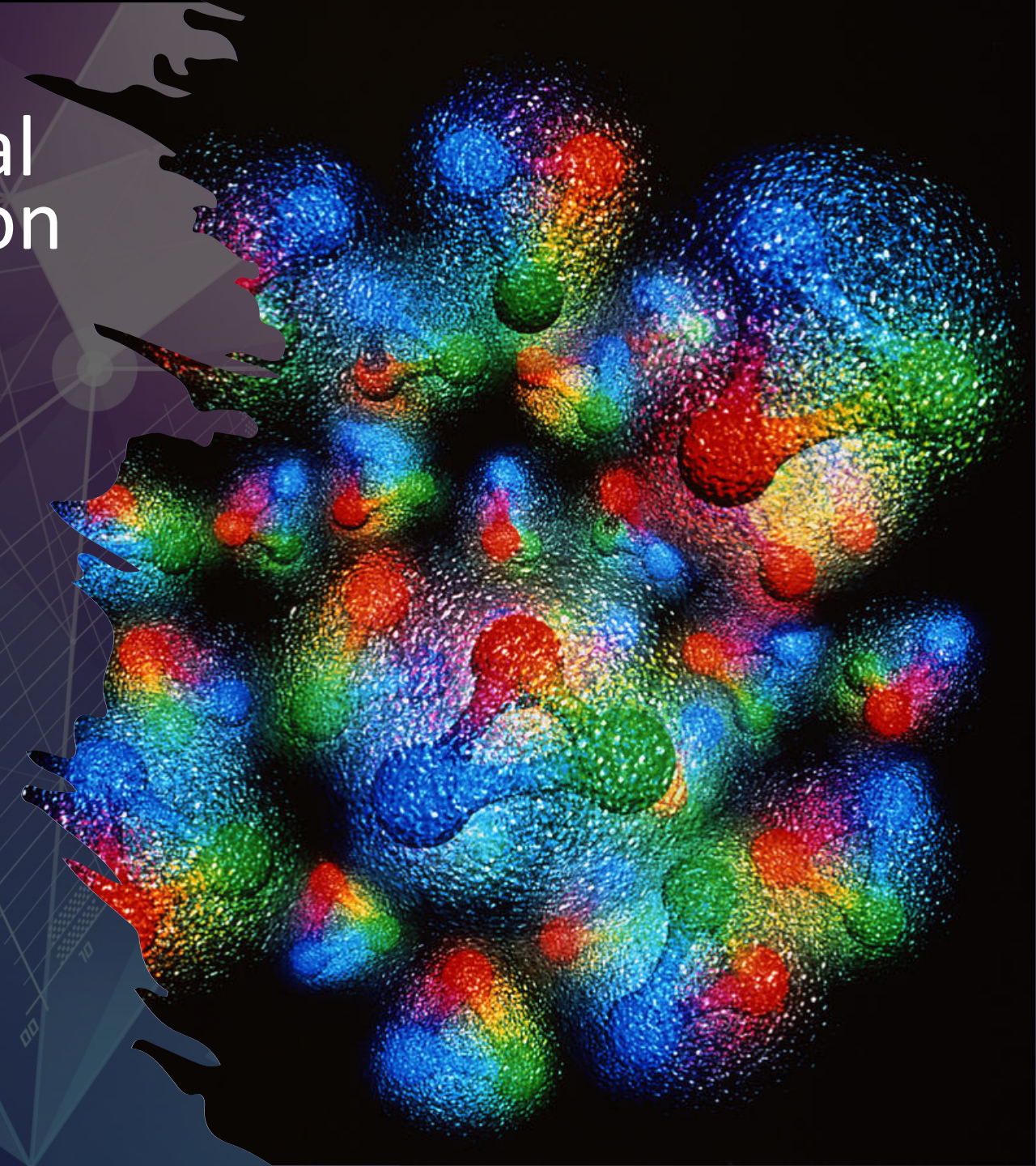
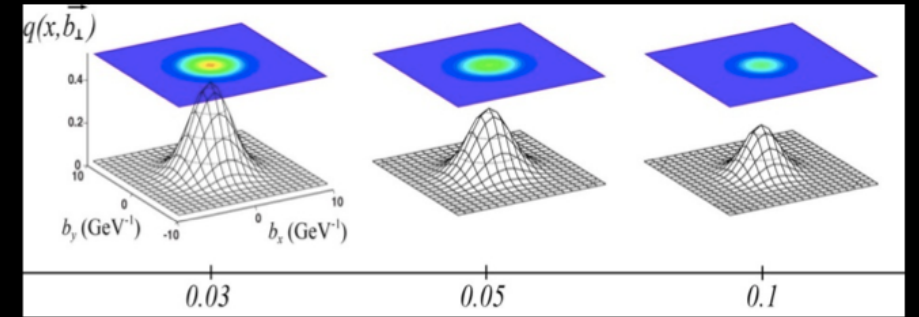


Benchmarks for a Global Extraction of Information from Deeply Virtual Exclusive Scattering Experiments

Simonetta Liuti



Motivation: Quark and Gluon Imaging



“Static” images

$$\rho_{\Lambda\lambda}^q(\mathbf{b}) = H_q(\mathbf{b}^2) + \frac{b^i}{M} \epsilon_{ij} S_T^j \frac{\partial}{\partial b} E_q(\mathbf{b}^2) + \Lambda\lambda \tilde{H}_q(\mathbf{b}^2),$$

D. Soper (1977), M. Burkardt (2000)

3D Coordinate Space Representation – Gluon Results

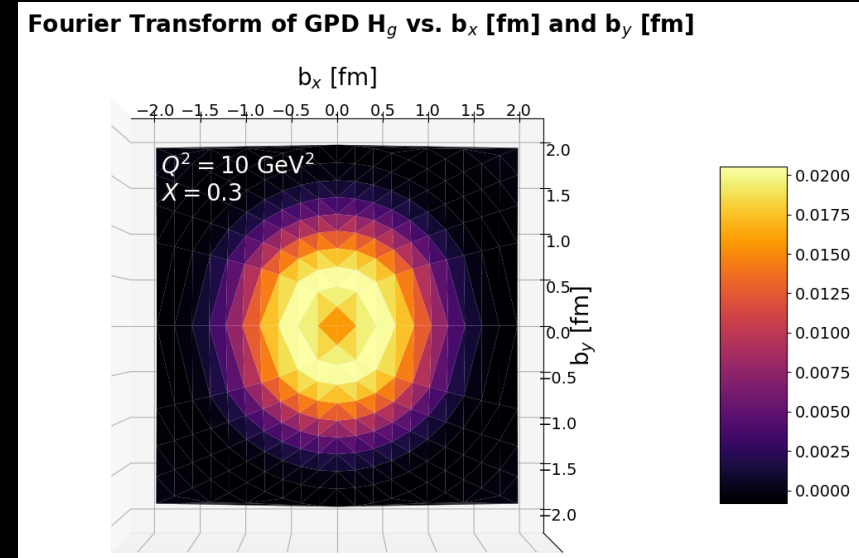
- GPDs can be Fourier transformed from momentum space into coordinate space, providing insight into matter, charge, and radial distributions of the quarks and gluons inside the proton.

M. Burkardt, Phys. Rev. D72, 094020 (2005), hep-ph/0505189.

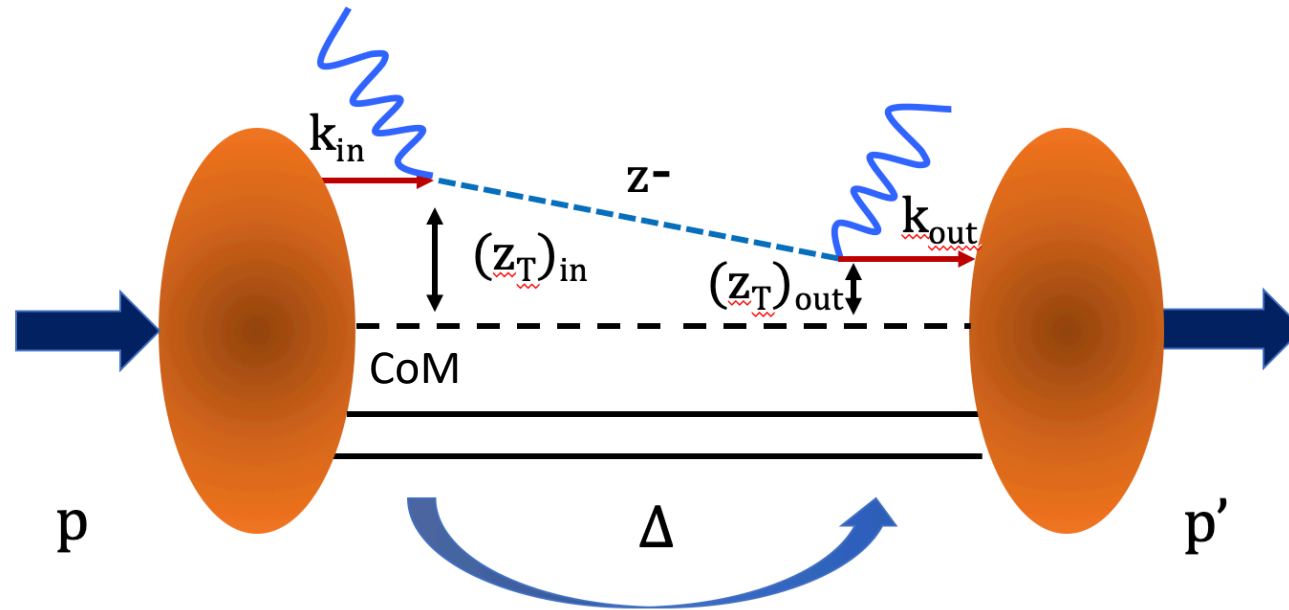
$$\mathcal{H}^q(X, 0, b_T) = \int \frac{d^2 \Delta_T}{(2\pi)^2} H^q(X, 0, \Delta_T) e^{-i\Delta_T \cdot b_T}$$

UVA's parametrization
constrained by lattice QCD and
experiment:

B. Kriesten, P. Velie, E. Yeats, F. Y. Lopez, & S. Liuti, *Phys.Rev.D* 105 (2022) 5, 056022



One body correlator



$$\left\{ \begin{array}{l} \underline{b}_T = \frac{z_{T \text{ in}} + z_{T \text{ out}}}{2} \end{array} \right.$$

