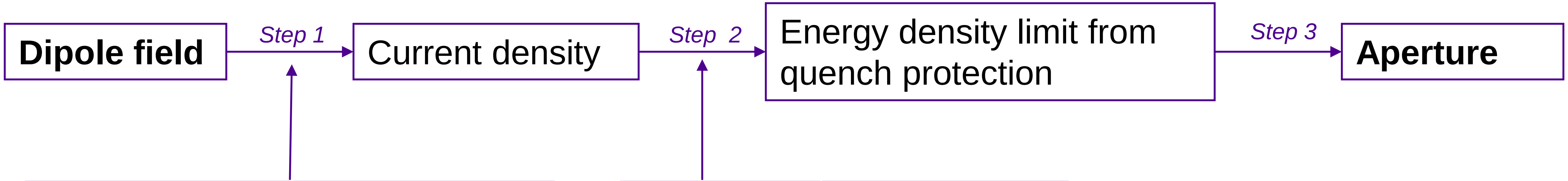
Analytical estimation of LTS accelerator magnet limits from quench protection

(proposal of a method, and application to Nb_3Sn dipoles)

Tiina Salmi, with Samuele Mariotto, Barbara Caiffi, Daniel Novelli, Stefania Farinon, Luca Bottura

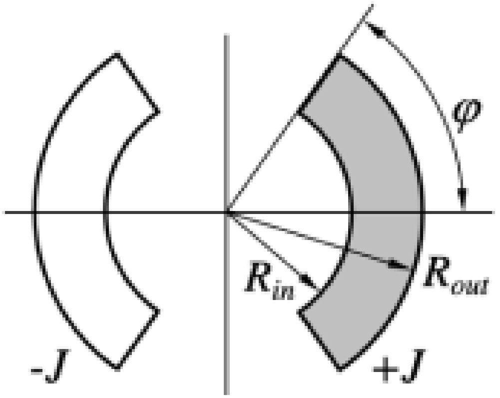
Muon magnets WG meeting 27.4.2023

Calculation process and input parameters



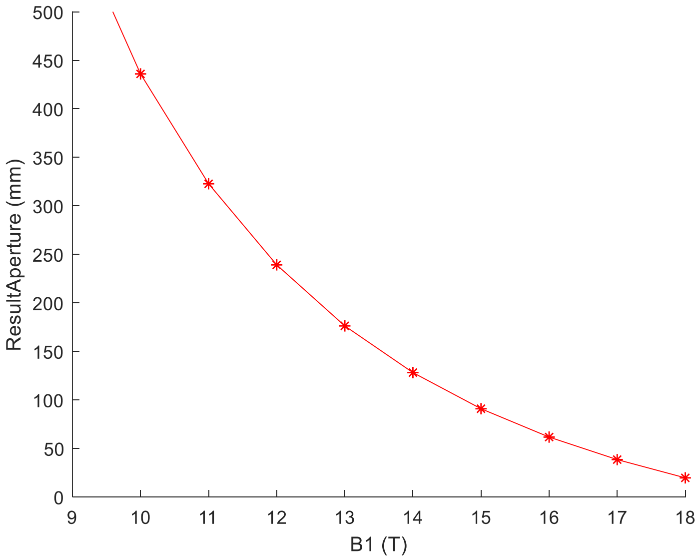
Dipole field → *Step 1* → Current density → *Step 2* → Energy density limit from quench protection → *Step 3* → **Aperture**

Input parameter	Value & Unit
$w = R_{out} - R_{in}$	45 mm
φ	60°



Input parameter	Value & Unit
Tmax	350 K
$t_{protdelay}$	40 ms
f_{Cu}	0.4
f_{SC}	0.3
f_{Ins}	0.3
copper RRR	150
Tcs	10 K
Bave	Bpeak / 2

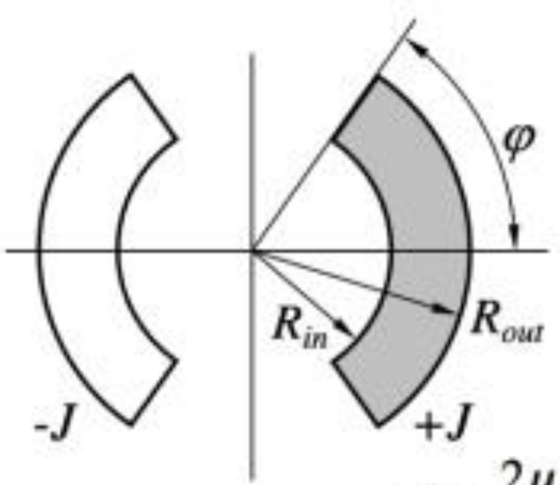
Result plot:



Equations for step 1

Dipole field → *Step 1* → Current density

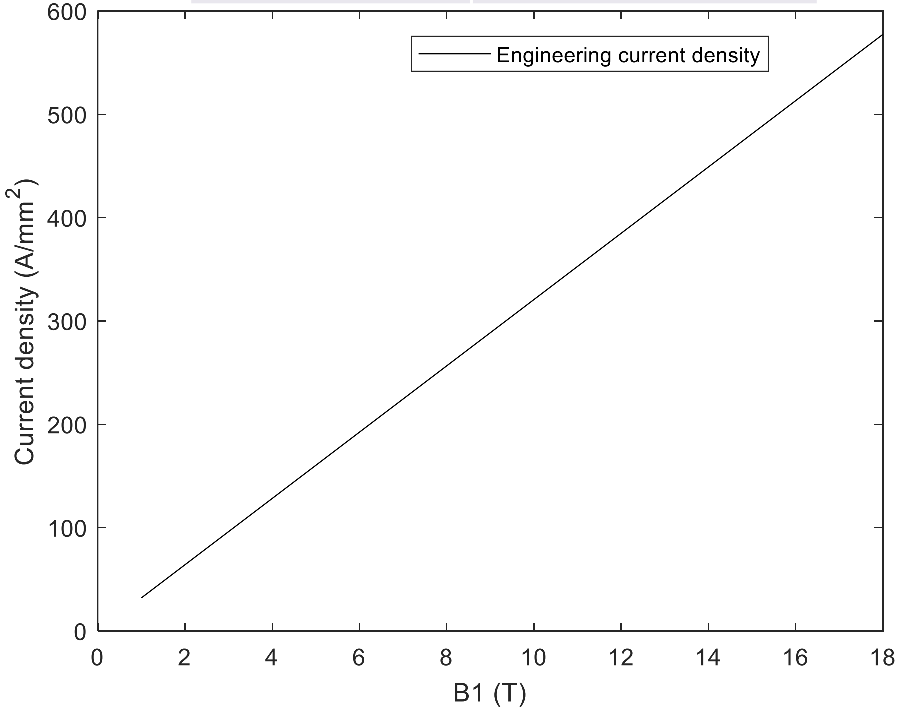
Equations from Barbara & Samuele presentation, citing E. Todesco Masterclass:



$$B_1 = \frac{2\mu_0}{\pi} J \left(R_{out} - R_{in}\right) \sin(\varphi)$$

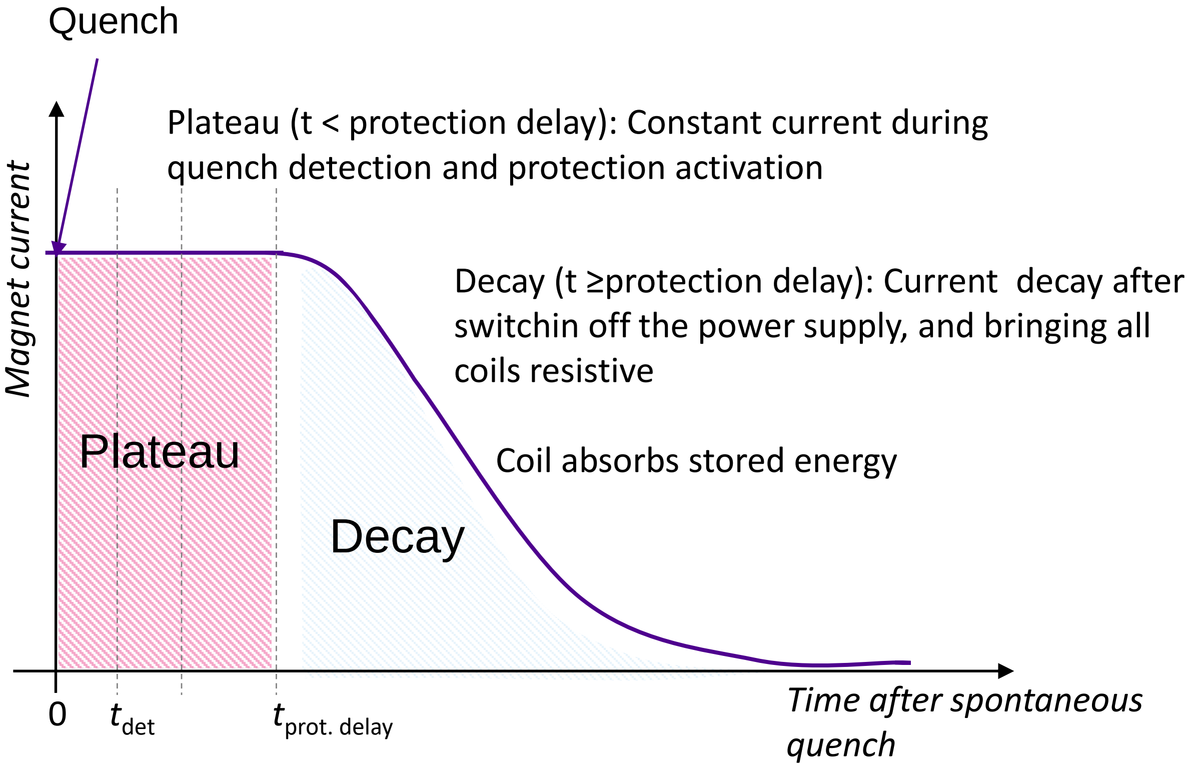
Result of step 1

Input parameter	Value & Unit
R_{out} - R_{in}	45 mm
φ	60°



Equations for step 2

Current density → *Step 2* → Energy density limit from quench protection



Plateau (t < protection delay): Constant current during quench detection and protection activation

Decay (t ≥protection delay): Current decay after switchin off the power supply, and bringing all coils resistive

Coil absorbs stored energy

Adiabatic heating at the quenched cable cross-section:

$$J_{Cu}^2 \rho_{Cu}(T,B,RRR) f_{Cu} \Delta t = C_{v,cable}(T)\Delta T$$

After re-arranging and integrating we get the commonly used quench load equation for hotspot temperature:

$$\int_{t_{quench}}^{t_\infty} J_{Cu}^2(t)dt = \int_{T_{cs}}^{Tmax} \frac{C_{v,cable}(T)}{f_{Cu}\rho_{Cu}(T,B,RRR)} dT$$

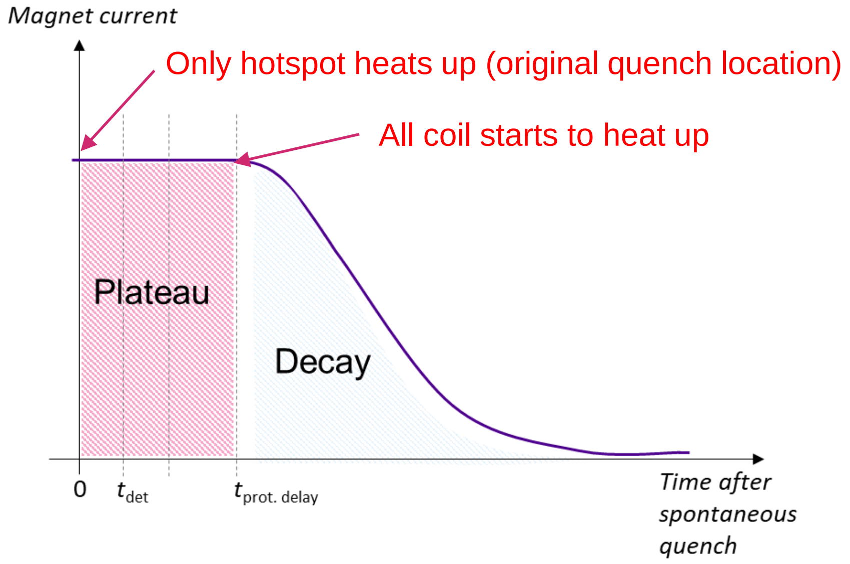
J_{Cu}: Copper current density (A/m^2), ρ_{Cu}: Copper electrical resistivity of copper (Ωm), which is a function of temperature, T, magnetic field, B, and its residual resistivity ratio, *RRR*, f_{Cu}: Fraction of copper area in cable cross-section, $C_{v,cable}$: Volumetric heat capacity of the cable (J/m^3), which is computed based on its material fractions *f* of copper, Nb3Sn and G10: $C_{v,cable}(T) = f_{Cu}\gamma_{Cu}c_{p,cu}(T) + f_{Nb_3Sn}\gamma_{Nb_3Sn}c_{p,Nb_3Sn}(T) + f_{G10}\gamma_{G10}c_{p,G10}(T)$, with the γ_{Cu}, γ_{Nb3Sn}, γ_{G10} are the material-specific mass densities in m^3/kg and c_p's the specific heat capacity in J/K/kg.

