The QCD shockwave approach at NLO: towards precision physics in gluonic saturation

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longstanding program, in collaboration with
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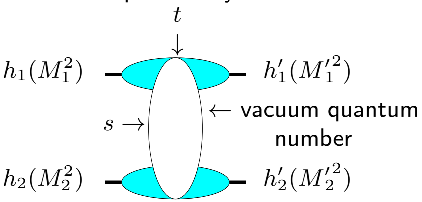
QCD in the perturbative Regge limit

- One of the important longstanding theoretical questions raised by QCD is its behaviour in the perturbative Regge limit $s \gg -t$
- Based on theoretical grounds, one should identify and test suitable observables in order to test this peculiar dynamics



hard scales: $M_1^2, M_2^2 \gg \Lambda^2_{QCD}$ or $M'^2_1, M'^2_2 \gg \Lambda^2_{QCD}$ or $t \gg \Lambda^2_{QCD}$ where the $t-$channel exchanged state is the so-called hard Pomeron

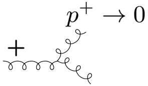
- Inclusive processes: the above picture applies at the level of cross-sections (optical theorem $\Rightarrow t = 0$)
- Diffractive processes: gap in rapidity between two clusters in the detector. The above picture applies at the level of amplitudes

How to test QCD in the perturbative Regge limit?

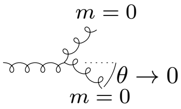
What kind of observable?

- perturbation theory should be applicable:
  selecting external or internal probes with transverse sizes $\ll 1/\Lambda_{QCD}$ (*hard* γ^*, *heavy* meson $(J/\Psi,\ \Upsilon)$, *energetic* forward jets) or by choosing large t in order to provide the hard scale.

- governed by the *rapidity divergences* of perturbative QCD



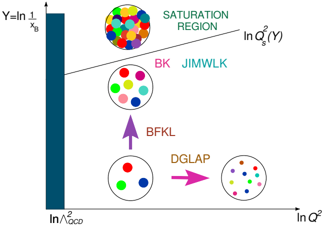
and *not* by its *collinear* dynamics



$\Longrightarrow$ select semi-hard processes with $s \gg p_{Ti}^2 \gg \Lambda_{QCD}^2$ where p_{Ti}^2 are typical transverse scale, all of the same order.

Quark and gluon content of proton

The various regimes governing the perturbative content of the proton



e.g.: DIS

- "usual" regime: x_B moderate ($x_B \gtrsim .01$):
  Evolution in Q governed by the QCD renormalization group
  (Dokshitser, Gribov, Lipatov, Altarelli, Parisi equation)

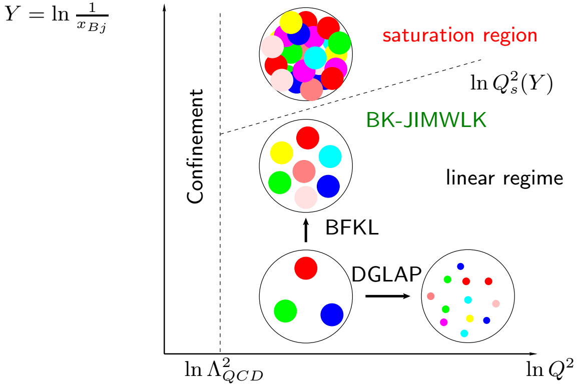
$$\underbrace{\sum_n (\alpha_s \ln Q^2)^n}_{\text{LLQ}} + \underbrace{\alpha_s \sum_n (\alpha_s \ln Q^2)^n}_{\text{NLLQ}} + \cdots$$

- perturbative Regge limit: $s_{\gamma^* p} \to \infty$ i.e. $x_B \sim Q^2/s_{\gamma^* p} \to 0$
  in the perturbative regime (hard scale Q^2)
  (Balitski Fadin Kuraev Lipatov equation)

$$\underbrace{\sum_n (\alpha_s \ln s)^n}_{\text{LLs}} + \underbrace{\alpha_s \sum_n (\alpha_s \ln s)^n}_{\text{NLLs}} + \cdots$$

Quark and gluon content of proton

The various regime governing the perturbative content of the proton



to handle with saturation effects:

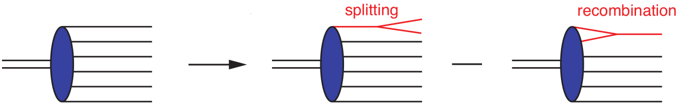
one should resum powers of $\alpha_s^2 A^{1/3}$

typical order of magnitude of dipole-dipole scattering between a dipole probe and a dipole inside a large nucleus A through 2-gluon exchange

