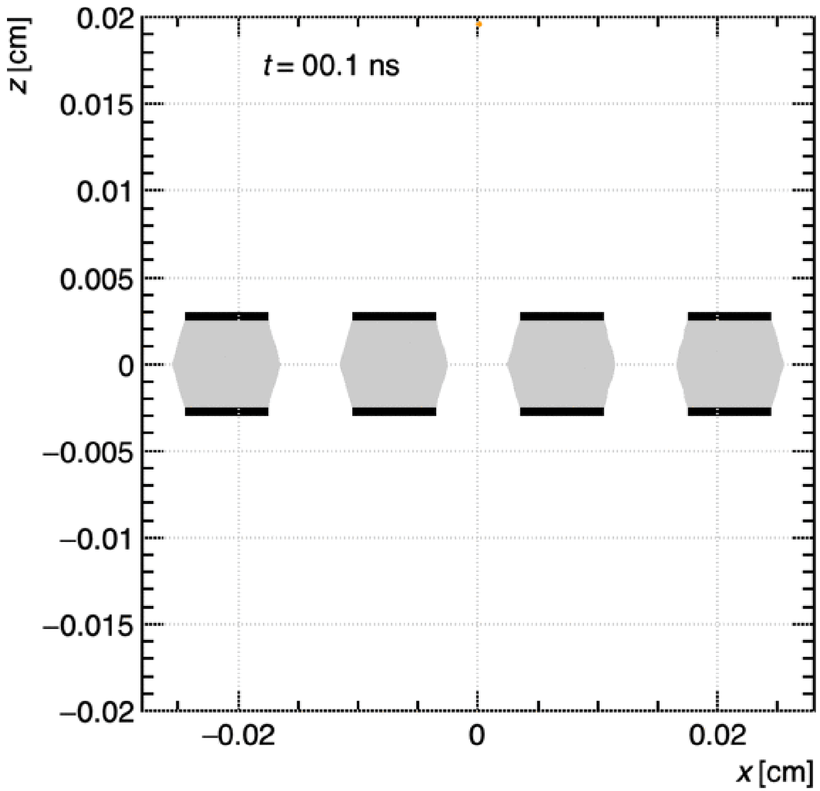
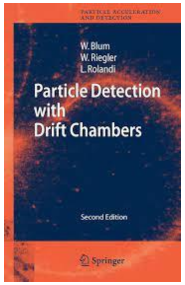
Signal Induction



- **Starting from an avalanche in a MPGD - or more general – a Gaseous Detector**
- **How is the signal induced in the readout electrodes?**

- *Nota Bene: Signal induction not correctly treated in many textbooks*
- *This lecture is a summary of a series of lectures given by W.Riegler*
  - *I will give you an "introduction" – the basics "ma non troppo"*
  - *Interested persons can dive deeper in references provided*

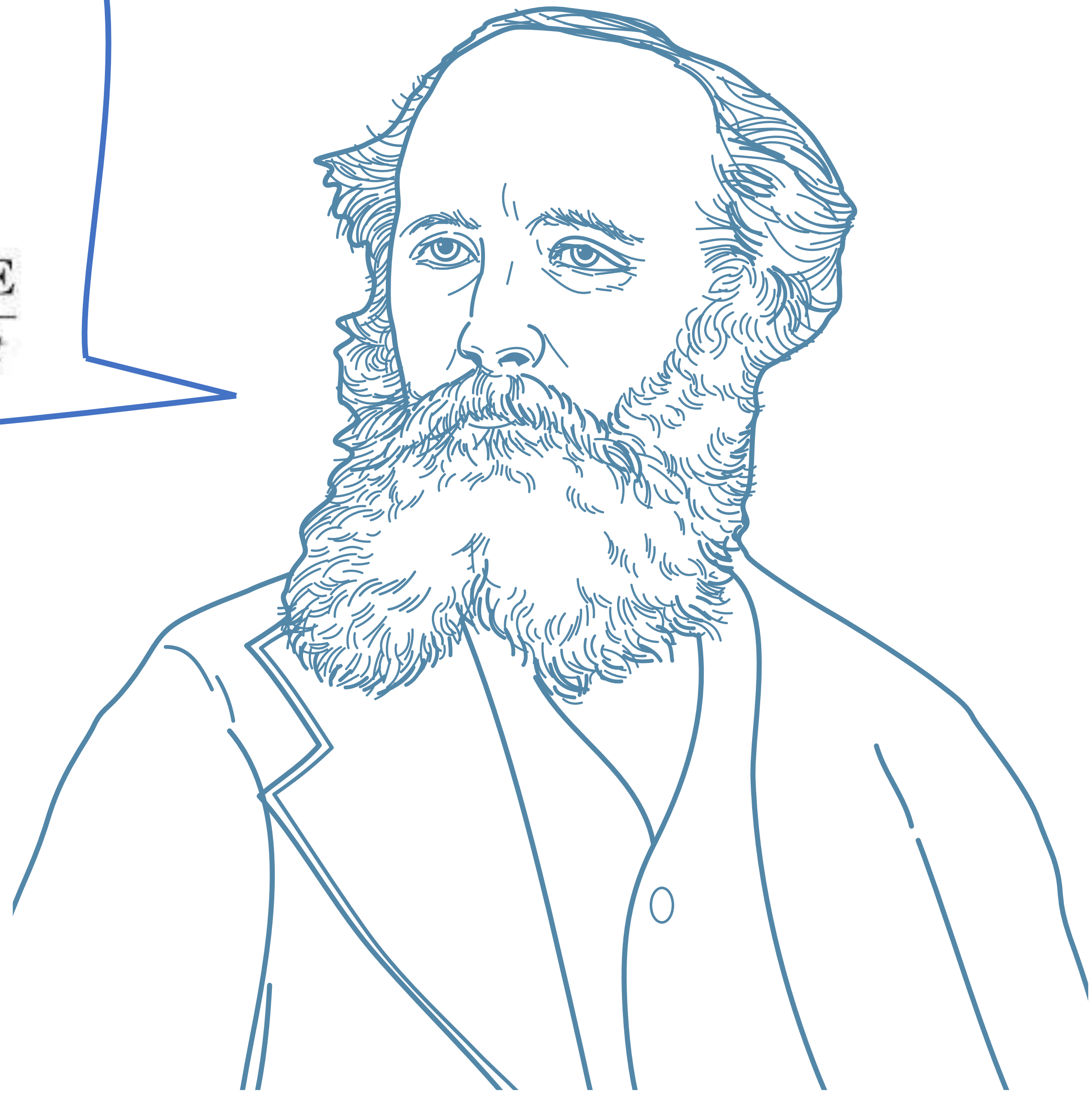


$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$



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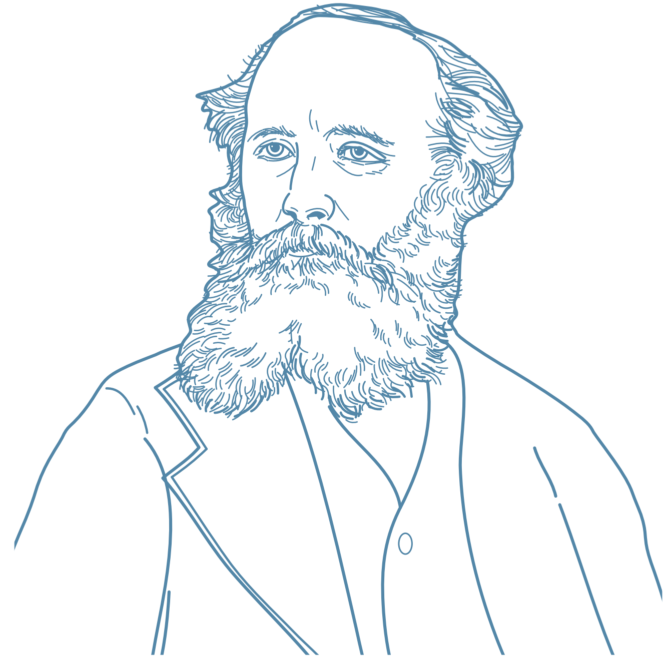
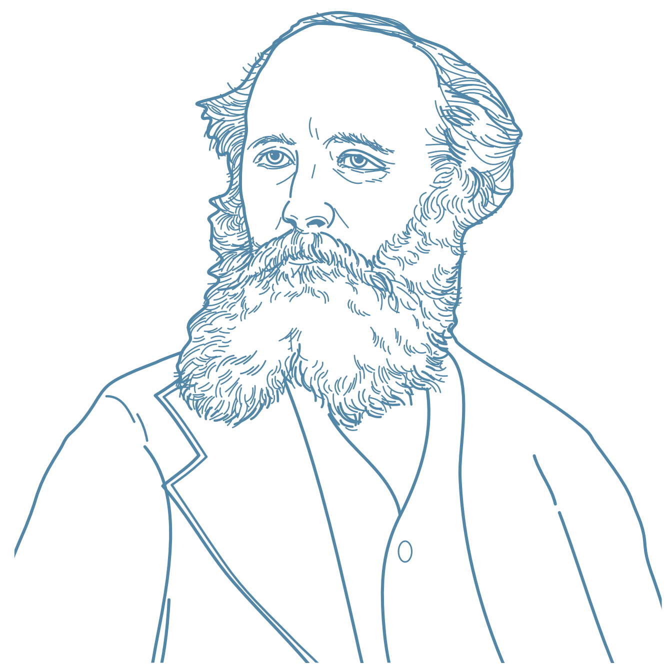


?

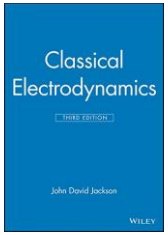
Can you simplify that

$$\mathcal{L} = -\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} - A_\mu J^\mu$$



I. Electrostatics

… let us first assume a charge at rest …



- **Gauss' Law:**
  assume point-charge q and a volume V with closed surface S => obtain Electric field $\boldsymbol{E}(\boldsymbol{x})$ given charge distribution $\rho(\boldsymbol{x})$

$$\oint_S \mathbf{E} \cdot \mathbf{n} \, da = \frac{1}{\epsilon_0} \int_V \rho(\mathbf{x}) \, d^3x$$

$$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$$

- **Generalized Coulomb's Law:**
  the vector field $E(\boldsymbol{x})$ is derived from a scalar potential $\phi(\boldsymbol{x})$
  - *$\phi(\boldsymbol{x})$ is arbitrarily defined*
  - *Physical interpretation: work done on charge q moving it from A to B:*
    $W_{AB} = q\phi(\boldsymbol{x}_{\boldsymbol{B}}) - q\phi(\boldsymbol{x}_A)$

assumption $\nabla \times \mathbf{E} = 0$

$$\mathbf{E} = -\nabla\Phi$$

$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} \, d^3x'$$

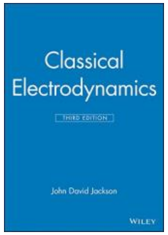
- **Poisson Equation:**
  combining $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ *and* $\mathbf{E} = -\nabla\Phi$
  solution of Poisson equation is unique

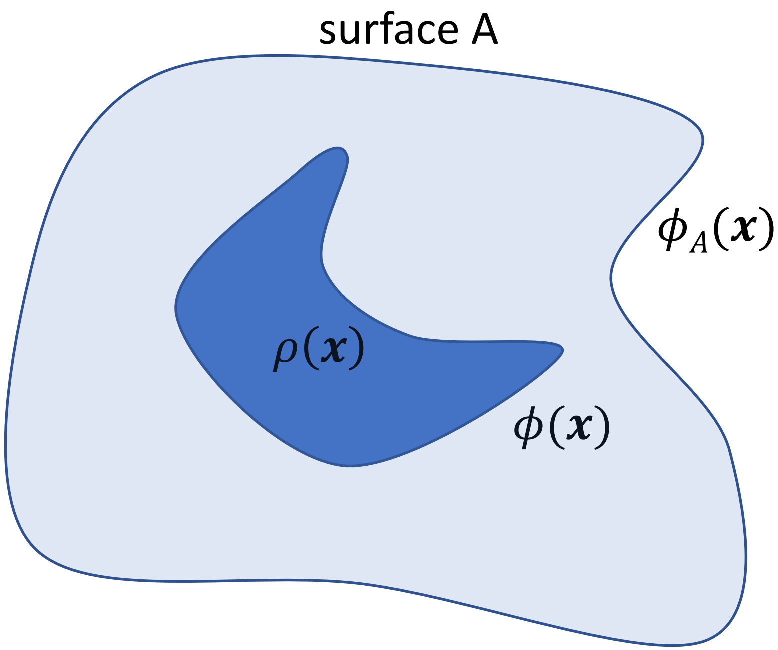
$$\nabla^2\Phi = -\rho/\epsilon_0$$

- **Laplace Equation:** *when* $\rho(\boldsymbol{x}) = 0$

$$\nabla^2\Phi = 0$$

I. Electrostatics





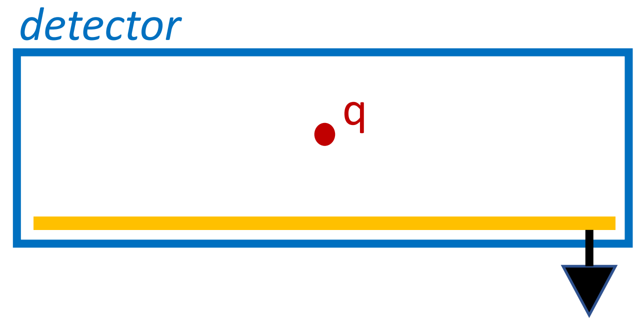
- Given charge distribution $\rho(\boldsymbol{x})$
- Potential on the surface: $\phi(\boldsymbol{x})$
  - $\phi(\boldsymbol{x}) = \frac{1}{4\pi\varepsilon}\int \frac{\rho(\boldsymbol{x})}{|\boldsymbol{x} - \boldsymbol{x}'|} d\boldsymbol{x}'$

- On another surface A: $\phi_A(\boldsymbol{x})$
- One can prove that if there are 2 solutions, $\phi_1(\boldsymbol{x})$ and $\phi_2(\boldsymbol{x})$, of the same Poisson equation, then the solutions are equal: $\phi_1(\boldsymbol{x}) = \phi_2(\boldsymbol{x})$ (uniqueness)

Defining the potential $\phi_A(\boldsymbol{x})$ on the entire (closed) surface therefore uniquely defines the electric field in the volume enclosed by the closed surface

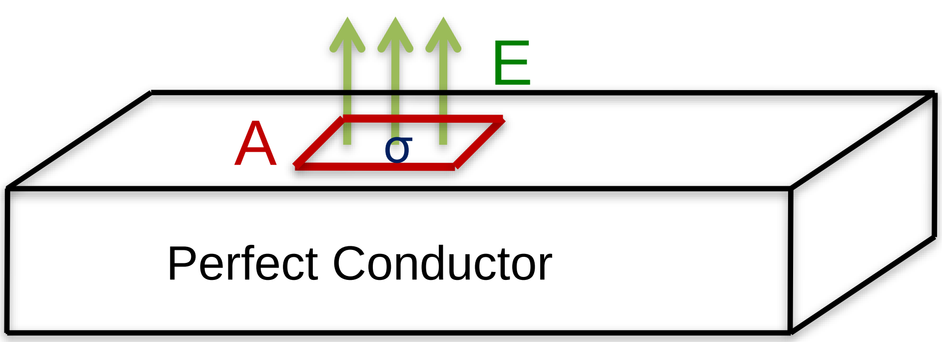
I. Electrostatics

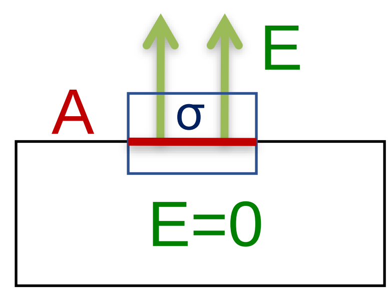
- **Given:**
  - Charge distribution $\rho(x)$ in detector
  - Readout Electrode = Perfect Conductor
- **We know:**
  - $\rho(x)$ induces an Electric Field $E(\boldsymbol{x})$
  - Perfect Conductor:
    - Inside conductor $E(\boldsymbol{x}) = \mathbf{0}$
    - *Field lines are perpendicular to surface*
- **We can calculate:**
  - Electric field on boundary of conductor
  - Surface charge density $\sigma(\boldsymbol{x})$ conductor
  - $\sigma(\boldsymbol{x}) = \epsilon_0 E(\boldsymbol{x})$



$$\oint_S \mathbf{E} \cdot \mathbf{n} \, da = \frac{1}{\epsilon_0} \int_V \rho(\mathbf{x}) \, d^3x$$