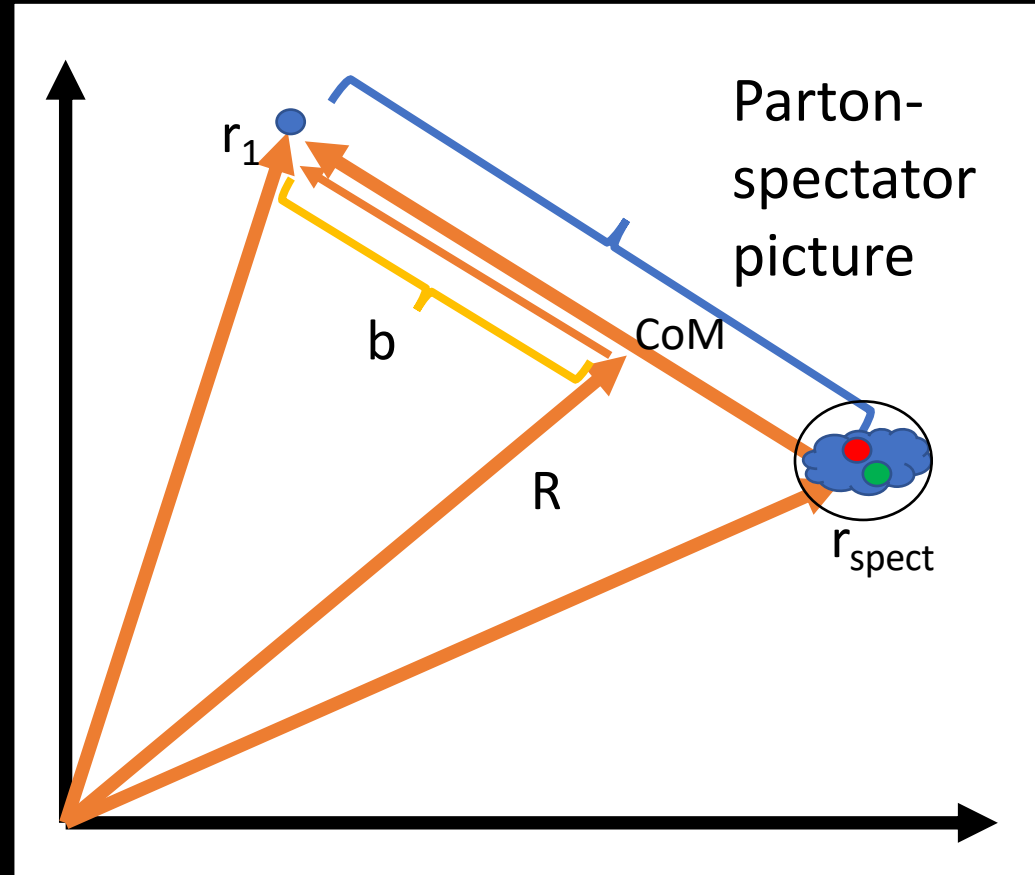
$$\Delta = k_{\text{in}} - k_{\text{out}} = p - p'$$

$$\left\{ \begin{array}{l} z = z_{\text{in}} - z_{\text{out}} \end{array} \right.$$

$$k = \frac{k_{\text{in}} + k_{\text{out}}}{2}$$

Two sets of conjugate **space-momentum** variables

Spatial Coordinates



One body densities

GPD

$$H_q(x, 0, t) = \int \frac{dz^-}{2\pi} e^{ixp^+ z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z) | p \rangle \Big|_{z^+ = z_T = 0}$$

$$t = \Delta^2$$

At twist-2 ...

$$\begin{aligned} H_q(x, 0, t) &= \int d^2 \mathbf{k}_X dk_X^+ \delta(k_X^+ - (1-x)P^+) \langle p' | \bar{\psi}_+(0) | X \rangle \langle X | \psi_+(0) | p \rangle \\ &= \int d^2 \mathbf{k} \phi^*(x, \mathbf{k} - \Delta) \phi(x, \mathbf{k}), \end{aligned}$$

Non-diagonal in \mathbf{k}

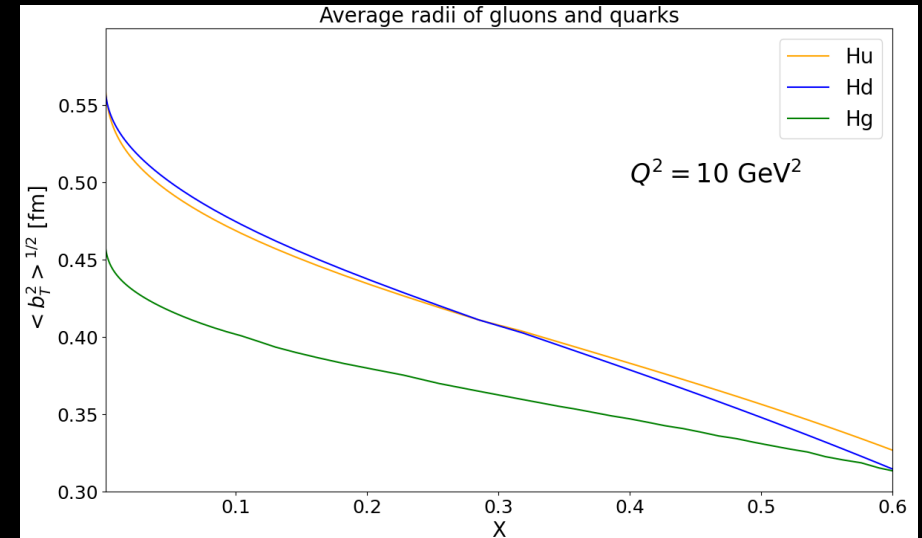
$$\begin{aligned}
H_q(x, 0, t) &= \int d^2\mathbf{k} \int d^2\mathbf{z}_T d^2\mathbf{z}'_T e^{-i\mathbf{z}'_T \cdot (\mathbf{k} - \Delta)} e^{i\mathbf{z}_T \cdot \mathbf{k}} \tilde{\phi}^*(x, \mathbf{z}'_T) \tilde{\phi}(x, \mathbf{z}_T) = \\
&= \int d^2\mathbf{k} \int d^2\mathbf{r} d^2\mathbf{b} e^{i\mathbf{r} \cdot \mathbf{k}} e^{i(\mathbf{b} - \mathbf{r}/2) \cdot \Delta} \tilde{\phi}^*\left(x, \mathbf{b} - \frac{\mathbf{r}}{2}\right) \tilde{\phi}\left(x, \mathbf{b} + \frac{\mathbf{r}}{2}\right) = \int d^2\mathbf{b} e^{i\mathbf{b} \cdot \Delta} \rho(x, \mathbf{b})
\end{aligned}$$

-
- Fourier transforming the vertex functions leads to a the one-body parton density distribution in the transverse coordinate

Average Distance spanned by quarks and gluons

- Expectation value of the transverse impact parameter distance
- The radius of the gluon matter density is smaller than the quark radius

$$\langle b_T^2 \rangle^q (X) = \frac{\int_0^\infty d^2 b_T b_T^2 \mathcal{H}^q(X, 0, b_T)}{\int_0^\infty d^2 b_T \mathcal{H}^q(X, 0, b_T)}$$

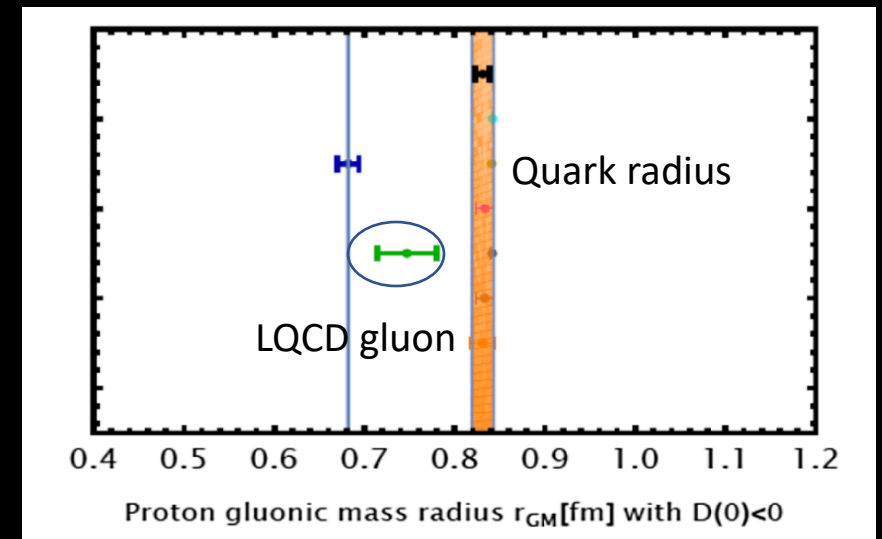


Compare to lattice and AdS/CFT results

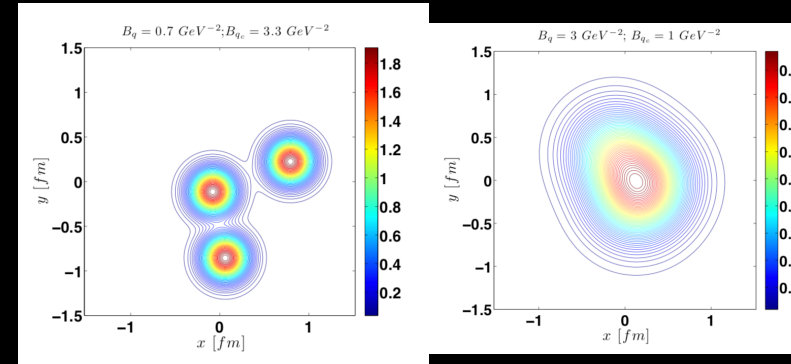
K. Mamo and I. Zaeed

PRD106, 086004 (2022)

LQCD: Detmold and Shanahan



Quark and Gluon Imaging



Traini and Blaizot, PRD(2019)

Recent development

A more *differential* imaging, describing the *event-by-event* quantum fluctuations in the wave function of the colliding hadron

H. Mantysaari, B. Schenke, F. Salazar et al.

arXiv 2001.10705 [hep-ph]

“Dynamic” images of gluon distributions forming hot-spots: can we connect them to GPDs?

B. Kriesten, SL, Z. Panjsheeri, P. Velie in preparation

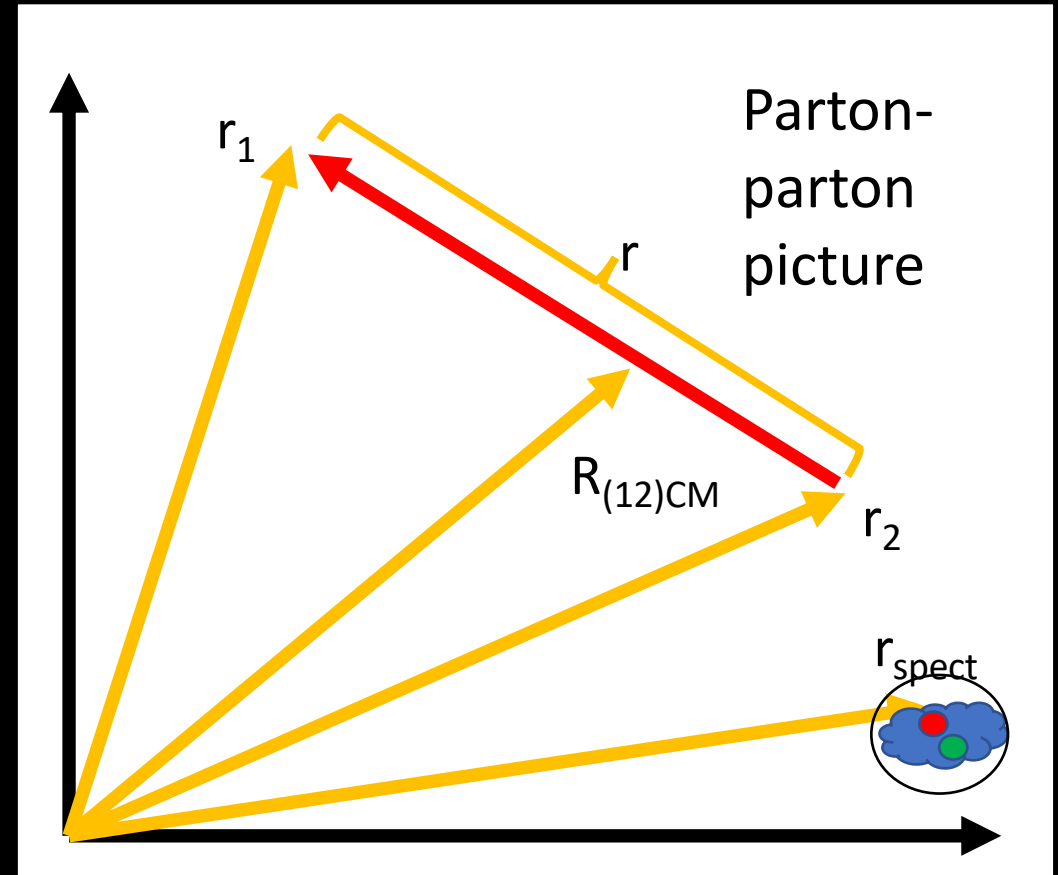
- This emerging picture supports the idea of the gluons being at the core of the nucleon and carrying baryon number

D. Kharzeev



From one-body to two-body densities

- We can see a lot just from the **one-body densities**, but is that enough for imaging the proton's internal structure?
- We want to also understand how partons are situated relative to one another.



