Nuclear structure of exotic nuclei

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n-TOF Nuclear Physics Winter School 2024

Outline

1. **Some general features: binding energies, Sn, Sp…**

2. **What's an exotic nucleus from a theoretical point of view ?**

- **Mean field calculations of nuclei far from stability,**
- **Shell closure evolution with N-Z,**
- **Charge radii staggering.**

2. **A mean field approach for excited/vibrational states:**

- **the QRPA approach,**
- **Neutron skin and pygmy states ; astrophysical interest.**

3. **Beta decay:**

- **the QRPA approach with charge exchange.**

1. Some general features:

binding energies, Sn, Sp...

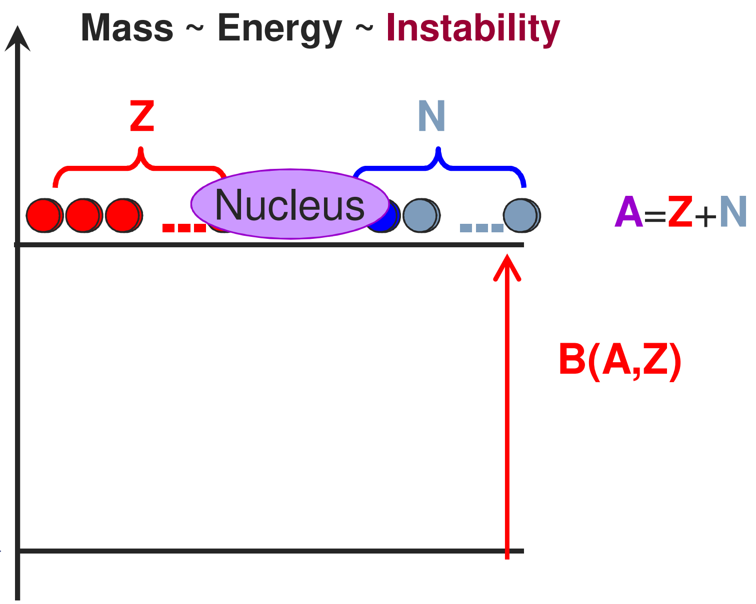
Nucleus features: binding energy

The nucleus is a linked system, so the sum of the masses of the nucleons is different from the mass of the nucleus.

Free nucleons≈ A× 1000 MeV

B(A,Z) ≈ A× 8 MeV

nucleus : M(A,Z) ≈ A× (1000 - 8) MeV

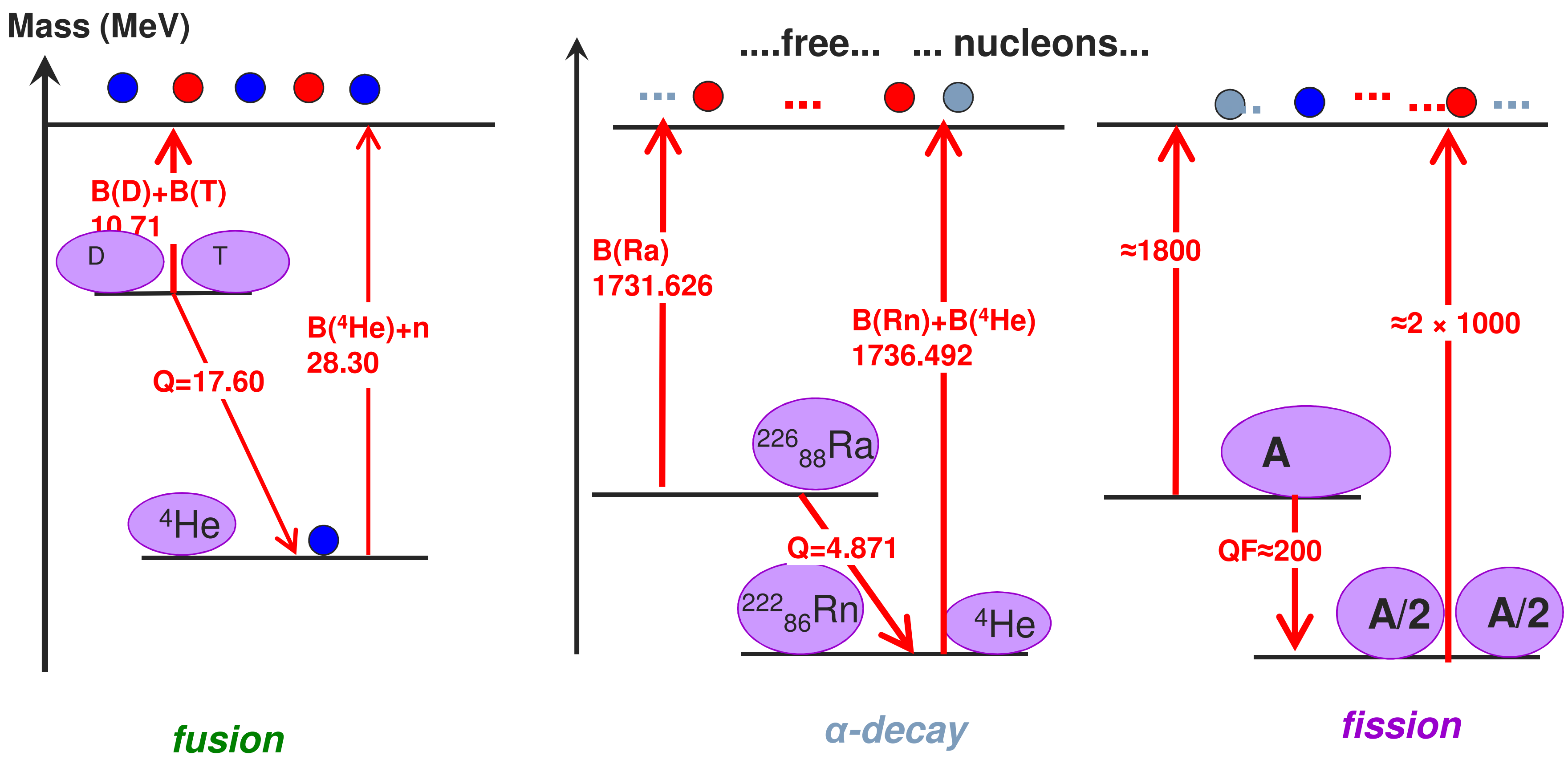


$M(A,Z) = N\,M_n + Z\,M_p - B(A,Z)$

Note : M(A,Z)=A (1uma) + Δm Δm=« mass exces »

Δm= 0 for $^{12}_{6}C$ => 1 uma = 931,500 MeV

Nucleus features: stability versus instability



Nucleus features: separation energies

$M(A,Z) = N\,M_n + Z\,M_p - B(A,Z)$

Separation energy

For a neutron:

$Sn = M(A-1,N-1,Z) + m_n - M(A,N,Z)$

$Sn = B(A,N,Z) - B(A-1,N-1,Z)$

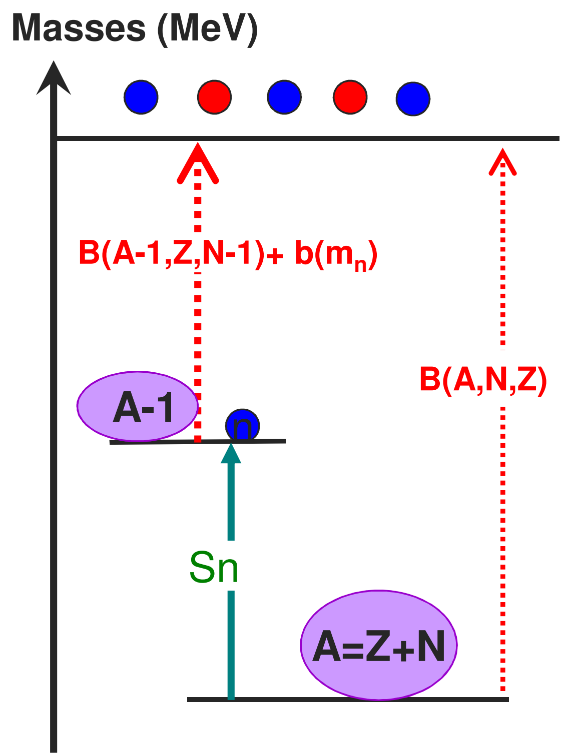
For a proton

$Sp = B(A,N,Z) - B(A-1,N,Z-1)$

α particle

$S\alpha = B(A,N,Z) - [B(A-4,N-2,Z-2) + B(\alpha)]$

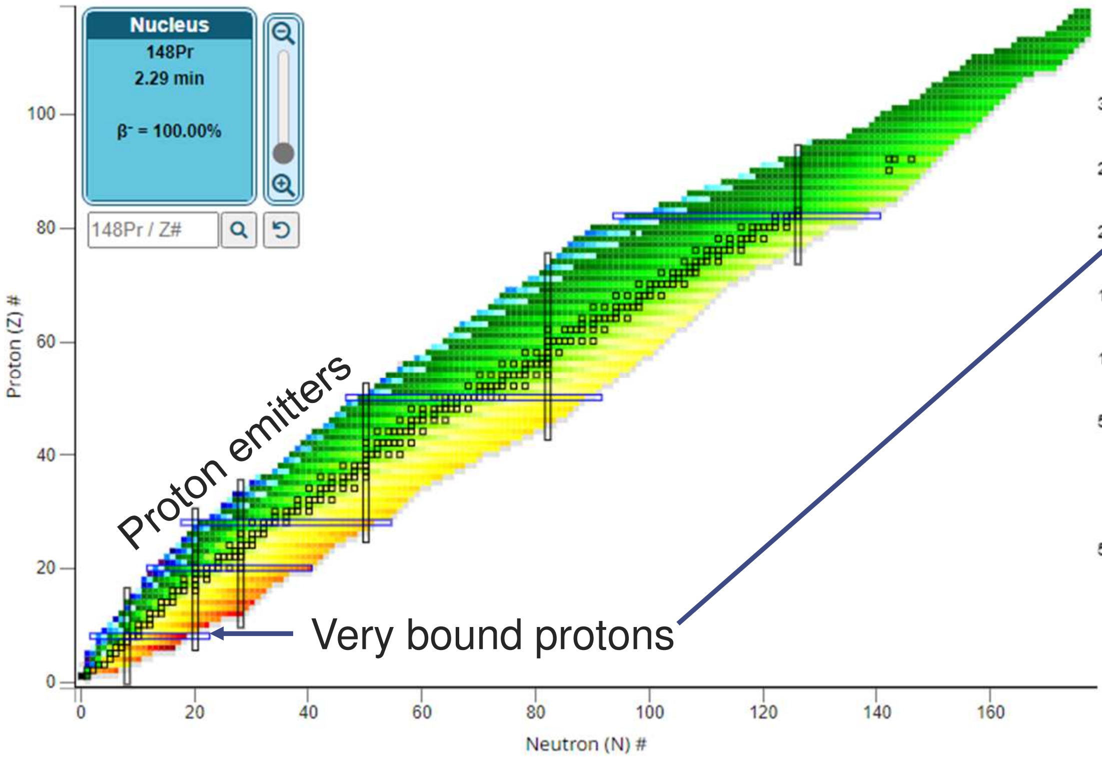
$= -Q\alpha$

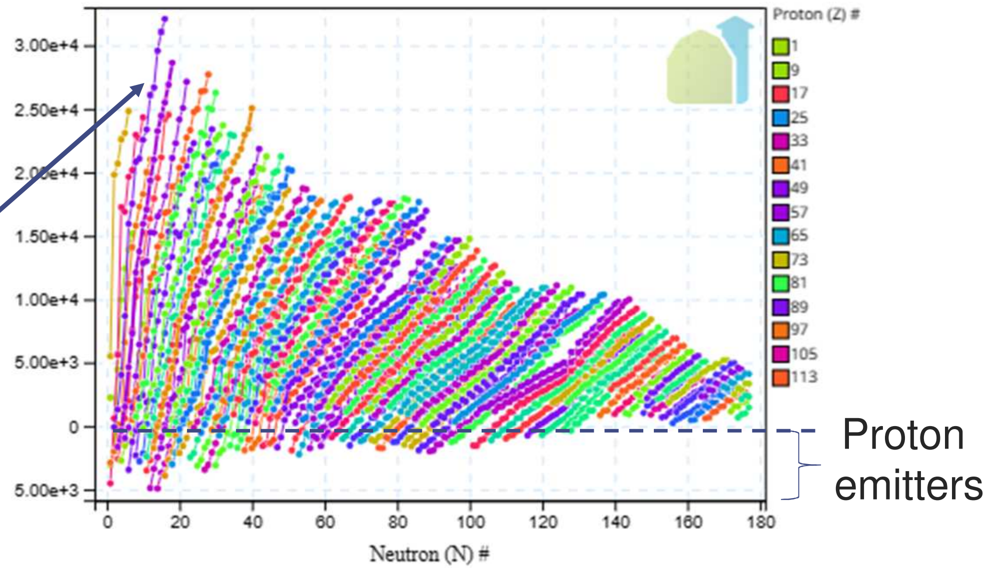


Sn (A) = neutron separation energy in the A nucleus

https://www.nndc.bnl.gov/nudat3/

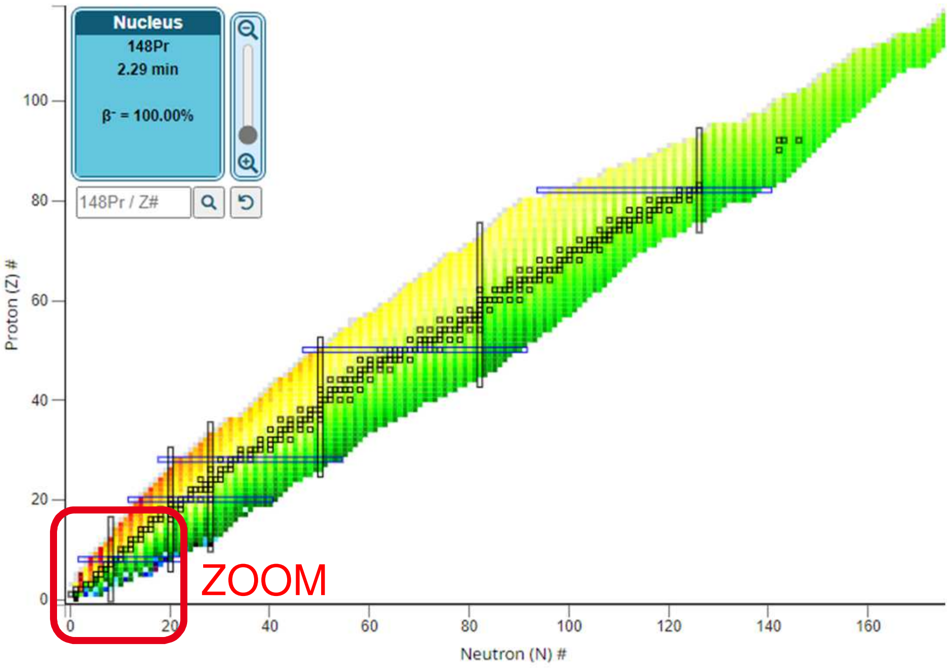
Experimental values of S_p

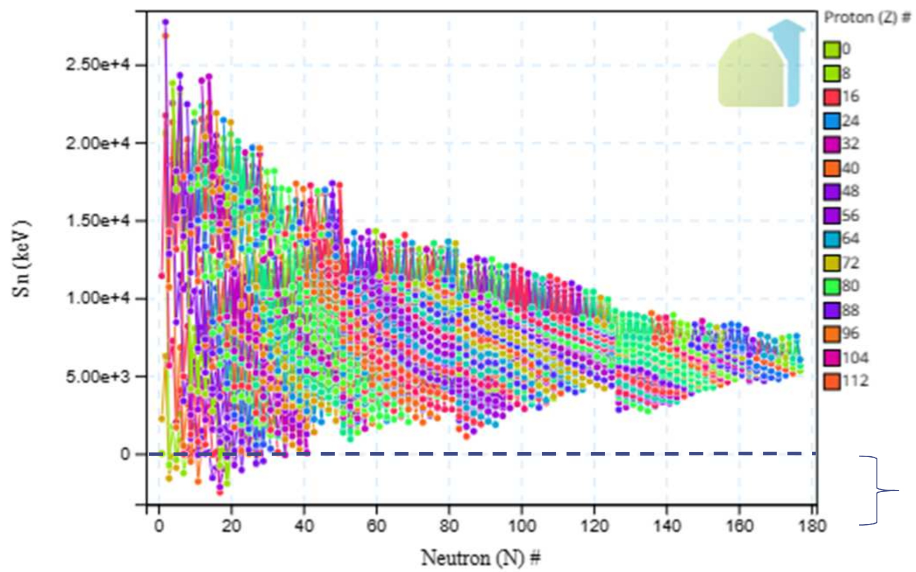




https://www.nndc.bnl.gov/nudat3/

Experimental values of S_n

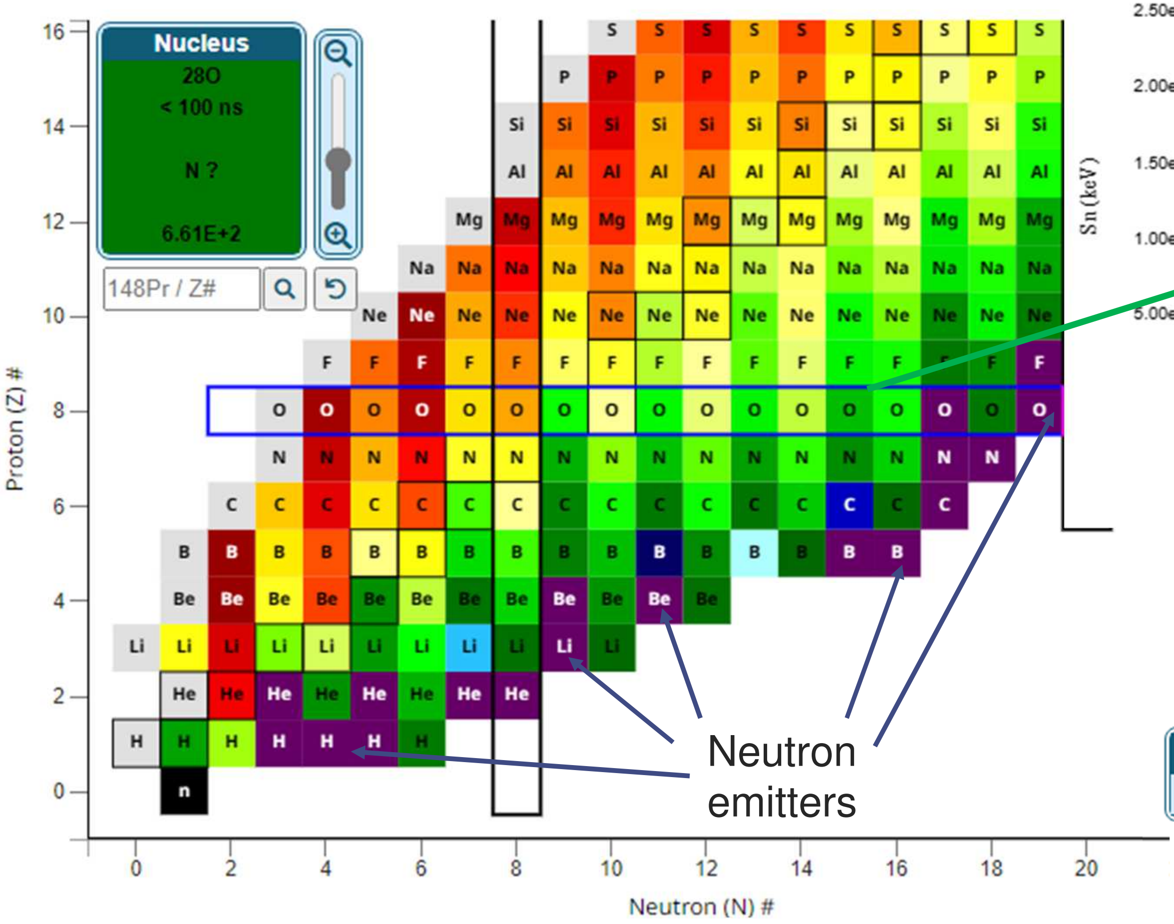


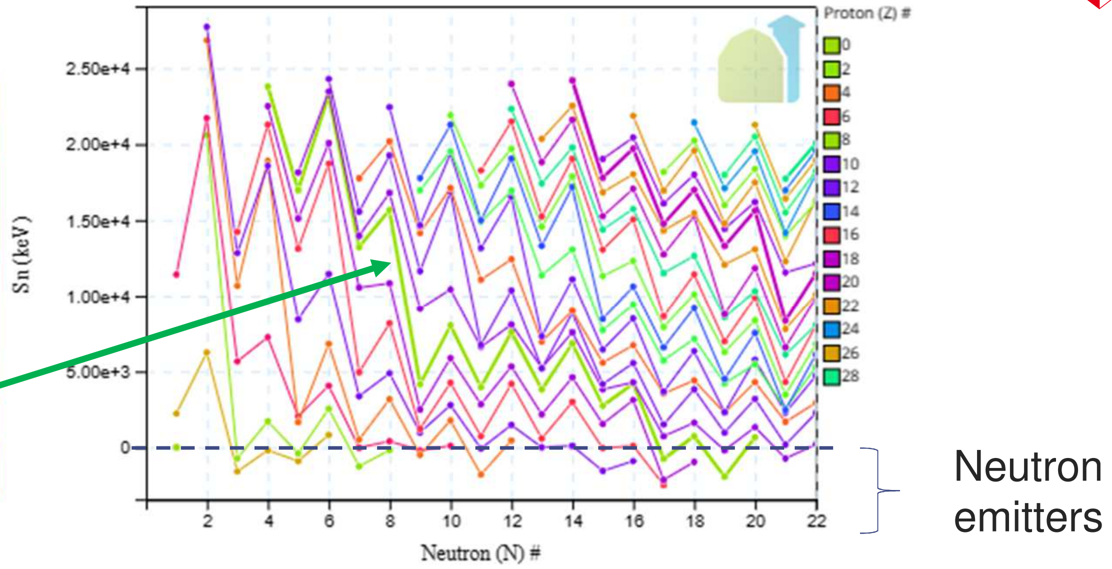


Neutron emitters

https://www.nndc.bnl.gov/nudat3/

Experimental values of S_n





Ground and isomeric state information for $^{26}_{8}O$

E(level) (MeV)	Jπ	Mass Excess (keV)	$T_{1/2}$	Decay Modes	S_n (keV)
0.0	0+	3.466E+4 *16*	4.5 ps +*32*-*34*	2N = 100.00% N?	739 10

Ground and isomeric state information for $^{25}_{8}O$

E(level) (MeV)	Jπ	Mass Excess (keV)	$T_{1/2}$	Decay Modes	S_n (keV)
0.0	(3/2+)	2.733E+4 *17*	6.3E-9 ps +*11*-*6*	N = 100.00%	-757 8

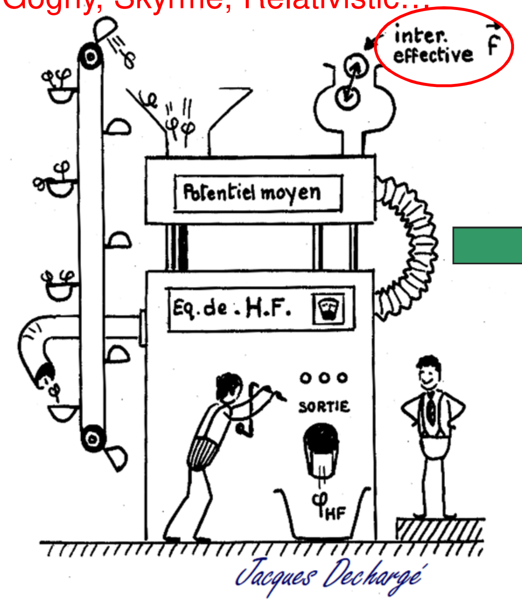
Ground and isomeric state information for $^{24}_{8}O$

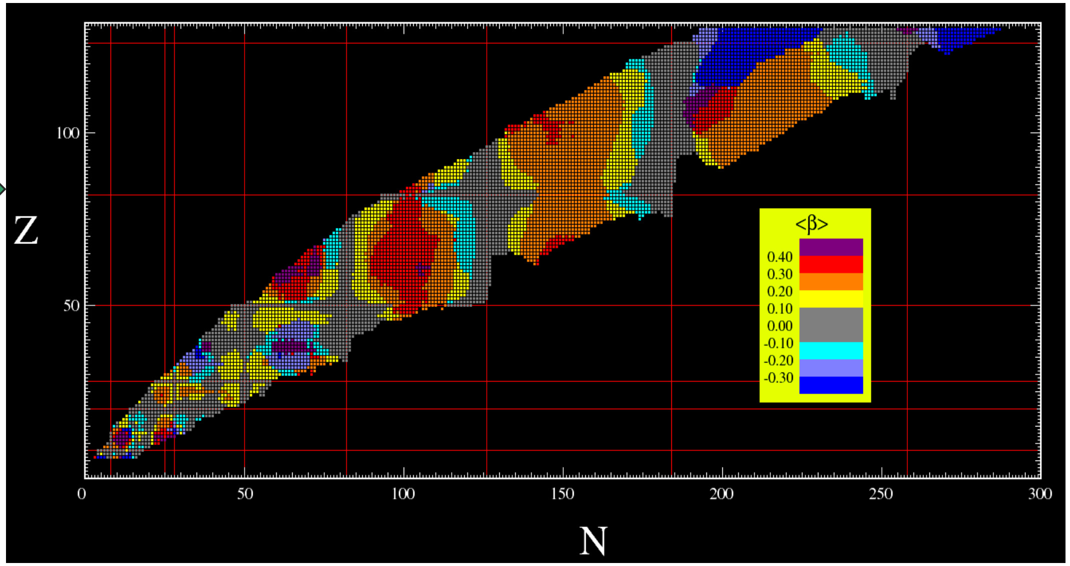
E(level) (MeV)	Jπ	Mass Excess (keV)	$T_{1/2}$	Decay Modes	S_n (keV)
0.0	0+	1.850E+4 *16*	72 ms *5*	β^- = 100.00% β^-n = 41.00%	4.19E+3 20

2. Mean field calculations of nuclei far from stability

Mean field framework

Gogny, Skyrme, Relativistic…





*http://www-phynu.cea.fr/HFB-Gogny_eng.htm; S. Hilaire & M. Girod, EPJ **A33** (2007) 237*

Static mean field (HFB)
for Ground State Properties:

- Masses
- Deformation
- Radii
- (Single particle levels)
- Magnetic moment

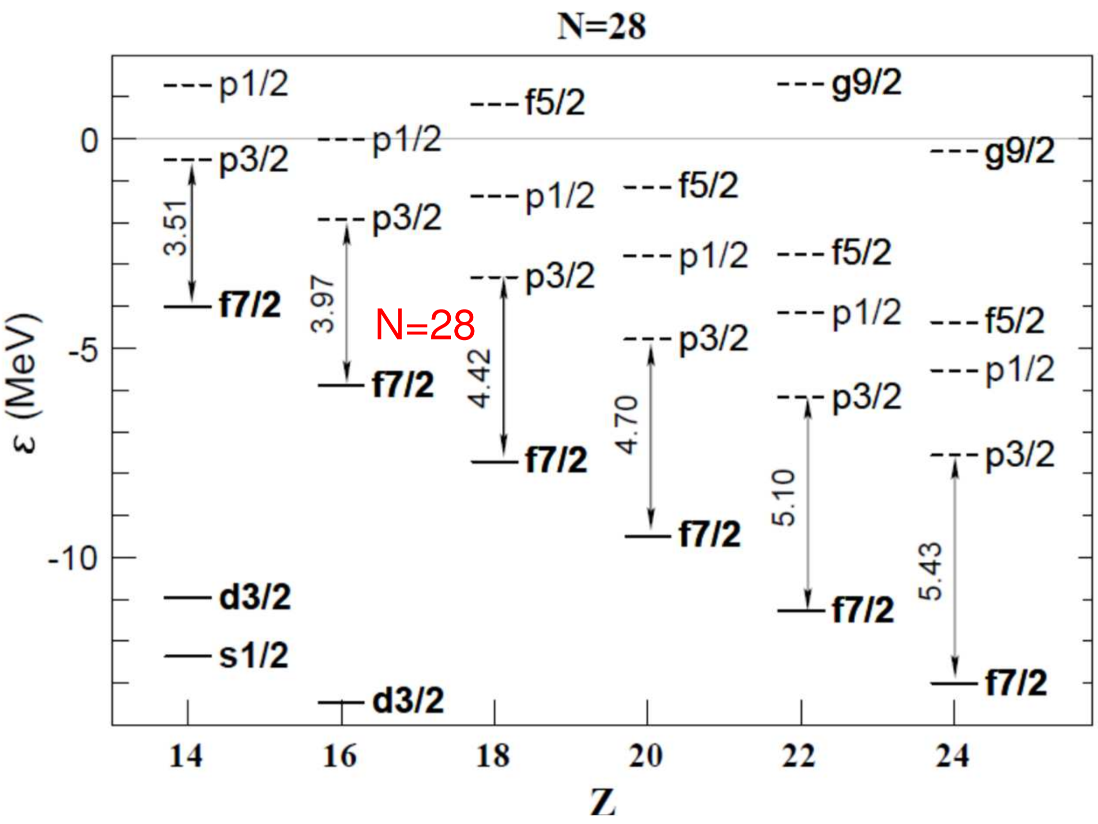
Beyond static mean field approximation (for exple QRPA or GCM like approaches)
for description of Excited State Properties

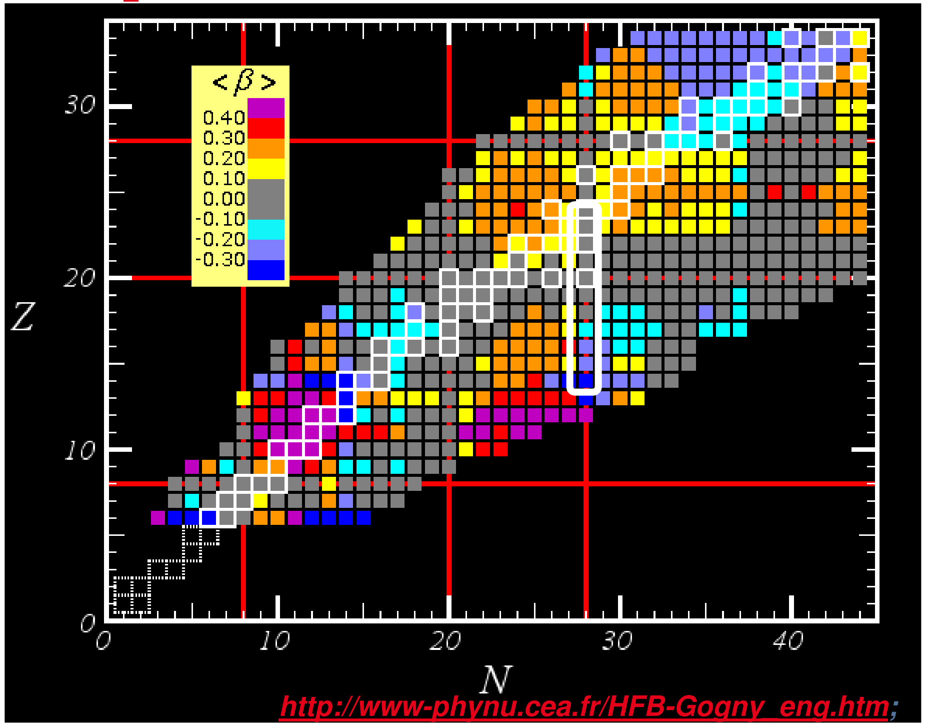
- Shape coexistence
- Low-energy collective levels
- Giant Resonances
- Beta decay

Or description of exotic aspect of ground states as shape mixing

Theoretical definition of the dripline

Using the "chemical potential:
Theoretical binding energy of the less bound nucleons





http://www-phynu.cea.fr/HFB-Gogny_eng.htm;
*S. Hilaire & M. Girod, EPJ **A33** (2007) 237*

