School

Piet Verwilligen
Rob Veenhof

Why computer models ?

- Less work than building actual detectors.
- Cheaper:
  - in particular when studying noble gases.
- Easier to modify the parameters of the model:
  - makes it e.g. easy to understand how the gas composition influences the transport parameters of electrons in gas mixtures.

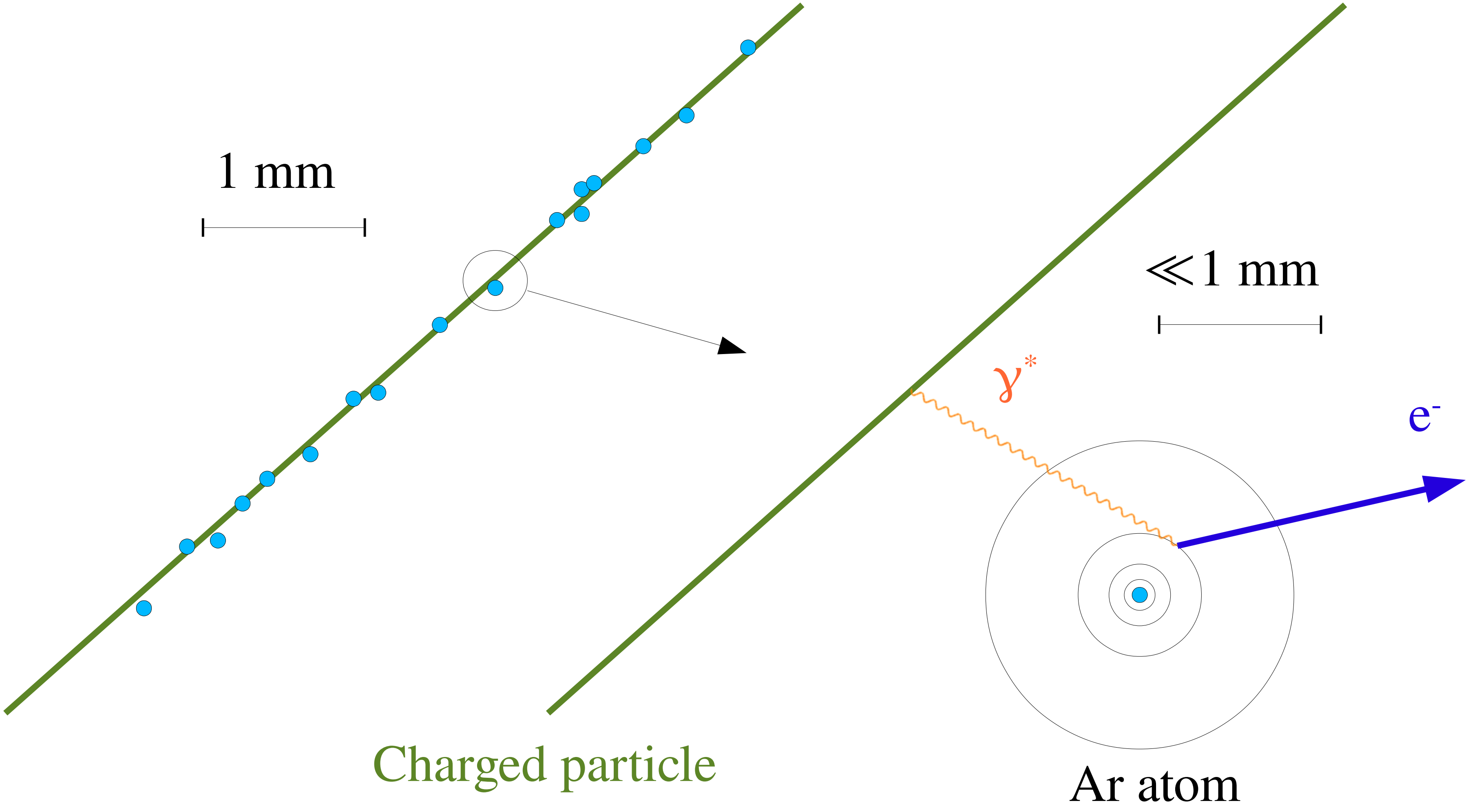
Which models ?

- What do we currently model and simulate ?
  - ionisation trace from charged particles;
  - electric and magnetic fields, govern motion of ions & electrons;
  - properties of gas mixtures;
    - velocity of electrons and ions;
    - diffusion of electrons (for ions see Mason & McDaniel et al.);
    - electron attachment (2-body and 3-body);
    - ionisation and excitation;
    - electron avalanches;
  - ion chemistry;
  - calculation of signals, now also with resistive layers;
  - wire sagging;
  - replacement of environment-contaminating gases.

Ionisation - How

SRIM	low energy charged particles;
Bethe	common for charged particles;
PAI	idem, only energy loss;
Heed	idem, with shell structury.

Virtual photon exchange



Basic formulae of the PAI model

- Key ingredient: photo-absorption cross section $\sigma_\gamma(E)$

$$\frac{\beta^2\pi}{\alpha}\frac{d\sigma}{dE}=\frac{\sigma_\gamma(E)}{E}\log\left(\frac{1}{\sqrt{(1-\beta^2\epsilon_1)^2+\beta^4\epsilon_2^2}}\right)+$$

Relativistic rise

Cross section to transfer energy E

$$\frac{1}{N\hbar c}\left(\beta^2-\frac{\epsilon_1}{|\epsilon|^2}\right)\theta+$$

Черенко́в

$$\frac{\sigma_\gamma(E)}{E}\log\left(\frac{2m_ec^2\beta^2}{E}\right)+$$

Resonance region

$$\frac{1}{E^2}\int_0^E\sigma_\gamma(E_1)\,dE_1$$

Rutherford scattering

With:

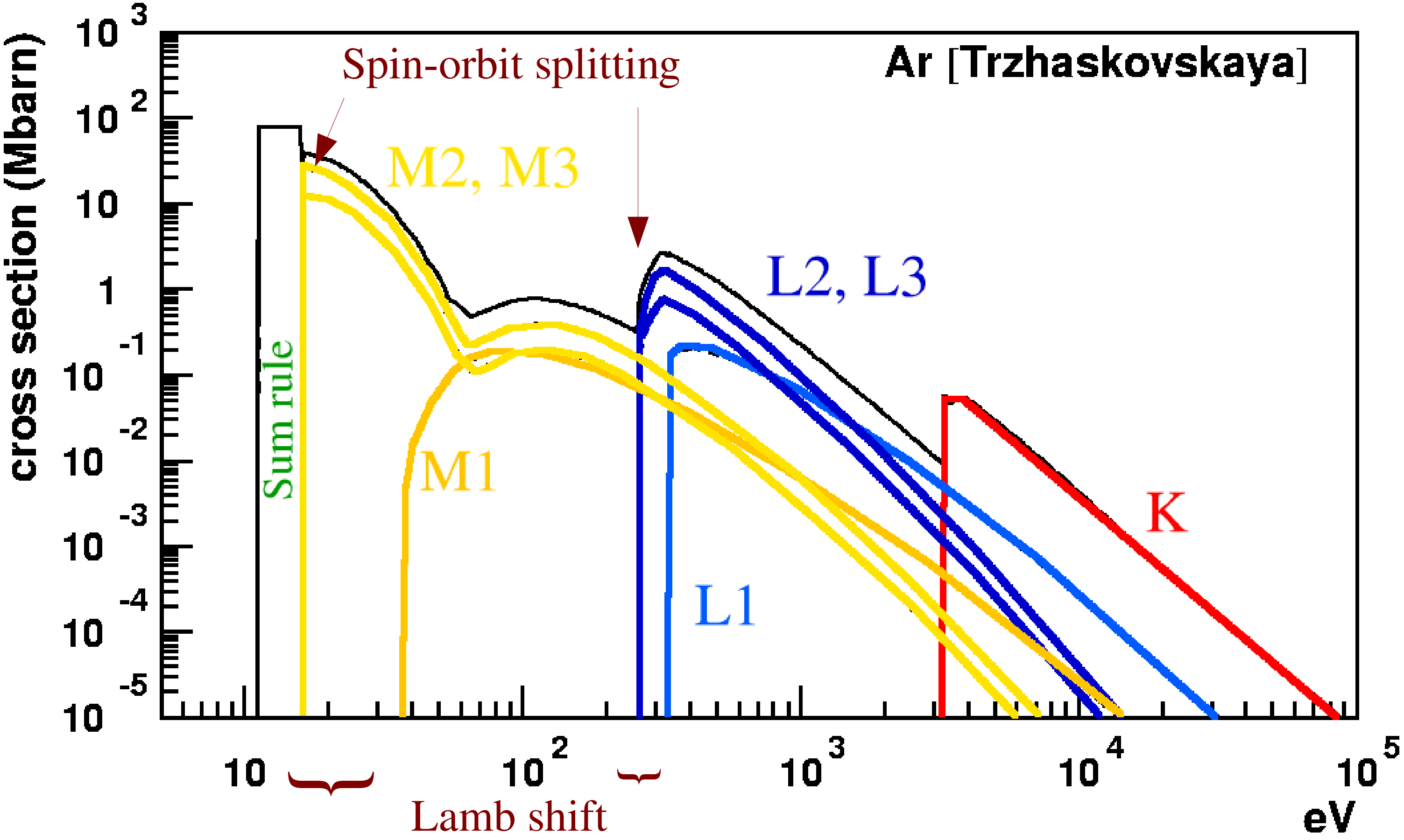
$$\epsilon_2(E)=\frac{N_e\hbar c}{EZ}\sigma_\gamma(E)$$

$$\epsilon_1(E)=1+\frac{2}{\pi}\mathrm{P}\int_0^\infty\frac{x\epsilon_2(x)}{x^2-E^2}\,dx$$

$$\theta=\arg(1-\epsilon_1\beta^2+i\epsilon_2\beta^2)=\frac{\pi}{2}-\arctan\frac{1-\epsilon_1\beta^2}{\epsilon_2\beta^2}$$

Photo-absorption in argon

- Argon has 3 shells, hence 3 groups of lines:



[Plot from Igor Smirnov]

K = 1s

L1 = 2s
L2 = 2p 1/2
L3 = 2p 3/2

M1 = 3s
M2 = 3p 1/2
M3 = 3p 3/2

Transport in gases – How ?

- Over simplified approach, collision to collision.
- Traditionally the Boltzmann equation is solved by Legendre polynomial expansion to 2nd or 3rd order.
- Around 1962, Art Phelps introduced the numeric approach.
- This is also what Magboltz (Steve Biagi) does.

Mean free path in argon

- We know (e.g. from literature) that:
  - Cross section of 1 atom: $\sigma \approx 1.5\ 10^{-16}$ cm^2
  - Atoms per volume: $n_0 \approx 2.7\ 10^{19}$ $\mathrm{atoms/cm}^3$

- Mean free path for an electron ?
  - An electron hits all atoms of which the centre is less than a cross section σ radius from its path;
  - over a distance L, the electron hits $n_0\ \sigma\ L$ atoms;
  - mean free path = distance over which it hits 1 atom;

    $$\lambda_e = 1/(\sigma\ n_0) \approx 2.5\ \mu\mathrm{m}$$

  - much larger than
    - 4 nm distance between atoms, and
    - 140-600 pm typical gas molecule diameters.

MPGDs and the mean free path

- Recall:
  - Mean free path of electrons in Ar: 2.5 µm,
- Compare with:
  - Micromegas mesh pitch: 63.5 µm
  - GEM polyimide thickness: 50 µm
  - Micromegas wire thickness: 18 µm
  - GEM conductor thickness: 5 µm
- Hence:
  - mean free path approaches small structural elements;
  - such devices should be treated at a molecular level.

Drift velocity in argon

- Compare with a Magboltz calculation for pure argon:
  - *E* dependence is not too far off (although linear is more common at low fields),

    BUT

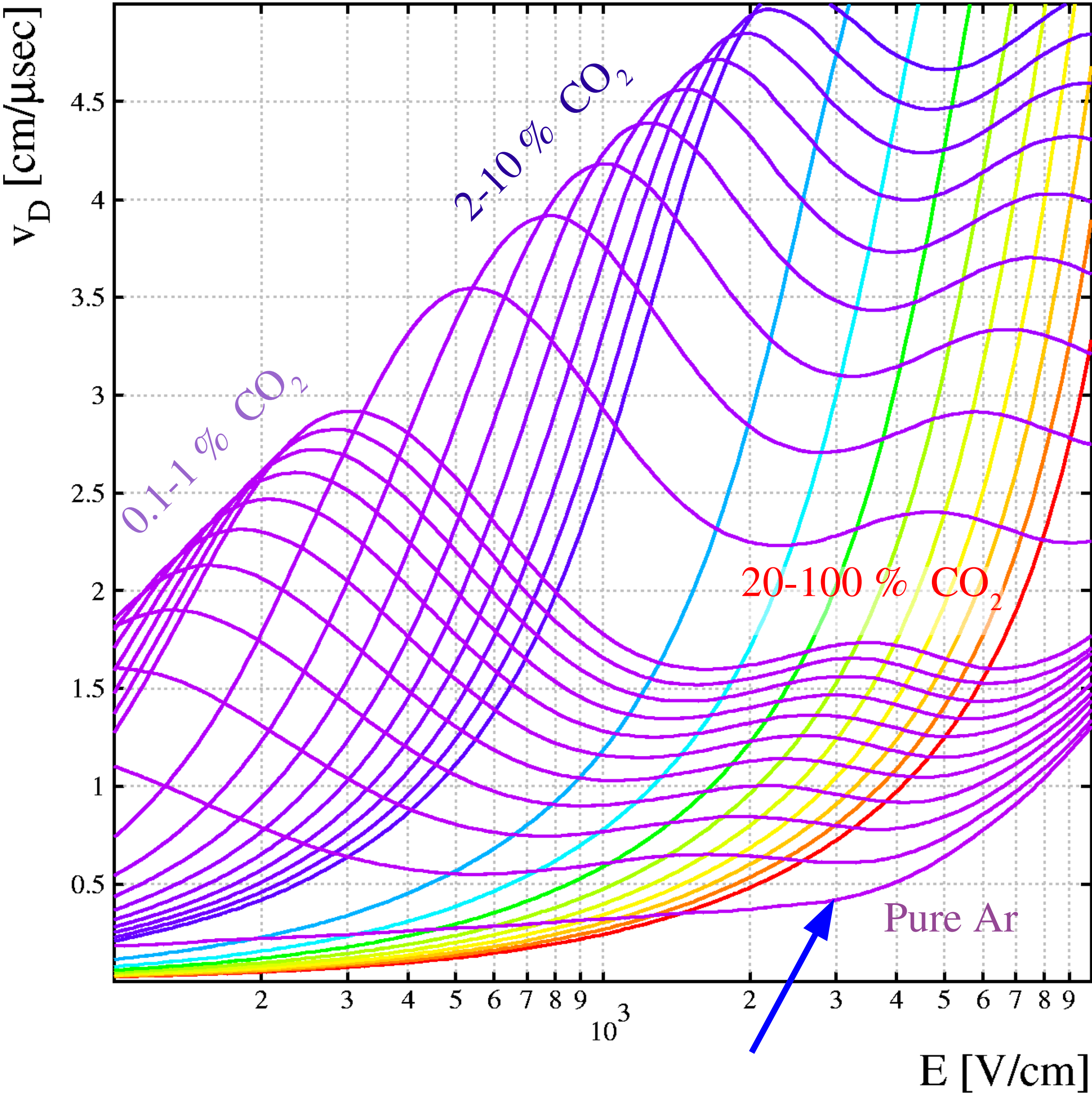
  - the velocity is *vastly* overestimated ! Magboltz finds a velocity that is *30 times* smaller ...

    WHY ?

Ar - CO_2

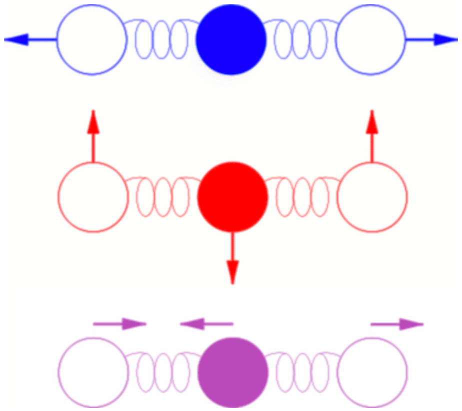
- CO_2 makes the gas faster, dramatically.

- Calculated by Magboltz for Ar/CO_2 at 3 bar.
  (Note where the arrow is !)