High energy: Regge limit

Non-linear perturbative regime and Color Glass Condensate

Gluonic saturation

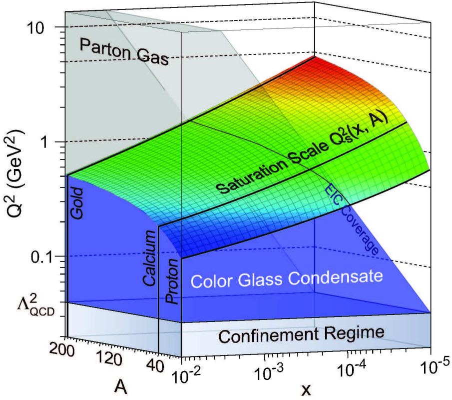


- $\alpha_s \ll 1$: weak coupling $\Rightarrow$ perturbative approach
- very dense system: very high occupation numbers
  $\Rightarrow$ gluons can recombine
- characteristic scale: saturation for $Q^2 \lesssim Q_s^2(x)$
  - number of gluons per surface unit:
    $$\rho \sim \frac{xG_A(x,Q^2)}{\pi R_A^2}$$
  - recombination cross-section:
    $$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$
  - effects are important when $\rho\,\sigma_{gg\to g} \gtrsim 1$

$$\text{i.e. } Q^2 \lesssim Q_s^2 \quad \text{with} \quad Q_s^2 \sim \frac{\alpha_s\; xG_A(x,Q_s^2)}{\pi R_A^2} \sim A^{1/3} x^{-0.3}$$

Gluonic saturation
Experimental future

Gluonic saturation with a perturbative control



- At EIC, the saturation scale Q_s will be in the perturbative range

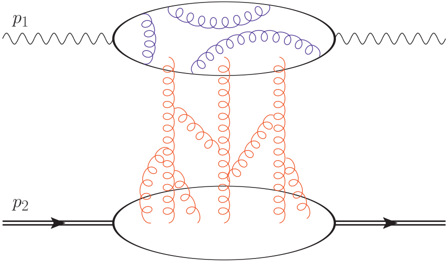
$$Q_s^2 \sim \left(\frac{A}{x}\right)^{1/3}$$

  - Moderate center of mass energy
  - Compensated by large A
- Large perturbative domain

$$\Lambda_{QCD}^2 \ll Q^2 \ll Q_s^2$$

in which saturation is under control

Kinematics



$$p_1 = p^+ n_1 - \frac{Q^2}{2p^+} n_2$$

$$p_2 = \frac{m_t^2}{2p_2^-} n_1 + p_2^- n_2$$

$$p^+ \sim p_2^- \sim \sqrt{\frac{s}{2}}$$

Lightcone Sudakov vectors

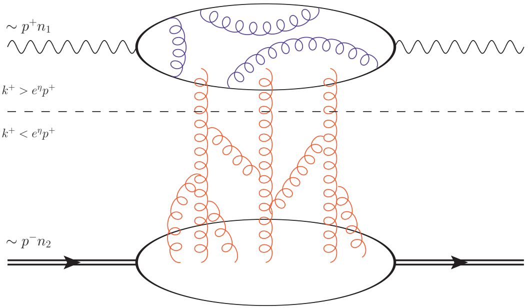
$$n_1 = \sqrt{\frac{1}{2}}(1, 0_\perp, 1), \quad n_2 = \sqrt{\frac{1}{2}}(1, 0_\perp, -1), \quad (n_1 \cdot n_2) = 1$$

Lightcone coordinates:

$$x = (x^0, x^1, x^2, x^3) \rightarrow (x^+, x^-, \vec{x}\,)$$

$$x^+ = x_- = (x \cdot n_2) \quad x^- = x_+ = (x \cdot n_1)$$

Rapidity separation



Let us split the gluonic field between "fast" and "slow" gluons

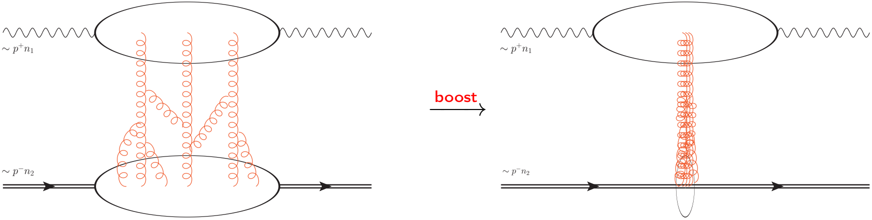
$$\mathcal{A}^{\mu a}(k^+, k^-, \vec{k}\,) = A_\eta^{\mu a}(|k^+| > e^\eta p^+, k^-, \vec{k}\,) \quad \text{quantum part}$$
$$+\, b_\eta^{\mu a}(|k^+| < e^\eta p^+, k^-, \vec{k}\,) \quad \text{classical part}$$

$e^\eta \ll 1$

$\Rightarrow$ effective field theory   I. Balitsky 1996

Large longitudinal boost to the projectile frame

Large longitudinal boost: $\Lambda \propto \sqrt{s}$



$$b^\mu(x) \quad \longrightarrow \quad b^-(x)\, n_2^\mu \simeq \delta(x^+)\, \mathbf{B}(\vec{x}\,)\, n_2^\mu$$

just like usual boost of $(\vec{E}, \vec{B})$ or A^μ in electrodynamics

Shockwave approximation

$b^-(x)\, n_2^\mu$: background field

Light-cone gauge: $n_2 \cdot A = 0$

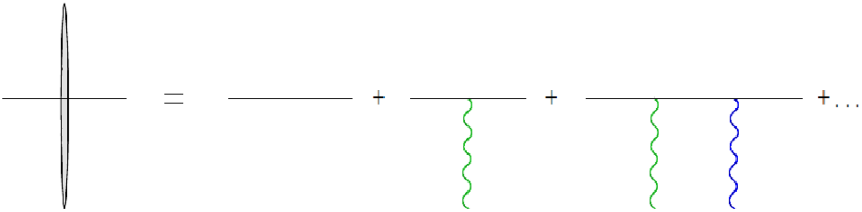
$\Rightarrow b \cdot A = 0$ which leads to simple Feynman rules in this effective field theory.