S. Péru, M. Girod and J.-F. Berger, Eur. Phys. Jour. A **9**, 35 (2000)

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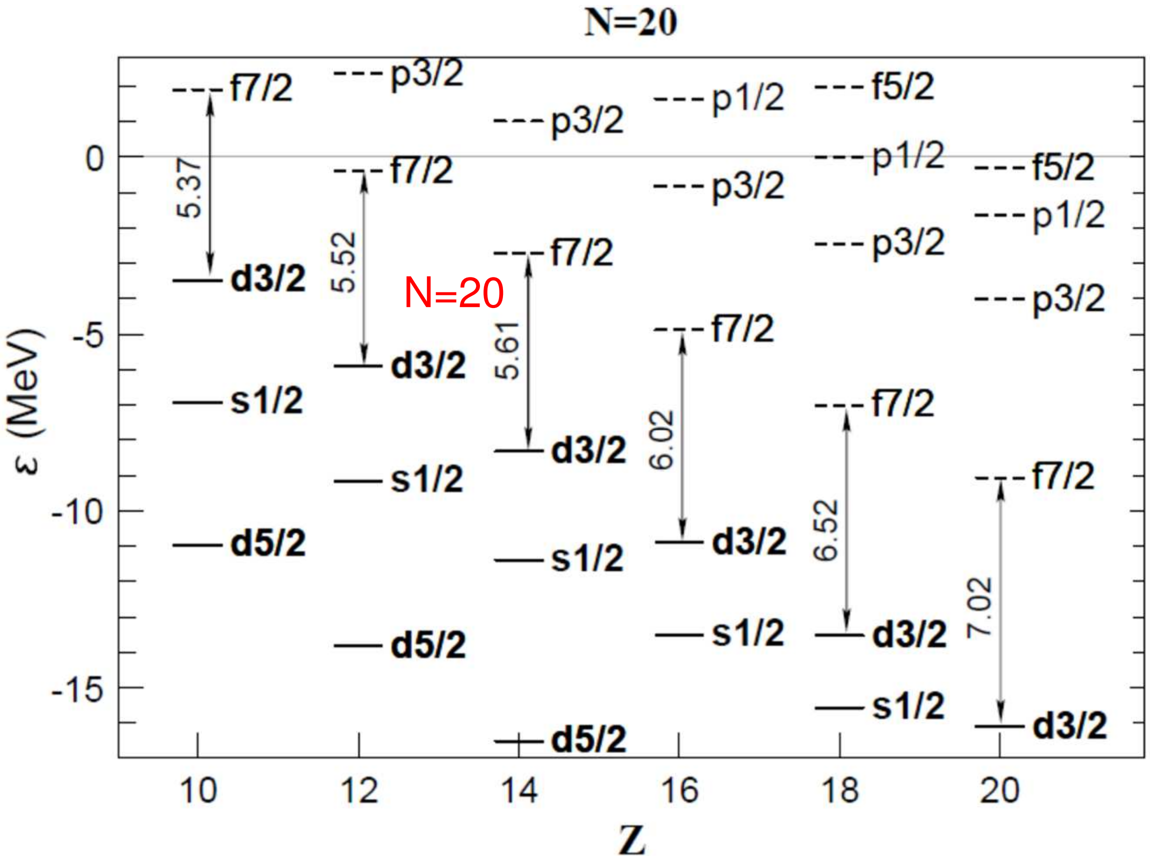
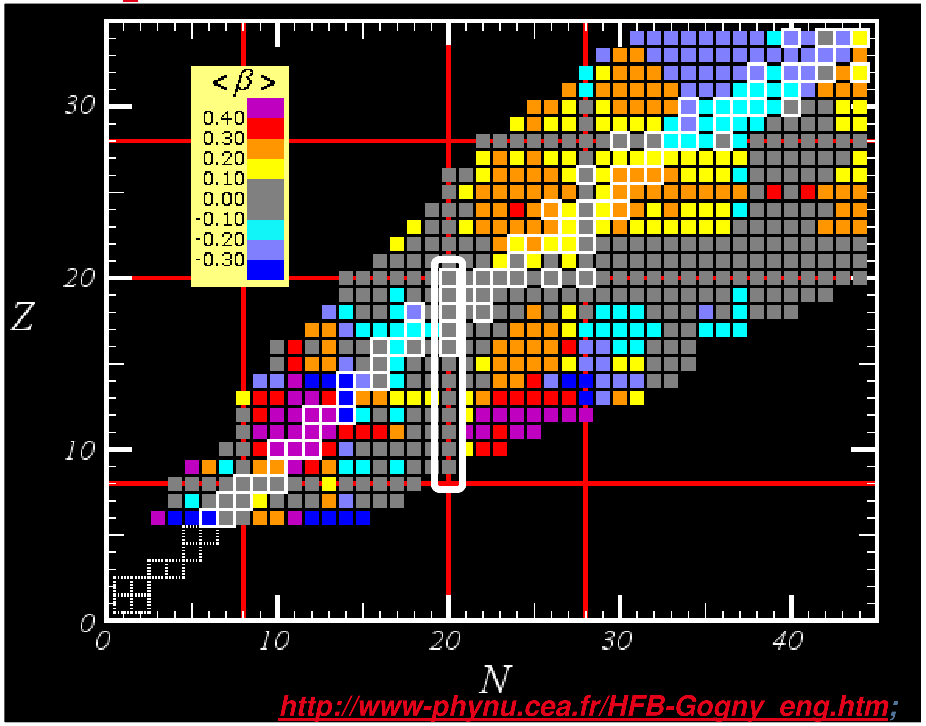
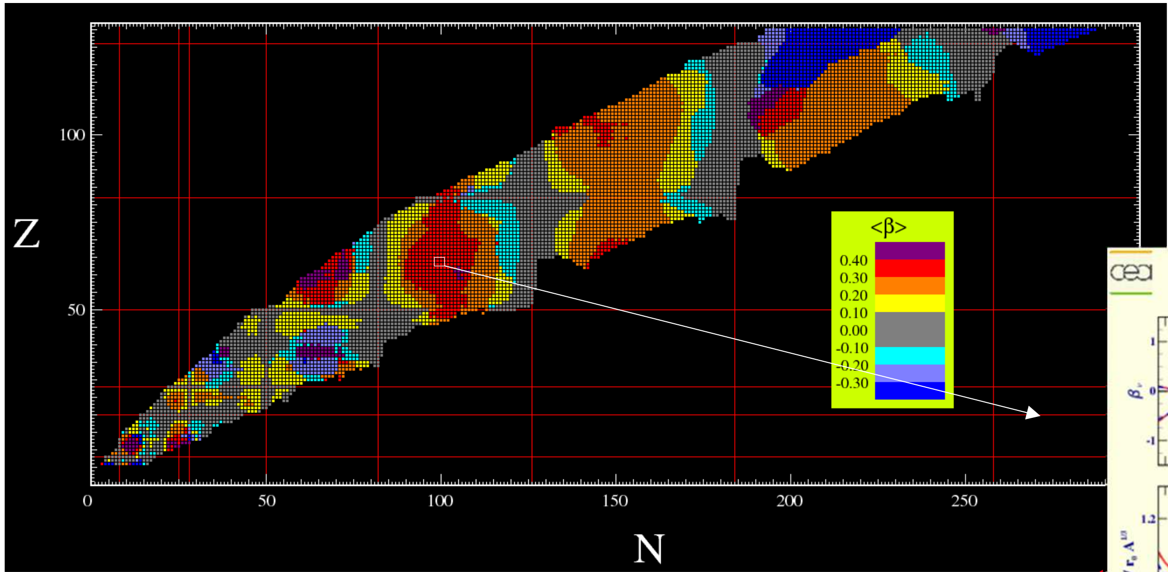


S. Péru, M. Girod and J.-F. Berger, Eur. Phys. Jour. A **9**, 35 (2000)

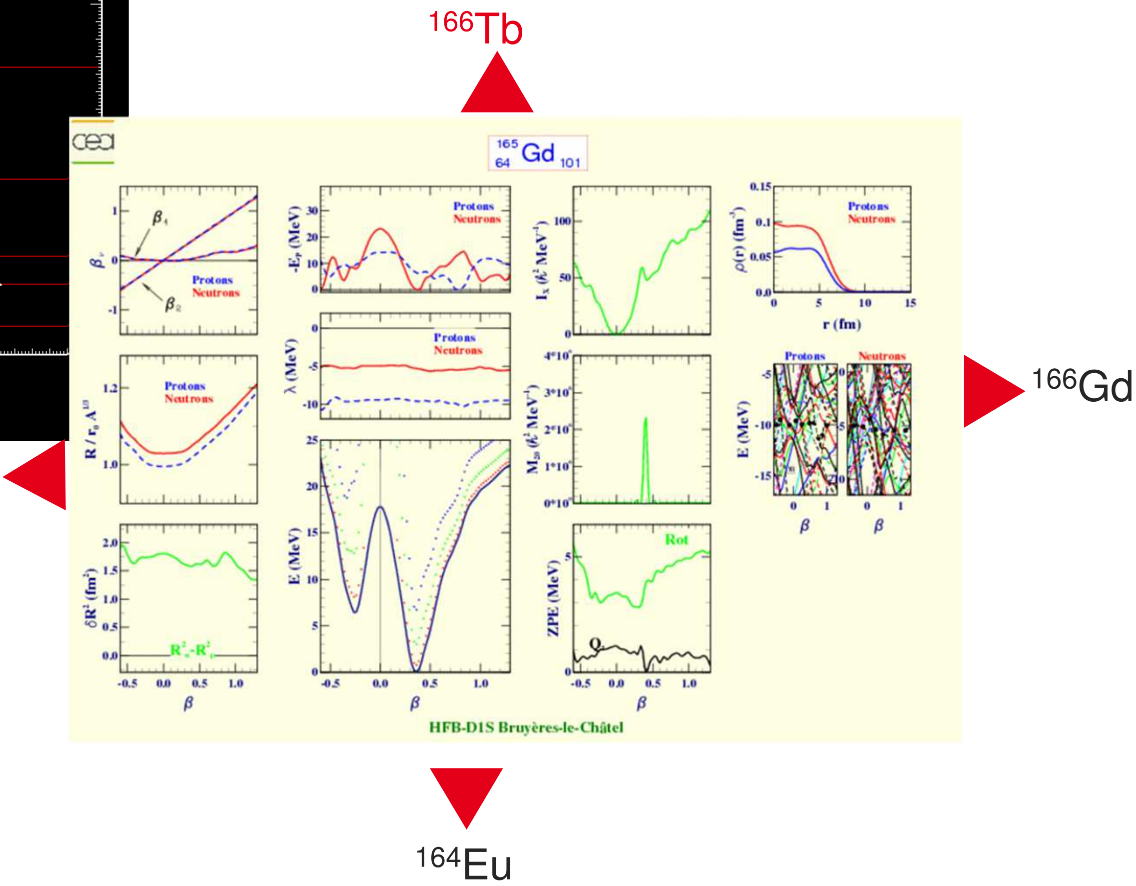


http://www-phynu.cea.fr/HFB-Gogny_eng.htm;
*S. Hilaire & M. Girod, EPJ **A33** (2007) 237*

A standard nucleus



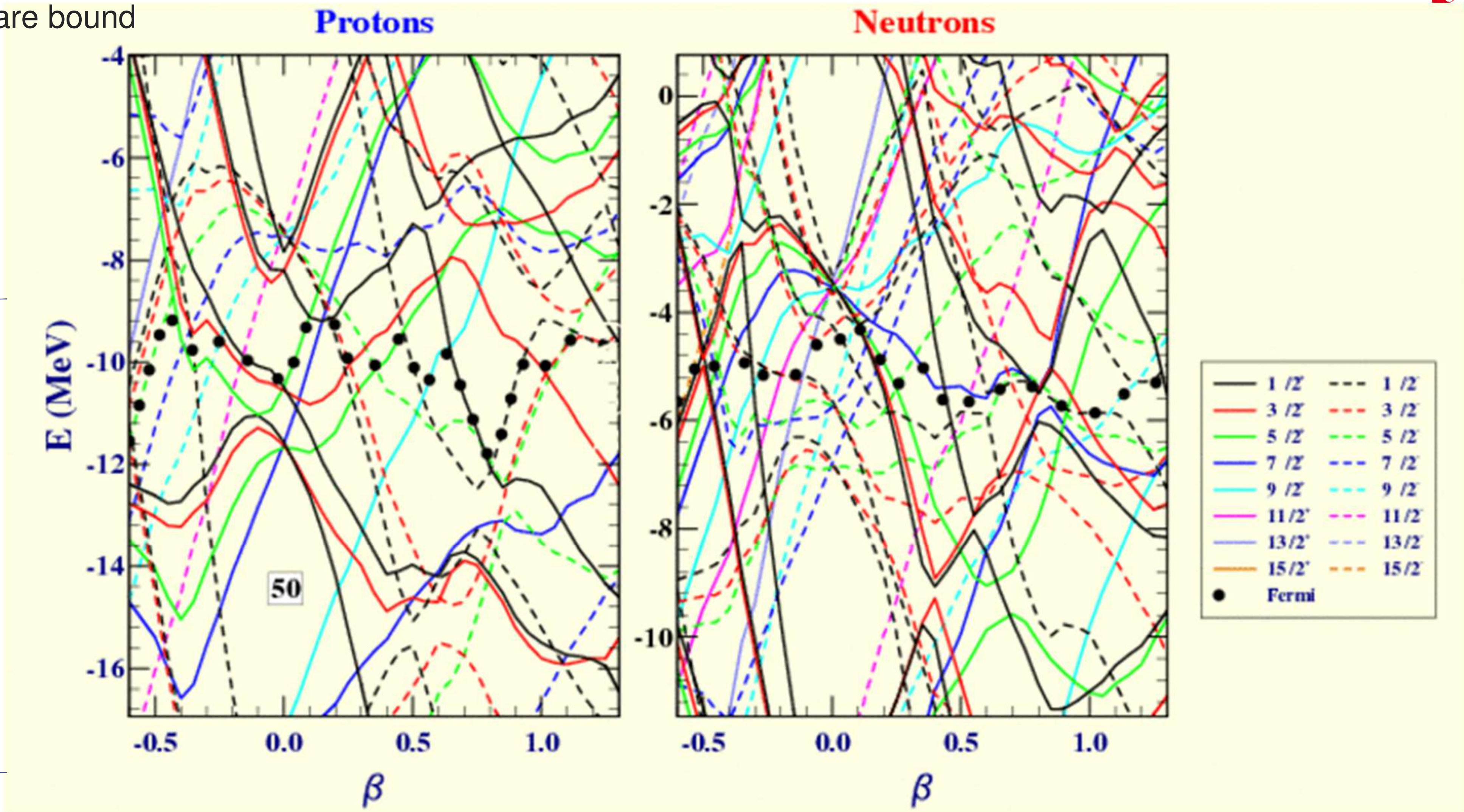
http://www-phynu.cea.fr/HFB-Gogny_eng.htm;
*S. Hilaire & M. Girod, EPJ **A33** (2007) 237*



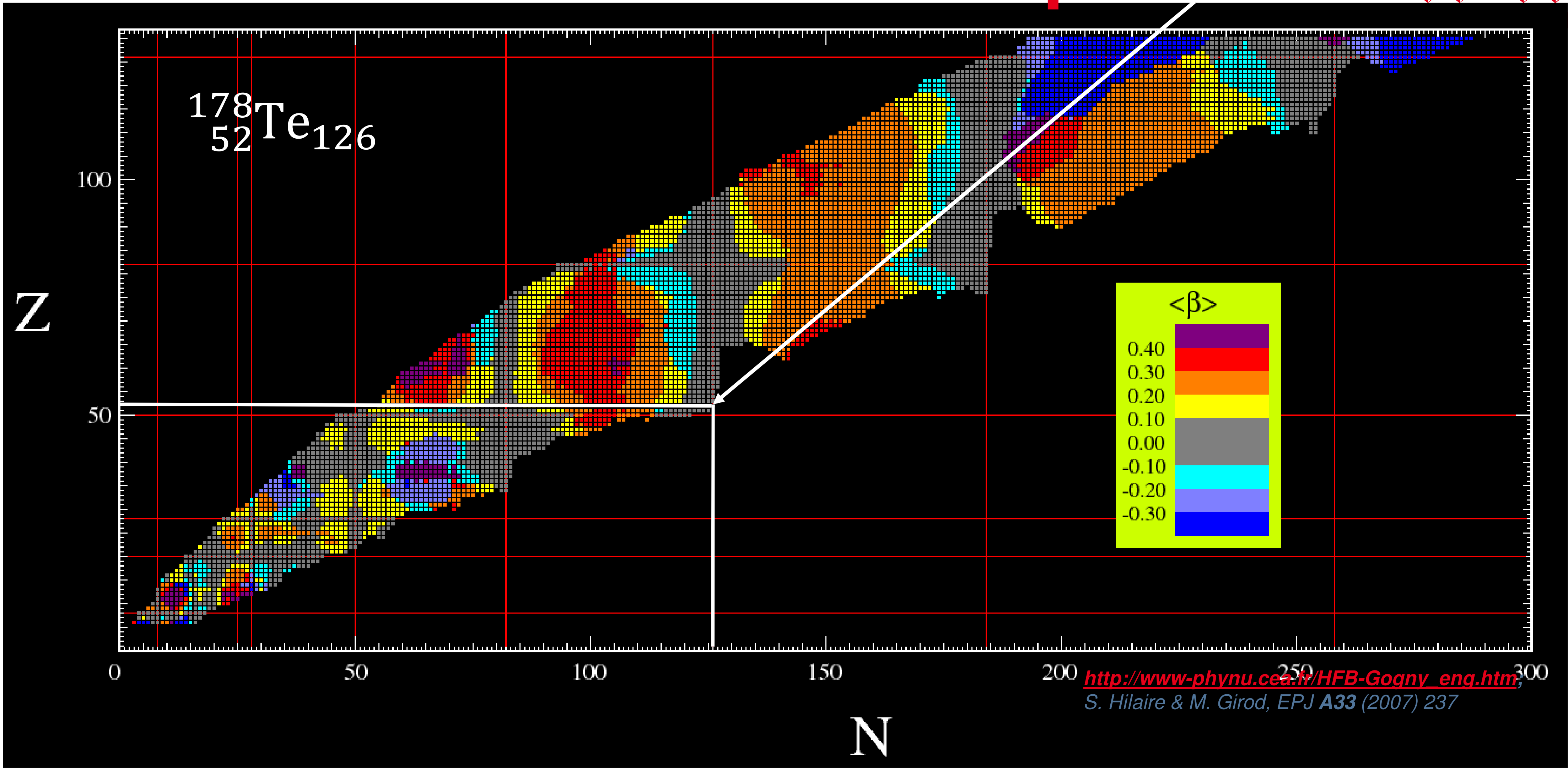
All occupied levels have negative energy;
All nucleons are bound

http://www-phynu.cea.fr/HFB-Gogny_eng.htm;
*S. Hilaire & M. Girod, EPJ **A33** (2007) 237*

