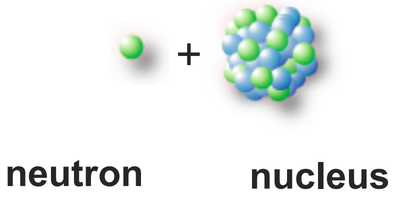
Introduction
neutron-induced resonance reactions

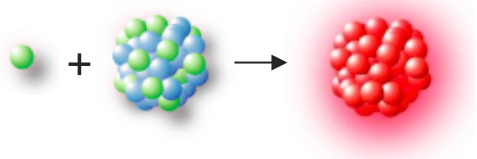
Frank Gunsing
CEA Irfu, University Paris-Saclay
France

Neutron induced nuclear reactions



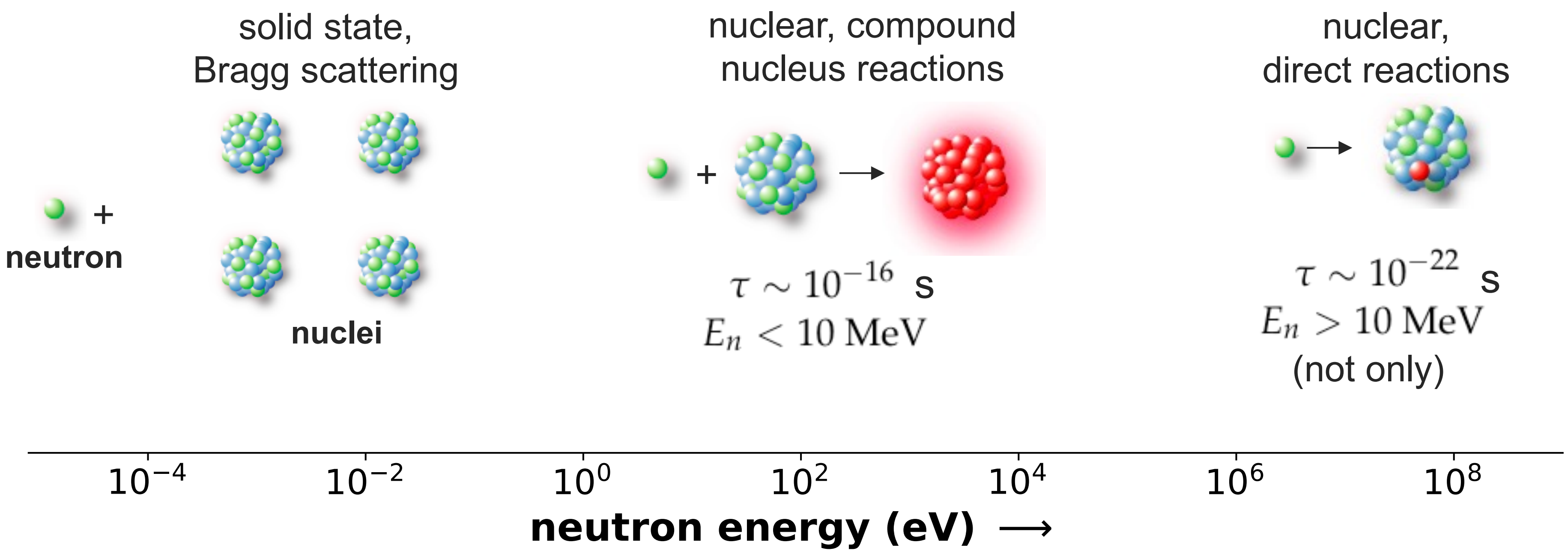
Neutron induced nuclear reactions

nuclear, compound
nucleus reactions



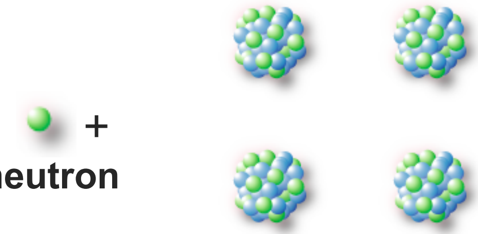
$\tau \sim 10^{-16}$ s
$E_n < 10$ MeV

Neutron induced nuclear reactions



Neutron induced nuclear reactions

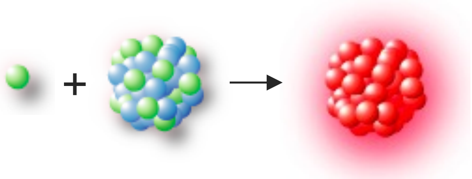
solid state,
Bragg scattering



neutron

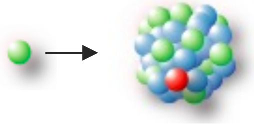
nuclei

nuclear, compound
nucleus reactions



$\tau \sim 10^{-16}$ s
$E_n < 10$ MeV

nuclear,
direct reactions



$\tau \sim 10^{-22}$ s
$E_n > 10$ MeV

10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4} 10^{6} 10^{8}

neutron energy (eV) ⟶

10^{-8} 10^{-9} 10^{-10} 10^{-11} 10^{-12} 10^{-13} 10^{-14} 10^{-15}

⟵ **wave length (m)**

1 Å = 10^{-10} m

de Broglie wavelength: $\lambda = \frac{h}{\sqrt{2mE_k}}$

Neutron induced nuclear reactions

- Reaction notations:

$^{10}B + {}^{1}n \rightarrow {}^{7}Li + {}^{4}He$ $\quad$ $^{238}U + n \rightarrow {}^{239}U^*$

$^{10}B + n \rightarrow {}^{7}Li + \alpha$ $\quad$ $^{238}U + n \rightarrow {}^{239}U + \gamma$

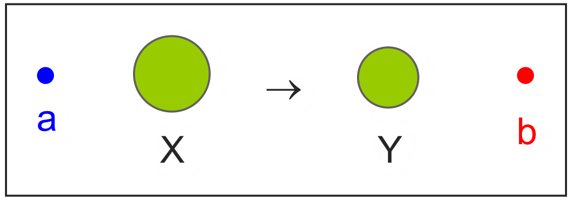
$^{10}B(n,\alpha)$ $\quad$ $^{238}U(n,\gamma)$

- Neutron induced nuclear reactions:
  - elastic scattering (n,n)
  - inelastic scattering (n,n')
  - capture (n,γ)
  - fission (n,f)
  - particle emission (n,α), (n,p), (n,xn)
  - total cross section σ_{tot}: sum of all partial reactions

- Cross section σ, expressed in barns, $1\ b = 10^{-28}\ m^2$

Neutron induced nuclear reactions

- neutron reaction X(a,b)Y



- neutron cross section:

  function of the kinetic energy of the particle a

  caveat: "differential measurements"

$$\sigma(E_a) = \int\int \frac{d^2\sigma(E_a, E_b, \Omega)}{dE_b d\Omega} dE_b d\Omega$$

- differential cross section:

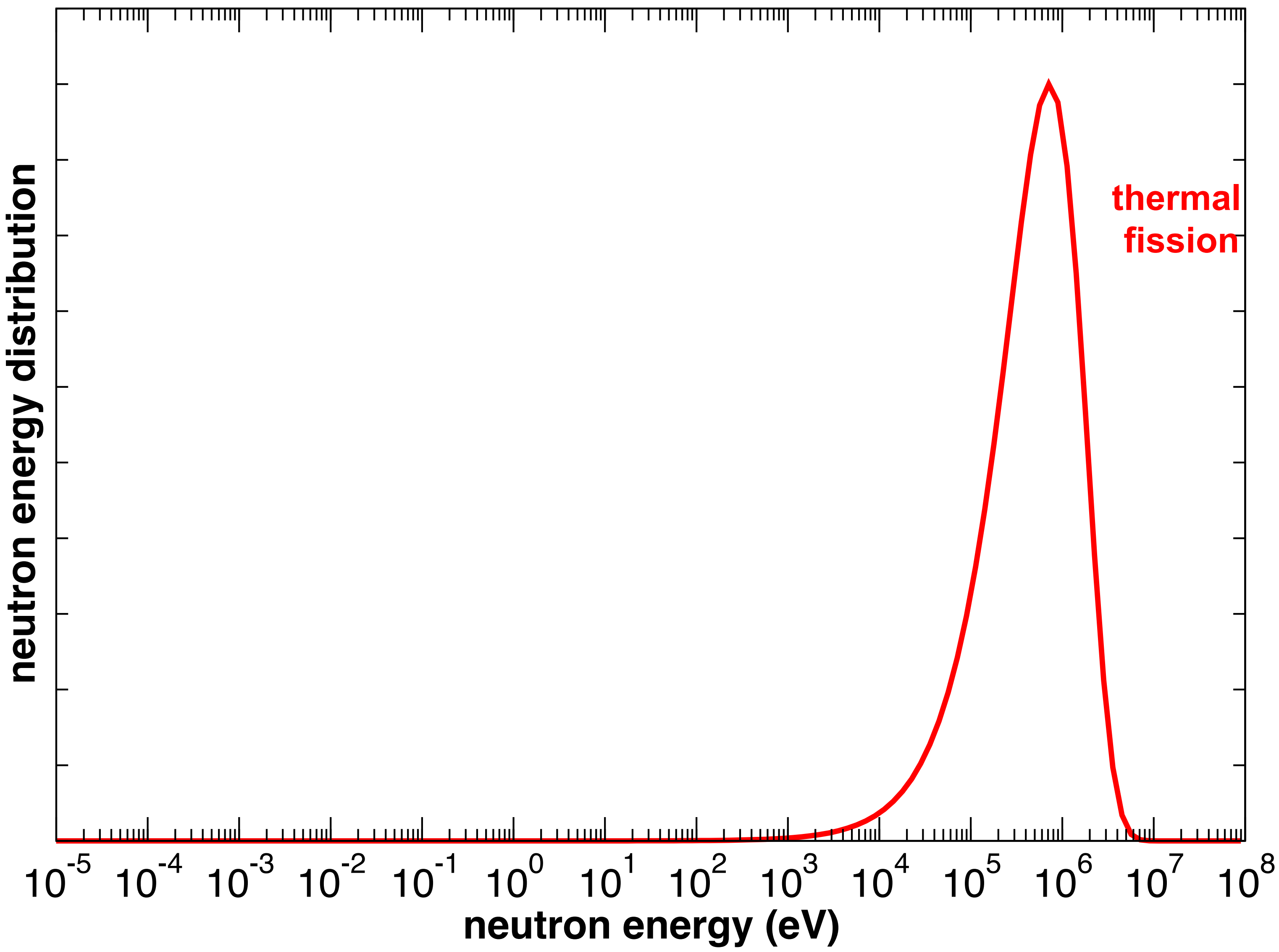
  function of the kinetic energy of the particle a
  and function of the kinetic energy or the angle
  of the particle b

$$\frac{d\sigma(E_a, E_b)}{dE_b} \qquad \frac{d\sigma(E_a, \Omega)}{d\Omega}$$

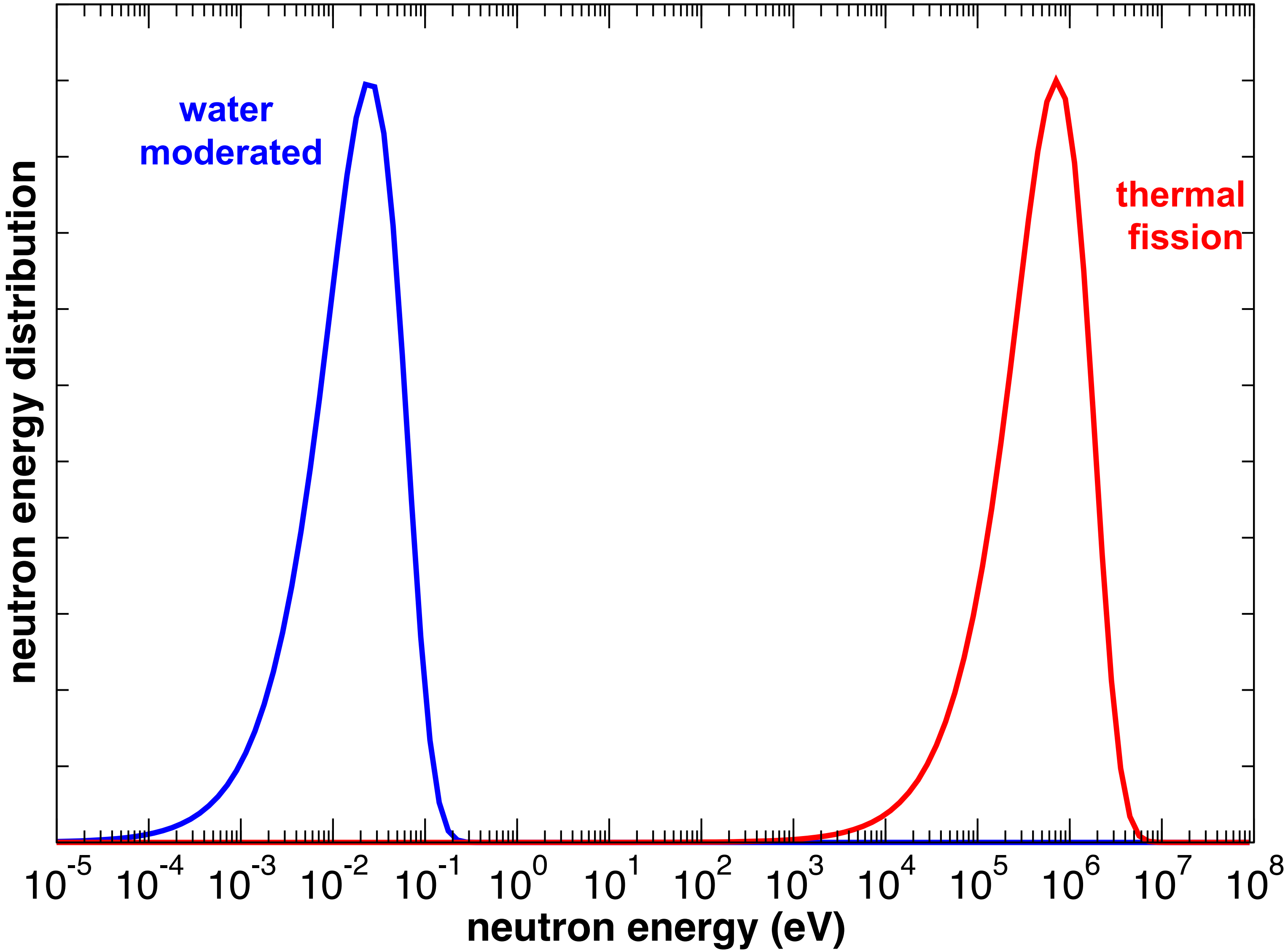
- double differential cross section:
  function of the kinetic energy of the particle a
  and function of the kinetic energy and the angle
  of the particle b

$$\frac{d^2\sigma(E_a, E_b, \Omega)}{dE_b d\Omega}$$

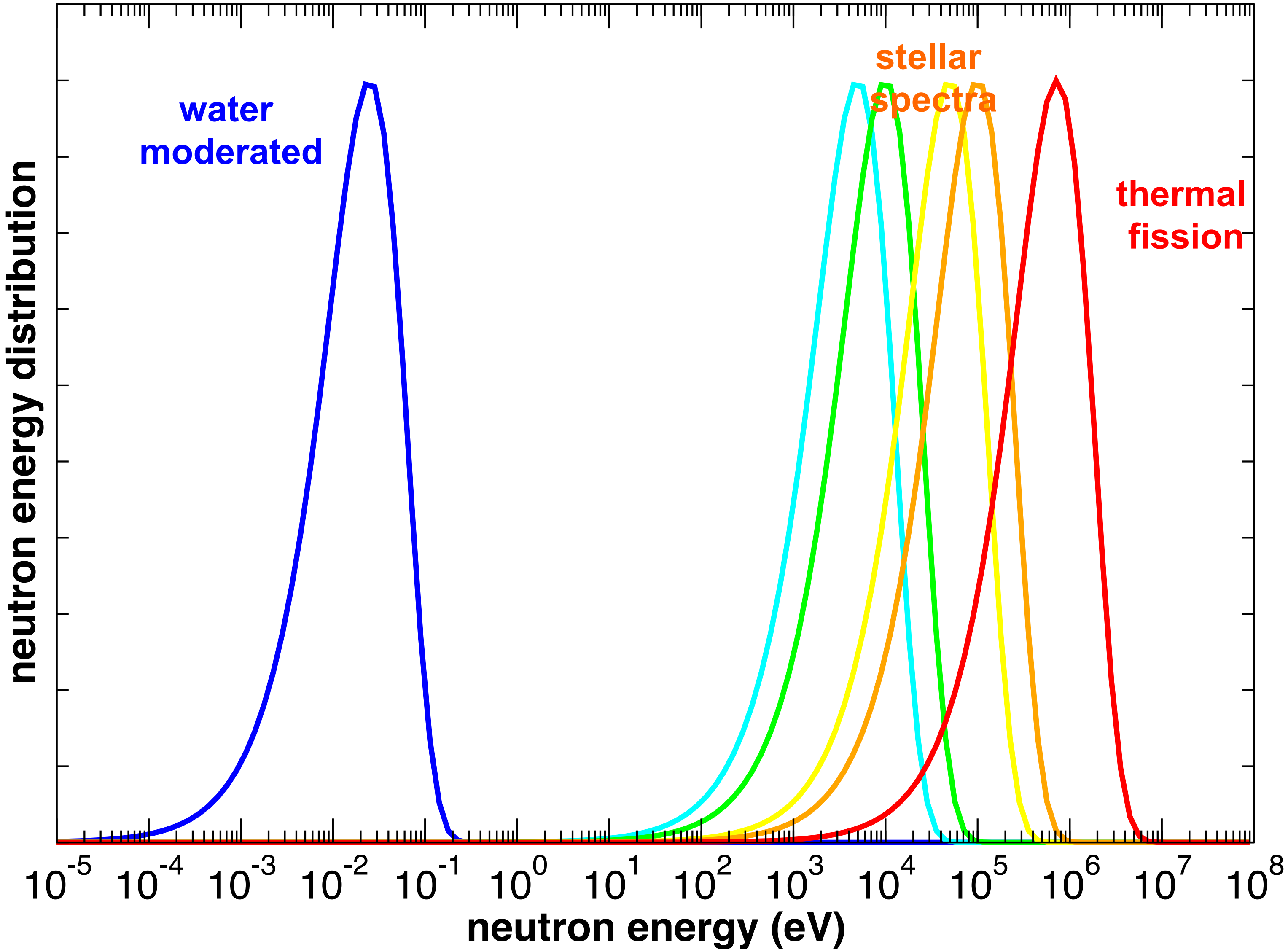
Neutron energy distributions



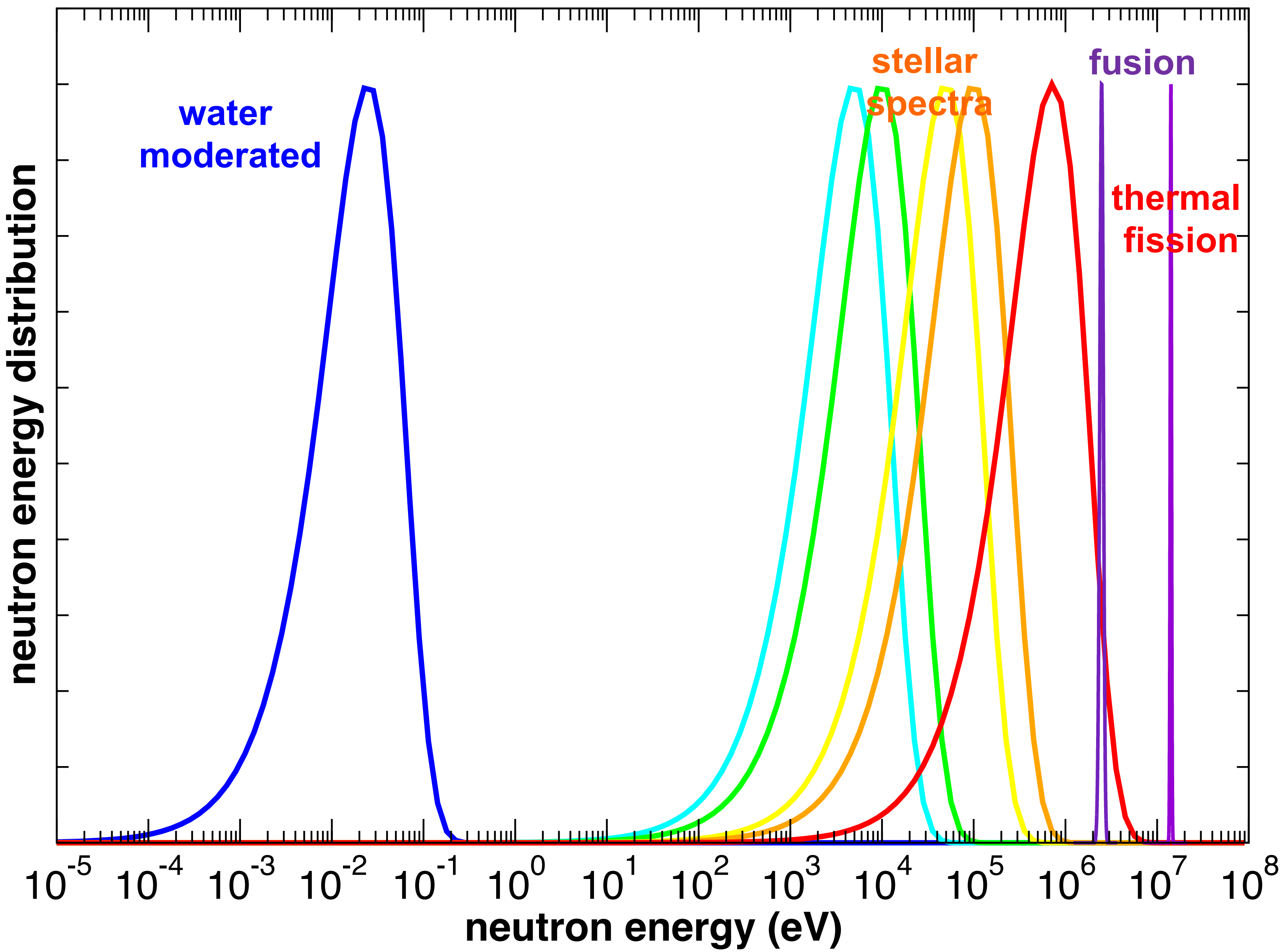
Neutron energy distributions



Neutron energy distributions



Neutron energy distributions



Maxwell-Boltzmann distribution

- Maxwell-Boltzmann statistics describe neutron spectra from
  - thermal-neutron induced fission
  - water moderated neutrons (infinite moderator)
  - stellar spectra (sources $^{22}Ne(\alpha,n)^{25}Mg$, $^{13}C(\alpha,n)^{16}O$)

- Velocity distribution at temperature *T*, Boltzmann constant *k*

$$n_v(v) = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right)$$

has maximum at

$$v_{\max} = \sqrt{2kT/m}$$

- At velocity v = 2200 m/s (definition, used as thermal neutron reference)
  E_{max} = **25.3** meV, *T* = **293.6** K, λ = **0.18** nm