Multiple interactions with the target = Propagator in the shockwave field

Multiple interactions with the target can be resummed into path-ordered Wilson lines attached to each parton crossing lightcone time 0:

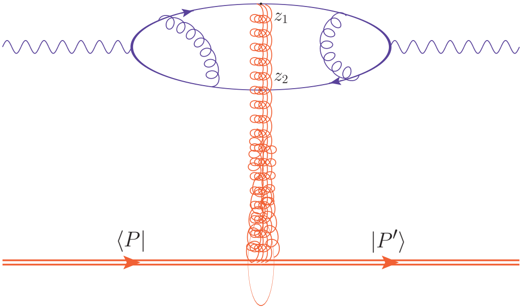
$$U_i = U_{\vec{z}_i} = U(\vec{z}_i, \eta) = P \exp\left[ig \int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i)\, dz_i^+\right]$$

$$U_i = 1 + ig \int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i) dz_i^+ + (ig)^2 \int_{-\infty}^{+\infty} b_\eta^-(z_i^+, \vec{z}_i) b_\eta^-(z_j^+, \vec{z}_j) \theta\left(z_{ji}^+\right) dz_i^+ dz_j^+ + \cdots$$



Factorized picture in the projectile frame

Factorized amplitude



$$\mathcal{A}^\eta = \int d^{D-2}\vec{z}_1 d^{D-2}\vec{z}_2\, \Phi^\eta(\vec{z}_1, \vec{z}_2)\, \langle P'|[\mathrm{Tr}(U^\eta_{\vec{z}_1} U^{\eta\dagger}_{\vec{z}_2}) - N_c]|P\rangle$$

Dipole operator $\mathcal{U}^\eta_{ij} = \frac{1}{N_c}\mathrm{Tr}(U^\eta_{\vec{z}_i} U^{\eta\dagger}_{\vec{z}_j}) - 1$

Written similarly for any number of Wilson lines in any color representation

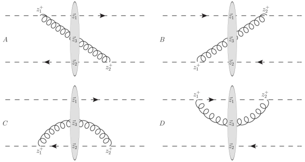
Evolution for the dipole operator

B-JIMWLK hierarchy of equations

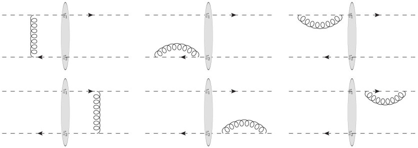
[I. Balitsky, J. Jalilian-Marian, E. Iancu, L. McLerran, H. Weigert, A. Leonidov, A. Kovner]

$$\frac{\partial \mathcal{U}^{\eta}_{12}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\mathcal{U}^{\eta}_{13} + \mathcal{U}^{\eta}_{32} - \mathcal{U}^{\eta}_{12} + \mathcal{U}^{\eta}_{13}\mathcal{U}^{\eta}_{32} \right]$$

$$\frac{\partial \mathcal{U}^{\eta}_{13}\mathcal{U}^{\eta}_{32}}{\partial \eta} = \ldots$$



double dipole contribution

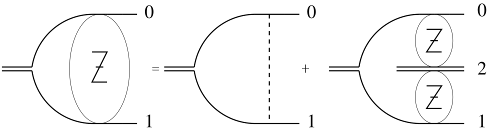


dipole contribution

Evolution for the dipole operator
Mean field approximation

Mean field approximation

$\leftrightarrow$ close connection with Mueller dipole's model 1994-1995 (large N_C)
(obtained using light-front quantization)



$\Rightarrow$ BK equation [I. Balitsky, 1995] [Y. Kovchegov, 1999]

$$\frac{\partial \langle \mathcal{U}_{12}^{\eta} \rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d\vec{z}_3 \, \frac{\vec{z}_{12}^{\,2}}{\vec{z}_{13}^{\,2}\vec{z}_{23}^{\,2}} \left[\langle \mathcal{U}_{13}^{\eta} \rangle + \langle \mathcal{U}_{32}^{\eta} \rangle - \langle \mathcal{U}_{12}^{\eta} \rangle - \langle \mathcal{U}_{13}^{\eta} \rangle \, \langle \mathcal{U}_{32}^{\eta} \rangle \right]$$

Non-linear term : saturation

The JIMWLK Hamiltonian

Hamiltonian formulation of the hierarchy of equations

For an operator built from n Wilson lines, the JIMWLK evolution is given at LO accuracy by

$$\frac{\partial}{\partial\eta}\left[U^{\eta}_{\vec{z}_1}...U^{\eta}_{\vec{z}_n}\right] = \sum_{i,j=1}^{n} H_{ij}\cdot\left[U^{\eta}_{\vec{z}_1}...U^{\eta}_{\vec{z}_n}\right],$$