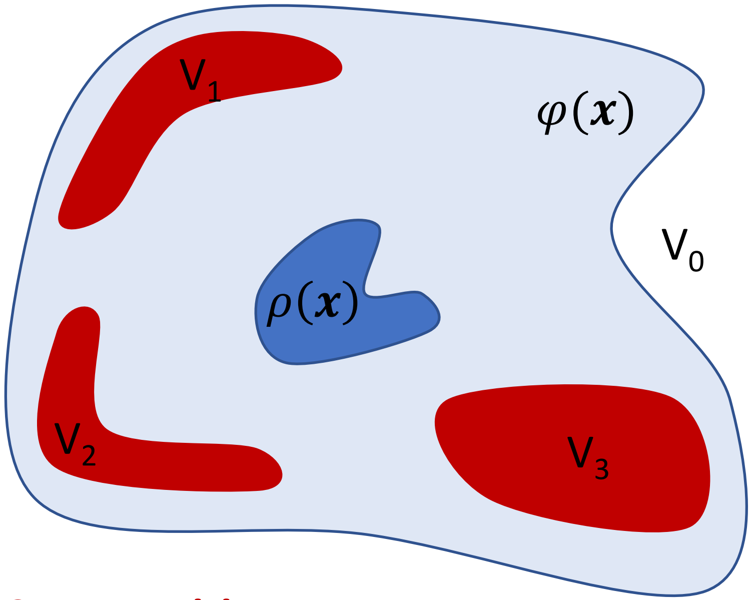
$$EA = \frac{1}{\epsilon_0}\sigma A \quad => \quad \sigma = \epsilon_0 E$$





Figures courtesy: W. Riegler

II. Induced Charge on metal electrode

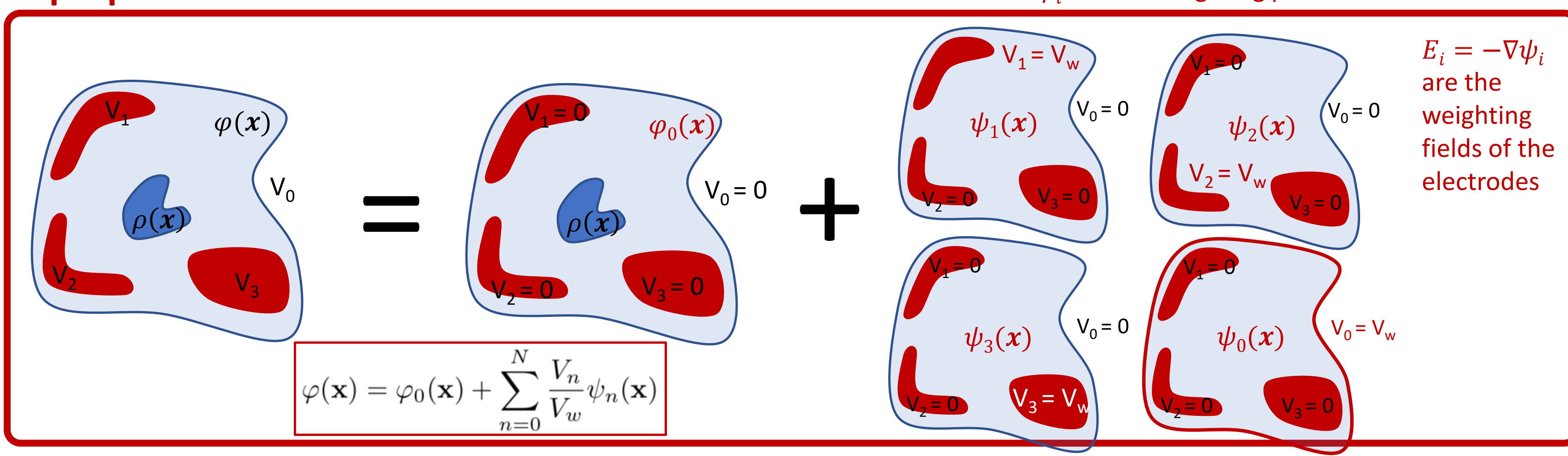


- Imagine a volume 0, with a charge density $\rho(\boldsymbol{x})$ and some electrodes 1,2,3
- Poisson equation: $\nabla^2\varphi = -\rho/\epsilon_0$
- Potential on each of the surface boundaries:
  - $\varphi(x)|_i = Vi$
- Charge on electrode:

$$Q_i = -\epsilon_0 \oint_{A_i} \nabla\varphi(\boldsymbol{x})dA$$

Superposition:

ψ_i are the weighting potentials of the electrodes

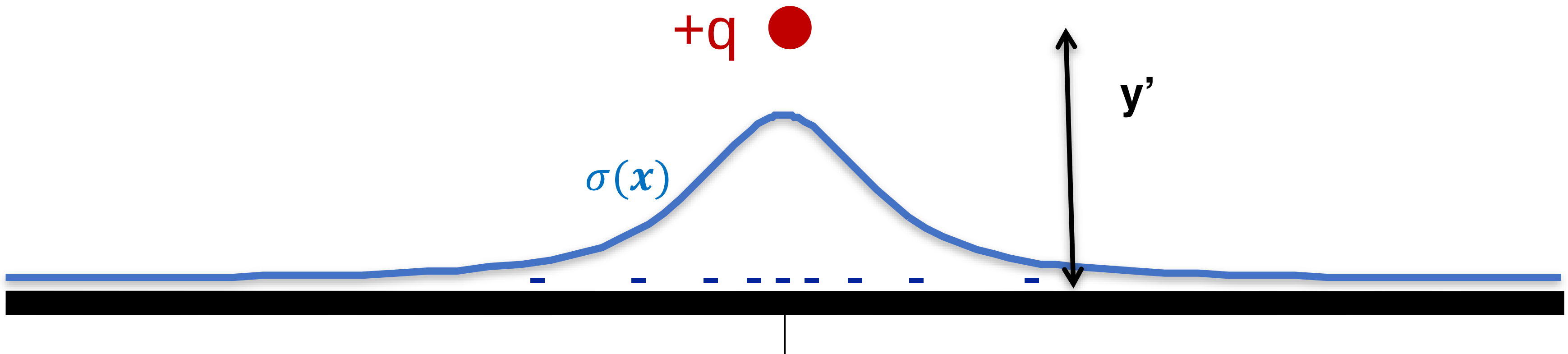


III. Induced Charge on infinite plate

A point charge q at a distance y' above a grounded metal plate induces a surface charge Q (with density $\sigma(\boldsymbol{x})$)

To do: *find charge Q induced on the electrode*

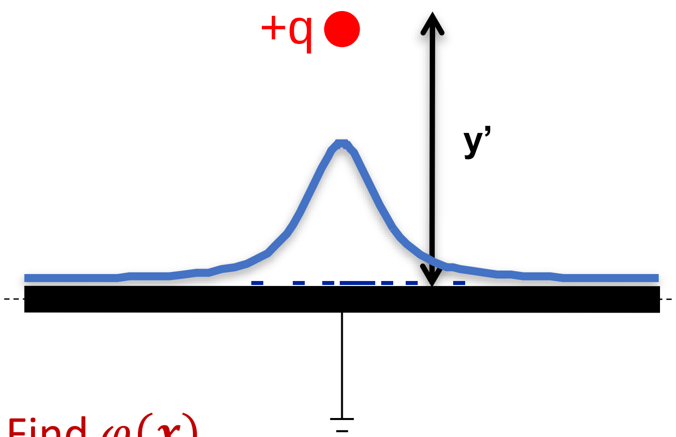
1) Solve Poisson equation with boundary condition $\varphi = 0$ on conductor surface
   $\nabla^2\varphi = -\rho/\epsilon_0$ & $\varphi(x)|_{conductor} = 0$ => $\varphi(\boldsymbol{x})$
2) Calculate the electric field E on the surface of the conductor: $E(\boldsymbol{x}) = -\nabla\varphi(\boldsymbol{x})$
3) Integrate over the electrode surface: $Q = \int \sigma(\boldsymbol{x})dA = \epsilon_0 \int E(\boldsymbol{x})dA$



Slide courtesy: W. Riegler

III. Induced Charge on infinite plate

- Solution for field of Point Charge above metal plate (left) is equal to solution for the charge & mirror charge (right)



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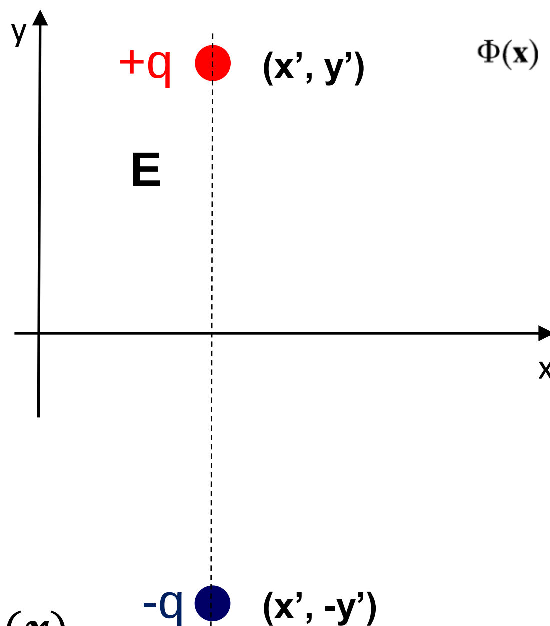


$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\, d^3x'$$

1st Step: Find $\varphi(\boldsymbol{x})$

- Superposition: $\varphi_{tot}(x) = \varphi_1(x) + \varphi_2(x)$
- Coulomb: $E(Q,r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ and $E(\boldsymbol{x}) = -\nabla\varphi(\boldsymbol{x})$

$$\varphi(x,y,z) = \frac{q}{4\pi\varepsilon_0}\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}} - \frac{q}{4\pi\varepsilon_0}\frac{1}{\sqrt{(x-x')^2+(y+y')^2+(z-z')^2}}$$

Slide courtesy: W. Riegler

III. Induced Charge on infinite plate

- 2nd step $E(\boldsymbol{x}) = -\nabla\varphi(\boldsymbol{x})$
- 3rd step $\sigma(\boldsymbol{x}) = \epsilon_0 E(\boldsymbol{x})$

$$\sigma(\boldsymbol{x}) = -\epsilon_0\nabla\varphi(\boldsymbol{x})$$

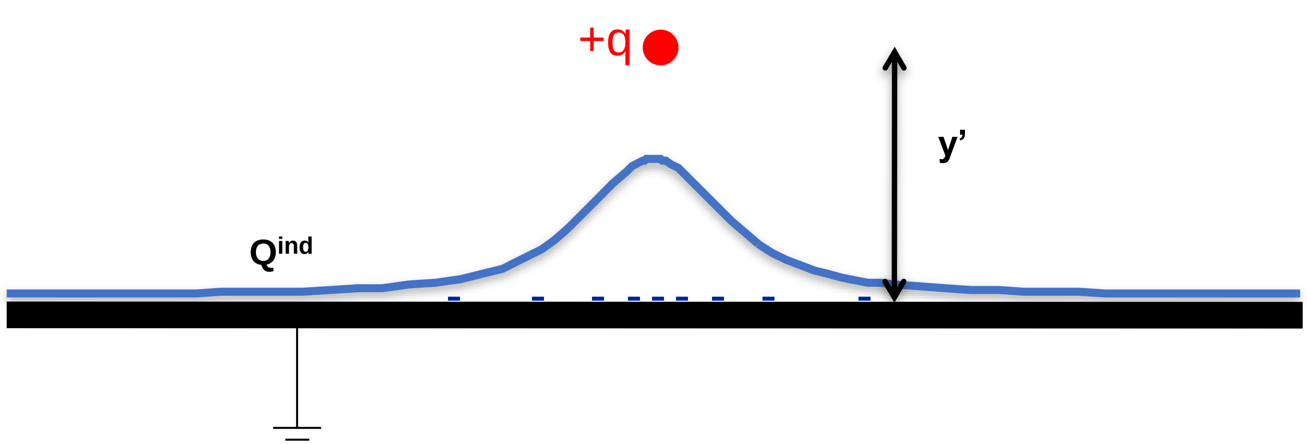
- **Surface Charge Density** $\sigma(\boldsymbol{x})$ on metallic plate ($y = 0$):

$$\sigma(x,y) = -\varepsilon_0 \frac{\partial\varphi}{dy}|_{y=0} = -\frac{q}{2\pi}\frac{y'}{((x-x')^2 + y'^2 + (z-z')^2)^{3/2}}$$

- **Induced Charge Q($\boldsymbol{x}$):**

$$Q^{ind} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sigma(x,y)dxdy = -q$$

- **Important:** Total charge induced by a point charge q on an infinitely large grounded metal plate is equal to –q, independent of the distance of the charge from the plate
  - *However, charge density depends on distance z*

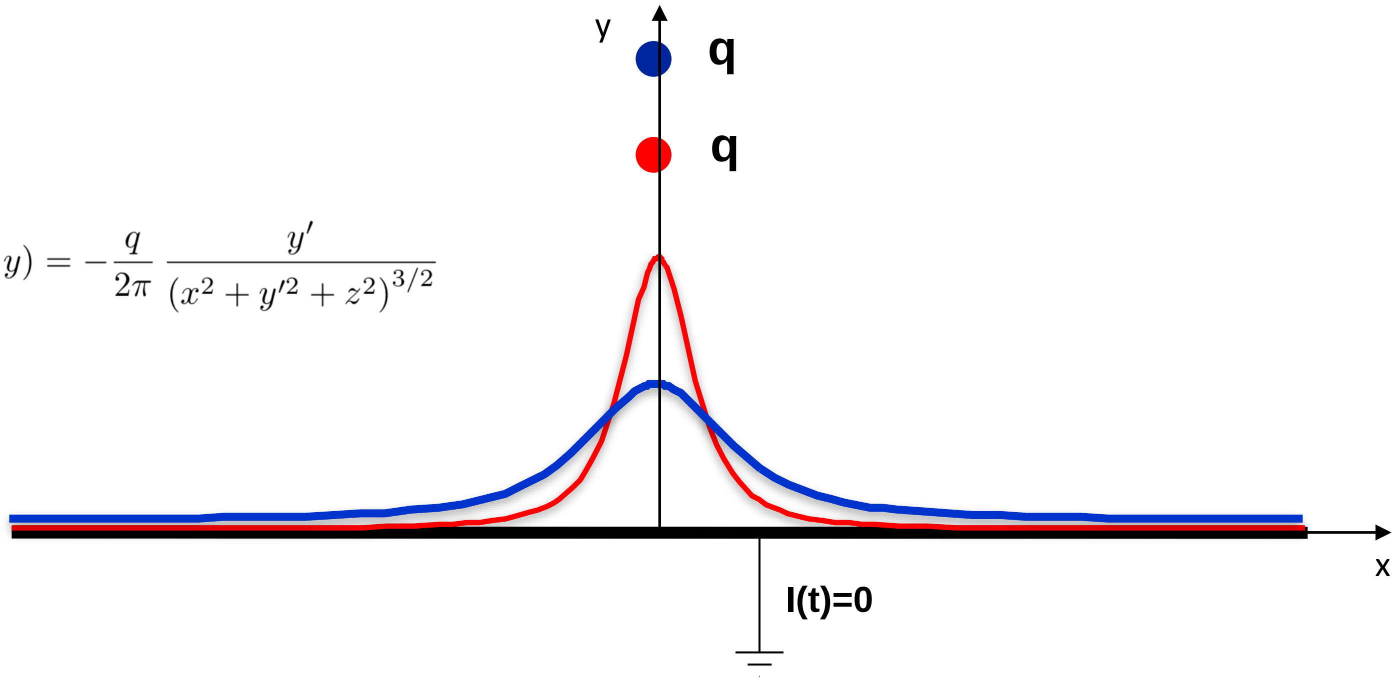


Slide courtesy: W. Riegler

III. Induced Charge on infinite plate

- Moving charge from point far away to point closer to metal plate, the surface charge density becomes more peaked
- Total induced charge always equal to –q
- Charge is rearranged on surface – no current flowing to GND

