

AZIMUTHAL ASYMMETRY IN J/ψ **PRODUCTION IN** e - p**COLLISION AT THE EIC**

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The 3-D nucleon structure in momentum space is still not known yet!

Parton intrinsic transverse momentum?

Spin and k_{\perp} correlations?

Angular momentum of partons?

Spatial distribution?

PLAN OF TALK

 $Gluon\,TMDs$

Azimuthal asymmetries in $J/\psi - jet$ and $J/\psi - \gamma$ pair production in ep scattering at EIC

Numerical estimates

Conclusion



Gluon TMDs



Parameterizations: for the unpolarized (U) and transversely polarized (T) target



A BACK-TO-BACK $J/\psi - jet$ and $J/\psi - \gamma$ production in ep scattering

➤ Consider the electroproduction processes: $e(l) + p(P) \rightarrow e(l') + J/\psi(P_{\psi}) + \gamma(p_{\gamma}) + X$ $e(l) + p(P) \rightarrow e(l') + J/\psi(P_{\psi}) + jet(p_j) + X$



 $\gamma^* - p$ c.m. frame

 $d\sigma \propto [TMD - PDF] \otimes d\hat{\sigma} \otimes [Hadronization]$



A BACK-TO-BACK $J/\psi - jet$ and $J/\psi - \gamma$ production in ep scattering

- > Assume TMD factorization.
- > $p \gamma^*$ center of mass frame which move along z direction
- > $P_{\psi\perp}$ and $P_{j\perp}$ are transverse momentum of J/ψ and *jet* respectively in the plane orthogonal to the proton momentum.
- > We define sum and difference of transverse momenta

$$q_T = P_{\psi \perp} + P_{j \perp}$$
, $K_{\perp} = \frac{P_{\psi \perp} - P_{j \perp}}{2}$ ϕ_T denotes azimuthal angle of q_T

> In the case where $|q_T| \ll |K_{\perp}|$, the J/ψ and jet are almost back-to-back in the transverse plane.

TMD Factorization



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CROSS SECTION:
$$ep \rightarrow e + J/\psi + jet + X$$

$$d\sigma = \frac{1}{2s} \frac{d^{3}l'}{(2\pi)^{3}2E_{l'}} \frac{d^{3}P_{\psi}}{(2\pi)^{3}2E_{\psi}} \frac{d^{3}P_{j}}{(2\pi)^{3}2E_{j}} \int dx d^{2}p_{T} (2\pi)^{4} \delta^{4} \left(q + p_{g} - P_{\psi} - P_{j}\right) \times \frac{1}{Q^{4}} L^{\mu\mu'} (l, q) \Phi_{g}^{\nu\nu'} \left(x, p_{T}^{2}\right) M_{\mu\nu}^{g\gamma^{*} \to J/\psi g} M_{\mu'\nu'}^{*g\gamma^{*} \to J/\psi g}$$

Lepton tensor:
$$L^{\mu\mu'}(l,q) = e^2(-g^{\mu\mu'}Q^2 + 2(l^{\mu}l'^{\mu'} + l^{\mu'}l^{\mu}))$$

Parameterization of gluon correlator for **unpolarized proton** target at 'Leading Twist'

$$\Phi_{g}^{\nu\nu'}(x, \boldsymbol{p}_{T}^{2}) = \frac{1}{2x} \left[-g_{\perp}^{\nu\nu'} f_{1}^{g}(x, \boldsymbol{p}_{T}^{2}) + \left(\frac{p_{T}^{\nu} p_{T}^{\nu'}}{M_{p}^{2}} + g_{\perp}^{\nu\nu'} \frac{\boldsymbol{p}_{T}^{2}}{2M_{p}^{2}} \right) h_{1}^{\perp g}(x, \boldsymbol{p}_{T}^{2}) \right]$$

Unpolarized gluon distribution
Linearly polarized gluon distribution



QUARKONIUM PRODUCTION

> Quarkonium is a bound state of heavy quark and anti-quark ($Q\bar{Q}$)



FEYNMAN DIAGRAMS

Gluon initiated hard process: $\gamma^* g \to Q\bar{Q} g$, contributes significantly over the quark(anti-quark) initiated hard process: $\gamma^* q(\bar{q}) \to Q\bar{Q} q(\bar{q})$, in the small-x domain.