Maxwell-Boltzmann distribution

- Distributions of kinetic energy, wavelength or time-of-flight can be converted into each other

$$n_v(v)dv = n_E(E)dE = n_t(t)dt = n_\lambda(\lambda)d\lambda$$

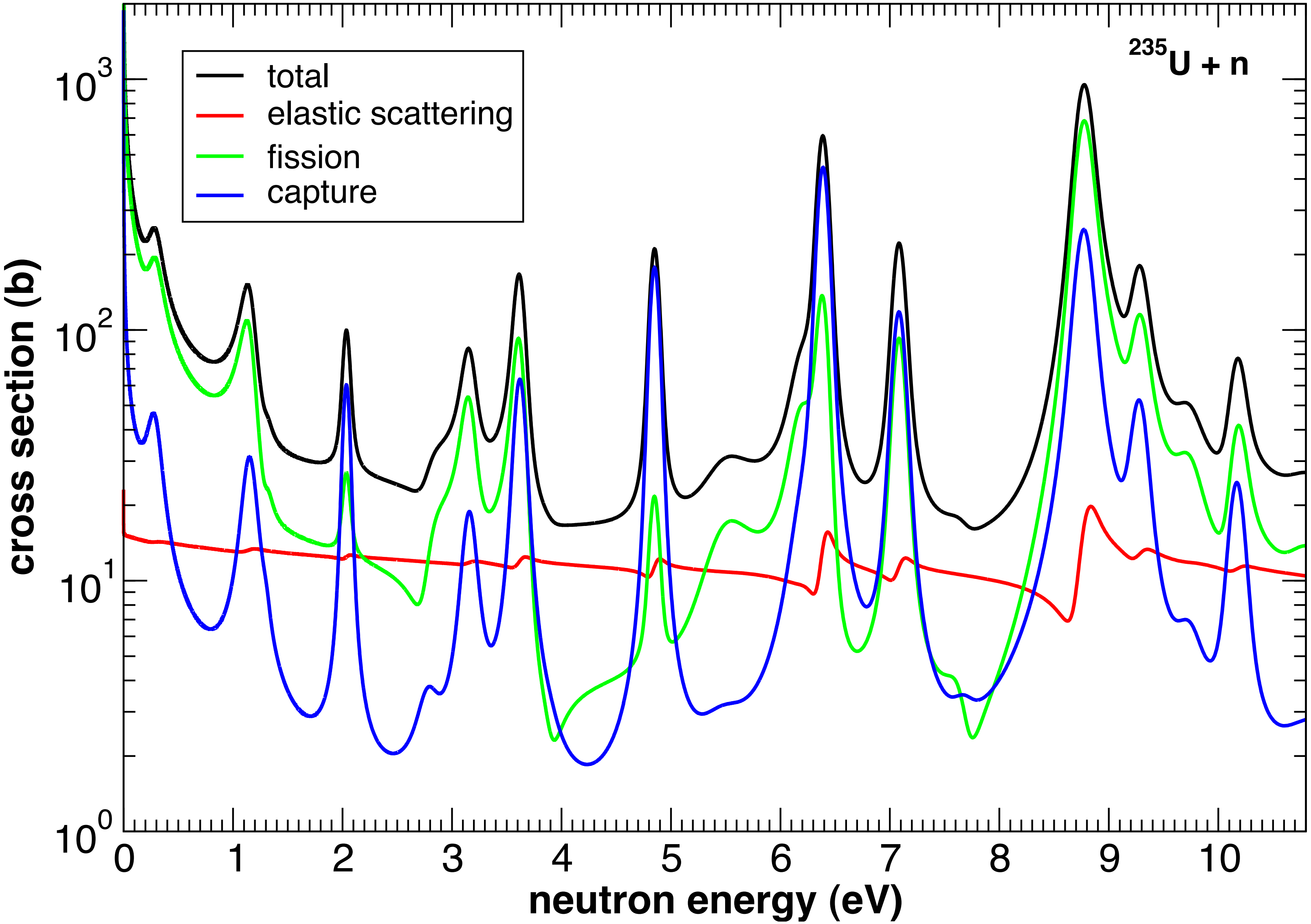
- For neutron beams, a “flux”-like distribution is more appropriate

$$\varphi_v(v) \propto v \times n_v(v)$$

- with conversions

$$\varphi_v(v)dv = \varphi_E(E)dE = \varphi_t(t)dt = \varphi_\lambda(\lambda)d\lambda$$

Neutron cross sections



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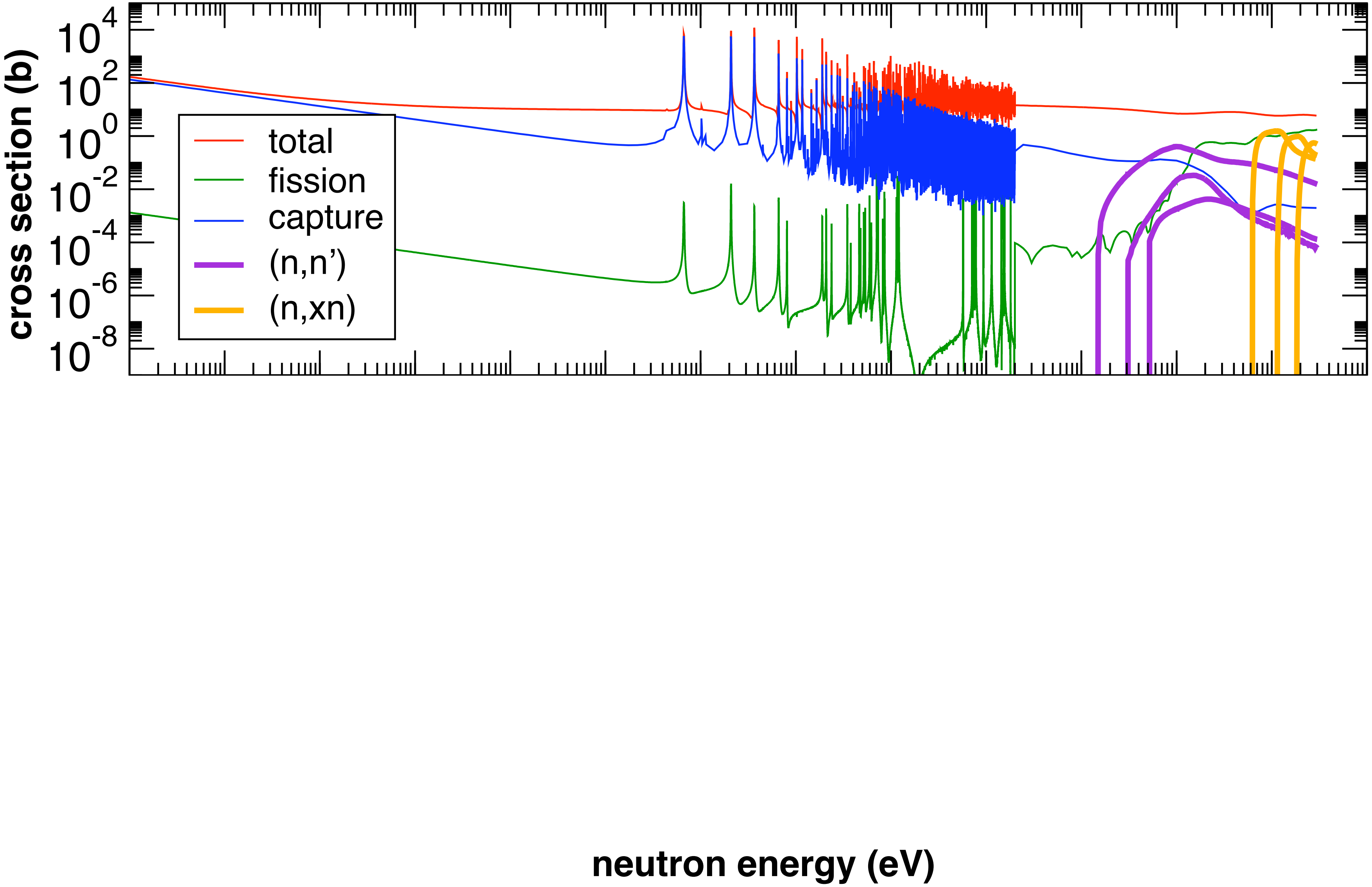

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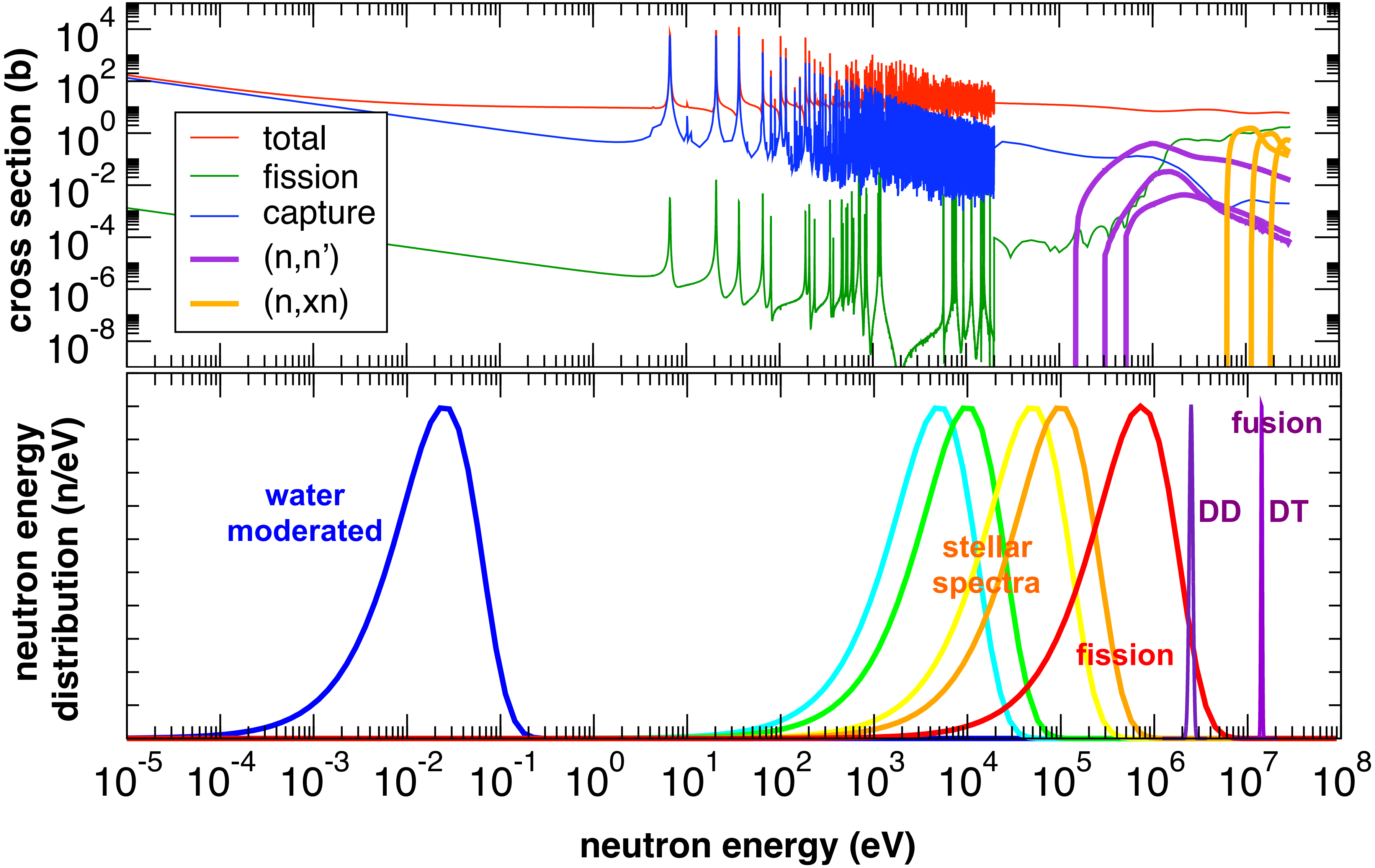
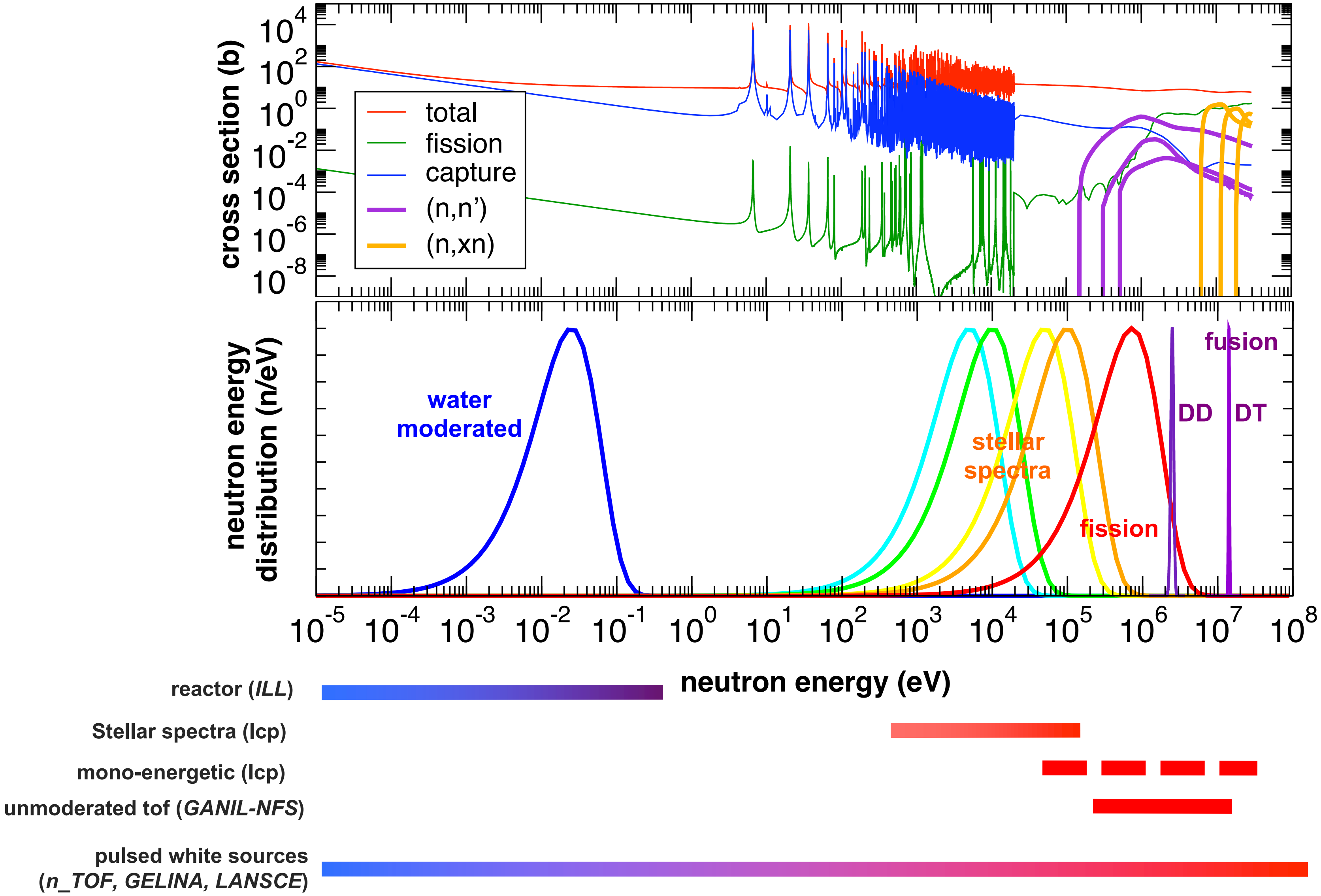


Neutron cross sections



Introduction R-matrix theory

- Formalism to decribe (neutron) reactions

- For resolved resonances, full cross sections can be constructed from only a few resonance parameters

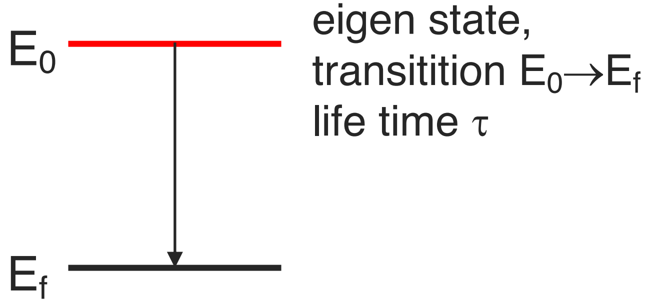
- Standard way of storage for evaluated nuclear data

- Complicated theory, but can be understood more easily in a global way

Decay of a quantum state

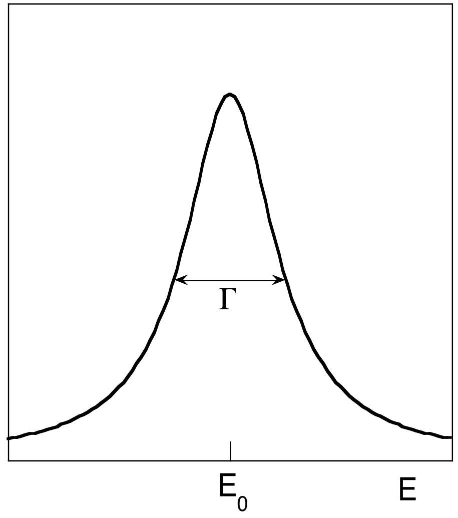
state with a life time τ:

$$\Psi(t) = \Psi_0 e^{-iE_0 t/\hbar} e^{-t/2\tau}$$



definition (Heisenberg):

$$\Gamma = \frac{\hbar}{\tau}$$



Fourier transform gives energy profile:

$$I(E) = \frac{\Gamma / 2\pi}{(E - E_0)^2 + \Gamma^2 / 4}$$

The Schrödinger equation

Time-independent Schrödinger equation for a spinless, one-dimensional particle:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

wave function

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\psi(x) = E\psi(x)$$

potential

energy

$$\hat{H}\psi(x) = E\psi(x)$$

Hamiltonian

Solutions: ψ, E

$\psi^*\psi$ Interpreted as probabitliy

Quantum system (1D): the infinite well

Solve Schrödinger equation for a spinless, onedimensional particle

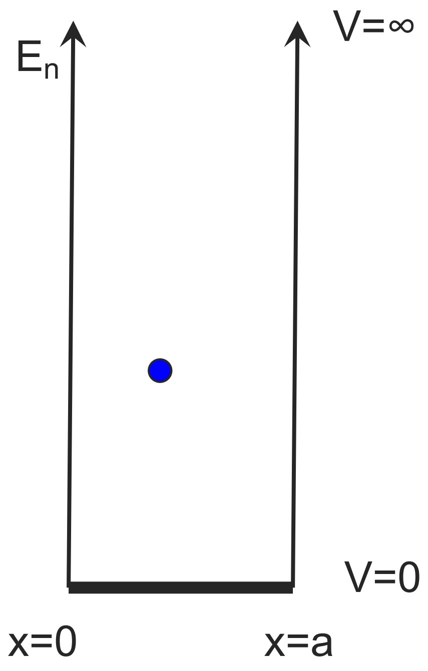
$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E(x)$$

Potential:

$$V(x) = 0 \quad \text{for} \quad 0 < x < a$$

$$V(x) = \infty \quad \text{elsewhere}$$

Solution:



Quantum system (1D): the infinite well

Solve Schrödinger equation
for a spinless, one-dimensional particle

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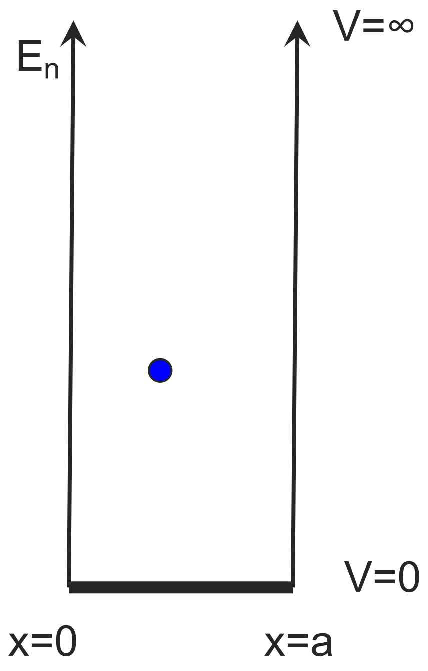


Solution:
Only solutions for $0 < x < a$

General solution: $\psi(x) = A\exp(ikx) + B\exp(-ikx)$

with $k = \sqrt{2mE/\hbar^2}$

Boundary conditions:

$$\psi(0) = \psi(a) = 0$$

Quantum system (1D): the infinite well

Solve Schrödinger equation
for a spinless, one-dimensional particle

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E(x)$$

Potential:

$V(x) = 0$ for $0 < x < a$

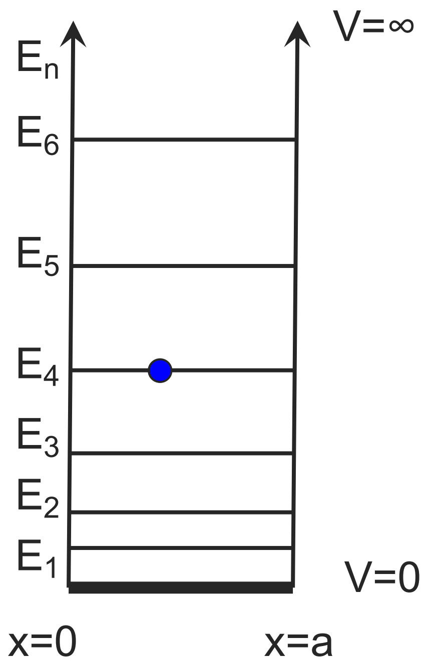
$V(x) = \infty$ elsewhere

Solution:

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

$$E_n = \frac{\hbar^2 \pi^2}{2ma^2} n^2$$

$$n = 1, 2, 3, \ldots$$



Quantum system (1D): the infinite well

Solve Schrödinger equation
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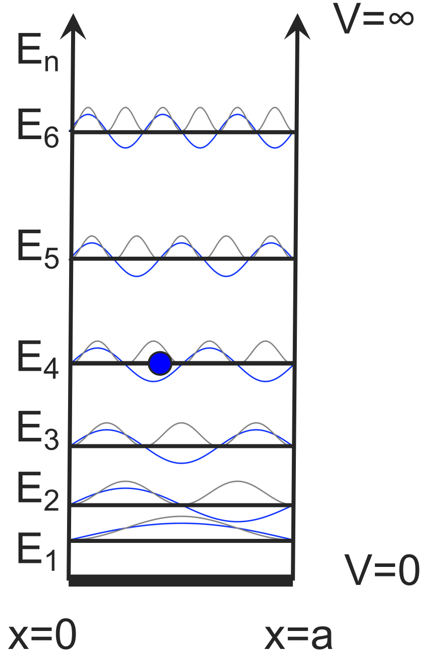


wave function: ψ
probability: $\psi^*\psi$

Quantum system (1D): the finite well

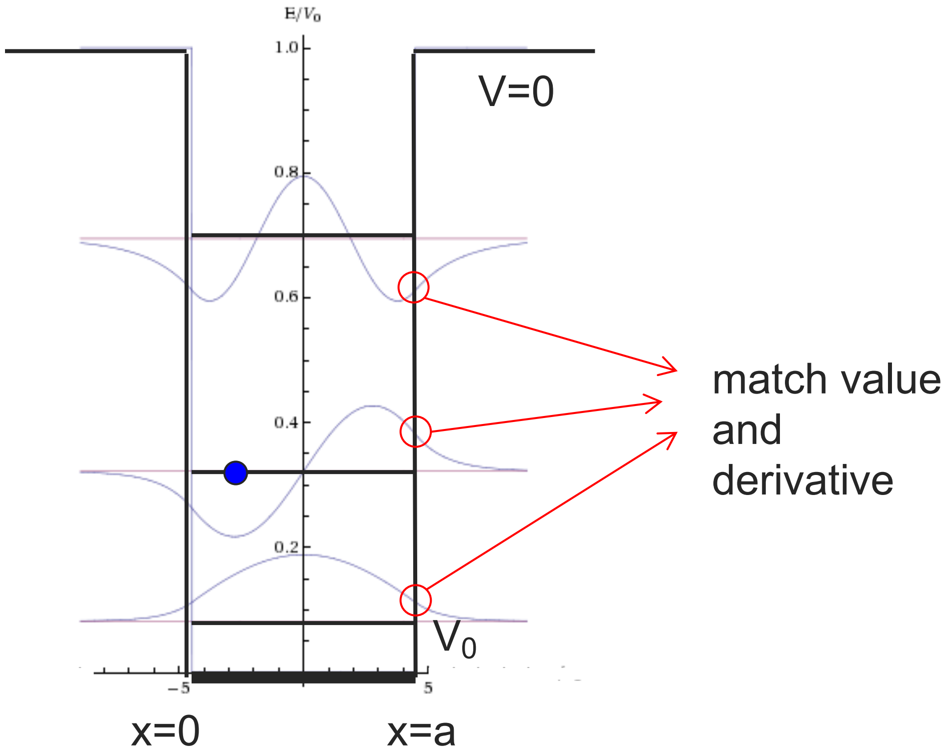
Solve Schrödinger equation in three regions and two energy ranges:

- E < 0, bound states
  wave function must vanish towards ± infinity
- E > 0, unbound states

$E < 0$	$E > 0$
$\psi_1(x) = A_1 e^{k_1 x}$	$\psi_1(x) = A_1 e^{ik_1 x} + B_1 e^{-ik_1 x}$
$\psi_2(x) = A_2 e^{ik_2 x} + B_2 e^{-ik_2 x}$	$\psi_2(x) = A_2 e^{ik_2 x} + B_2 e^{-ik_2 x}$
$\psi_3(x) = B_3 e^{-k_3 x}$	$\psi_3(x) = A_3 e^{ik_3 x}$

$$k_{1,3} = \sqrt{2mE/\hbar^2}$$

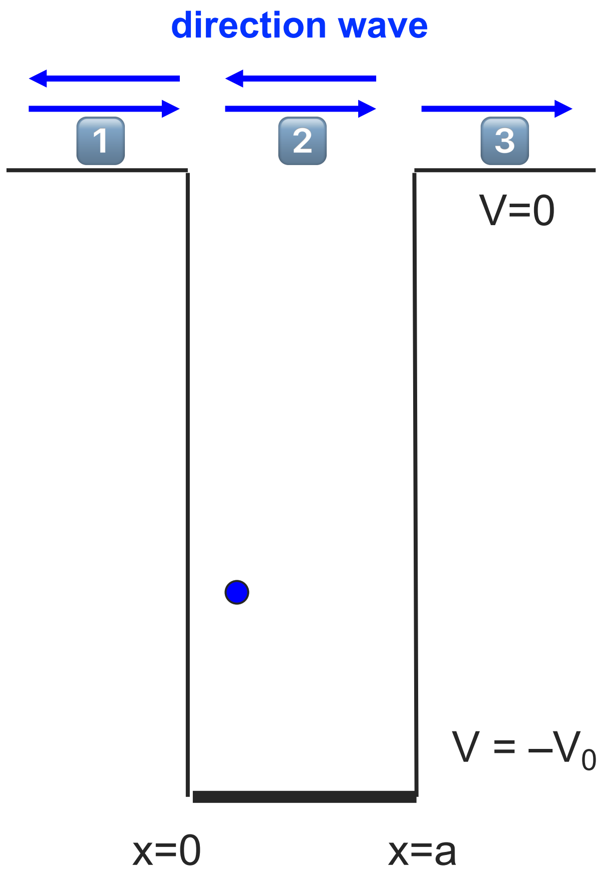
$$k_2 = \sqrt{2m(E+V_0)/\hbar^2}$$



Quantum system (1D): the finite well

Solve Schrödinger equation in three regions and two energy ranges:

- require **continuity** of ψ and $d\psi/dx$ at boundaries
- E < 0, bound states
  wave function must vanish towards ± infinity
- E > 0, unbound states
  assume particle travelling from left to right



$$E < 0$$
$$\psi_1(x) = A_1 e^{k_1 x}$$
$$\psi_2(x) = A_2 e^{ik_2 x} + B_2 e^{-ik_2 x}$$
$$\psi_3(x) = B_3 e^{-k_3 x}$$

$$E > 0$$
$$\psi_1(x) = A_1 e^{ik_1 x} + B_1 e^{-ik_1 x}$$
$$\psi_2(x) = A_2 e^{ik_2 x} + B_2 e^{-ik_2 x}$$
$$\psi_3(x) = A_3 e^{ik_3 x}$$

$$k_{1,3} = \sqrt{2mE/\hbar^2}$$
$$k_2 = \sqrt{2m(E+V_0)/\hbar^2}$$

Quantum system (1D): the finite well

Solve Schrödinger equation in three regions and two energy ranges:

- require **continuity** of ψ and $d\psi/dx$ at boundaries
- $E < 0$, bound states
  wave function must vanish towards ± infinity
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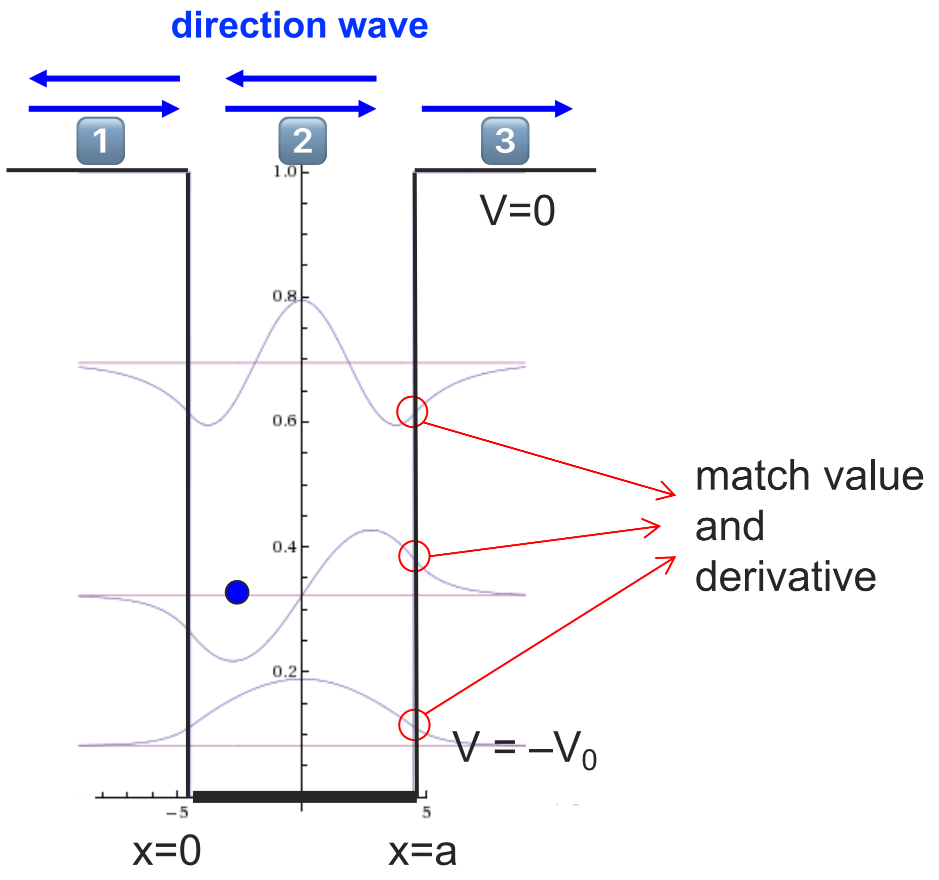
  assume particle travelling from left to right

Solutions E<0:
only implicit solutions, transcendental equations

$$k_1 = k_2 \tan(k_2 a)$$

$$k_1 = -k_2 \cot(k_2 a)$$

- solve graphically as function of E,
- results in limited discrete values of E,
- number of solutions (states) depend on a and V_0



Quantum system (1D): the finite well

Solve Schrödinger equation in three regions and two energy ranges:

- require **continuity** of ψ and $d\psi/dx$ at boundaries
- $E < 0$, bound states
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- $E > 0$, unbound states
  assume particle travelling from left to right

Solutions E>0:

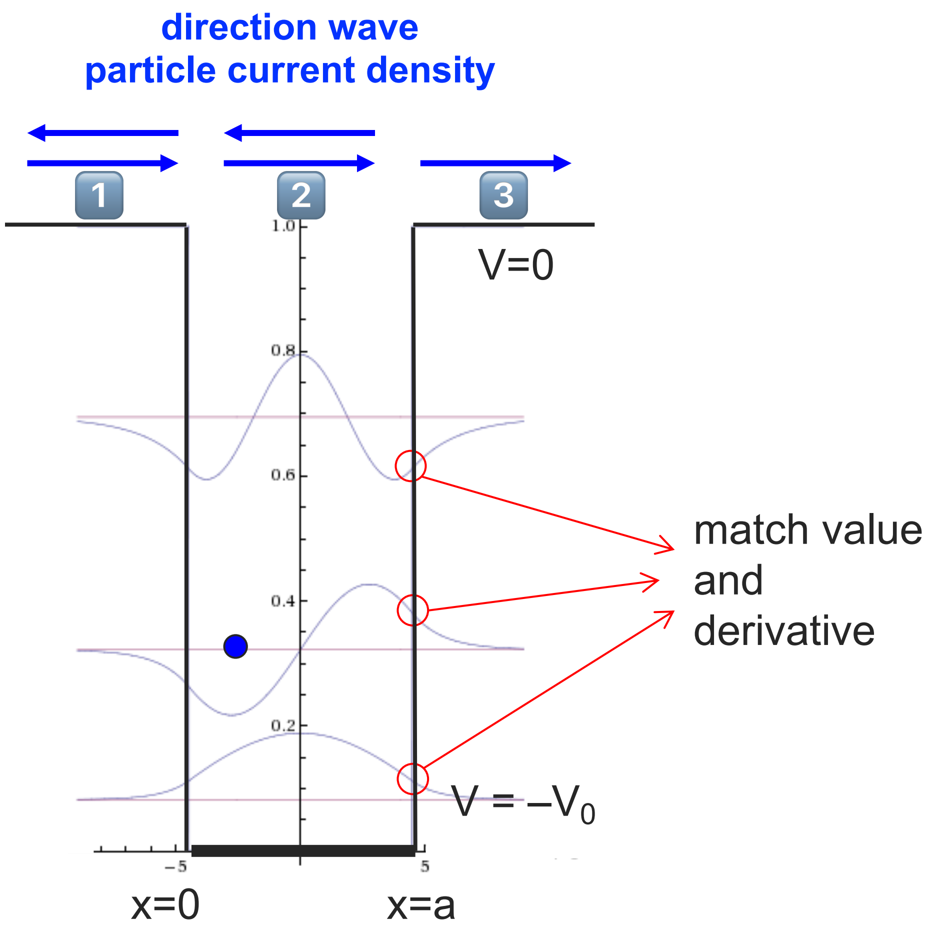
- infinite number of solutions, plain waves of particle of any E
- boundaries can reflect or transmit waves
- use particle current density

$$j = \frac{\hbar}{2mi}\left(\psi^* \nabla \psi - \nabla \psi^* \psi\right)$$

To get transmission and reflection

$$R = \frac{\|B_1\|^2}{\|A_1\|^2}$$

$$T = \frac{\|A_3\|^2}{\|A_1\|^2}$$



Quantum system (1D): the finite well

Solve Schrödinger equation in three regions and two energy ranges:

- require **continuity** of ψ and $d\psi/dx$ at boundaries
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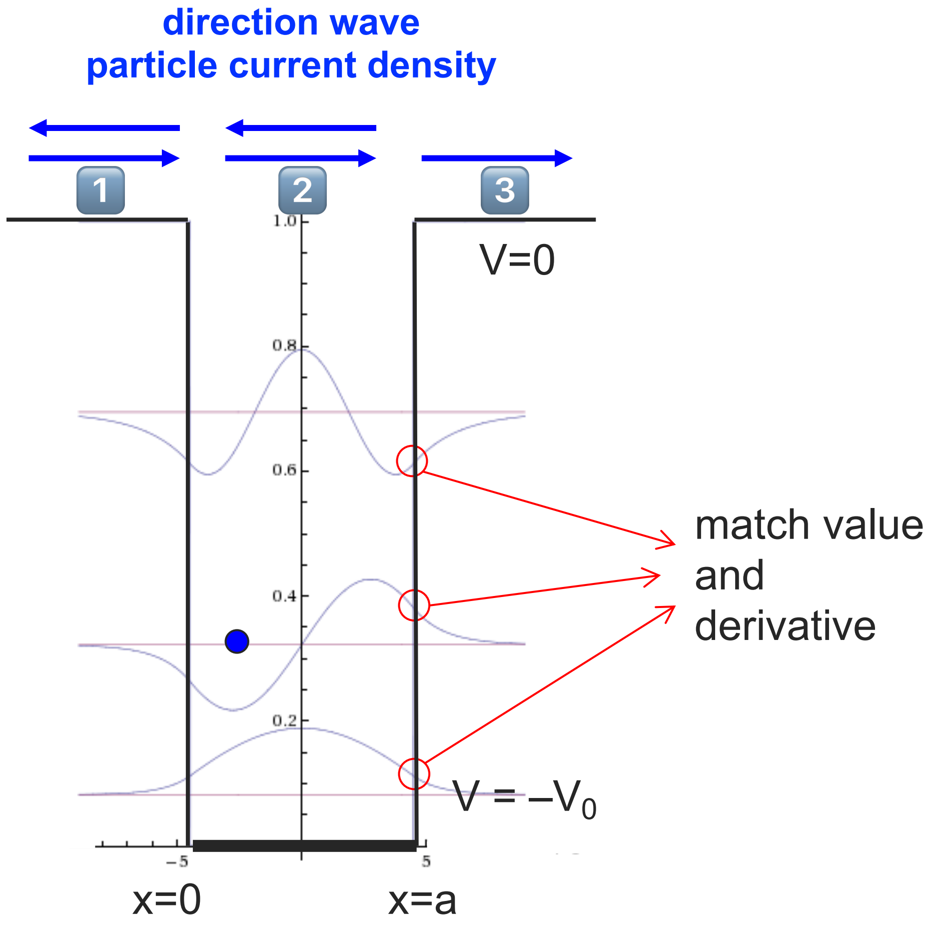


Solutions E>0:

- set arbitrarily A_1=1, then solution is

$$R = \frac{\frac{V_0^2}{4E(E+V_0)} \sin^2\left(a\sqrt{\left(\frac{2m}{\hbar^2}(E+V_0)\right)}\right)}{1 + \frac{V_0^2}{4E(E+V_0)} \sin^2\left(a\sqrt{\left(\frac{2m}{\hbar^2}(E+V_0)\right)}\right)}$$

$$T = \frac{1}{1 + \frac{V_0^2}{4E(E+V_0)} \sin^2\left(a\sqrt{\left(\frac{2m}{\hbar^2}(E+V_0)\right)}\right)}$$

If potential is real, then $R+T=1$

Quantum system (1D): the finite well

- For (E>0), still values where

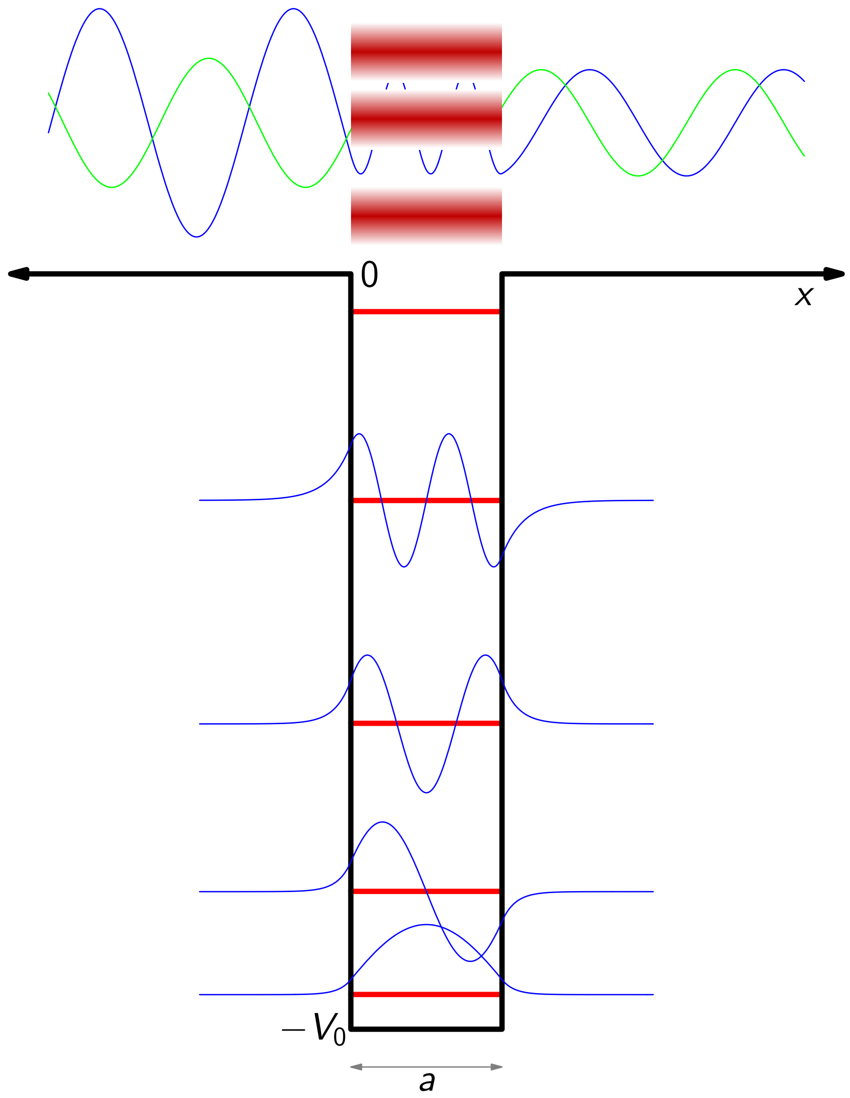
$$\sin^2(k_2 a) = 0$$

i.e. $$k_2 = \frac{n\pi}{a}$$

At those values of E,
T = 1 and R = 0

These are quasi-bound or unbound states.

To account for absorption, an imaginary potential is needed



Quantum systems

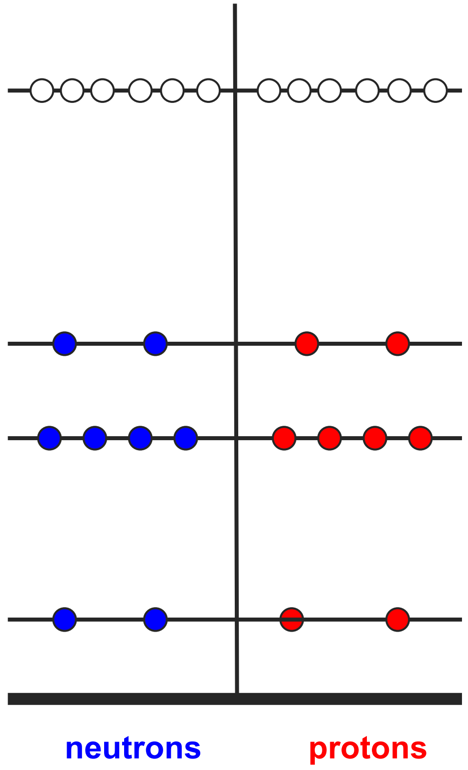
Other useful excercises in 1D:
- barrier potential
- finite potential barrier
- harmonic oscillator

More complicated in 3D, V=V(r), more particles, degeneracy:
- cartesian well
- spherical well
- harmonic oscillator
- realistic potentials (Woods-Saxon),

No analytical solution possible
→ numerical solutions

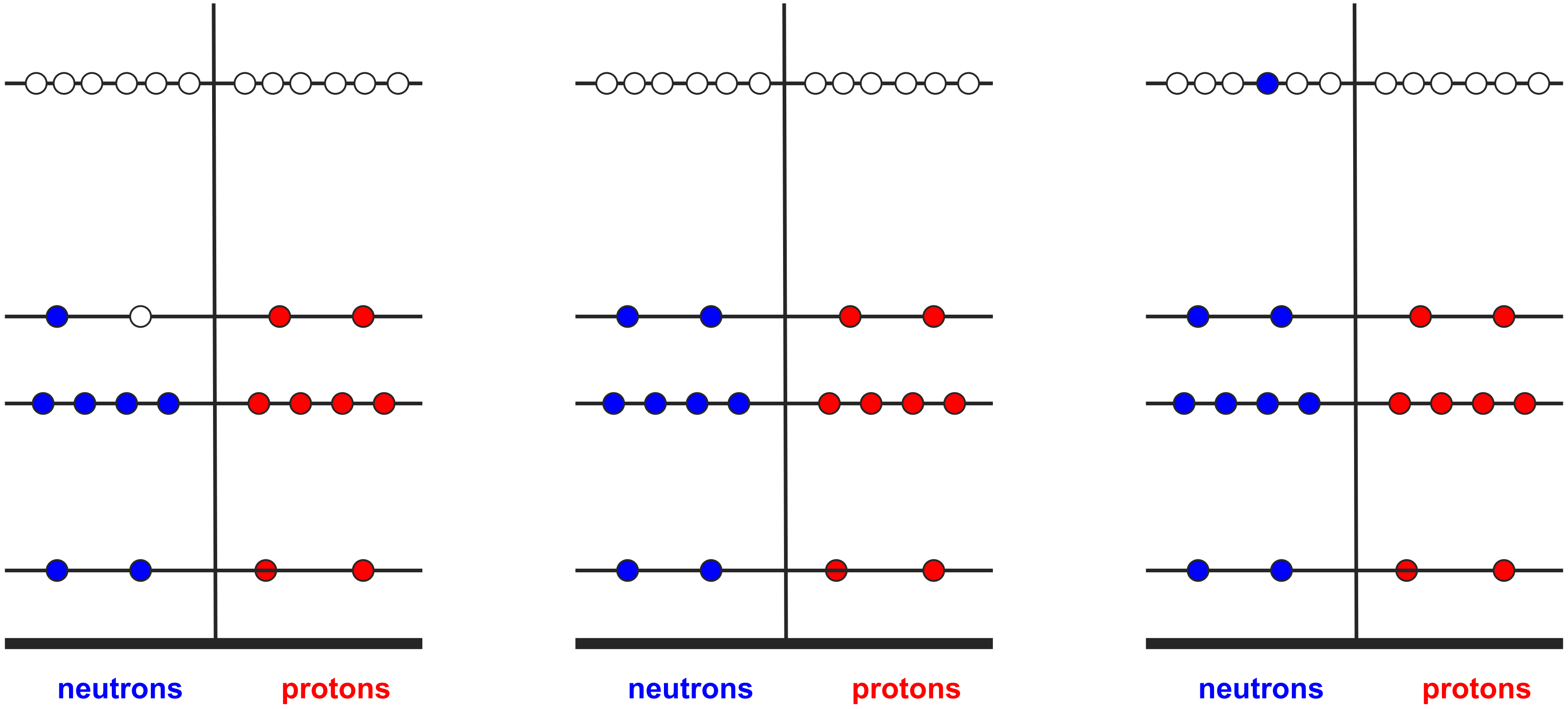
Applies to real quantum systems
atoms, nuclei.