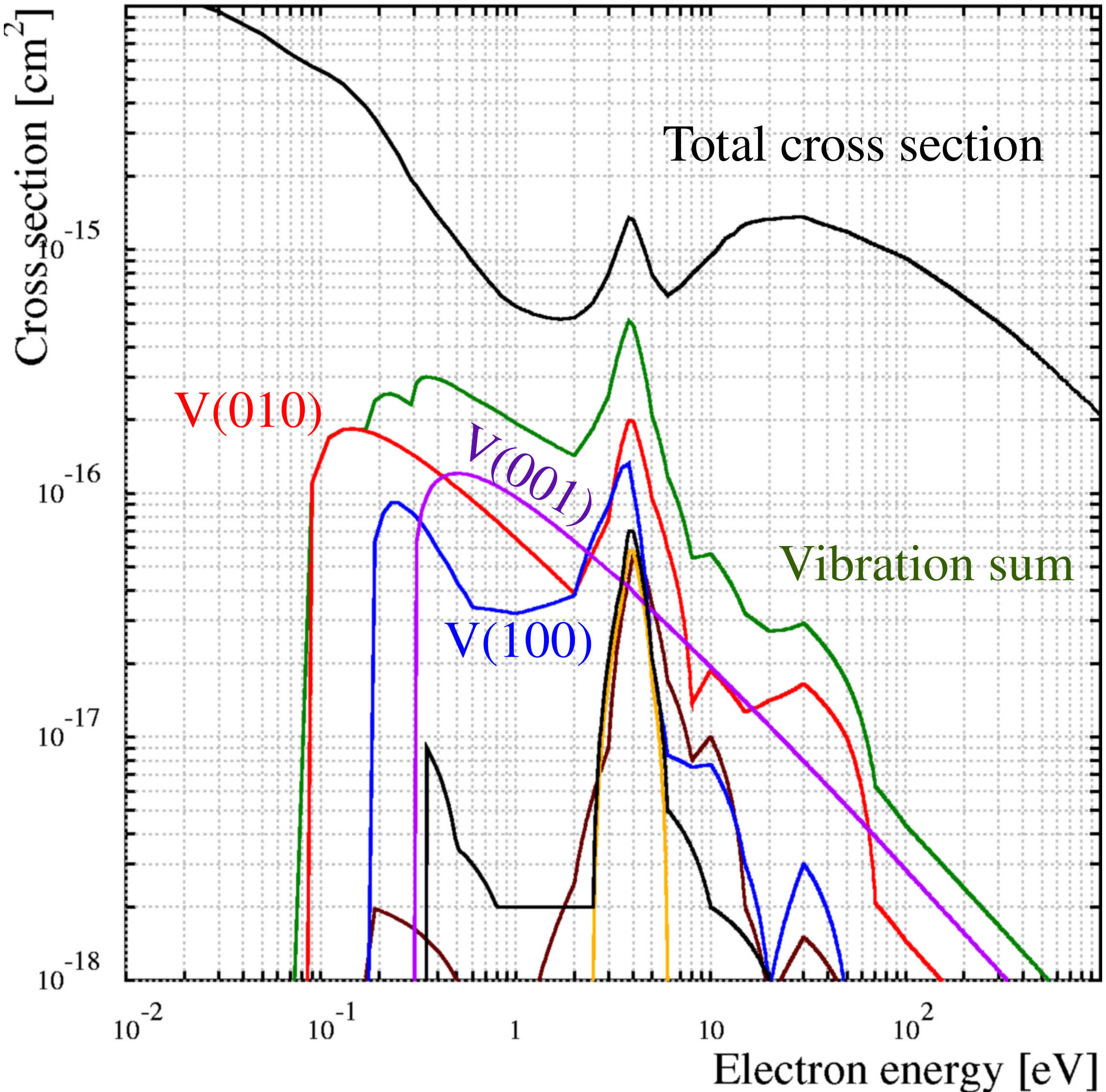


CO_2 – vibration modes

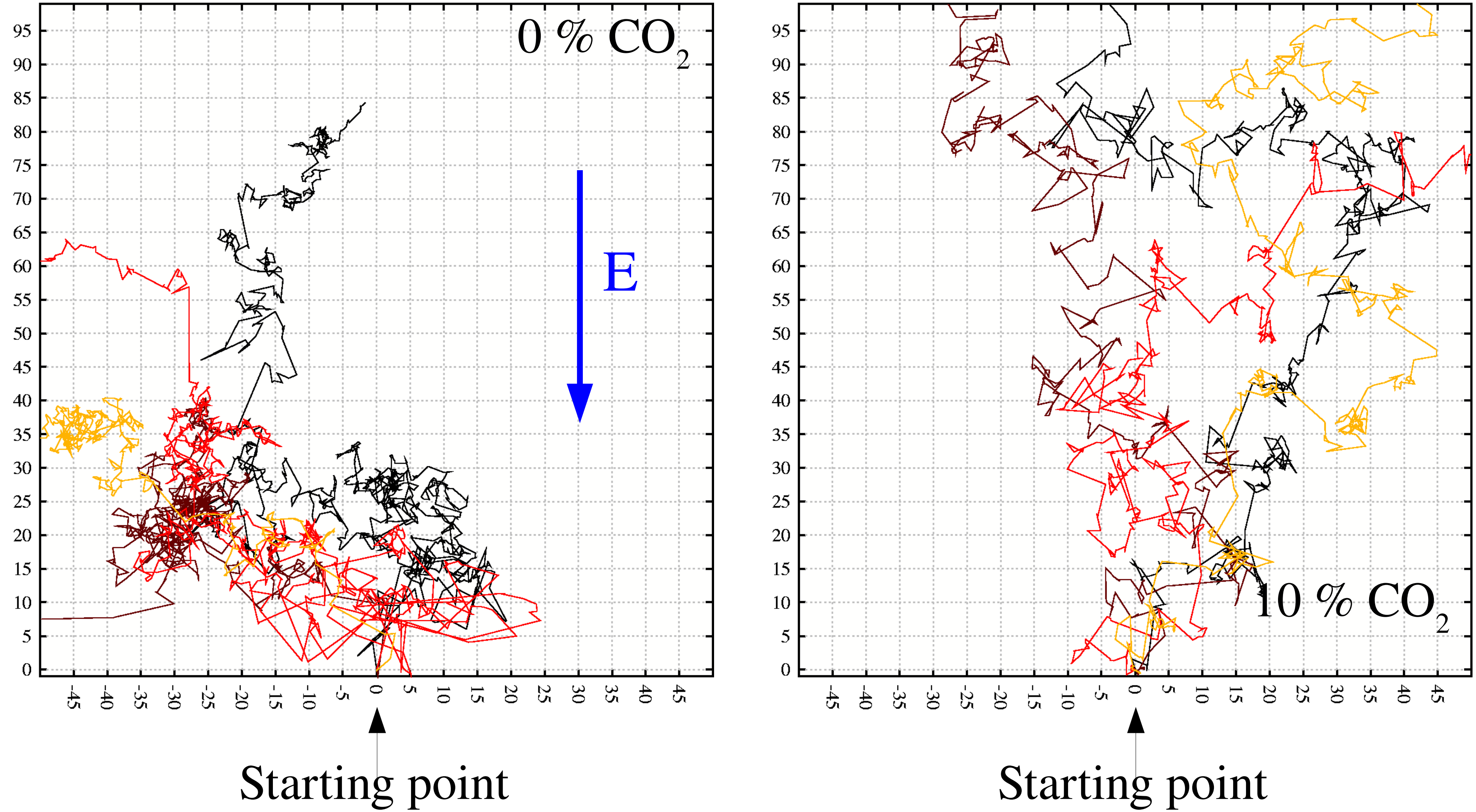
- CO_2 is linear:
  - O – C – O
- Vibration modes are numbered V(*ijk*)
  - *i*: symmetric,
  - *j*: bending,
  - *k*: anti-symmetric.



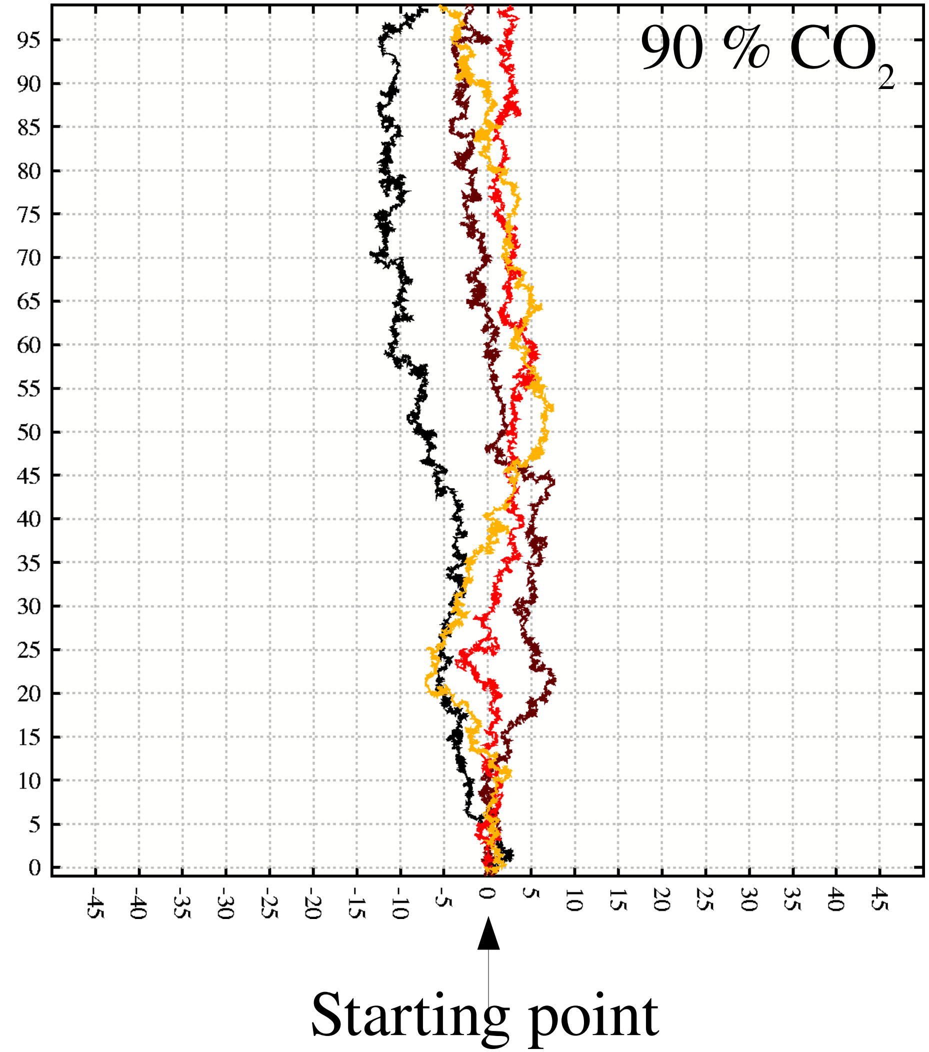
Vibrations V(ijk)

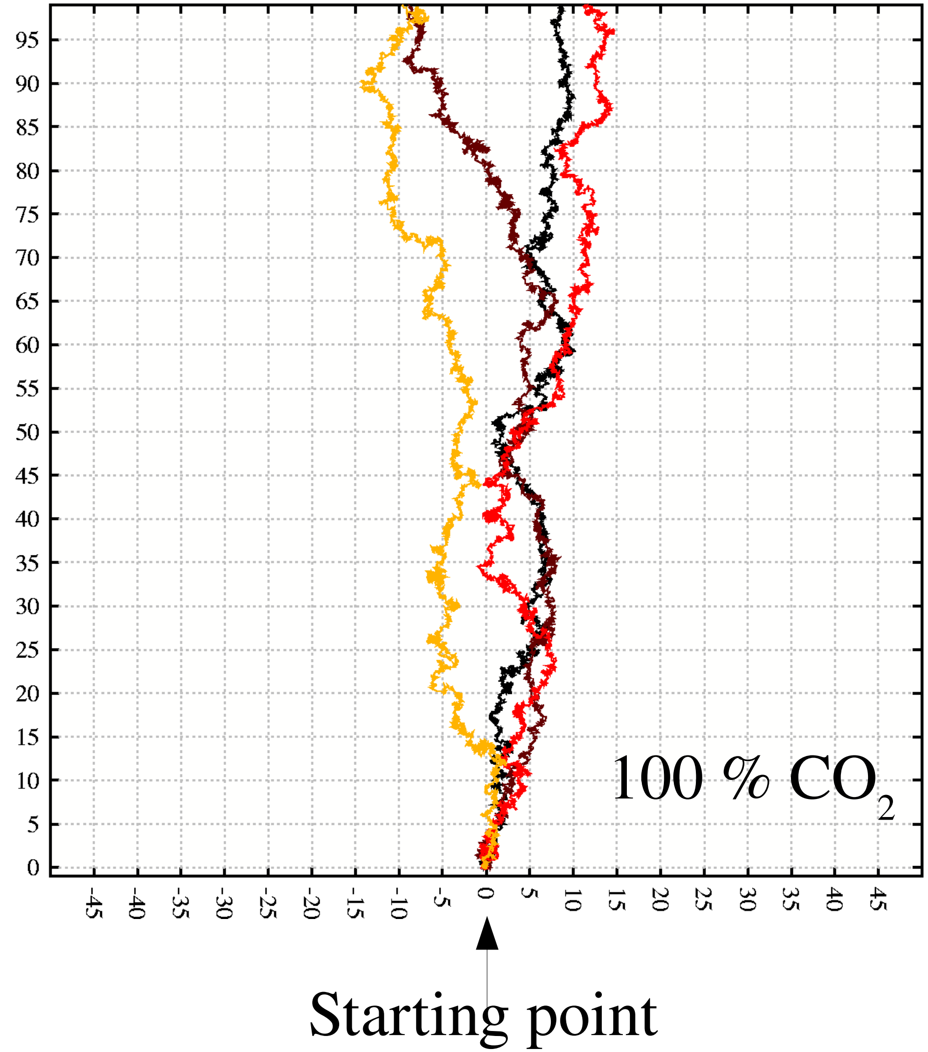


Electrons in Ar/CO$_2$ at E=1 kV/cm



Electrons in Ar/CO$_2$ at *E*=1 kV/cm



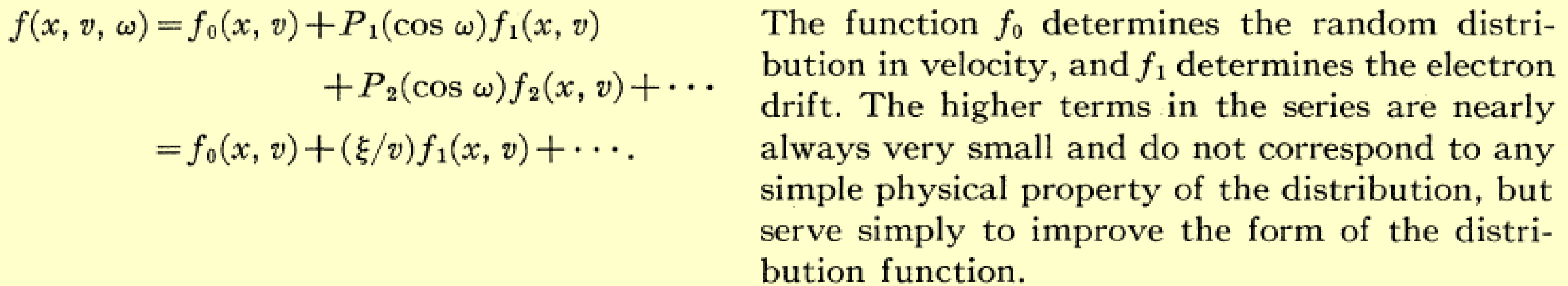


Calculating transport properties

- One can of course measure every mixture one needs …
- … but it would be far more efficient if one could compute the transport properties of arbitrary mixtures.

1935: Electron energy distribution

- Calculation of the electron energy distribution
  - allowing for energy loss in elastic collisions;
  - detailed balancing of energy and momentum, gain (E-field, diffusion) and loss (elastic collision);
  - velocity dependent cross section;
  - use of Legendre expansion (crediting H.A. Lorentz, 1916):



$$\begin{aligned} f(x, v, \omega) &= f_0(x, v) + P_1(\cos\omega) f_1(x, v) \\ &\quad + P_2(\cos\omega) f_2(x, v) + \cdots \\ &= f_0(x, v) + (\xi/v) f_1(x, v) + \cdots. \end{aligned}$$

The function f_0 determines the random distribution in velocity, and f_1 determines the electron drift. The higher terms in the series are nearly always very small and do not correspond to any simple physical property of the distribution, but serve simply to improve the form of the distribution function.

(P_1, P_2: Legendre polynomials)

[Philip M. Morse, W.P. Allis and E.S. Lamar, *Velocity Distributions for Elastically Colliding Electrons*, Phys. Rev. **48** (1935) 412–419]

Arthur V. Phelps
(1923 - 2012)

1962: Numerical e^- transport

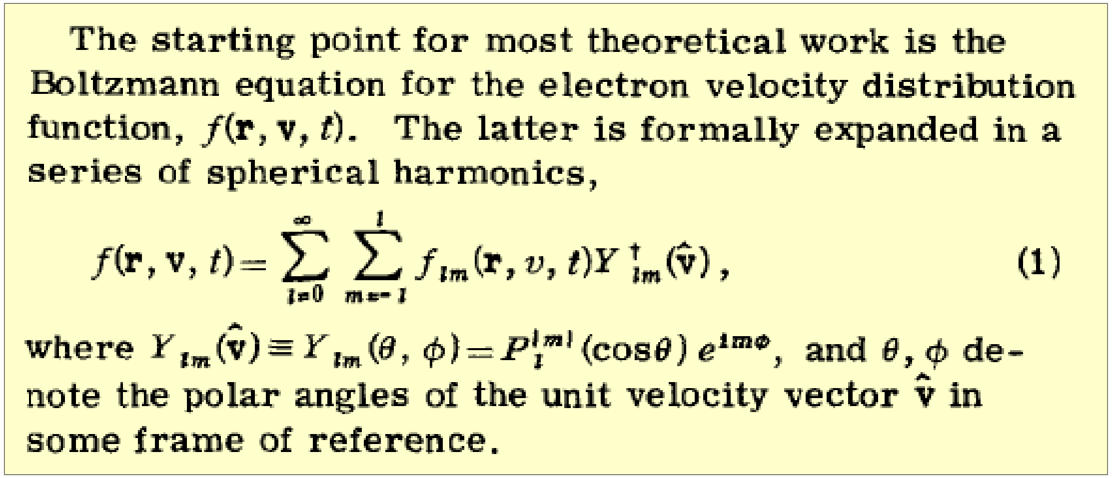
- Iterative approach, allowing for inelastic cross section terms:
  - educated guess of cross sections (elastic & inelastic);
  - numerically solve the Boltzmann equation (no moments);
  - compare calculated and measured mobility and diffusion;
  - adjust cross sections.