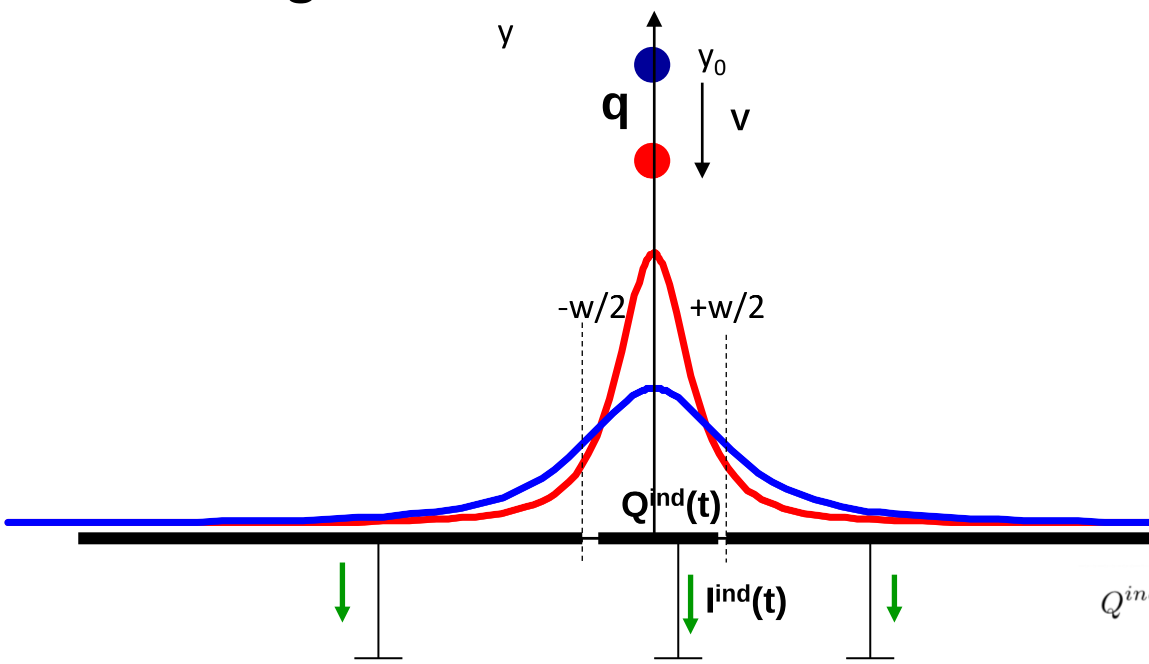
$$\sigma(x,y) = -\frac{q}{2\pi}\frac{y'}{(x^2+y'^2+z^2)^{3/2}}$$

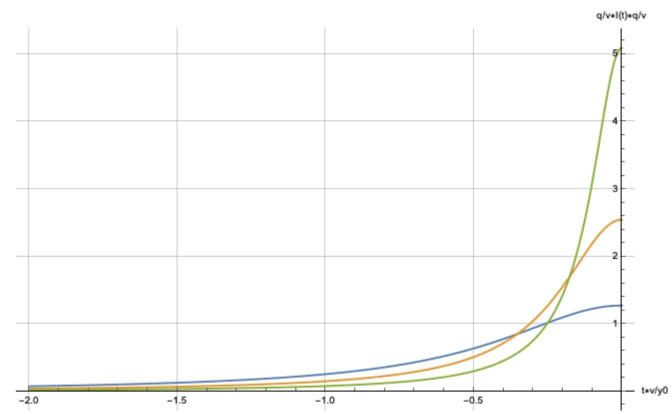


Slide courtesy: W. Riegler

III. Induced Charge on Strip Electrode

- Now segment the grounded metal plate & ground individual strips
- Surface charge density $\sigma(x)$ does not change
- Induced charge on strips now depends on position
- If charge now moves currents are induced - sum $\sum I = 0$





$$Q^{ind}(t) = q\frac{2}{\pi}\arctan\frac{w}{2vt} \qquad t < 0 \qquad y'(t) = -vt$$

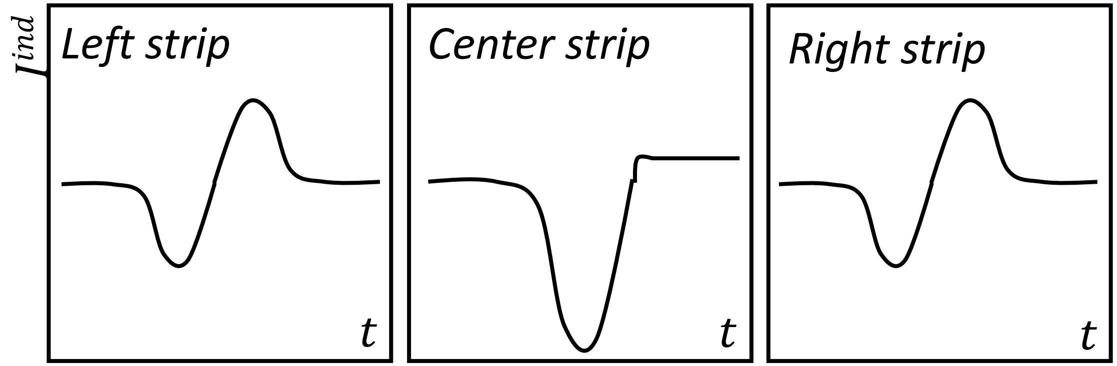
$$I^{ind}(t) = -\frac{dQ^{ind}(t)}{dt} = q\frac{4w}{\pi\,(w^2 + 4v^2t^2)}\,v \qquad t < 0$$

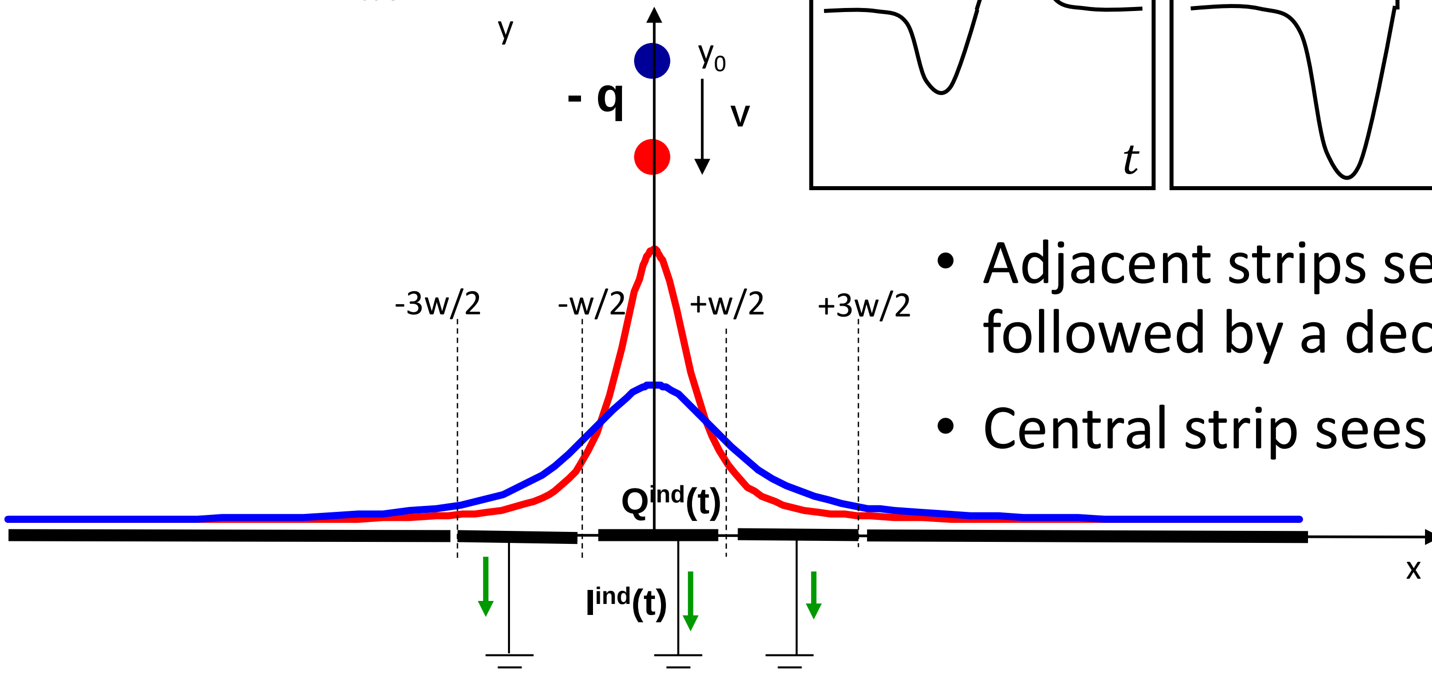
Slide courtesy: W. Riegler

III. Induced Charge on Strip Electrode

- Signal pulses in adjacent electrodes
- Assuming a negative charge now ☺

$$I^{ind}(t) = \frac{dQ^{ind}(t)}{dt}$$



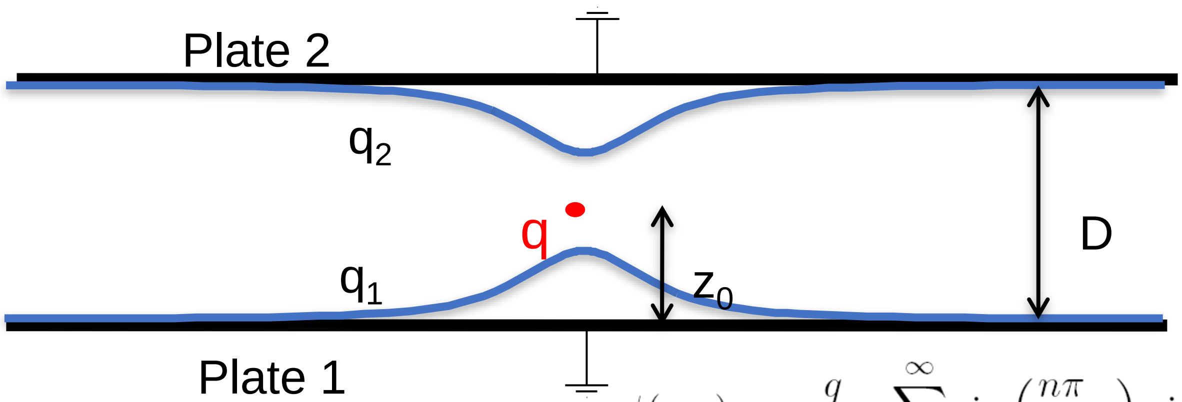


- Adjacent strips see first increase followed by a decrease in $\sigma(x)$
- Central strip sees increase until

Figure courtesy: W. Riegler

III. Induced Charge on Parallel Plates

- Calculation of Potentials $\varphi(\mathbf{x})$ becomes rapidly complicated



$$\phi(r,z) = \frac{q}{\varepsilon_0 \pi D} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{D}\,z\right)\,\sin\left(\frac{n\pi}{D}\,z_0\right)\,K_0\left(\frac{n\pi}{D}r\right)$$

$$E(r,z) = \frac{q}{\varepsilon_0 \pi D} \sum_{n=1}^{\infty} \frac{n\pi}{D}\cos\left(\frac{n\pi}{D}\,z\right)\,\sin\left(\frac{n\pi}{D}\,z_0\right)\,K_0\left(\frac{n\pi}{D}r\right)$$

$$\sigma_1(r) = \varepsilon_0 E(r, z=0) = \frac{q}{\pi D} \sum_{n=1}^{\infty} \frac{n\pi}{D}\,\sin\left(\frac{n\pi}{D}\,z_0\right)\,K_0\left(\frac{n\pi}{D}r\right)$$

$$q_1 = \int_0^{\infty} 2r\pi\sigma(r)dr = \frac{q}{\pi D}\sum_{n=1}^{\infty}\frac{n\pi}{D}\sin\left(\frac{n\pi}{D}\,z_0\right)\int_0^{\infty} 2r\pi K_0\left(\frac{n\pi}{D}r\right)dr = \frac{2q}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{D}\,z_0\right) =$$

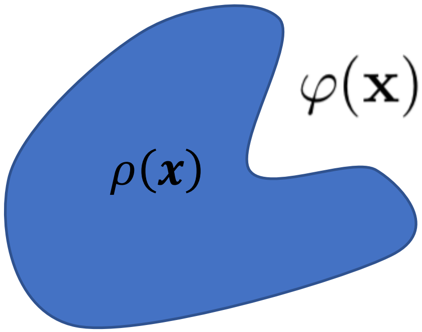
$$= -q\left(1-\frac{z_0}{D}\right) \qquad q_2 = \ldots = -q\,\frac{z_0}{D} \qquad q_1 + q_2 = -q$$

- Geometry looks “simple”
- Potentials are “complicated”
- But formula for induced charge is much simpler
- Is there an easier way to calculate the signals?

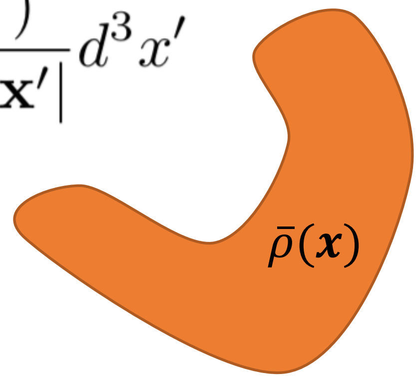
Slide courtesy: W. Riegler

IV. Reciprocity Theorem

- Assume Two arbitrary charge distributions $\rho(\boldsymbol{x})$ and $\bar{\rho}(\boldsymbol{x})$:



$$\varphi(\mathbf{x}) = \int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x'$$



$$\overline{\varphi}(\mathbf{x}) = \int \frac{\overline{\rho}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x'$$

- Then:

$$W = \int \overline{\rho}(\mathbf{x})\varphi(\mathbf{x})d^3x = \int\int \frac{\overline{\rho}(\mathbf{x})\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x d^3x' = \int \rho(\mathbf{x}')\overline{\varphi}(\mathbf{x}')d^3x'$$

- Or:

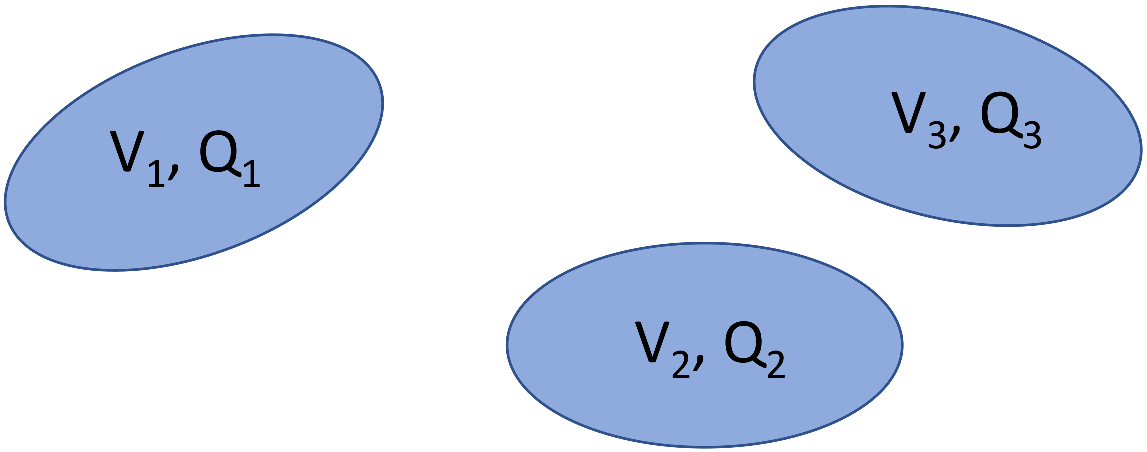
$$\int \overline{\rho}(\mathbf{x})\varphi(\mathbf{x})d^3x = \int \rho(\mathbf{x})\overline{\varphi}(\mathbf{x})d^3x$$

Reciprocity Theorem

- Interpretation: Work needed to move one charge distribution in field of other charge distribution

Slide courtesy: W. Riegler

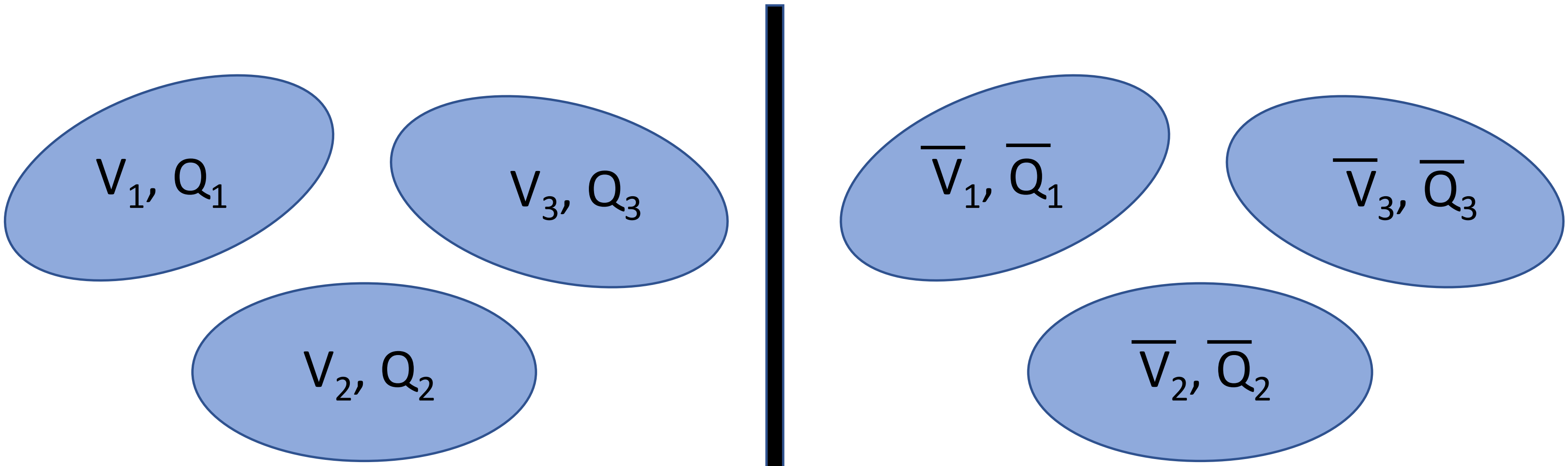
IV. Reciprocity Theorem



- Assume three electrodes with potentials V_1, V_2, V_3 .
- The potentials will result in charges Q_1, Q_2, Q_3 .
- Assume no external charge distribution. To find Q_i we solve the Laplace equation $\Delta\varphi = 0$ with boundary conditions $\varphi = V_i$ on electrode surface
- We can then calculate: $Q_i = -\epsilon_0 \oint_{A_i} \nabla\varphi(\boldsymbol{x}) dA$

Slide courtesy: W. Riegler

IV. Reciprocity Theorem



- Assume now two electric states
- Reciprocity Theorem states: $\sum Q_i \overline{V}_i = \sum \overline{Q}_i V_i$
- Let's use this to calculate our signals!