Tree level Feynman diagrams for the hard process: $\gamma^* + g \rightarrow c + \bar{c} + g$

AMPLITUDE CALCULATIONS USING NRQCD

The amplitude can be written as

$$M(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}](P_{\psi}) + g(p_j))$$

= $\sum_{L_Z S_Z} \int \frac{d^3k}{(2\pi)^3} \Psi_{LL_Z}(k) \langle LL_Z; SS_Z | JJ_Z \rangle \operatorname{Tr}[\mathcal{O}(q, p, P_{\psi}, k) \mathcal{P}_{SS_Z}(P_{\psi}, k)]$
D. Boer and C. Pisano (2012)

 $\mathcal{O}(q, p, P_{\psi}, k)$: amplitude for production of $Q\bar{Q}$ pair.

$$\mathcal{O}(q, p, P_{\psi}, k) = \sum_{m=1}^{8} C_m \mathcal{O}_m(q, p, P_{\psi}, k)$$

The spin projection operator, $\mathcal{P}_{SS_z}(P_{\psi}, k)$, projects the spin triplet and spin singlet states of $Q\bar{Q}$ pair

$$\mathcal{P}_{SS_{z}}(P_{\psi},k) = \sum_{s_{1}s_{2}} \left\langle \frac{1}{2}s_{1}; \frac{1}{2}s_{2} \middle| SS_{z} \right\rangle v \left(\frac{P_{\psi}}{2} - k, s_{1} \right) \bar{u} \left(\frac{P_{\psi}}{2} + k, s_{2} \right) \qquad \Pi_{SS_{z}} = \gamma^{5} \text{ for spin singlet } (S = 0)$$

$$= \frac{1}{4M_{\psi}^{3/2}} \left(-\not\!\!\!\!/\psi + 2\not\!\!\!/k + M_{\psi} \right) \Pi_{SS_{z}} \left(\not\!\!\!/\psi + 2\not\!\!\!/k + M_{\psi} \right) + O(k^{2}) \qquad \Pi_{SS_{z}} = \epsilon_{S_{z}}^{\mu} \left(P_{\psi} \right) \gamma_{\mu} \text{ for spin triplet } (S = 1)$$

Amplitude Calculations

Since, $k \ll P_h$, amplitude expanded in Taylor series about k = 0

First term in the expansion gives the S-states (L = 0, J = 0, 1). The linear term in k gives the P-states (L = 1, J = 0, 1, 2).

The S-states amplitude :
$$M[^{2S+1}S_J^{(1,8)}](P_{\psi},k) = \frac{1}{\sqrt{4\pi}}R_0(0)\operatorname{Tr}[\mathcal{O}(q,p,P_{\psi},k)\mathcal{P}_{SS_z}(P_{\psi},k)\Big|_{k=0}$$

The P-states amplitude :

$$M[^{2S+1}P_{J}^{(1,8)}](P_{\psi},k) = -i\sqrt{\frac{3}{4\pi}}R_{1}^{\prime}(0)\sum_{L}\epsilon_{L_{z}}^{\alpha}(P_{\psi})\langle LL_{z};SS_{z}|JJ_{z}\rangle\mathrm{Tr}[\mathcal{O}_{\alpha}(0)\mathcal{P}_{SS_{z}}(0) + \mathcal{O}(0)\mathcal{P}_{SS_{z}\alpha}(0)]$$
$$\mathcal{O}_{\alpha}(0) = \frac{\partial}{\partial k^{\alpha}}\mathcal{O}(q,p,P_{\psi},k)\Big|_{k=0} \qquad \mathcal{P}_{SS_{z}\alpha}(0) = \frac{\partial}{\partial k^{\alpha}}\mathcal{P}_{SS_{z}}(q,p,P_{\psi},k)\Big|_{k=0}$$