Occupied nucleons levels

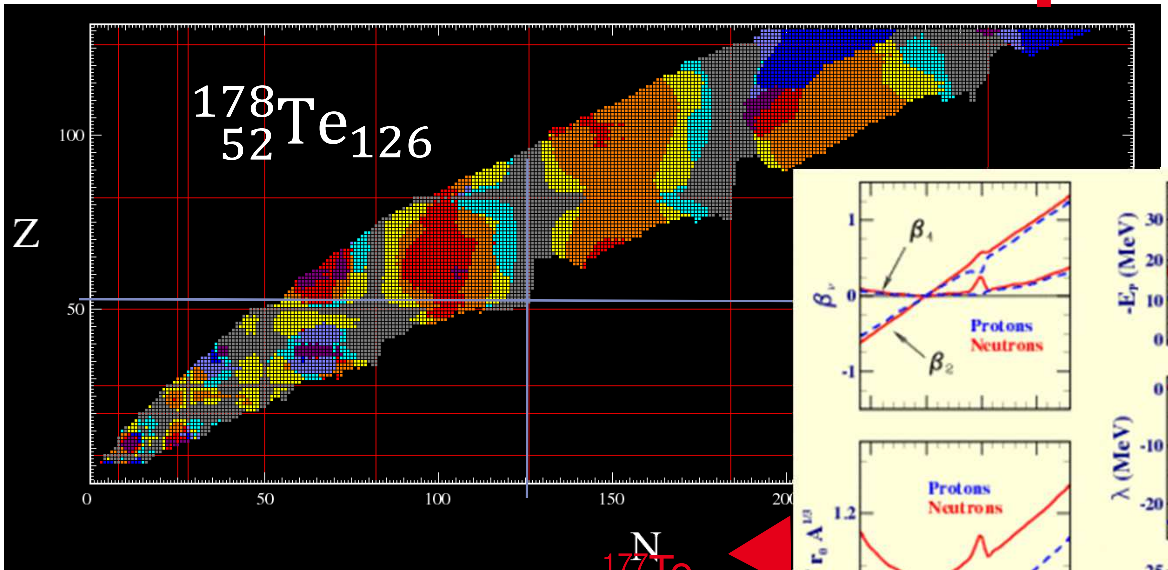


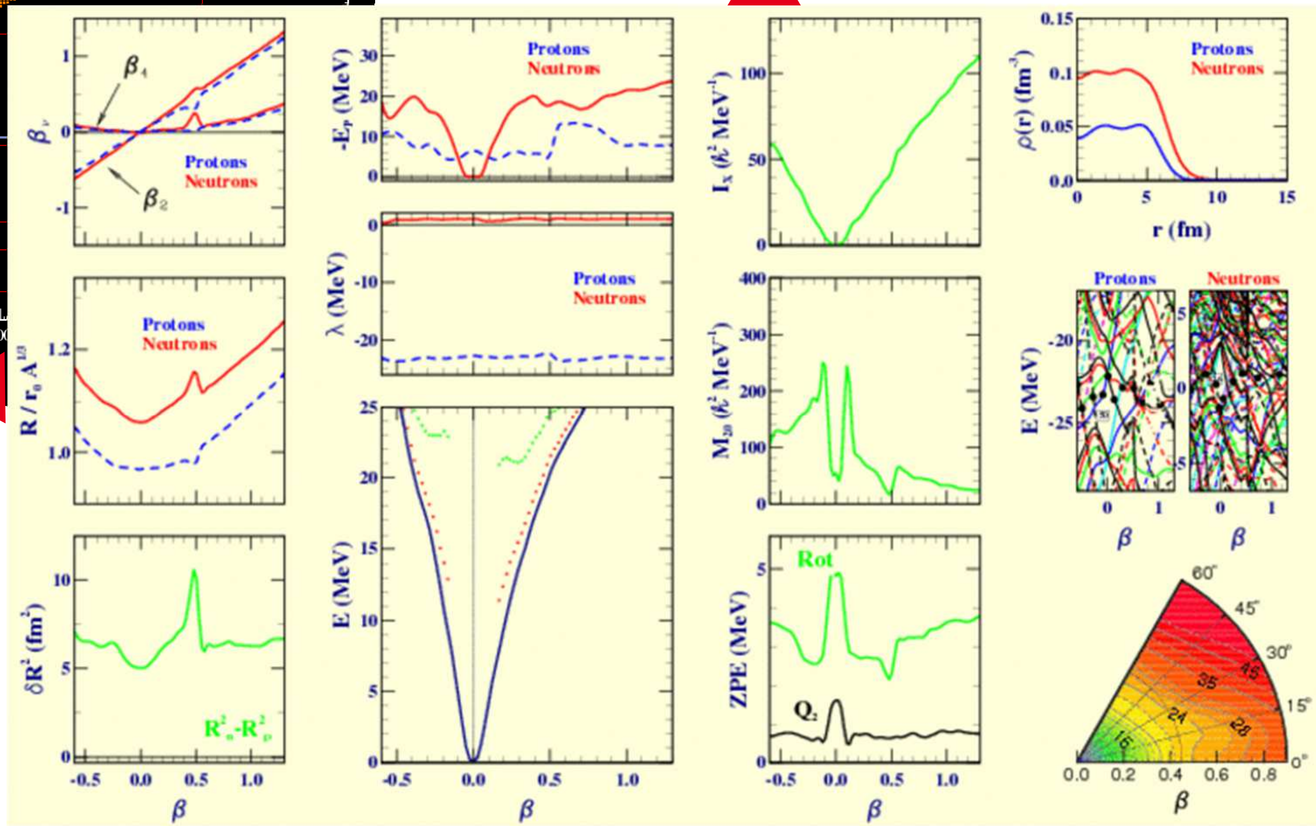
A exotic nuclei at the neutron dripline:



http://www-phynu.cea.fr/HFB-Gogny_eng.htm;
S. Hilaire & M. Girod, EPJ **A33** (2007) 237

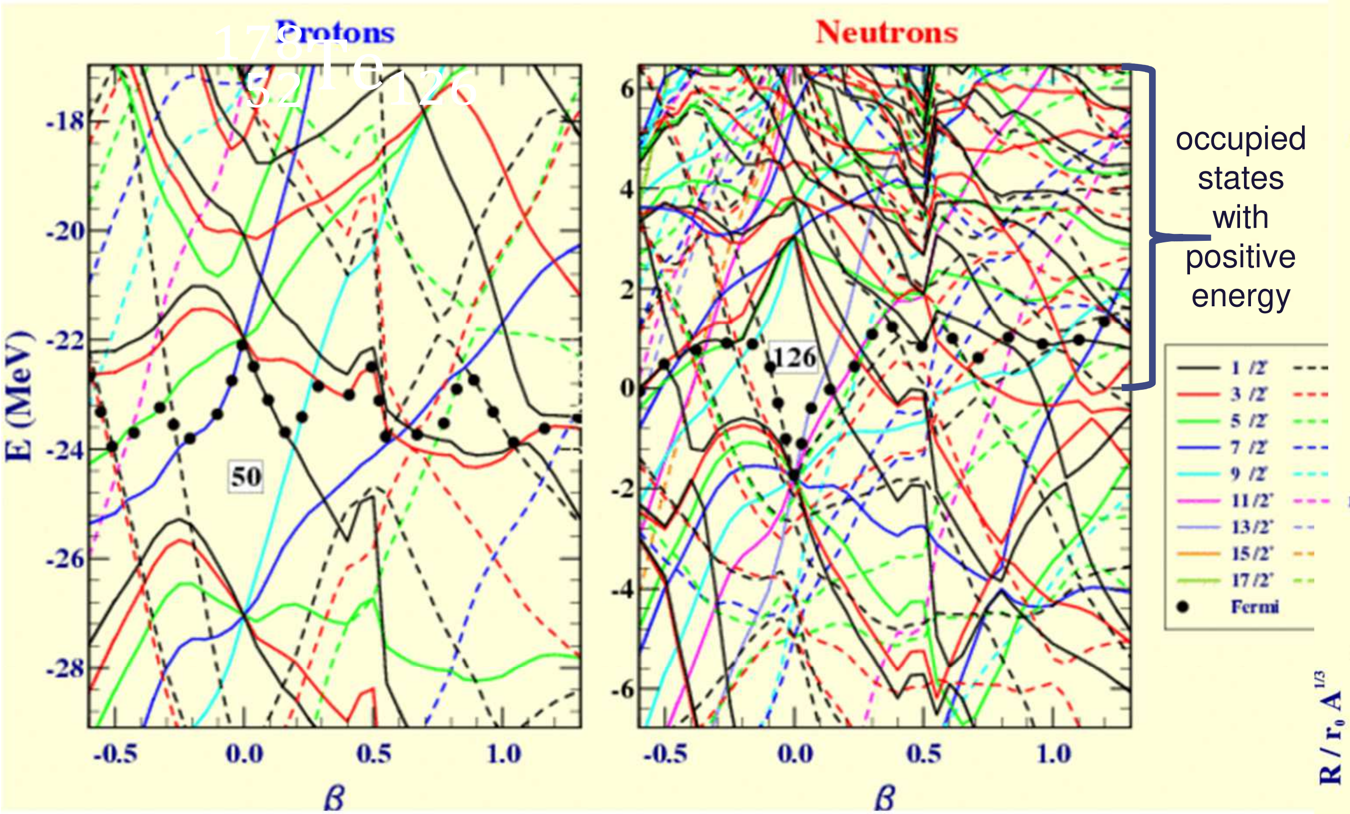
A exotic nuclei at the dripline:

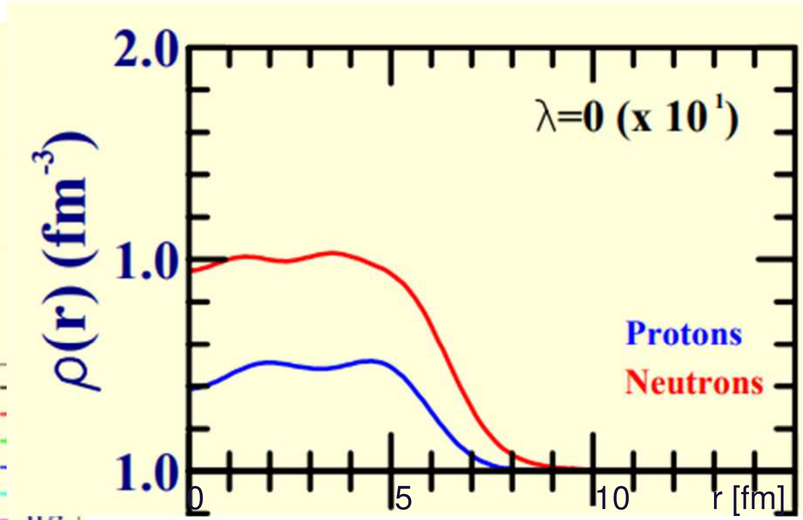


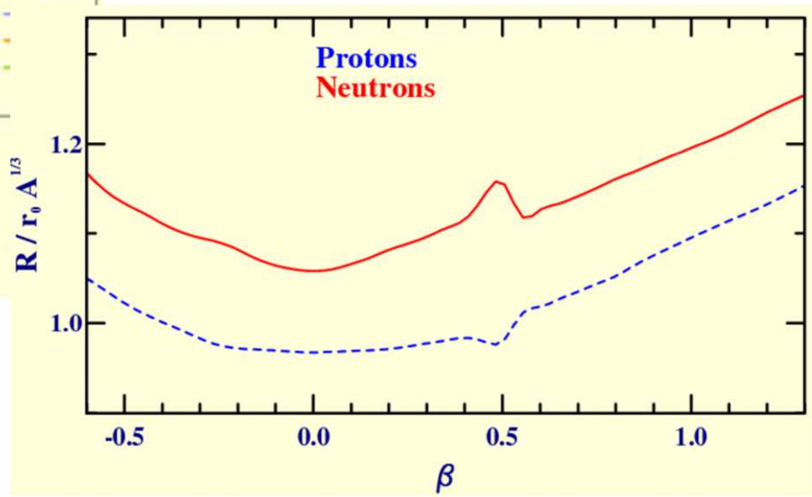


http://www-phynu.cea.fr/HFB-Gogny_eng.htm;
*S. Hilaire & M. Girod, EPJ **A33** (2007) 237*

A exotic nuclei at the dripline : $^{178}_{52}\mathrm{Te}_{126}$



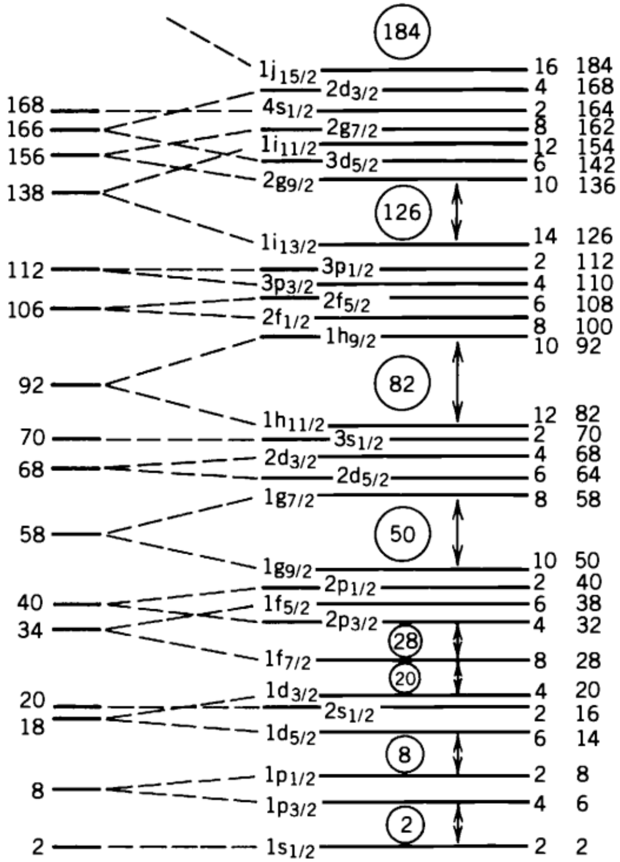


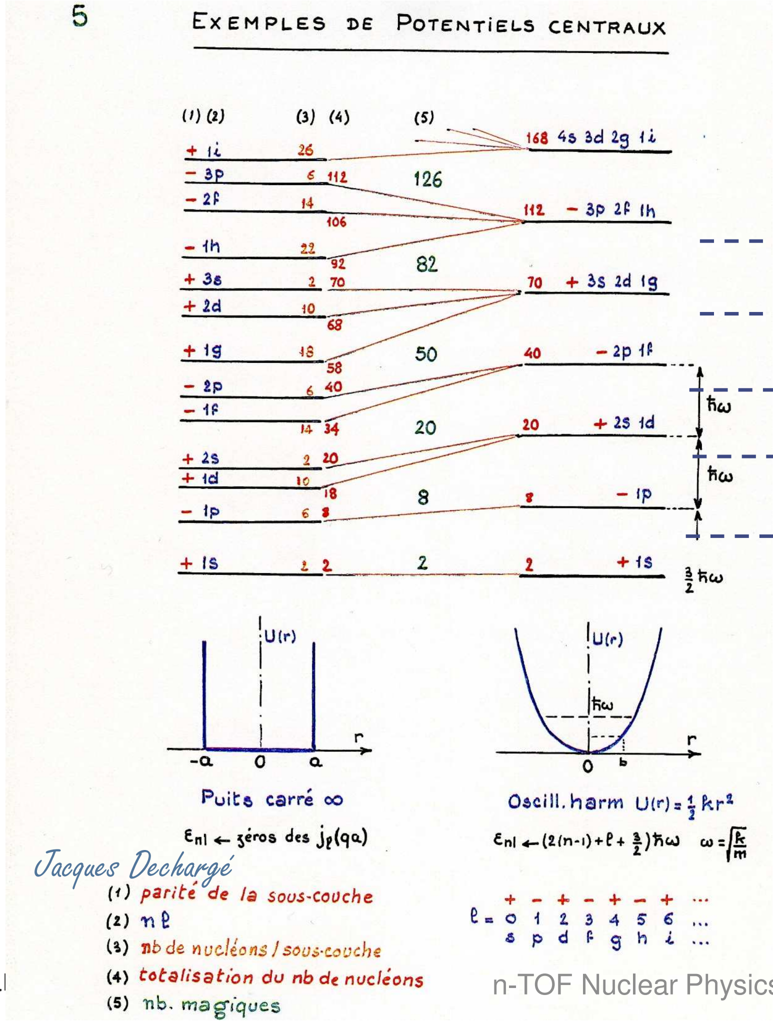


Occurrence of neutron skin for very N-Z values,
its impact will be discussed in the following

Some technical points of Harmonic oscillator basis

HO Major shell number convergence : what does this mean?