JIMWLK Hamiltonian

$$H_{ij} = \frac{\alpha_s}{2\pi^2}\int d\vec{z}_k \frac{\vec{z}_{ik}\cdot\vec{z}_{kj}}{\vec{z}_{ik}^{\,2}\vec{z}_{kj}^{\,2}}[T^a_{i,L}T^a_{j,L} + T^a_{i,R}T^a_{j,R} - U^{ab}_{\vec{z}_k}(T^a_{i,L}T^b_{j,R} + T^a_{j,L}T^b_{i,R})]$$

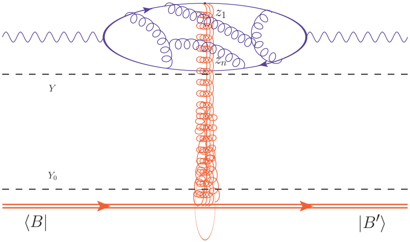
Theoretical status
Evolution equations

Kwown pieces of the evolution beyond leading accuracy

- Explicit NLO dipole operator evolution [I. Balitsky, G. Chirilli 2007]
- Explicit NLO 3-point operator evolution [I. Balitsky, Grabovsky 2014]
- Explicit NLO 4-point operator evolution [A. Grabovsky 2015]
- Complete NLO JIMWLK Hamiltonian [A. Kovner, M. Lublinsky, Y. Mulian 2013]
- Additionnal resummation of collinear logarithms [E. Iancu, J. Madrigal, A. Mueller, G. Soyez, D. Triantafyllopoulos 2015], improved kinematics [Beuf 2015]
- Progress towards a more moderate-x extension [I. Balitsky, B. Tarasov 2015]
- Progress towards "Next-to-Eikonal" and "Next-to-Next-to-Eikonal" corrections for nucleus targets [T. Altinoluk, N. Armesto, G. Beuf, M. Martinez, A. Moscoso, C. Salgado 2014]
- Extensions to spin dependent distributions [F. Cougoulic, Y. Kovchegov, B. Tarasov, Y. Tawabutr 2022]

Practical use of the formalism

- Compute the upper impact factor using the effective Feynman rules
- Build non-perturbative models for the matrix elements of the Wilson line operators acting on the target states
- Solve the B-JIMWLK evolution for these matrix elements with such non-perturbative initial conditions at a typical target rapidity Y_0.
- Evaluate the solution at a typical projectile rapidity Y, or at the rapidity of the slowest gluon
- Convolute the solution and the impact factor



$$\mathcal{A} = \int d\vec{z}_1 ... d\vec{z}_n \; \Phi(\vec{z}_1, ..., \vec{z}_n)$$
$$\times \langle P'|U_{\vec{z}_1} ... U_{\vec{z}_n}|P\rangle$$

Exclusive diffraction allows one to probe the $b_\perp$-dependence of the non-perturbative scattering amplitude

Probing QCD in the Regge limit and towards saturation

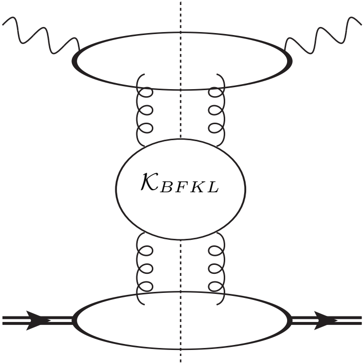
Observables to probe small$-x$ QCD and saturation physics

- Perturbation theory should apply : a hard scale Q^2 is required
- One needs semihard kinematics : $s \gg p_T^2 \gg \Lambda_{QCD}^2$ where all the typical transverse scales p_T are of the same order
- Saturation is reached when $Q^2 \sim Q_s^2 \propto \left(\frac{A}{x}\right)^{\frac{1}{3}}$: the smaller $x \sim \frac{Q^2}{s}$ is and the heavier the target ion, the easier saturation is reached.

Known NLO impact factors

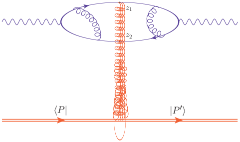
Known NLO BFKL impact factors

- $\gamma^* \to \gamma^*$ [J. Bartels, D. Colferai, S. Gieseke, A. Kyrielis, C. Qiao 2001]
- Forward jet production [J. Bartels, D. Colferai, G. Vacca 2003; F. Caporale, D. Ivanov, B. Murdaca, A. Papa, A. Perri 2011; G. Chachamis, M. Hentschinski, J. Madrigal, A. Sabio Vera 2012]
- Inclusive production of a pair of hadrons separated by a large interval of rapidity [D. Ivanov, A. Papa 2012]
- Diffractive $\gamma_L^* \to V_L$ in the forward limit [D. Ivanov, I. Kotsky, A. Papa 2004]
- Higgs production [F. G. Celiberto, D. Ivanov, M. Fucilla, M. Mohammed, A. Papa 2022]