"... more than 50,000 transistors plus extremely fast magnetic core storage. The new system can simultaneously read and write electronically at the rate of 3,000,000 bits of information a second, when eight data channels are in use. In 2.18 millionths of a second, it can locate and make ready for use any of 32,768 data or instruction numbers (each of 10 digits) in the magnetic core storage. The 7090 can perform any of the following operations in one second: 229,000 additions or subtractions, 39,500 multiplications, or 32,700 divisions. " (IBM 7090 documentation)

[L.S. Frost and A.V. Phelps, *Rotational Excitation and Momentum Transfer Cross Sections for Electrons in* H_2 *and* N_2 *from Transport Coefficients*, Phys. Rev. **127** (1962) 1621–1633.]

1980s: Higher moments, high precision

- Expansion in spherical harmonics;
- An accuracy of 1 % (and better) becomes routine.



The starting point for most theoretical work is the Boltzmann equation for the electron velocity distribution function, $f(\mathbf{r}, \mathbf{v}, t)$. The latter is formally expanded in a series of spherical harmonics,

$$f(\mathbf{r}, \mathbf{v}, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_{lm}(\mathbf{r}, v, t) Y^{\dagger}_{lm}(\hat{\mathbf{v}}), \tag{1}$$

where $Y_{lm}(\hat{\mathbf{v}}) \equiv Y_{lm}(\theta, \phi) = P_l^{|m|}(\cos\theta)\, e^{im\phi}$, and θ, ϕ denote the polar angles of the unit velocity vector $\hat{\mathbf{v}}$ in some frame of reference.

S.L. Lin, R.E. Robson and E.A. Mason, *Moment theory of electron drift and diffusion in neutral gases in an electrostatic field,* J. Chem. Phys. **71** (1979) 3483-3498 (the “LRM” paper).

R.E. Robson and K.F. Ness, *Velocity distribution function and transport coefficients of electron swarms in gases: Spherical-harmonics decomposition of Boltzmann’s equation*, Phys. Rev. A **33** (1986) 2068–2077.

K.F. Ness and R.E. Robson, *Velocity distribution function and transport coefficients of electron swarms in gases. II. Moment equations and applications*, Phys. Rev. A **34** (1986) 2185–2209.

Magboltz: microscopic e^- transport

- A large number of cross sections for 60 molecules...
  - Numerous organic gases, additives, *e.g.* CO_2:
    - elastic scattering,
    - 44 inelastic cross sections (5 vibrations and 30 rotations + super-elastic and 9 polyads),
    - attachment,
    - 6 excited states and
    - 3 ionisations.
  - noble gases (He, Ne, Ar, Kr, Xe):
    - elastic scattering,
    - 44 excited states and
    - 7 ionisations.

LXcat

- LXcat (pronounced *elecscat*) is an open-access website for collecting, displaying, and downloading ELECtron SCATtering cross sections and swarm parameters (mobility, diffusion coefficient, reaction rates, etc.) required for modeling low temperature plasmas. [...]”

[http://www.lxcat.laplace.univ-tlse.fr/]

LXcat people

- Art Phelps,
- Leanne Pitchford – Toulouse,
- Klaus Bartschat – Iowa,
- Oleg Zatsarinny – Iowa,
- Michael Allan – Fribourg,
- Steve Biagi
- ...

Art Phelps

Leanne Pitchford

Michael Allan

Klaus Bartschat

Greenhouse gases - How

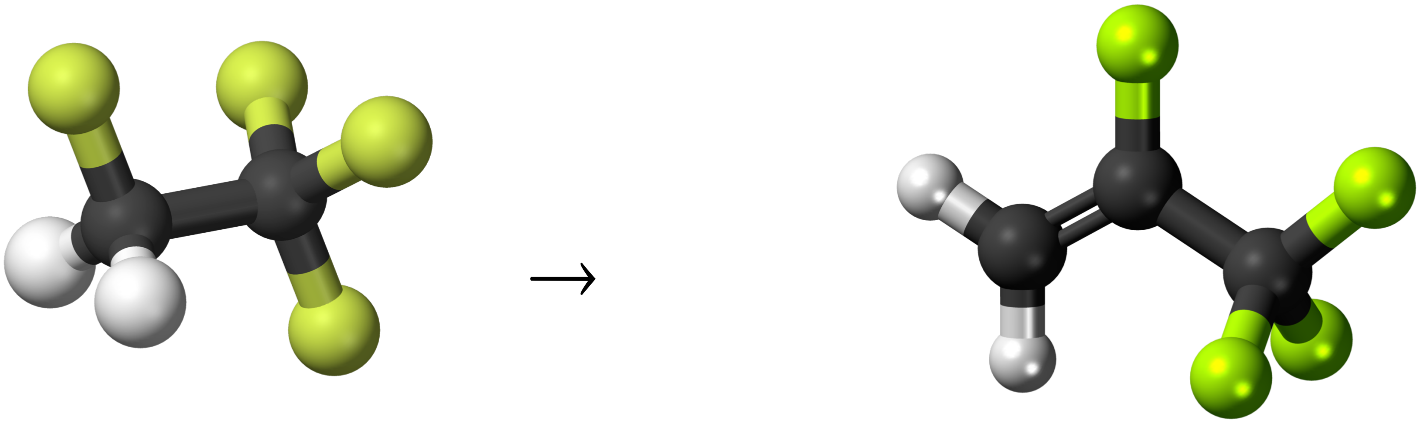
- Besides Ar and CO_2 other gases are used at present.

- Some of these have undesirable properties, damaging the atmosphere and groundwater. The Kyoto-7 list:
  - carbon dioxide (CO_2),
  - methane (CH_4),
  - nitrous oxide (N_2O),
  - hydrofluorocarbons (HFCs eg R134a),
  - perfluorocarbons (PFCs),
  - sulphur hexafluoride (SF_6), and
  - nitrogen trifluoride (NF_3).
- CF_4 has long life and high GWP.

- These are being phased out. HFOs are candidates.

HFO

- HFO stands for hydrofluoroolefin.
- They are HFCs which have a double C=C bond.
- This makes them more reactive and gives them a shorter lifetime.
- They are slated to replace R134a (CH_2F-CF_3):



1,1,1,2-tetrafluoroethane → 2,3,3,3-tetrafluoropropene

Good and bad properties of HFOs

- HFO-1234yf $CH_2=CF-CF_3$
  - air conditioning in cars, to replace R134a

- Good:
  - low GWP: < 1,
  - short atmospheric lifetime: 10-20 days,
  - ODP ~ 0 in gases made only with fluorine; a chlorinated gas serves as reference (R11); and ODP is large with brominated substances.
- Bad:
  - concern about flammability,
  - produces HF when burning,
  - atmospheric decay to trifluoroacetic acid (phytotoxic, aquatic life).

Lifetime, GWP, ODP

- Properties of some common compounds:

Gas	Lifetime [years]	GWP [100 years]	ODP
CO_2	10-100	reference	0
CCl_3F (R11)	45	4660	reference
CH_4	12.4	28	
CH_2F-CF_3 (R134a)	13.4	1300	0
CHF_2-CHF_2 (R134)	9.7	1120	0
CH_3-CF_3 (R143a)	47.1	4800	0
CHF_2-CF_3 (R125)	28.2	3170	0
CF_4	50000	6630	0
C_2F_6	10000	11100	0
C_3F_8	2600	8900	0
SF_6	3200	23500	0 ?
CF_3Br (Halon 1301)	65	6290	16
1234yf	10.5 days	<1	0
(Z) 1234ze	10 days	<1	0
(E) 1234ze	16.4 days	<1	0

[from IPCC WG1/AR5]

Field calculation techniques – How ?

- Closed expressions, “analytic method”:
  - almost all 2d structures of wires, planes + periodicities;
  - dielectrics and space/surface charge are laborious;
  - fast and precise, if applicable.

- Finite element method:
  - 2d and 3d structures, with or without dielectrics;
  - several major intrinsic shortcomings.

- Integral equations or Boundary element methods:
  - equally comprehensive without the intrinsic flaws;
  - technically challenging and emerging;
  - consumes more CPU time than FEM, but catching up.

- Finite differences:
  - used for iterative, time-dependent calculations.

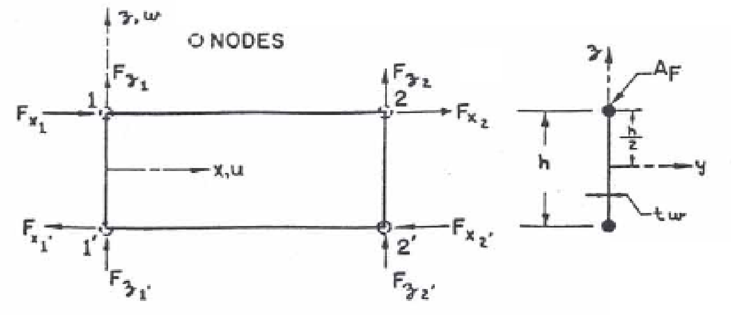
Analytic field calculations (2d only)

- Analytic calculations rely on complex functions because of two remarkable properties:
  - Cauchy-Riemann equations:
    - The real part of *any* complex analytic function is a valid potential function.
  - Conformal mapping:
    - Almost *every* analytic geometric transformation of a valid complex potential, is a valid complex potential too.
- Applicability:
  - drift chambers, TPCs, MWPCs, drift tubes, hexagonal counters and other 2d detectors.

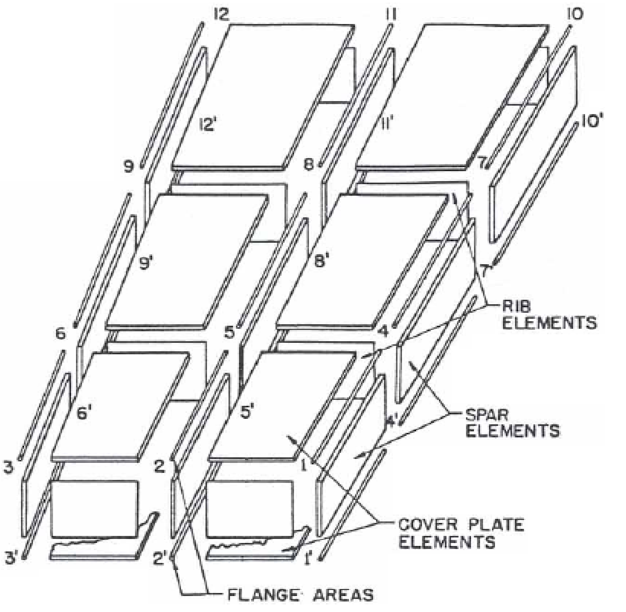
Ιωάννης Αργύρης
(1913-2004)

Aircraft wings – finite elements

- “*Stiffness and Deflection Analysis of Complex Structures*”, a study in the use of the finite element technique (then called “direct stiffness method”) for aircraft wing design.



$$[K] = \frac{6EI}{Lh^2(1+4n)} \begin{array}{c} \begin{array}{cccccc} u_1 & v_1 & w_1 & u_2 & v_2 & w_2 \end{array} \\ \begin{bmatrix} (4/3)\,(1+n) & & & & & \\ 0 & 0 & & & & \\ -(h/L) & 0 & h^2/L^2 & & & \\ (2/3)\,(1-2n) & 0 & -(h/L) & (4/3)\,(1+n) & & \\ 0 & 0 & 0 & 0 & 0 & \\ h/L & 0 & -(h^2/L^2) & h/L & 0 & h^2/L^2 \end{bmatrix} \end{array}$$



[M.J. Turner, R.W. Clough, H.C. Martin and L.J. Topp, *Stiffness and Deflection Analysis of Complex Structures*, J. Aero. Sc. **23** (1956), 805-824. MJT & LJT with Boeing.]

Electron transport – How ?

- RKF (Runge Kutta with automatic step adjustment).
- Microscopic – routine for MPGDs.
- Monte Carlo – deprecated because the step length is not correctly simulated.
- Can be combined with gas gain.

Ion transport and chemistry – How ?

- Using the ion mobility – beware of unphysical approaches, but commonly used !

- E.g. Ar^+ does not drift long distances in CO_2.

Avalanche ions in Ar-CO_2 (93-7)

	Ion	Energy [eV]	Rate [GHz]	Fraction
Ar	Ar^+	15.75961	57.67	93.3 %
CO_2	CO_2^+	13.776	3.234	5.2 %
	CO_2^{+*}	17.314	0.3922	
	CO_2^{+*}	18.077	0.2272	
	O^+	19.07	0.1374	
	CO^+	19.47	0.1593	

Magboltz 11.2bis,
E = 100 kV/cm
1 atm, 20 C

Reactions in Ar-CO_2

$Ar^+ + Ar + M \rightarrow Ar_2^+ + M$	100 %	$k = 2.3\ 10^{-31}$	± 10 %	cm^6/s
$Ar^+ + CO_2 \rightarrow CO_2^+ + Ar$	100 %	$k = 5\ 10^{-10}$	± 10 %	cm^3/s
$CO_2^+ + CO_2 + M \rightarrow CO_2 \bullet CO_2^+ + M$	100 %	$k = 2.4\ 10^{-28}$	+-	cm^6/s

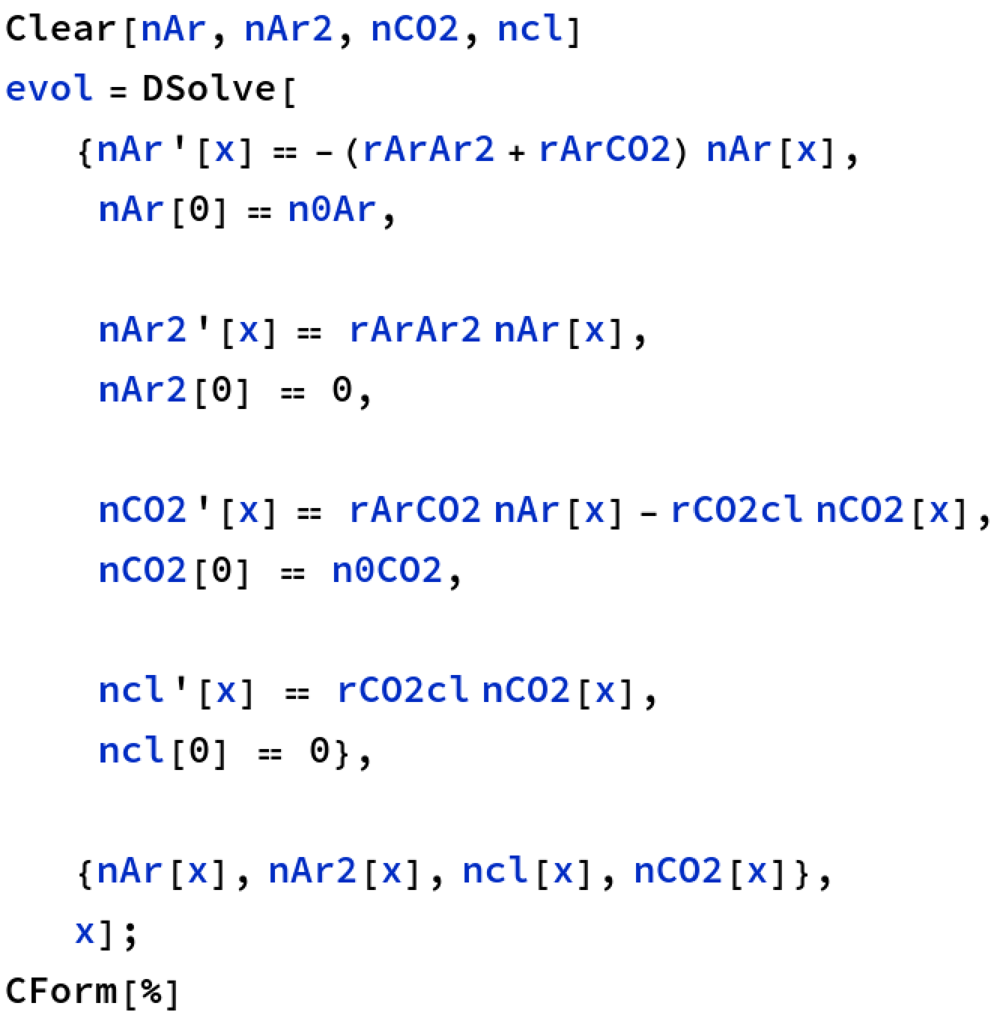
[B.M. Smirnov, Cluster Ions and Van Der Waals Molecules]
[V.G. Anicich and W.T. Huntress Jr., Astrophys. J. Suppl. **62** (1986) 553-672.]

