Odderon Mechanism for Transverse Single Spin Asymmetry in pp and pA collisions

Eric Andreas Vivoda (University of Zagreb, Faculty of Science) Workshop: REVESTRUCTURE workshop, Zagreb, July 10.-12. 2023.







Transverse Single Spin Asymmetry

- Left-right asymmetry of produced particles in collisions involving polarized hadrons
- Known to be largest in forward region \rightarrow small x effects in target



 Spin vector comes with a factor of *i*, so to make cross section real, one has to find another factor of *i* from diagrams!

A.V. Eferemov and O.V. Teryaev, Sov. J. Nucl. Phys. 36, 140 (1982).

S. Benić talk

Transverse Single Spin Asymmetry

- Left-right asymmetry of produced particles in collisions involving polarized hadrons
- Known to be largest in forward region \rightarrow small x effects in target



 Spin vector comes with a factor of *i*, so to make cross section real, one has to find another factor of *i* from diagrams!

A.V. Eferemov and O.V. Teryaev, Sov. J. Nucl. Phys. 36, 140 (1982).

S. Benić talk

TSSA in pA (data):



PHENIX Collaboration, C. Aidala et. al., Phys.Rev.Lett. **123**, 122001 (2019).

Star Collaboration, J. Adam et. al., Phys.Rev.D. **103**, 072005 (2021).

 $1.8 < P_{hT} < 7.0$ GeV (integrated)

 $0.004 \le x \le 0.1$

$$A_N \sim A^{-\frac{1}{3}}$$



 $2.0 < P_{hT} < 3.0 \text{ GeV}...$

x < 0.005

 $A_N \sim A^{-0.027 \pm 0.005}$

TSSA in pA (data):



PHENIX Collaboration, C. Aidala et. al., Phys.Rev.Lett. 123, 122001 (2019).

Star Collaboration, J. Adam et. al., Phys.Rev.D. 103, 072005 (2021).

A dependance! $0.004 \le x \le 0.1$ $A_N \sim A^{-\frac{1}{3}}$



CGC in pA collisions:

- Small x in nuclei (Color Glass Condensate (CGC) framework)
 - > Effects of small-x and spin physics combined!
 - Small x gluons dominate in the description of collision



$$\Gamma_{gg \to g} = \frac{\alpha_S N_C}{(N_C^2 - 1)Q^2} \frac{x f_g(x, Q^2)}{\pi R^2} \approx 1 \longrightarrow Q_S \text{-Saturation scale}$$

$$(Q_S^A)^2 = A^{\frac{1}{3}} (Q_S^p)^2 \longrightarrow A \text{ dependance!}$$

F. Gelis, 1211.3327





CGC in pA collisions:

- Small x in nuclei (Color Glass Condensate (CGC) framework)
 - > Effects of small-x and spin physics combined!
 - Small x gluons dominate in the description of collision



$$\Gamma_{gg \rightarrow g} = \frac{\alpha_{S}N_{C}}{(N_{C}^{2} - 1)Q^{2}} \xrightarrow{xf_{g}(x,Q^{2})}{\pi R^{2}} \approx 1 \longrightarrow Q_{S}$$
- Saturation scale

$$Q_{S}^{A} = \frac{(Q_{S}^{A})^{2}}{(Q_{S}^{A})^{2}} = A^{\frac{1}{3}}(Q_{S}^{p})^{2} \longrightarrow A$$
 dependance!

$$(Q_{S}^{A})^{2} = A^{\frac{1}{3}}(Q_{S}^{p})^{2} \longrightarrow A$$
 dependance!

$$(V(x_{\perp}) - 1) \longrightarrow Wilson line$$

ETQS mechanism in pA

k

dood

 k_1

Hybrid approach:

- Eferemov-Teryaev-Qiu-Sterman (ETQS) functions for projectile
- Unintegrated gluon distribution for target

$$\frac{\mathrm{d}\sigma^{SGP}}{\mathrm{d}y_{h}\mathrm{d}^{2}P_{h\perp}} = \frac{\pi M x_{F}}{2(N_{C}^{2}-1)} \epsilon^{\alpha\beta} S_{\perp\beta} \int_{x_{F}}^{1} \frac{\mathrm{d}z}{z^{3}} D(z) \left\{ -\frac{1}{(P_{h\perp}/z)^{2}} \right. \\ \left. \times \frac{\partial}{\partial(P_{h}^{\alpha}/z)} \left(\frac{P_{h\perp}^{2}}{z^{2}} F(x_{g}, P_{h\perp}/z) \right) G_{F}(x, x) \right. \\ \left. + \frac{2P_{h\alpha}/z}{(P_{h\perp}/z)^{2}} F(x_{g}, P_{h\perp}/z) x \frac{\mathrm{d}}{\mathrm{d}x} G_{F}(x, x) \right\}$$

Y. Hatta, B.-W. Xiao, S. Yoshida and F. Yuan, Phys. Rev. D 94, 054013 (2016).J. W. Qiu and G.F. Sterman, Phys. Rev. Lett. 67, 2264 (1991).

A.V. Efremov and O.V. Teryaev, Phys. Lett. B **150**, 383 (1985).

SGP = Soft-gluon pole

$$G_F \propto \langle \psi F \bar{\psi} \rangle$$

 $\times \int_{x_F} \frac{\mathrm{d}z}{z^2} D_{h/q}(z) G_F(x_p, x_p) \otimes F(x_g, P_{h\perp}/z)$

 $\frac{\mathrm{d}^{3}\Delta\sigma(p^{\uparrow}A \to hX)}{\mathrm{d}y_{h}d^{2}P_{h\perp}} = \epsilon^{\alpha\beta}P_{h\alpha}S_{\perp\beta}$



 $k_2 - k_1$

 k_2



Phenix data can't be explained

4

Odderon mechanism for TSSA

Odderon = imaginary part of dipole distribution function

$$\mathcal{S}(\boldsymbol{x}_{\perp}, \boldsymbol{x}'_{\perp}) \equiv \frac{1}{N_C} \operatorname{tr} \langle V(\boldsymbol{x}_{\perp}) V^{\dagger}(\boldsymbol{x}'_{\perp}) \rangle \qquad \qquad \mathcal{S}(\boldsymbol{x}_{\perp}, \boldsymbol{x}'_{\perp}) \equiv \mathcal{P}(\boldsymbol{x}_{\perp}, \boldsymbol{x}'_{\perp}) + \boldsymbol{i} \mathcal{O}(\boldsymbol{x}_{\perp}, \boldsymbol{x}'_{\perp})$$

- Wilson lines come from the CGC propagator
- Odderon can supply necessary phase!
- Calculated at parton level (qA collisions)

• We need interference diagrams to get non-zero contributions to TSSA (there is no LO contribution)

• Result:
$$A_N \propto A^{-\frac{7}{6}}$$

 $k_q \frac{d\Delta\sigma}{d^3q} \propto i\alpha_S \int_{k_\perp k_{2\perp}} \int_{r_\perp b_\perp r'_\perp} E_q \frac{d\Delta\sigma}{d^3q} \propto i\alpha_S \int_{k_\perp k_{2\perp}} \int_{r_\perp b_\perp r'_\perp} \nabla (r_\perp, b_\perp) \mathcal{P}(r'_\perp, b'_\perp) \mathcal{P}(r'_\perp, b'_\perp)$

- What will change when we go from partonic to hadronic
- What will change when we go from partonic to hadronic level?
- What is responsible PDF?

Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D 86, 034028 (2012).

 $r_{\perp} = x_{\perp} - y_{\perp}$

 $\boldsymbol{b}_{\perp} = \frac{1}{2}(\boldsymbol{x}_{\perp} + \boldsymbol{y}_{\perp})$ $\boldsymbol{b}_{\perp} - \boldsymbol{b}'_{\perp} = \frac{1}{2}(\boldsymbol{r}_{\perp} + \boldsymbol{r}'_{\perp})$

Polarized cross section in pA collisions

All order formula for twist-3 cross section:

S. Benić, Y. Hatta, H. Li, D.J. Yang, Phys. Rev. D **100**, 094027 (2019).

$$E_{h} \frac{d\Delta\sigma}{d^{3}P_{h}} = \frac{1}{2(2\pi)^{3}} \int \frac{dz_{h}}{z_{h}^{2}} D(z_{h}) \left\{ \frac{M_{N}}{2} \int dx_{p} g_{T}(x_{p}) \operatorname{Tr}[\gamma_{5} \$_{\perp} S^{(0)}(p_{1})] + \right\}$$
 Intrinsic

$$\frac{M_{N}}{2} \int dx_{p} \operatorname{Tr}\left[\left(\gamma_{5} p_{p}^{b} S_{\perp}^{\lambda} g_{1T}^{(1)}(x_{p}) + e^{\lambda \bar{n}nS_{\perp}} p_{p}^{b} f_{1T}^{(1)}(x_{p}) \right) \left(\frac{\partial S^{(0)}(k_{1})}{\partial k_{\perp}^{\lambda_{\perp}}} \right)_{k_{1}=p_{1}} \right] +$$
 Kinematica

$$\frac{iM_{N}}{4} \int dx_{p} dx_{p}^{\prime} \operatorname{Tr}\left[\left(p_{p}^{b} e^{\bar{n}n\lambda S_{\perp}} \frac{G_{F}(x_{p}, x_{p}^{\prime})}{x_{p} - x_{p}^{\prime}} + i\gamma_{5} p_{p} S_{\perp}^{\lambda} \frac{\tilde{G}_{F}(x_{p}, x_{p}^{\prime})}{x_{p} - x_{p}^{\prime}} \right) S_{\lambda}^{(1)}(x_{p}P_{p}, x_{p}^{\prime}P_{p}) \right] \right\}$$
 Dynamical

$$p_{h} \frac{g_{0}}{g_{0}} \frac{g_{0}(k_{1})}{y_{0}} \frac{p_{h}}{k_{1}} \frac{g_{0}(k_{1})}{k_{1} - k_{1}^{2}} \frac{g_{0}(k_{1},k_{1})}{k_{1} - k_{1}^{2}} \frac{g_{T}(x)}{x_{1}^{\prime}} = \int_{x}^{1} \frac{dx'}{x'} \Delta_{q}(x') + \cdots$$
Helicity quark PDF

Wandzura – Wilczek approximation

Neglect all genuine twist-3 contributions

 $xg_T(x) \approx g_{1T}^{(1)}(x) + \text{Twist-3}$

$$E_h \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3 P_h} \simeq \frac{1}{2(2\pi)^3} \frac{M_N}{2} \int \frac{\mathrm{d}z_h}{z_h^2} D(z_h) \int dx_p g_T(x_p) \times \left(S_\perp^\lambda \frac{\partial}{\partial k_{\perp}^\lambda} \operatorname{tr}[\gamma_5 k_1 S^{(0)}(k_1)]\right)_{k_1 = p_1}$$

- This is our MASTER formula
- $S^{(0)}(k_1)$ is calculated in perturbation theory
- First two contributions
 - 1. $q \rightarrow q$ Real contribution (integration over final state gluon)
 - *2.* $q \rightarrow q$ Virtual contribution

Wandzura – Wilczek approximation

Neglect all genuine twist-3 contributions

 $xg_T(x) \approx g_{1T}^{(1)}(x) + \text{Twist-3}$

$$E_h \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3 P_h} \simeq \frac{1}{2(2\pi)^3} \frac{M_N}{2} \int \frac{\mathrm{d}z_h}{z_h^2} D(z_h) \int dx_p g_T(x_p) \times \left(S_\perp^\lambda \frac{\partial}{\partial k_{1\perp}^\lambda} \operatorname{tr}[\gamma_5 \not k_1 S^{(0)}(k_1)]\right)_{k_1 = p_\perp}$$

- This is our MASTER formula
- $S^{(0)}(k_1)$ is calculated in perturbation theory
- First two contributions
 - 1. $q \rightarrow q$ Real contribution (integration over final state gluon)
 - *2.* $q \rightarrow q$ Virtual contribution



$q \rightarrow q$ Real contribution

S. Benić, D. Horvatić, A. Kaushik and E. A. Vivoda, Phys. Rev D **106**, 114025 (2022).

We need interference diagrams:

• Extraction of $S^0(k_1)$:

 $S^{(0)}(k_{1}) = \frac{q^{+}}{P_{p}^{+}} \frac{g^{2}C_{F}}{4q^{+}k_{g}^{+}} \int_{k_{g\perp}k_{\perp}k'_{\perp}} \int_{x_{\perp}x'_{\perp}y_{\perp}y'_{\perp}} e^{ik_{\perp}\cdot x_{\perp}} e^{i(k_{2\perp}-k_{\perp})\cdot y_{\perp}} \times e^{-ik'_{\perp}\cdot x'_{\perp}} e^{-i(k_{2\perp}-k'_{\perp})\cdot y'_{\perp}} d_{\mu\mu'}(k_{g})$ $\times \left[S(\boldsymbol{x}_{\perp}, \boldsymbol{x}'_{\perp}) \overline{T}_{q}^{\mu'} \not q T_{q}^{\mu} + S_{qqg}(\boldsymbol{x}'_{\perp}, \boldsymbol{x}_{\perp}, \boldsymbol{y}'_{\perp}) \overline{T}_{qg}^{\mu'}(\boldsymbol{k}'_{\perp}) \not q T_{q}^{\mu} \right]$ $+ S_{qqg}(\boldsymbol{x}'_{\perp}, \boldsymbol{x}_{\perp}, \boldsymbol{y}_{\perp}) \overline{T}_{q}^{\mu'} \not q T_{qg}^{\mu}(\boldsymbol{k}_{\perp}) + S_{qgqg}(\boldsymbol{x}'_{\perp}, \boldsymbol{y}'_{\perp}, \boldsymbol{x}_{\perp}, \boldsymbol{y}_{\perp}) \overline{T}_{qg}^{\mu'}(\boldsymbol{k}'_{\perp}) \not q T_{qg}^{\mu}(\boldsymbol{k}_{\perp})$

• After final state gluon integration (and due to C-parity) only interference terms survive, so the trace takes following form:

$$\operatorname{tr}[\gamma_{5} \not{k}_{1} S^{(0)}(k_{1})]$$

$$= \frac{q^{+}}{P_{p}^{+}} g^{2} C_{F} \int_{k_{g\perp} k_{\perp} k_{\perp}'} \int_{x_{\perp} x_{\perp}' y_{\perp} y_{\perp}'} e^{i k_{\perp} \cdot x_{\perp}} e^{i (k_{2\perp} - k_{\perp}) \cdot y_{\perp}} e^{-i k_{\perp}' \cdot x_{\perp}'} e^{-i (k_{2\perp} - k_{\perp}') \cdot y_{\perp}'}$$

$$\times [-S_{qqg}(x_{\perp}', x_{\perp}, y_{\perp}') \mathcal{H}(k_{\perp}', k_{1\perp}) + S_{qqg}(x_{\perp}', x_{\perp}, y_{\perp}) \mathcal{H}(k_{\perp}, k_{1\perp})]$$

$$\mathcal{H}(\boldsymbol{k}_{\perp}, \boldsymbol{k}_{1\perp}) \equiv \frac{1}{4q^+ k_g^+} d_{\mu\mu'}(k_g) \operatorname{Tr}[\gamma_5 \boldsymbol{k}_1 \overline{T}_q^{\mu} \, q \overline{T}_{qg}^{\mu'}(\boldsymbol{k}_{\perp})]$$

$$S_{qqg}(\mathbf{x}'_{\perp}, \mathbf{x}_{\perp}, \mathbf{y}'_{\perp}) = \frac{1}{2C_F N_C} (N_C^2 S(\mathbf{y}'_{\perp}, \mathbf{x}'_{\perp}) S(\mathbf{x}_{\perp}, \mathbf{y}'_{\perp}) - S(\mathbf{x}_{\perp}, \mathbf{x}'_{\perp}))$$
 Dipole distribution