HO basis size

n= quantum number = 0,1, 2..

No=n+1number of major shells

With l=0,1,…n

And π = (-)^l

n = 4, No = 5

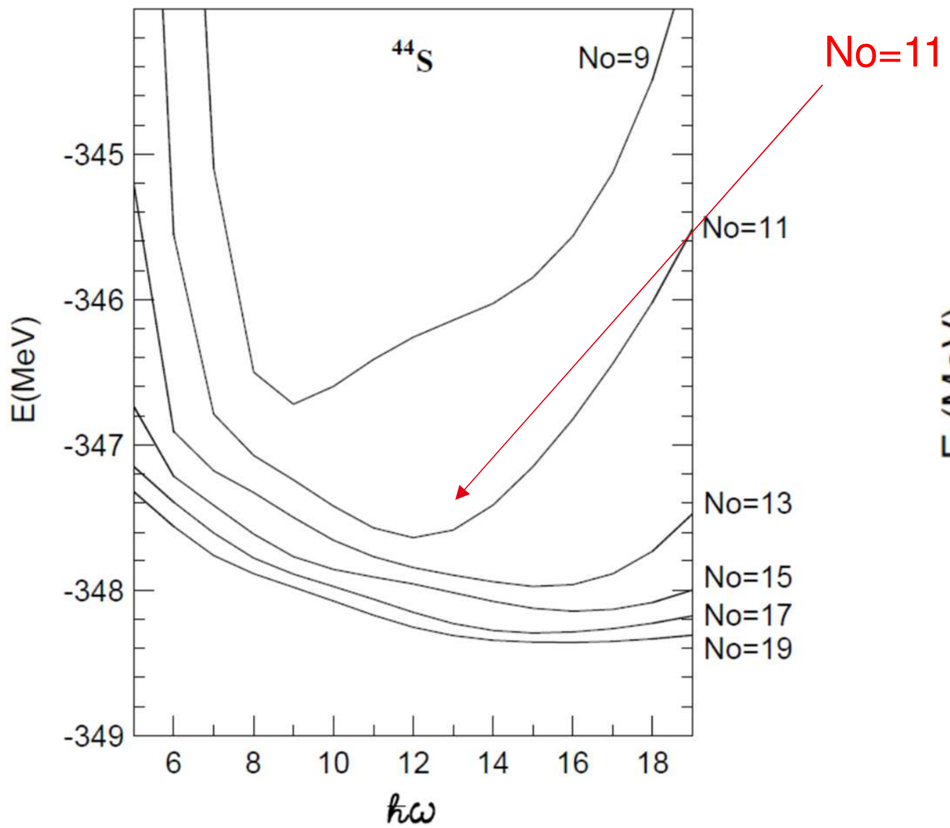
n = 3, No = 4

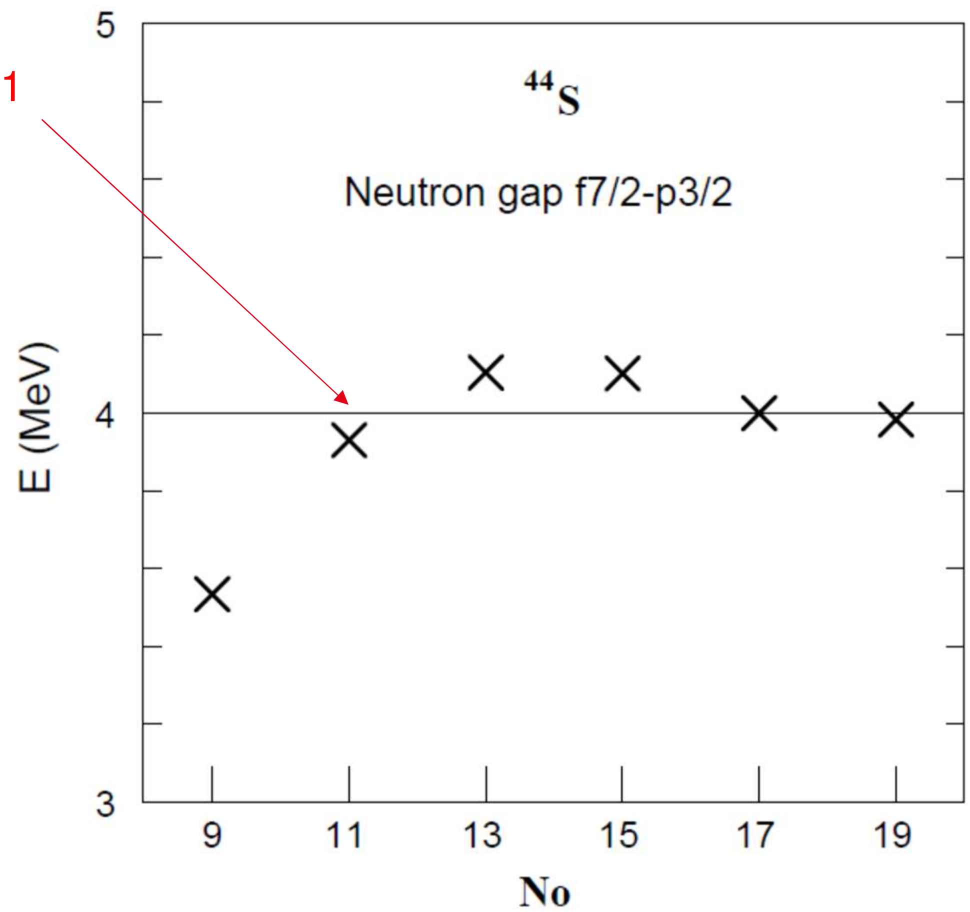
n = 2, No = 3

n = 1, No = 2

n = 0, No = 1

Convergence with respect to HO basis size - HFB with D1S interaction

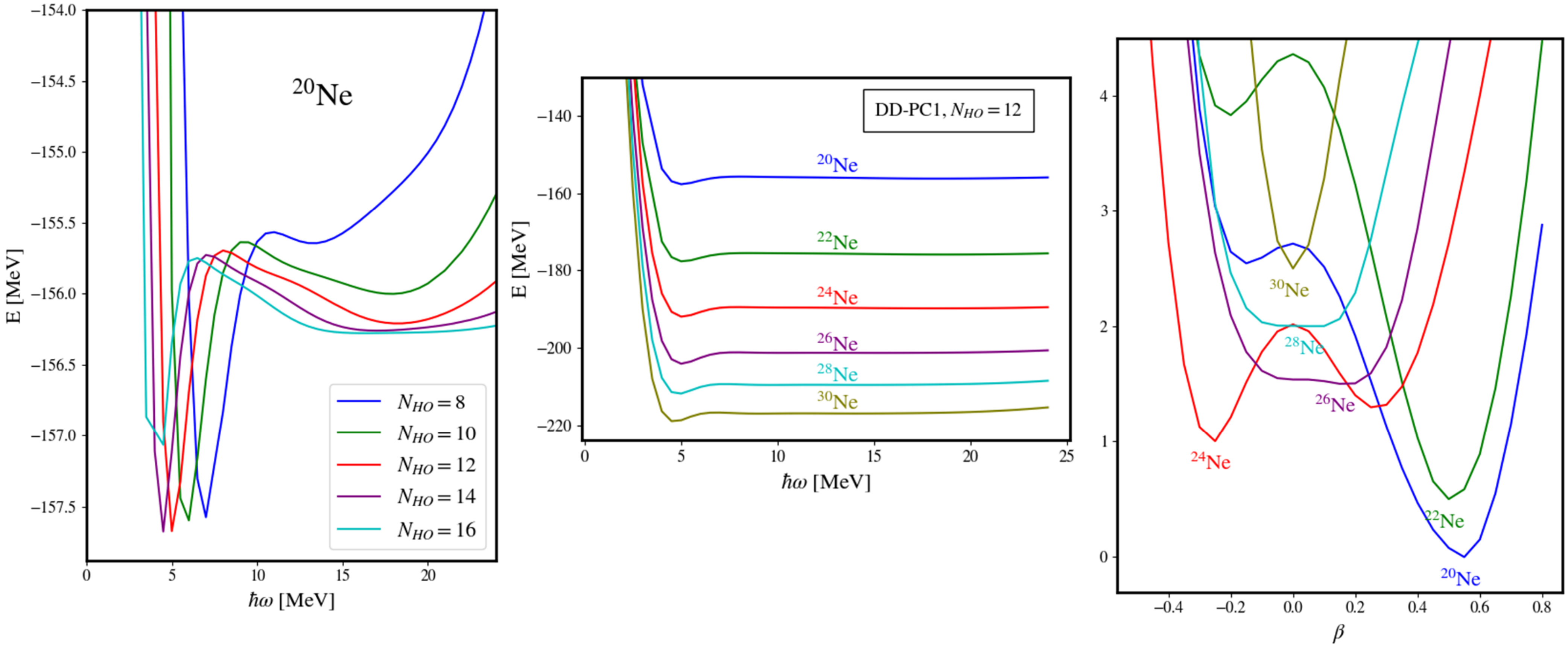




The base size must be large enough to initiate convergence of the quantities of interest.
Need to choose a base size as small as possible for a reasonable calculation time.

S. Péru, M. Girod and J.-F. Berger, Eur. Phys. Jour. A **9**, 35 (2000)

Convergence with respect to HO basis size - HB with DD-PC1 Covariant interaction



Luis González-Miret thesis, defense scheduled for December 2024

Shell closure evolutions: phenomema specific to exotic nuclei

Some words of context:

Magic numbers : N=8,20,28, 40 ?, 50 etc…

- But there exists some experimental evidence of deformation in ^{32}Mg isotope (Z=12, N20):

*"More generally, experimental mass measurements of N = 20 isotones yield S2n curves that strongly indicate that the N = 20 magic gap vanishes in exotic nuclei [*4. D.J. Vieira et al., Phys. Rev. Lett. **57**, 3253 (1986); 5. N.A. Orr et al., Phys. Lett. B **258**, 29 (1991); 6. X.G. Zhou et al., Phys. Lett. B **260**, 285 (1991).] …the very low excitation energy (*Ex*= 885 keV) of the first 2+ state [8. C. Detraz et al., Nucl. Phys. A **394**, 378 (1983); 9. D. Guillemaud-Mueller et al., Nucl. Phys. A **426**, 37(1984).]"

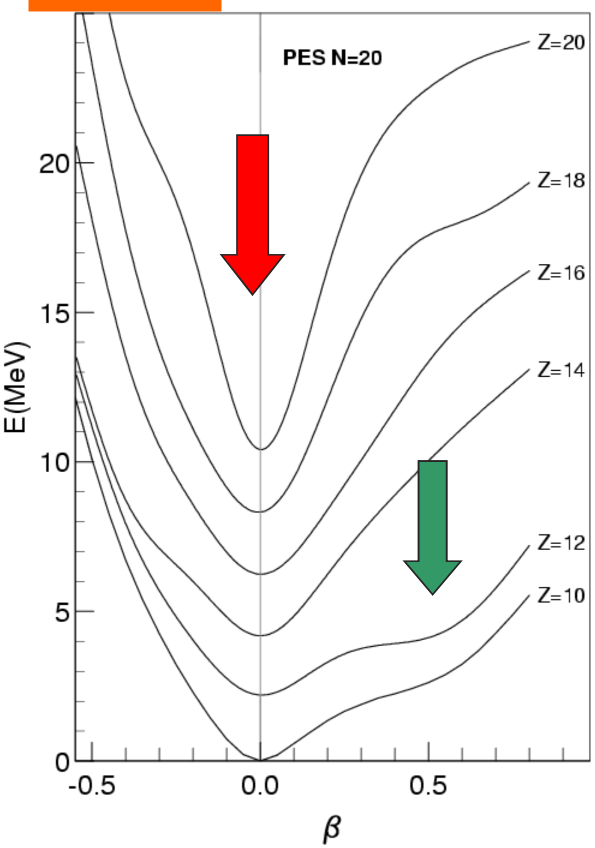
- How to characterize a shell closure in a mean field framework ?

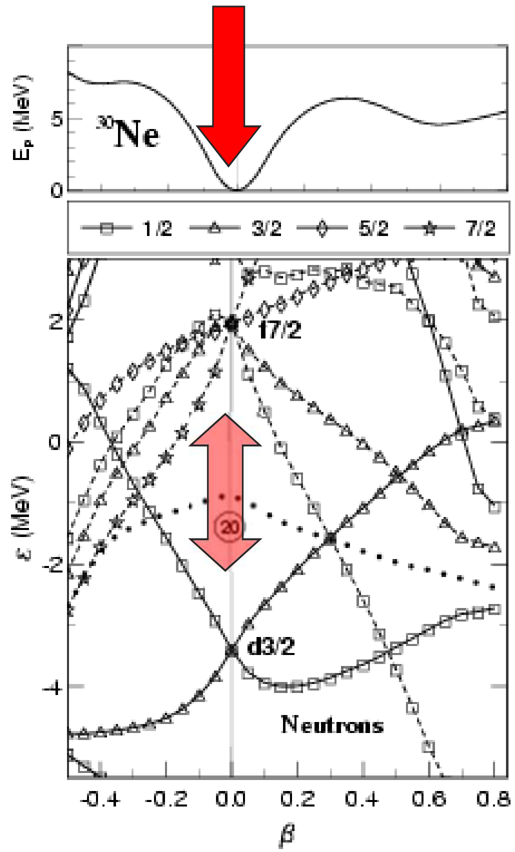
We defined some criteria:

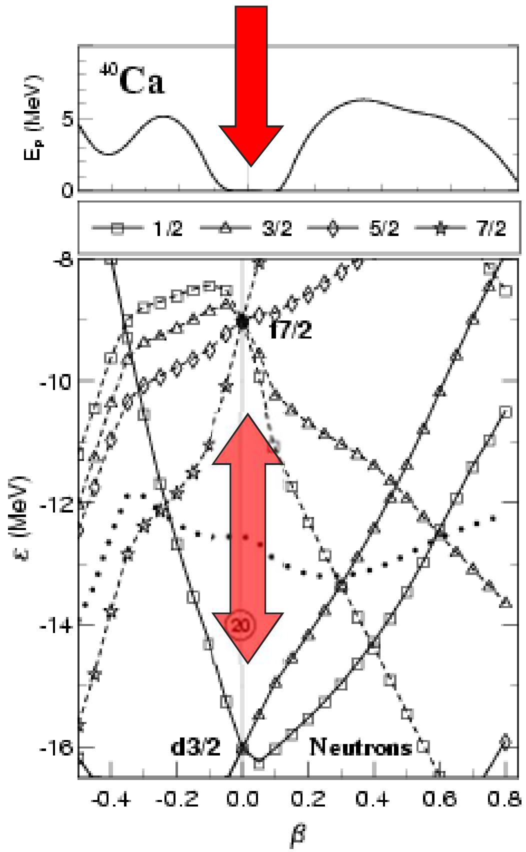
- ✓ Shell gap (ph gap) versus pairing gap,
- ✓ "Magic" shell closure when shell closure occurs for spherical shape.