Reaction rate constant

- Consider a charge transfer reaction $A^+ B \rightarrow A B^+$:
    - rate ∝ density of B molecules N_B [1/cm^3];
- The proportionality factor is called rate constant k:
    - $\text{rate}[1/\text{s}] = k\,[\text{cm}^3/\text{s}]\ N_B[1/\text{cm}^3]$

mathematica

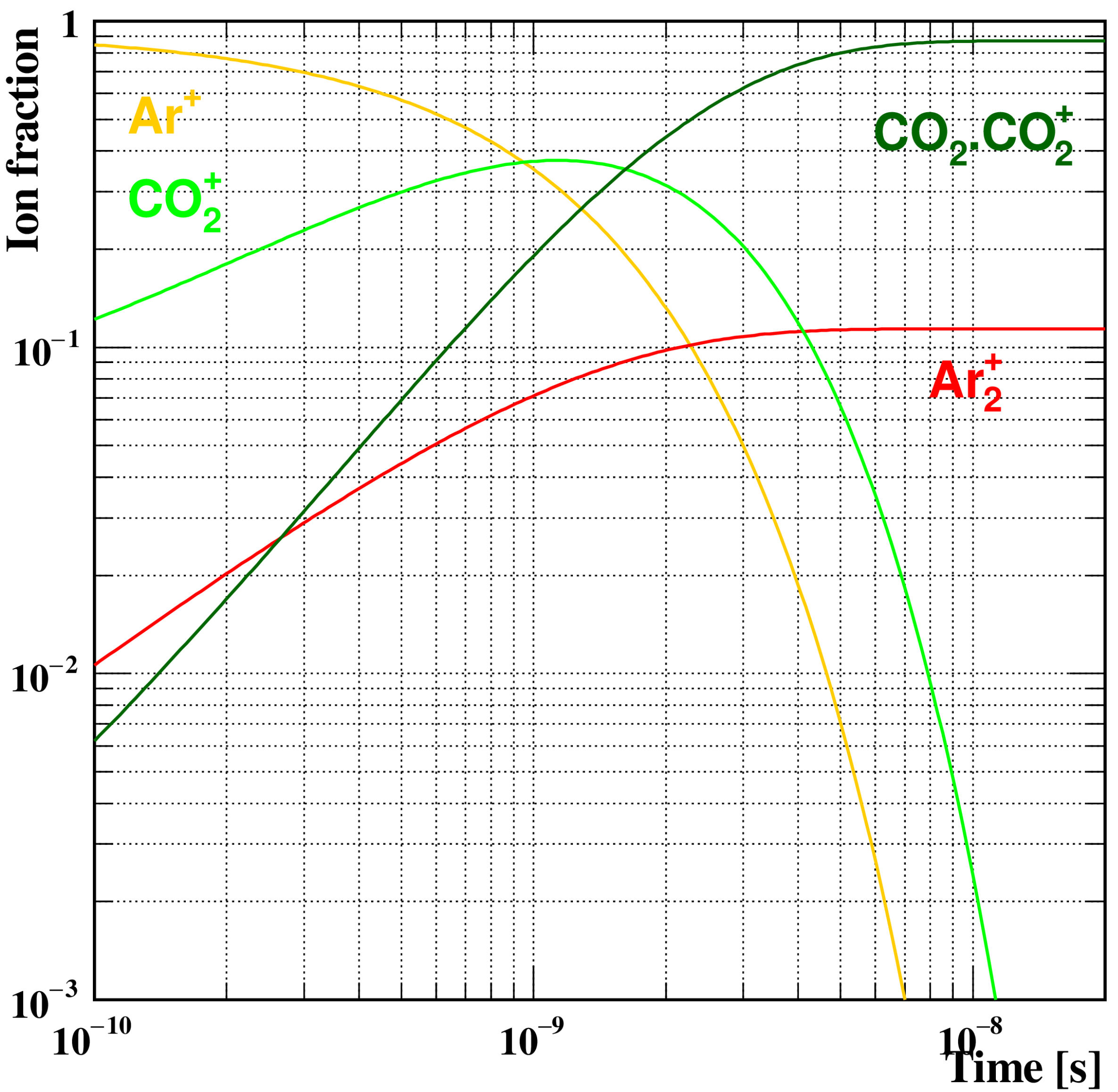
- Start from the number of Ar^+ and CO_2^+ produced in the avalanche.
- Calculate evolution of Ar^+, CO_2^+, $CO_2^+ \cdot CO_2$ clusters and Ar_2^+ dimers.



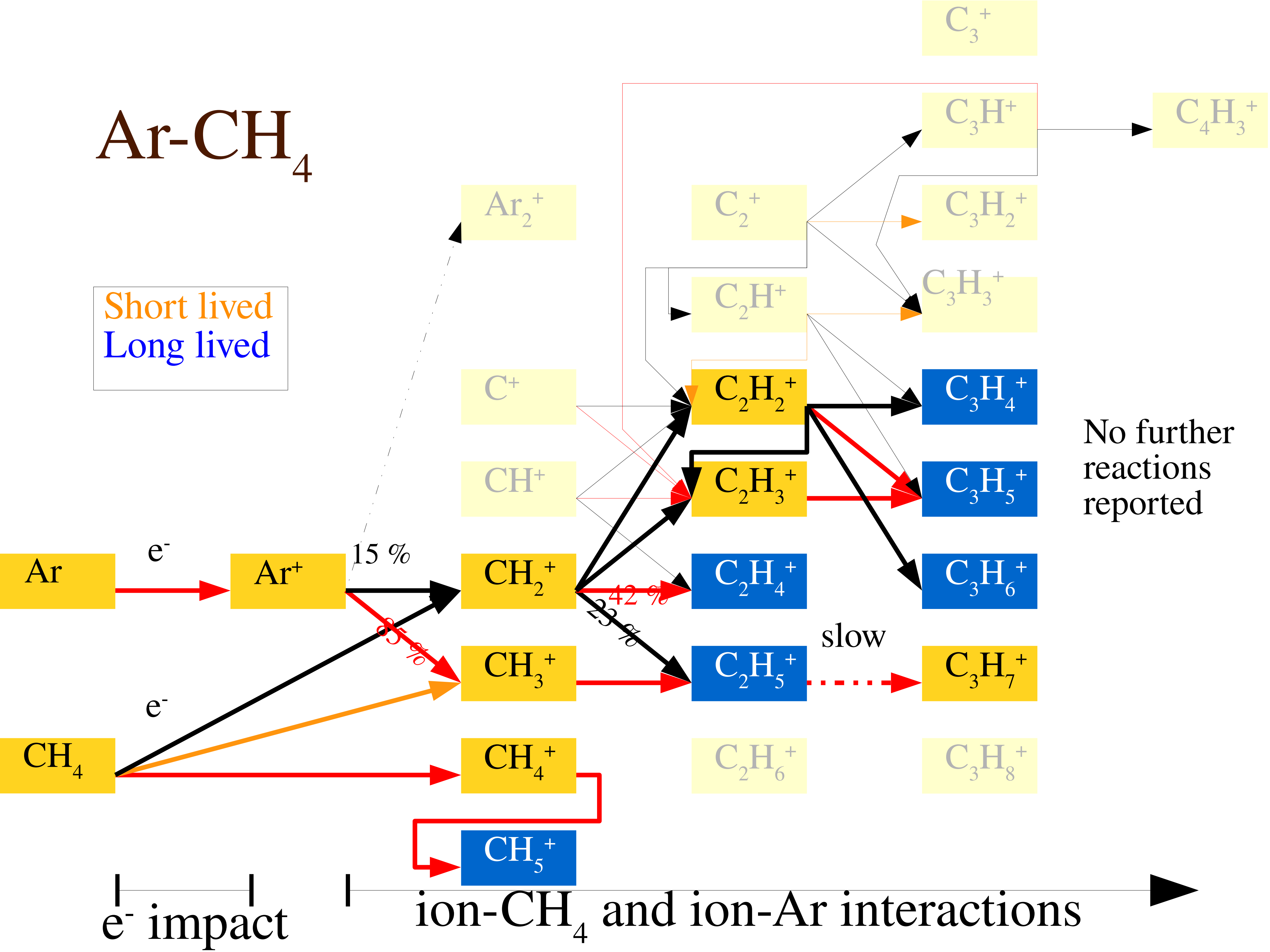
```
Clear[nAr, nAr2, nCO2, ncl]
evol = DSolve[
    {nAr'[x] == -(rArAr2 + rArCO2) nAr[x],
     nAr[0] == n0Ar,

     nAr2'[x] == rArAr2 nAr[x],
     nAr2[0] == 0,

     nCO2'[x] == rArCO2 nAr[x] - rCO2cl nCO2[x],
     nCO2[0] == n0CO2,

     ncl'[x] == rCO2cl nCO2[x],
     ncl[0] == 0},

    {nAr[x], nAr2[x], ncl[x], nCO2[x]},
    x];
CForm[%]
```

Evolution in Ar-CO_2 (93-7)

- Initial ions:
  - Ar^+ →
    - CO_2^+
    - Ar_2^+
  - CO_2^+ →
    - $CO_2^+.CO_2$

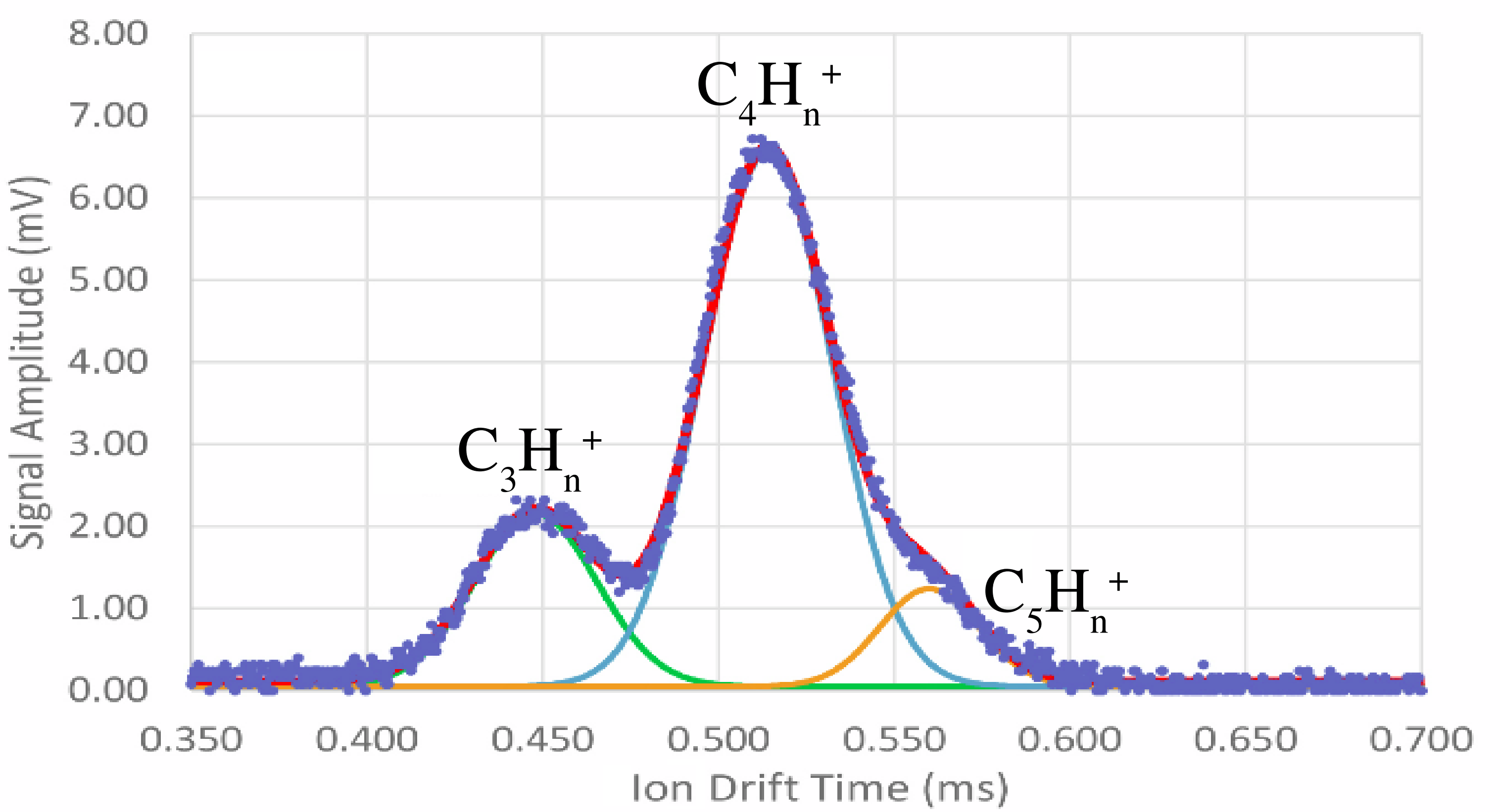


Ar-CH_4


Short lived
Long lived
Ar
e^-
Ar^+
15 %
85 %
CH_4
e^-
Ar_2^+
C^+
CH^+
CH_2^+
CH_3^+
CH_4^+
CH_5^+
C_2^+
C_2H^+
$C_2H_2^+$
$C_2H_3^+$
$C_2H_4^+$
42 %
23 %
$C_2H_5^+$
$C_2H_6^+$
C_3^+
C_3H^+
$C_3H_2^+$
$C_3H_3^+$
$C_3H_4^+$
$C_3H_5^+$
$C_3H_6^+$
slow
$C_3H_7^+$
$C_3H_8^+$
$C_4H_3^+$
No further reactions reported
e^- impact
ion-CH_4 and ion-Ar interactions

How about alkanes ?

- Ar 90 % - C_2H_6 10 %, at low pressure.
- Expect Ar^+ or C_2H_6 but ... none are seen – why ?



[André Cortez et al. 10.1088/1748-0221/8/12/P12012]

Avalanche ions in Ar-CO_2-CH_4 (90-7-3)

	Ion	Energy [eV]	Rate [GHz]	Fraction
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CO_2	CO_2^+	13.776	3.119	5.1 %
	CO_2^{+*}	17.314	0.3758	
	CO_2^{+*}	18.077	0.2218	
	O^+	19.07	0.1324	
	CO^+	19.47	0.1525	
CH_4	CH_4^+	12.65	1.959	3.2 %
	CH_3^+	14.25	1.115	1.8 %
	CH_2^+	15.2	0.07018	