Known NLO impact factors

Known NLO CGC impact factors



- $\gamma^* \to \gamma^*$ [I. Balitsky, G. Chirilli, 2011]; in the wave function approach [G. Beuf 2016]
- Single inclusive particle production [G. Chirilli, B.-W. Xiao, F. Yuan 2012]
- Exclusive diffractive electro- and photo- production of a forward dijet [R. Boussarie, A. Grabovsky, L. Szymanowski, S.W. 2016]
- $\gamma_{L,T}^{(*)} \to V_L$ [R. Boussarie, A. Grabovsky, D. Ivanov, L. Szymanowski, S.W. 2017]
- inclusive photon+dijet production in e+A DIS [K. Roy, R. Venugopalan 2019]
- Dijet impact factor in DIS [R. Venugopalan, F. Salazar, P. Caucal 2021]
- $\gamma^* \to \gamma^*$ with massive quarks in the wave function approach [G. Beuf, T. Lappi, R. Paatelainen 2021]
- Dijet impact factor in DIS [R. Venugopalan, F. Salazar, P. Caucal 2021]
- Semi-inclusive diffractive electro- and photo- production of a pair of hadrons at large p_T [M. Fucilla, A. Grabovsky, E. Li, L. Szymanowski, S.W. 2022]
- Semi-inclusive diffractive electro- and photo- production of a single hadron at large p_T [M. Fucilla, A. Grabovsky, E. Li, L. Szymanowski, S.W. 2023, to appear]

Practical implementation for diffractive processes

Framework

We are using the following framework:

- Regge-Gribov limit : $s \gg (\text{hard scale})^2 \gg \Lambda_{QCD}^2$

  hard scale:
  - Q^2
  - t
  - Diffractive mass of a dijet system
  - p_T of produced hadrons

- Otherwise completely general kinematics $\Rightarrow$ connection with Wigner distributions

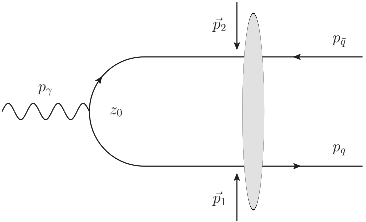
- Shockwave (CGC) Wilson line approach

- Longitudinal cutoff: $|p_g^+| > \alpha p_\gamma^+$

- Transverse dimensional regularization: $d = 2 + 2\varepsilon$

Various diffractive processes in the shockwave approach

LO open $q\bar{q}$ production



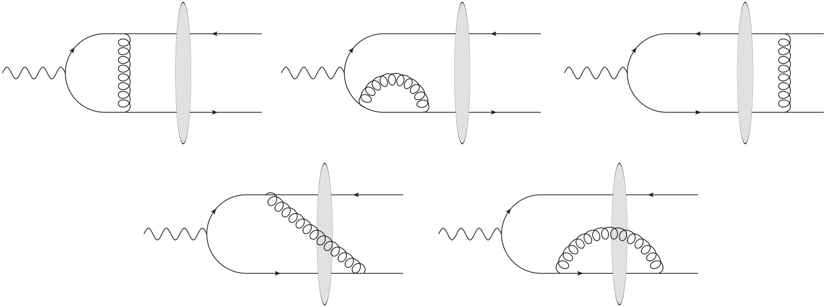
$$\mathcal{A} = \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\, \Phi_0(\vec{p}_1, \vec{p}_2)$$
$$\times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

$$\tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) = \int d^d\vec{z}_1 d^d\vec{z}_2\, e^{-i(\vec{p}_1\cdot\vec{z}_1) - i(\vec{p}_2\cdot\vec{z}_2)} [\tfrac{1}{N_c}\mathrm{Tr}(U^\alpha_{\vec{z}_1} U^{\alpha\dagger}_{\vec{z}_2}) - 1] \qquad \text{Target}$$

Various diffractive processes in the shockwave approach

NLO open $q\bar{q}$ production

Virtual corrections

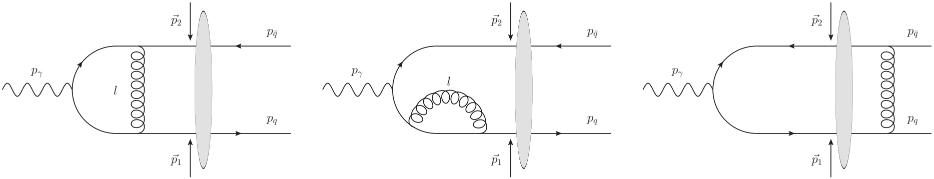


Diagrams contributing to the NLO correction

Various diffractive processes in the shockwave approach

First kind of virtual corrections

Virtual corrections (1)

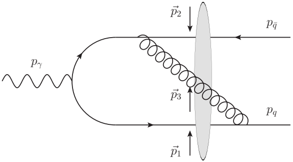


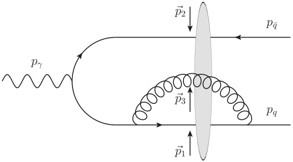
$$\mathcal{A}_{NLO}^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)\, \Phi_{V1}(\vec{p}_1, \vec{p}_2\,)$$

$$\times C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha (\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

Various diffractive processes in the shockwave approach
Second kind of virtual corrections

Virtual corrections (2)