N=20

HFB D1S







N=20
remains "magic"

But
occurrence
of secondary minimum
for ^{32}Mg and ^{30}**Ne**

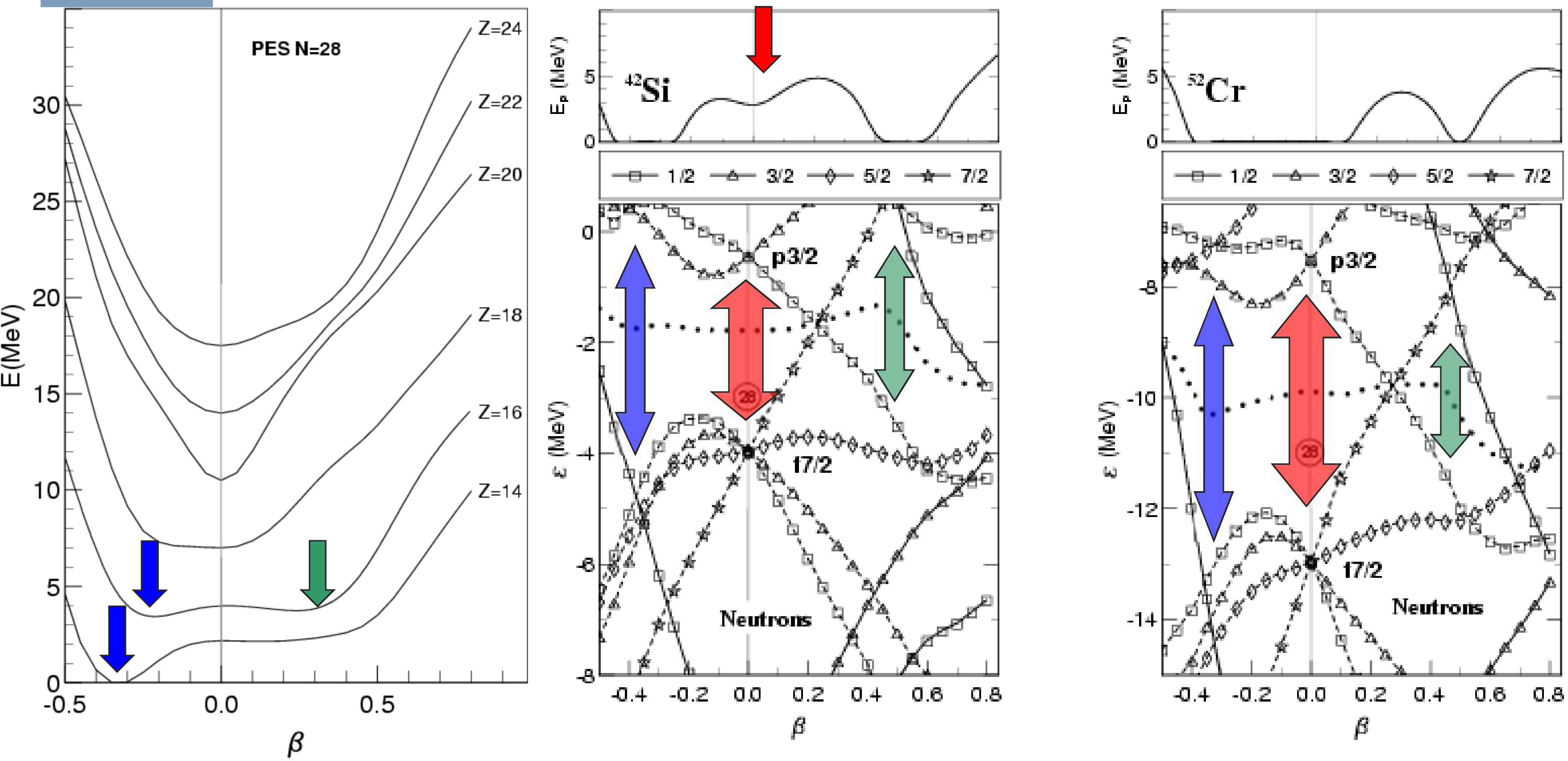
For all nuclei under study the mean field minimum is still spherical

Large ph gap for spherical shape.

Sufficient ph gap to cancel pairing correlations at spherical shape.

S. Péru, M. Girod and J.-F. Berger, Eur. Phys. Jour. A **9**, 35 (2000)

N=28



HFB D1S

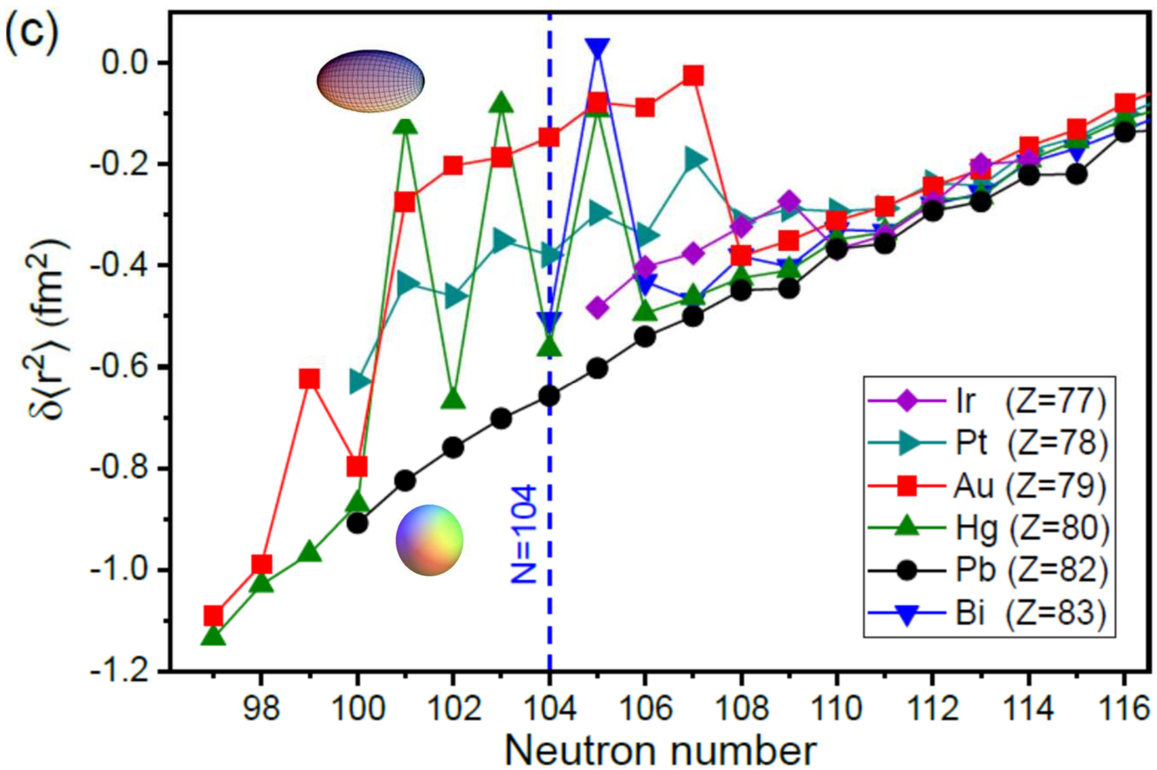
N=28
is no longer
a “magic” number.

The potential energy curves for 44S and 42Si have deformed minima.

For spherical shape, the ph gap reduces with Z and pairing correlations appear.

S. Péru, M. Girod and J.-F. Berger, Eur. Phys. Jour. A **9**, 35 (2000)

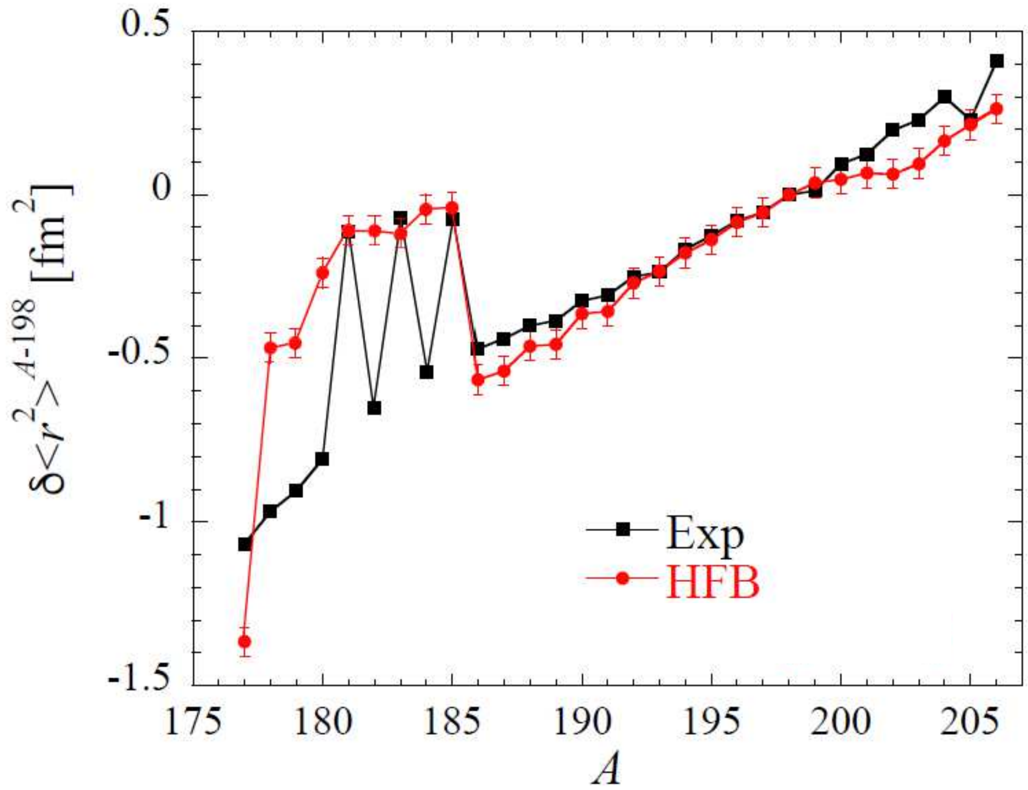
Charge radii and isotopic shift



Theoretical challenge to identify
ground state and isomeric state structure
Some observables as magnetic moments are helpful

J. Cubiss et al, PHYSICAL REVIEW LETTERS 131, 202501 (2023)

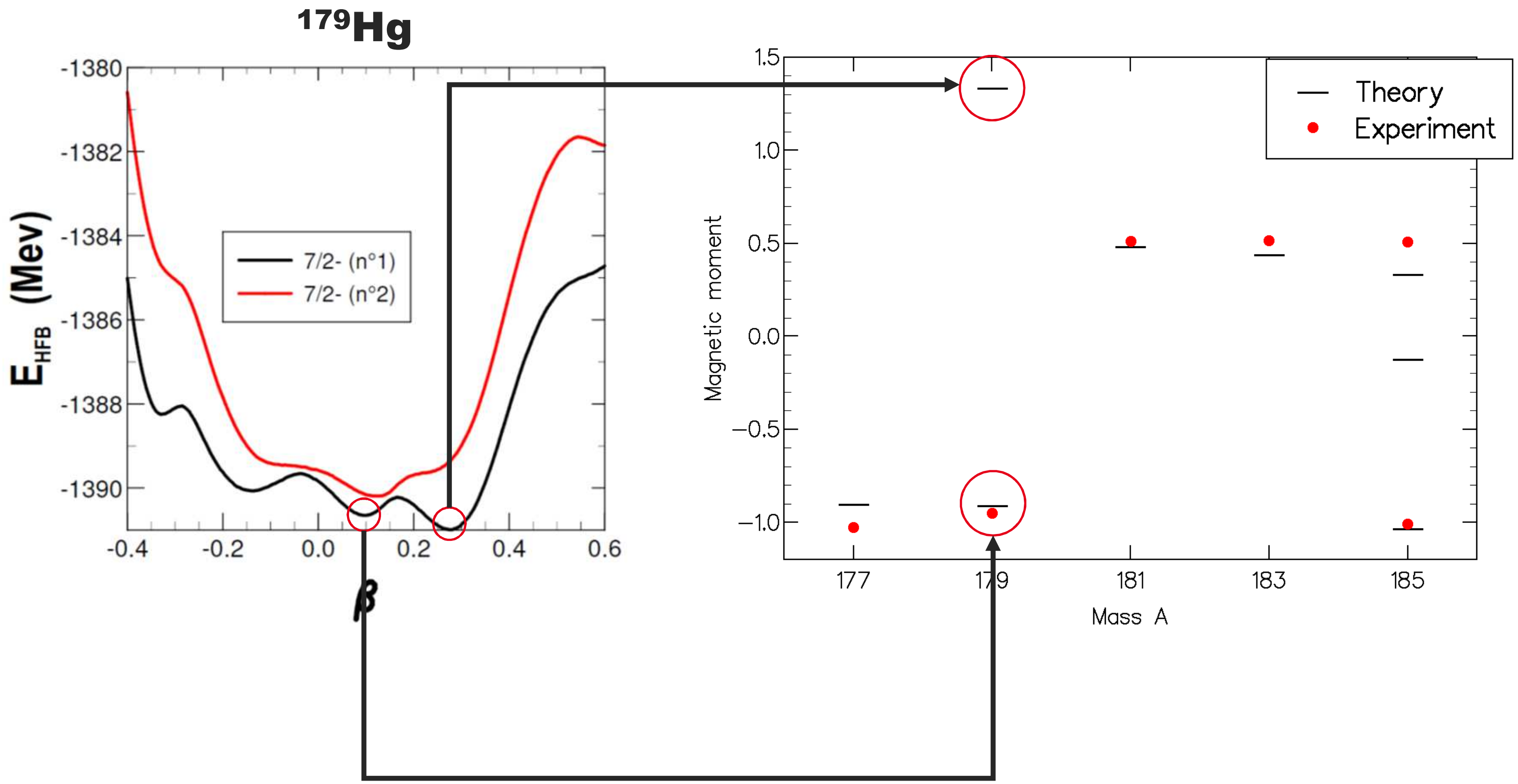
Raw isotopic shifts in Mercury isotopes



⇒ HFB Ground states not correct

Magnetic moments used as constraint

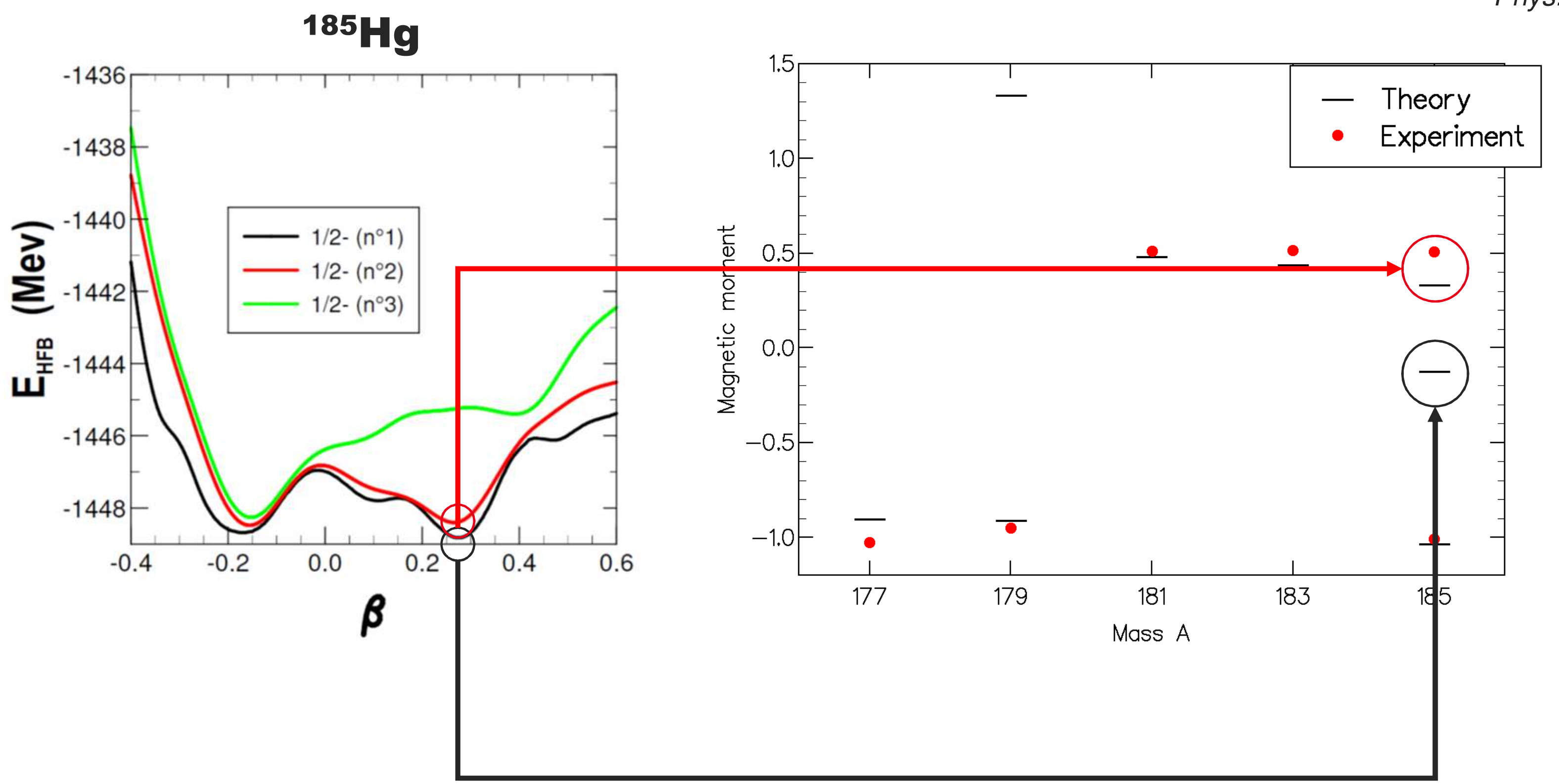
Phys. Rev. C 104, 024328 (2021)