Magboltz 11.2bis,
E = 100 kV/cm
1 atm, 20 C

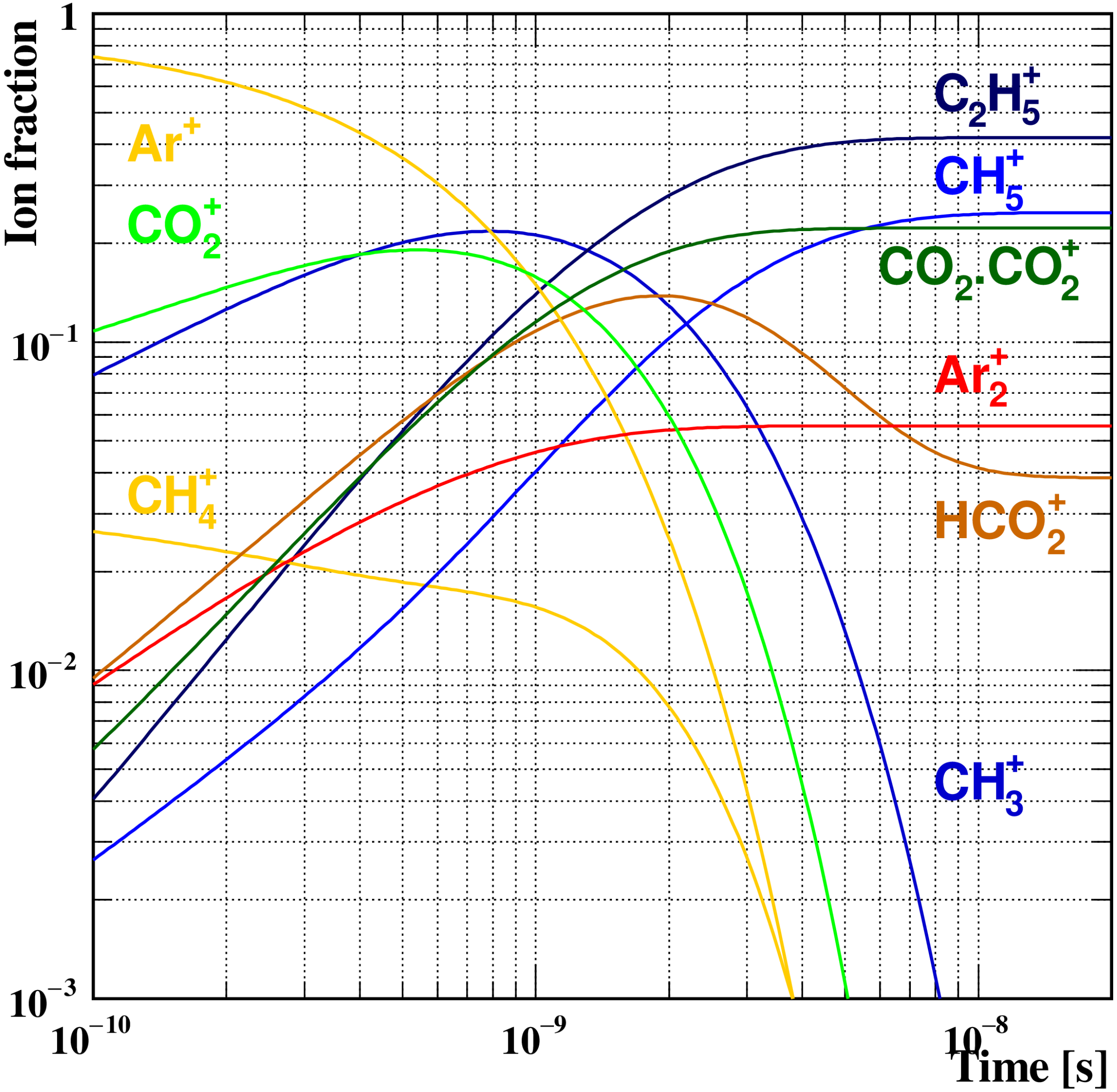
Reactions in Ar-CO_2-CH_4

$Ar^+ + Ar + M$	$\rightarrow Ar_2^+ + M$	100 %	$k = 2.3\ 10^{-31}$	± 10 %
$Ar^+ + CO_2$	$\rightarrow CO_2^+ + Ar$	100 %	$k = 5\ 10^{-10}$	± 10 %
$Ar^+ + CH_4$	$\rightarrow CH_3^+ + Ar + H$	85 %	$k = 1.1\ 10^{-9}$	± 10 %
	$\rightarrow CH_2^+ + Ar + H_2$	15 %		
$CO_2^+ + CO_2 + M$	$\rightarrow CO_2{\bullet}CO_2^+ + M$	100 %	$k = 2.4\ 10^{-28}$	
$CO_2^+ + CH_4$	$\rightarrow CH_4^+ + CO_2$	25 %	$k = 1.1\ 10^{-9}$	± 10 %
	$\rightarrow HCO_2^+ + CH_3$	75 %		
$CH_3^+ + CH_4$	$\rightarrow C_2H_5^+ + H_2$	100 %	$k = 1.1\ 10^{-9}$	± 10 %
$C_2H_5^+ + CH_4$	$\rightarrow C_3H_7^+ + H_2$	100 %	$k = 1\ 10^{-14}$	± 10 %
$CH_4^+ + CH_4$	$\rightarrow CH_5^+ + CH_3$	100 %	$k = 1.15\ 10^{-9}$	± 10 %
$CH_5^+ + CH_4$	$\rightarrow$ products	100 %	$k = 3\ 10^{-11}$	± 30 %
$CH_4^+ + CO_2$	$\rightarrow HCO_2^+ + CH_3$	99 %	$k = 9.6\ 10^{-10}$	± 10 %
	$\rightarrow CH_3CO^+ + OH$	1 %		
$CH_5^+ + CO_2$	$\rightarrow HCO_2^+ + CH_4$	100 %	$k = 4\ 10^{-11}$	± 10 %
$HCO_2^+ + CH_4$	$\rightarrow CH_5^+ + CO_2$	100 %	$k = 6\ 10^{-10}$	± 15 %

[B.M. Smirnov, Cluster Ions and Van Der Waals Molecules]
[V.G. Anicich and W.T. Huntress Jr., Astrophys. J. Suppl. **62** (1986) 553-672.]

Evolution in Ar-CO_2-CH_4 (90-7-3)

- Initial ions:
  - Ar^+ →
    - CO_2^+,
    - CH_3,
    - Ar_2^+
  - CO_2^+ →
    - CH_4^+
    - $CO_2^+.CO_2$
    - HCO_2^+
  - CH_4^+ →
    - CH_5^+,
    - HCO_2^+
    - CH_3CO^+
  - CH_3^+ →
    - $C_2H_5^+$



Products after 10 ns

- $C_2H_5^+ \rightarrow C_3H_7^+$ slow reaction, $k = 1\ 10^{-14}$

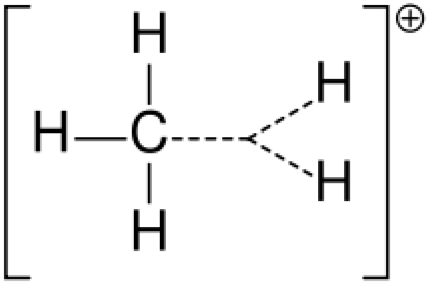
[Hiraoka & Kebarle https://doi.org/10.1063/1.431116]

- $CH_5^+ \rightarrow$ "products" slow reaction, $k = 3\ 10^{-11}$
- $CO_2 \cdot CO_2^+ \rightarrow$ grows to $n = 3\text{-}4$, reacts with CH_4 to HCO_2 ?

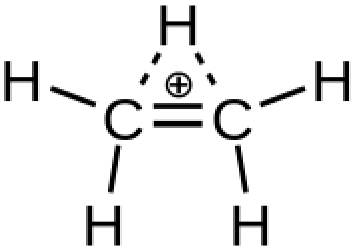
[Y. Kalkan et al. 2015 JINST 10 P07004]

- $Ar^+ \rightarrow Ar_2^+$ drifts faster than Ar^+

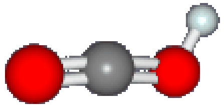
- $HCO_2^+ \rightarrow CH_5^+$ HCO_2^+ decays at $k = 6\ 10^{-10}$



methanium



ethenium

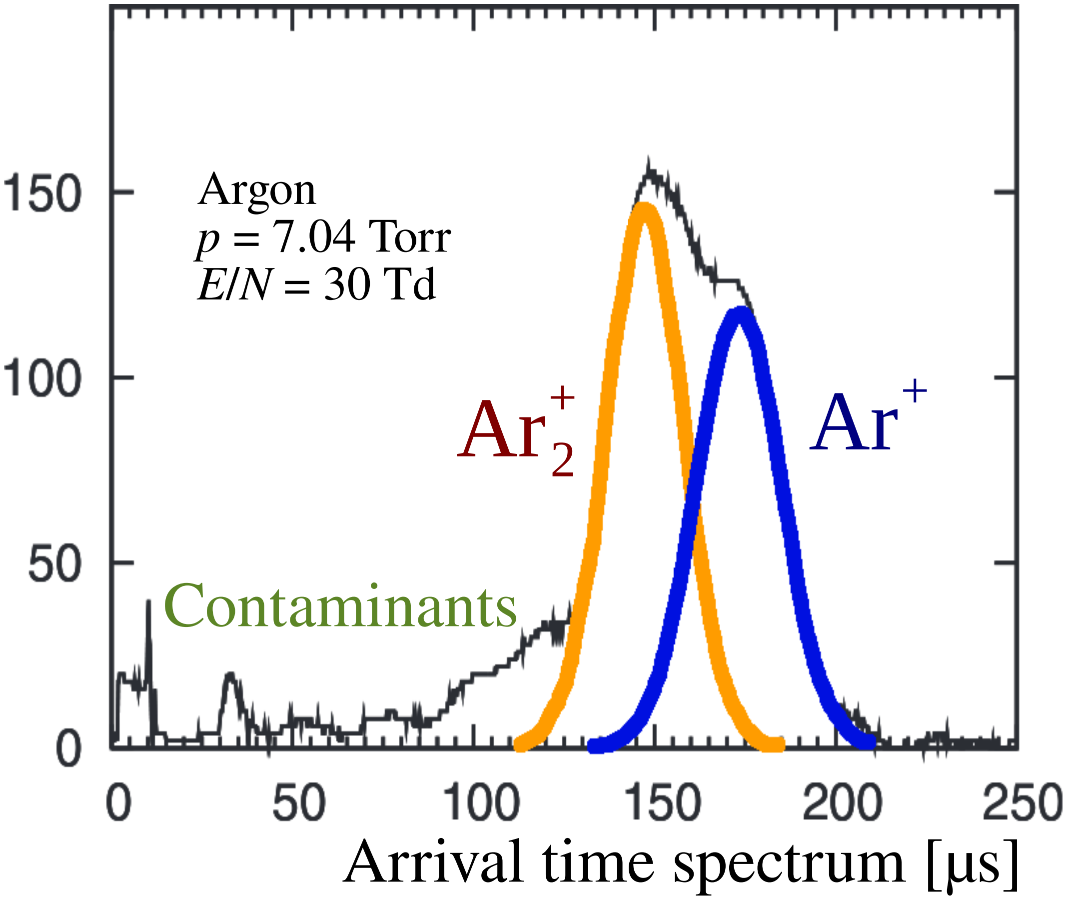


protonated
carbon dioxide

Ions drifting in pure Ar

- In pure argon, dimers are formed:
  - $Ar^+(^2P^o_{3/2}) + 2Ar \rightarrow Ar^+\bullet Ar + Ar$ ($k = 2.3 \pm 0.1\ 10^{-31}$ cm^6/s, 7 ns)
  - Note: dimers move *faster* than ions due to $Ar \leftrightarrow Ar^+$ resonant charge exchange

[PNB Neves et al. 10.1063/1.3497651]



Gain – How ?

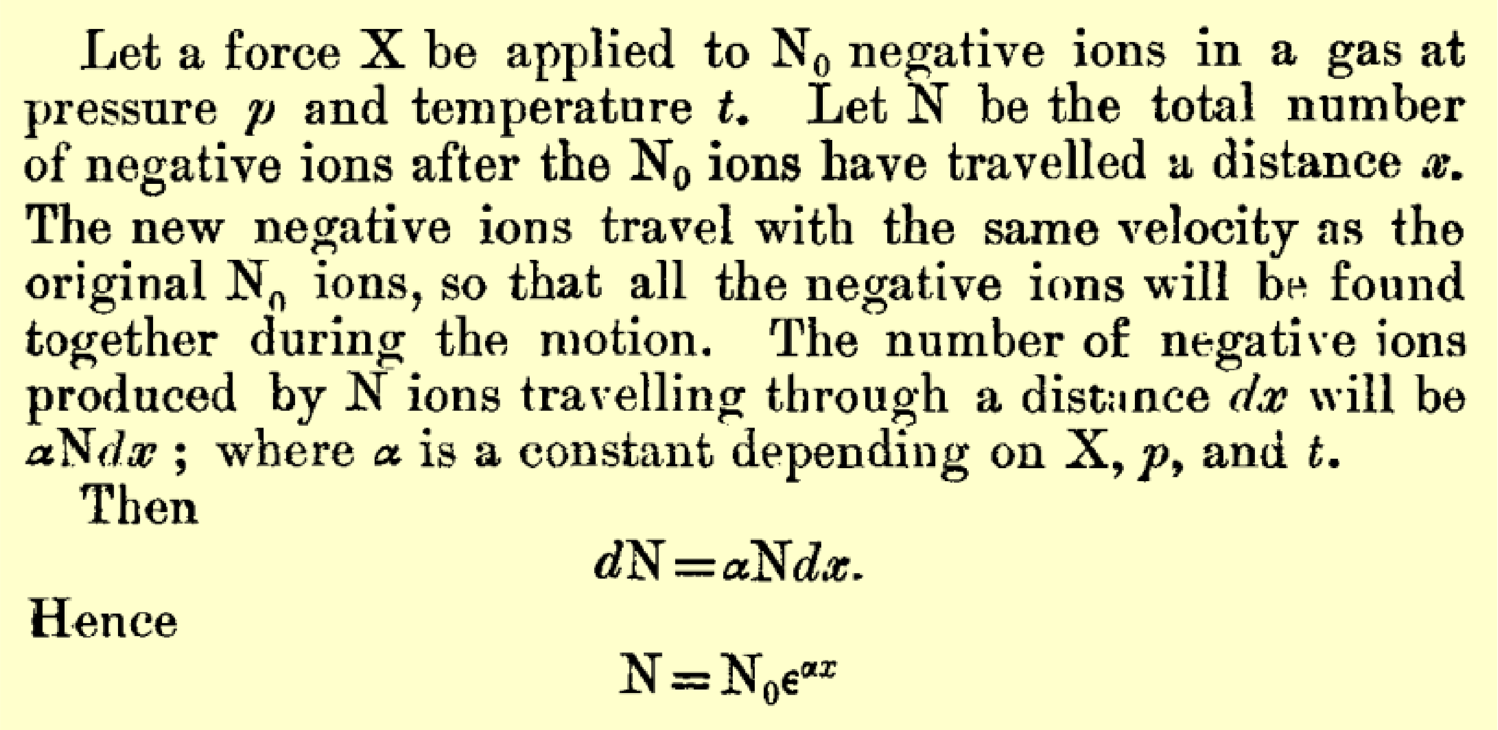
- Take into account:
  - ionisation, and also
  - excitation;
  - Penning effect.

- Study:
  - fluctuations;
  - hump.

Sir John Sealy Edward Townsend
(1868-1957)

1901: Gas multiplication

- John Townsend:



Let a force X be applied to N_0 negative ions in a gas at pressure p and temperature t. Let N be the total number of negative ions after the N_0 ions have travelled a distance x. The new negative ions travel with the same velocity as the original N_0 ions, so that all the negative ions will be found together during the motion. The number of negative ions produced by N ions travelling through a distance dx will be $\alpha N dx$; where α is a constant depending on X, p, and t.

Then

$$dN = \alpha N dx.$$

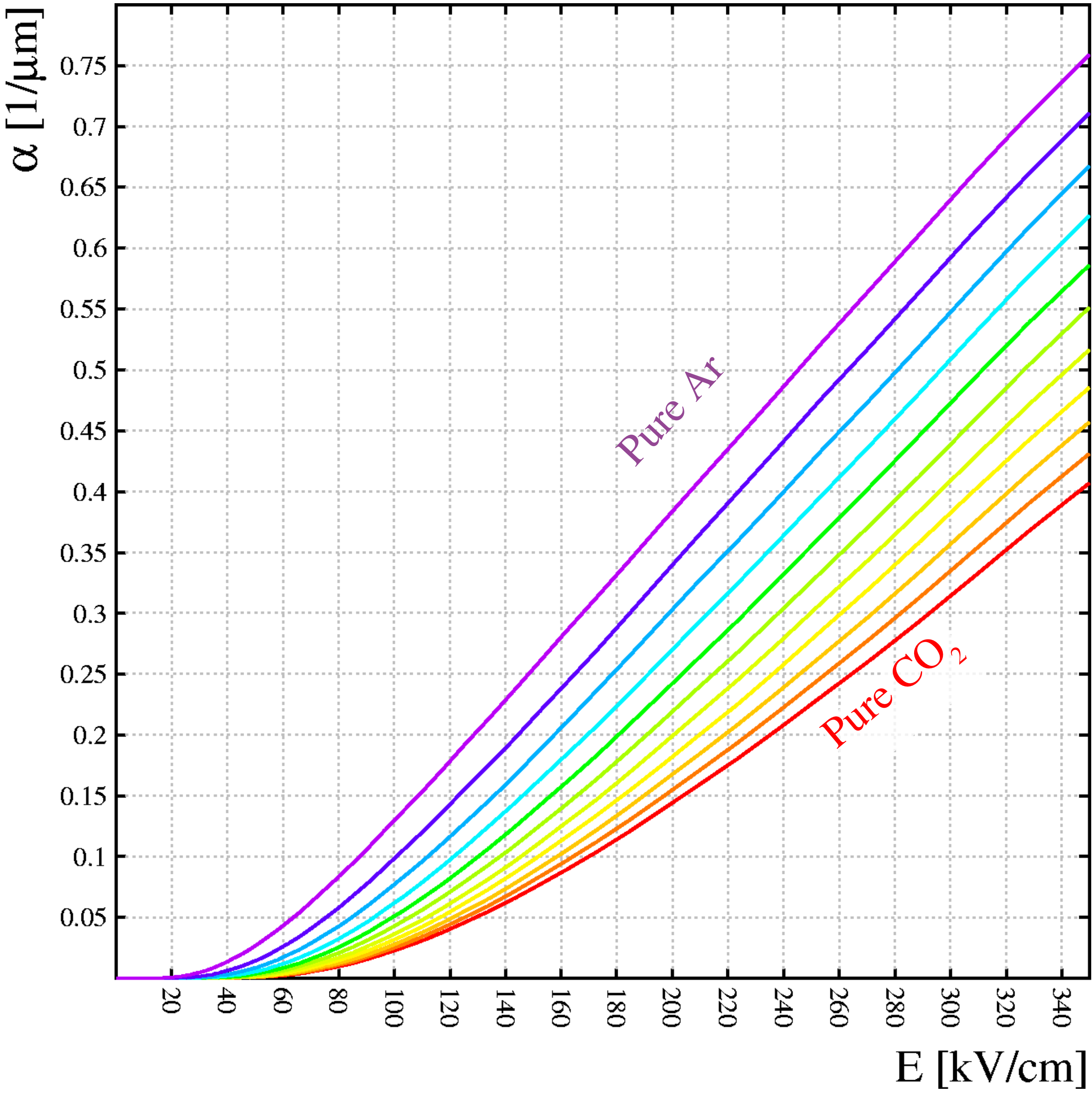
Hence

$$N = N_0 \epsilon^{\alpha x}$$

[J.S. Townsend, “*The conductivity produced in gases by the motion of negatively charged ions*”, Phil. Mag. **6-1** (1901) 198-227. If access to the Philosophical Magazine is restricted, then consult a German-language abstract at http://jfm.sub.uni-goettingen.de/.]