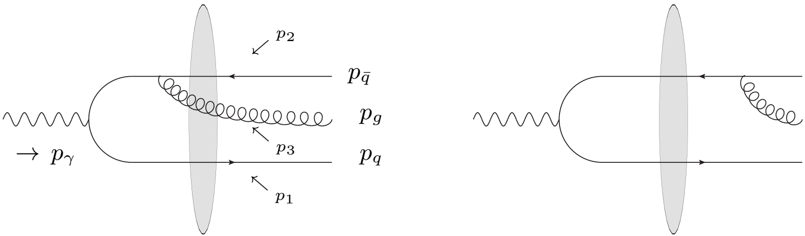
$$\mathcal{A}_{NLO}^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$

$$\times [\Phi'_{V1}(\vec{p}_1, \vec{p}_2) C_F \left\langle P' \right| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \left| P \right\rangle$$ dipole contribution

$$+\Phi_{V2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \right| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left| P \right\rangle]$$ double dipole contribution

Various diffractive processes in the shockwave approach
LO open $q\bar{q}g$ production

Real corrections



$$\mathcal{A}_R^{(2)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 d^d\vec{p}_3 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2 - \vec{p}_3)$$
$$\times [\Phi'_{R1}(\vec{p}_1, \vec{p}_2)\, C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$
$$+ \Phi_{R2}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \left\langle P' \middle| \tilde{\mathcal{W}}(\vec{p}_1, \vec{p}_2, \vec{p}_3) \middle| P \right\rangle]$$

$$\mathcal{A}_R^{(1)} \propto \delta(p_q^+ + p_{\bar{q}} + p_g^+ - p_\gamma^+) \int d^d\vec{p}_1 d^d\vec{p}_2 \delta(\vec{p}_q + \vec{p}_{\bar{q}} + \vec{p}_g - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2)$$
$$\times \Phi_{R1}(\vec{p}_1, \vec{p}_2)\, C_F \left\langle P' \middle| \tilde{\mathcal{U}}^\alpha(\vec{p}_1, \vec{p}_2) \middle| P \right\rangle$$

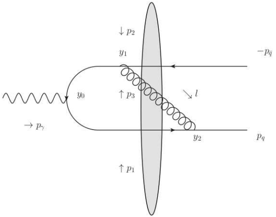
Various diffractive processes in the shockwave approach

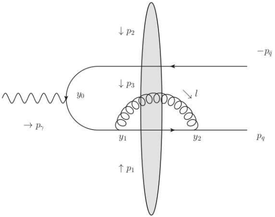
Various type of divergences

Divergences

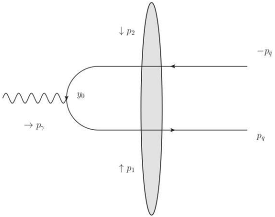
- Rapidity divergence $p_g^+ \to 0$ — $\Phi_{V2}\Phi_0^* + \Phi_0\Phi_{V2}^*$
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ — $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ — $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$, $\Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ — $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ — $\Phi_{R1}\Phi_{R1}^*$

Various diffractive processes in the shockwave approach
Rapidity divergence





Double dipole virtual correction Φ_{V2}



B-JIMWLK evolution of the LO term : $\Phi_0 \otimes \mathcal{K}_{BK}$

Various diffractive processes in the shockwave approach

Rapidity divergence

B-JIMWLK equation for the dipole operator

$$\frac{\partial \tilde{\mathcal{U}}^{\alpha}_{12}}{\partial \ln \alpha} = 2\alpha_s N_c \mu^{2-d} \int \frac{d^d \vec{k}_1 d^d \vec{k}_2 d^d \vec{k}_3}{(2\pi)^{2d}} \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_1 - \vec{p}_2) \left(\tilde{\mathcal{U}}^{\alpha}_{13} \tilde{\mathcal{U}}^{\alpha}_{32} + \tilde{\mathcal{U}}^{\alpha}_{13} + \tilde{\mathcal{U}}^{\alpha}_{32} - \tilde{\mathcal{U}}^{\alpha}_{12} \right)$$

$$\times \left[2 \frac{(\vec{k}_1 - \vec{p}_1)\cdot(\vec{k}_2 - \vec{p}_2)}{(\vec{k}_1 - \vec{p}_1)^2 (\vec{k}_2 - \vec{p}_2)^2} + \frac{\pi^{\frac{d}{2}} \Gamma(1 - \frac{d}{2}) \Gamma^2(\frac{d}{2})}{\Gamma(d-1)} \left(\frac{\delta(\vec{k}_2 - \vec{p}_2)}{\left[(\vec{k}_1 - \vec{p}_1)^2\right]^{1-\frac{d}{2}}} + \frac{\delta(\vec{k}_1 - \vec{p}_1)}{\left[(\vec{k}_2 - \vec{p}_2)^2\right]^{1-\frac{d}{2}}} \right) \right]$$

η rapidity divide, which separates the upper and the lower impact factors

$$\Phi_0 \tilde{\mathcal{U}}^{\alpha}_{12} \to \Phi_0 \tilde{\mathcal{U}}^{\eta}_{12} + 2 \ln \left(\frac{e^{\eta}}{\alpha} \right) \mathcal{K}_{BK} \Phi_0 \tilde{\mathcal{W}}_{123}$$

Provides a counterterm to the $\ln \alpha$ divergence in the virtual double dipole impact factor:

$\Phi_0 \tilde{\mathcal{U}}^{\alpha}_{12} + \Phi_{V2} \tilde{\mathcal{W}}^{\alpha}_{123}$ is finite and independent of α

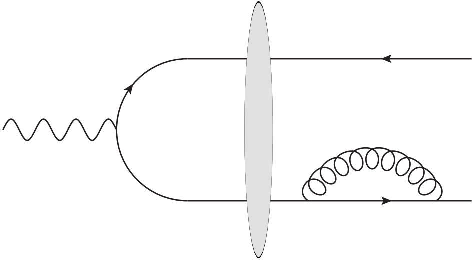
Various diffractive processes in the shockwave approach

Various type of divergences

- Rapidity divergence
- UV divergence $\vec{p}_g^{\,2} \to +\infty$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*$
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

Various diffractive processes in the shockwave approach
UV divergence

Dressing of the external lines



Some null diagrams just contribute to turning UV divergences into IR divergences

$$\Phi = 0 \propto \left(\frac{1}{2\epsilon_{IR}} - \frac{1}{2\epsilon_{UV}} \right)$$

Various diffractive processes in the shockwave approach

Various type of divergences

- Rapidity divergence
- UV divergence
- Soft divergence $p_g \to 0$ $\Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence $p_g = \frac{p_g^+}{p_q^+} p_q$ or $\frac{p_g^+}{p_{\bar{q}}^+} p_{\bar{q}}$, $p_g^+ \to 0$ $\Phi_{R1}\Phi_{R1}^*$

At this stage, the treatment of collinear and soft and collinear divergence are process dependent

3 examples

- exclusive dijet diffractive production: jet algorithm [R. Boussarie, A. Grabovsky, L. Szymanowski, S.W., JHEP 11 (2016)]
- exclusive meson diffractive production: renormalisation of the meson distribution amplitude
  [R. Boussarie, A. Grabovsky, D. Ivanov, L. Szymanowski, S.W., PRL 119 (2017)]
- semi-inclusive dihadron production: renormalisation of the parton fragmentation functions
  [M. Fucilla, A. Grabovsky, D. Ivanov, E. Li, L. Szymanowski, S.W., JHEP 03 (2023)]

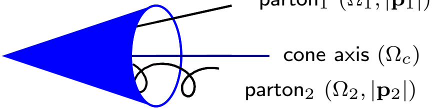
Various diffractive processes in the shockwave approach

Soft and collinear divergence: dijet case

Cone jet algorithm at NLO (Ellis, Kunszt, Soper)

- Should partons $(|\mathbf{p}_1|, \phi_1, y_1)$ and $(\mathbf{p}_2|, \phi_2, y_2)$ combined in a single jet?
  $|\mathbf{p}_i|$ =transverse energy deposit in the calorimeter cell i of parameter
  $\Omega = (y_i, \phi_i)$ in $y - \phi$ plane
- define transverse energy of the jet: $p_J = |\mathbf{p}_1| + |\mathbf{p}_2|$
- jet axis:

$$\Omega_c \left\{ \begin{array}{l} y_J = \dfrac{|\mathbf{p}_1|\, y_1 + |\mathbf{p}_2|\, y_2}{p_J} \\[2ex] \phi_J = \dfrac{|\mathbf{p}_1|\, \phi_1 + |\mathbf{p}_2|\, \phi_2}{p_J} \end{array} \right.$$



$\Omega = (y_i, \phi_i)$ in $y - \phi$ plane

If distances $|\Omega_i - \Omega_c|^2 \equiv (y_i - y_c)^2 + (\phi_i - \phi_c)^2 < R^2$ $(i = 1$ and $i = 2)$

$\Longrightarrow$ partons 1 and 2 are in the same cone Ω_c

Applying this (in the small R^2 limit) cancels our soft *and* collinear divergence

Exclusive dijet diffractive production

Various type of divergences: dijet case

Dijet case

- Rapidity divergence
- UV divergence
- Soft divergence $p_g \to 0$ $\qquad \Phi_{V1}\Phi_0^* + \Phi_0\Phi_{V1}^*, \Phi_{R1}\Phi_{R1}^*$
- Collinear divergence $p_g \propto p_q$ or $p_{\bar{q}}$ $\qquad \Phi_{R1}\Phi_{R1}^*$
- Soft and collinear divergence

The remaining divergences cancel the standard way:
virtual corrections and real corrections cancel each other

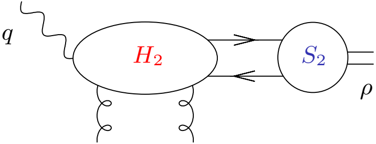
This is done after combining:

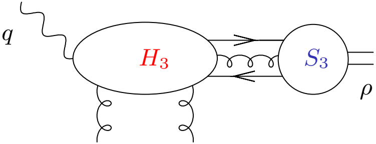
- the (LO + NLO) contribution to $q\bar{q}$ production
- the part of the contribution of the $q\bar{q}g$ production where the gluon is either soft or collinear to the quark or to the antiquark, so that they both form a single jet

