_\perp) - 3\mathcal{O}^4(\mathbf{r}_\perp, \boldsymbol{b}_\perp) \geq 0\,,$$

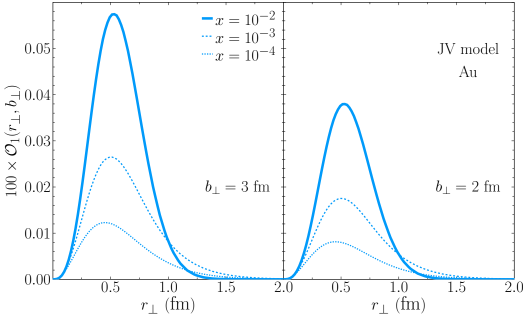
$$\lambda^{197}_{\mathrm{max}} = 1.143\lambda^{197}_{\mathrm{JV}}\,, \qquad \lambda^{63}_{\mathrm{max}} = 1.553\lambda^{63}_{\mathrm{JV}}\,, \qquad \lambda^{27}_{\mathrm{max}} = 2.26\lambda^{27}_{\mathrm{JV}}$$

Solutions of BK evolution

- Negligible higher harmonics induced in the odderon by non-linear terms

  Yao, Hagiwara, Hatta PLB 790 (2019) 361 Motyka, PLB 637 (2006) 185

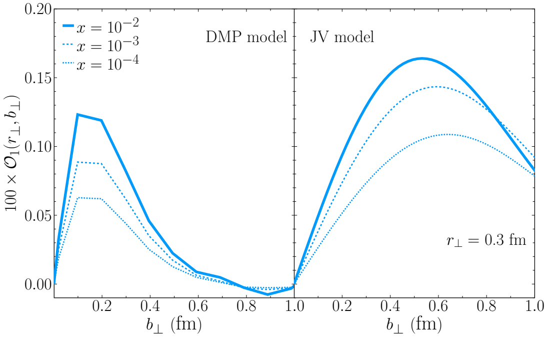
$$\mathcal{O}(\mathbf{r}_\perp, \boldsymbol{b}_\perp) = \mathcal{O}_1(r_\perp, b_\perp)\cos(\phi_{rb}) + \mathcal{O}_3(r_\perp, b_\perp)\cos(3\phi_{rb}) + \ldots$$



- Odderon decreases significantly with evolution
- Peak position around $r_\perp \sim Q_S$. Changes slowly with evolution
- No geometric scaling

  Yao, Hagiwara, Hatta PLB 790 (2019) 361
  Motyka, PLB 637 (2006) 185 Hagiwara, Hatta, Ueda PRD 94 (2016) 094036

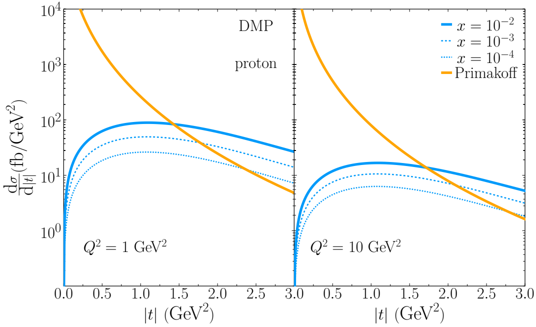
Solutions of BK evolution



- Peak position dictated by $\frac{dT_A(\boldsymbol{b}_\perp)}{db}$ in JV model, close to the edge of the system, increases slowly with evolution
  $\rightarrow$ Gluon radius $\uparrow$ as x $\downarrow$

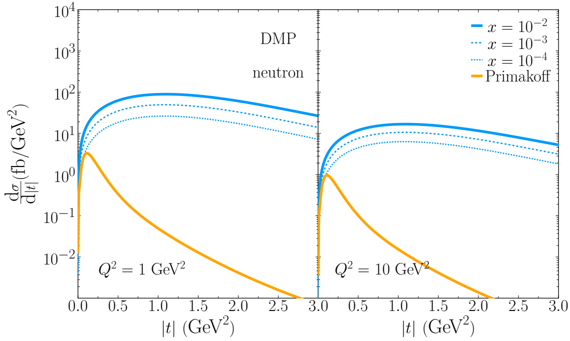
Results: $ep \to e + \eta_c + p$ with DMP odderon

Important QED background: Primakoff process. Photon ($C = -1$) from proton can also result in η_c. Can be calculated from well known electromagnetic charge form factor.