Two Body Densities

$$\rho_2^{q,q}(x, \mathbf{b}_1, \mathbf{b}_2) = \frac{1}{2} \left[\rho(\mathbf{b}_1)\rho(\mathbf{b}_2) - \frac{1}{2}\rho(\mathbf{b}_1, \mathbf{b}_2) \right]$$

We can now observe the **granularity** of the distributions by comparing the average relative distance of the particles and the distance of the pair from the proton CoM

$$\langle r^2(x) \rangle^{1/2} = \left[\int d^2b_1 \int d^2b_2 r^2 H_{q,g}(x_1, b_1) H_{q,g}(x_2, b_2) \delta^2(r - |\mathbf{b}_1 - \mathbf{b}_2|) \right]^{1/2}$$

$$\langle R_{CM}^2(x) \rangle^{1/2} = \left[\int d^2b_1 \int d^2b_2 R_{CM}^2 H_{q,g}(x_1, b_1) H_{q,g}(x_2, b_2) \delta^2(r - |\mathbf{b}_1 - \mathbf{b}_2|) \right]^{1/2}$$

Overlap between two partons

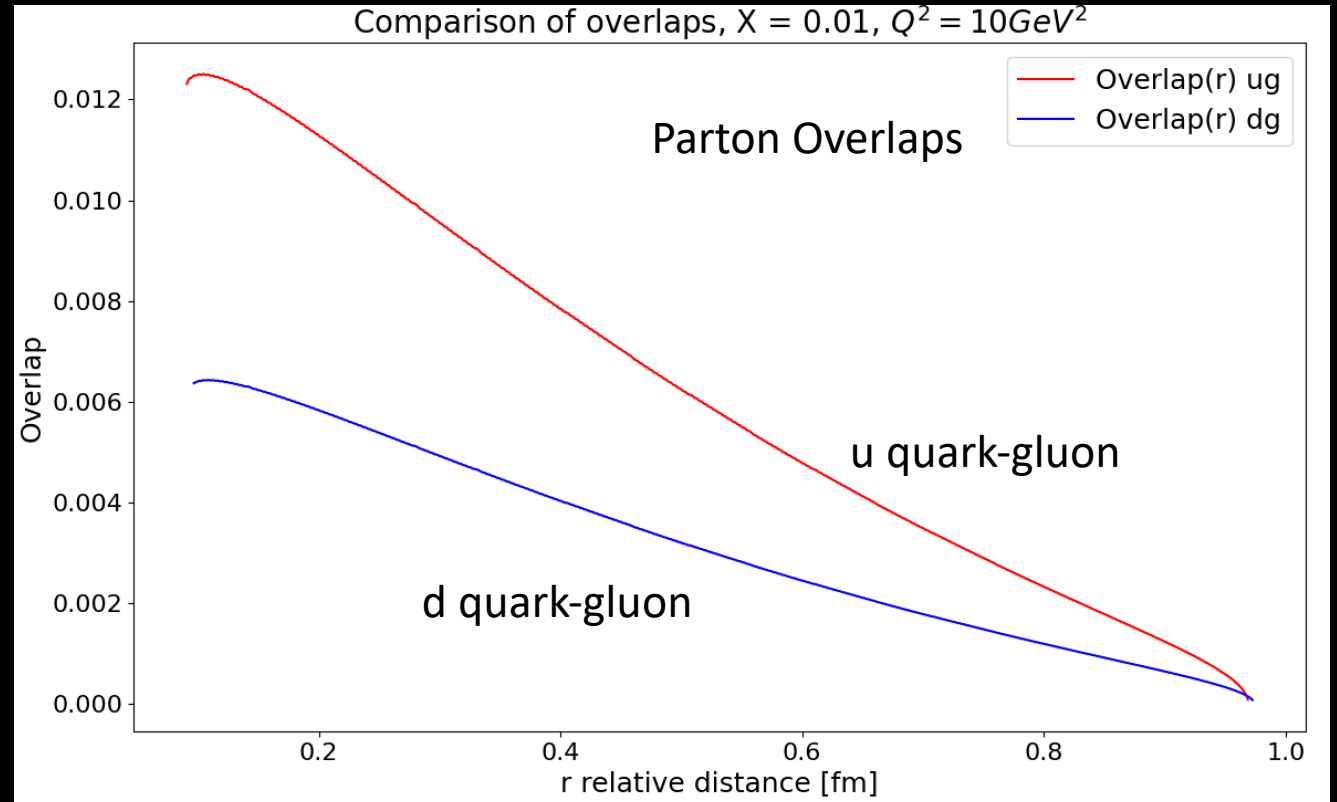
$$O_{q_1, q_2}(x) = \int d^2 R_{CM} A_o(R_{q_1}, R_{q_2}, r) H_{q_1}(x_1, R_{CM} + \frac{r}{2}) H_{q_2}(x_2, R_{CM} - \frac{r}{2})$$

A_o refers to the overlap between two circles as a function of their radii and the distance between their centers.

$$A_o(r) = R_1^2 \cos^{-1} \left(\frac{r^2 + R_1^2 - R_2^2}{2rR_1} \right) + R_2^2 \cos^{-1} \left(\frac{r^2 + R_2^2 - R_1^2}{2rR_2} \right) - \frac{1}{2} \sqrt{(-r + R_1 + R_2)(r + R_1 - R_2)(r - R_1 + R_2)(r + R_1 + R_2)}$$

Our preliminary observations

1. Overlap between gluons and a **u quark** seems more likely at this value of X than for the overlap between gluons and a **d quark**
2. R_q is always larger than R_g : we are never in the small qq overlap case, which means that the valence quarks span a large area defining the proton's size
3. Large qg, gg overlaps. The gluons are all together and the gluons follow the quarks, but the quarks are distinct from one another. **"Gluon sea and flooded quark islands."**



Z. Panjsheeri, in progress

J. Bautista, Z. Panjsheeri, B. Kriesten, SL, in preparation

Work in progress: Connection to the observables

$$D_{q_1 q_2}(x, \mathbf{r}, \mathbf{R}) = \int d^2 \mathbf{r}' \int \int dz^- e^{ixp^+ z^-} \langle p', \Lambda' | \mathcal{O}(y, z) \mathcal{O}(y, z) | p, \Lambda \rangle$$

$$\mathcal{O}(z, \mathbf{r}) = [\bar{\psi}_{q_1}(z^-, \mathbf{r}) \gamma^+ \psi_{q_1}(0, \mathbf{r})]$$

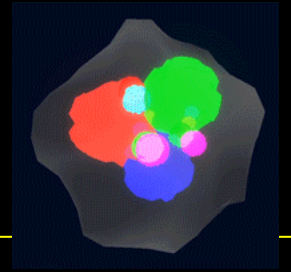
$$\mathcal{O}(z, 0) = [\bar{\psi}_{q_2}(z^-, 0) \gamma^+ \psi_{q_2}(0, 0)]$$

- Different from standard Double-Parton Distributions (DPDs)
- Accessible in multi-parton final state processes

Defining the Benchmarks for a Global Analysis of Deeply Virtual Exclusive Experiments:

M. Almaeen et al. arXiv 2207.10766

The EXCLAIM project
(EXCLUSives with Artificial Intelligence and
Machine learning)



Co-Pis:

Computer Science: Gia Wei Chern, Yaohang Li

Experiment: Marie Boer

Lattice QCD: Michel Engelhardt, Huey Wen Lin

Phenomenology/ Theory: Gary Goldstein, S.L., Matt Sievert

Affiliates:

Aurore Courtoy, Tanja Horn, Dennis Sivers

UVA students: Joshua Bautista, Adil Khawaja, Zaki Panjsheeri

In the process of hiring several postdocs!

PRELIMINARY STEP

- Define a set of established **benchmarks** to provide a solid common ground that will allow practitioners to conduct quantitative comparisons
- Are results are compatible within error?(identifying possible outliers)

ML EXAMPLE:

- NNPDF uses **closure** and **future tests** corresponding to interpolation/extrapolation methods.
- Femtonet uses different metrics to validate our results

a common benchmark needs to be defined in order to be able to compare results including the **uncertainty** in the predictions

OUR PROGRAM: To develop *Physics Informed* networks including theory constraints in deep learning models.

Hard constraints

“built into the architecture of the network”

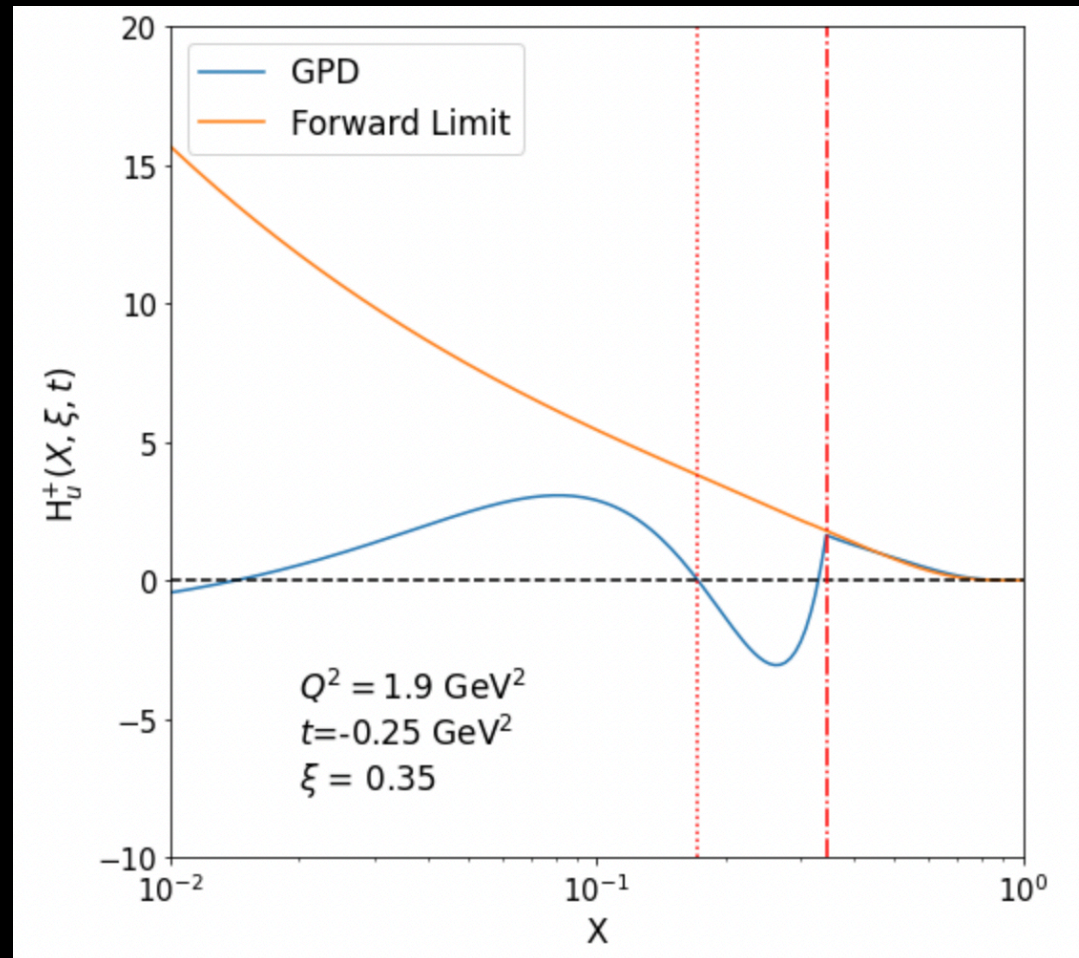
- network invertibility
- choice of activation functions
- defining customized neural network layers