The nucleus as a quantum system



$$^{16}_{8}O$$

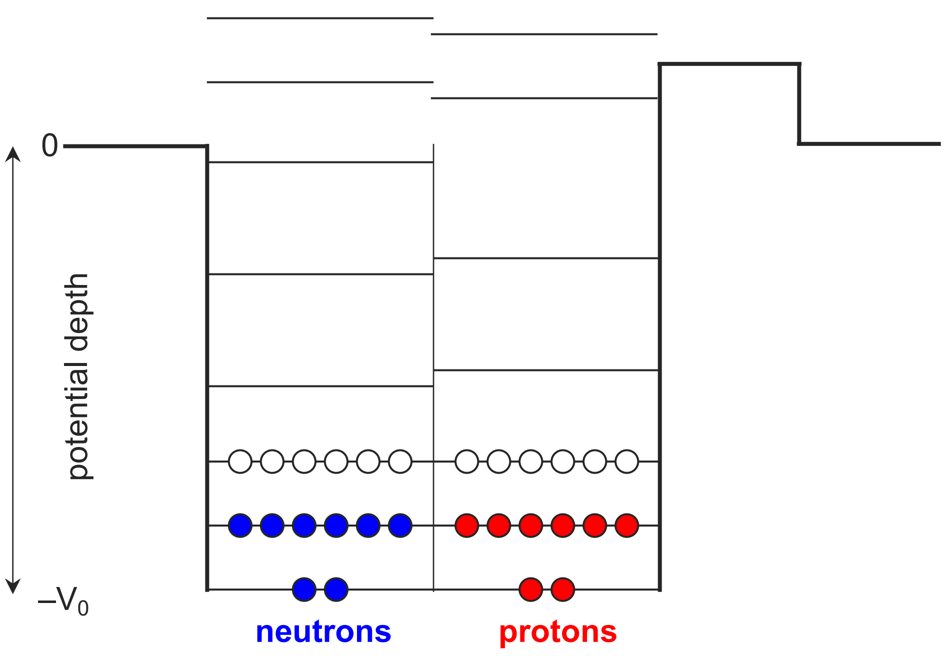
The nucleus as a quantum system



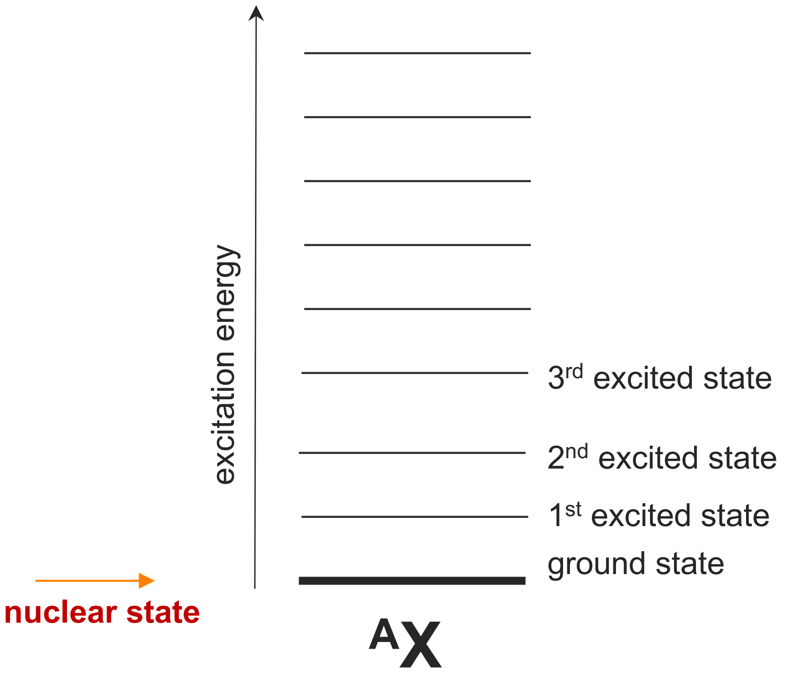
$^{15}_{8}O$ $^{16}_{8}O$ $^{17}_{8}O$

The nucleus as a quantum system

shell model representation:
configuration of nucleons in their potential



level scheme representation:
excited states of a nucleus
(shell model and other states)



The nucleus as a quantum system

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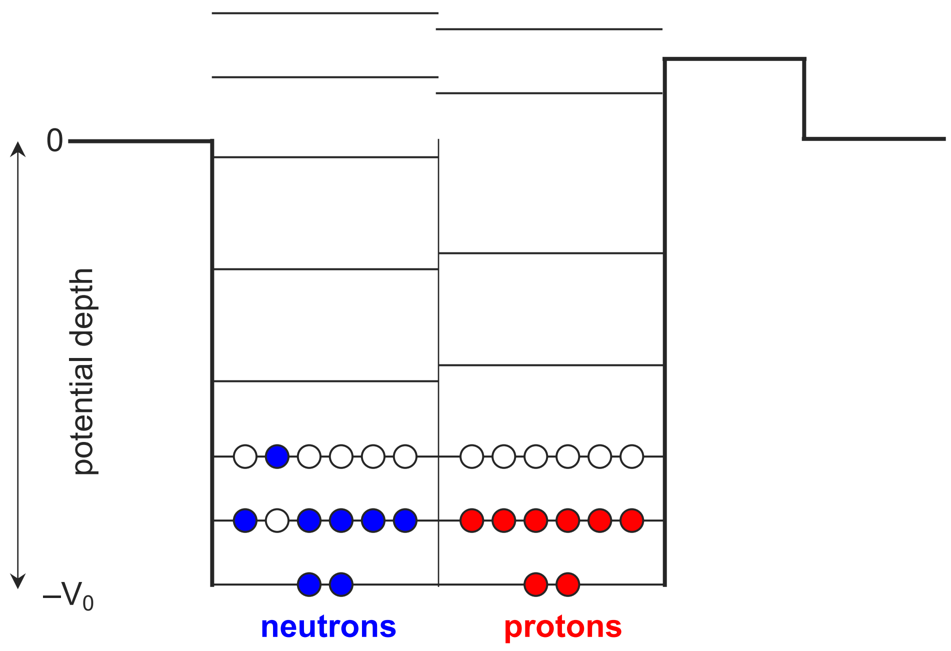

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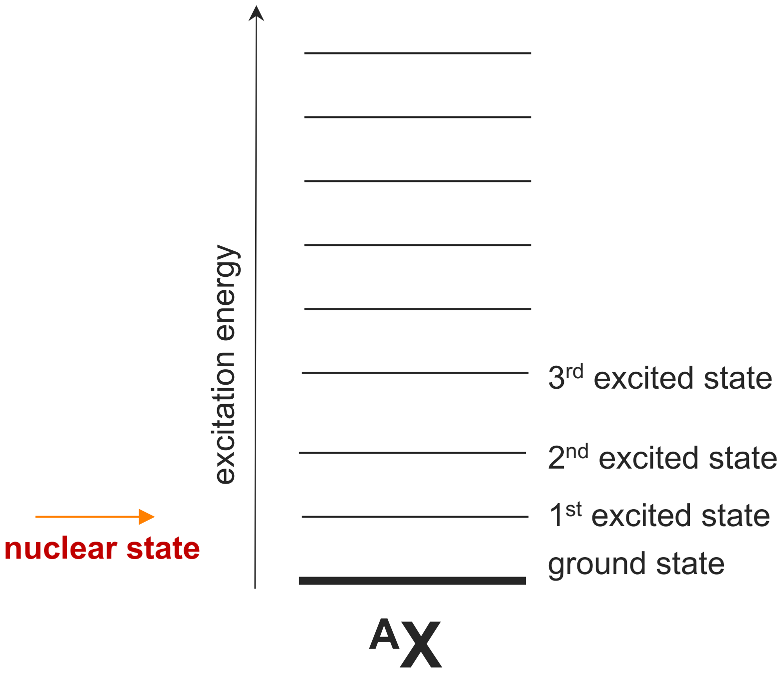

The nucleus as a quantum system

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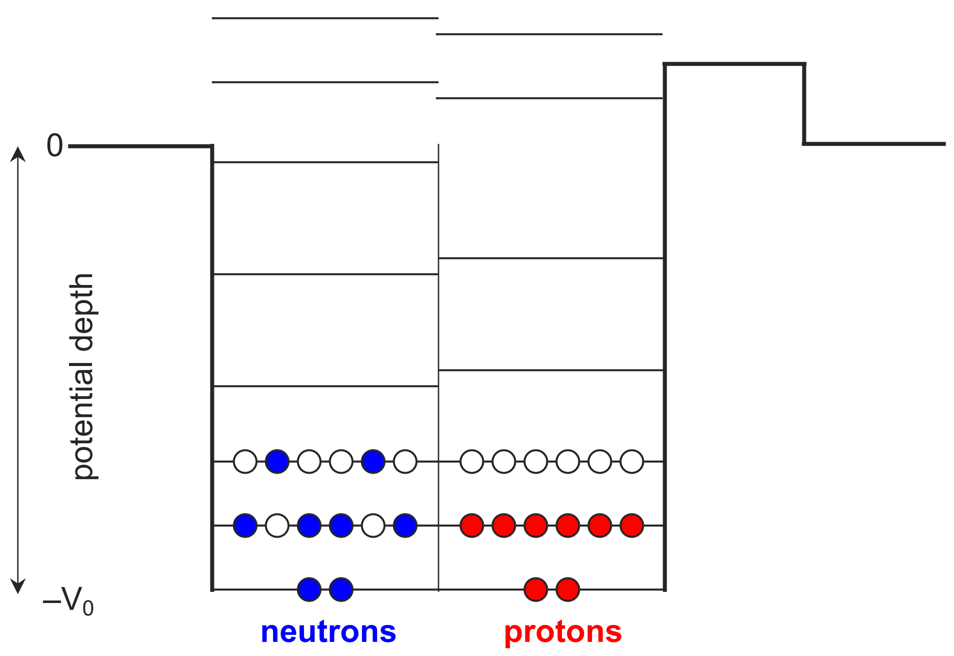

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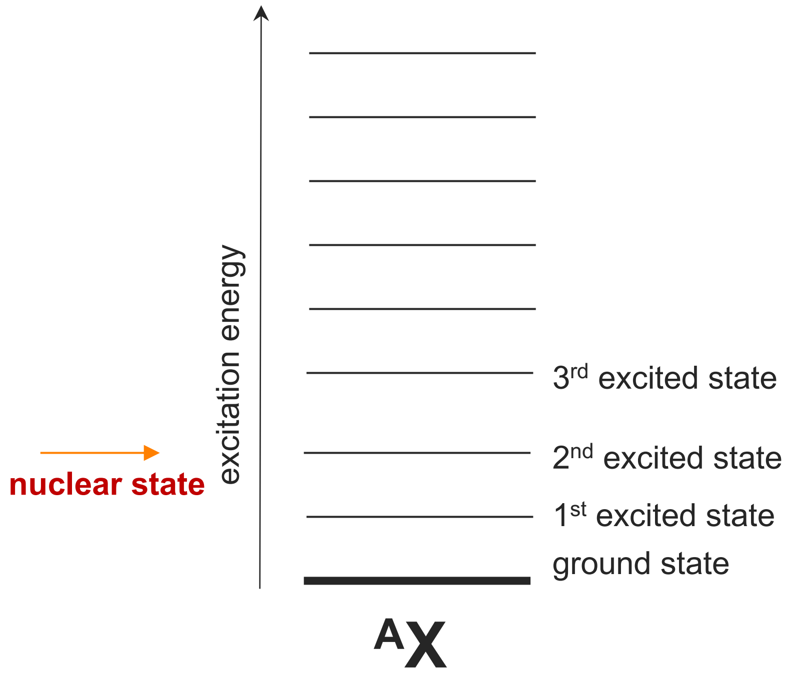

The nucleus as a quantum system

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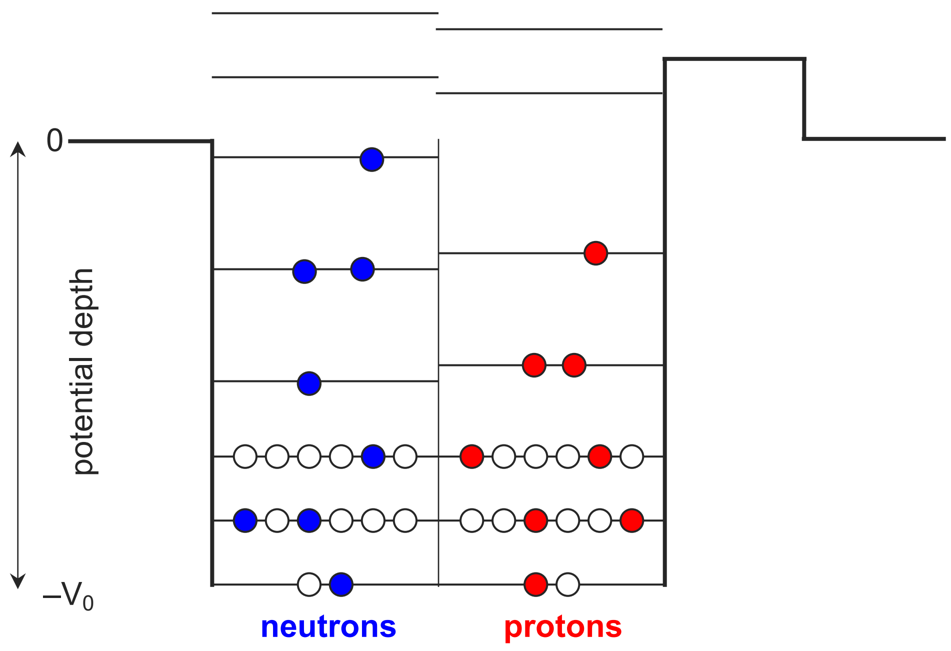

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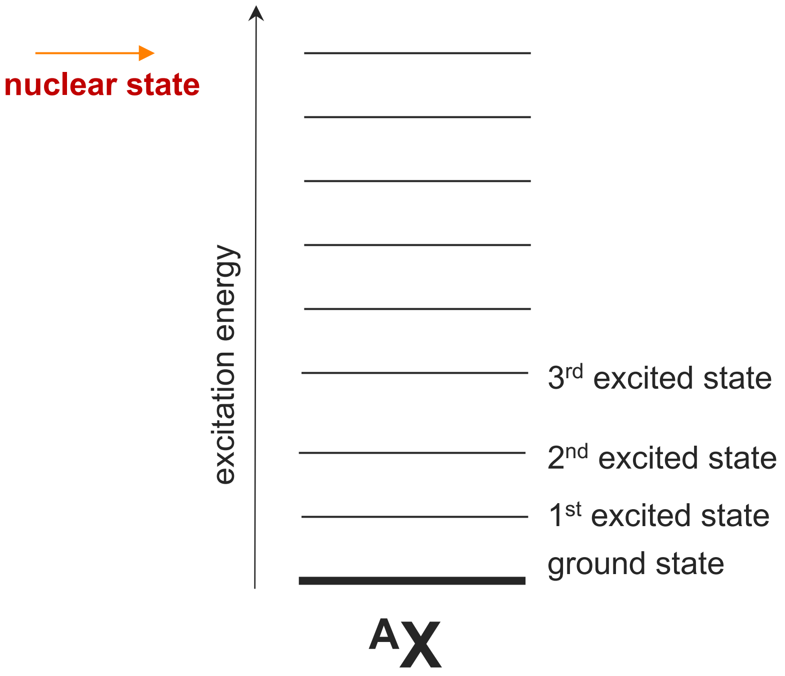
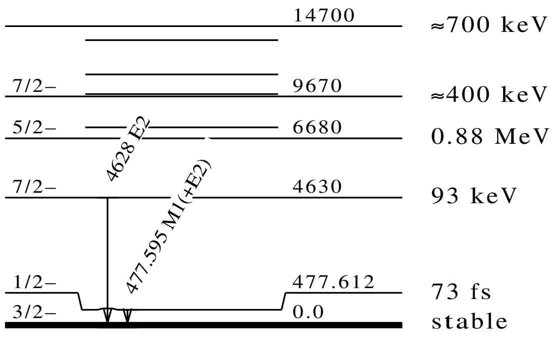
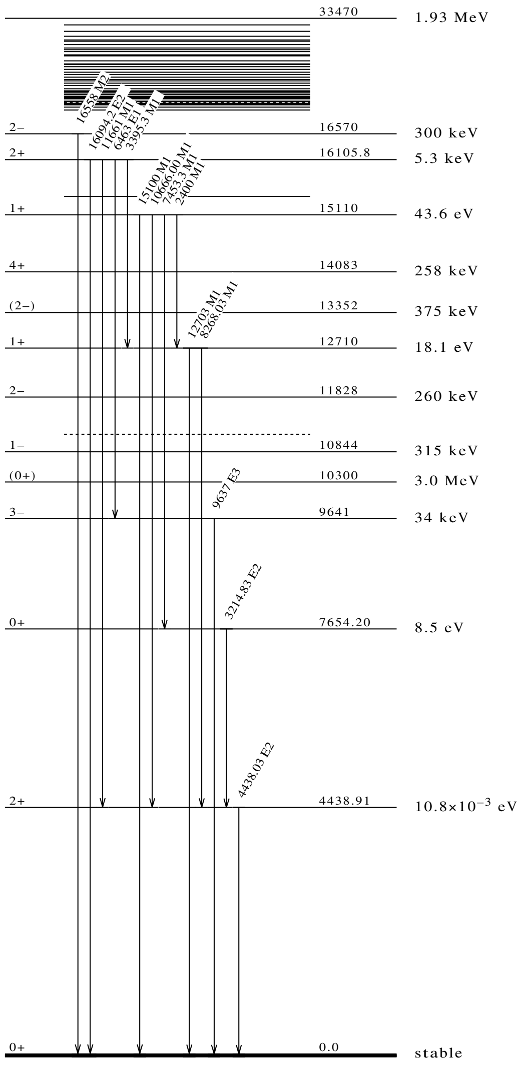