Contribution: ${}^{3}S_{1}^{(1)}$, ${}^{3}S_{1}^{(8)}$, ${}^{1}S_{0}^{(8)}$, ${}^{3}P_{j(=0,1,2)}^{(8)}$

 R_0 and R'_1 are related with the LDMEs

FINAL CROSS SECTION: $ep \rightarrow e + J/\psi + jet + X$

 $\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}y\mathrm{d}x_B\mathrm{d}^2\boldsymbol{q}_T\mathrm{d}^2\boldsymbol{K}_{\perp}} \equiv \mathrm{d}\sigma(\phi_S,\phi_T,\phi_{\perp}) = \mathrm{d}\sigma^U(\phi_T,\phi_{\perp}) + \mathrm{d}\sigma^T(\phi_S,\phi_T,\phi_{\perp})$

$$d\sigma^{U} = \mathcal{N} \left[(\mathcal{A}_{0} + \mathcal{A}_{1} \cos \phi_{\perp} + \mathcal{A}_{2} \cos 2\phi_{\perp}) f_{1}^{g}(x, \boldsymbol{q}_{T}^{2}) + (\mathcal{B}_{0} \cos 2\phi_{T} + \mathcal{B}_{1} \cos(2\phi_{T} - \phi_{\perp}) + \mathcal{B}_{2} \cos 2(\phi_{T} - \phi_{\perp}) + \mathcal{B}_{3} \cos(2\phi_{T} - 3\phi_{\perp}) + \mathcal{B}_{4} \cos(2\phi_{T} - 4\phi_{\perp})) \frac{\boldsymbol{q}_{T}^{2}}{M_{p}^{2}} h_{1}^{\perp g}(x, \boldsymbol{q}_{T}^{2}) \right]$$

$$d\sigma^{T} = \mathcal{N}|S_{T}| \left[\sin(\phi_{S} - \phi_{T})(\mathcal{A}_{0} + \mathcal{A}_{1}\cos\phi_{\perp} + \mathcal{A}_{2}\cos2\phi_{\perp}) \frac{|q_{T}|}{M_{p}} f_{1T}^{\perp g}(x, q_{T}^{2}) + \cos(\phi_{S} - \phi_{T})(\mathcal{B}_{0}\sin2\phi_{T} + \mathcal{B}_{1}\sin(2\phi_{T} - \phi_{\perp}) + \mathcal{B}_{2}\sin2(\phi_{T} - \phi_{\perp})) + \mathcal{B}_{3}\sin(2\phi_{T} - 3\phi_{\perp}) + \mathcal{B}_{4}\sin(2\phi_{T} - 4\phi_{\perp})) \frac{|q_{T}|^{3}}{M_{p}^{3}} h_{1T}^{\perp g}(x, q_{T}^{2}) + (\mathcal{B}_{0}\sin(\phi_{S} + \phi_{T}) + \mathcal{B}_{1}\sin(\phi_{S} + \phi_{T} - \phi_{\perp}) + \mathcal{B}_{2}\sin(\phi_{S} + \phi_{T} - 2\phi_{\perp})) + \mathcal{B}_{3}\sin(\phi_{S} + \phi_{T} - 3\phi_{\perp}) + \mathcal{B}_{4}\sin(\phi_{S} + \phi_{T} - 4\phi_{\perp})) \frac{|q_{T}|}{M_{p}} h_{1T}^{g}(x, q_{T}^{2}) \right],$$

ASYMMETRY CALCULATIONS

Weighted azimuthal asymmetry:

$$A^{W(\phi_S,\phi_T)} \equiv 2 \frac{\int d\phi_S d\phi_T d\phi_\perp W(\phi_S,\phi_T) d\sigma(\phi_S,\phi_T,\phi_\perp)}{\int d\phi_S d\phi_T d\phi_\perp d\sigma(\phi_S,\phi_T,\phi_\perp)}$$