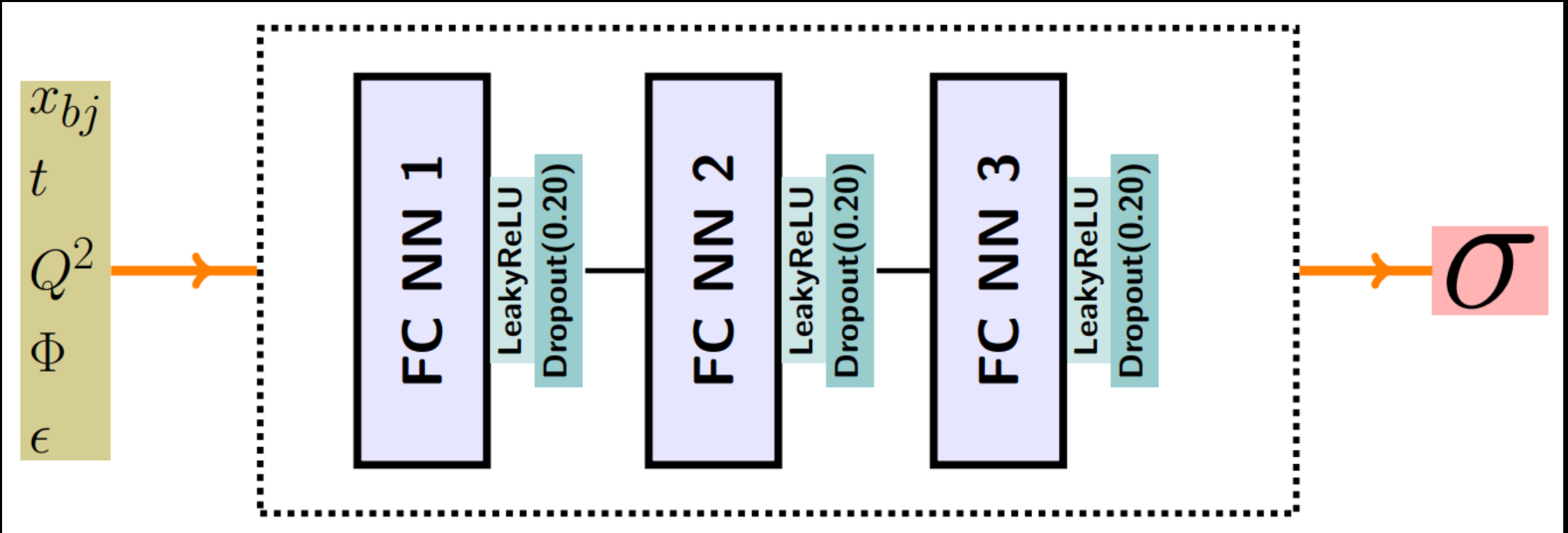
Soft constraints

“adding additional terms to the loss function that can be learned to minimize and generate physics weighted parameters”

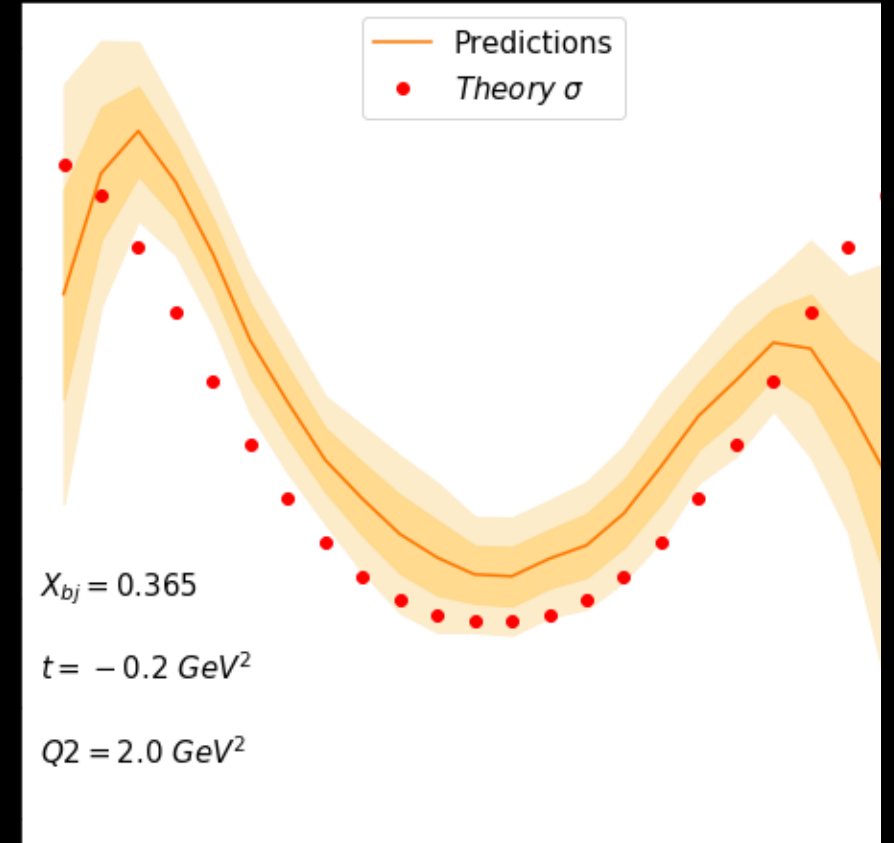
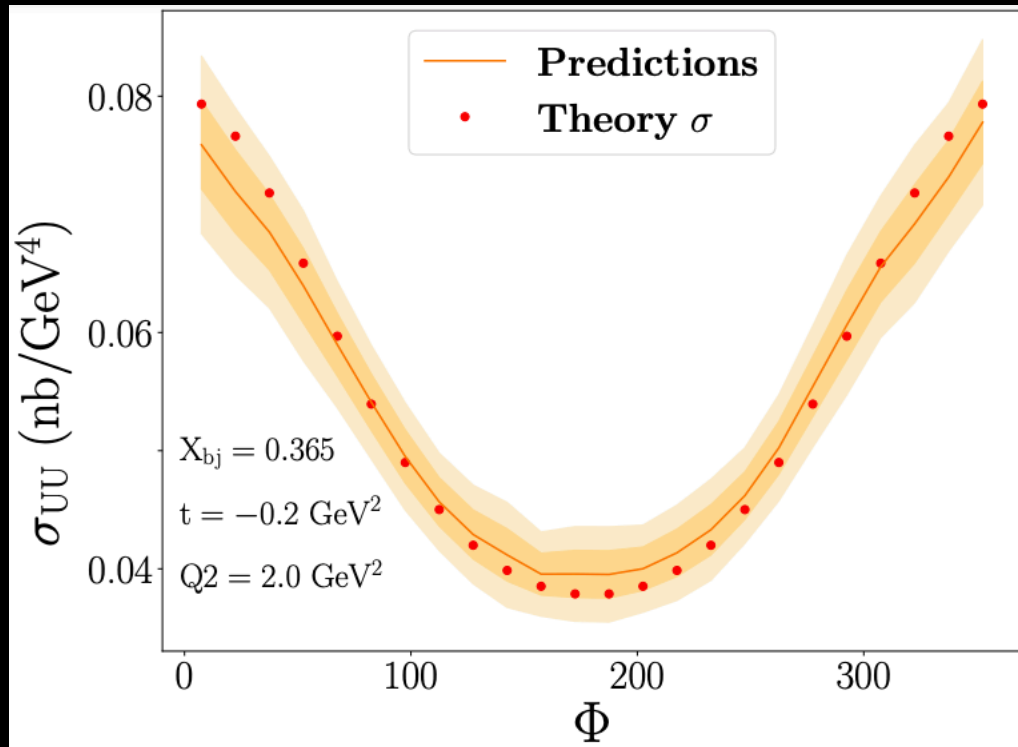
1. Cross section structure
2. Lorentz invariance
3. Positivity constraints
4. Forward kinematic limit, defined by $\xi, t \rightarrow 0$, to PDFs, when applicable
5. \Re - \Im connection of CFFs through dispersion relations with proper consideration of threshold effects