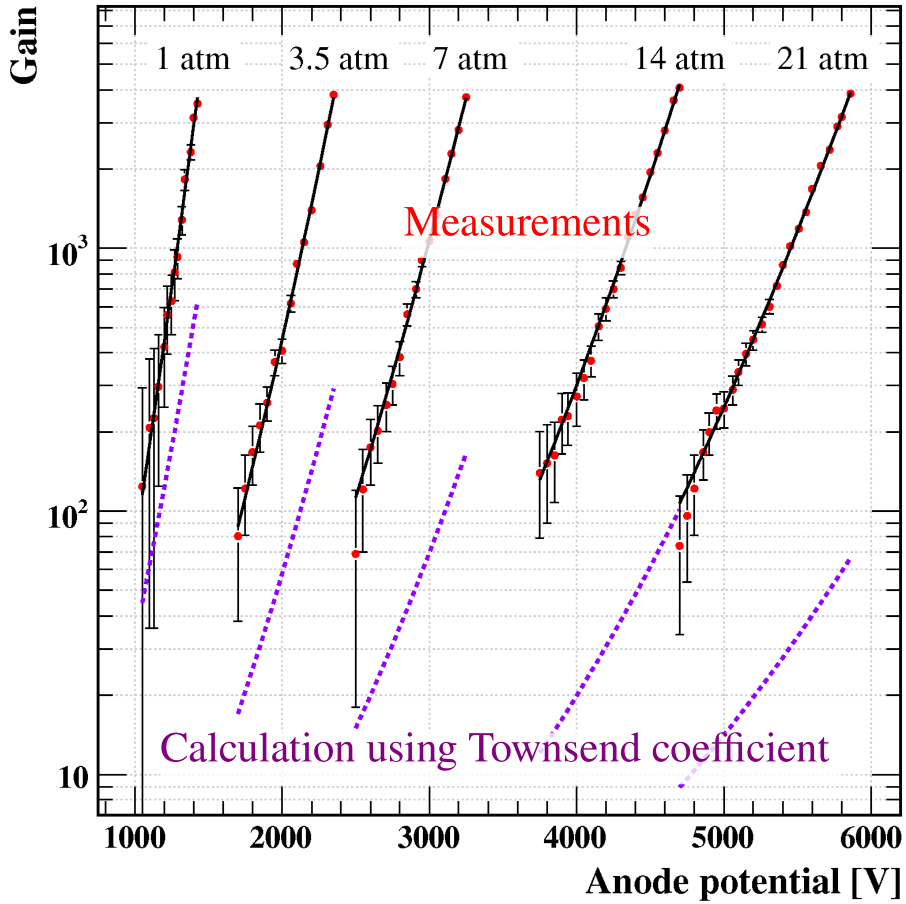
$\alpha(Ar\text{-}CO_2)$

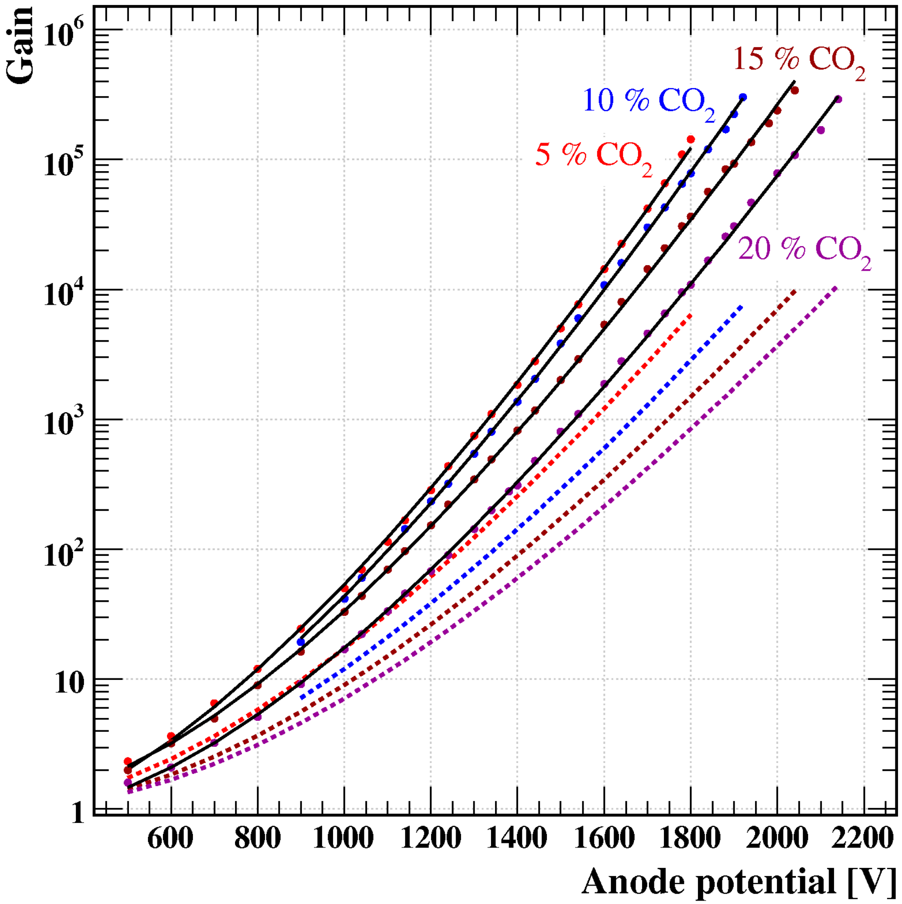
- α = number of e^- an avalanche e^- creates per cm.
- Adding CO_2 reduces the gain.
- Calculated by Magboltz for Ar/CO_2 at 3 bar.

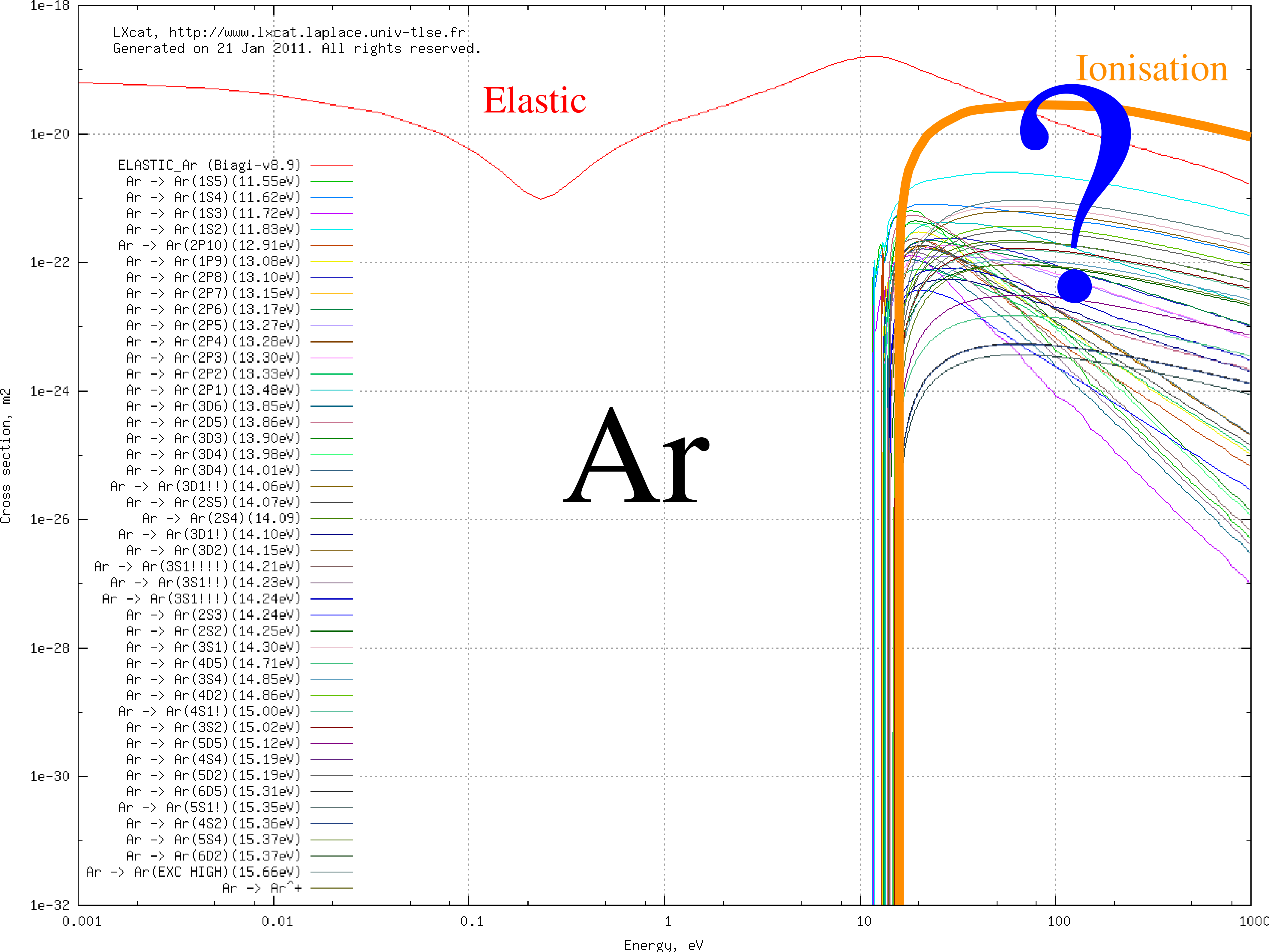


Does this reproduce the measurements ?

- Ar - CH_4
- Ar - CO_2



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Elastic
Ionisation
?
Ar
ELASTIC_Ar (Biagi-v8.9)
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Ar -> Ar(1S4)(11.62eV)
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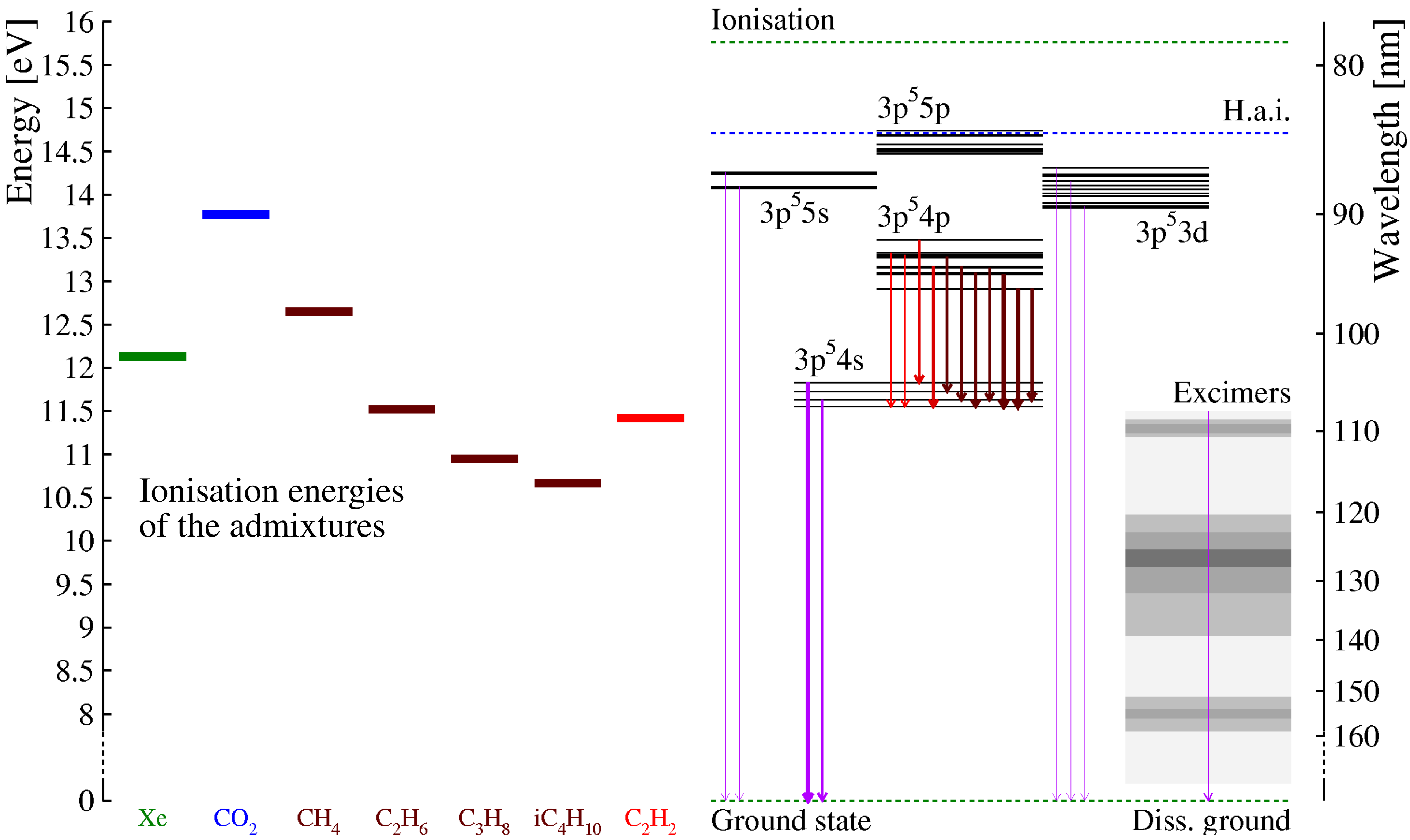
Cross section, m2
1e-18
1e-20
1e-22
1e-24
1e-26
1e-28
1e-30
1e-32
Energy, eV
0.001
0.01
0.1
1
10
100
1000

Level diagram argon and admixtures



Frans Michel Penning
(1894-1953)

Penning effect

- Ar* 3$p^5$4s can transfer to iC_4H_{10}, C_3H_8 and C_2H_6;
    - two 4s are metastable, the two others live 2.6 ns and 8.6 ns;

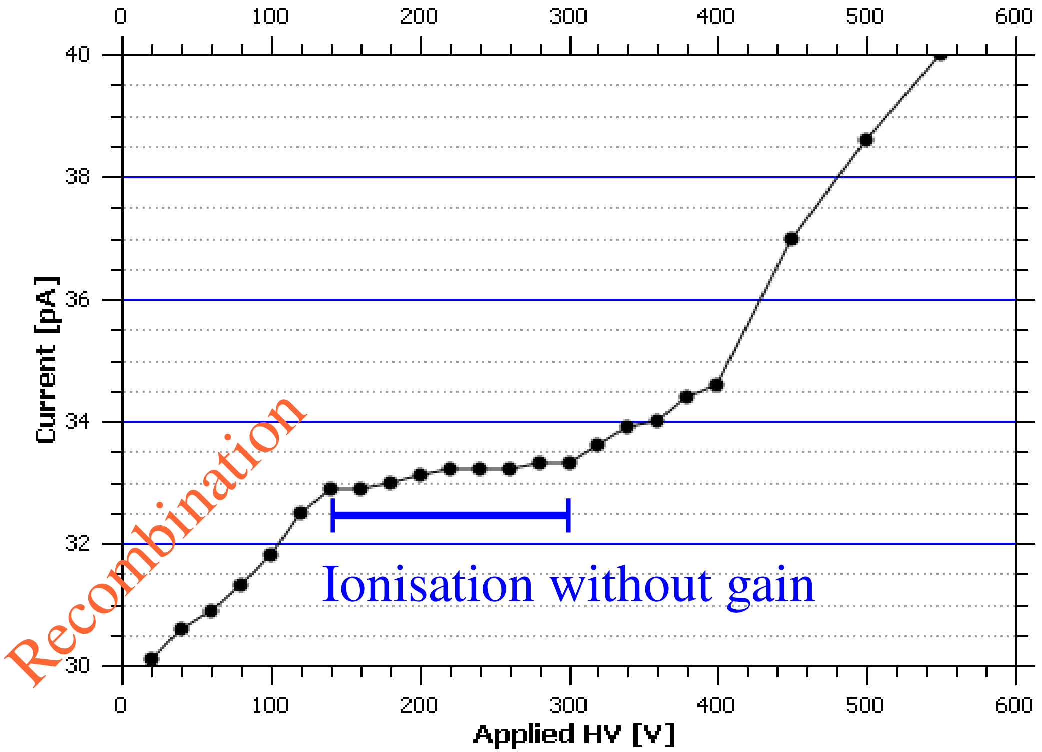
- Ar* 3$p^5$4p can also ionise CH_4;
    - 4p decays to 4s with a lifetime of 20-40 ns;

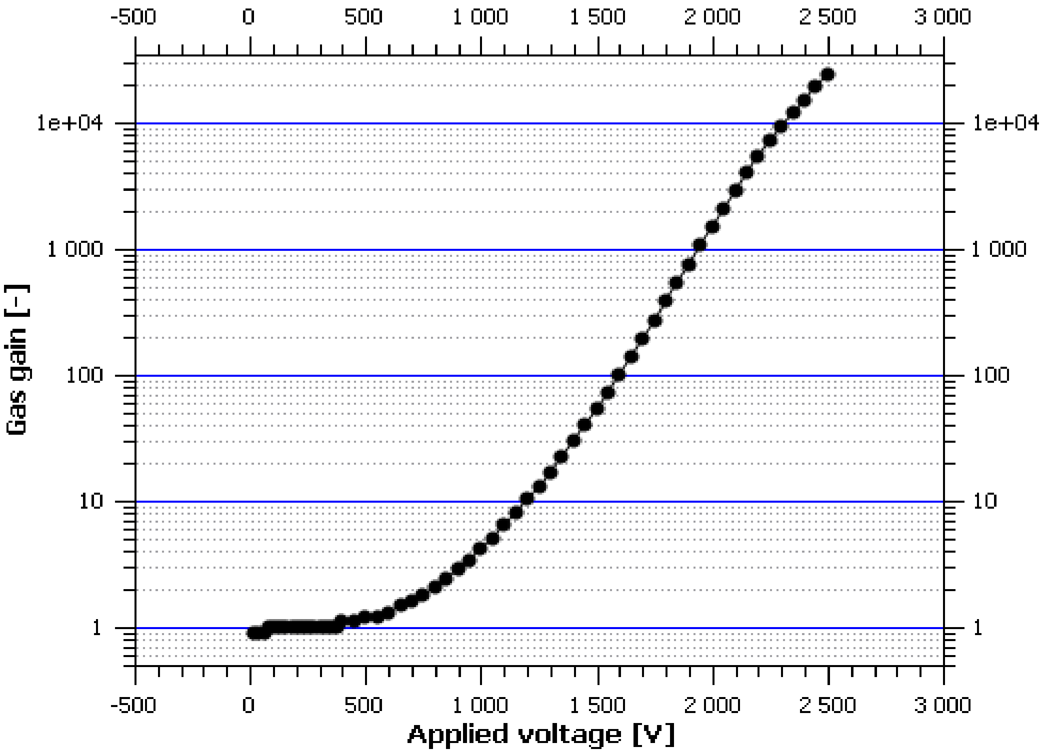
- Ar* 3$p^5$3d can in addtion transfer to CO_2;
    - radiative 3d decays take ~3.5 ns, the others ~50 ns.