⇒ The PES minimum does not correspond to the observed GS

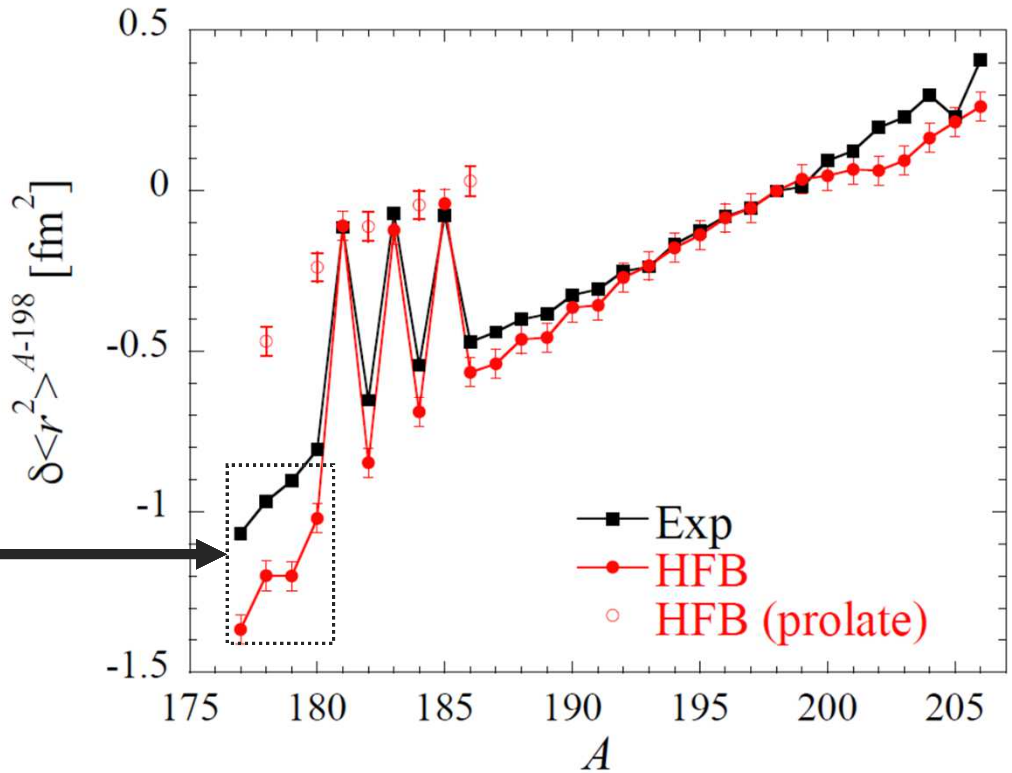
Magnetic moments used as constraint

Phys. Rev. C 104, 024328 (2021)



⇒ The first blocked state is does not correspond to the observed GS !

Final isotopic shifts in Mercury isotopes

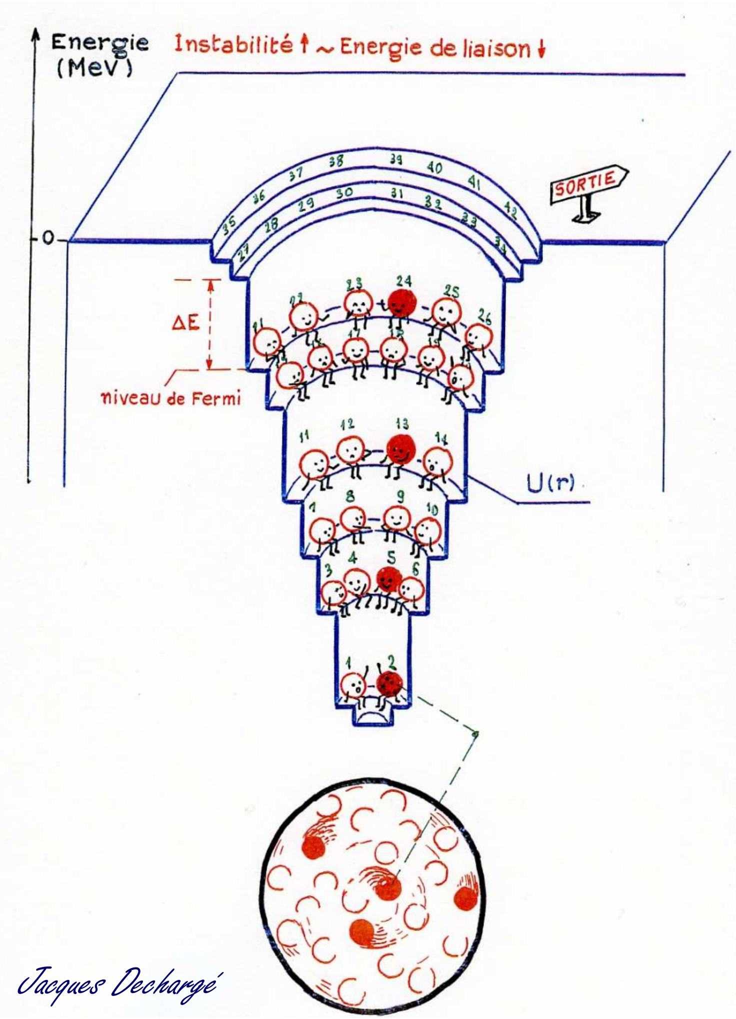


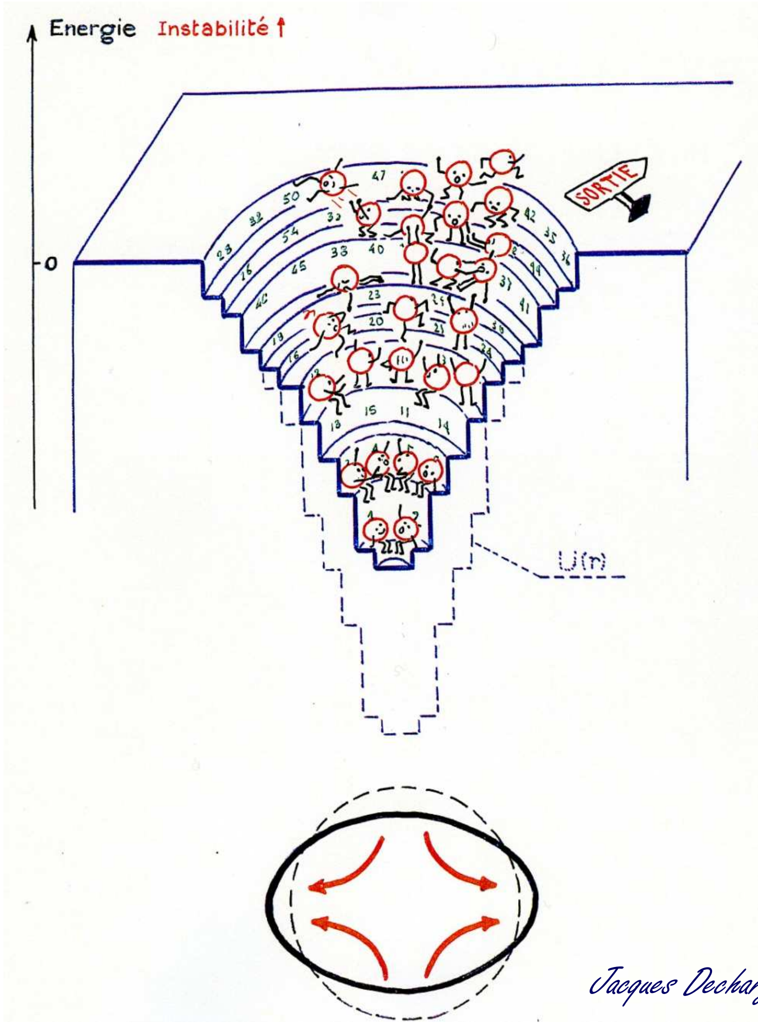
Phys. Rev. C 104, 024328 (2021)

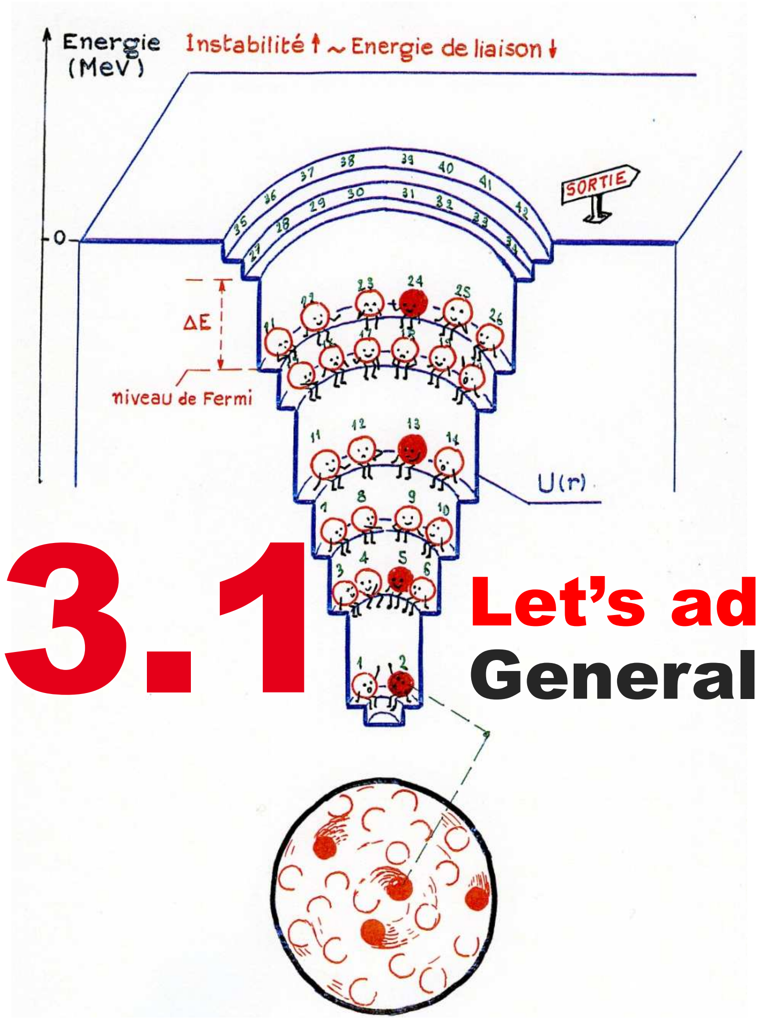
⇒ For even-even nuclei, the GS can only be constrained by the isotopic shift (or alternatively by b-decay properties)

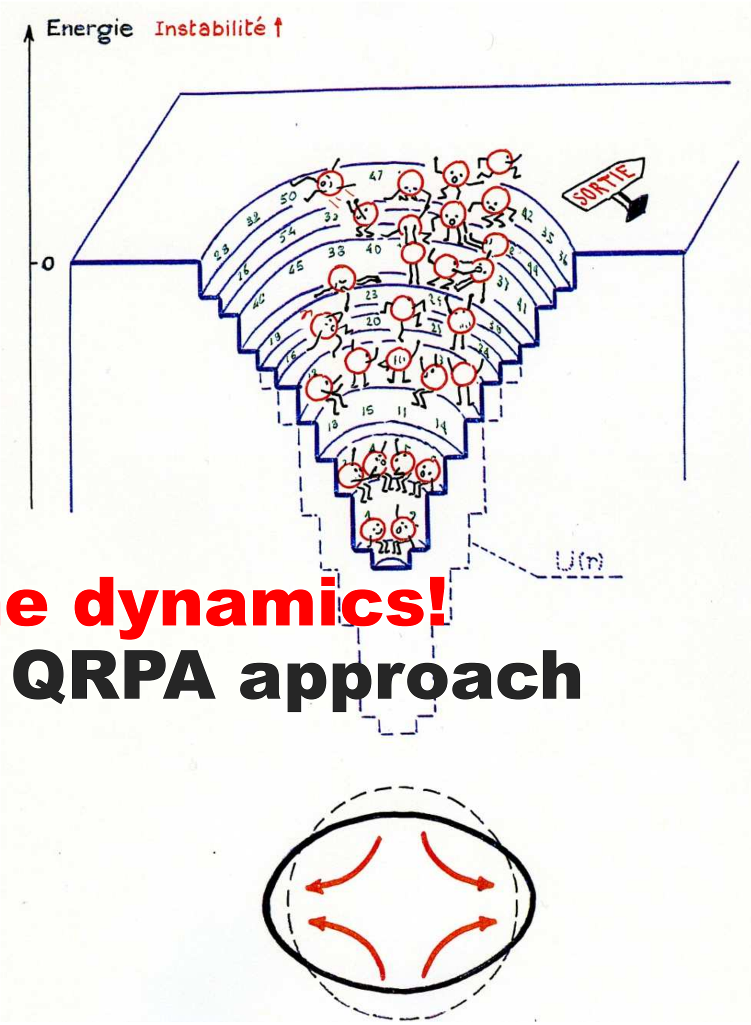
⇒ Discrepancy for light isotopes

Nuclear structure: Let's add some dynamics!







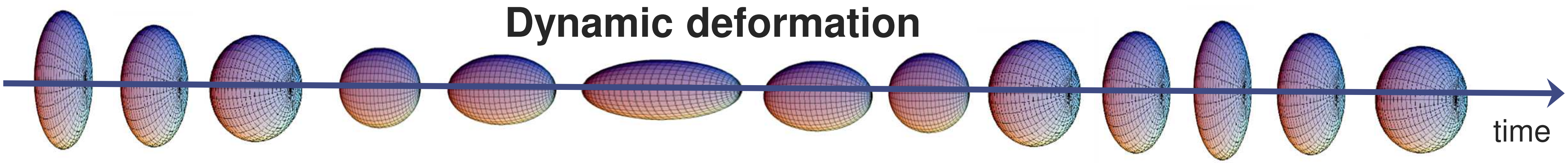


3.1 Let's add some dynamics!
Generality on QRPA approach

What is the standard QRPA approach ?

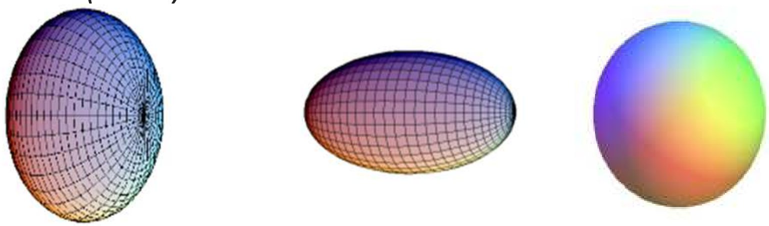
The QRPA methods describe nuclear excited states for all multipoles and both parities whatever the intrinsic deformation of the ground state.

Quadrupole, octupole and higher multipolarities can be obtained even on top of spherical HFB calculations. But standard QRPA approaches don't describe rotational motion.



What can we do with standard matrix QRPA codes?