Example of a soft constraint

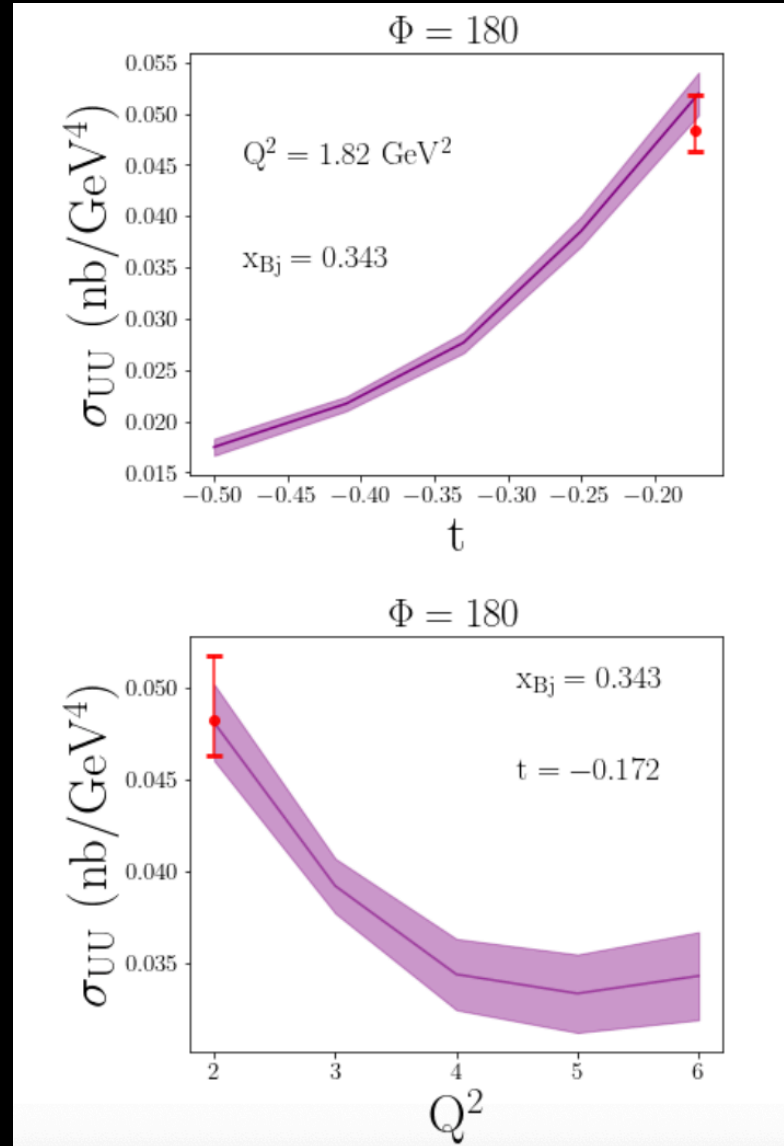




Bethe-Heitler contribution with and without parity constraint



How our DNN generalizes trends in t and Q^2

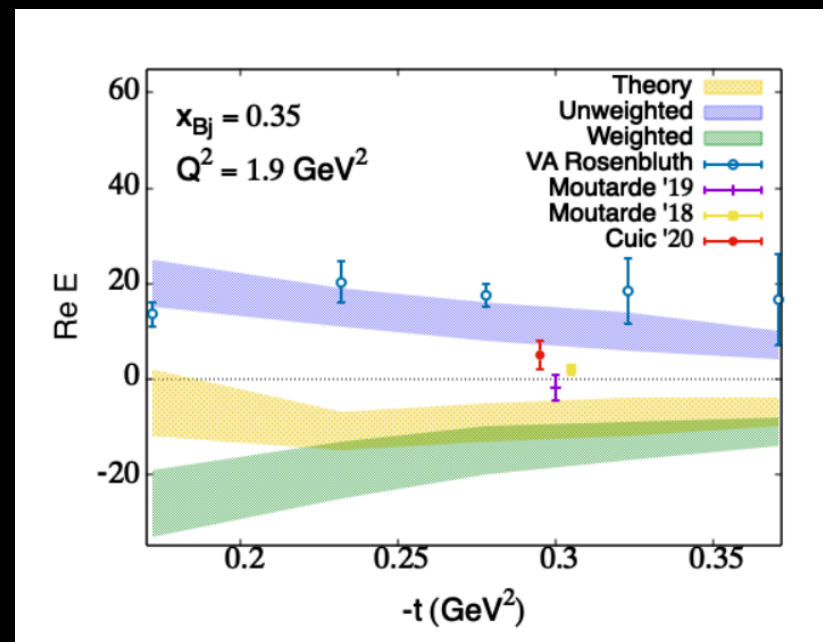
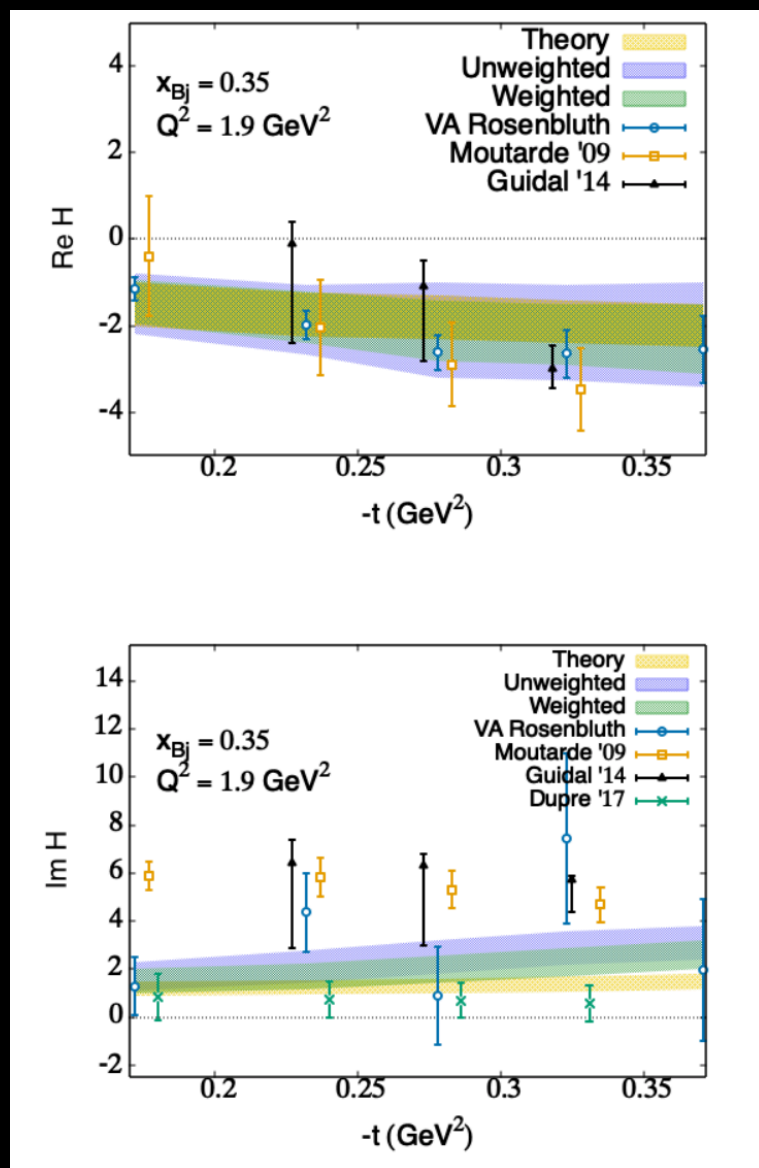


Parametrization of DVCS cross section

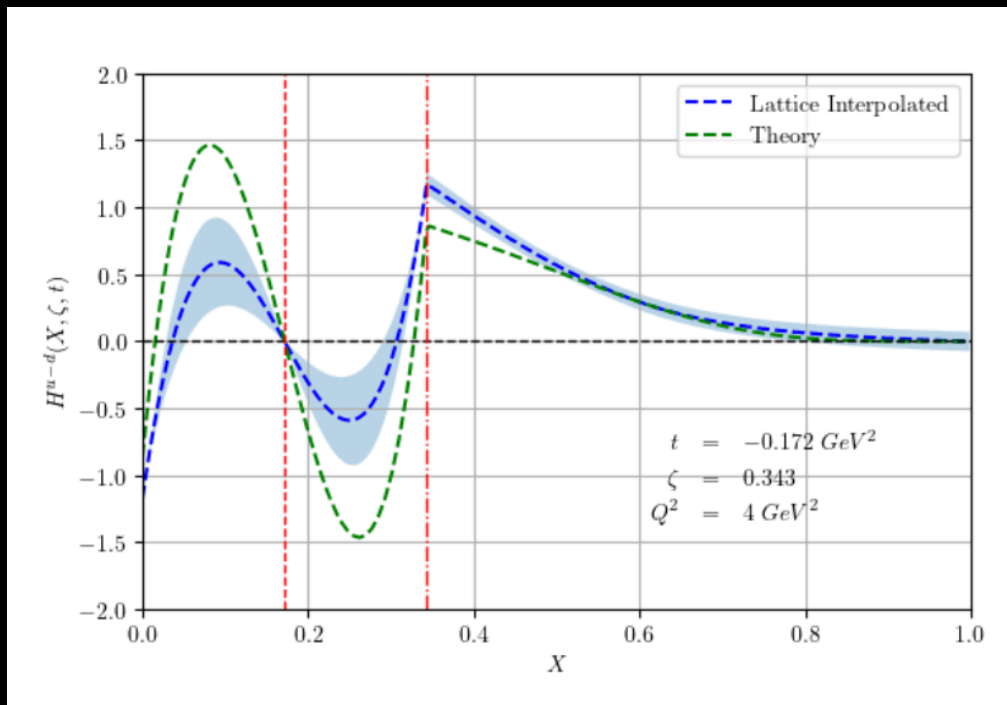
$$\begin{aligned}
 |T_{UU}^{BH}|^2 &= \frac{\Gamma}{t} \left[A_{UU}^{BH} (F_1^2 + \tau F_2^2) + B_{UU}^{BH} \tau G_M^2(t) \right] \\
 |T_{UU}^{\mathcal{I}}|^2 &= \frac{\Gamma}{Q^2 t} \left[A_{UU}^{\mathcal{I}} \Re (F_1 \mathcal{H} + \tau F_2 \mathcal{E}) + B_{UU}^{\mathcal{I}} G_M \Re (\mathcal{H} + \mathcal{E}) + C_{UU}^{\mathcal{I}} G_M \Re \tilde{\mathcal{H}} \right] \\
 |T_{LU}^{\mathcal{I}}|^2 &= \frac{\Gamma}{Q^2 t} \left[A_{LU}^{\mathcal{I}} \Im (F_1 \mathcal{H} + \tau F_2 \mathcal{E}) + B_{LU}^{\mathcal{I}} G_M \Im (\mathcal{H} + \mathcal{E}) + C_{LU}^{\mathcal{I}} G_M \Im \tilde{\mathcal{H}} \right] \\
 |T_{UU}^{DVCS}|^2 &= \frac{\Gamma}{Q^2} \frac{2}{1-\epsilon} \left[(1-\xi^2) \left[(\Re \mathcal{H})^2 + (\Im \mathcal{H})^2 + (\Re \tilde{\mathcal{H}})^2 + (\Im \tilde{\mathcal{H}})^2 \right] \right. \\
 &\quad + \frac{t_o - t}{4M^2} \left[(\Re \mathcal{E})^2 + (\Im \mathcal{E})^2 + \xi^2 (\Re \tilde{\mathcal{E}})^2 + \xi^2 (\Im \tilde{\mathcal{E}})^2 \right] \\
 &\quad \left. - 2\xi^2 \left(\Re \mathcal{H} \Re \mathcal{E} + \Im \mathcal{H} \Im \mathcal{E} + \Re \tilde{\mathcal{H}} \Re \tilde{\mathcal{E}} + \Im \tilde{\mathcal{H}} \Im \tilde{\mathcal{E}} \right) \right]
 \end{aligned}$$

- B. Kriesten et al, *Phys.Rev. D* 101 (2020)
- B. Kriesten and S. Liuti, *Phys.Rev. D*105 (2022), arXiv [2004.08890](https://arxiv.org/abs/2004.08890)
- B. Kriesten and S. Liuti, *Phys. Lett.* B829 (2022), arXiv:2011.04484

Extraction of CFFs

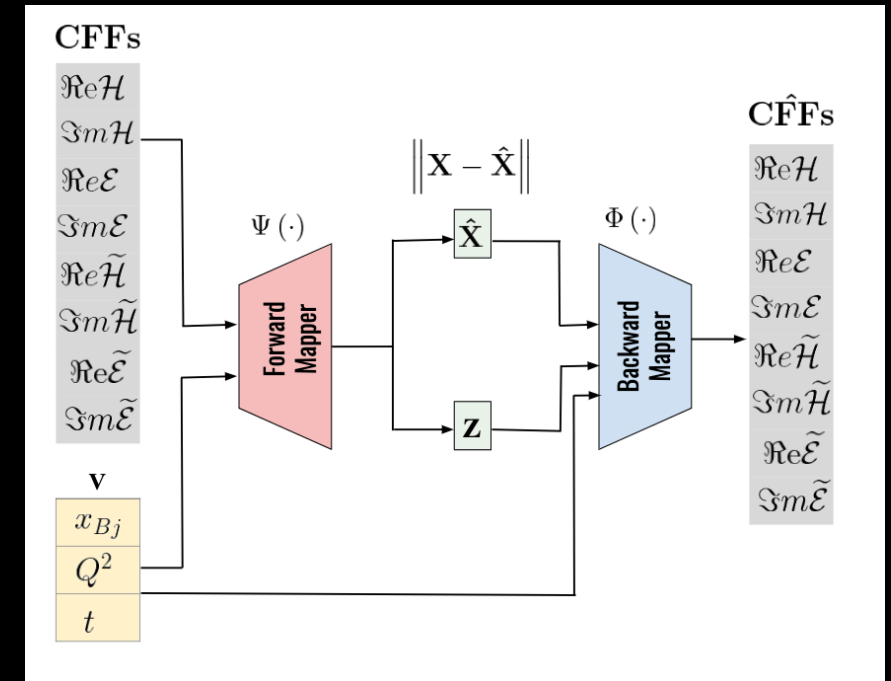
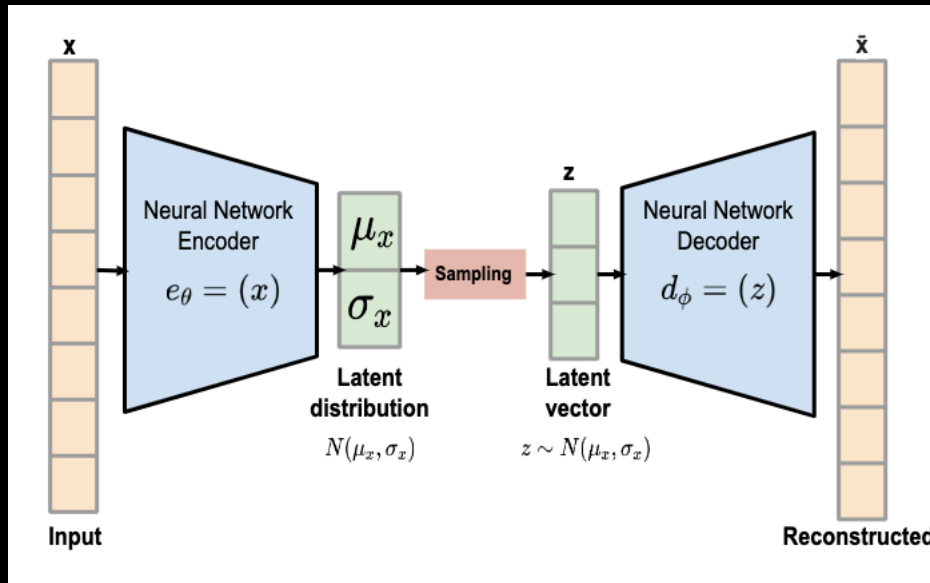