- For comparison, collision frequencies of Ar* in pure quencher are ~100 ps.

Data covers 5 orders of magnitude !

- Current reference is taken at the ionisation level.
- Main source of error: ~5 %.



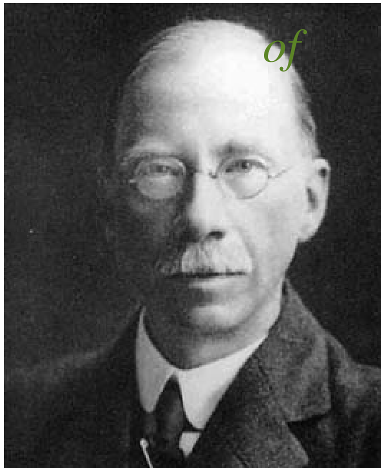


G.U. Yule (1923), W.H. Furry (1937), R.A. Wijsman (1949) & others

- If the distance between ionisations fluctuates exponentially with a mean of 1/α (reciprocal of the Townsend coefficient),
- then, the avalanche size *n* fluctuates (nearly) exponentially:

$$p(n) = \frac{1}{\bar{n}}\left(1-\frac{1}{\bar{n}}\right)^{n-1}$$

[G. Udny Yule, *A Mathematical Theory of Evolution, based on the Conclusions of Dr. J.C. Willis, F.R.S.*, Phil. Trans. Roy. Soc. London B **213** (1925) 21-87.
W.H. Furry, *On Fluctuation Phenomena in the Passage of High Energy Electrons through Lead*, Phys. Rev. **52** (1937) 569-581.
Robert A. Wijsman, *Breakdown Probability of a Low Pressure Gas Discharge*, Phys. Rev. **75** (1949) 833-838.]



George Udny Yule (1871-1951)

Statistics Yule-Furry

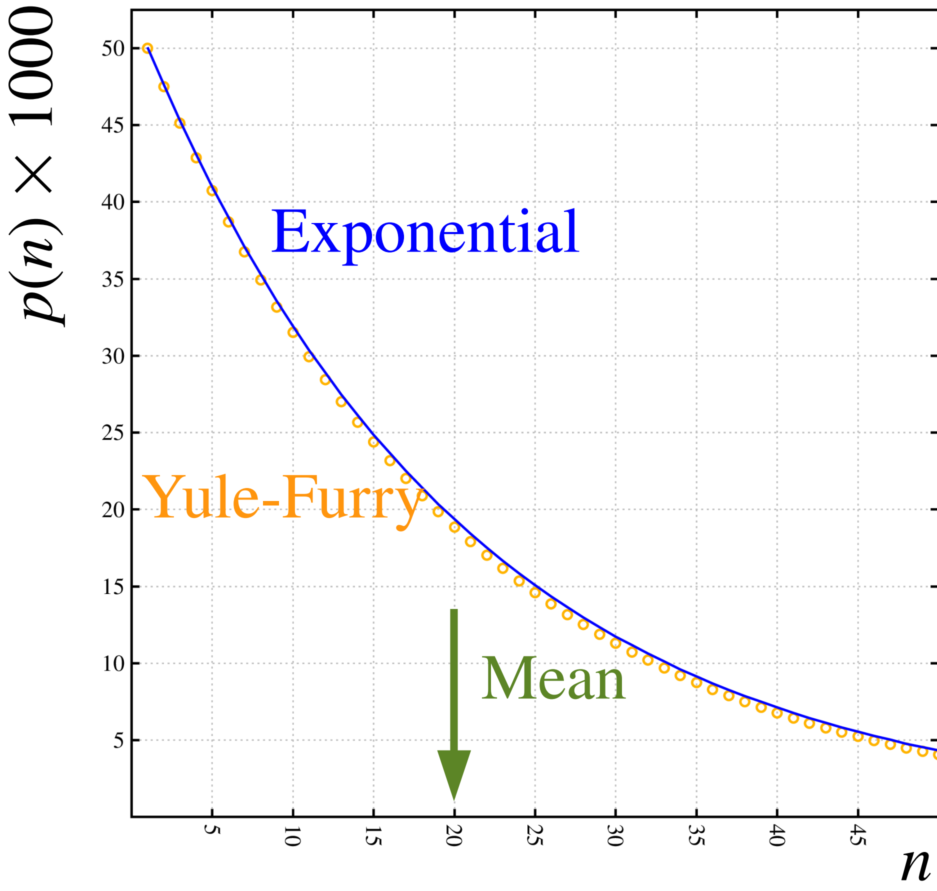
- Yule-Furry is exponential for large mean avalanche sizes:

$$
\begin{aligned}
p(n) &= \frac{1}{\bar n}\left(1-\frac{1}{\bar n}\right)^{n-1} \\
&= \frac{1}{\bar n}\,\frac{1}{1-1/\bar n}\left(1-\frac{1}{\bar n}\right)^{\bar n \frac{n}{\bar n}}
\end{aligned}
$$

- $$\approx \frac{\mathrm{e}^{-n/\bar n}}{\bar n - 1}$$

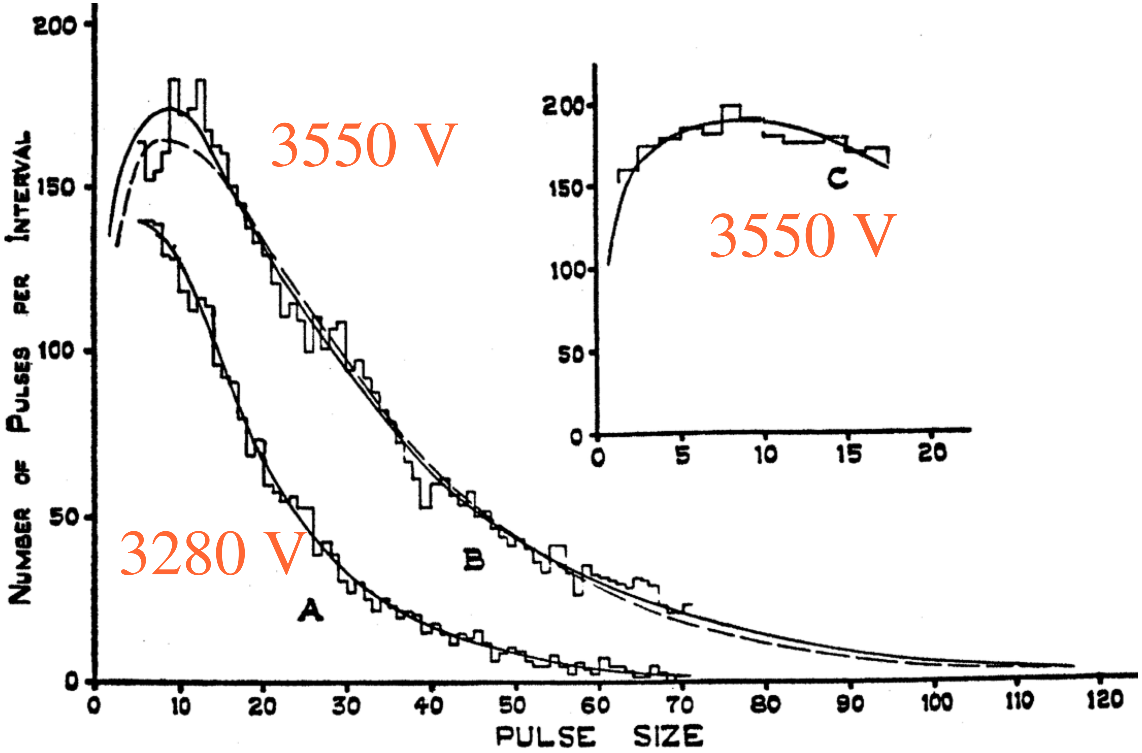
Mean: $\bar p$

RMS: $\bar p\sqrt{1-1/\bar p} \approx \bar p$



S.C. Curran (1949)

- S.C. Curran *et al*. measure the pulse height distribution in a cylindrical counter (d = 150 μm wire, Ar 50 % CH_4 50 %, p = 670 mbar) at G ~10^4-10^5:



Fit curve:

$$p(n) = \sqrt{n}\, e^{-n}$$

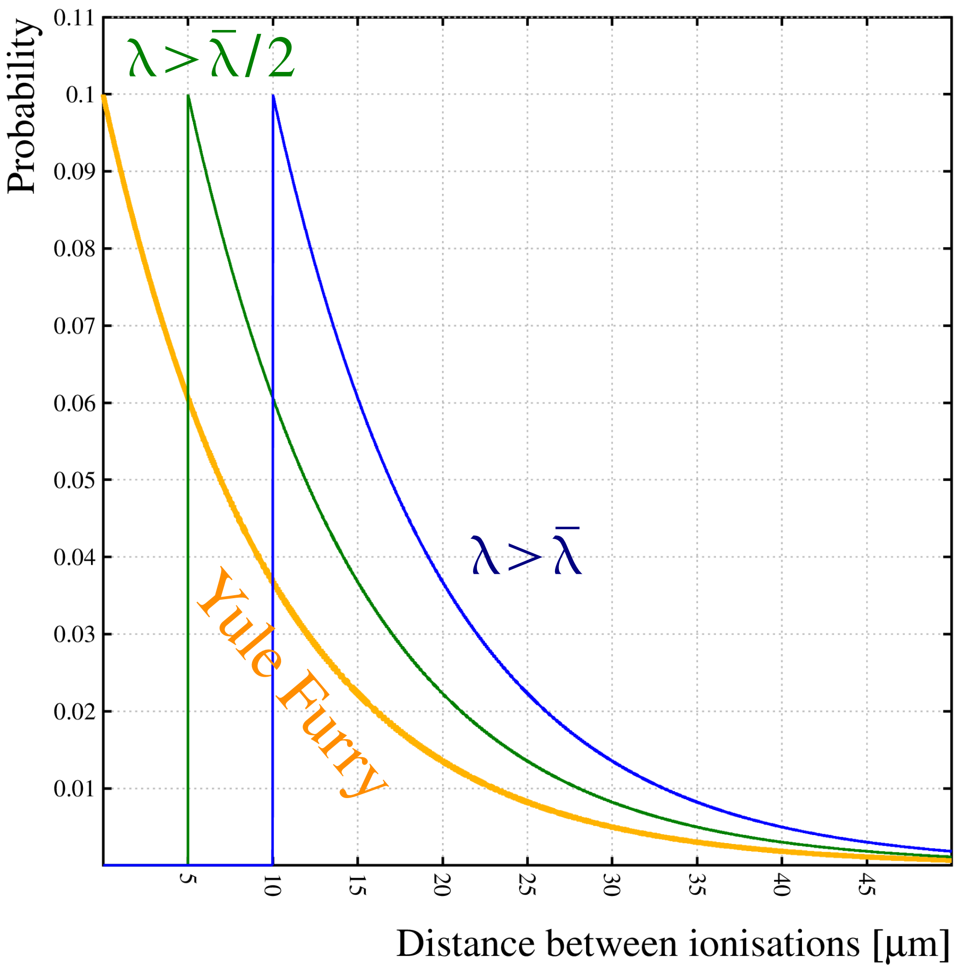
$$\left(\frac{\sigma}{\mu}\right)^2 \approx \frac{2}{3}$$

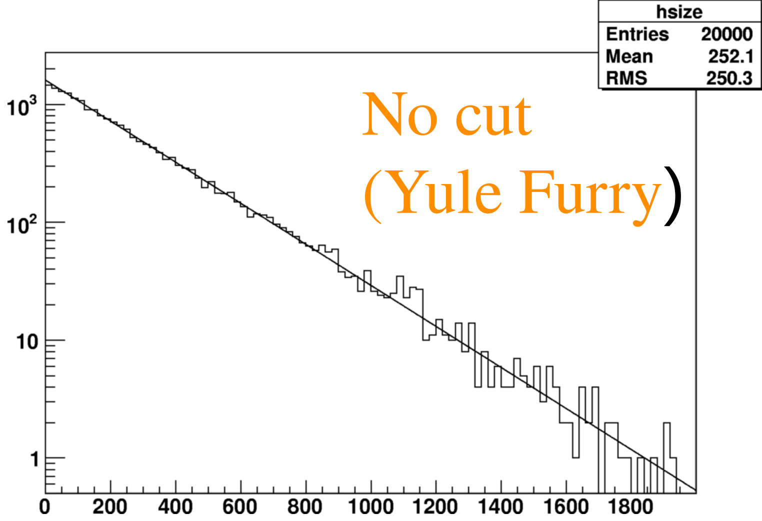
Two schools of thought ...

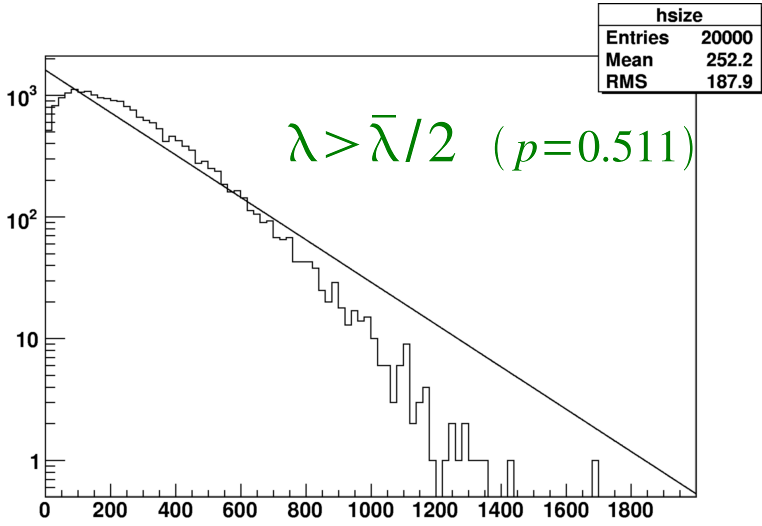
- The distance between ionisations does not simply vary exponentially (e.g. the Raether group).
- The Townsend coefficient is not constant (e.g. Byrne, Lansiart & Morucci).

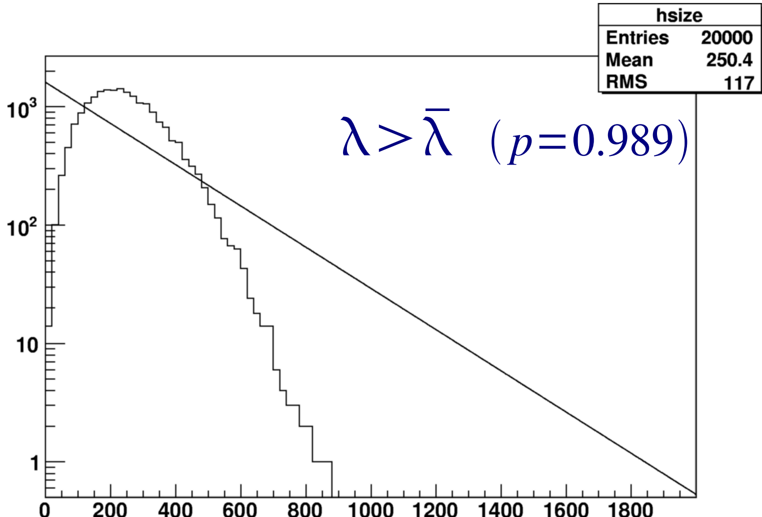
Minimum step length

- Imposing a minimum distance between ionisations adds a hump to the avalanche size spectrum.