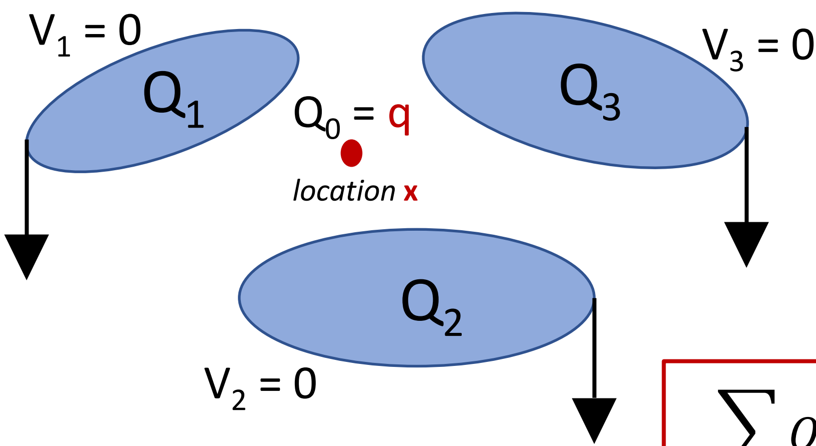
Discrete version

$$\int \overline{\rho}(\mathbf{x})\varphi(\mathbf{x})d^3x = \int \rho(\mathbf{x})\overline{\varphi}(\mathbf{x})d^3x$$

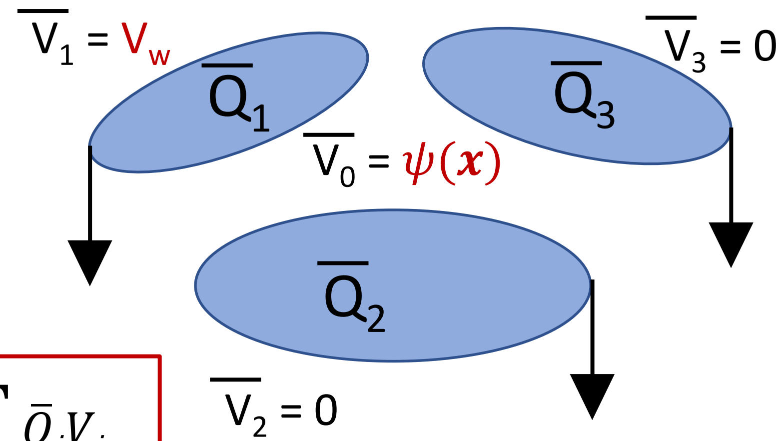
Slide courtesy: W. Riegler

IV. Reciprocity Theorem

- Our problem: *Three grounded electrodes with a point charge*
  - *??? What are the charges induced on the grounded electrodes ???*
- Approach:
  - Treat point charge q as 4th electrode with $Q_0 = q$ and V_0
  - Assume another set of voltages and charges, where we remove the point charge (set it zero) and put one electrode to voltage V_w



$$\sum Q_i \bar{V}_i = \sum \bar{Q}_i V_i$$



- Solution:
  - Use reciprocity theorem: $q\bar{V}_0 + Q_1 V_w = 0$ => $Q_1 = -q\frac{\psi(\boldsymbol{x})}{V_w}$
  - Calculate weighting potential $\psi(\boldsymbol{x})$ by removing charge & set electrode 1 to V_w
  - Voltages V_i & charges Q_i are related trough capacitance matrix $Q_i = \sum_j c_{ij} V_j$

Slide courtesy: W. Riegler

IV. Reciprocity Theorem

- The charge induced by a point charge q on a grounded conducting electrode can be calculated in the following way:
  1. Remove point charge q and put the electrode on potential V_w and all other electrodes on ground potential (0)
  2. Calculate the weighting potential $\psi(\boldsymbol{x})$ of this configuration
  3. Induce charged charge is now calculated as $Q_{ind} = -\frac{q}{V_w}\psi(\boldsymbol{x})$
- This way we do not need to solve the Poisson Equation for a point charge, but we solve the Laplace equation
  - Solve a 2D Laplace equation instead of 3D Poisson equation
  - Numerically more stable
  - Simplify by $\psi(\boldsymbol{x})$ for simple $V_w = 1$: $Q_{ind} = -q\,\psi(\boldsymbol{x})$
  - Moving charge with trajectory $\boldsymbol{x}(\boldsymbol{t})$: $Q_{ind}(t) = -q\,\psi(\boldsymbol{x}(t))$
- This is the Ramo-Shockley Theorem:
  - $I_{ind} = \frac{dQ_{ind}(\boldsymbol{x}(t))}{dt} = \frac{q}{V_w}\nabla\psi(\boldsymbol{x}) \cdot \dot{\boldsymbol{x}}(t) = -\mathrm{q}\mathbf{v} \cdot \mathbf{E}$ *(with E the weighting field)*

IV. Ramo-Shockley Theorem
Reciprocity theorem

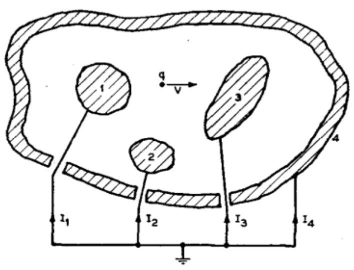
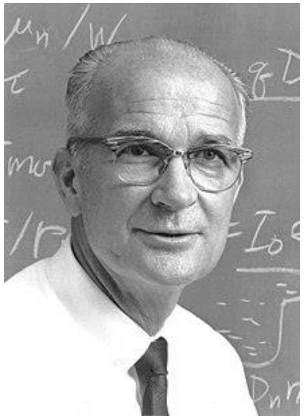


FIG. 1. Schematic representation of conductors and currents.

Currents to Conductors Induced by a Moving Point Charge

W. SHOCKLEY
Bell Telephone Laboratories, Inc., New York, N. Y.
(Received May 14, 1938)

General expressions are derived for the currents which flow in the external circuit connecting a system of conductors when a point charge is moving among the conductors. The results are applied to obtain explicit expressions for several cases of practical interest.



William Shockley

584 *Proceedings of the I.R.E.* *September, 1939*

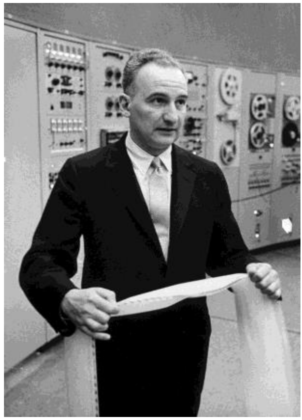
Currents Induced by Electron Motion*

SIMON RAMO†, ASSOCIATE MEMBER, I.R.E.

Summary—*A method is given for computing the instantaneous current induced in neighboring conductors by a given specified motion of electrons. The method is based on the repeated use of a simple equation giving the current due to a single electron's movement and is believed to be simpler than methods previously described.*

METHOD OF COMPUTATION

The method is based on the following equation, whose derivation is given later:

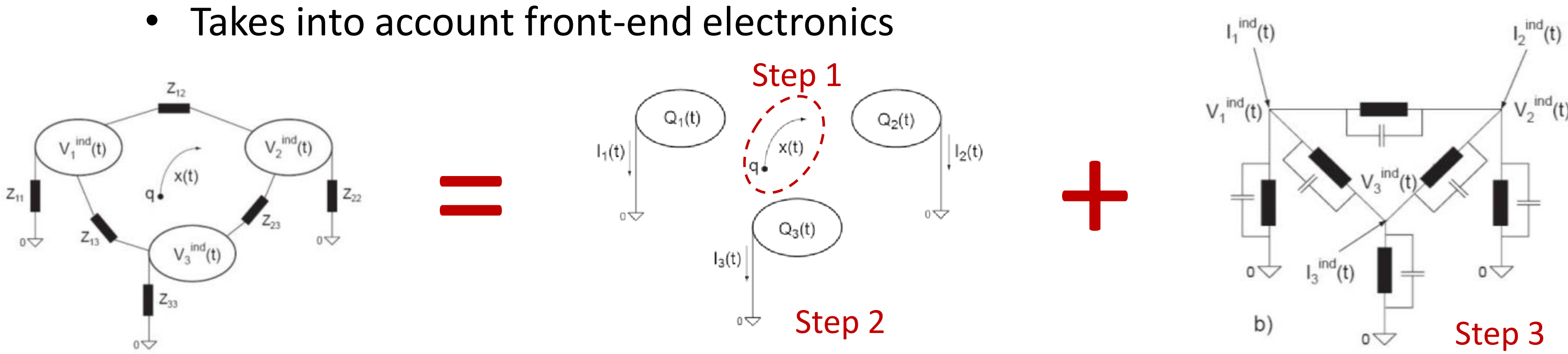


Simon Ramo

*Shockley, W. (1938) Journal of Applied Physics **9** (10), doi:10.1063/1.1710367*
*Ramo, S. (1939) Proceedings of the IRE **27** (9) 584-585, doi:10.1109/JRPROC.1939.228757*

V. Recipe for Simulation

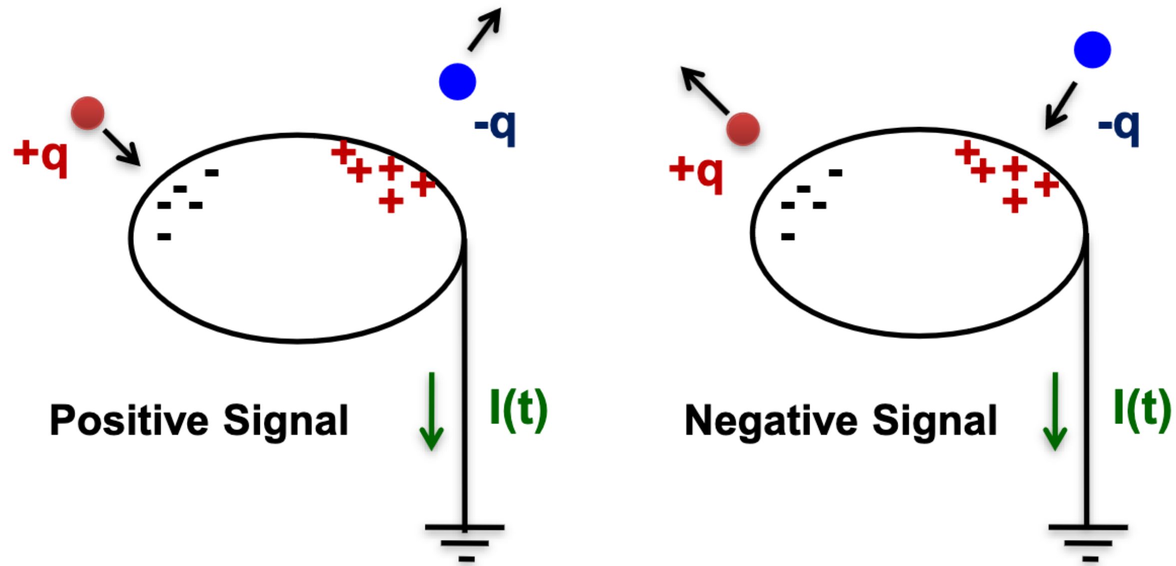
- We now have a recipe to calculate induced signals
  - *And that is exactly what we need to simulate signals!*
- In 3 steps:
  1. Calculate the particle trajectory in the real electric field
     - OK: Avalanche simulation inside our detector (e.g. Garfield++)
  2. Connect all electrodes to ground and calculate the currents induced by the moving charge on those electrodes
     - Requires the calculation of weighting fields
       - Remove the charge; put 1 electrode to 1V and other electrodes to GND
       - Result is weighting field for that electrode you put to 1V
       - To be repeated for each readout electrode (e.g. strip/pixel) in your geo
     - OK: done with Finite Element Method or Boundary Element Method
  3. Feed currents into network simulator (e.g. spice) or apply Transfer-function (e.g. Garfield++)
     - Takes into account capacitive couplings between electrodes
     - Takes into account front-end electronics



Slide courtesy: W. Riegler

V. Generalities: Signal Polarity

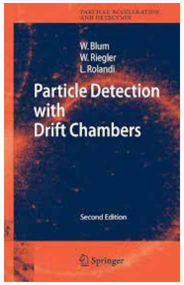
$$I(t) = -\frac{dQ(t)}{dt}$$



- A positive charge moving towards the electrode
  - attracts negative Q moving from GND to electrode => Positive signal, $I_{ind} > 0$
- A negative charge moving towards the electrode
  - Attracts positive Q moving from GND to electrode => Negative signal, $I_{ind} < 0$
- Likewise for Q moving away from electrode => *they also induce signal!*

Slide courtesy: W. Riegler

V. Generalities: Theorems



- Consequences for the Induced Charges and Currents:

1. *The charge induced on an electrode for a charge that have been moved from point x_0 to point x_1 is:*

$$Q_{ind} = \int_{t0}^{t1} I_{ind}(t)dt = -\frac{q}{V_w}\int_{t_0}^{t_1} E\left(\boldsymbol{x}(t)\right)\dot{\boldsymbol{x}}(t)dt = \frac{q}{V_w}[\psi(\boldsymbol{x}_1) - \psi(\boldsymbol{x}_0)]$$

   and is <u>independent</u> on the actual path

2. *Once all charges have arrived at the electrodes, then the total induced charge in the electrodes is equal to the charge that has arrived at this electrode*
   - *Consequence: once all charges have arrived at the electrode => Induced current is zero*
   - *Consequence: current signals on electrodes that do not receive any charge => strictly bipolar*

3. *In case there is one electrode enclosing all other electrodes, the sum of all induced currents is zero at any time*

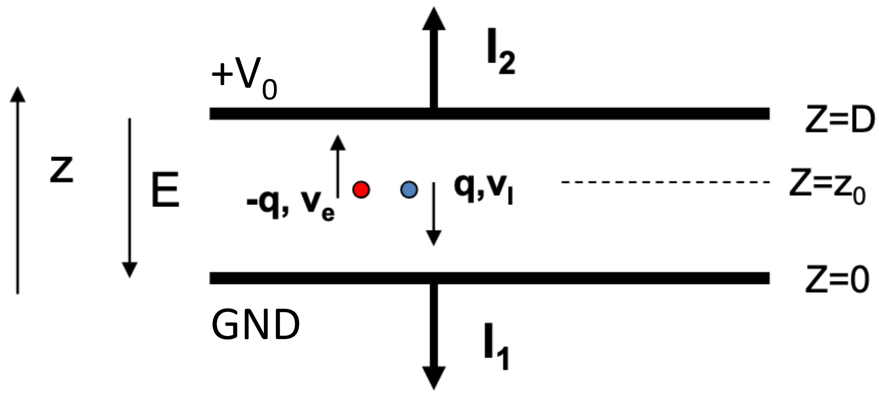
VI. Examples
Parallel Plate Avalanche Counter (PPAC) - I

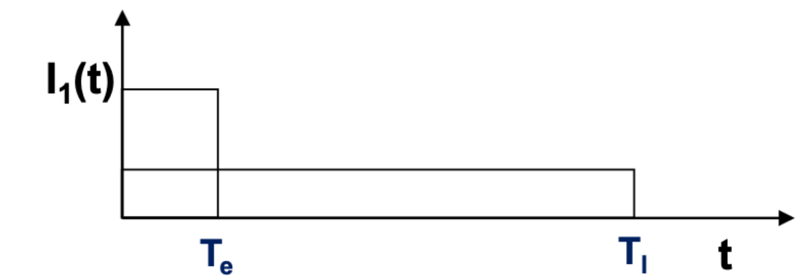
- **Simple geometry - many applications**
  - Electron-ion pair in gas (parallel-plate)
  - Electron-hole pair in liquid (e.g. Liquid Ar)
  - Electron-ion pair in solid (e.g. silicon strip)