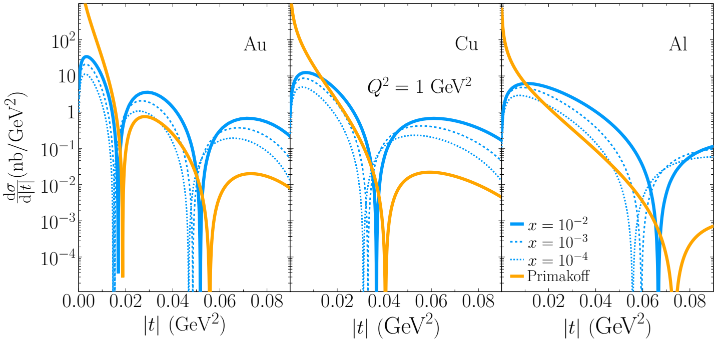
- Odderon contribution has rather small slope in $|t|$ since the odderon peak is well within the proton
- Primakoff contribution dominates at small $|t|$. Need $|t| \gtrsim 1.5$ GeV^2 to access odderon
- Similar to earlier results by Dumitru and Stebel
  Dumitru, Stebel, PRD 99 (2019) 094038

Results: $en \to e + \eta_c + n$ with DMP odderon



- Primakoff contribution negligible
- Odderon accesible even at low momentum transfers
- In practice, could be done with deuteron or He^3 target with spectator proton tagging in the near forward region
  CLAS, PRL 108, 142001 (2012)
  Friscic et al., PLB 823, 136726 (2021)

Results: $eA \to e + \eta_c + A$ with JV odderon



- Diffractive patterns of geometric origin (c.f. leading twist estimates)
- Multiple scattering effects $\implies$ diffractive dips shifted to smaller $|t|$ w.r.t Primakoff case
- Shifts more pronounced as $x \downarrow$ or $|t| \uparrow$
- $d\sigma/d|t|$ upto 10 nb/GeV^2 with $\lambda = \lambda_{\text{max}}$ / as low as 5 pb/GeV^2 with $\lambda = 0.026\lambda_{\text{JV}}$ (normalisation set by DMP vs JV amplitude ratio)

Leading twist estimates

- Odderon:

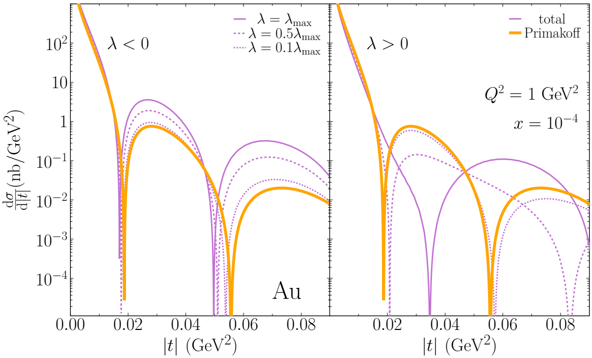
$$\frac{d\sigma}{d|t|} \propto |t| T_A^2(\sqrt{|t|}) \, .$$

- QED (Primakoff):

$$\frac{\mathrm{d}\sigma}{\mathrm{d}|t|} \propto \frac{\mathcal{T}_A^2(\sqrt{|t|})}{|t|}$$

We use same Woods-Saxon for both nuclear electromagnetic distribution and strong interaction distribution. Experiment and theory indicate strong radii are somewhat larger.
STAR, Sci. Adv. 9, eabq3903 (2023)
Mantysaari, Salazar, Schenke, PRD 106, 074019 (2022)

Results: $eA \to e + \eta_c + A$ with JV odderon



Odderon and Primakoff contributions will interfere:

1. $\lambda < 0$ ($\mathcal{O}(r_\perp, b_\perp) > 0$ as in JV and DMP): Interference is mostly constructive. Depending on size of λ, Odderon can shift diffractive pattern relative to Primakoff component.
2. $\lambda > 0$: Interference destrucutive. Depending on size of λ, Odderon can severely distort diffractive pattern relative to Primakoff pattern.

Conclusions

For proton target:

- Isolating odderon requires large momentum transfer $|t| \gtrsim 1.5\text{-}3\ \text{GeV}^2$ for $x \sim 10^{-2} - 10^{-4}$.
- Similar to conclusions drawn for the dilute regime.
- Small-x evolution does not alter $|t|$ slope, but cross-section reduces in magnitude.

For neutron target:

- Negligible Primakoff component. Can probe odderon at low $|t|$.
- Feasible at EIC for He^3 targets with spectator protons tagged in the near forward direction.

For nuclear targets:

- Saturation effects in Odderon distribution shift/distort diffractive pattern w.r.t known QED contributions
- Effect $\sim$ few percent and accumulates for small-x/large momentum transfers.

Thank you!