U. D'Alesio (2019)

Azimuthal modulations probe Boer-Mulder gluon TMD:

$$A^{\cos 2\phi_T} = \frac{q_T^2}{M_p^2} \frac{\mathcal{B}_0}{\mathcal{A}_0} \frac{h_1^{\perp g}(x, q_T^2)}{f_1^g(x, q_T^2)} \qquad A^{\cos 2(\phi_T - \phi_\perp)} = \frac{q_T^2}{M_p^2} \frac{\mathcal{B}_2}{\mathcal{A}_0} \frac{h_1^{\perp g}(x, q_T^2)}{f_1^g(x, q_T^2)}$$

Azimuthal modulations which can be exploited to extract polarized TMDs $f_{1T}^{\perp g}$, h_1^g and $h_{1T}^{\perp g}$

$$A^{\sin(\phi_{S}-\phi_{T})} = \frac{|\boldsymbol{q}_{T}|}{M_{p}} \frac{f_{1T}^{\perp g}(x, \boldsymbol{q}_{T}^{2})}{f_{1}^{g}(x, \boldsymbol{q}_{T}^{2})}$$
$$A^{\sin(\phi_{S}+\phi_{T})} = \frac{|\boldsymbol{q}_{T}|}{M_{p}} \frac{\mathcal{B}_{0}}{\mathcal{A}_{0}} \frac{h_{1}^{g}(x, \boldsymbol{q}_{T}^{2})}{f_{1}^{g}(x, \boldsymbol{q}_{T}^{2})}$$

GAUSSIAN PARAMETERIZATION (GP)

 $f_1^g(x, q_T^2)$ and $h_1^{\perp g}(x, q_T^2)$ are assumed to be factorized as function of x, i.e. collinear PDFs and a Gaussina function of the transverse momentum q_t .

$$f_{1}^{g}(x, \boldsymbol{q}_{T}^{2}) = f_{1}^{g}(x, \mu) \frac{1}{\pi \langle q_{T}^{2} \rangle^{2}} e^{-\frac{q_{T}^{2}}{\langle q_{T}^{2} \rangle}}$$
$$h_{1}^{\perp g}(x, \boldsymbol{q}_{T}^{2}) = \frac{M_{P}^{2} f_{1}^{g}(x, \mu)}{\pi \langle q_{T}^{2} \rangle^{2}} \frac{2(1-r)}{r} e^{1-\frac{q_{T}^{2}}{r \langle q_{T}^{2} \rangle}}$$

r (0 < r < 1) and $\langle q_T^2 \rangle$ are parameters We took, r = 1/3 and $\langle q_T^2 \rangle = 0.25$

$$\begin{split} \Delta^{N} f_{g/p^{\uparrow}}(x,q_{T}) &= \left(-\frac{2|\boldsymbol{q}_{T}|}{M_{P}}\right) f_{1T}^{\perp g}(x,q_{T}) \\ &= 2\frac{\sqrt{2e}}{\pi} \mathcal{N}_{g}(x) f_{g/p}(x) \sqrt{\frac{1-\rho}{\rho}} q_{T} \frac{e^{-\boldsymbol{q}_{T}^{2}/\rho \langle q_{T}^{2} \rangle}}{\langle q_{T}^{2} \rangle^{3/2}} \\ \mathcal{N}_{g}(x) &= N_{g} x^{\alpha} (1-x)^{\beta} \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^{\alpha} \beta^{\beta}} \\ N_{g} &= 0.25, \quad \alpha = 0.6, \quad \beta = 0.6, \quad \rho = 0.1 \end{split}$$

U. D'Alesio (2019)

Linearly polarized gluon TMD satisfies a positivity bound

$$\frac{\boldsymbol{q}_T^2}{2M_P} \left| h_1^{\perp g} \left(x, \boldsymbol{q}_T^2 \right) \right| \le f_1^g \left(x, \boldsymbol{q}_T^2 \right)$$

D. Boer and C. Pisano (2012)

PARAMETERIZATION: SPECTATOR MODEL (SM)

Nucleon is assumed to emit a gluon and the remaining is treated as a single on-shell particle called spectator particle.