Using lattice moments to constrain ERBL rregion



$$\begin{aligned}
 M_2^q(\zeta, t) &= \int_{\zeta/2}^1 \frac{dX}{1 - \zeta/2} \left(\frac{X - \zeta/2}{1 - \zeta/2} \right) H^+(X, \zeta, t) \\
 &= A_{2,0}^q(t) + 4 \left(\frac{\zeta}{2 - \zeta} \right)^2 C_{2,0}^q(t)
 \end{aligned}$$

Moving forward: introducing unsupervised methods with Variational Autoencoder

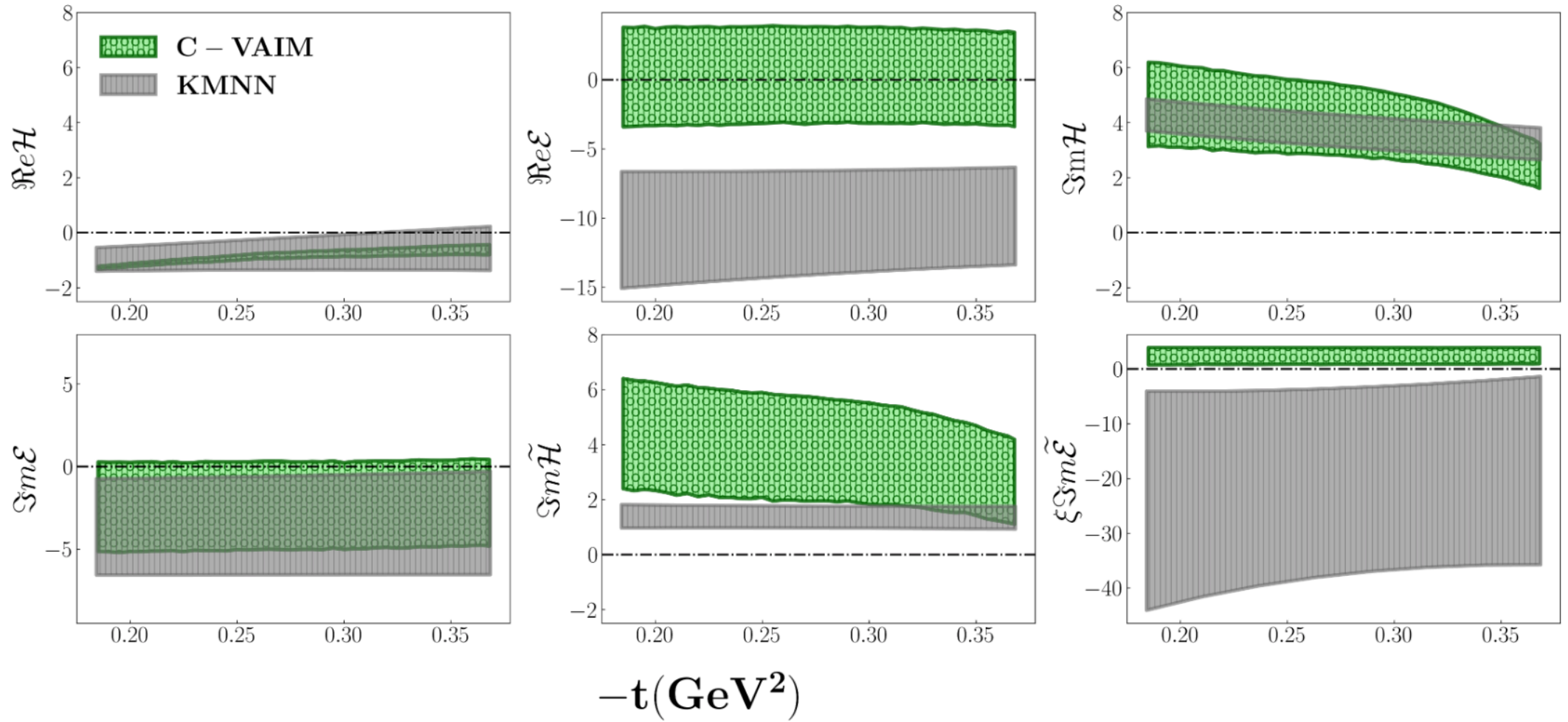


Unsupervised methods already tried for PDFs

New avenue to the parton distribution functions: Self-organizing maps

H. Honkanen, S. Liuti, J. Carnahan, Y. Loitiere, and P. R. Reynolds
 Phys. Rev. D **79**, 034022 – Published 20 February 2009

CFFs

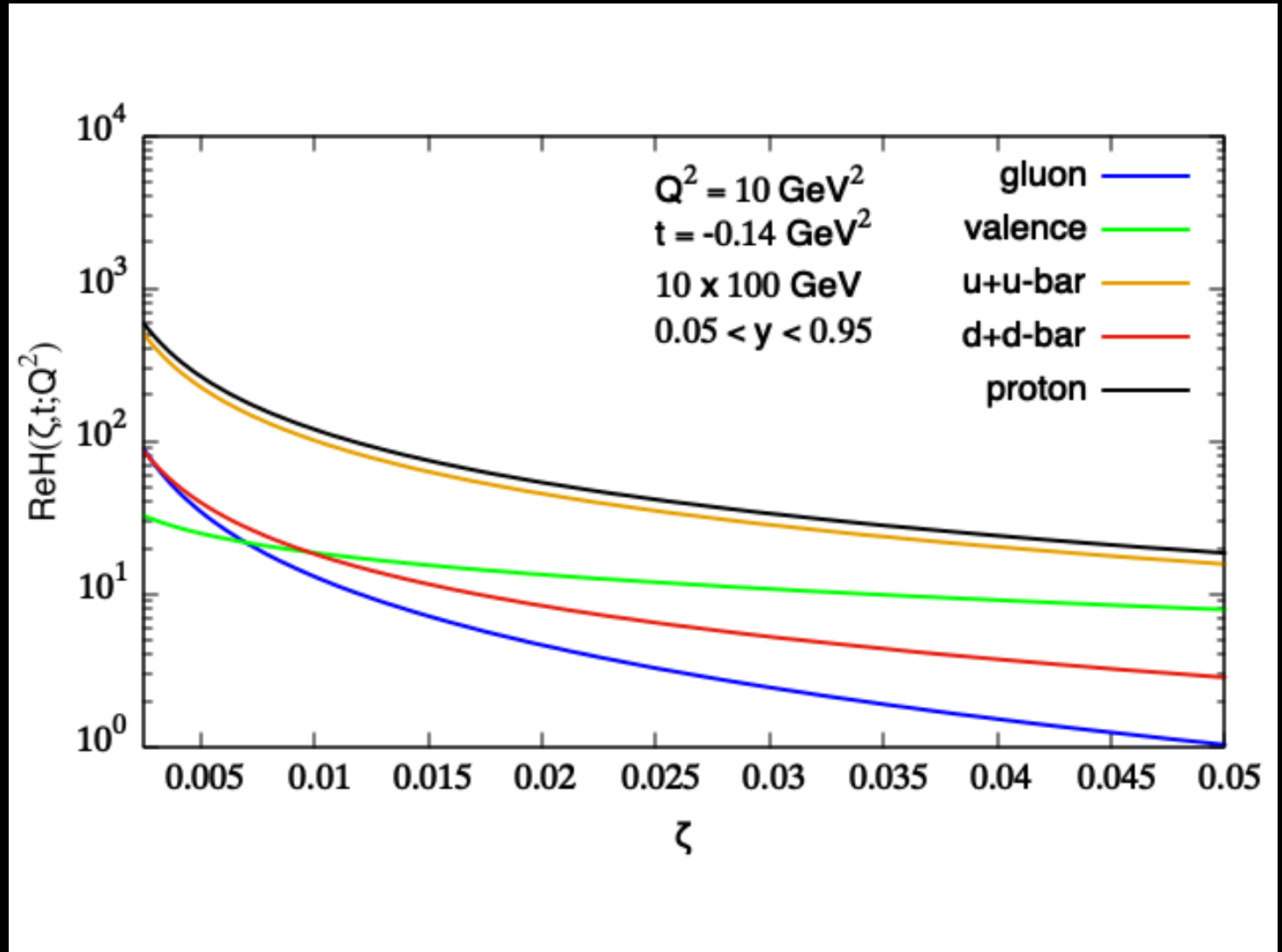




- Outstanding questions we want to clarify on GPDs

CFFs

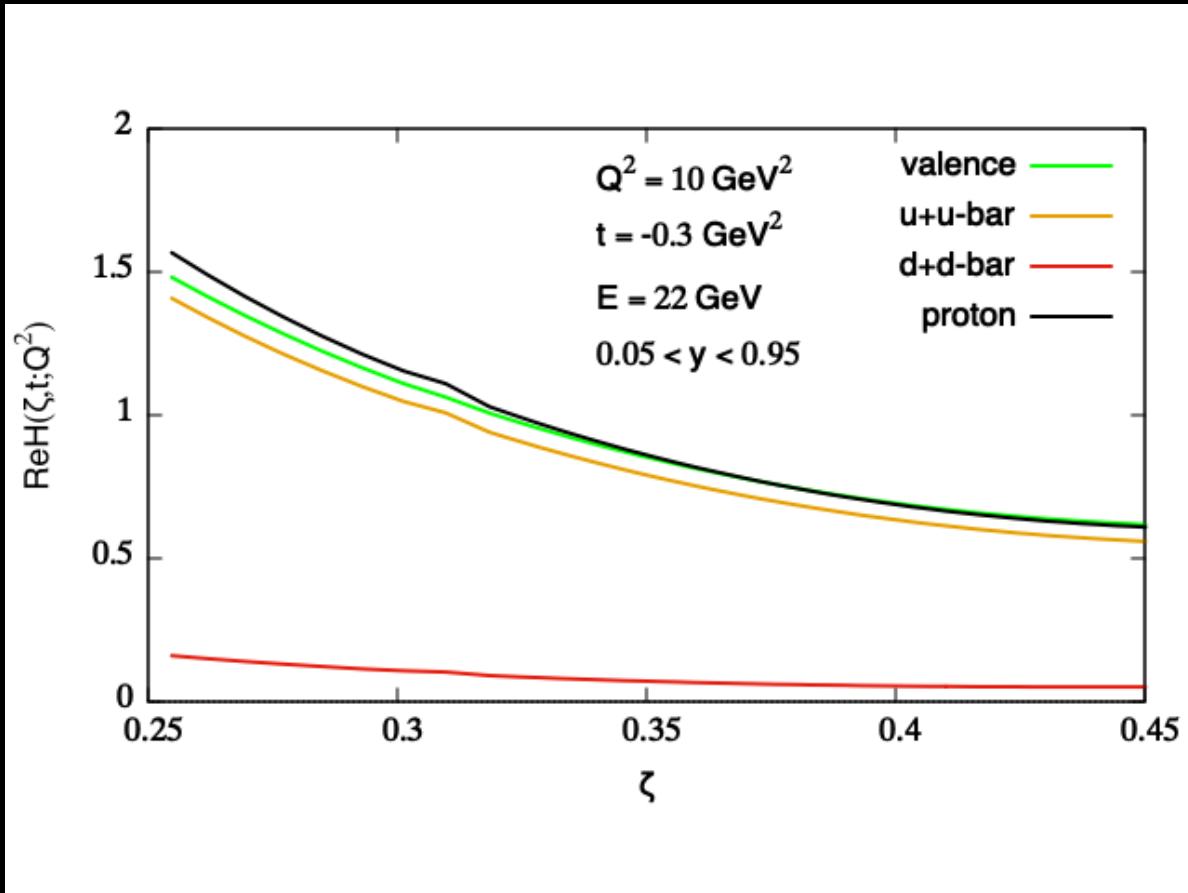
1. How do we separate the various flavor components at leading order (twist)?