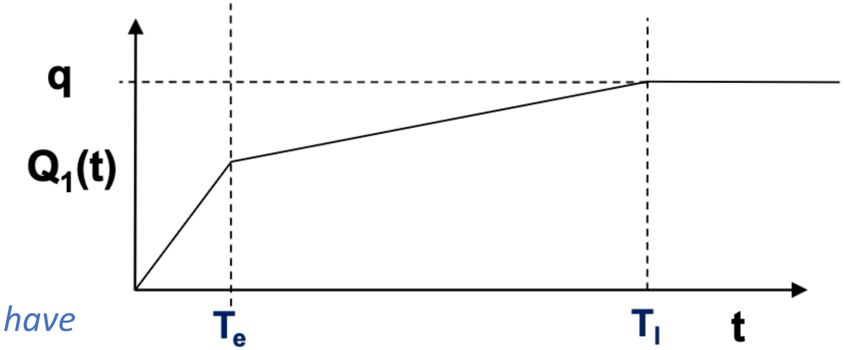
- **Weighting Fields:**
  - Electrode 1: $E_1 = \frac{V_0}{D}$
  - Electrode 2: $E_2 = -\frac{V_0}{D}$

- **Induced Currents:**
  - Elec 1: $I_1 = -\frac{-q}{V_0}\frac{V_0}{D}v_e - \frac{+q}{V_0}\frac{V_0}{D}(-vi)$
  - Elec 2: $I_2 = -\frac{-q}{V_0}\frac{-V_0}{D}v_e - \frac{+q}{V_0}\frac{-V_0}{D}(-vi) = -I_1$

- **Induced Charges:**
  - Elec 1: $Q_1 = \frac{q}{D}v_e t_e + \frac{q}{D}v_i t_i = \underbrace{q\frac{D-z_0}{D}}_{q_e^{ind}} + \underbrace{q\frac{z_0}{D}}_{q_i^{ind}} = q$
  - Elec 2: $Q_2 = -q$





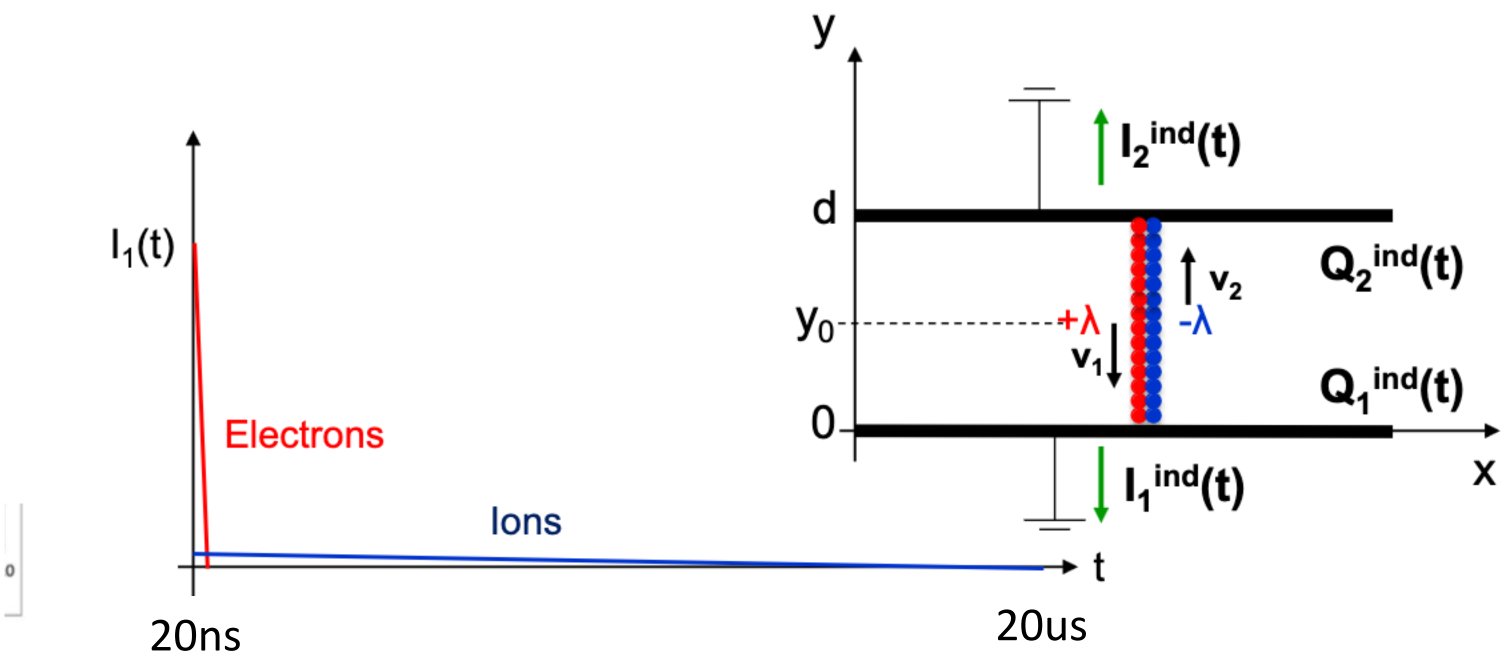


Total Induced charge on a given electrode, once all charges have arrived is equal to the charge that has arrived at this electrode

Slide courtesy: W. Riegler

VI. Examples
Parallel Plate Avalanche Counter (PPAC) - II

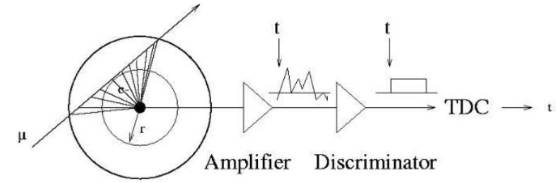
- **Often not a single energy deposit**
  - *E.g. Ionization along the trail of a passing MIP*
  - *Cluster density of λ / mm*
  - *1-few mm gap, v_e = ~50um/ns, v_i = ~0.05um/ns*



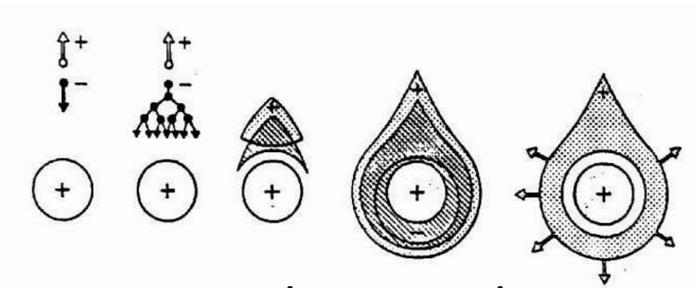
Slide courtesy: W. Riegler

VII. Examples

Signal in Wire Chambers – Drift Tubes

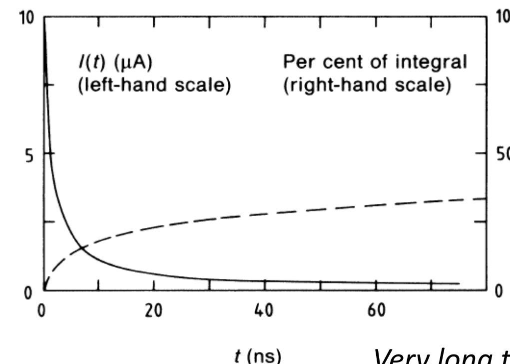


- Many textbooks provide (correctly) the electric field: $E(r) = \frac{V}{r \ln\left(\frac{b}{a}\right)}$
    - a = Wire radius (10, 25 or 50um)
    - b =Tube radius (1-3cm)
    - V = postive voltage applied to wire



Electron avalanche very close to the wire. First multiplication only at r = 2a

Current signal for t_0 = 1.25ns, b/a = 500 and q = 10^6 e



Very long tail

- Say G= 10^4, then 10^4 electrons arrive to the wire within 1ns and do not move
- *Ions close to the wire have opposite charge => in begin zero charge induced*
- *Only once ions move away from the wire the signal is induced*
    - Signal can take up to 100us

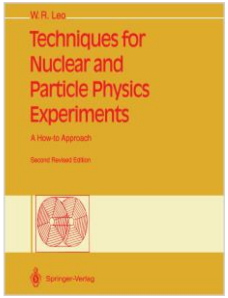
Weighting field: $E_w(r) = \frac{V_w}{r \ln\left(\frac{b}{a}\right)}$ Ion trajectory: $r(t) = a\sqrt{1 + 1/t_0}$ $t_0 = \frac{a^2 \ln\left(\frac{b}{a}\right)}{2\mu V}$

Induced signal: $I_{ind}(t) = -\frac{N_{tot}\, e}{V_w} E\big(r(t)\big)\dot{r}(t) = -\frac{N_{tot}\, e}{2\ln\left(\frac{b}{a}\right)} \frac{1}{t + t_0}$ $t_{max} = t_0\left(\frac{b^2}{a^2} - 1\right)$

ATLAS MDTs: V=3500V, a = 25um, b = 1.46cm, t_0 = 11ns, t_{max}=3.73ms

VII. Examples

Signal in Wire Chambers – Energy Argument



6.5.1 Pulse Formation and Shape

Contrary to what might be inferred from the brief description of ionization counters in Sect. 6.1, the pulse signal on the electrodes of ionization devices is formed by induction

due to the movement of the ions and electrons as they drift towards the cathode and anode, rather than by the actual collection of the charges itself. Let us see how this occurs. For the cylindrical proportional counter, the electric field and potential can be written as

$$E(r) = \frac{CV_0}{2\pi\varepsilon}\frac{1}{r}\,,$$

$$\varphi(r) = -\frac{CV_0}{2\pi\varepsilon}\ln\left(\frac{r}{a}\right)\,, \qquad (6.25)$$

where r is the radial distance from the wire, V_0 the applied voltage, ε the dielectric constant of the gas, and

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} \qquad (6.26)$$

is the capacitance per unit length of this configuration.

Suppose that there is now a charge q located at a distance r from the central wire. The potential energy of the charge is then

$$W = q\varphi(r)\,. \qquad (6.27)$$

If now the charge moves a distance dr, the change in potential energy is

$$dW = q\frac{d\varphi(r)}{dr}dr\,. \qquad (6.28)$$

For a cylindrical capacitor, however, the electrostatic energy contained in the electric field is $W = \frac{1}{2}lCV_0^2$, where l is the length of the cylinder. If the movement of the charges is fast relative to the time that an external power supply can react to changes in the energy of the system, we can consider the system as closed. Energy is then conserved, so that

$$dW = lCV_0\,dV = q\frac{d\varphi(r)}{dr}dr\,. \qquad (6.29)$$

Thus there is a voltage change,

$$dV = \frac{q}{lCV_0}\frac{d\varphi(r)}{dr}dr \qquad (6.30)$$

induced across the electrodes by the displacement of the charge. Equation (6.30) is a general result, in fact, and can be used for any configuration.

For our cylindrical proportional counter, let us assume that an ionizing event has occurred and that multiplication takes place at a distance r' from the anode. The total induced voltage from the electrons is then

$$V^- = \frac{-q}{lCV_0}\int_{a+r'}^{a}\frac{d\varphi}{dr}dr = -\frac{q}{2\pi\varepsilon l}\ln\left(\frac{a+r'}{a}\right) \qquad (6.31)$$

while that from the positive ions is

$$V^+ = \frac{-q}{lCV_0}\int_{a+r'}^{b}\frac{d\varphi}{dr}dr = -\frac{q}{2\pi\varepsilon l}\ln\frac{b}{a+r'}\,. \qquad (6.32)$$

The sum of the two contributions is then $V = V^- + V^+ = -q/lC$ and their ratio of the contributions is

$$\frac{V^-}{V^+} = \frac{\ln\dfrac{a+r'}{a}}{\ln\dfrac{b}{a+r'}}\,. \qquad (6.33)$$

Since the multiplication region is limited to a distance of a few wire radii, it is easy to see that the contribution of the electrons is small compared to the positive ions. Taking some typical values of $a = 10\,\mu m$, $b = 10$ mm and $r' = 1\,\mu m$, V^- turns out to be less than 1% of V^+. The induced signal, therefore, is almost entirely due to the motion of the positive charges and one can ignore the motion of the electrons[1].

With this simplification we can now calculate the time development of the pulse. Thus,

$$V(t) = \int_{r(0)}^{r(t)}\frac{dV}{dr}dr = -\frac{q}{2\pi\varepsilon l}\ln\frac{r(t)}{a}\,. \qquad (6.34)$$

To find $r(t)$, we have the definition (6.19)

$$\frac{dr}{dt} = \mu E(r) = \frac{\mu CV_0}{2\pi\varepsilon}\frac{1}{r} \qquad (6.35)$$

so that

$$r\,dr = \frac{\mu CV_0}{2\pi\varepsilon}dt\,. \qquad (6.36)$$

Since the positive ions all come from the region close to the anode, we can set $r(0) = a$ for simplicity. Integration then yields

$$r(t) = \left(a^2 + \frac{\mu CV_0}{\pi\varepsilon}t\right)^{1/2}\,. \qquad (6.37)$$

[1] The contribution of the electrons can be ignored only if they are all created near the anode. In some high gain gases, such as the magic gas to be discussed later, this is not always the case. Indeed, ultraviolet photons emitted in avalanches near the anode can extend the avalanche radially outward where the process is finally halted by the low field. In such cases the path length of the electrons is long and their contribution to the induced signal becomes significant [6.14].

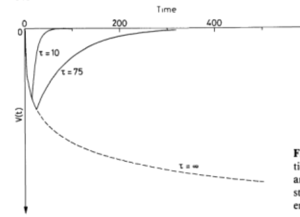


Fig. 6.6. Pulse signal from a cylindrical proportional counter. The pulse is usually cut short by an RC differentiating circuit with a time constant τ. The figure shows the effect of two different constants

Substituting into (6.34), we find

$$V(t) = -\frac{q}{4\pi\varepsilon l}\ln\left(1 + \frac{\mu CV_0}{\pi\varepsilon a^2}t\right) = -\frac{q}{4\pi\varepsilon l}\ln\left(1 + \frac{t}{t_0}\right)\,, \qquad (6.38)$$

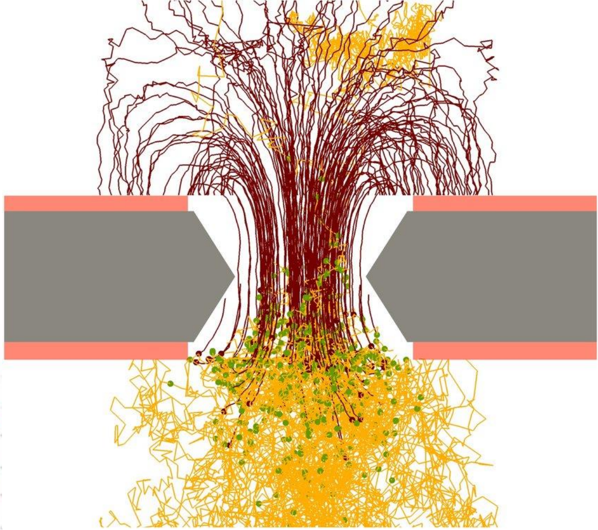
where $t_0 = a^2\pi\varepsilon/\mu CV_0$. For this distance the total drift time T is

$$T = \frac{t_0}{a^2}(b^2 - a^2)\,. \qquad (6.39)$$

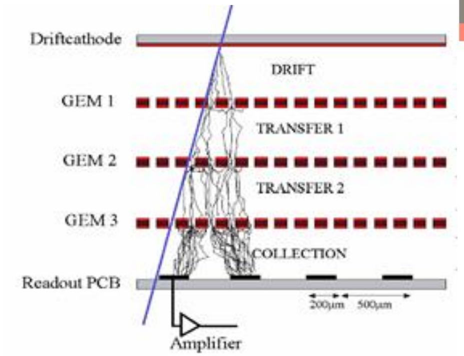
This function is graphed in Fig. 6.6 for some typical values. Since it is not necessary to use the entire signal, the pulse is usually differentiated (see Sect. 14.23.2) to shorten its duration. In this manner only the faster rising part of the pulse is exploited. Depending on the time constant of the differentiator, the fall time of the resulting pulse will vary.