Mass of the spectator particle is allowed to take a continuous values described by a spectral function, $\rho_X(M_X)$

Gluon TMDs:
$$F^{g}(x, q_{t}^{2}) = \int_{M}^{\infty} dM_{X} \rho_{X}(M_{X}) \hat{F}^{g}(x, q_{t}^{2}; M_{X})$$
 $\rho_{X}(M_{X}) = \mu^{2a} \left[\frac{A}{B + \mu^{2b}} + \frac{C}{\pi \sigma} e^{-\frac{(M_{X} - D)^{2}}{\sigma^{2}}} \right]$

where, A, B, C, D, a, b, σ are free parameters

At leading-twist, T-even unpolarized and linearly polarized gluon TMDs can be written as

$$\hat{f}_{1}^{g}(x,\boldsymbol{q}_{t}^{2};M_{X}) = \left[(2Mxg_{1} - x(M + M_{X})g_{2})^{2} \left[\left(M_{X} - M(1 - x) \right)^{2} + \boldsymbol{q}_{t}^{2} \right] + 2\boldsymbol{q}_{t}^{2} \left(\boldsymbol{q}_{t}^{2} + xM_{X}^{2} \right) g_{2}^{2} + 2\boldsymbol{q}_{t}^{2} M^{2} (1 - x) \left(4g_{1}^{2} - xg_{2}^{2} \right) \right] \times \left[(2\pi)^{3} 4x M^{2} \left(L_{X}^{2}(0) + \boldsymbol{q}_{t}^{2} \right)^{2} \right]^{-1}$$

$$\hat{h}_1^{\perp g}(x, \boldsymbol{q}_t^2; M_X) = \left[4M^2(1-x)g_1^2 + \left(L_X^2(0) + \boldsymbol{q}_t^2\right)g_2^2\right] \times \left[(2\pi)^3 x \left(L_X^2(0) + \boldsymbol{q}_t^2\right)^2\right]^{-1}$$

Where $g_{1,2}(p^2)$ are model-dependent form factors, given as: $g_{1,2}(p^2) = \kappa_{1,2} \frac{p^2(1-x)^2}{(q_t^2 + L_X^2(\Lambda_X^2))^2}$

 p^2 is gluon momentum, $\kappa_{1,2}$ and Λ_X are normalization and cut-off parameters respectively, and $L_X^2(\Lambda_X^2) = xM_X^2 + (1-x)\Lambda_X^2 - x(1-x)M^2$

RESULTS: $J/\psi - jet$ **PRODUCTION**

Asymmetry hardly change with \sqrt{s}

We used CSMWZ set of LDME

Chao (2012)

RESULTS: $J/\psi - jet$ **PRODUCTION**

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(A) Comparing $|A^{\cos 2\phi_t}|$ calculated in SM, GP and TMD evolution with the upper bound on the asymmetry.

(B) Contribution to $|A^{\cos 2\phi_t}|$ from all the color singlet and color octet states for two sets of LDMEs, CMSWZ (left) and SV (right).

Chao (2012)

Sharma (2013)

AMPLITUDE CALCULATIONS USING NRQCD

$$\succ e(l) + p(P) \rightarrow e(l') + J/\psi(P_{\psi}) + \gamma(p_{\gamma}) + X$$

The amplitude can be written as

$$M(\gamma^* g \to Q\bar{Q}[^{2S+1}L_J^{(1,8)}](P_{\psi}) + \gamma(p_{\gamma}))$$

$$= \sum_{L_Z S_Z} \int \frac{d^3k}{(2\pi)^3} \Psi_{LL_Z}(k) \langle LL_Z; SS_Z | JJ_Z \rangle \operatorname{Tr}[\mathcal{O}(q, p, P_{\psi}, k) \mathcal{P}_{SS_Z}(P_{\psi}, k)]$$
D. Boer and C. Pisano (2012)

Contribution: ${}^{3}S_{1}^{(8)}$

UNPOLARIZED CROSS SECTION: $J\psi - \gamma$ PRODUCTION

Z < 0.9: avoid contribution from diffractive process and prevents hitting ultraviolet divergences.