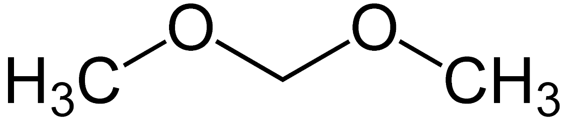




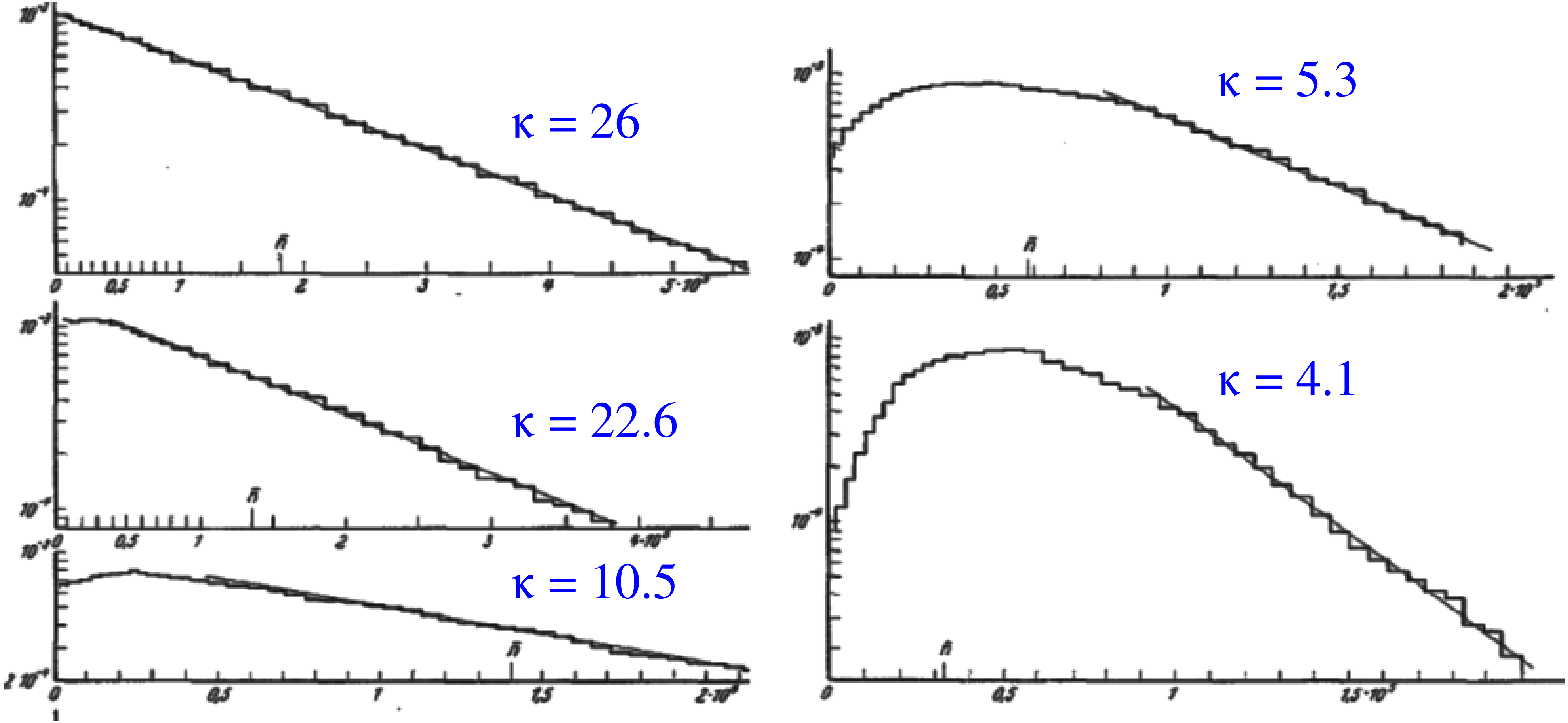




Hans Schlumbohm (1958)



- Dimethoxymethane gain spectra: increasing *E*, decreasing *p d* and ~ equal mean gain.



A. Lansiart & J.P. Morucci (1962)

- Small avalanches are composed of electrons that
  - have ionised less, hence
  - have more energy, hence
  - will ionise more easily
- They modeled this with an avalanche size-dependent α:

$$\alpha(n)=\alpha(0)\left(1+\frac{k}{n}\right)$$

- Implies that $(\sigma/\mu)^2 = 1/(1+k) < 1$, in agreement with Curran's measurements.

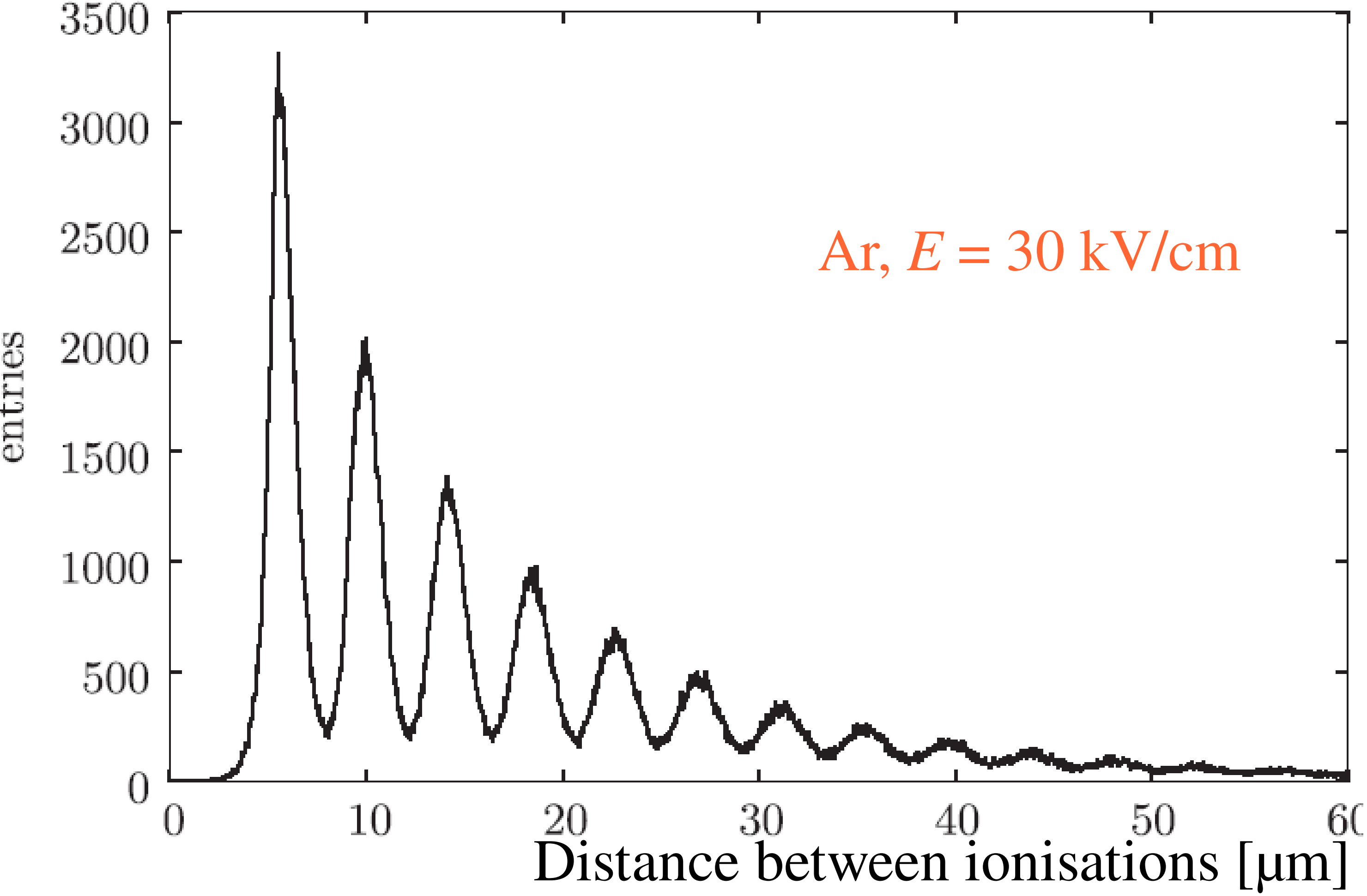
- In such a mechanism, the electron energy distribution continues to decrease, without reaching an equilibrium.

Monte Carlo approach

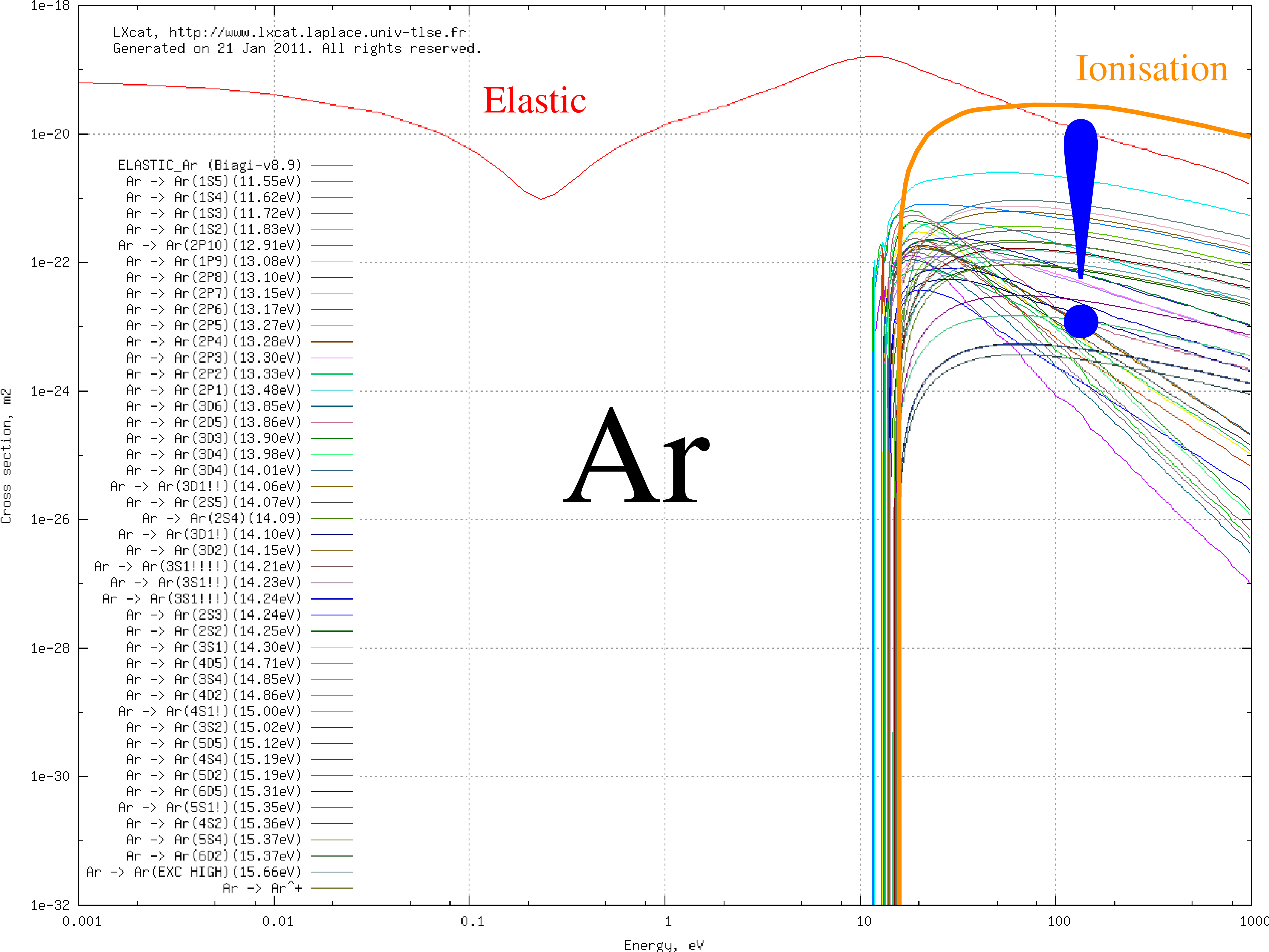
- Analytic models are precious for the insight they afford.
- But ... the complexity of real gases and electric fields in real detectors, make realistic models unwieldy:
  - inelastic collisions (vibrations, rotations, polyads);
  - excitations and Penning transfers;
  - attachment;
  - intricate, position-dependent *E* and *B* fields.
- Predictions for experiments are more practical using a Monte Carlo approach, here based on Magboltz.

Distance between ionisation: MC

- The distance between successive ionisations oscillates, shown here for Ar (also happens in e.g. CH_4). Why ?



[Magboltz calculations by Heinrich Schindler]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Energy, eV
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0.01
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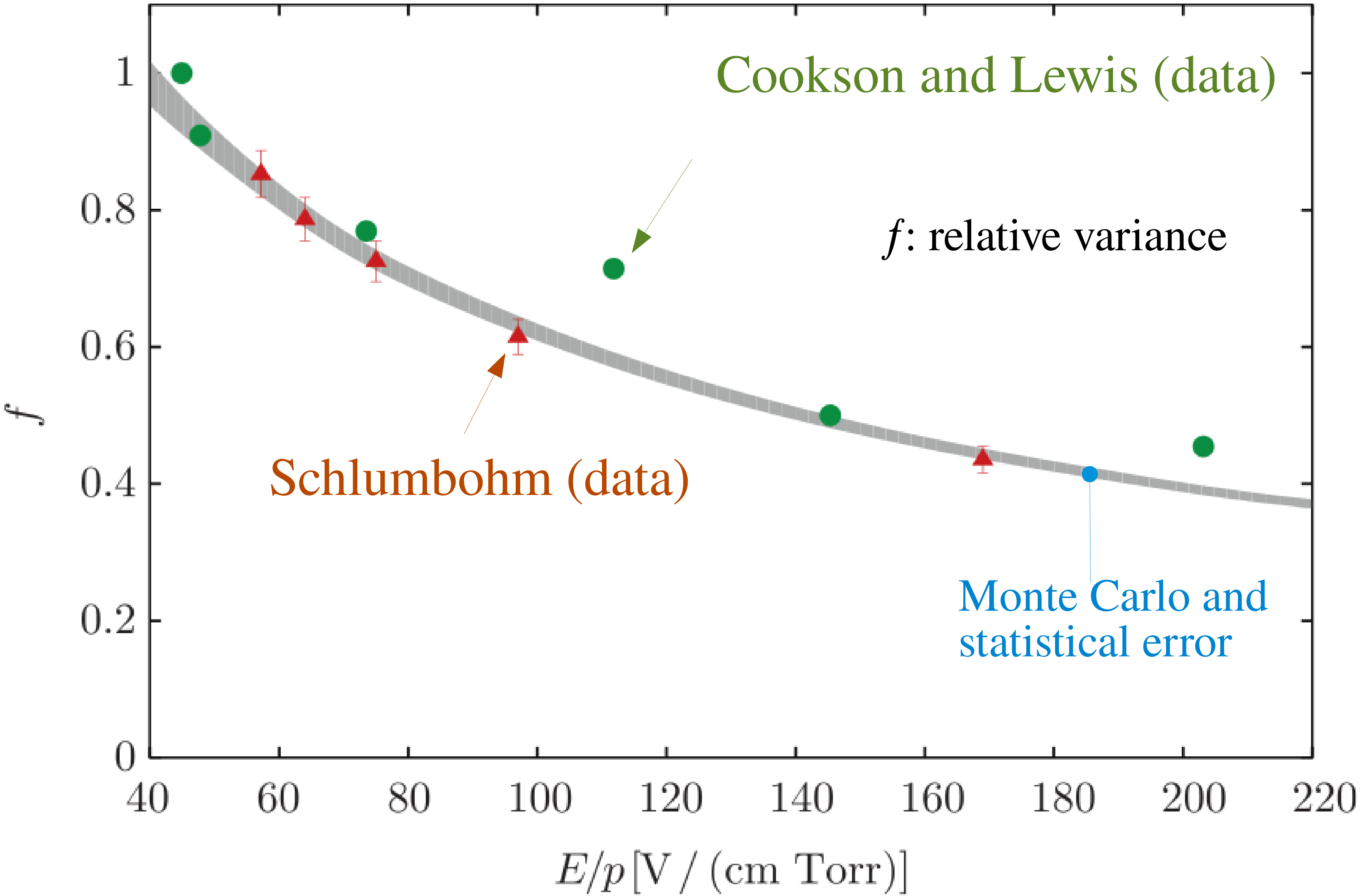
1
10
100
1000

MC verification: methane



Signals – How ?

- Weighting field
- quasi-static