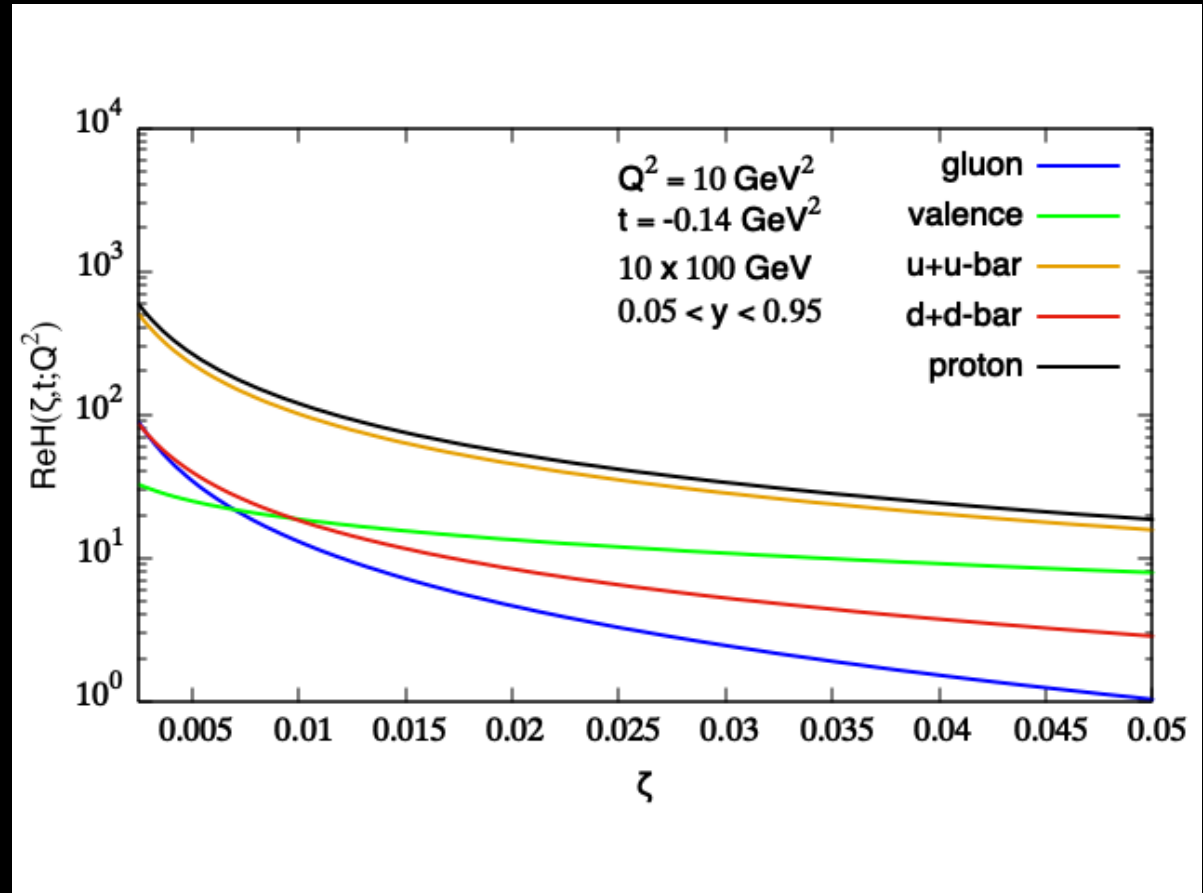
B. Kriesten and S. Liuti, *in preparation*

Two different machines/two different regimes

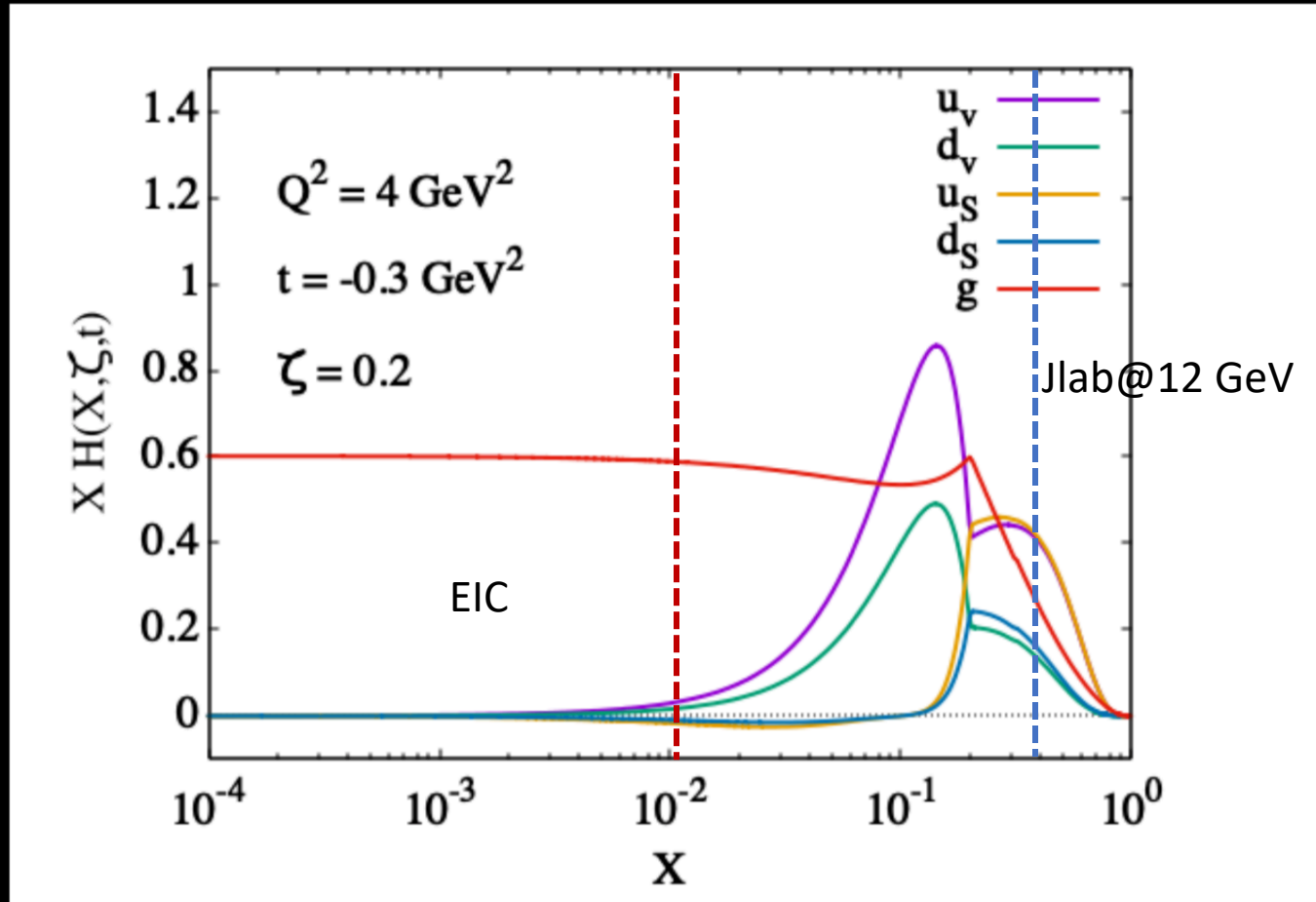
Jlab



EIC



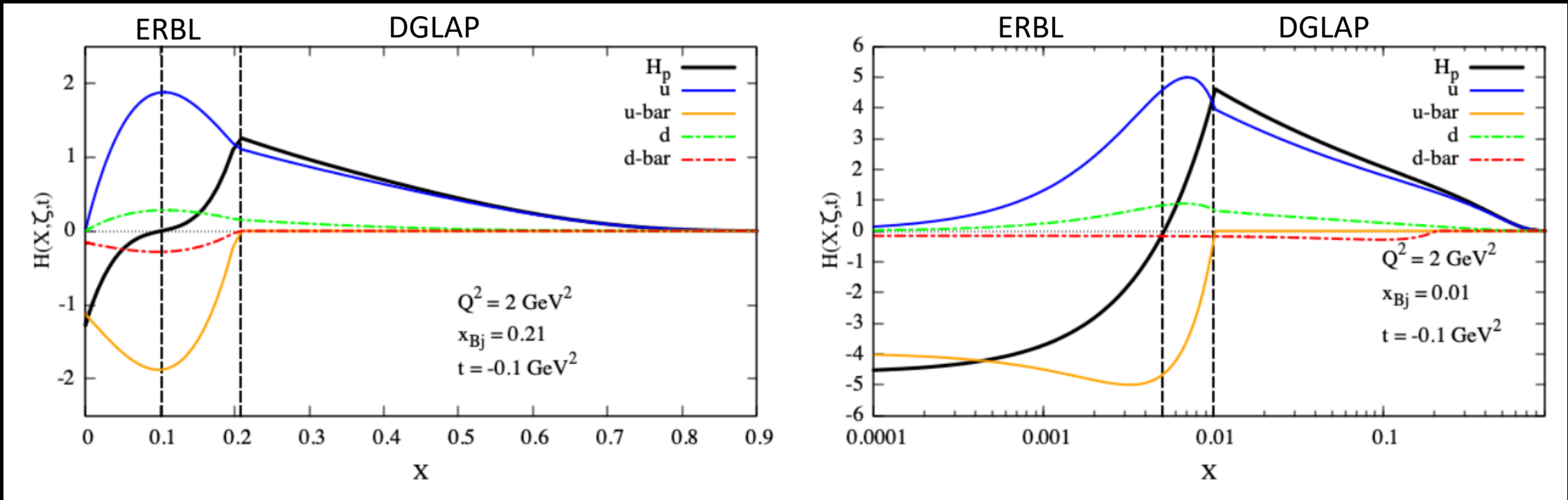
GPDs



All GPDs

- B. Kriesten, P. Velie, E. Yeats, F. Y. Lopez and S. Liuti, *Phys. Rev D* 105 (2022), arXiv:2101.01826