… Equations for step 2

Current density → Step 2 → **Energy density limit from quench protection**

- If we fix the maximum allowed hotspot temperature, we can compute the allowed "quench load" Γ_{Tmax} :

$$\int_{t_{quench}}^{t_\infty} J_{Cu}^2(t)dt = \frac{1}{f_{Cu}}\int_{T_{cs}}^{T_{max}} \frac{C_{v,cable}(T)}{\rho_{Cu}(T,B_{peak},RRR)}dT = \Gamma_{Tmax}$$



- It can be divided to two parts, reflecting the current decay profile:

$$\Gamma_{\mathrm{Tmax}} = \Gamma_{\mathrm{plateau}} + \Gamma_{decay}$$

- If we fix the protection delay, $\Gamma_{plateau}$ becomes trivial:

$$\Gamma_{plateau} = t_{protdelay} J_{Cu}^2$$

- Γ_{decay} is related to average temperature rise in coil that was not heated during plateau:

$$\Gamma_{decay}(T_{bulk}) = \int_{t_{protdelay}}^{t_\infty} J_{Cu}^2(t)dt = \frac{1}{f_{Cu}}\int_{Tc(s)}^{Tbulk} \frac{C_{v,cable}(T)}{\rho_{Cu}(T,B_{ave},RRR)}dT$$

- The average temperature can be obtained from the stored energy density (J/m^3):

$$\boxed{E_{vol}} = \int_{T_0}^{T_{bulk}} C_{v,cable}(T)dT$$

- We can solve for Jcu:

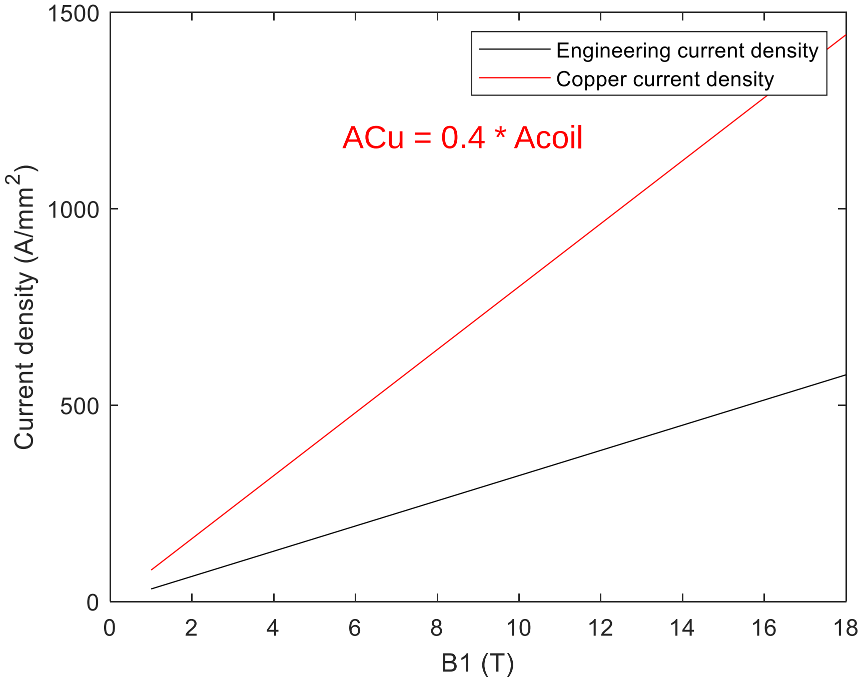
$$\boxed{J_{Cu}(T_{bulk})} = \sqrt{\frac{\Gamma_{plateau}}{t_{protdelay}}} = \sqrt{\frac{\Gamma_{Tmax}-\Gamma_{decay}}{t_{protdelay}}} = \sqrt{\frac{\frac{1}{f_{Cu}}\left(\int_{T_{cs}}^{T_{max}} \frac{C_{v,cable}(T)}{\rho_{Cu}(T,B_{peak},RRR)}dT - \int_{T_{cs}}^{\boxed{T_{bulk}}} \frac{C_{v,cable}(T)}{\rho_{Cu}(T,B_{ave},RRR)}dT\right)}{t_{protdelay}}}$$

L. Bottura, Contribution to: WAMSDO 2013, 1-9, e-Print: 1401.3927 [physics.acc-ph],
T. Salmi and D. Schoerling, IEEE TAS 29(4), 2019, doi: 10.1109/TASC.2018.2880340