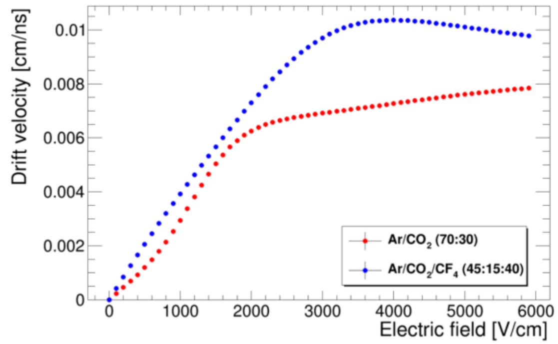
- Energy Argument gives correct result only in the case of two electrodes
- Electrons lose energy due to scattering collisions in the gas (= heating of the gas)

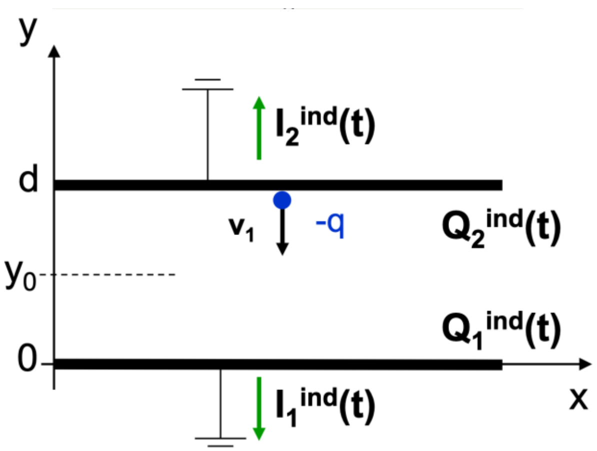
VIII. Examples
Signal in Triple-GEM

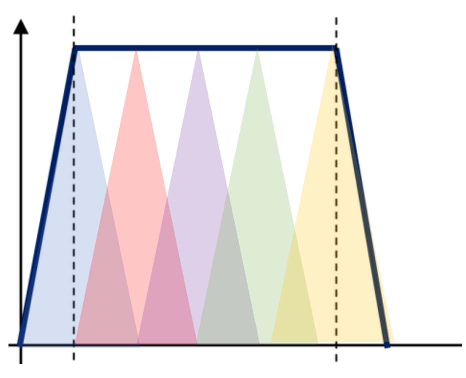


- Triple Amplification Stage
  - Signal induced in last Gap
- Gaseous Electron Multiplier
  - Ions move to top (shielded)
  - Electrons move down (signal)
- Geom equiv to Parallel-Plate & e- only
  - v_e in drift: ~50um/ns ($Ar:CO_2$)
  - 3mm drift gap: primary ionization spread over 60ns
  - 1mm induction gap: raising and falling edge ~20ns









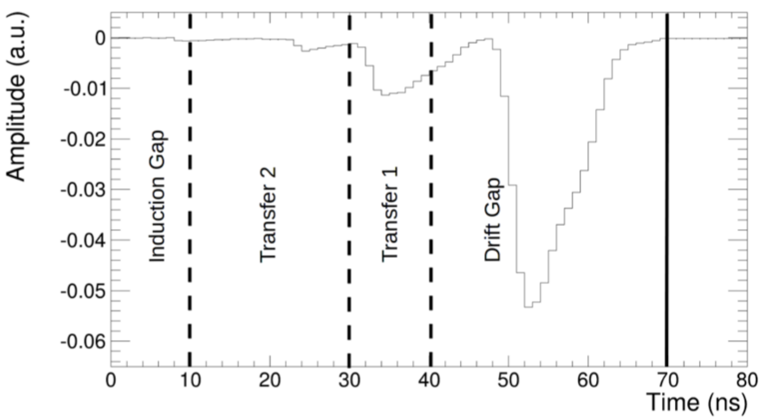


Figure courtesy: W. Riegler

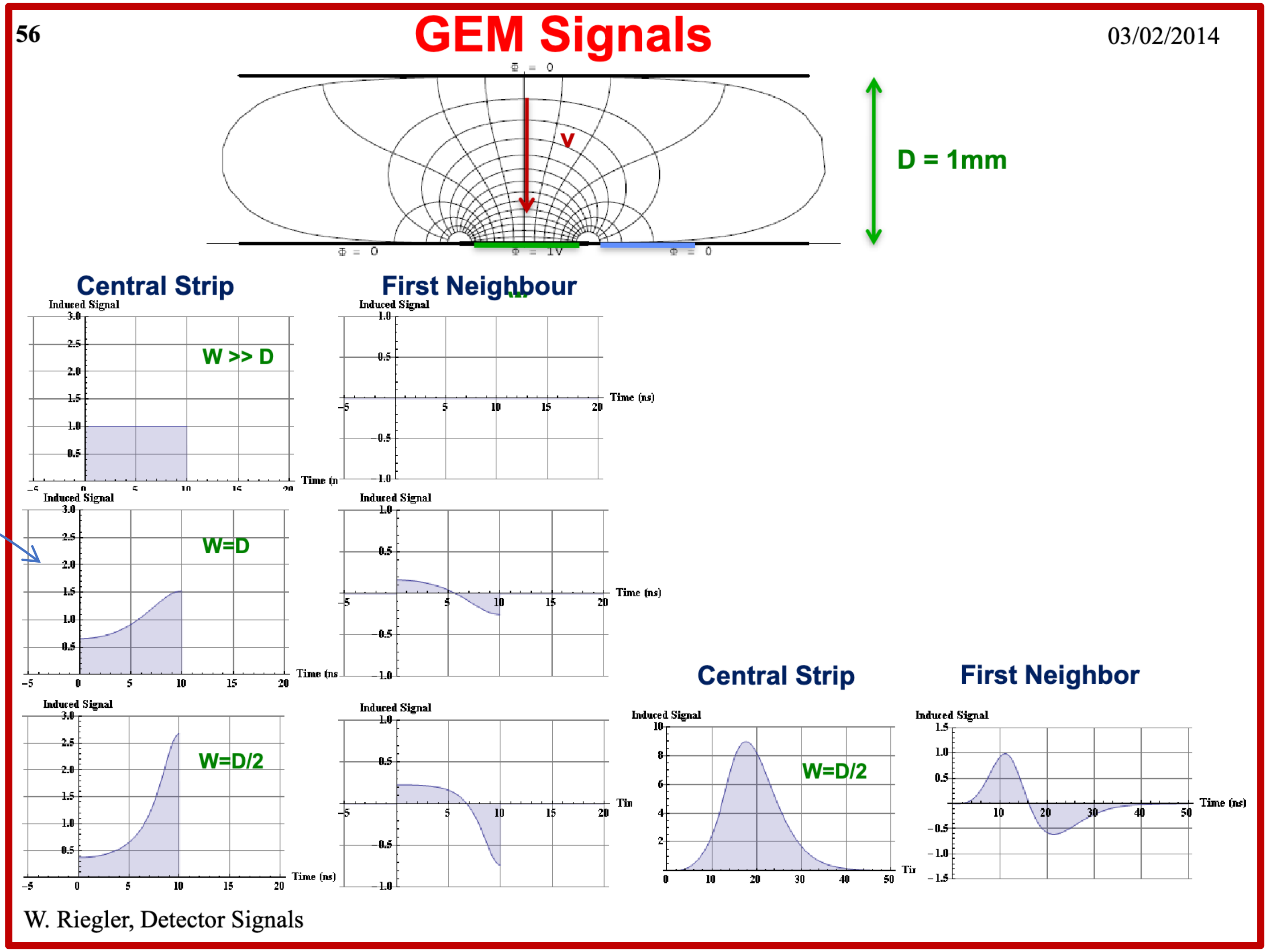
Figure courtesy: T. Maerschalck

VIII. Examples
Signal in Triple-GEM

Signal shape for different Stripwidth

CMS GEMs
GE21: 1.2mm
GE11: 0.6mm

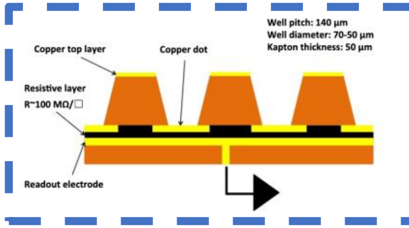
10x10 Triple-GEMs: 0.4mm



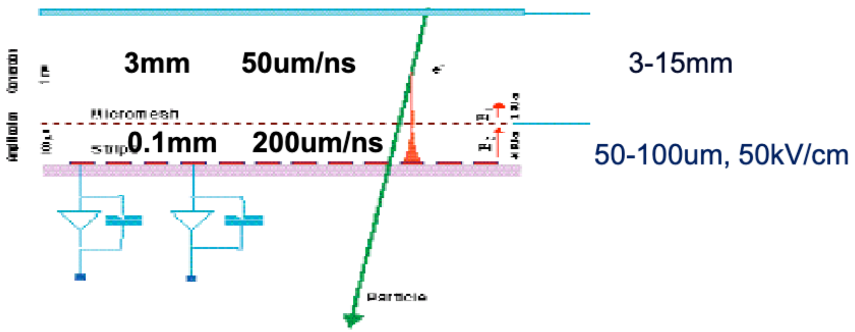
Slide courtesy: W. Riegler

VIII. Examples
Signal in Micromegas / uRWELL

uRWELL



Micromegas



Electrons movement in the induction gap takes about 0.1mm/v_1=0.5ns.

Collecting all electrons from the drift gap takes e.g. 3mm/v_1=60ns.

The MICROMEGA electron signal has a length of about 60ns.

Ion movement – e.g. Argon Ions take 130ns for 50kV/cm and 100um gap, so the total length of the ions component is around 180ns.

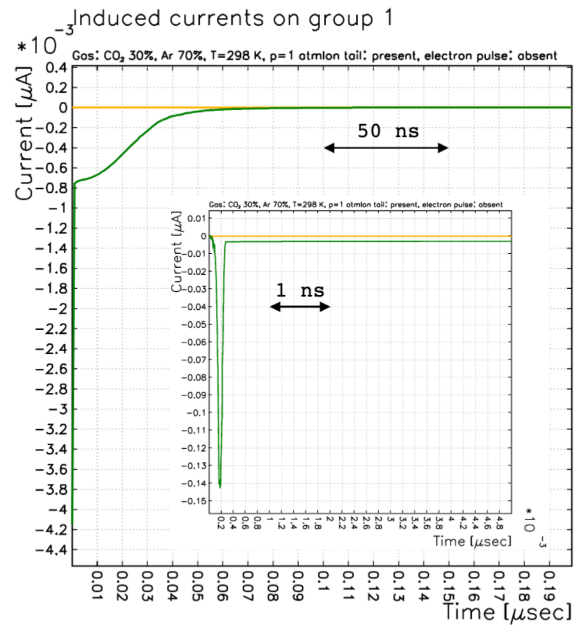
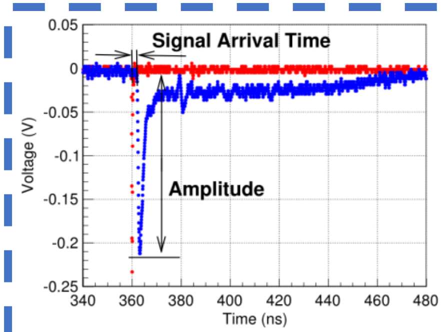


Figure 4. GARFIELD simulation of a signal from a single ionization electron in a µ-RWELL in Ar:CO_2 70:30 gas mixture. The absence of the induction gap is responsible for the fast initial spike, about 200 ps, induced by the motion and fast collection of the electrons and followed by a 50 ns ion tail.

Slide courtesy: W.Riegler
Figure courtesy: picosec
NIM A 903 (2018) 317-325



Figures courtesy: M. Poli Lener
JINST 10 (2015) P02008

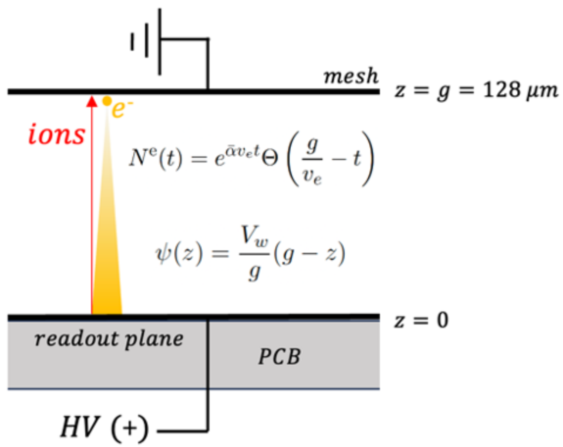
VIII. Examples

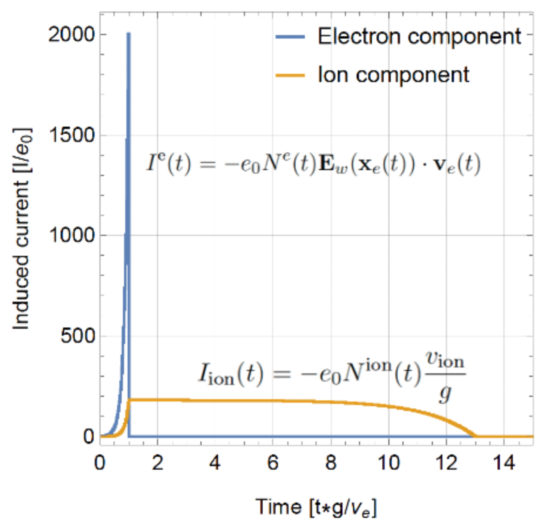
Signal in Micromegas / uRWELL

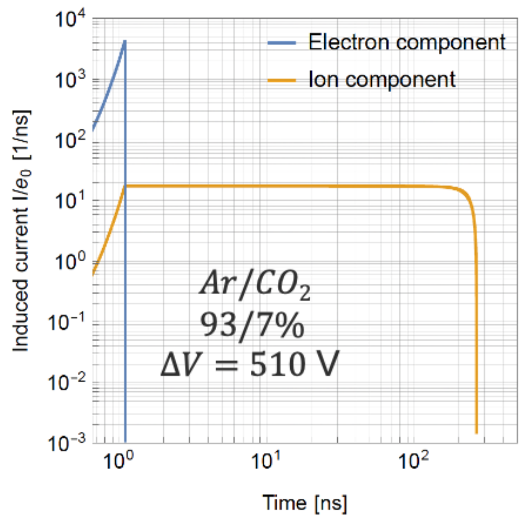
Signal in a non-resistive Micromegas

Simulation

Let us consider a Townsend avalanche inside the amplification gap of a Micromegas detector that induces a signal on the anode plane.







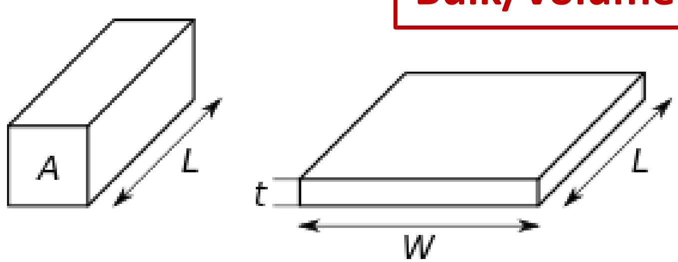
Slide courtesy: D. Janssens

IX. Signals in Resistive Detectors

- So far we treated only configurations with
  - *Electrodes as perfect (metallic) conductors*
  - *Electrodes at GND (measure I^{ind}) or insulated (measure V^{ind})*
  - *Non-Electrode detector materials are perfect insulators*
- Need extension of Ramo-Shockley theorem
  - *Detector materials with finite conductivity (RPC)*
  - *Detectors with resistive layers (uRWELL, resistive-MM, …)*

Slide courtesy: W. Riegler

Bulk/Volume Resistivity & Surface Resistivity



R = Resistance [Ω]

ρ = (Bulk) Resistivity [$\Omega \cdot m$]

R_s = Sheet resistance [$\Omega/\boxdot$]

σ = Conductivity (ρ^{-1}) [S/m]

$$R = \rho \frac{L}{A} = \rho \frac{L}{Wt}$$

$$R = \frac{\rho}{t} \frac{L}{W} = R_s \frac{L}{W}$$

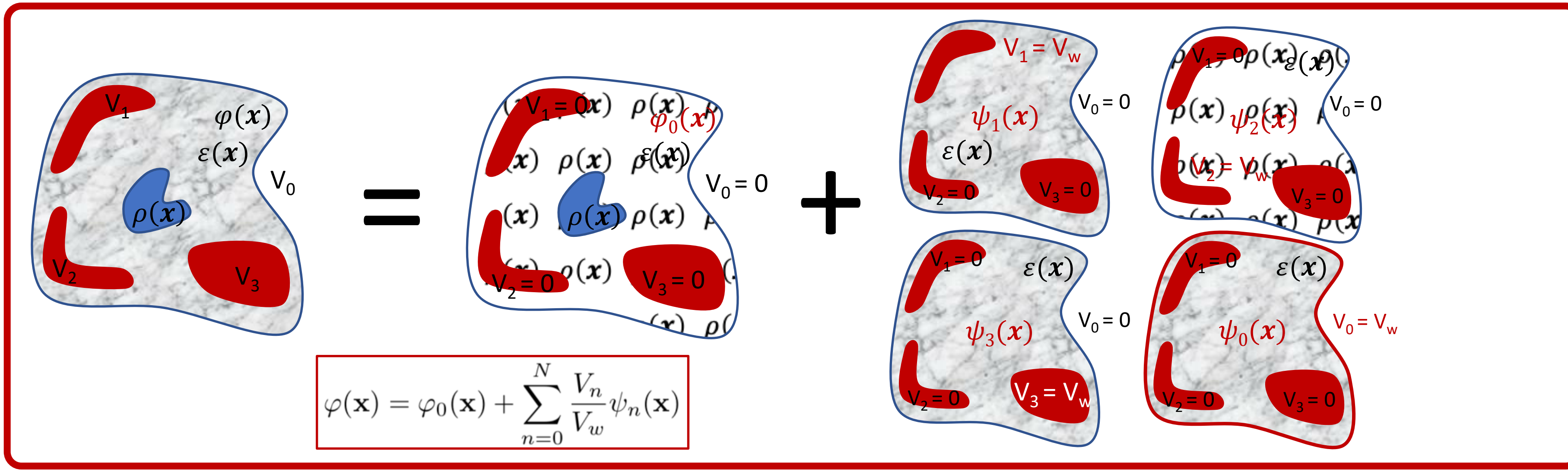
$$\rho = R_s \cdot t$$

Current density j($\boldsymbol{x}$) related to electric field E($\boldsymbol{x}$) by:

$$\mathbf{j}(\mathbf{x}) = \sigma(\mathbf{x})\mathbf{E}(\mathbf{x}) = -\sigma(\mathbf{x})\nabla\varphi(\mathbf{x})$$