The nucleus as a quantum system

Level schemes from ENSDF
www.nndc.bnl.gov/ensdf

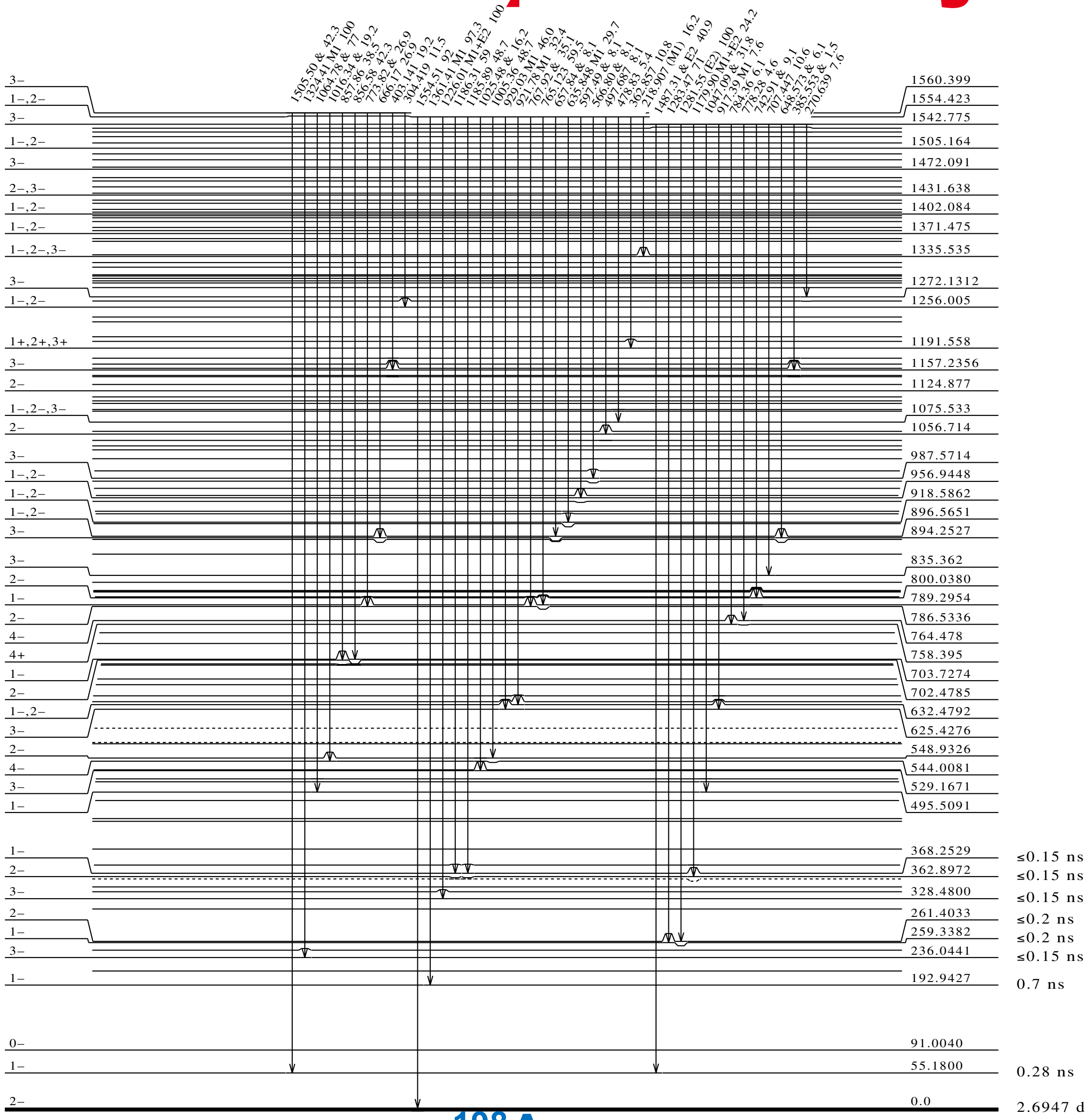


^{7}Li



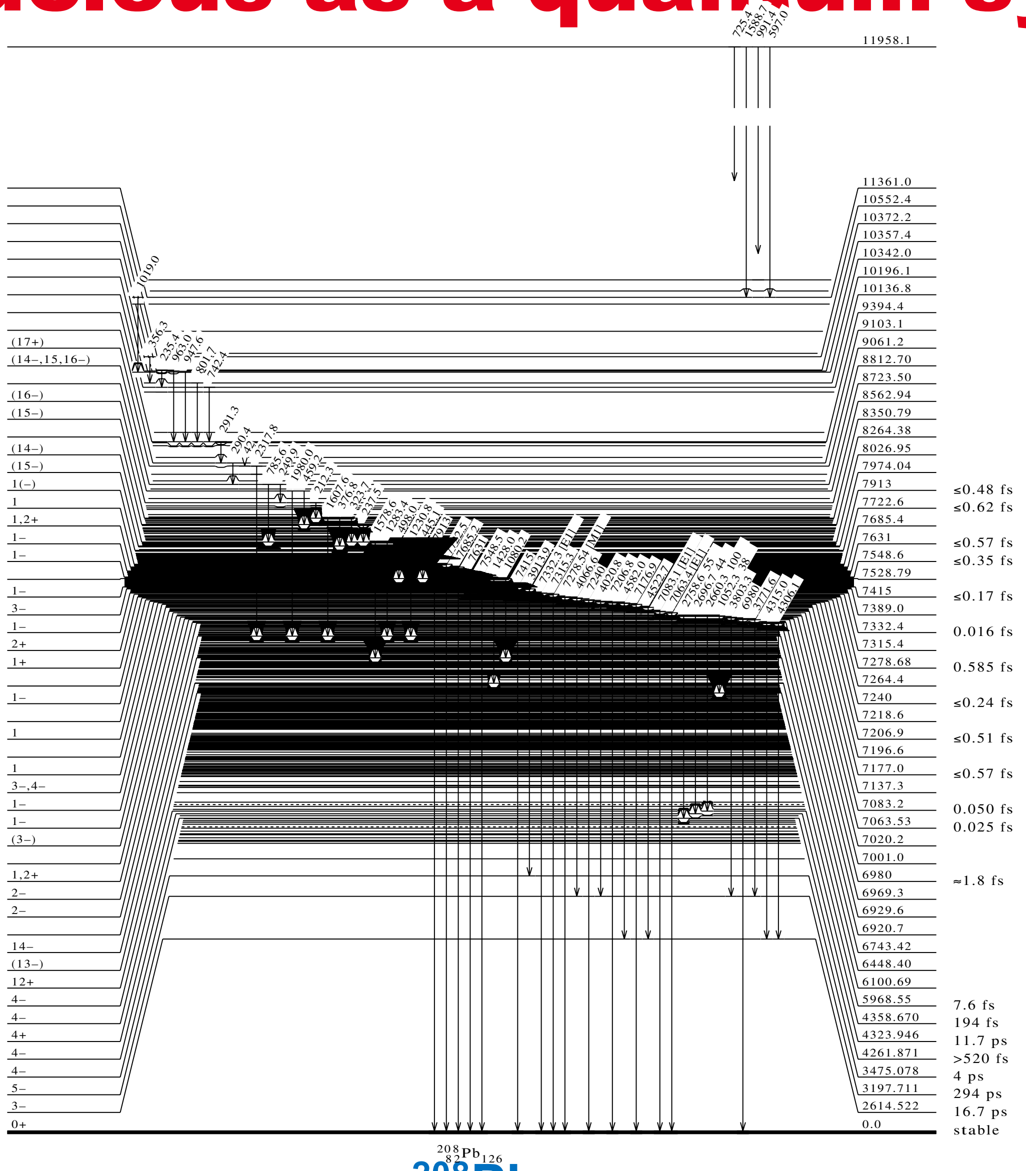
^{12}C

The nucleus as a quantum system



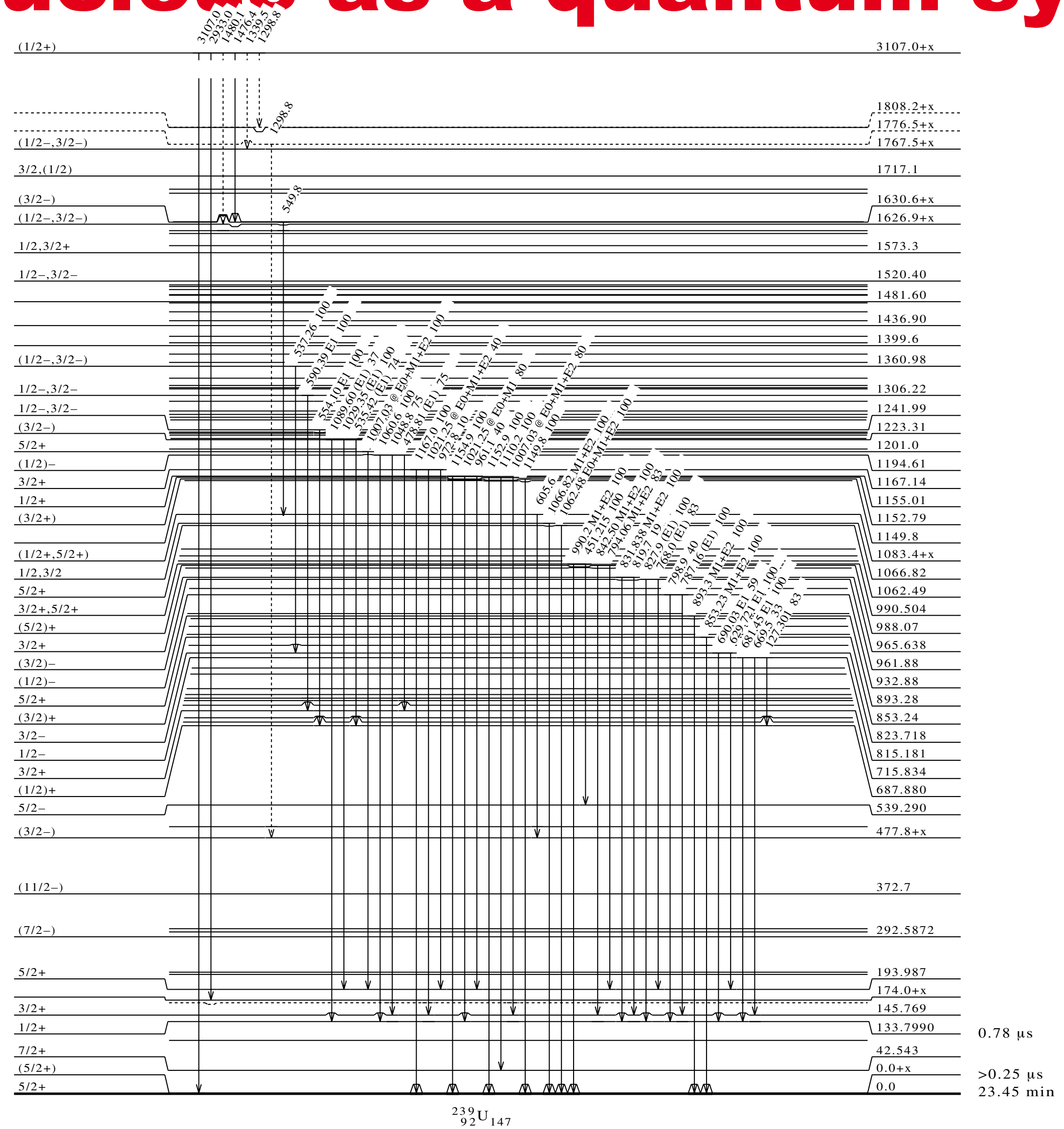
^{198}Au

The nucleus as a quantum system



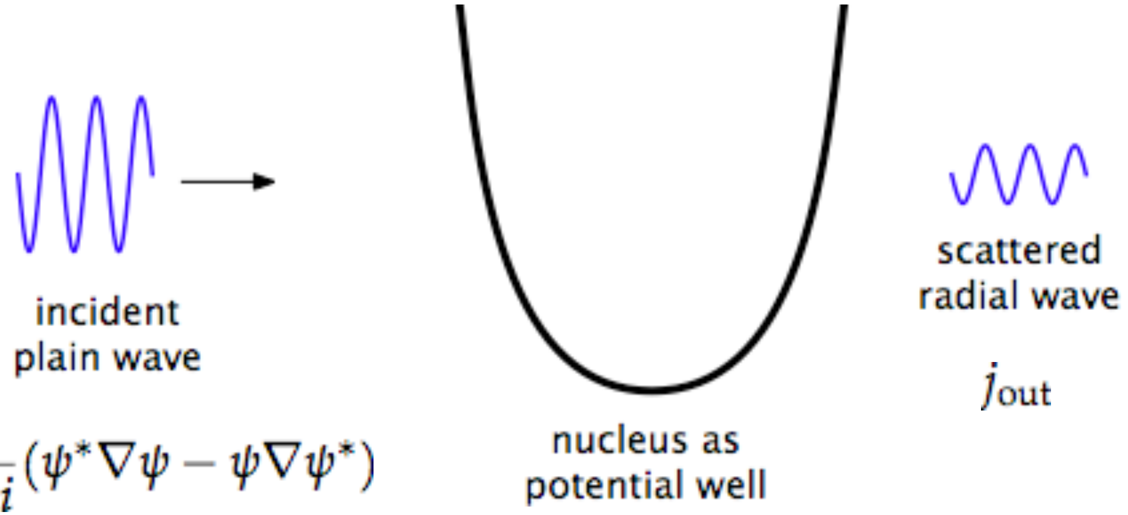
^{208}Pb

The nucleus as a quantum system



^{239}U

Nuclear scattering



$$j_{inc} = \frac{\hbar}{2mi}(\psi^* \nabla \psi - \psi \nabla \psi^*)$$

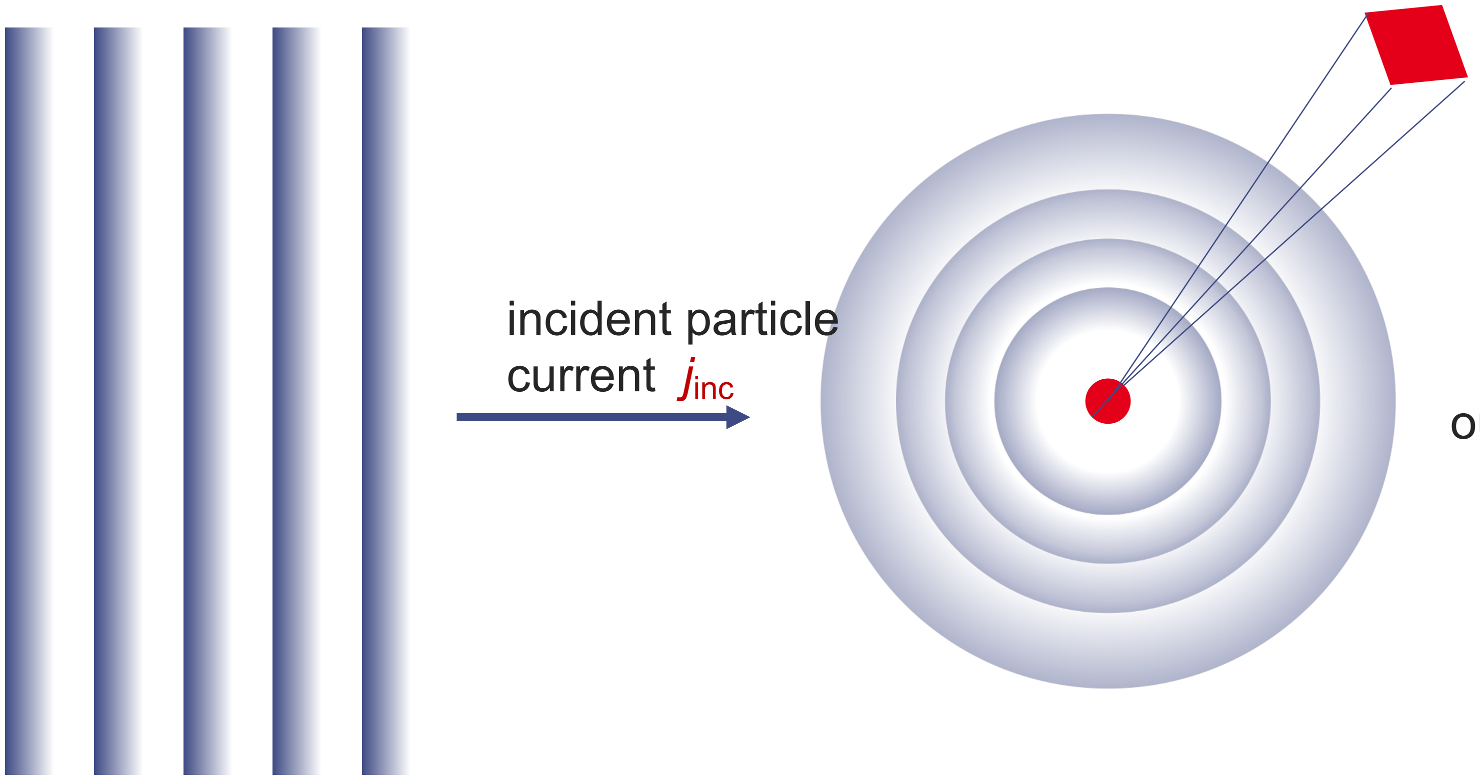
Conservation of probability density gives: $\sigma(\Omega) = \frac{r^2 j_{out}(r,\Omega)}{j_{inc}}$

Solve Schrödinger equation of system to get cross sections.
Shape of wave functions of in- and outgoing particles are known,
potential is unknown. Two approaches:

- **calculate potential (optical model calculations, smooth cross section)**
- **use eigenstates (R-matrix, resonances)**

Nuclear scattering 3D

incident plain wave



incident particle current j_{inc}

outgoing particle current j_{out} in direction $d\Omega$

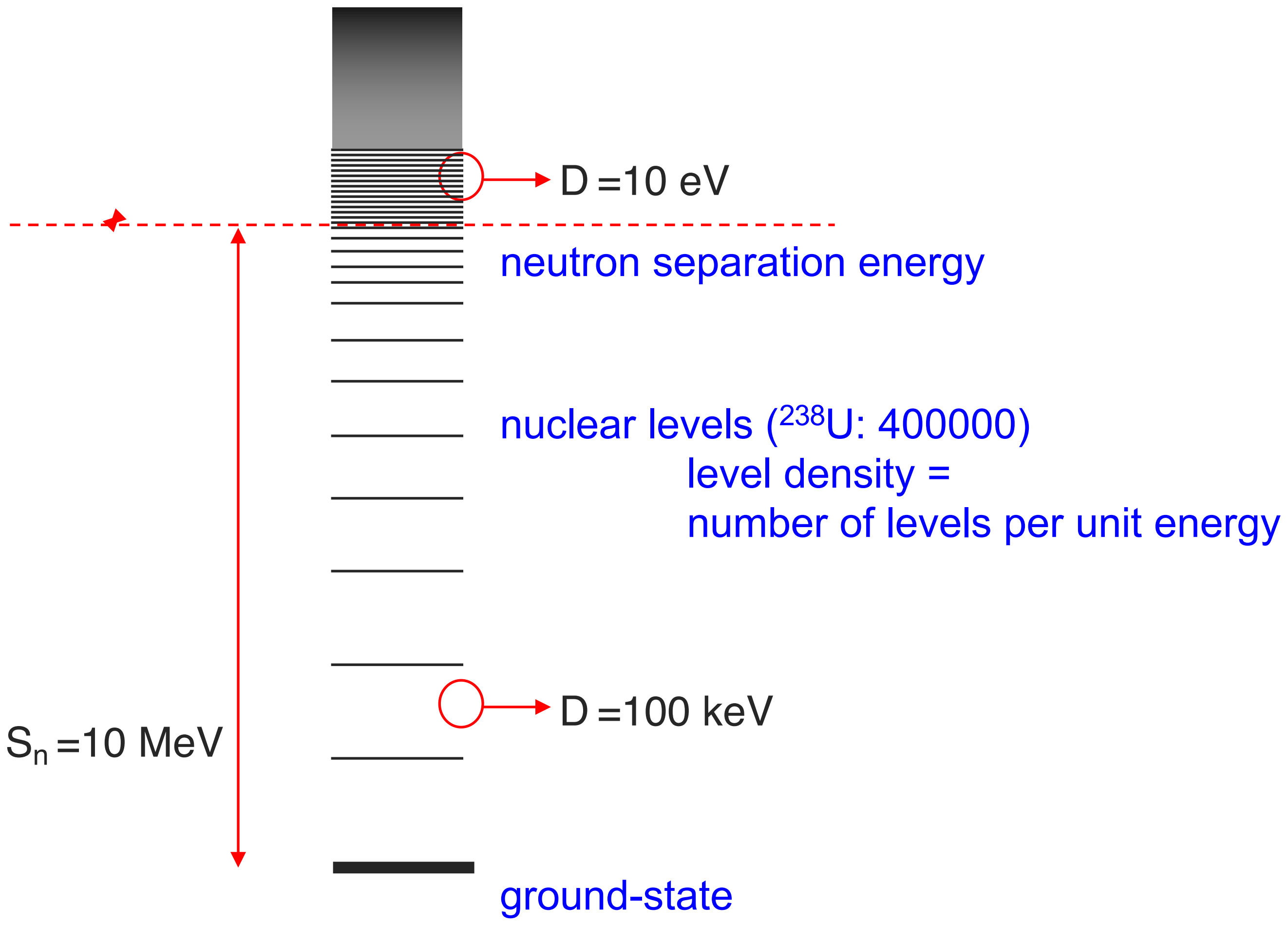
$$d\sigma = \frac{j_{\mathrm{sc}}}{j_{\mathrm{inc}}} r^2 d\Omega$$

outgoing radial wave

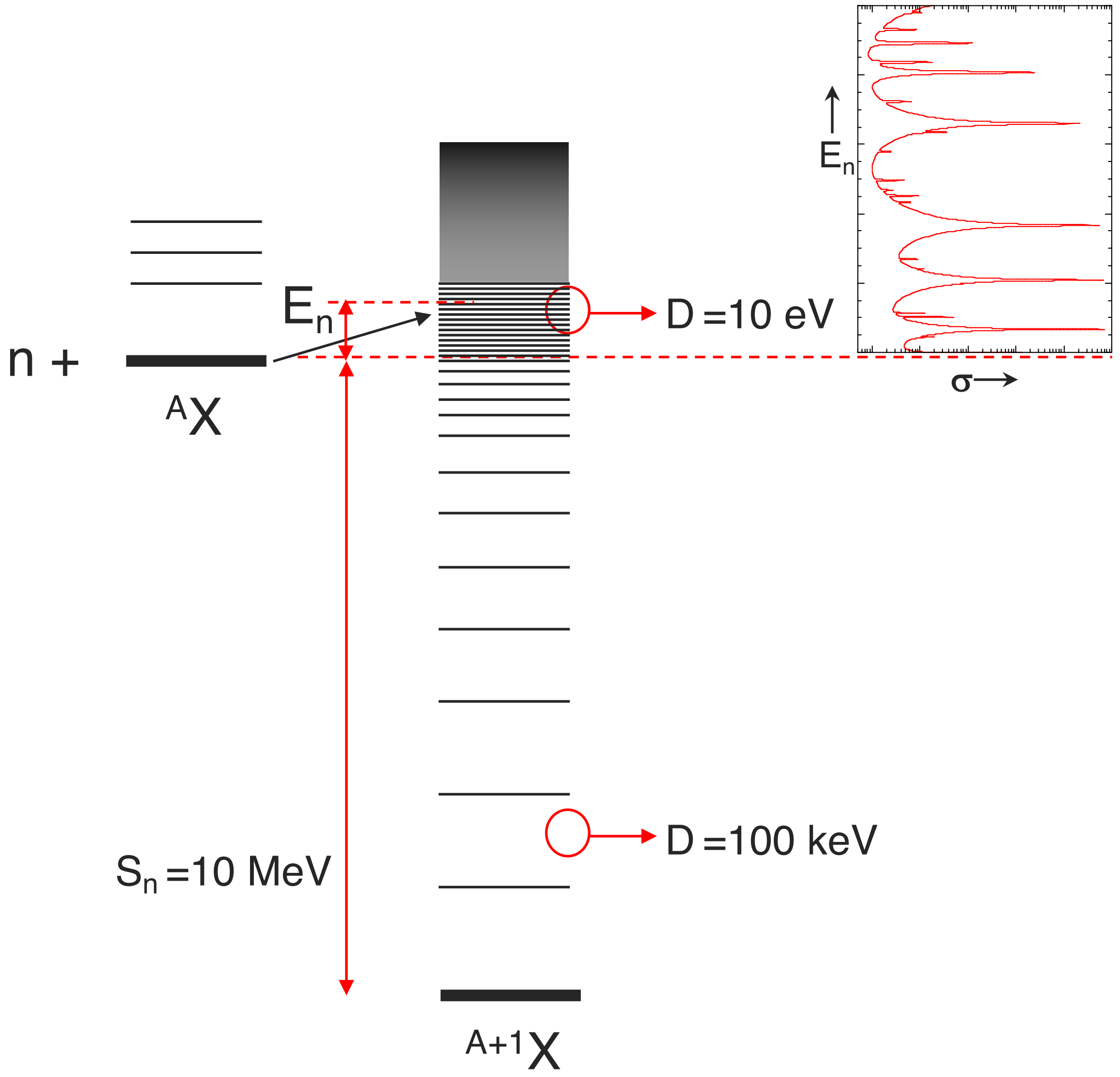
Blatt Biedenharn (1952): expansion plain wave into infinite sum of radial waves

$$\frac{d\sigma_{\alpha\alpha'}}{d\Omega} = \lambda^2 \sum_{L=0}^{\infty} B_{L\alpha\alpha'} P_L(\cos\theta)$$