Result of step 2

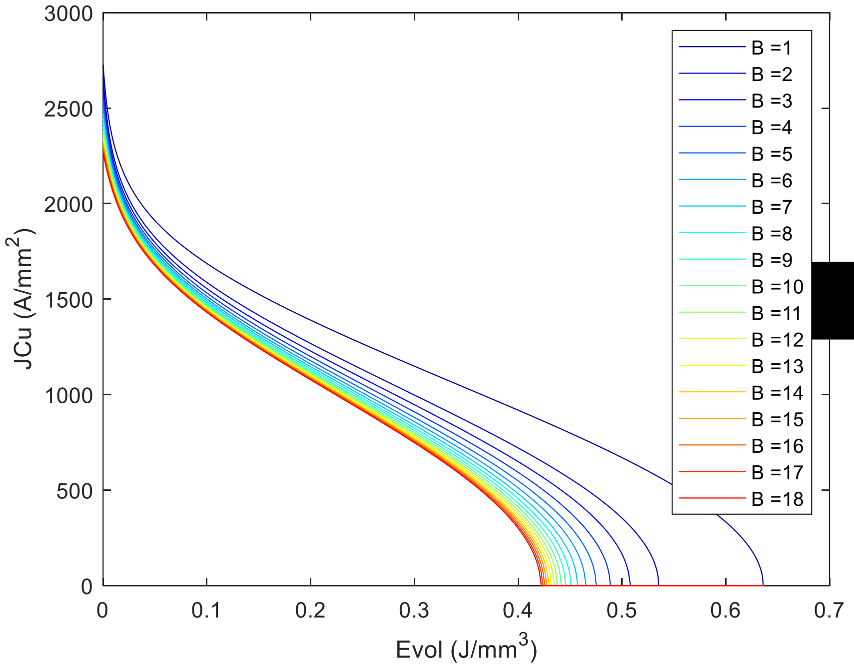
Input parameter	Value & Unit	Justification / reference
f_{Cu}	0.4	Cable material fractions based on average values of some recent cables designed for Nb3Sn accelerator magnets
f_{SC}	0.3	
f_{Ins}	0.3	
copper RRR	150	
*T*max	350 K	Ambrosio, WAMSDO 2013
$t_{protdelay}$	40 ms	Salmi et al. 2017
*T*cs	10 K	
Bave	Bpeak / 2	

Copper current density vs. dipole field from step 1

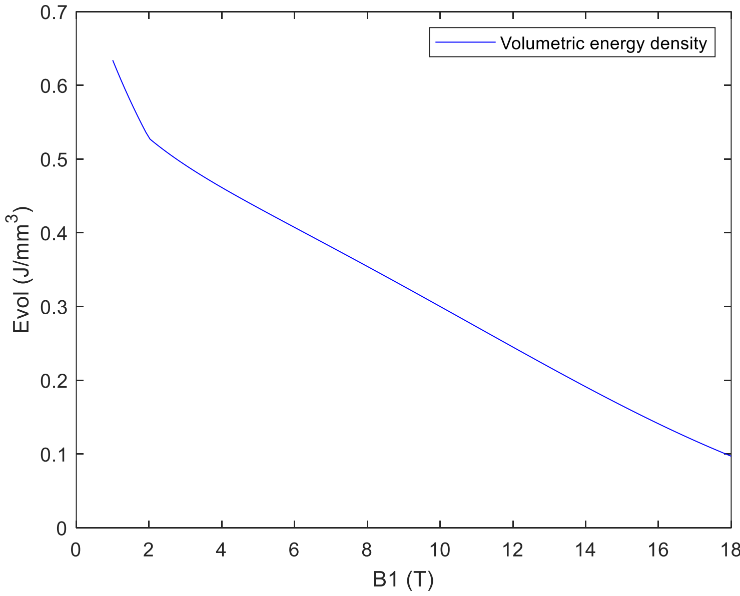


ACu = 0.4 * Acoil

Max volumetric energy density vs. copper current density (Tmax = 350 K)



Max volumetric energy density vs. dipole field (Tmax = 350 K)

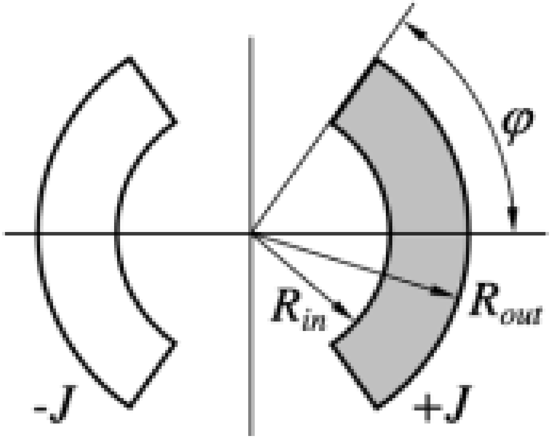


Equations for step 3

Energy density limit from quench protection → *Step 3* → **Aperture**

Stored energy per unit length, equation from Luca Bottura:

$$\frac{E}{l} = \frac{\pi B_1^2 R_{in}^2}{\mu_0}\left\{1+\frac{2}{3}\left(\frac{R_{out}}{R_{in}}-1\right)+\frac{1}{6}\left(\frac{R_{out}}{R_{in}}-1\right)^2\right\}$$



$$A_{coil} = \left(\pi R_{out}^2 - \pi R_{in}^2\right)\frac{2\varphi}{\pi}$$
$$w = R_{out} - R_{in}$$

Stored energy (E) has to match the volumetric energy density (Evol) given by quench protection requirement:

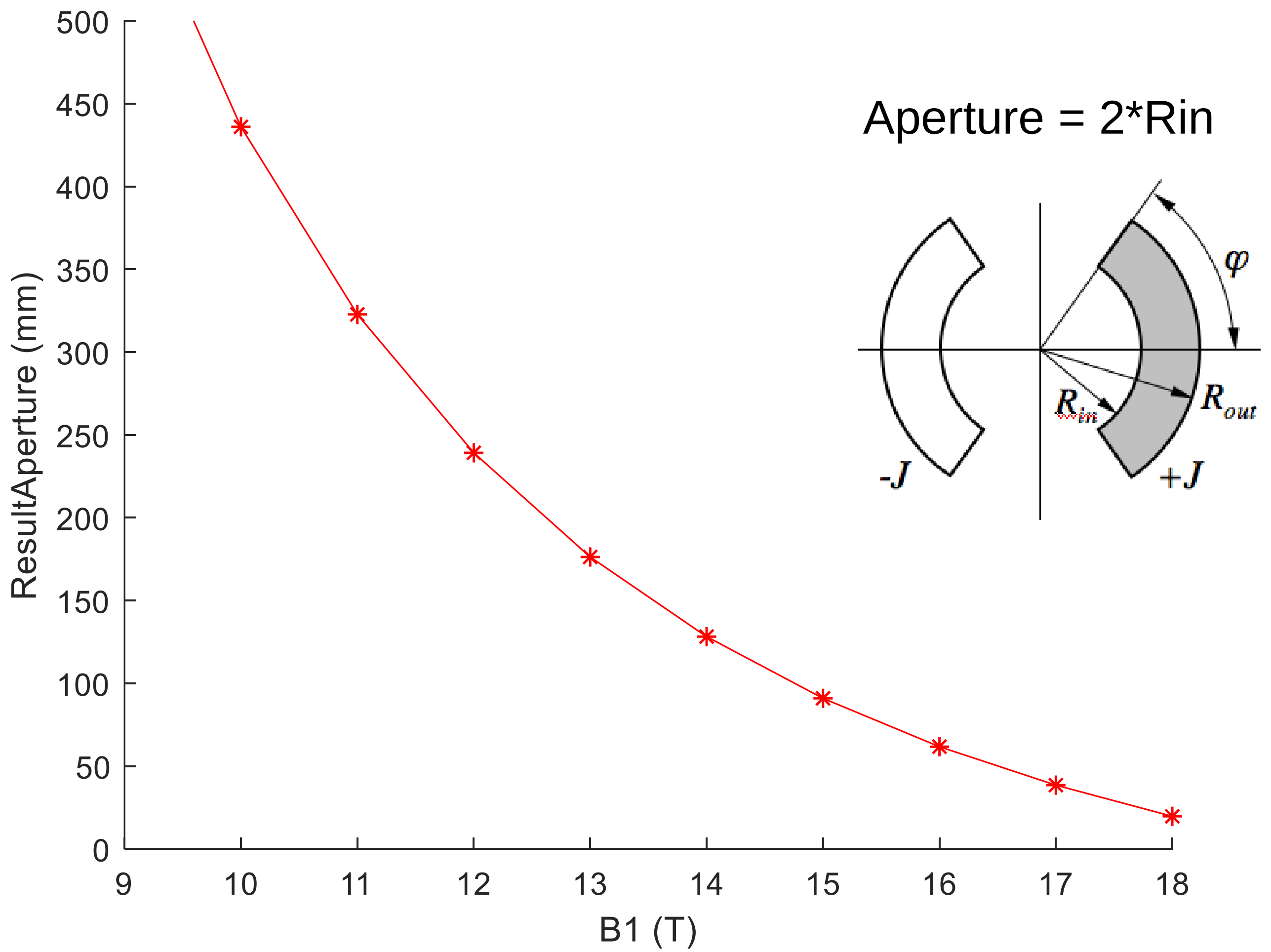
$$\frac{E(B,r_{in})}{l} = E_{vol}A_{coil} = E_{vol}\left((R_{in}+w)^2 - R_{in}^2\right)2\varphi$$
$$= \frac{\pi B_1^2 R_{in}^2}{\mu_0}\left\{1+\frac{2}{3}\left(\frac{R_{out}}{R_{in}}-1\right)+\frac{1}{6}\left(\frac{R_{out}}{R_{in}}-1\right)^2\right\}$$

$$\vdots$$

➡ $$R_{in} = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}$$

$$\frac{\pi B_1^2}{\mu_0 E_{vol} 2\varphi} = \alpha,$$
$$2w\left(\frac{1}{3}\alpha - 1\right) = \beta,$$
$$w^2\left(\frac{1}{6}\alpha - 1\right) = \gamma$$

Result of step 3: Aperture vs. dipole field



Aperture = 2*Rin

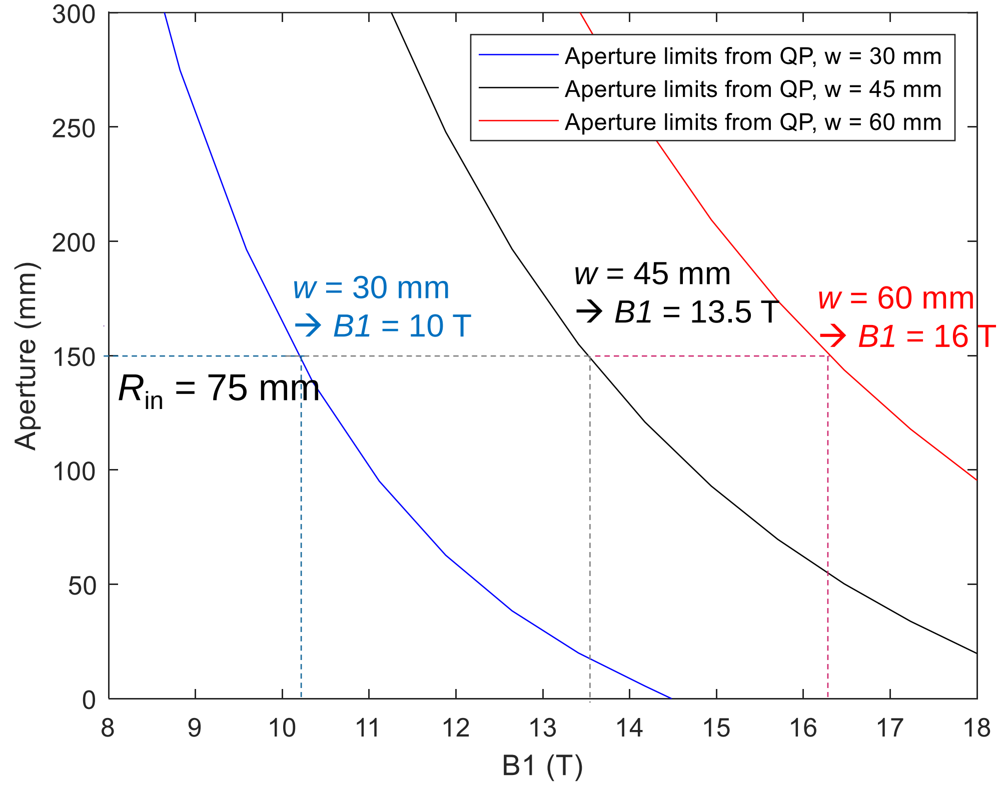
Reminder of used input parameters:

Parameter	Value & Unit
R_{out} -R_{in}	45 mm
φ	60°
*T*max	350 K
$t_{protdelay}$	40 ms
f_{Cu}	0.4
f_{SC}	0.3
f_{Ins}	0.3
copper RRR	150
*T*cs	10 K
Bave	Bpeak / 2