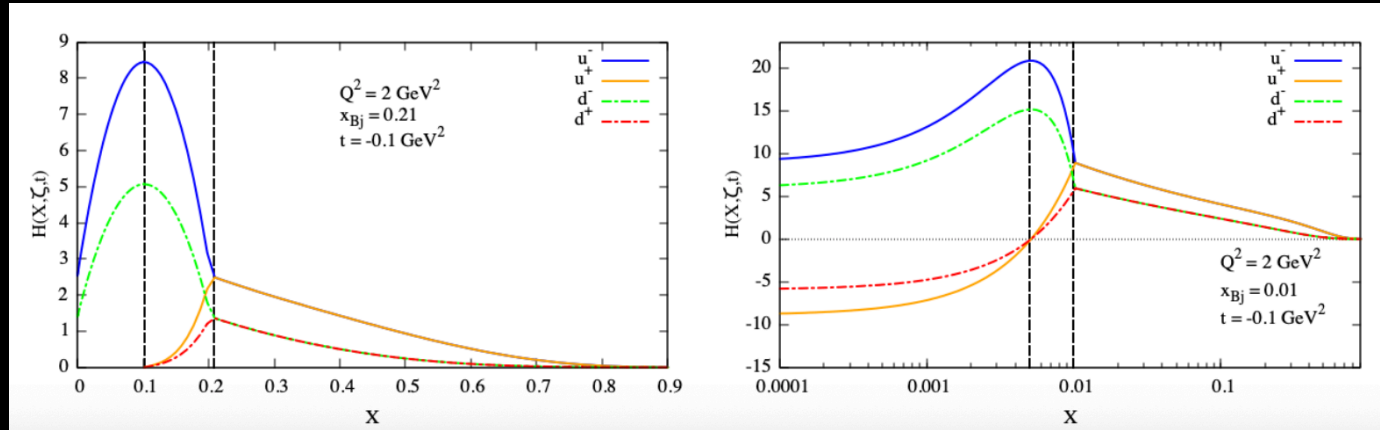
10/31/20



3. What type of information is in the Compton Form Factors?

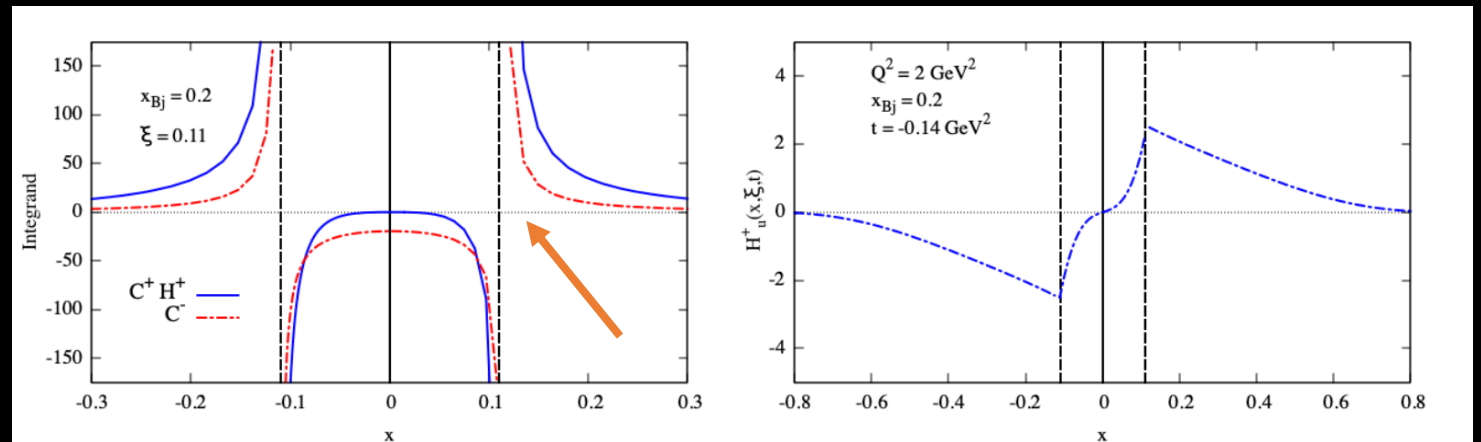
The way the symmetries play for DVCS

GPDs



CFFs kernels

$$C^\pm(x, \xi) = \frac{1}{x - \xi + i\epsilon} \pm \frac{1}{x + \xi - i\epsilon}.$$

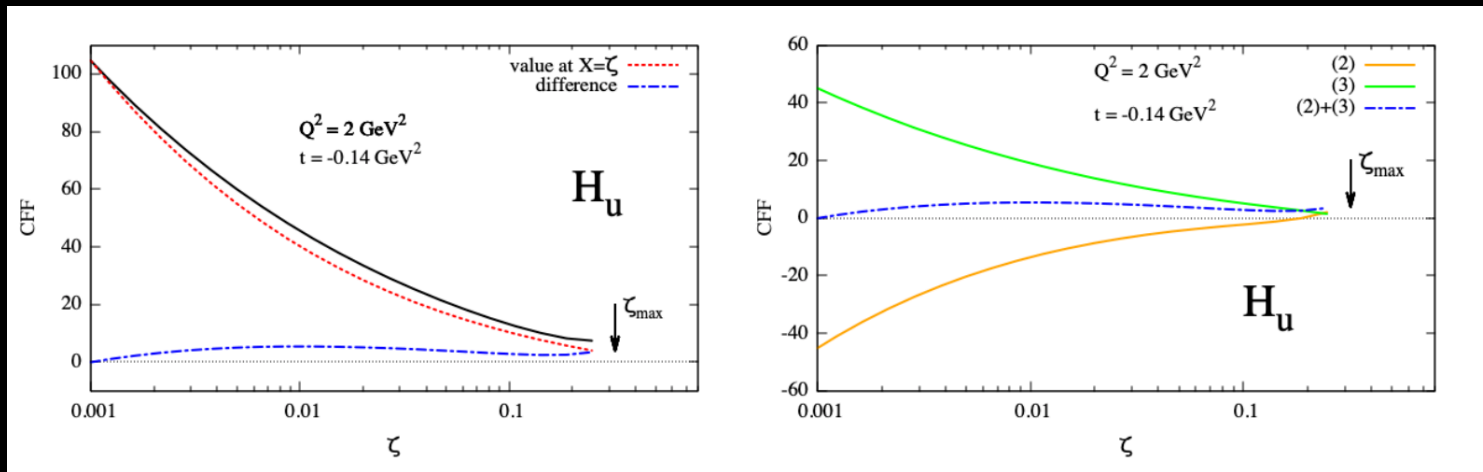


From our initial study $\text{Re } H$ does not carry much information on the region away from $x=\xi$

Inverse problem might be trivial !!

$$\Re \mathcal{H}_q = H_q^+(\zeta, \zeta, t) \ln \frac{1-\zeta}{\zeta/2} + \left[\left(\frac{\partial H_q^+}{\partial X} \right)_{X=\zeta}^{\leftarrow} (1-\zeta) - \left(\frac{\partial H_q^+}{\partial X} \right)_{X=\zeta}^{\rightarrow} (\zeta/2) \right] + \int_{\zeta/2}^1 dx \frac{H_q^+(X, \zeta, t)}{X}$$

(1) (2) (3)



Twist -3 GPDs

	GPD	$P_q P_p$	TMD	Ref. [1]
	H^\perp	UU	f^\perp	$2\tilde{H}_{2T} + E_{2T}$
J_L	\tilde{H}_L^\perp	LL	g_L^\perp	$2\tilde{H}'_{2T} + E'_{2T}$
	H_L^\perp	UL	$f_L^\perp^{(*)}$	$\tilde{E}_{2T} - \xi E_{2T}$
	\tilde{H}^\perp	LU	$g^\perp^{(*)}$	$\tilde{E}'_{2T} - \xi E'_{2T}$
J_T	$H_T^{(3)}$	UT	$f_T^{(*)}$	$H_{2T} + \tau \tilde{H}_{2T}$
	$\tilde{H}_T^{(3)}$	LT	g'_T	$H'_{2T} + \tau \tilde{H}'_{2T}$

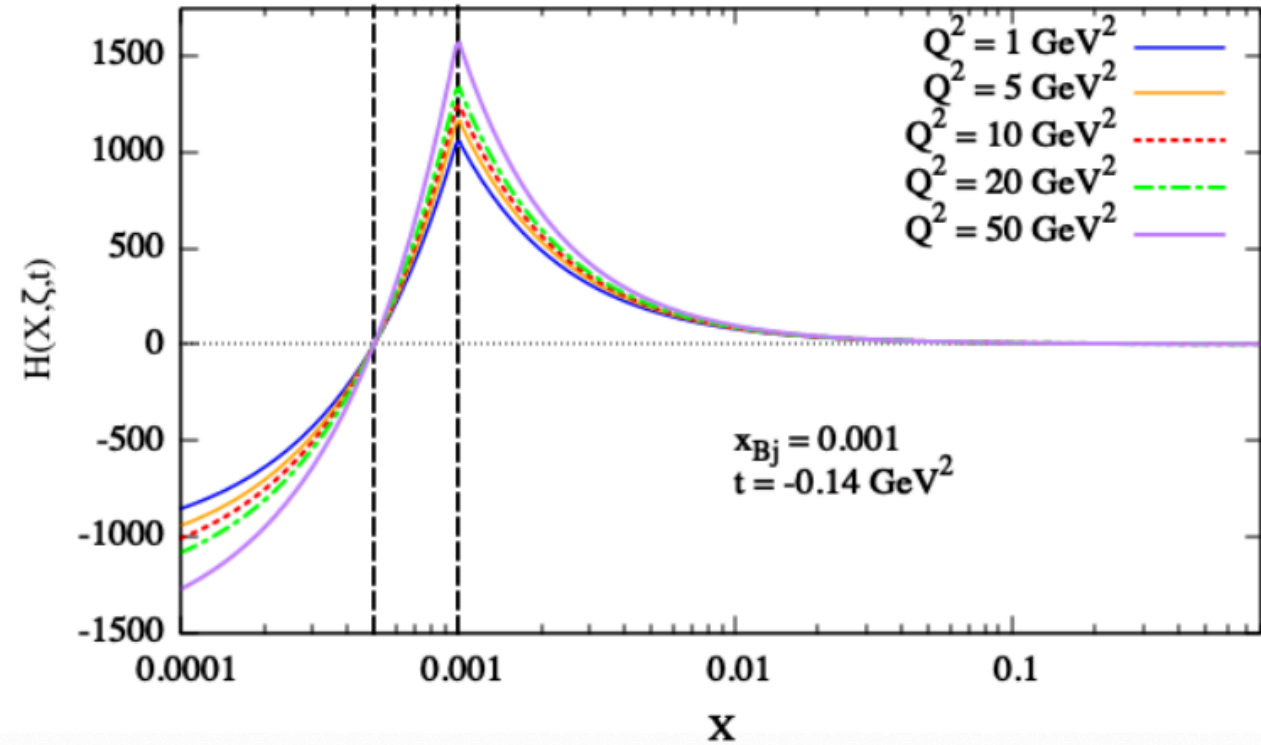
(*) T-odd

B. Kriesten and S. Liuti, *Phys.Rev. D105* (2022), arXiv [2004.08890](https://arxiv.org/abs/2004.08890)

[1] Meissner, Metz and Schlegel, *JHEP*(2009)

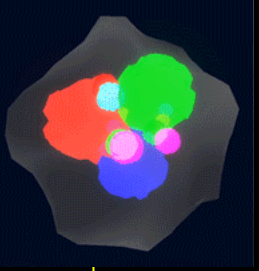
2. How do we separate twist two and twist three components?

- Q^2 dependence



Conclusions and Outlook

- One-body densities have provided a great insight into the internal structure of the proton, but we can go beyond them
- Using the two-body density formalism for studying the distribution of quarks and gluons inside the proton at various values of X and Q^2
- Exploring new set of exclusive observables connected to double parton distributions
- Extracting 3D information from data is an unprecedented challenging problem which is uniquely highly-dimensional with respect to what done in DIS and SIDIS: it is important to keep developing ML-based approaches and to build a platform with benchmarks for the community to compare results with both epistemic and aleatory uncertainties



- More refined statistical analyses are needed including Machine Learning methods:

EXCLAIM – Collaboration

EXCLusives via Artificial Intelligence and Machine learning

- ✓ EXPERIMENT: M. Boer, T. Horn
- ✓ LATTICE QCD: M. Engelhardt, H-W Lin
- ✓ ML: G-W Chern, Y. Li, M. Almaeen, P. Alonzi, J. Hoskins,
- ✓ PHENOMENOLOGY: G. Goldstein, SL, M. Sievert, A. Courtoy, B. Kriesten