0.3 < Z: avoid contribution via resolved-photon channel

A sizable cross section, expected to be detected at upcoming EIC.

Its measurement can provide a clean probe CO mechanism within NRQCD framework.

Provide clean extraction of a CO LDME

ASYMMETRY AT EIC: $J/\psi - \gamma$ production

Model independent Upper bounds on the asymmetries:

Asymmetries are independent of center of mass energy.

Asymmetries are independent of LDMEs.

ASYMMETRY AT EIC: $J/\psi - \gamma$ production

Upper bounds on the Sivers asymmetry is 1.

 q_T dependent of Sivers asymmetry is obtained using Gaussian parameterization. Asymmetry depends on center of mass energy. Asymmetry hardly depends on virtuality.

CONCLUSION

We calculated the azimuthal asymmetry in $J/\psi - jet$ and $J/\psi - \gamma$ electroproductions where the pair produced in a almost back-to-back in the transverse plane.

We used NRQCD framework for J/ψ production.

 $A^{\cos(2\phi_T)}$ and $A^{\cos 2(\phi_T - \phi_\perp)}$ azimuthal asymmetries probes the linearly polarized gluon TMD, where as the $A^{\sin(\phi_S - \phi_T)}$ can probe gluon Sivers function.

We show the numerical estimates of the asymmetries in a model dependent parameterizations of the TMDs and model independent upper bounds on these asymmetries using the positivity bounds on TMDs. We found a sizable azimuthal asymmetries both in $J/\psi - jet$ and $J/\psi - \gamma$ productions.

Back-to-back $J/\psi - jet$ and $J/\psi - \gamma$ electroproduction could be a promising channel to probe poorly known gluon TMDs at the future proposed EIC.

LINEARLY POLARIZED GLUON DISTRIBUTION FUNCTION (BOER-MULDERS) $h_1^{\perp g}(x, k_T^2)$

- > Interpret $h_1^{\perp g}(x, k_T^2)$ as "azimuthal correlated" gluon distribution function.
- > It affects the unpolarized cross section and cause azimuthal asymmetries: $(\cos(2\phi))$
- > It's a time-reversal even function and in the small-x domain, can be Weizsäcker-Williams(WW) or dipole distribution depending on type of Wilson line.
 - Wilson lines: + + or - WW distribution
 - Wilson lines: + or + dipole distribution

Linearly polarized gluon distributions were first introduced in

Mulders and Rodrigues, PRD 63, 094021 (2001)

It can be probed in Drell-Yan and semi-inclusive deep inelastic scattering (SIDIS) processes. Though it has not been extracted from the data yet, but lot of theoretical studies has been done.

CONCLUSION

We calculated the $\cos 2\phi_t$ azimuthal asymmetry in a J/ψ and a *jet* electroproduction where the $J/\psi - jet$ pair produced in a almost back-to-back in the transverse plane.

We consider the full NRQCD framework for J/ψ production

We show the numerical estimates of the $A^{\cos 2\phi_t}$ using the parameterizations of the TMDs in the Spectator model and the Gaussian parameterizations.

We also show the effect of TMD evolution on the asymmetry, where we see that the magnitude of the asymmetry is small as compare with the asymmetries calculated using the TMD parameterizations.

We obtained a significant asymmetry both in the Spectator model and Gaussian parameterizations of TMDs.

Back-to-back J/ψ and *jet* electroproduction could be a promising channel to probe poorly known linearly polarized gluon TMDs at the future proposed EIC.