Tampere University

Impact of coil width

Max aperture vs field for *w* = 30, 45, and 60 mm



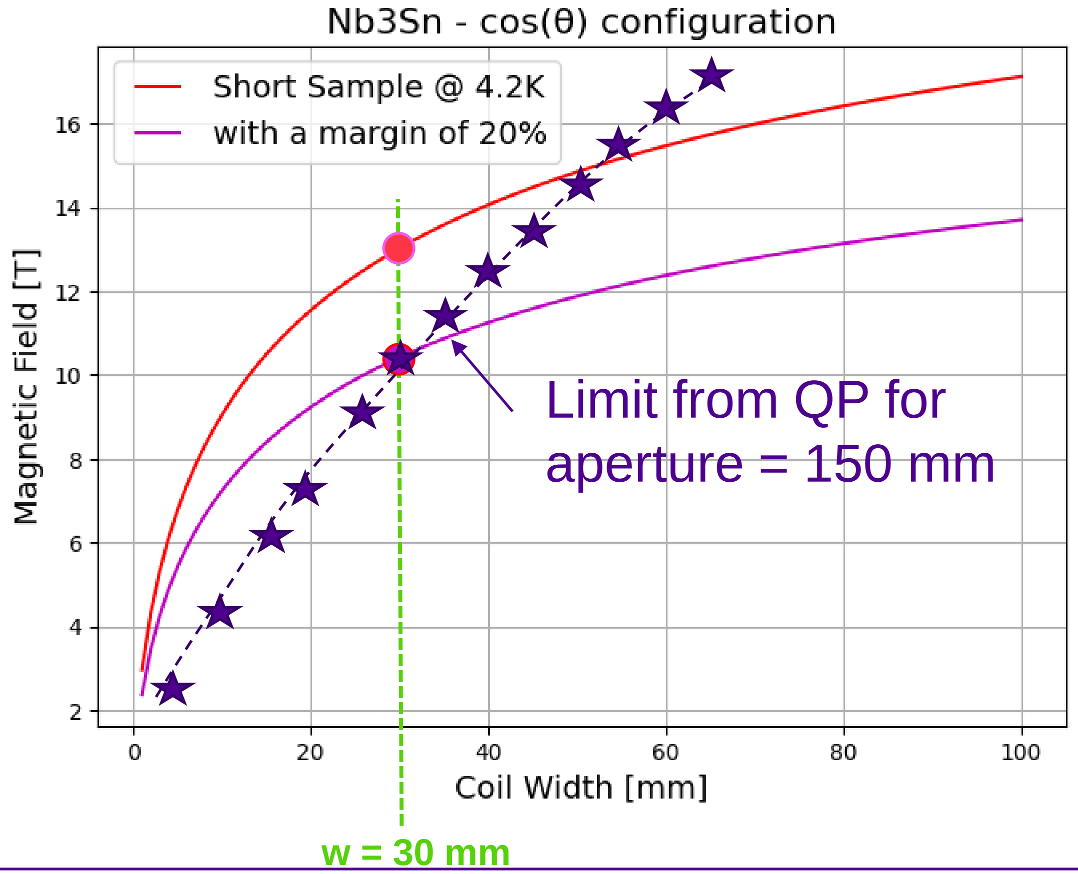
w = 30 mm
→ *B1* = 10 T

w = 45 mm
→ *B1* = 13.5 T

w = 60 mm
→ *B1* = 16 T

R_{in} = 75 mm

Inserting QP induced limit to the picture from Daniel Novelli :