• Hard factor is easy to calculate:

$$\operatorname{tr}[\gamma_{5} k_{1} S^{(0)}(k_{1})] = i g^{2} N_{C} \frac{q^{+}}{P_{p}^{+}} \int_{k_{2\perp} k_{\perp}} \int_{x_{\perp} x_{\perp}' y_{\perp}} e^{i k_{\perp} \cdot (x_{\perp} - y_{\perp})} e^{-i k_{2\perp} \cdot (x_{\perp}' - y_{\perp})} \\ \times [\mathcal{P}(x_{\perp}, y_{\perp}) \mathcal{O}(x_{\perp}', y_{\perp}) - \mathcal{O}(x_{\perp}, y_{\perp}) \mathcal{P}(x_{\perp}', y_{\perp})] \mathcal{H}(k_{\perp}, k_{1\perp})$$

$$\mathcal{H}(\boldsymbol{k}_{\perp}, \boldsymbol{k}_{1\perp}) = 4i(\bar{z}+1) \frac{\boldsymbol{v}_{1\perp} \times \boldsymbol{v}_{2\perp}}{\boldsymbol{v}_{1\perp}^2 \boldsymbol{v}_{2\perp}^2} \longrightarrow \begin{array}{c} \text{Manifestly} \\ \text{finite} \end{array}$$

$$\boldsymbol{v}_{1\perp} \equiv \begin{vmatrix} \boldsymbol{q}_{\perp} - \bar{z} \boldsymbol{k}_{1\perp} \\ \boldsymbol{v}_{2\perp} \equiv \begin{vmatrix} \boldsymbol{q}_{\perp} - \bar{z} \boldsymbol{k}_{1\perp} \\ \boldsymbol{q}_{\perp} - \bar{z} \boldsymbol{k}_{1\perp} \end{vmatrix} - \boldsymbol{k}_{\perp}$$
 Same vectors!

There is no polarized cross section!

• To get a usual sine modulation we need a **reference vector**:

 $q_{\perp} \times S_{\perp}$

- Our proxy for S_{\perp} is $k_{1\perp}$ (because of the derivative)
- In our calculation q_{\perp} and $k_{1\perp}$ appear in unique combination:

$$q_{1\perp} = q_\perp - \bar{z}k_{1\perp}$$



> There is no polarized cross section in this channel!

$q \rightarrow q$ Virtual contribution

• Interference with leading order amplitude:



$$\boldsymbol{v}_{1\perp} \equiv y\boldsymbol{q}_{\perp} - \boldsymbol{k}_{g\perp}$$
$$\boldsymbol{v}_{2\perp} \equiv \boldsymbol{k}_{\perp} + \bar{y}\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{g\perp} - \boldsymbol{q}_{\perp}$$

$$y = \frac{z}{\bar{z}}$$

• Same manipulations as before lead to:

$$\mathcal{H}(\boldsymbol{k}_{\perp},\boldsymbol{k}_{1\perp}) = -4i(\bar{y}+1)\frac{\boldsymbol{v}_{1\perp}\times\boldsymbol{v}_{2\perp}}{\boldsymbol{v}_{1\perp}^2\boldsymbol{v}_{2\perp}^2}$$

S. Benić, D. Horvatić, A. Kaushik and E. A. Vivoda, Phys. Rev D 106, 114025 (2022).

Integration over virtual gluon transverse momentum:



Gluon initiated channels



- There is no $g \rightarrow gg$ contribution because adjoint Wilson lines are real
- In Wandzura Wilczek approximation there is no TSSA in above channels



Is there any odderon contribution?

- 1. Going beyond WW approximation
 - a) Taking the real distribution in target (pomeron) and the phase from the cut of the propagator
 - b) Principal value of the propagator and phase from the odderon

$$\frac{1}{k^2 + i\epsilon} = P \frac{1}{k^2} - i\pi\delta(k^2)$$

2. NNLO -> competing mechanisms (lensing vs. odderon)



Y.V. Kovchegov and M.G. Santiago, Phys. Rev D **102**, 014022 (2020).

Is there any odderon contribution?

- 1. Going beyond WW approximation
 - a) Taking the real distribution in target (pomeron) and the phase from the cut of the propagator
 - b) Principal value of the propagator and phase from the odderon

$$\frac{1}{k^2 + i\epsilon} = P \frac{1}{k^2} - i\pi\delta(k^2)$$

2. NNLO -> competing mechanisms (lensing vs. odderon)



Y.V. Kovchegov and M.G. Santiago, Phys. Rev D **102**, 014022 (2020).