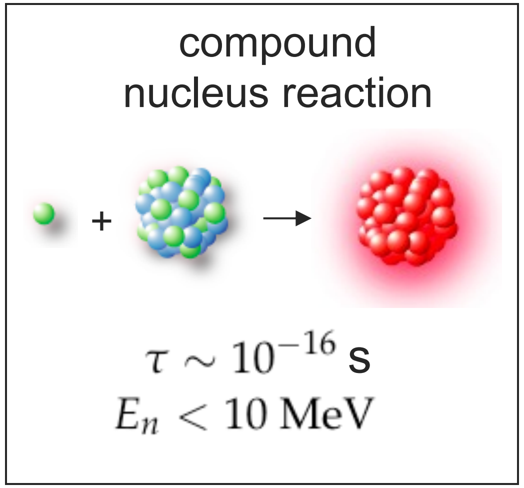
Compound neutron-nucleus reactions

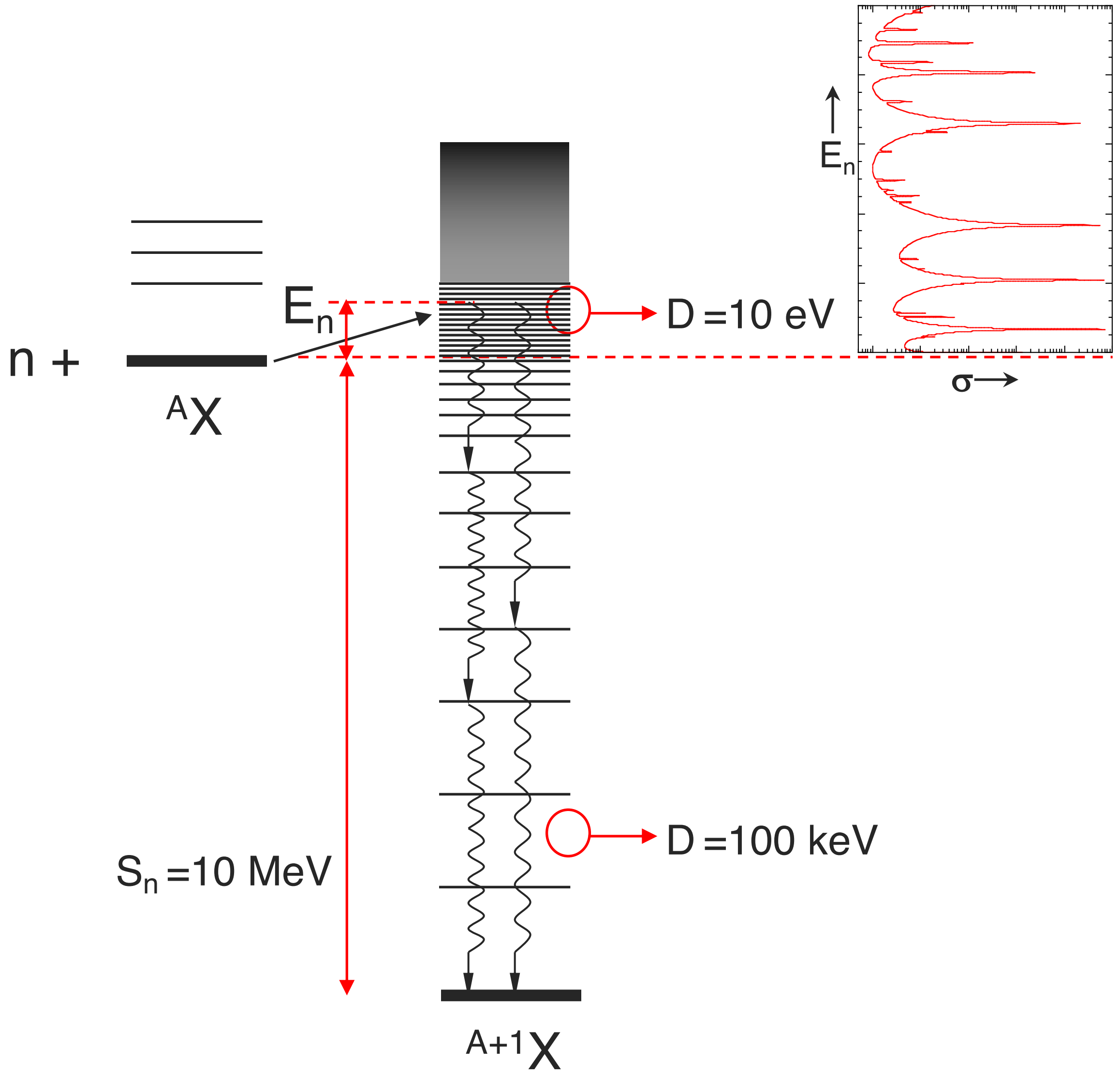


Compound neutron-nucleus reactions



Compound neutron-nucleus reactions





R-matrix formalism

partial incoming wave functions: $\mathcal{I}_c$

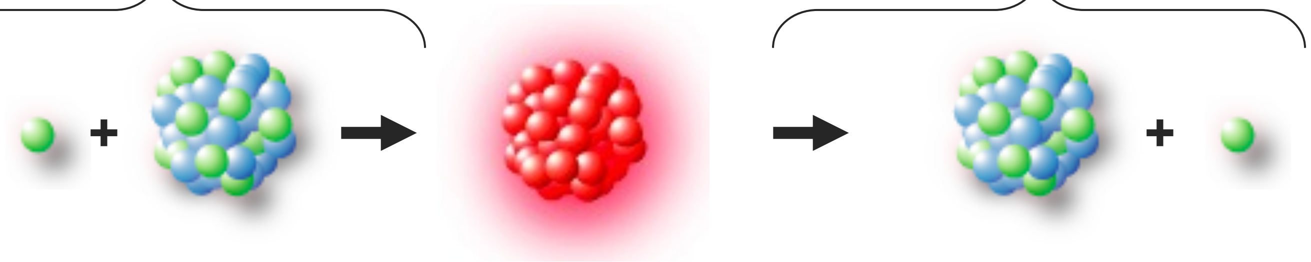
partial outgoing wave functions: $\mathcal{O}_{c'}$

related by collision matrix: $U_{cc'}$

cross section:

$$\sigma_{cc'} = \pi \lambda_c^2 |\delta_{c'c} - U_{c'c}|^2$$

External region ($r>a_c$, well separated particles):
• no interaction, Schrödinger equation solvable.



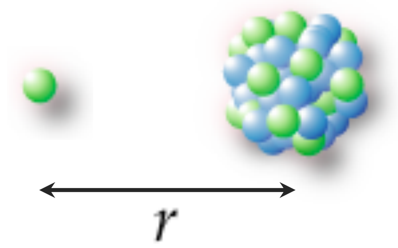
entrance channel
$c = \{\alpha, \ell, j, J, m_J\}$

compound nucleus

exit channel
$c' = \{\alpha', \ell', j', J', m_J'\}$

Internal region ($r<a_c$ compound nucleus):
• wave function is expansion of eigenstates λ.

R-matrix formalism



$r > a_c$ external region

$r < a_c$ internal region

$r = a_c$ match value and derivate of Ψ

$$\left[\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - \frac{2m_c}{\hbar^2}(V-E)\right] rR(r) = 0$$

External region: **easy**, solve Schrödinger equation
central force, separate radial and angular parts.

$$\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$$

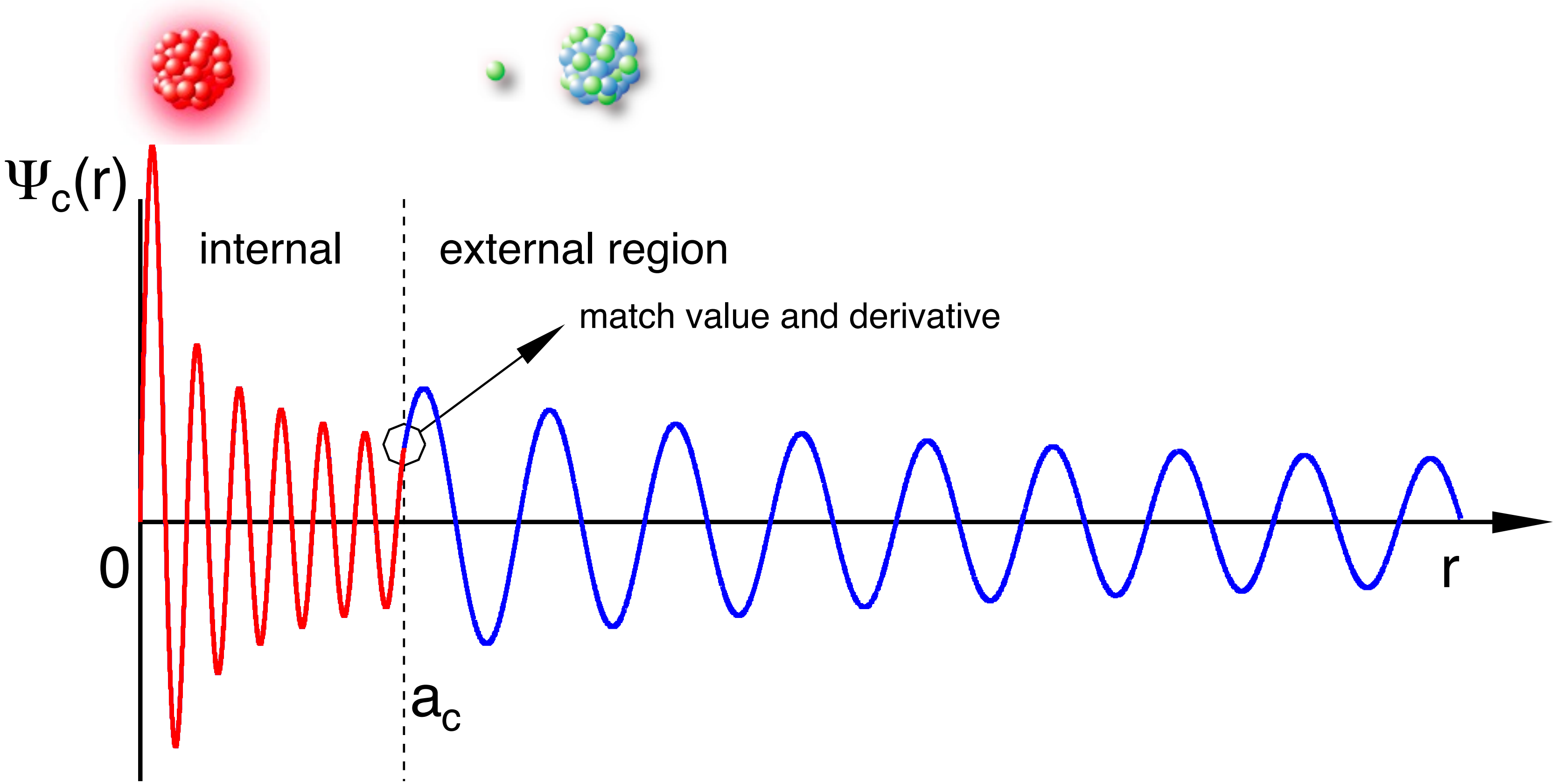
solution: solve Schrödinger equation of relative motion:

- Coulomb functions
- special case of neutron particles (neutrons): Bessel functions

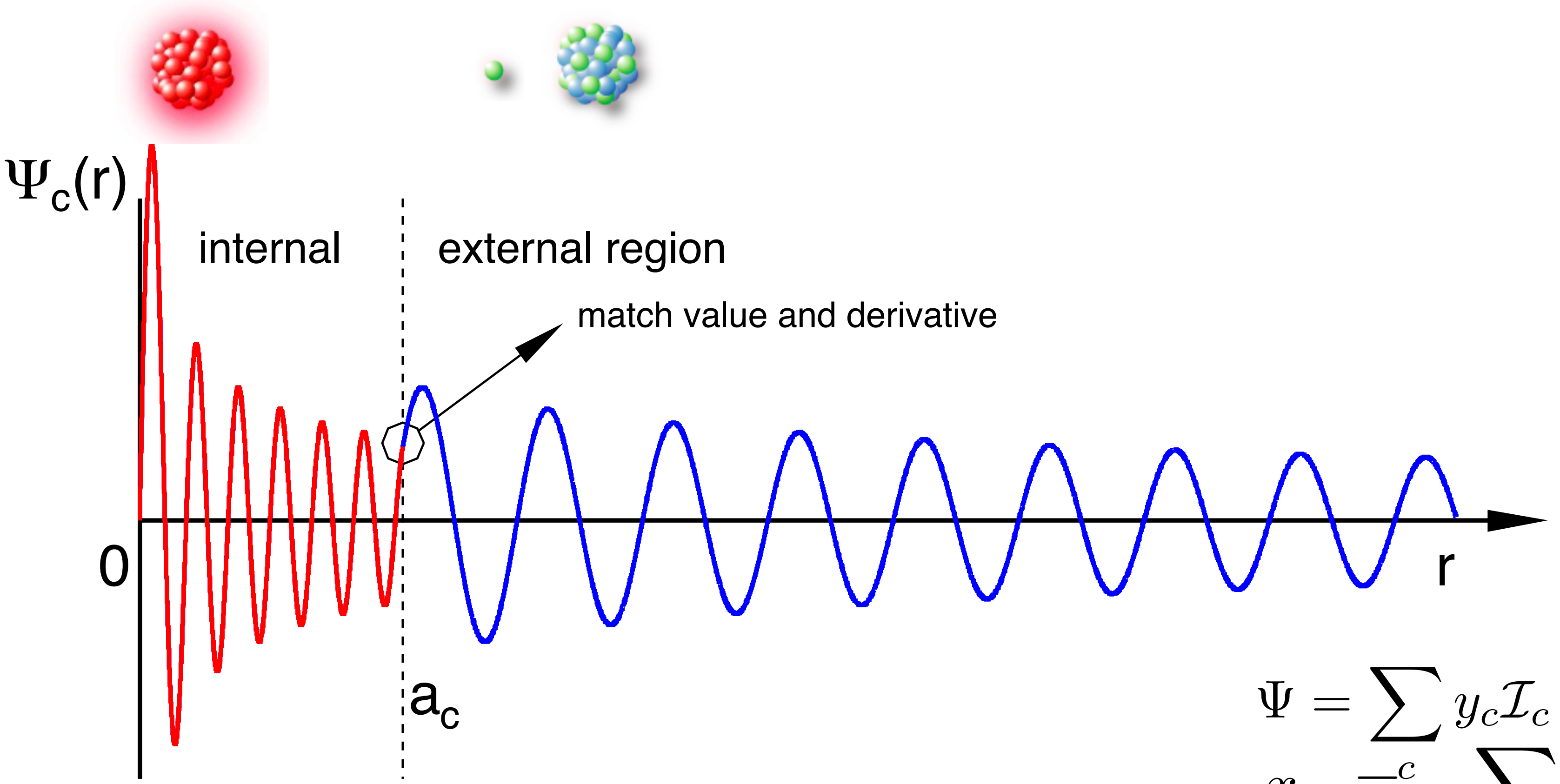
Internal region: **very difficult**, Schrödinger equation cannot be solved directly
solution: expand the wave function as a linear combination of its eigenstates.
using the R-matrix:

$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

R-matrix formalism



R-matrix formalism



$$\Psi = \Psi(R_{cc'})$$

$$R_{cc'} = \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

$$\Psi = \sum y_c \mathcal{I}_c + \sum x_{c'} \mathcal{O}'_c$$

$$x_{c'} \equiv^{c} - \sum U_{c'c}^{c'} y_c$$

$$\mathcal{I}_c = I_c r^{-1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta,\phi)/\sqrt{v_c}$$

$$\mathcal{O}_c = O_c r^{-1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta,\phi)/\sqrt{v_c}$$

R-matrix formalism

- The wave function of the system is a superposition of incoming and outgoing waves:

$$\Psi = \sum_c y_c \mathcal{I}_c + \sum_{c'} x_{c'} \mathcal{O}'_c$$

- Incoming and outgoing wavefunctions have form:

$$\mathcal{I}_c = I_c r^{-1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta, \phi)/\sqrt{v_c}$$

$$\mathcal{O}_c = O_c r^{-1} \varphi_c i^\ell Y^\ell_{m_\ell}(\theta, \phi)/\sqrt{v_c}$$

- The physical interaction is included in the collision matrix **U**:

$$x_{c'} \equiv -\sum_c U_{c'c} y_c$$

- The wave function depends on the R-matrix, which depends on the widths and levels of the eigenstates.

$$\Psi = \Psi(R_{cc'})$$

$$R_{cc'} = \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}$$

R-matrix formalism

- **The relation between the R-matrix and the collision matrix:**

$$\mathbf{U} = \mathbf{\Omega} \mathbf{P}^{1/2} [\mathbf{1} - \mathbf{R}(\mathbf{L} - \mathbf{B})]^{-1} [\mathbf{1} - \mathbf{R}(\mathbf{L}^* - \mathbf{B})] \mathbf{P}^{-1/2} \mathbf{\Omega}$$

with: $$L_c = S_c + iP_c = \left(\frac{\rho}{O_c} \frac{dO_c}{d\rho} \right)_{r=a_c}$$

- **The relation between the collision matrix and cross sections:**

channel to one other channel: $$\sigma_{cc'} = \pi \lambda\!\!\!^{-}_c{}^2 |\delta_{c'c} - U_{c'c}|^2$$

channel to any other channel: $$\sigma_{cr} = \pi \lambda\!\!\!^{-}_c{}^2 (1 - |U_{cc}|^2)$$

channel to same channel: $$\sigma_{cc} = \pi \lambda\!\!\!^{-}_c{}^2 |1 - U_{cc}|^2$$

channel to any channel (total): $$\sigma_{c,T} = \sigma_c = 2\pi \lambda\!\!\!^{-}_c{}^2 (1 - \mathrm{Re}\, U_{cc})$$

R-matrix formalism

The Breit-Wigner Single Level approximation:
total cross section:

$$\sigma_c = \pi \lambda\!\!\!^{-2}_c g_c \left(4\sin^2\phi_c + \frac{\Gamma_\lambda \Gamma_{\lambda c} \cos 2\phi_c + 2(E - E_\lambda - \Delta_\lambda)\Gamma_{\lambda c} \sin 2\phi_c}{(E - E_\lambda - \Delta_\lambda)^2 + \Gamma_\lambda^2/4} \right)$$

R-matrix formalism

The Breit-Wigner Single Level approximation:
total cross section:

$$\sigma_c = \pi \lambda_c^2 g_c \left(4 \sin^2\phi_c + \frac{\Gamma_\lambda \Gamma_{\lambda c} \cos 2\phi_c + 2(E - E_\lambda - \Delta_\lambda)\Gamma_{\lambda c} \sin 2\phi_c}{(E - E_\lambda - \Delta_\lambda)^2 + \Gamma_\lambda^2/4} \right)$$

neutron channel: $c = n$
only capture, scattering, fission: $\Gamma_\lambda = \Gamma = \Gamma_n + \Gamma_\gamma + \Gamma_f$
other approximations: $\ell = 0$ $\cos\phi_c = 1$ $\sin\phi_c = \rho = ka_c$ $\Delta_\lambda = 0$

total cross section:

$$\sigma_T(E) = \underbrace{4\pi R'^2}_{\text{potential}} + \pi \lambda^2 g \frac{\underbrace{4\Gamma_n (E - E_0) R'/\lambda}_{\text{interference}} + \underbrace{\Gamma_n^2}_{\text{elastic}} + \underbrace{\Gamma_n \Gamma_\gamma}_{\text{capture}} + \underbrace{\Gamma_n \Gamma_f}_{\text{fission}}}{(E - E_0)^2 + (\underbrace{\Gamma_n + \Gamma_\gamma + \Gamma_f}_{\text{total width}})^2/4}$$

Average cross sections

- From optical model calculations one can calculate $\overline{U_{cc}}$ but not $\overline{|U_{cc}|^2}$

- Therefore, only $\overline{\sigma_{c,T}}$, $\overline{\sigma_{cc}^{\mathrm{se}}}$, $\overline{\sigma_c}$ can be calculated, of which only the total average cross section can be compared with measurements.