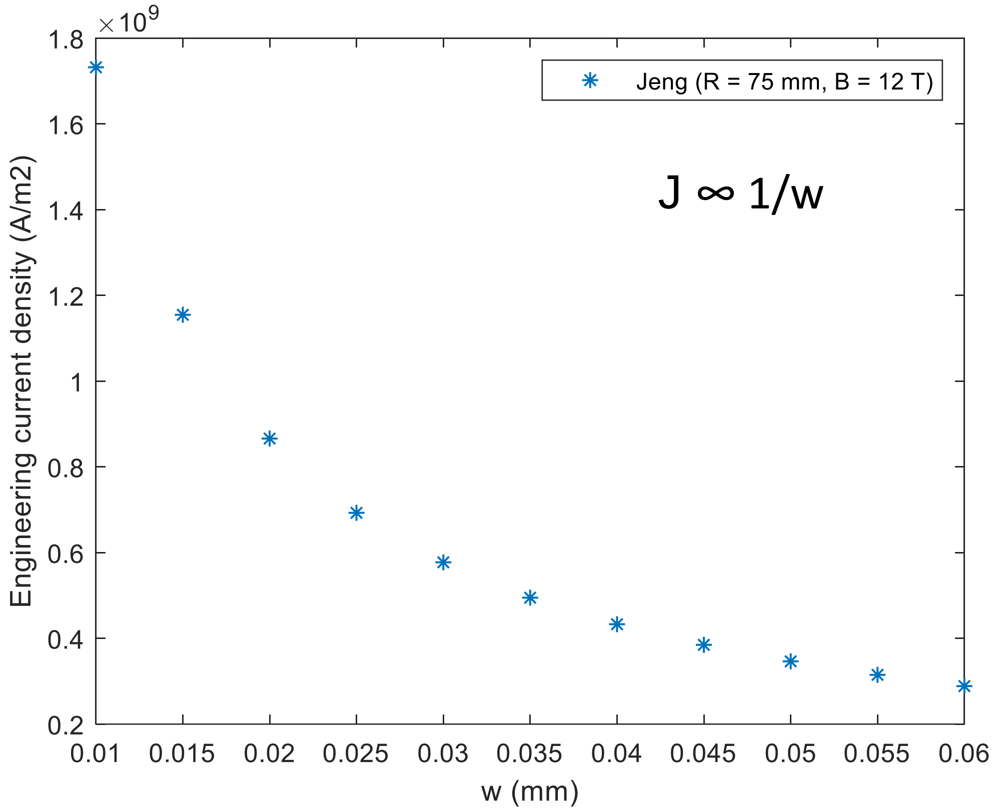

Nb3Sn - cos(θ) configuration

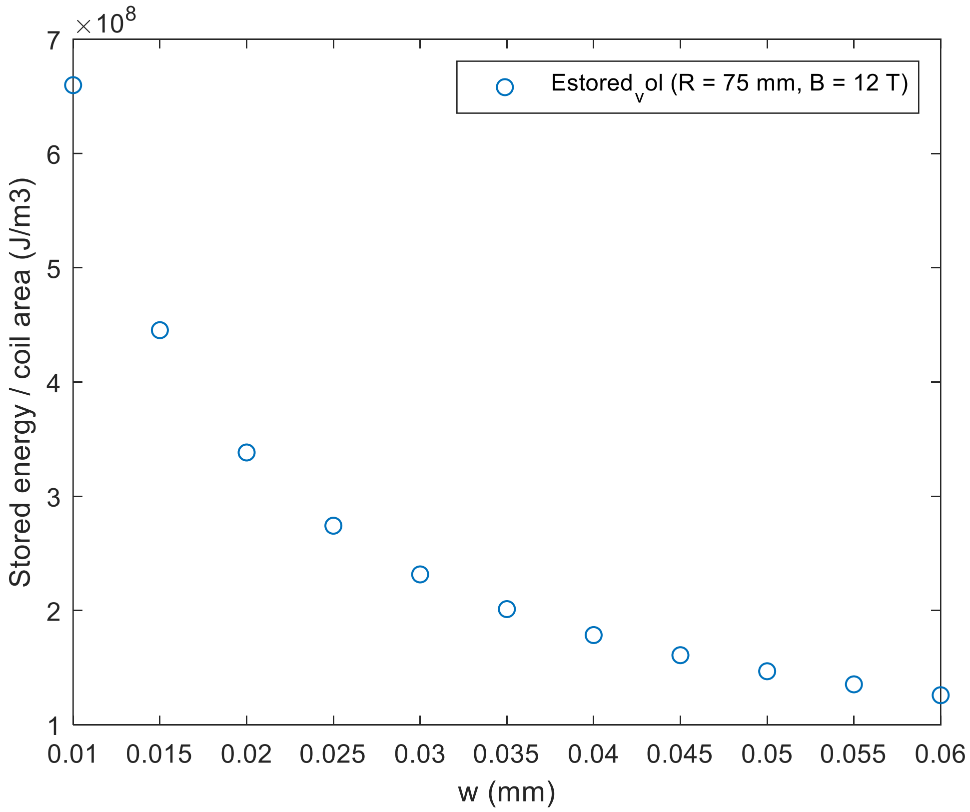
Limit from QP for aperture = 150 mm

w = 30 mm

Quench protection seems to be a limiting factor for coils with w < 30 mm, for thicker coils the limit comes from critical current density
→ When designing compact high current density/high energy density magnets, estimating the limits from QP very important

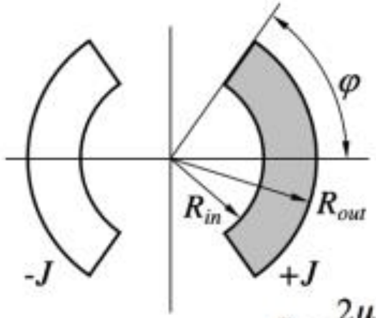
Strong increase of current density and stored energy density for coil w < 30 mm





Summary of equations…

Step 1: Current density vs. dipole field:



$$B_1 = \frac{2\mu_0}{\pi} J\left(R_{out} - R_{in}\right)\sin(\varphi)$$

Step 2. Maximum energy density vs. current density:

$$J_{Cu}(T_{bulk}) = \sqrt{\frac{\frac{1}{f_{Cu}}\left(\int_{T_{cs}}^{T_{max}} \frac{C_{v,cable}(T)}{\rho_{Cu}(T, B_{peak}, RRR)} dT - \int_{T_{cs}}^{T_{bulk}} \frac{C_{v,cable}(T)}{\rho_{Cu}(T, B_{ave}, RRR)} dT\right)}{t_{protdelay}}}$$

with

$$E_{vol} = \int_{T_0}^{T_{bulk}} C_{v,cable}(T) dT$$

Step 3. Aperture vs stored energy density:

$$\frac{E(B,r_{in})}{l} = E_{vol}A_{coil}$$

$$= E_{vol}\left((R_{in} + w)^2 - R_{in}^2\right)2\varphi$$

$$= \frac{\pi B_1^2 R_{in}^2}{\mu_0}\left\{1 + \frac{2}{3}\left(\frac{R_{out}}{R_{in}} - 1\right) + \frac{1}{6}\left(\frac{R_{out}}{R_{in}} - 1\right)^2\right\}$$

$$R_{in} = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}$$

With

$$\frac{\pi B_1^2}{\mu_0 E_{vol} 2\varphi} = \alpha,$$

$$w\left(\frac{1}{3}\alpha - 1\right) = \beta,$$

$$w^2\left(\frac{1}{6}\alpha - 1\right) = \gamma$$

Aperture vs. dipole field!

Next steps

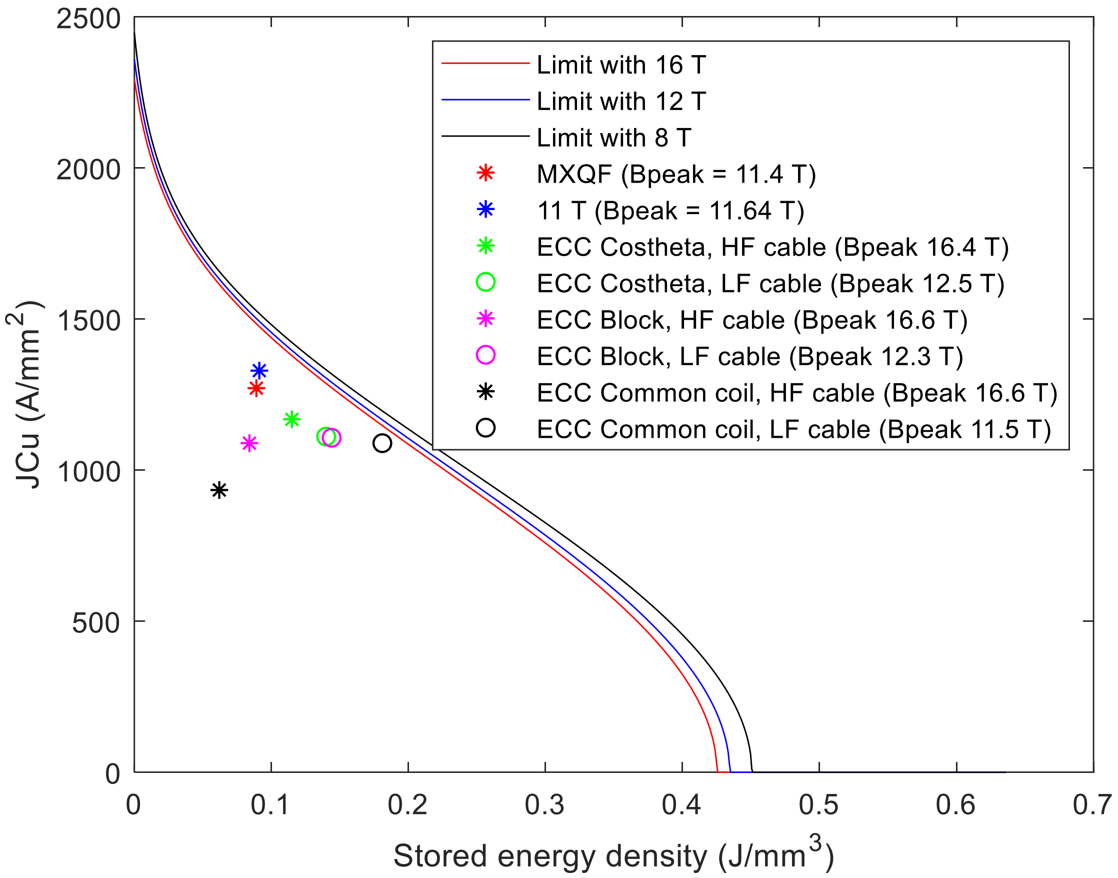
- Nb_3Sn quadrupole
- NbTi dipole and quadrupole
- HTS magnets, insulated
- HTS magnets, non-insulated…