Going beyond WW approximation

- Possible LO contribution
- In LO $S^{(0)}(k_1)$ can contribute only by Sievers term:

$$E_{h}\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^{3}P_{h}} = \frac{1}{2(2\pi)^{3}}\frac{M_{N}}{2}\int\frac{\mathrm{d}z_{h}}{z_{h}^{2}}D(z_{h})\int\mathrm{d}x_{p}\mathrm{d}x'_{p}\operatorname{Tr}\left[\epsilon^{\lambda\bar{n}nS_{\perp}}\not{p}_{p}f_{1T}^{(1)}(x_{p})\left(\frac{\partial S^{(0)}(k_{1})}{\partial k_{1\perp}^{\lambda}}\right)_{k_{1}=p_{1}}\right]$$
$$+\frac{i}{2}\left(\not{p}_{p}\epsilon^{\bar{n}n\lambda S_{\perp}}\frac{G_{F}(x_{p},x'_{p})}{x_{p}-x'_{p}}+i\gamma_{5}\not{p}_{p}S_{\perp}^{\lambda}\frac{\tilde{G}_{F}(x_{p},x'_{p})}{x_{p}-x'_{p}}\right)S_{\lambda}^{(1)}(x_{p}P_{p},x'_{p}P_{p})$$

• We need to calculate $S_{\lambda}^{(1)}(x_p P_p, x'_p P_p)$ from perturbation theory

Possible LO contributions:



Amplitudes:

$$T^{\mu}_{qg}(k_{\perp}) = q^{+}k^{+}_{g}\gamma^{\nu}(\not\!\!\!k_{1} + \not\!\!\!k)\gamma^{+} \times \frac{d^{\mu}_{\nu}(q - k_{1} - k)}{\left(k^{+}_{1}\not\!\!\!q_{\perp} - q^{+}(\not\!\!\!k_{1\perp} + \not\!\!\!k_{\perp})\right)}$$



 k'_1

000000 k_2



$$T^{\mu}_{qg}(k_{\perp}) = k_{1}^{+}k_{g}^{+}\gamma^{+}(\not q - \not k)\gamma^{\nu} \times \frac{d^{\mu}_{\nu}(k_{1} - q + k)}{\left(q^{+}k_{1\perp} + k_{1}^{+}(k_{\perp} - q_{\perp})\right)}$$

$$\mathcal{M}_{qg \to qg} = -2k_g^+ \gamma^+ \\ \times V(\mathbf{k}_\perp) U^{ab}(\mathbf{k}_{2\perp} - \mathbf{k}_\perp)$$

$$\mathcal{M}_{q \to qg}^{\mu} = ig \int_{\boldsymbol{k}_{\perp}} \int_{\boldsymbol{x}_{\perp}} \int_{\boldsymbol{y}_{\perp}} e^{i\boldsymbol{k}_{\perp} \cdot \boldsymbol{x}_{\perp}} e^{i(\boldsymbol{k}_{2\perp} - \boldsymbol{k}_{\perp}) \cdot \boldsymbol{y}_{\perp}} \\ \left[T_{q}^{\mu} t^{a} V(\boldsymbol{x}_{\perp}) + T_{qg}^{\mu}(\boldsymbol{k}_{\perp}) V(\boldsymbol{x}_{\perp}) t^{b} U^{ab}(\boldsymbol{y}_{\perp}) \right]$$



$$T_q^{\mu} = \gamma^{\mu} \frac{\not q + \not k_g}{\left(q + k_g\right)^2} \gamma^+$$

Further tasks:

1. Extract $S_{\lambda}^{(1)}(x_p P_p, x'_p P_p)$

2. Check for possible pole contribution and compare with old results

3. Calculate Odderon contribution

Conclusions:

- Odderon mechanism for TSSA at hadron level
- Wandzura Wilczek approximation (intrinsic and kinematical contribution)
- There is no TSSA at LO and NLO
- Going beyond WW approximation

THANK YOU!