IX. Signals in Resistive Detectors

Dielectric medium $\varepsilon(\boldsymbol{x})$



ψ_n are the weighting potentials of the electrodes

$E_n = -\nabla\psi_n$ are the weighting fields of the electrodes

$$\nabla[\varepsilon(\boldsymbol{x})\nabla\varphi_0(\boldsymbol{x})] = -\rho(\boldsymbol{x}) \qquad \varphi_0(\boldsymbol{x})|_n = Vn$$

$$\nabla[\varepsilon(\boldsymbol{x})\nabla\psi_n(\boldsymbol{x})] = 0 \qquad \psi_n(\boldsymbol{x})|_n = Vw\delta_{mn}$$

$$Q_n = \oint \varepsilon(\boldsymbol{x})\boldsymbol{E}(\boldsymbol{x})dA$$

$$c_{mn} = \frac{1}{V_w}\oint \varepsilon(\boldsymbol{x})\nabla\psi_n(\boldsymbol{x})dA$$

$$\varphi(\boldsymbol{x}) = \varphi_0(\boldsymbol{x}) + \sum_{n=0}^{N}\frac{V_n}{V_w}\psi_n(\boldsymbol{x})$$

Ramo-Shockley theoremo also holds for dielectric media

Slide courtesy: W. Riegler

X. Quasi-Static Approximation

- Introduction of resistive material => current density $j(\boldsymbol{x},t) = \sigma(\boldsymbol{x})E(\boldsymbol{x},t)$ (1)
- In addition to this current we have an externally impressed current $j_e(\boldsymbol{x},t)$
  - Related to an external charge density $\rho_e(\boldsymbol{x},t)$
  - Total current $j(\boldsymbol{x},t) = \underbrace{\sigma(\boldsymbol{x})E(\boldsymbol{x},t)} + j_e(\boldsymbol{x})$ (2)
- Assume externally impressed $j_e(\boldsymbol{x},t)$ is changing only slowly in time
  - => neglect Faraday's law and approximate: $\nabla \times E(\boldsymbol{x},t) \approx 0$ => $E(\boldsymbol{x},t) = -\nabla\varphi(\boldsymbol{x},t)$ (3)
- We obtain the Electro Quasi-Static (EQS) approximation:
  - $\nabla \cdot \varepsilon(\boldsymbol{x})E(\boldsymbol{x},t) = \rho(\boldsymbol{x},t)$ (4)
  - $\nabla \cdot j(\boldsymbol{x},t) + \frac{\partial\rho(\boldsymbol{x},t)}{\partial t} = 0$ (5)
- Ampere's Law: $\nabla \times B(\mathrm{x},\mathrm{t}) = \varepsilon\mu\frac{\partial E(\boldsymbol{x},t)}{\partial t} + \mu j(\boldsymbol{x},t)$ (6)
- Now taking the divergence of (6) and substituting (2)
  - $\nabla \cdot [\nabla \times B(\mathrm{x},\mathrm{t})] = 0 = \nabla \cdot [\varepsilon(\boldsymbol{x})\mu\frac{\partial E(\boldsymbol{x},t)}{\partial t} + \mu\sigma(\boldsymbol{x})E(\boldsymbol{x},t) + \mu j_e(\boldsymbol{x})]$
  - $\nabla \cdot \left[\varepsilon(\boldsymbol{x})\nabla\frac{\partial\varphi(\boldsymbol{x},t)}{\partial t} + \sigma(\boldsymbol{x})\nabla\varphi(\boldsymbol{x},t)\right] = j_e(\boldsymbol{x}) = -\frac{\partial\rho_e(\boldsymbol{x},t)}{\partial t}$ (7)

X. Quasi-Static Approximation

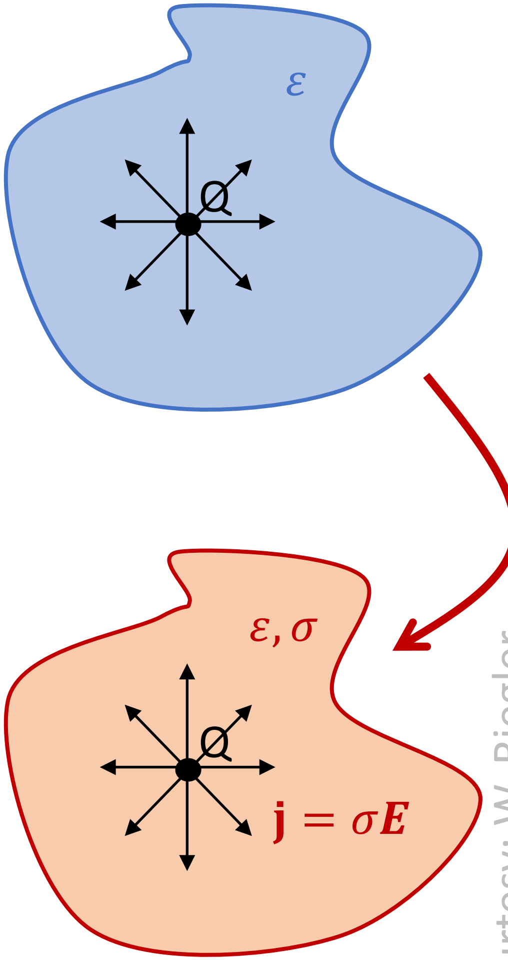
- To solve the equation: $\nabla \cdot \left[\varepsilon(\boldsymbol{x})\nabla \frac{\partial \varphi(\boldsymbol{x},t)}{\partial t} + \sigma(\boldsymbol{x})\nabla\varphi(\boldsymbol{x},t)\right] = -\frac{\partial \rho_e(\boldsymbol{x},t)}{\partial t}$
- Apply Laplace transform: $\mathcal{L}[f(t)] = F(s) = \int_{-\infty}^{+\infty} f(t)\exp(-st)dt$
- We find:
  - $\nabla \cdot [\varepsilon(\boldsymbol{x})\nabla s\varphi(\boldsymbol{x},s) + \sigma(\boldsymbol{x})\nabla\varphi(\boldsymbol{x},t)] = -s\rho_e(\boldsymbol{x},s)$
- Introduce:
  - $\varepsilon_{\boldsymbol{eff}}(\boldsymbol{x}) = \varepsilon(\boldsymbol{x}) + \sigma(\boldsymbol{x})/\boldsymbol{s}$
- We obtain Poisson equation with effective permittivity:
  - $\nabla \cdot [\varepsilon_{eff}(\boldsymbol{x})\nabla\varphi(\boldsymbol{x},s)] = \rho_e(\boldsymbol{x},s)$
- Therefore:
  - *We can find the time-dependent solutions for medium with conductivity by solving the "electrostatic" Poisson equation in Laplace Domain*
  - *Knowing the solution for $\varepsilon(\boldsymbol{x})$ we substitute $\varepsilon(\boldsymbol{x}) \rightarrow \varepsilon(\boldsymbol{x}) + \sigma(\boldsymbol{x})/\boldsymbol{s}$ and we perform the inverse Laplace transformation*

Slide courtesy: W. Riegler

X. Quasi-Static Approximation

… A *simple example* …

- Assume a point charge Q at $x = 0$ in a medium with constant permittivity ε
  - $\rho(\boldsymbol{x}) = Q\delta(\boldsymbol{x})$ & $\varphi(\boldsymbol{x}) = \frac{Q}{4\pi\varepsilon}\frac{1}{|\boldsymbol{x}|}$
- Assume now that this medium has also a constant conductivity σ
  - $\rho_e(\boldsymbol{x},t) = Q\delta(\boldsymbol{x})\mathbb{H}(t)$ => $\rho_e(\boldsymbol{x},s) = \frac{Q}{s}\delta(\boldsymbol{x})$
  - $\varphi(\boldsymbol{x},s) = \frac{Q/s}{4\pi(\varepsilon+\frac{\sigma}{s})}\frac{1}{|\boldsymbol{x}|} = \frac{Q}{(s+1/\tau)}\frac{1}{4\pi\varepsilon|\boldsymbol{x}|}$ $\tau = \frac{\varepsilon}{\sigma} = \varepsilon\rho$
- At $t = 0$ the potential is equal to the static potential in absense of conductivity, while in the limit for $t \to \infty$ the potential = 0
- Time-dependent potential is:
  - $\varphi(\boldsymbol{x},s) = \mathrm{L}^{-1}[\varphi(\boldsymbol{x},s)] = \frac{Q}{4\pi\varepsilon}\frac{1}{|\boldsymbol{x}|}\exp(-\mathrm{t}/\tau)$
- Charge density: $\rho(\boldsymbol{x}) = Q\delta(\boldsymbol{x})\exp(-\mathrm{t}/\tau)$
- Current density:
  - $j(\boldsymbol{x},s) = \sigma E(\boldsymbol{x},s) = -\sigma\nabla\varphi(\boldsymbol{x},s) = \frac{1}{\tau}\frac{Q}{\mathrm{s}+1/\tau}$
  - $j(r,t) = \mathrm{L}^{-1}[j(r,s)] = \frac{Q}{\tau}\exp(-\mathrm{t}/\tau)$ (spherical coords)



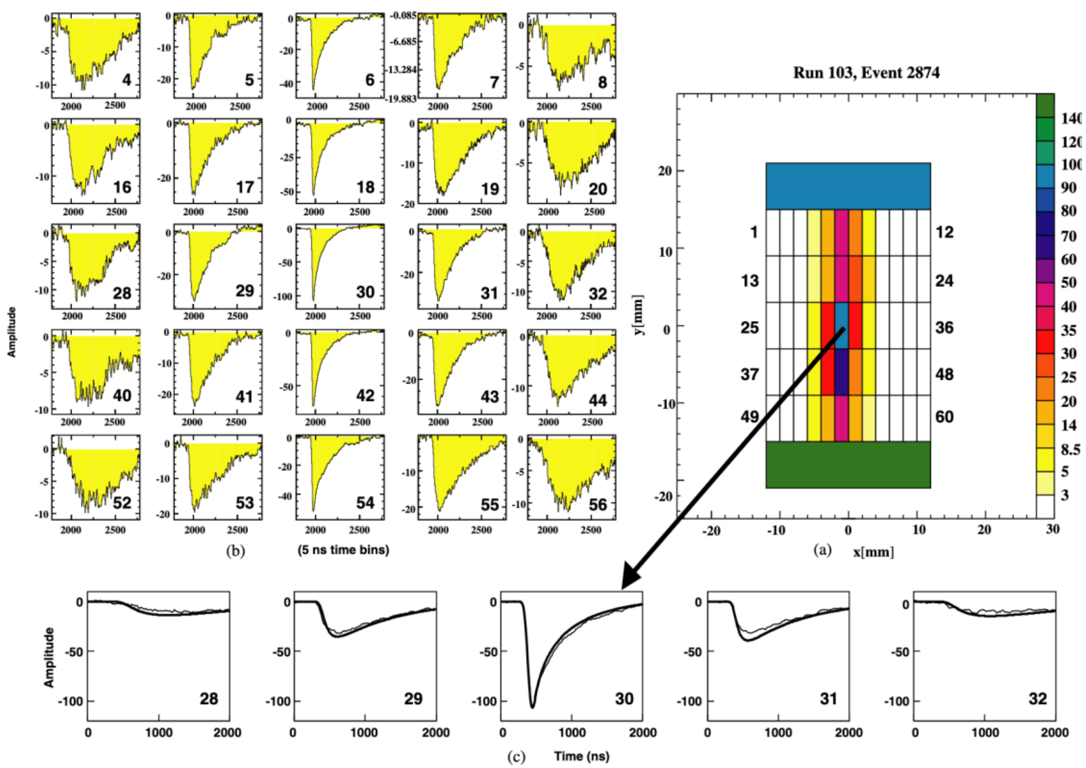
Slide courtesy: W. Riegler

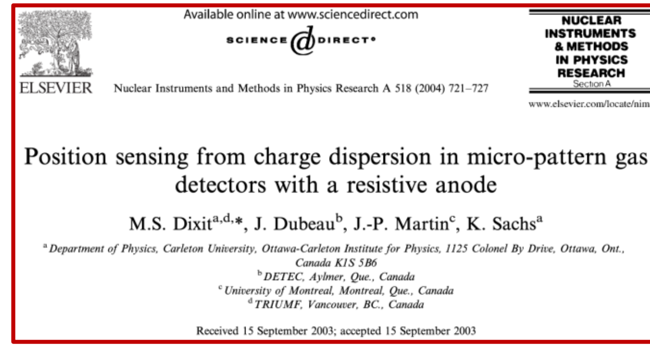
X. Quasi-Static Approximation

Charge spreading in resistive layers – Telegraph eqn

- Model Resistive layer as 2D R-C network
  - Solution given by 2D Telegraph equation ($h = \tau = 1/RC$)

$$\frac{\partial \rho}{\partial t} = h\left[\frac{\partial^2 \rho}{\partial r^2} + \frac{1}{r}\frac{\partial \rho}{\partial r}\right] \qquad \rho(r,t) = \frac{1}{2th}\exp(-r^2/4th).$$

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Position sensing from charge dispersion in micro-pattern gas detectors with a resistive anode

M.S. Dixit[a,d,*], J. Dubeau[b], J.-P. Martin[c], K. Sachs[a]
[a] Department of Physics, Carleton University, Ottawa-Carleton Institute for Physics, 1125 Colonel By Drive, Ottawa, Ont., Canada K1S 5B6
[b] DETEC, Aylmer, Que., Canada
[c] University of Montreal, Montreal, Que., Canada
[d] TRIUMF, Vancouver, BC., Canada

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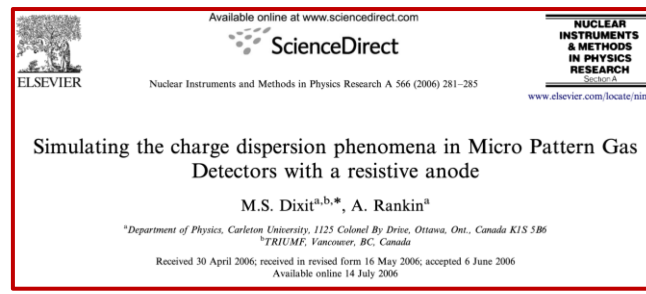
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Simulating the charge dispersion phenomena in Micro Pattern Gas Detectors with a resistive anode

M.S. Dixit[a,b,*], A. Rankin[a]
[a] Department of Physics, Carleton University, 1125 Colonel By Drive, Ottawa, Ont., Canada K1S 5B6
[b] TRIUMF, Vancouver, BC, Canada

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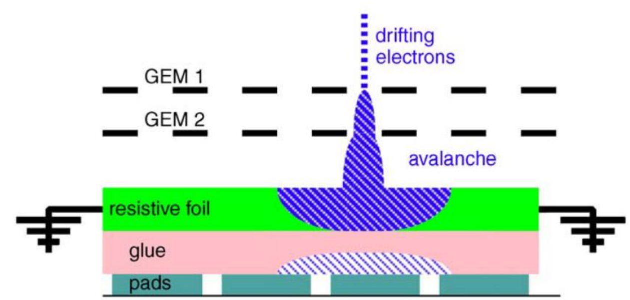

Figures : M.S Dixit – https://doi.org/10.1016/j.nima.2006.06.050

X. Quasi-Static *& Beyond …*

- You made it! – *I will stop now using mathematical formulas*
- With this introduction now you are ready to read this paper ☺

Reference work for

- Weighting fields for detector geometries with multiple parallel layers
- Charge spreading on thin resistive layers
- Fields and inducted signals for RPCs, resistive Micromegas, …
- How resistive layers affect signal shape and crosstalk

This work was further extended:

- detectors where finite propagation time of Electromagnetic waves and radiation effects cannot be neglected
- Full extend of Maxwell's equations
- Accounts for all electrodynamic effects
- All devices that detect fields / radiation

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Electric fields, weighting fields, signals and charge diffusion in detectors including resistive materials

W. Riegler
CERN EP,
CH-1211 Geneve 23, Switzerland
E-mail: werner.riegler@cern.ch

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Signals induced on electrodes by moving charges, a general theorem for Maxwell's equations based on Lorentz-reciprocity

W. Riegler [a,*], P. Windischhofer [b]
[a] CERN, Switzerland
[b] University of Oxford, United Kingdom of Great Britain and Northern Ireland

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XI. Beyond: Time dependent E_w

- Most general form of Maxwell's equations
  - 3x3 matrices for freq-dependent $\varepsilon(\boldsymbol{x},\omega)$, $\mu(\boldsymbol{x},\omega)$, $\rho(\boldsymbol{x},\omega)$

$$\mathbf{D} = \hat{\varepsilon}\mathbf{E} \qquad \mathbf{B} = \hat{\mu}\mathbf{H} \qquad \mathbf{J} = \hat{\sigma}\mathbf{E} \tag{1}$$

The source of the fields is an externally impressed current density $\mathbf{J}^e(\mathbf{x},\omega)$. In the Fourier domain, Maxwell's equations then read as

$$\nabla\cdot\hat{\varepsilon}\mathbf{E} = \rho \qquad \nabla\cdot\hat{\mu}\mathbf{H} = 0 \tag{2}$$

$$\nabla\times\mathbf{E} = -i\omega\hat{\mu}\mathbf{H} \qquad \nabla\times\mathbf{H} = \mathbf{J}^e + \hat{\sigma}\mathbf{E} + i\omega\hat{\varepsilon}\mathbf{E} \tag{3}$$

- Lorentz Reciprocity Theorem:

$$\int_V \overline{\mathbf{E}}(\mathbf{x},\omega)\mathbf{J}^e(\mathbf{x},\omega)dV = \int_V \mathbf{E}(\mathbf{x},\omega)\overline{\mathbf{J}}^e(\mathbf{x},\omega)dV$$

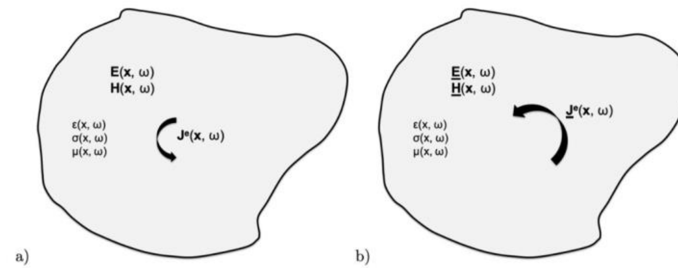


Figure 1: Two different current densities in the same geometry with the same material properties.