Various diffractive processes in the shockwave approach

Collinear factorization: meson case

The impact factor is the convolution of a hard part and the vacuum-to-meson matrix element of an operator





$$\int_x (H_2(x))_{ij}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^{\alpha}(x)\psi_j^{\beta}(0) \right| 0 \right\rangle \quad \int_{x_1,x_2} (H_3^{\mu}(x_1,x_2))_{ij,c}^{\alpha\beta} \left\langle \rho \left| \bar{\psi}_i^{\alpha}(x_1) A_{\mu}^{c}(x_2) \psi_j^{\beta}(0) \right| 0 \right\rangle$$

H and S are connected by:

- convolution
- summation over spinor and color indices

Once factorization in the t channel is done, now factorize in the s channel with collinear factorization: expand the impact factor in powers of the hard scale

Collinear factorization at twist 2

- Leading twist DA for a longitudinally polarized light vector meson

$$\langle \rho \left| \bar{\psi}(z) \gamma^{\mu} \psi(0) \right| 0 \rangle \to p^{\mu} f_{\rho} \int_{0}^{1} dx e^{ix(p\cdot z)} \varphi_{1}(x)$$

- Leading twist DA for a transversely polarized light vector meson

$$\langle \rho \left| \bar{\psi}(z) \sigma^{\mu\nu} \psi(0) \right| 0 \rangle \to i(p^{\mu} \varepsilon_{\rho}^{\nu} - p^{\nu} \varepsilon_{\rho}^{\mu}) f_{\rho}^{T} \int_{0}^{1} dx e^{ix(p\cdot z)} \varphi_{\perp}(x)$$

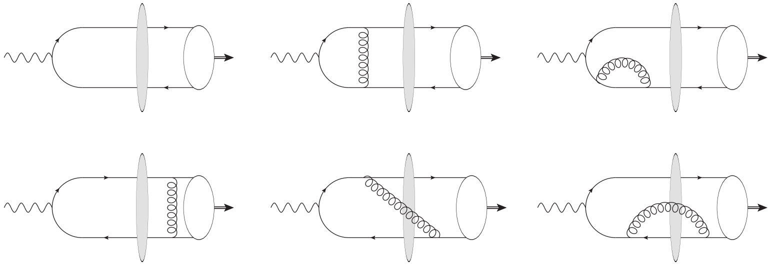
The twist 2 DA for a transverse meson is chiral odd, thus $\gamma^* A \to \rho_T A$ starts at twist 3

Various diffractive processes in the shockwave approach

Collinear factorization: meson case

NLO Deep Virtual Meson Production

with Pomeron (shockwave) exchange



Leading twist for a longitudinally polarized, C^- meson:

Make the $q\bar{q}$ pair collinear to the meson, and convolute with a Distribution Amplitude (vacuum-to-meson matrix element)

Additional divergence from the colinearity: canceled from the renormalization of the s-channel operator (ERBL evolution equation for the DA)

Probes gluon GPDs at low x, as well as twist 2 DAs

Various diffractive processes in the shockwave approach

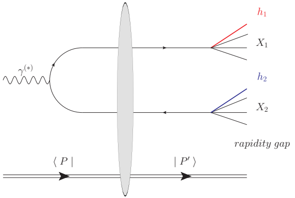
Collinear factorization: Diffractive di-hadron production

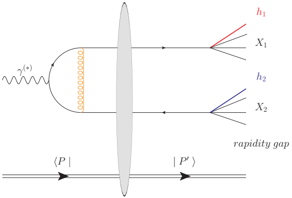
Diffractive di-hadron production at NLO

$$\gamma^{(*)}(p_\gamma) + P(p_0) \to h_1(p_{h1}) + h_2(p_{h2}) + X + P(p_0') \qquad (X = X_1 + X_2)$$

Rapidity gap between $(h_1 h_2 X)$ and $P'(p_0')$.

- General kinematics (t, Q^2) and arbitrary photon polarization: process could be either photo-production or electro-production





- Collinear factorization: Hard scale with $\Lambda^2_{QCD} \ll \vec{p}^{\,2}_{h_1} \sim \vec{p}^{\,2}_{h_2}$.

  Assume $\vec{p}^{\,2} \gg \vec{p}^{\,2}_{h_{1,2}}$ $\quad \vec{p} =$ relative transverse momentum of the two hadrons
  $\Rightarrow$ Use of single hadron fragmentation functions to describe hadronization
  $\Rightarrow$ Proof of cancellation of collinear and soft divergencies
  finite term computed analytically

- Saturation region : $\vec{p}^{\,2}_{h_1} \sim \vec{p}^{\,2}_{h_2} < Q_s^2$

Conclusion

- There have been very important progresses in the theoretical description of gluonic saturation:
  - Evolution kernels are now known at NLO
  - Many impact factor are now known at NLO
- Description of processes at a complete NLO level remains challenging in view of the complexity of the obtained analytical results:
  no full phenomenological NLO description of any process including saturation for the moment
- Understanding the way of including collinear logarithms effects is an important problem (to avoid negative cross-sections!)
- There is a clear hope that these various results should provide precise observables to reveal without ambiguity the saturation of gluons in nucleons and nuclei, and to study the Coor Glass Condensate
- Many precision observables could be studied at LHC in UPC and at the future EIC