- In OMP one uses **transmission coefficients** $T_c = 1 - |\overline{U_{cc}}|^2$

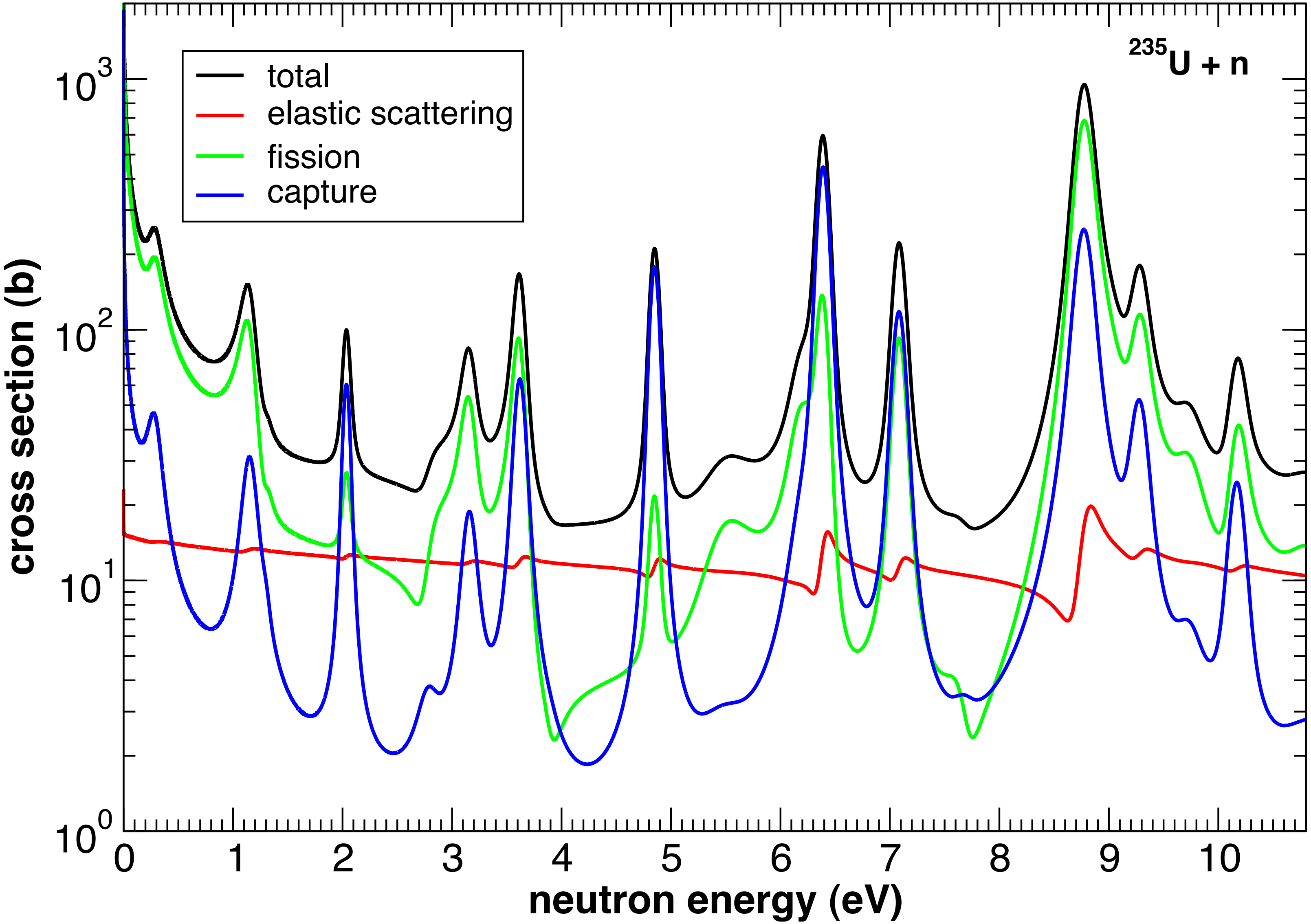
- Average single reaction cross section (Hauser-Feshbach):

$$\overline{\sigma_{cc'}} = \overline{\sigma_{cc}^{\mathrm{se}}}\delta_{cc'} + \pi\lambda_c^2 g_c \frac{T_c T_{c'}}{\Sigma T_i} W_{cc'}$$

- Average single reaction cross section (Hauser-Feshbach):

$$W_{cc'} = \overline{\left(\frac{\Gamma_c \Gamma_{c'}}{\Gamma}\right)} \frac{\overline{\Gamma}}{\overline{\Gamma_c}\,\overline{\Gamma_{c'}}}$$

Cross sections σ_T, σ_γ, σ_n and σ_f



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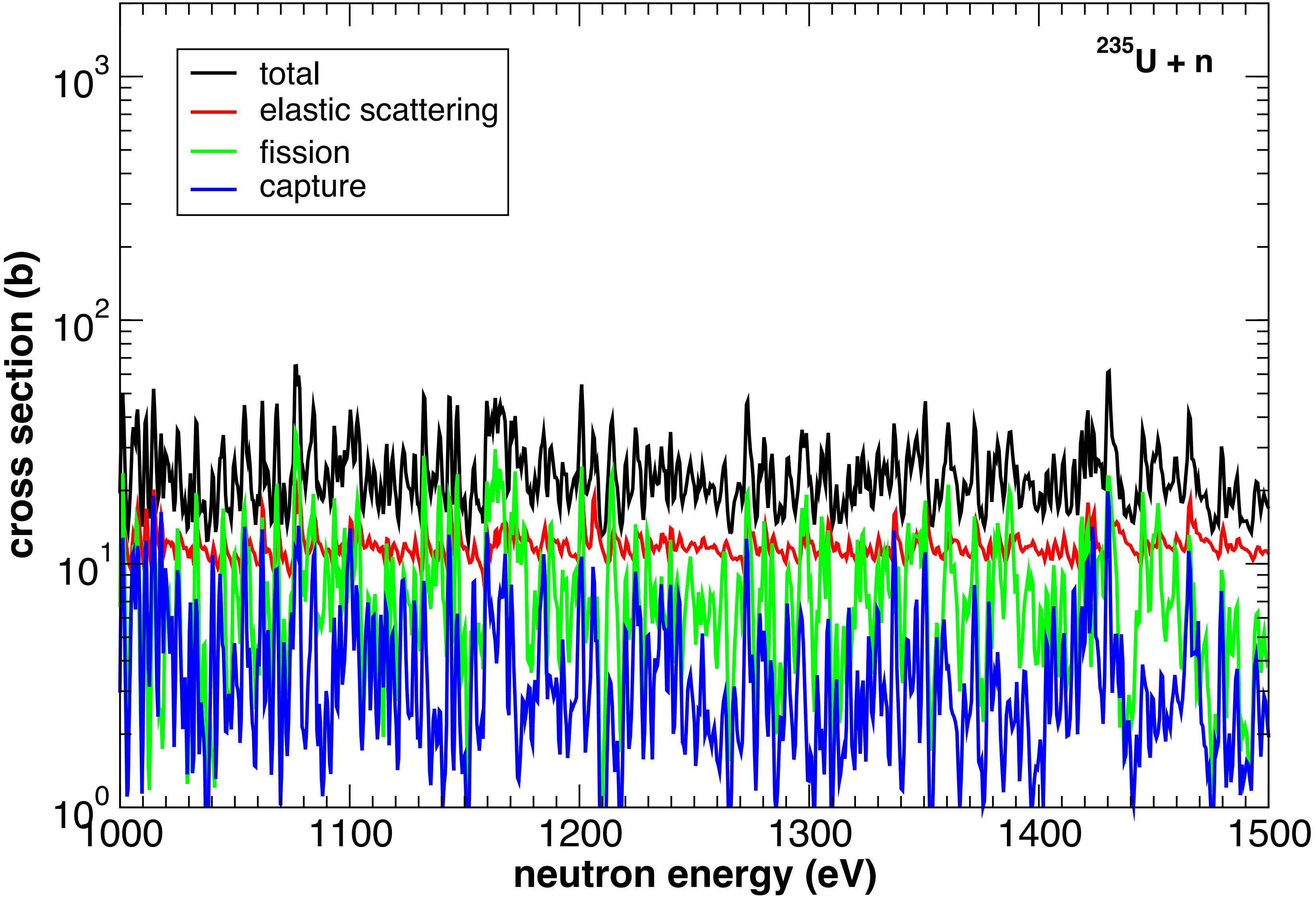

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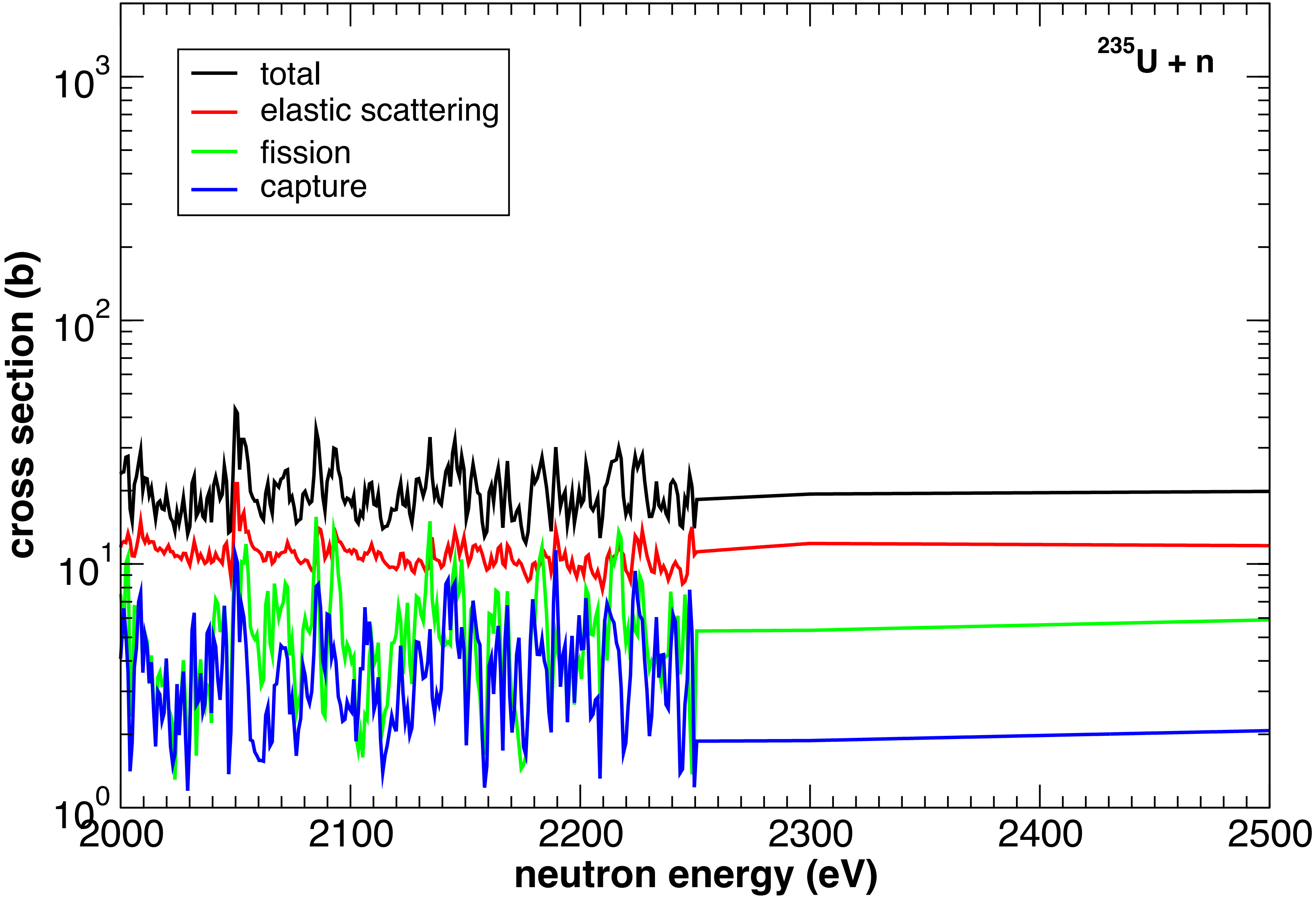
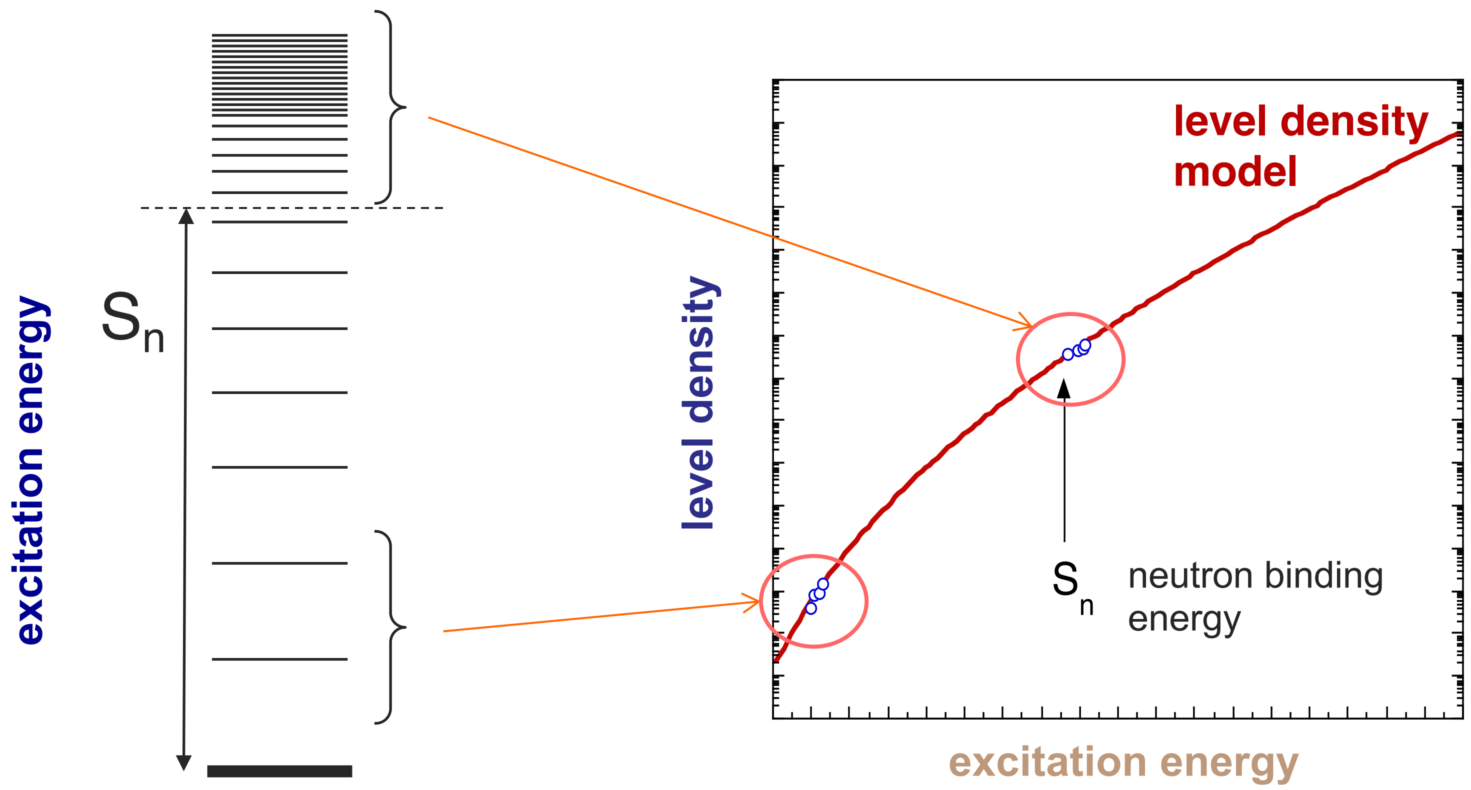


Nuclear level densities: level spacing D_0



low-lying levels:
Count levels, all J^π

neutron resonances:
Count levels, selected J^π,
extract $\mathbf{D_0}$

- **All level density models reproduce the low-lying levels and D_0 at S_n**

Nuclear level densities

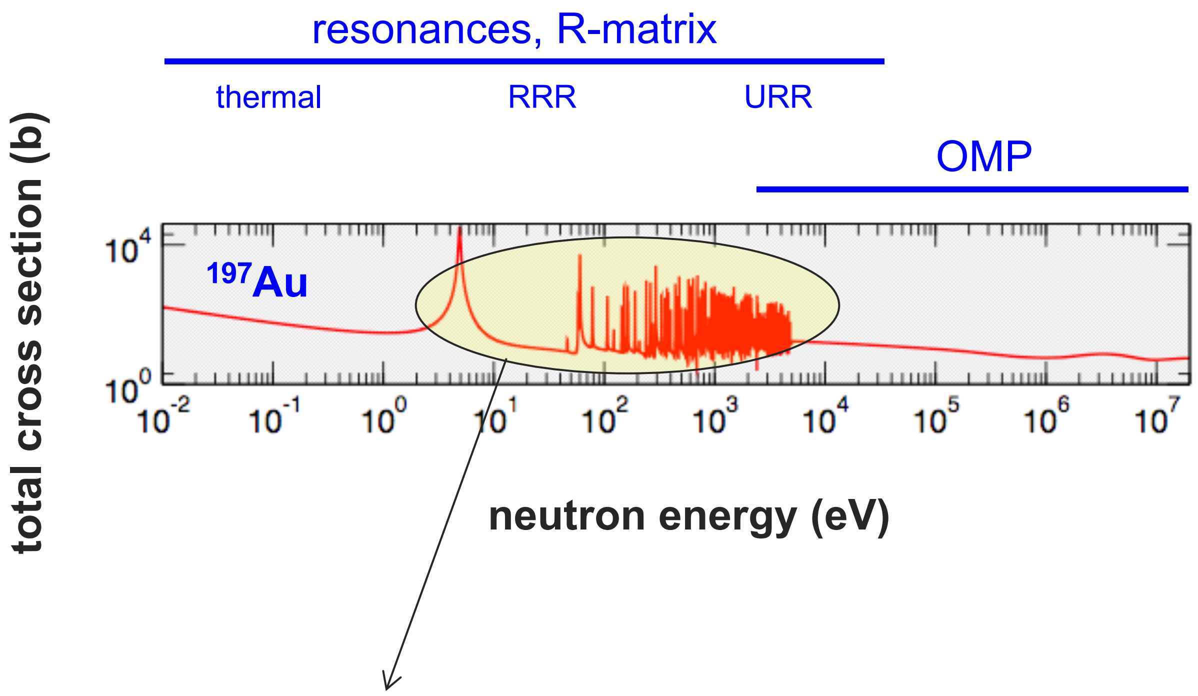
- The level spacing D_0 at the neutron binding energy is a crucial input parameter for calibrating level density models. Level density: $\rho = 1/D$.

- D_0 is the spacing between levels excited by neutrons on nuclei bringing in zero orbital momentum (s-wave resonances).

- Spacings from higher orbital momentum are equally important, but in general much more affected by missing levels.

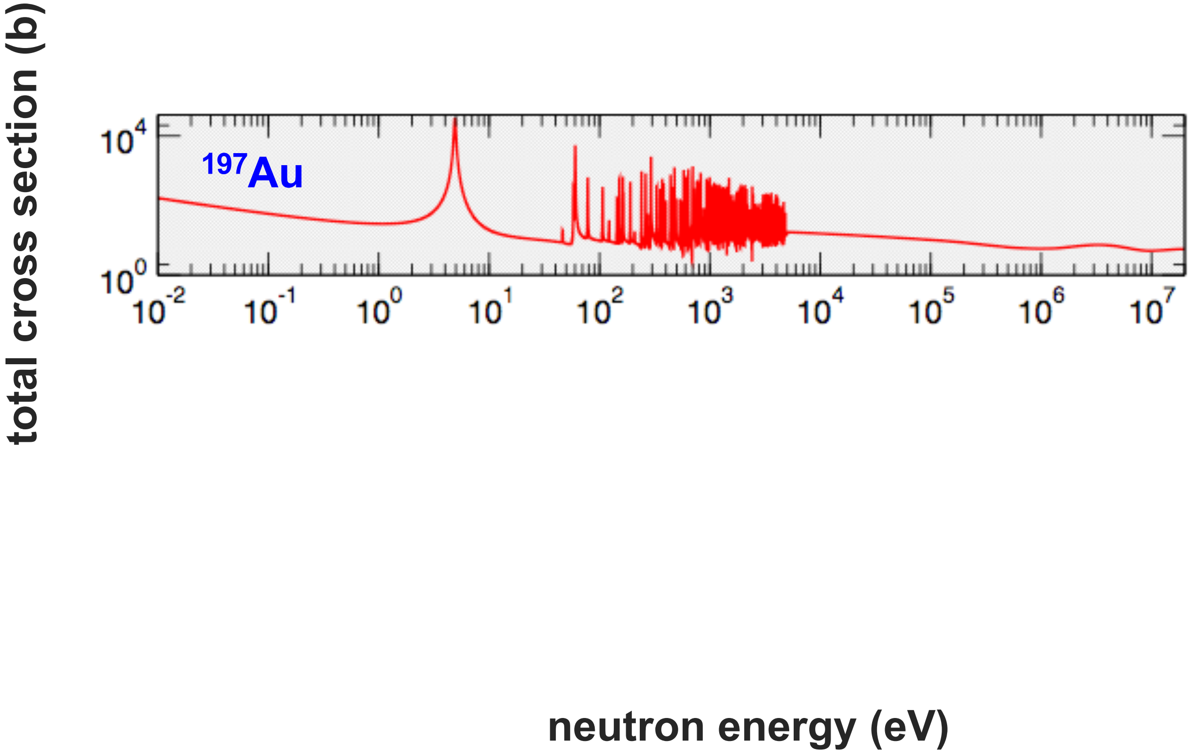
- Problems concerning the determination of D_0:
  - spin and parity assignment of levels
  - corrections for missing levels (those which are not observed experimentally)