Tampere University

Some more analysis

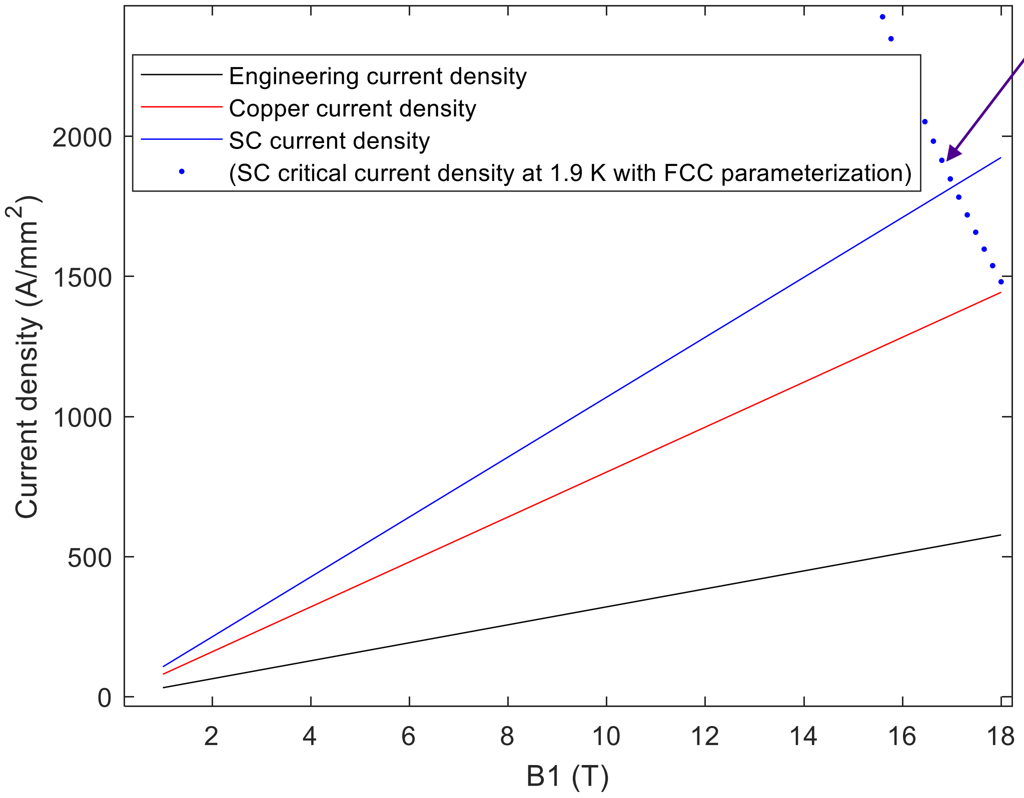
Examples of Jcu vs Estored in other magnets

Input parameter	Value & Unit	Justification / reference
f_{Cu}	0.4	Cable material fractions based on average values of some recent cables designed for Nb3Sn accelerator magnets
f_{SC}	0.3	
f_{Ins}	0.3	
copper RRR	150	
*T*max	350 K	Ambrosio, WAMSDO 2013
$t_{protdelay}$	40 ms	Salmi et al. 2017
*T*cs	10 K	
Bave	Bpeak / 2	



Additional analyses

1. Taking into account Nb3Sn critical current



Limit at 16 T for non-graded coil if SC fraction of cable is 0.3 and w = 45 mm

Jc fit by Bernardo Bordini, CERN
"Agressive parameters", based on Davide's email Oct 29, 2015

Tc0 (K)	16
Bc20 (T)	29.38
alpha	0.96
C0 (A/mm^2T)	267845

$$B_{c2}(T) = B_{c20}\cdot(1-t^{1.52})$$

$$J_c = \frac{C(t)}{B_p}\cdot b^{0.5}\cdot(1-b)^2$$

$$C(t) = C_0\cdot(1-t^{1.52})^{\alpha}\cdot(1-t^2)^{\alpha}$$

Where: $t = \frac{T}{T_{c0}}$; $b = \frac{B_p}{B_{c2}(t)}$ with B_p peak field on the conductor

Assumed current decay profile after quench

- Quench protection system detects a quench, considers needed validation delay times, and activates a protection system
- The protection system is based on quench heaters (or CLIQ) that heats the coil, and brings the entire coil to resistive state
- → The stored magnetic energy is dissipated in the windings via resistive heating, there is no external energy extraction
- In reality, the coil transition to resistive is gradual. For first calculations, we can consider only the average quench delay time in coil (entire coil becomes resistive instantaneously and uniformly), <u>the time between original quench and all coils becoming resistive is "protection delay"</u>
- The current decay is divided to parts:
  - Plateau (t < protection delay): Constant current before coils become resistive (neglect original normal one resistance)
  - Decay (t ≥protection delay): Current decay after switchin off the power supply, and bringing all coils resistive, current decay governed by $\tau = L_{mag}/r_{mag}(t)$