- Time-dependent weighting fields:

$$V^{\mathrm{ind}}(t) = \frac{q}{Q_0}\int_{-\infty}^{\infty}\mathbf{E}_w(\mathbf{x}_0(t'), t-t')\dot{\mathbf{x}}_0(t')dt'$$

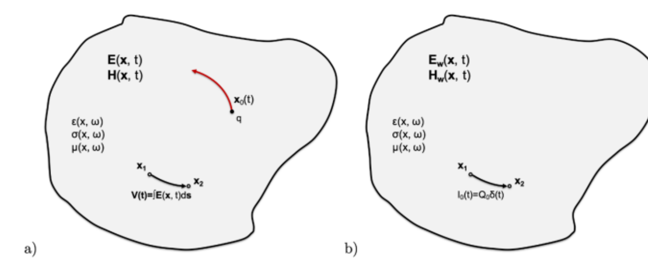


Figure 4: a) A moving point charge is creating an electric field and therefore a 'potential difference' between the points $\mathbf{x}_1$ to $\mathbf{x}_2$. b) A line current $I_0 = Q_0\delta(t)$ producing an electric field $\mathbf{E}_w(\mathbf{x},t)$, the so called 'weighting field'.

Slide courtesy: W. Riegler

XI. Time dependent E_w

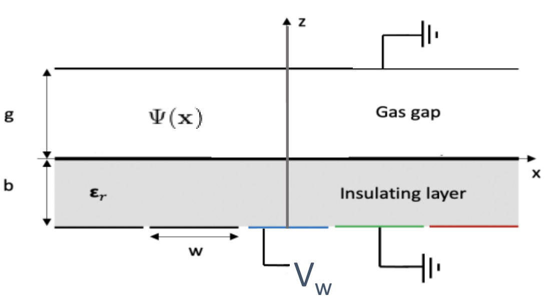
Solution is sum of static + time-dependent E_w

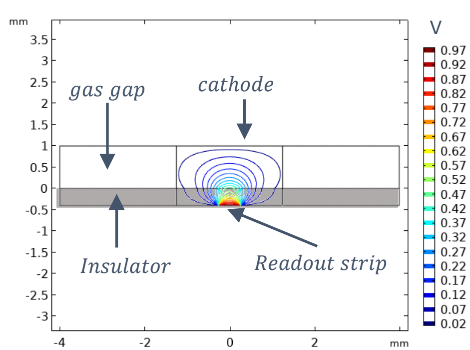
Static Weighting Field E_w

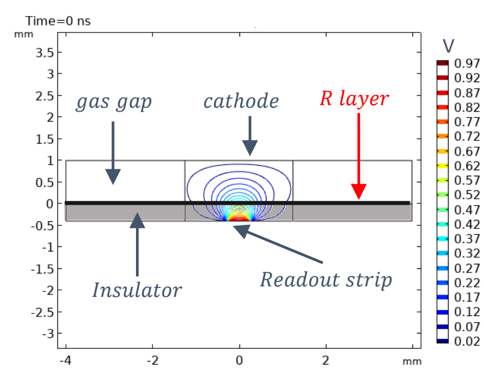
- $I_i(t) = -\frac{q}{V_w} \boldsymbol{E}_i(\boldsymbol{x}_q(t)) \cdot \dot{\boldsymbol{x}}_q(t)$
- $\boldsymbol{E}_i(\boldsymbol{x}) = -\nabla\psi_i(\boldsymbol{x})$
- The static $\psi_i(\boldsymbol{x})$ can be calculated for a grounded electrode using the following steps
  - Remove drifting charges
  - Put electrode at potential V_w
  - Ground all other electrodes

Time-dependent Weighting E_w

- $I_i(t) = -\frac{q}{V_w} \int_{-\infty}^{\infty} \boldsymbol{E}_w(\boldsymbol{x}_q(t'), t - t')\dot{\boldsymbol{x}}_q(t')dt'$
- $\boldsymbol{E}_i(\boldsymbol{x}, t) = -\nabla \frac{\partial\psi_i(\boldsymbol{x},\boldsymbol{t})}{\partial t}$
- The dynamic $\psi_i(\boldsymbol{x}, \boldsymbol{t})$ can be calculated for a grounded electrode using the following steps
  - Remove drifting charges
  - Put electrode at potential V_w at time t=0
  - Ground all other electrodes







Figures courtesy: D. Janssens

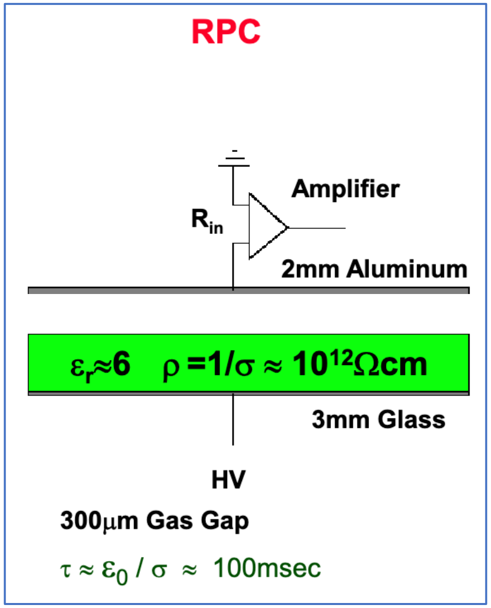
XI. Beyond: Time dependent E_w

- This is cutting-edge of simulation development:
  => Simulation of detectors with resistive layers
- $E_w(t)$ can be used to calculate signals in any detector due to movement of charged particles
- Use of $E_w(t)$ already implemented in GARFIELD++
  - **Difficulty:** *calculation of time-dependent weighting field $E_w(t)$*
    - *Some analytic expressions for specific geometries (e.g. RPC)*
    - *Fieldmaps from COMSOL, TCAD Synopsis, …*
  - **Progress:**
    - *Based on specific use-cases and dedication of student manpower!*
    - *Implement geometry in COMSOL, calculate fields, perform benchmark*
    - *Work, Present, Document*

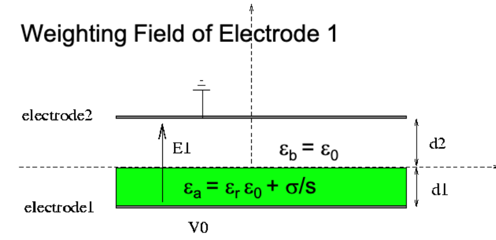
Slide courtesy: W. Riegler

XII. Examples

I. Analytic example: RPC (using Quasi-static)



Weighting Field of Electrode 1

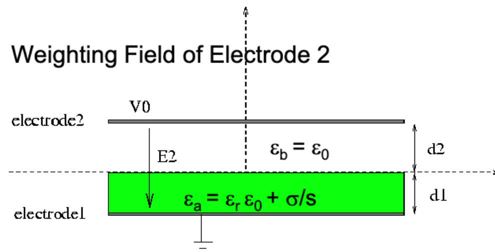


$$E_{1z}(s) = \frac{\varepsilon_a V_0}{\varepsilon_a d_2 + \varepsilon_b d_1} = \frac{V_0 \varepsilon_r}{(d_1 + d_2 \varepsilon_r)} \frac{s + \frac{1}{\tau_1}}{s + \frac{1}{\tau_2}} \quad z > 0$$

$$= \frac{\varepsilon_b V_0}{\varepsilon_a d_2 + \varepsilon_b d_1} = \frac{V_0}{(d_1 + d_2 \varepsilon_r)} \frac{s}{s + \frac{1}{\tau_2}} \quad z < 0$$

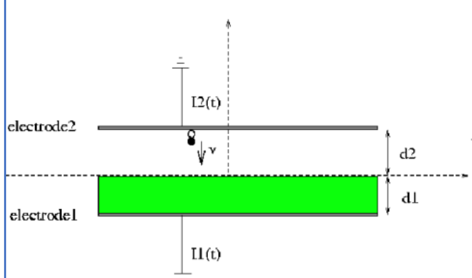
$$\tau_1 = \frac{\varepsilon_r \varepsilon_0}{\sigma} \qquad \tau_2 = \frac{\varepsilon_0}{\sigma} \left(\frac{d_1 + d_2 \varepsilon_r}{d_2} \right)$$

Weighting Field of Electrode 2



$$E_{2z}(s) = -E_{1z}(s)$$

At t=0 a pair of charges q, -q is created at $z=d_2$.
One charge is moving with velocity v to z=0
Until it hits the resistive layer at $T=d_2/v$.



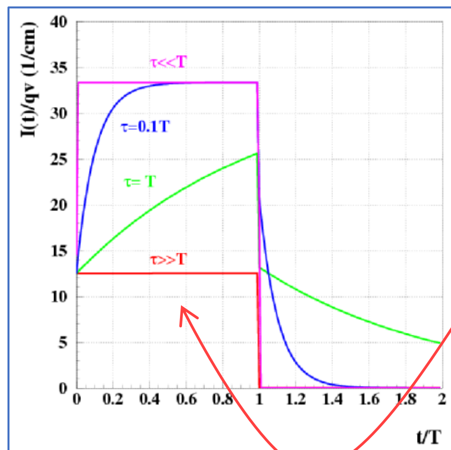
$$x_0(t) = d_2 - vt \quad t < T$$
$$= 0 \quad t > T$$

$$\dot{x}_0(t) = -v \quad t < T$$
$$= 0 \quad t > T$$

$$E_{1z}(\vec{x}, t) = \frac{\varepsilon_r V_0}{d_1 + \varepsilon_r d_2} \left[\delta(t) + \frac{\tau_2 - \tau_1}{\tau_1 \tau_2} e^{-\frac{t}{\tau_2}} \right] \quad z > 0$$

$$I_1(t) = qv \frac{\varepsilon_r}{d_1 + \varepsilon_r d_2} \left[1 + \frac{d_1}{d_2 \varepsilon_r} (1 - e^{-\frac{t}{\tau_2}}) \right] \quad t < T$$

$$= qv \frac{1}{d_1 + \varepsilon_r d_2} \frac{d_1}{d_2} \left(e^{\frac{T}{\tau_2}} - 1 \right) e^{-\frac{t}{\tau_2}} \quad t > T$$



T = time to travel to resistive electrode

$\tau = \varepsilon\rho$ = time cte of resistive electrode

ATLAS/CMS RPC:
v_d = 140um/ns
T = 14ns
τ = 10ms

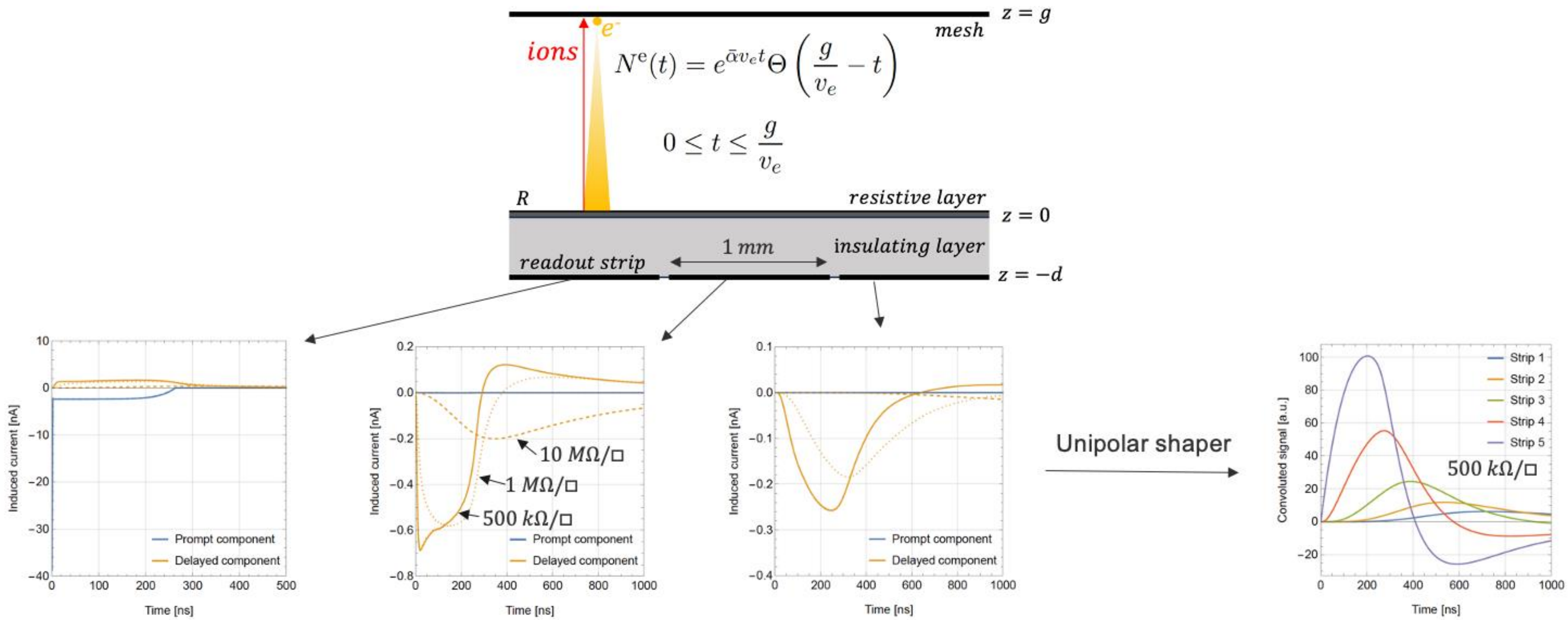
Slide courtesy: W. Riegler

XII. Examples

I. Simulation: Resistive-MM (using t-dependent E_w)

Signal 'spreading' over a thin resistive layer

Let's examine a Townsend avalanche occurring within the amplification gap of a Micromegas detector, resulting in a signal being generated on the readout strips.



RD51 Collaboration

As an example of a unipolar shaper, we took the APV25 with parameters taken from this link

17

Slide courtesy: D. Janssens

Further Reading

"If I have seen further than others, it is by standing upon the shoulders of giants"

- W. Riegler –Fundamentals of Particle Detectors and Developments in Detector Technologies for future Experiments – Academic Training Programme 2008
  https://indico.cern.ch/event/24765/
- W.Riegler – Signals in MPGDs, including resistive elements RD51 Open Lectures Dec 2017
  https://indico.cern.ch/event/676702
- W. Riegler – Signals in Particle Detectors Academic Training Programme 2019
  https://indico.cern.ch/event/843083/
- D. Janssens – Signal formation in detectors with resistive elements CERN EP-DT Seminar 2023
  https://indico.cern.ch/event/1339732/
- W. Blum, W. Riegler, L Rolandi – Particle Detection with Drift Chambers, 2nd Edition 2008 Springer – 85 EUR bookshop
  SCEM code: 90.10.03.002.8