Nuclear level densities

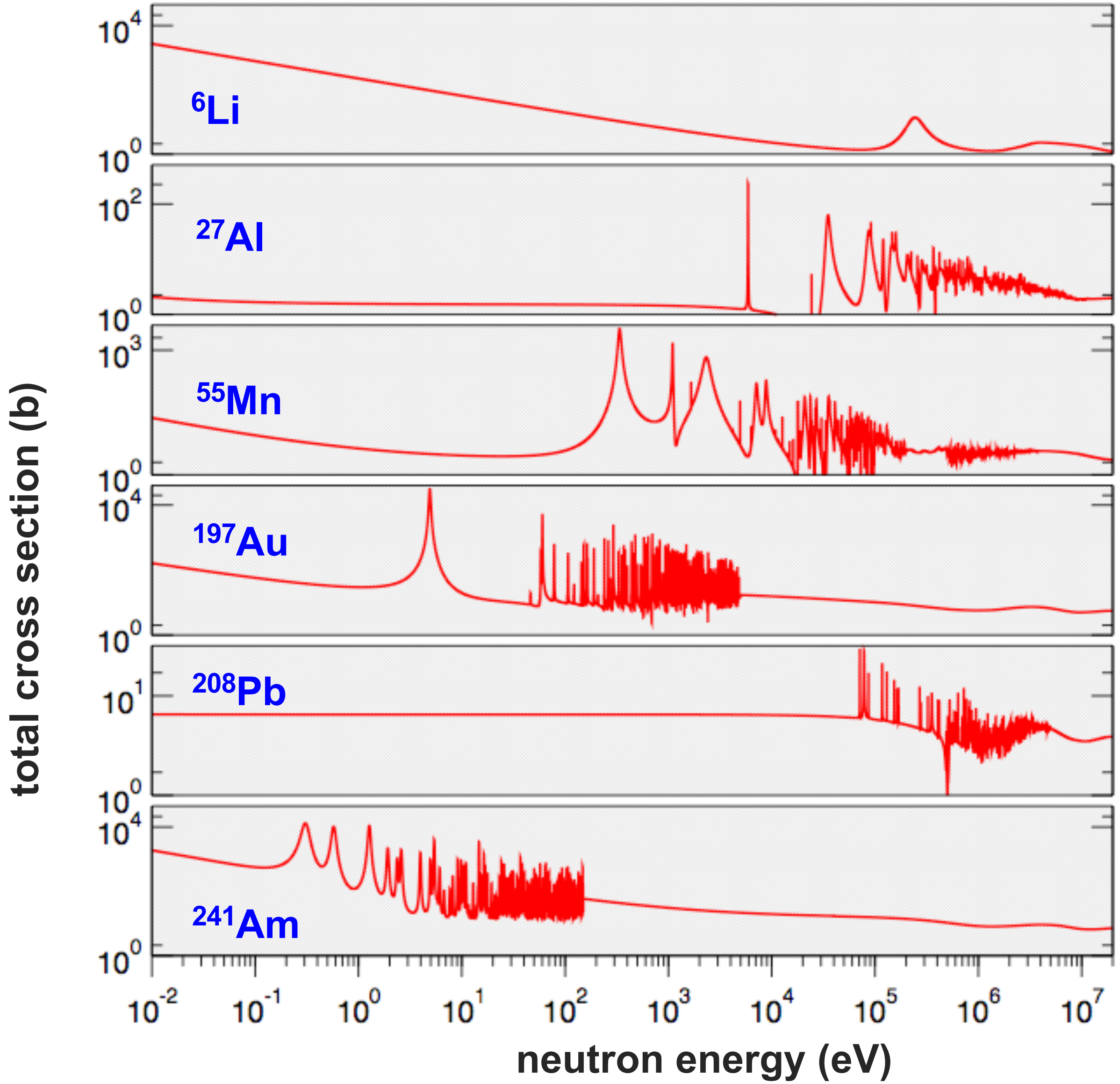


Count the number of levels in the energy interval → level density

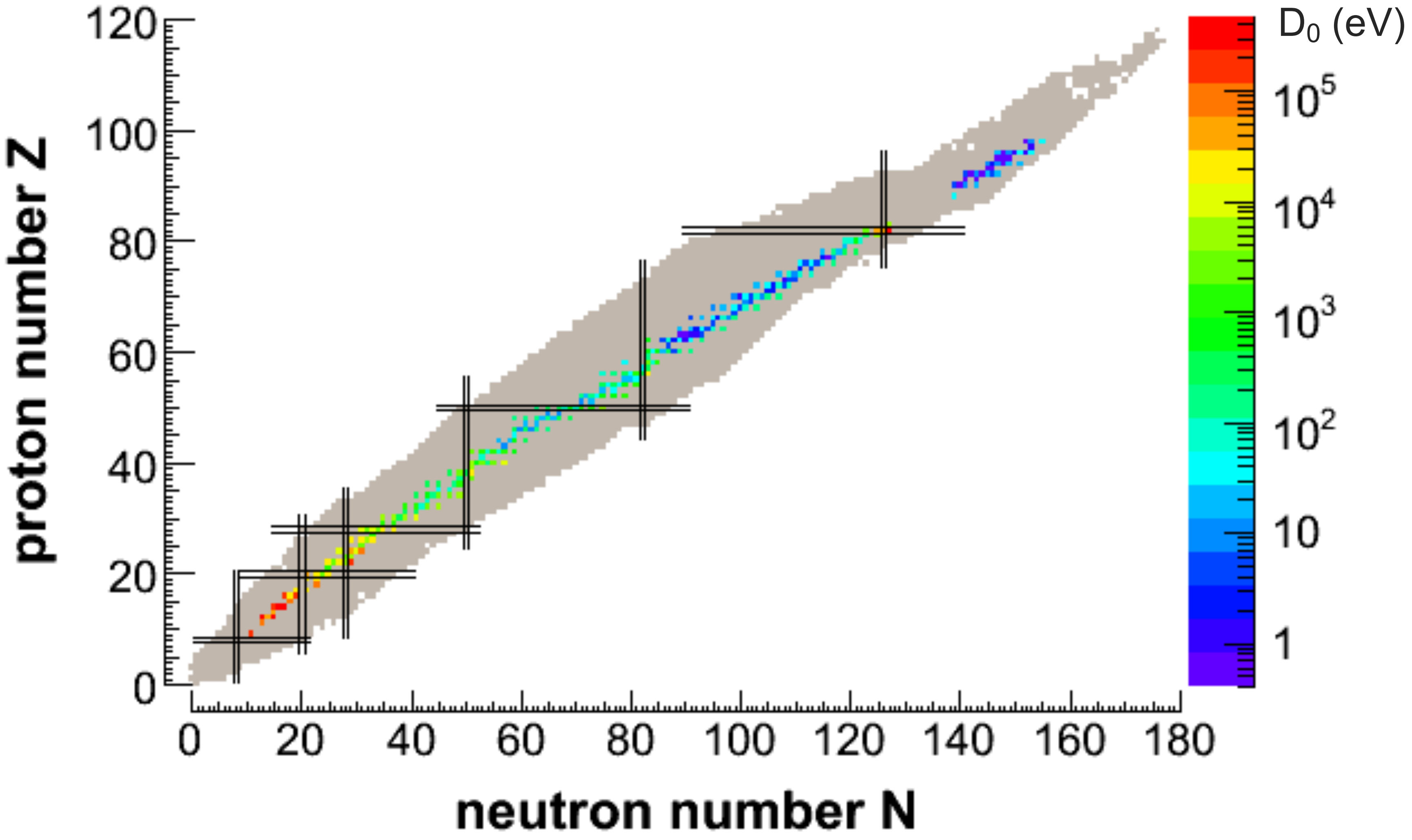
Nuclear level densities



Nuclear level densities



Nuclear level densities: level spacing D_0

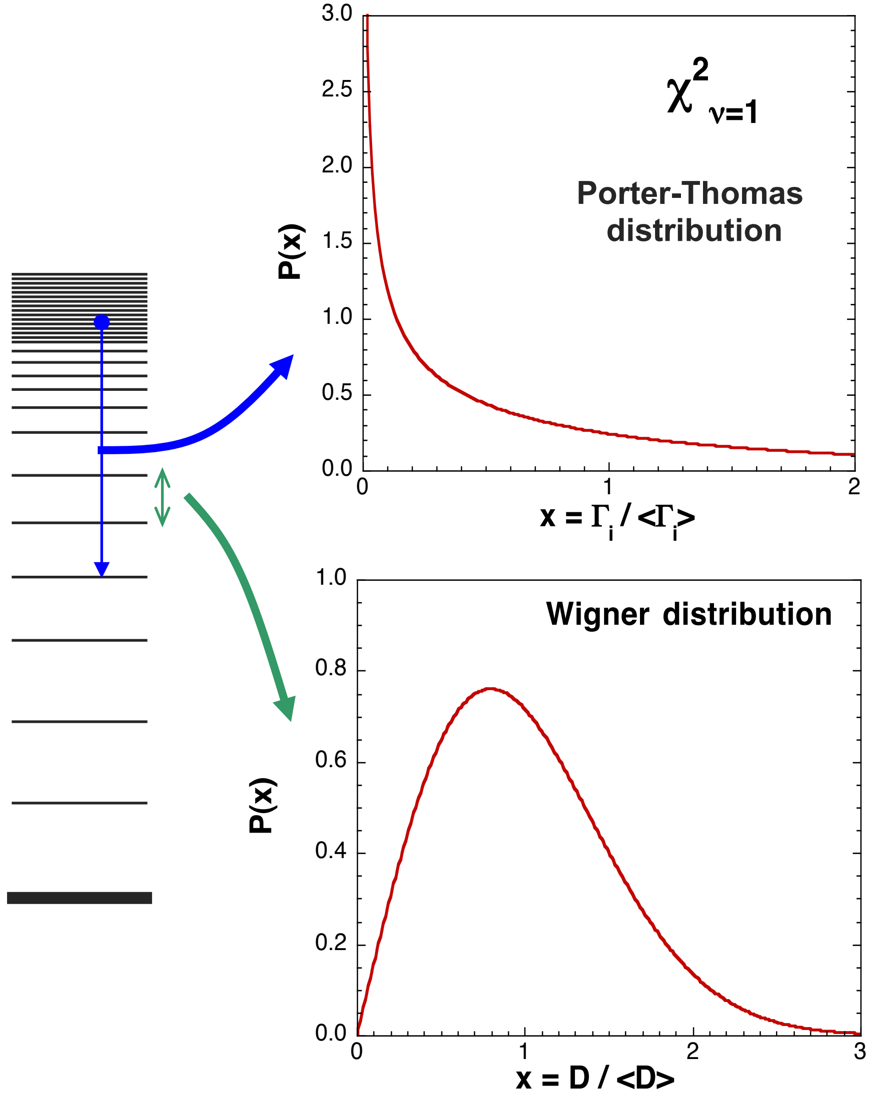


Level statistics from resonances

- The nucleus at energies around S_n can be described by the Gaussian Orthogonal Ensemble (GOE)

- The matrix elements governing the nuclear transitions are random variables with a Gaussian distribution with zero mean.

- **Consequences**:
  - The partial widths have a **Porter-Thomas** distribution.
  - The spacing of levels with the same J^{π} have approximately a **Wigner** distribution.



Level statistics from resonances

For different nuclei, use evaluated data to verify the statistical model.
Use levels of a same family (spin, parity, orbital momentum)

- neutron width distribution:
  Porter-Thomas distribution

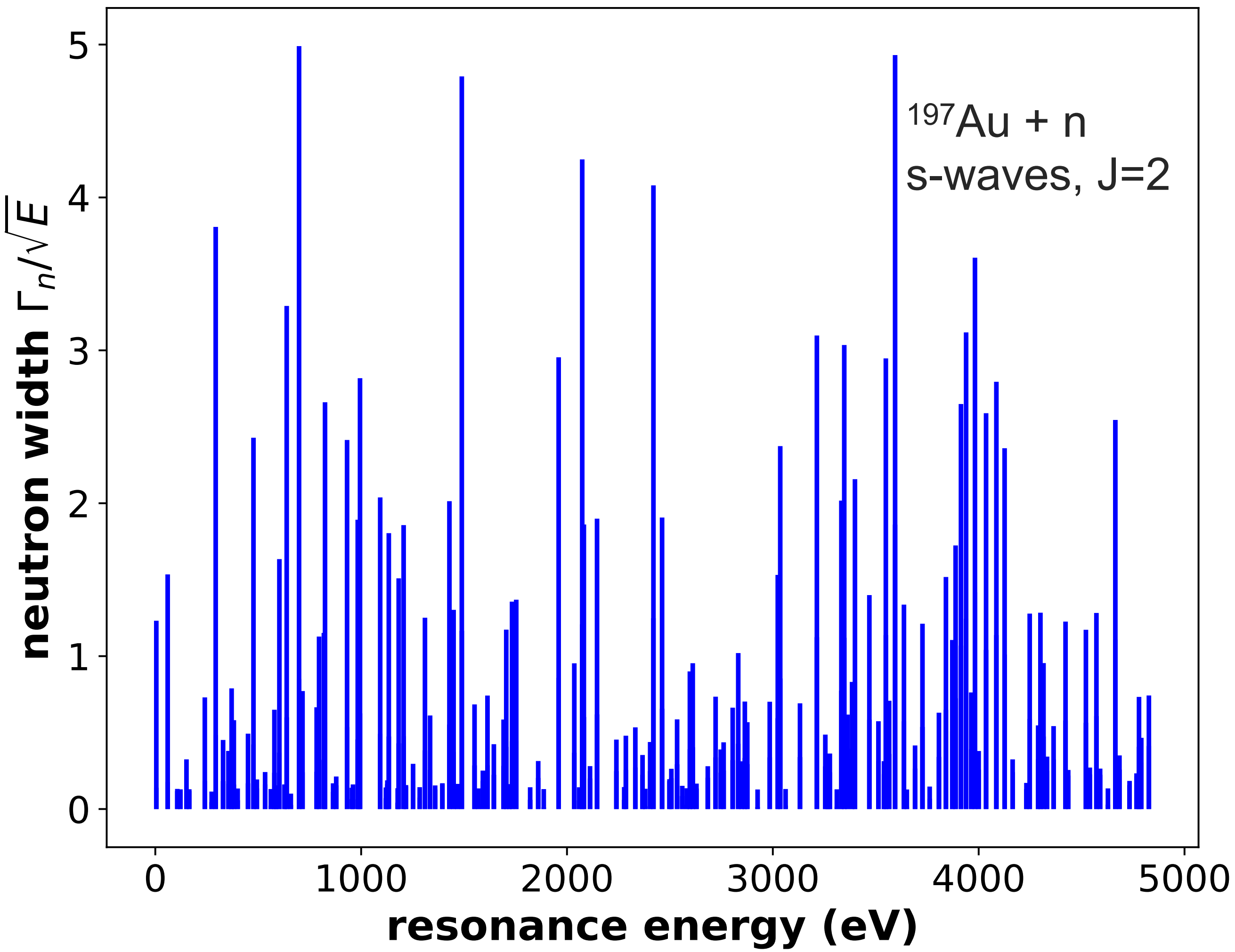
$$x = \gamma_c^2/\langle\gamma_c^2\rangle = \Gamma_c/\langle\Gamma_c\rangle$$

$$P_{\mathrm{PT}}(x) = \frac{1}{\sqrt{2\pi x}}\exp\left(-\frac{x}{2}\right)$$

- next-neighbour spacing:
  Wigner distribution

$$x = D/\langle D\rangle$$

$$P_{\mathrm{W}}(x) = \frac{\pi}{2}x\exp\left(-\frac{\pi}{4}x^2\right)$$



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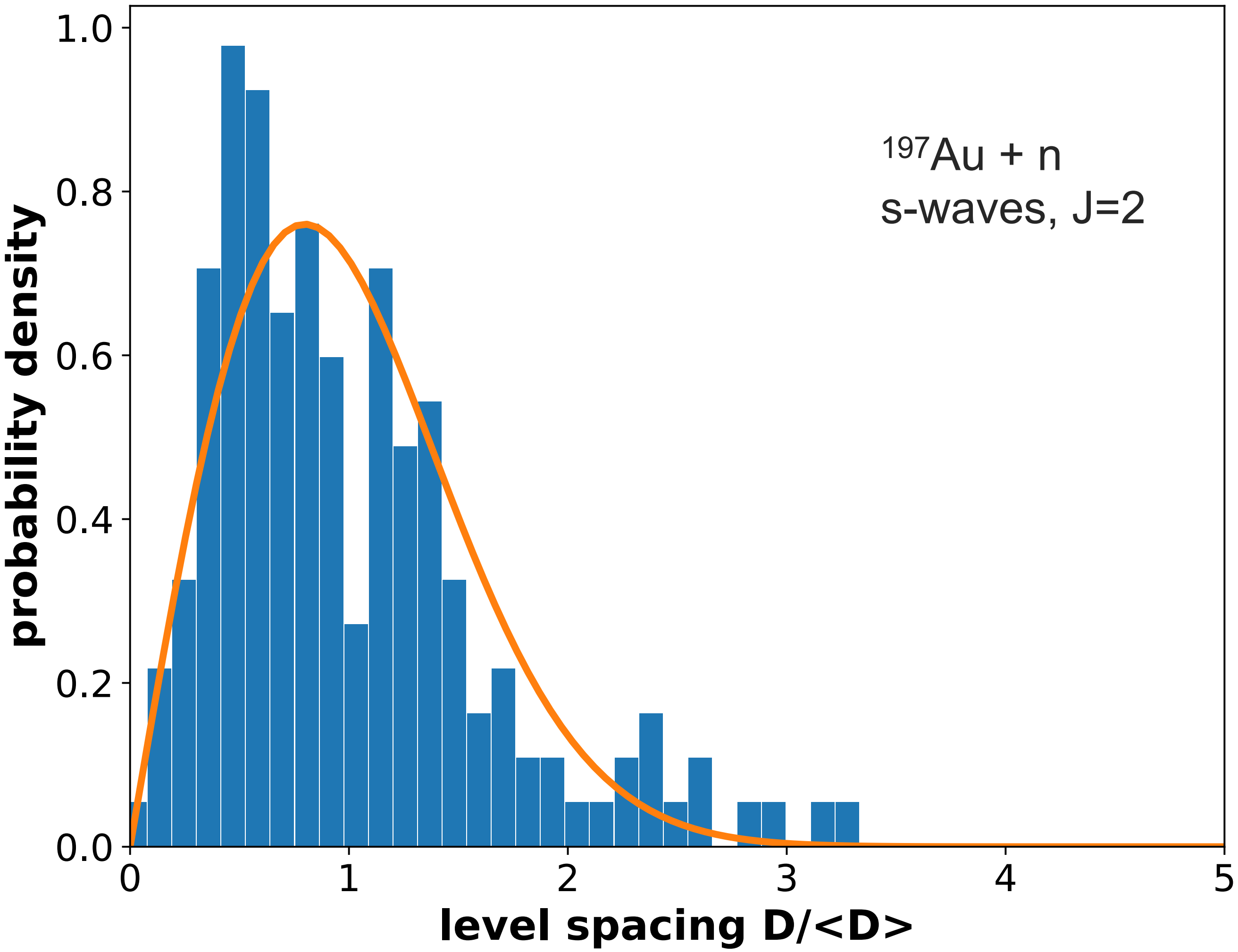

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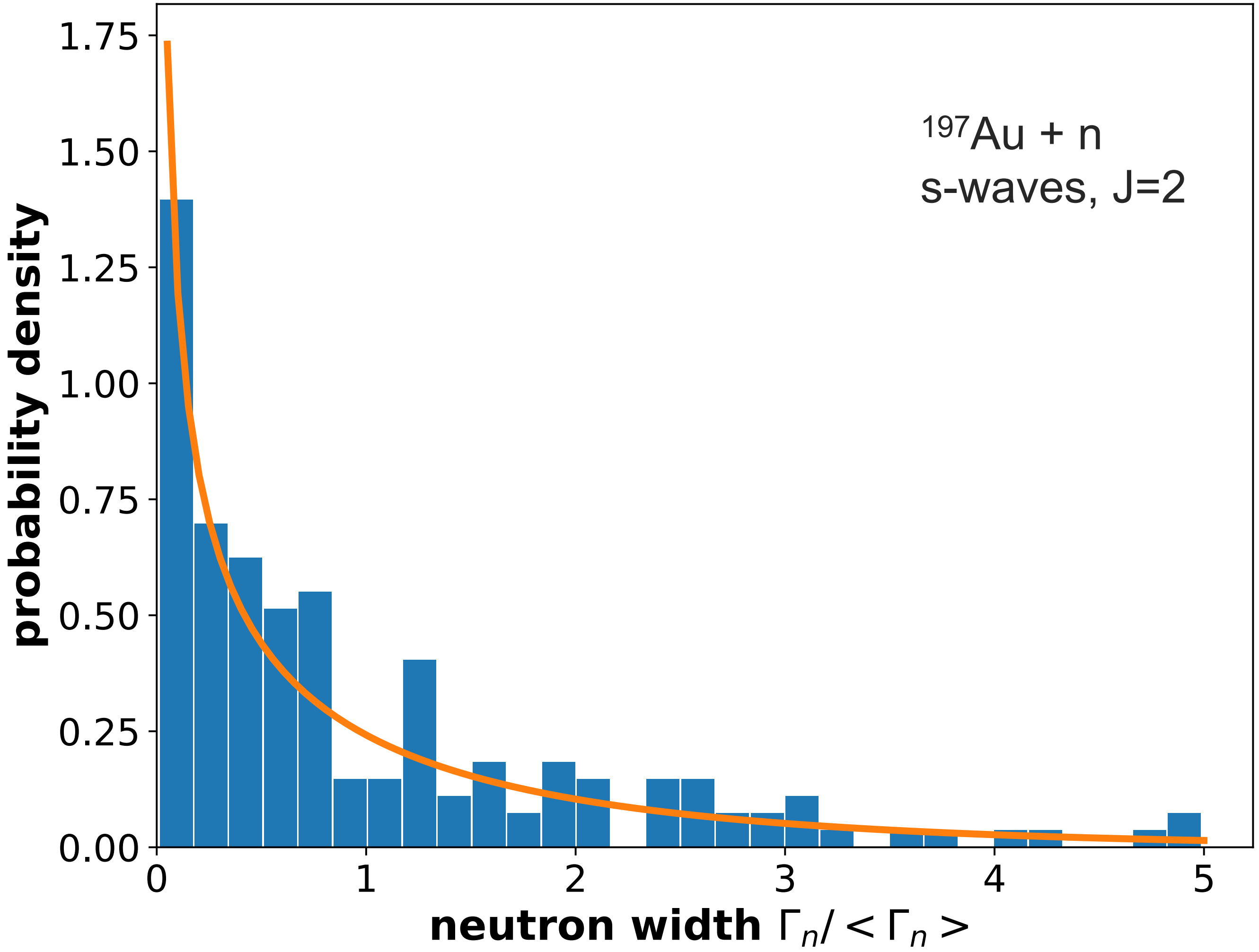

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  Wigner distribution

$$x = D/\langle D\rangle$$

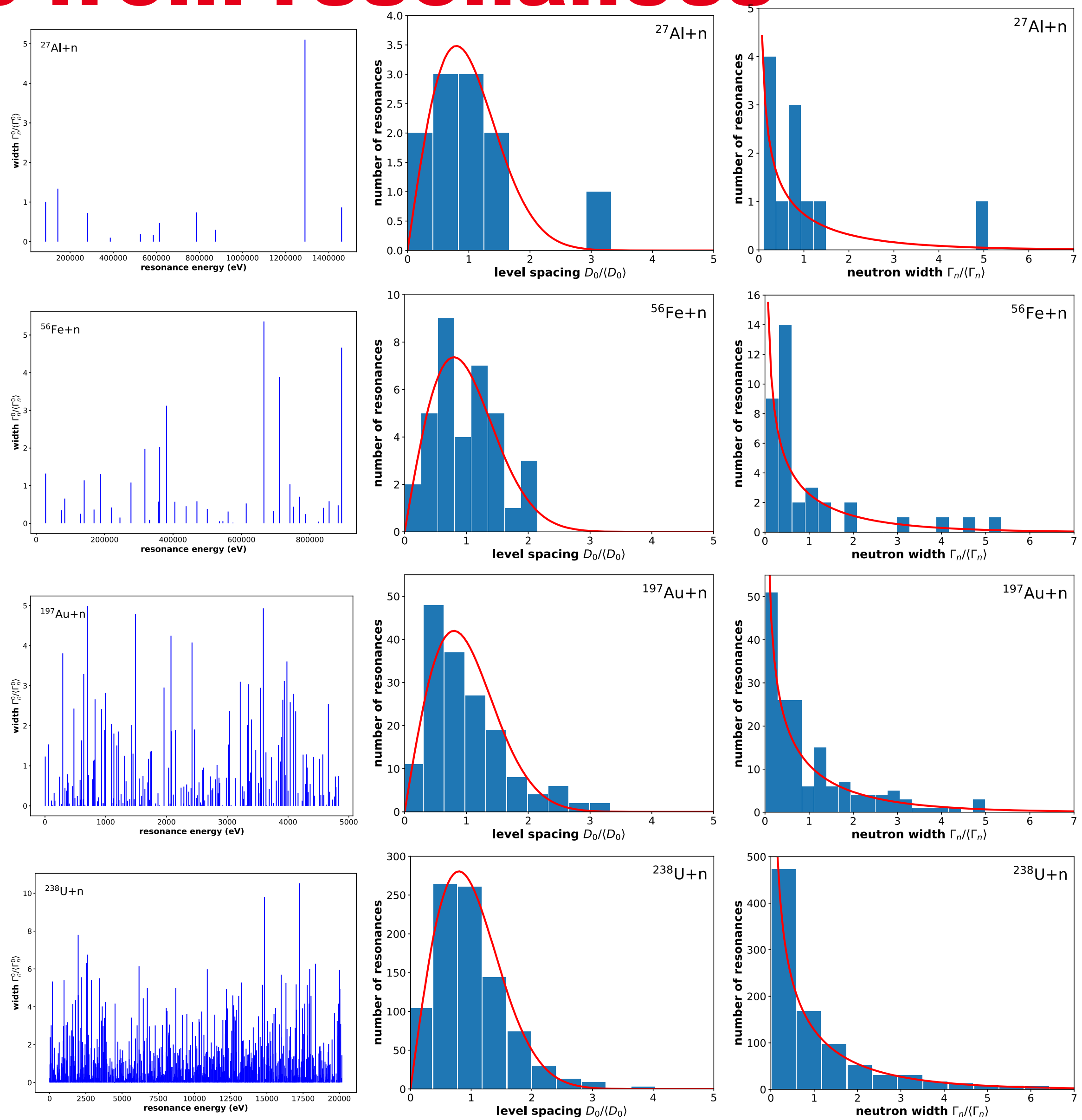
$$P_{\mathrm{W}}(x) = \frac{\pi}{2}x\exp\left(-\frac{\pi}{4}x^2\right)$$

^{27}Al + n

^{56}Fe + n

^{197}Au + n

^{238}U + n



Further reading

Books/articles

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Nuclear data sites

www.oecd-nea.org
nds.iaea.org
www.nndc.bnl.gov
www.cern.ch/ntof
jrc.ec.europa.eu/geel

Nuclear data codes

TALYS
SAMMY
REFIT