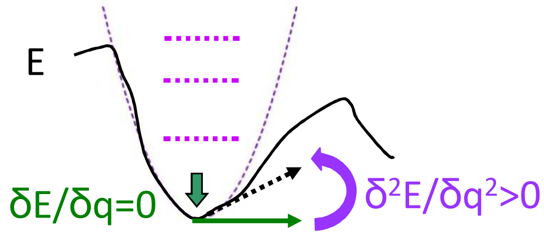
ISAAC describes excited states, transition probabilities for intrinsic deformed nuclei with axial symmetry.
S. Péru and M. Martini, Eur. Phys. J. A (2014) 50: 88



Intrinsic or static deformation

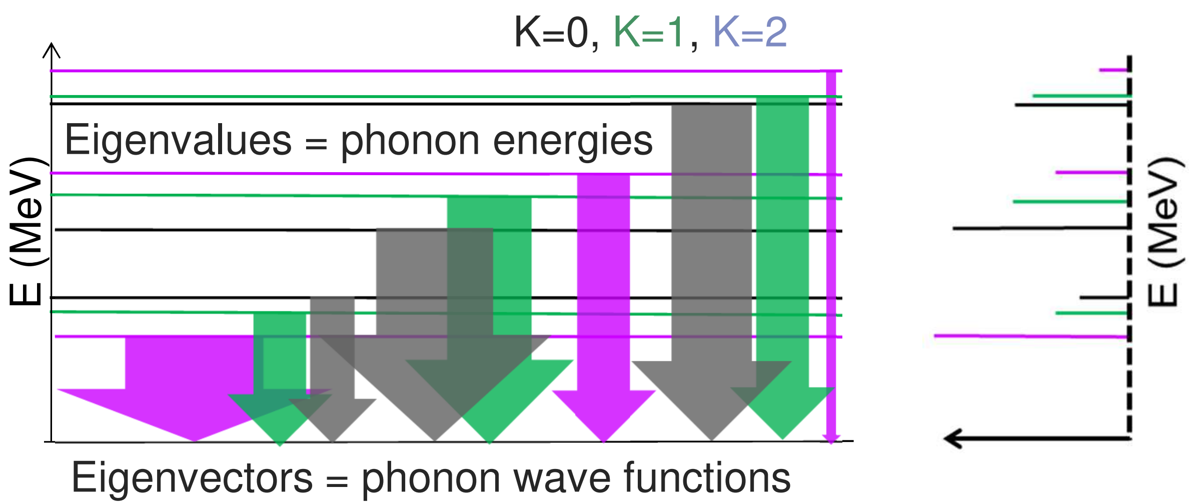
Main approximation: Linear response, i.e. harmonic potential approximation

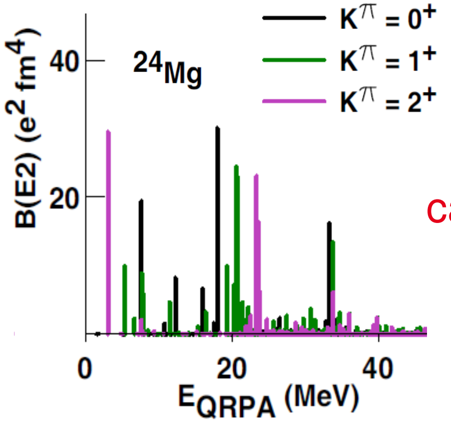
Main approximation:
Linear response, i.e. harmonic potential approximation



The present QRPA approach (ISAAC code) using matrix representation allows to provide excited state wave functions, excitation energies and transitions (probabilities and densities) from the GS for deformed nuclei with axial symmetry.

And the results ?



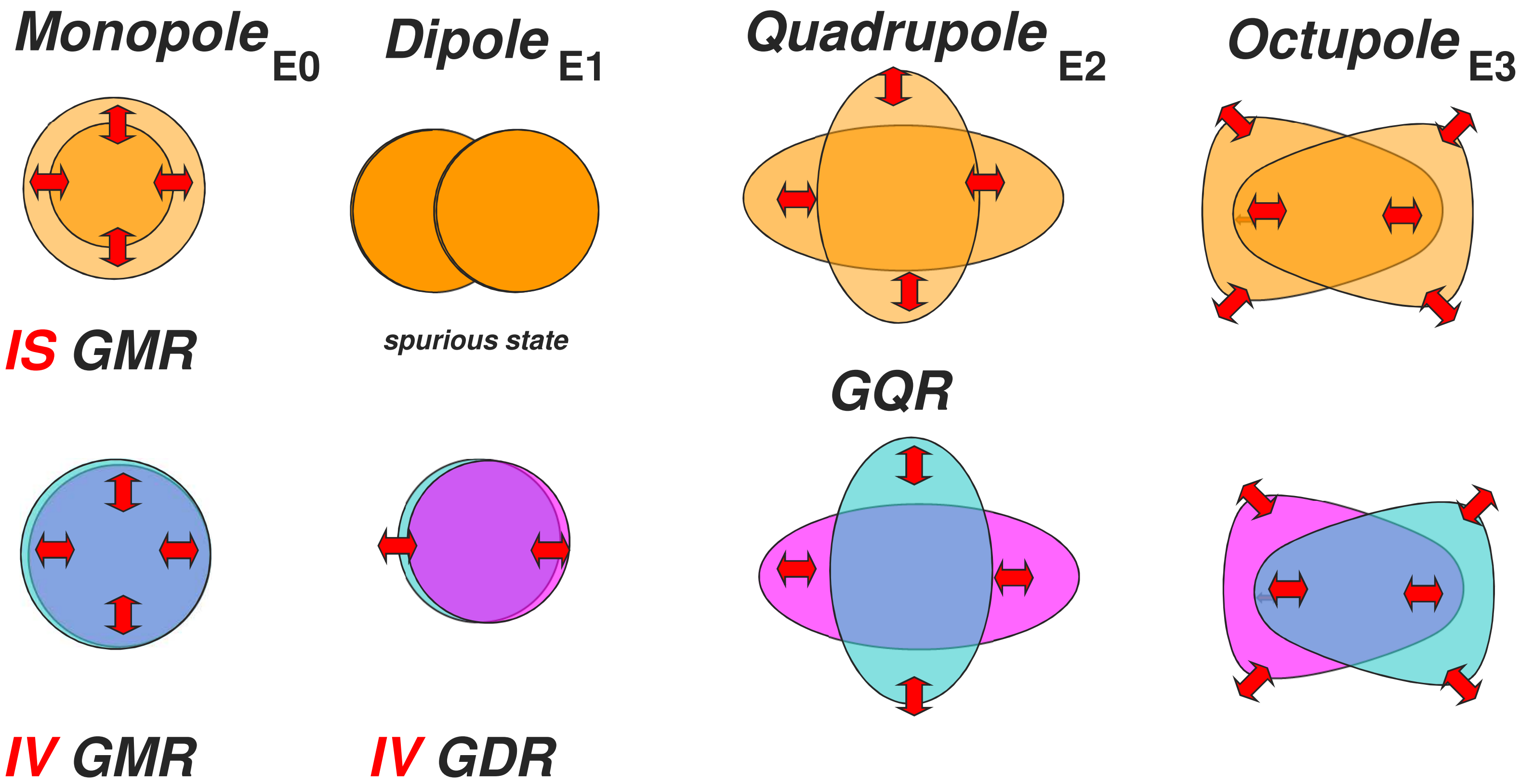


S. Péru and H. Goutte, PRC77, 044313 (2008)

What can be describe with it ?

Giant Nuclear resonances

Giant resonances are related to the properties of nuclear matter and are involved in reaction processes.



These resonances are called giant because almost all nucleons are involved and because they can be seen as a bump in cross sections.

QRPA provides good results for low energy collective states

PHYSICAL REVIEW C **98**, 014327 (2018)

Gogny-HFB+QRPA dipole strength function and its application to radiative nucleon capture cross section

S. Goriely,[1] S. Hilaire,[2] S. Péru,[2] and K. Sieja[3]
[1]*Institut d'Astronomie et d'Astrophysique, Université Libre de Bruxelles, CP-226, 1050 Brussels, Belgium*
[2]*CEA, DAM, DIF, F-91297 Arpajon, France*
[3]*Université de Strasbourg, IPHC, 23 rue du Loess 67037 Strasbourg, France CNRS, UMR7178, 67037 Strasbourg, France*

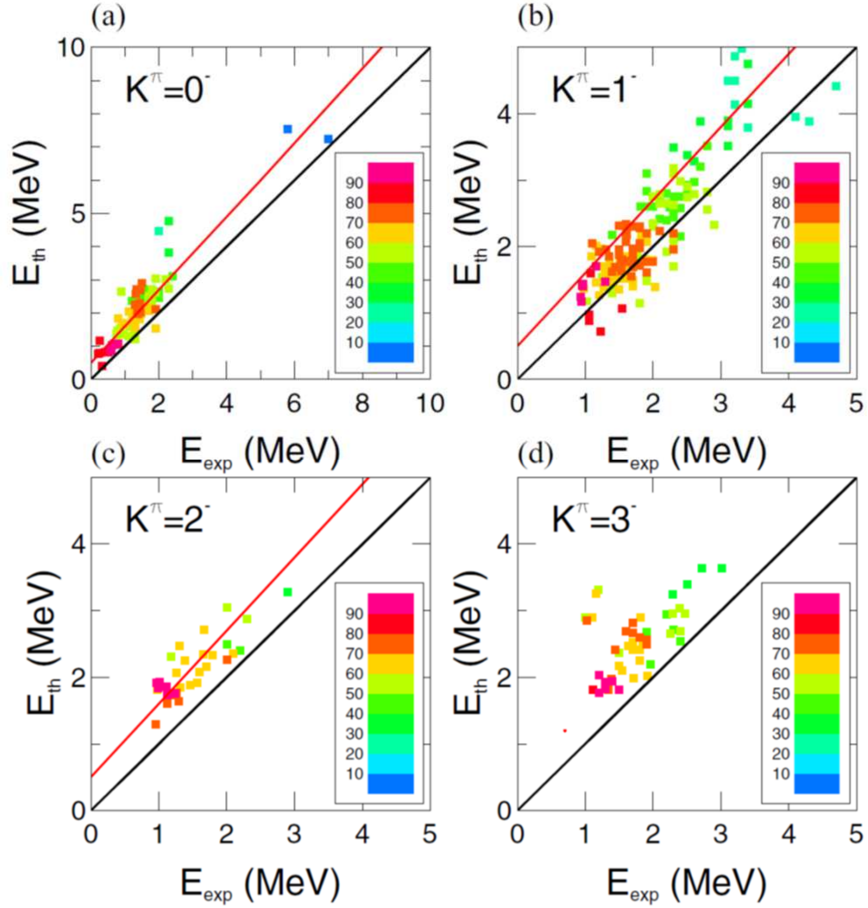


FIG. 1. Experimental–theoretical comparison of the energy of the low-energy vibrational states (a) $K^\pi = 0^-$, (b) $K^\pi = 1^-$, (c) $K^\pi = 2^-$, and (d) $K^\pi = 3^-$. The red curve corresponds to the energy shift applied to the HFB+QRPA strength. The color code refers to the atomic mass range of the nuclei.

For Vibrational modes !

3.2 Pygmy states

- Impact on reaction models (…on astrophysics studies)
- Nature of pygmy states, nuclear structure analysis

Nuclear Physics A 706 (2002) 217–232

Large-scale QRPA calculation of E1-strength and its impact on the neutron capture cross section

S. Goriely [a,*], E. Khan [b]

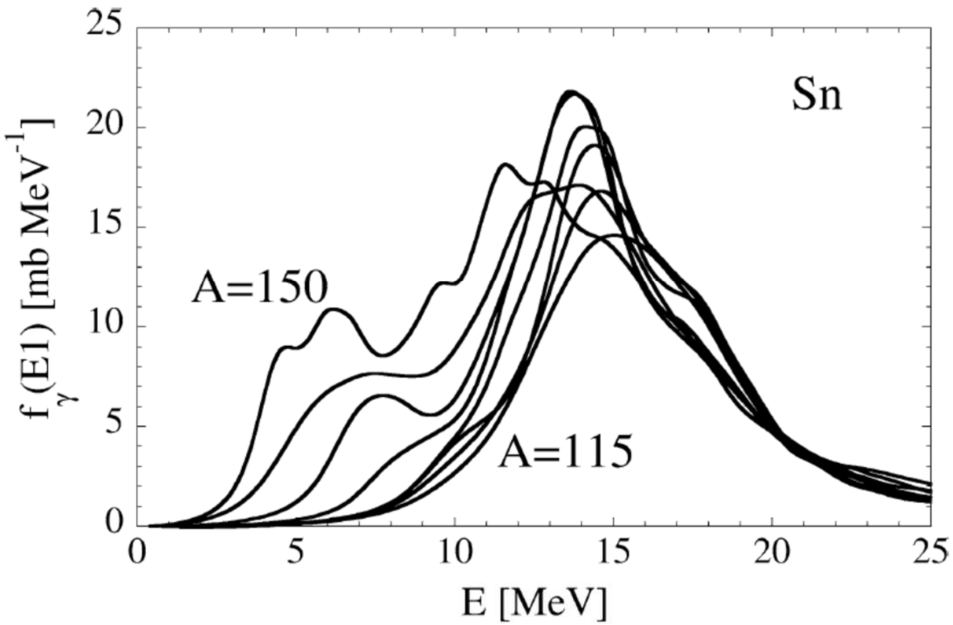


Fig. 5. E1-strength function for the Sn isotopic chain predicted by the QRPA with the SLy4 Skyrme force. Only isotopes ranging between $A = 115$ and $A = 150$ by steps of $\Delta A = 5$ are displayed.

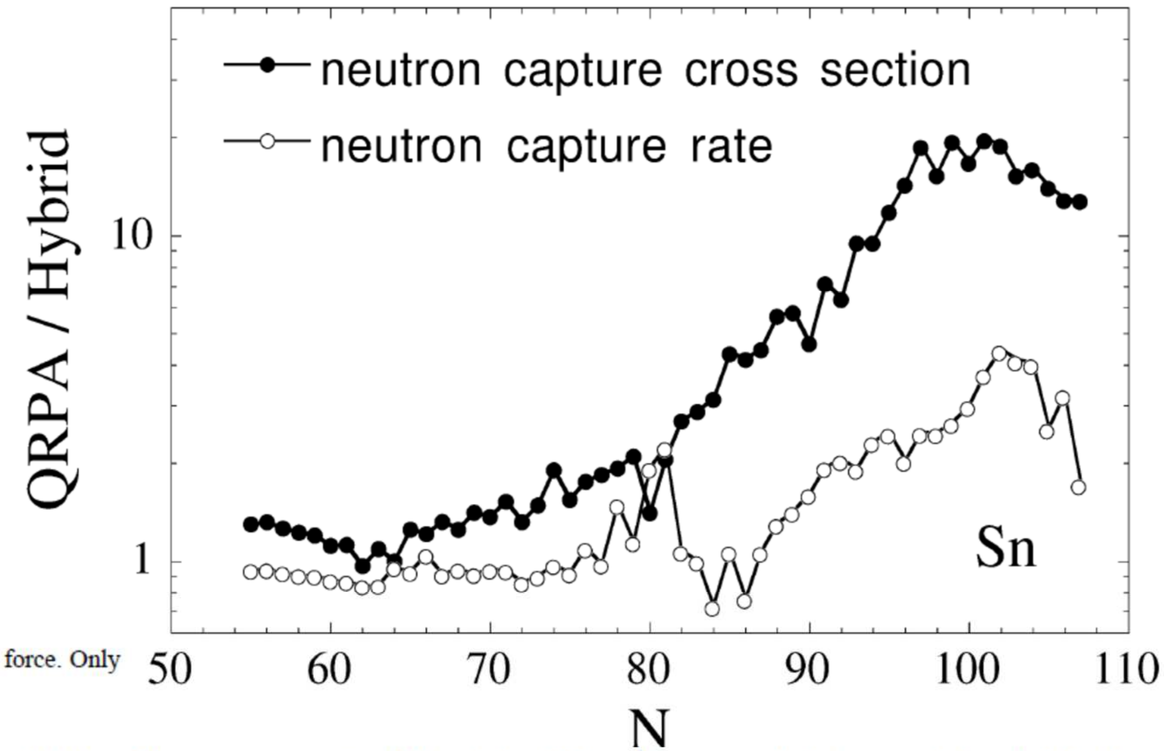


Fig. 6. Ratios of the (n, γ) cross section (full circles) and Maxwellian-averaged rate (open circles) predicted with the E1-strength taken from the QRPA to the one taken from the Hybrid formula [1] for the Sn isotopes. The ratios are plotted as a function of the neutron number. The cross section is estimated at an incident energy $E = 10$ MeV and the Maxwellian-averaged rate at a temperature of 1.5×10^9 K.

Impact of low energy part of the strength function in reaction rate

Maxwellian-averaged capture cross sections (MACS)

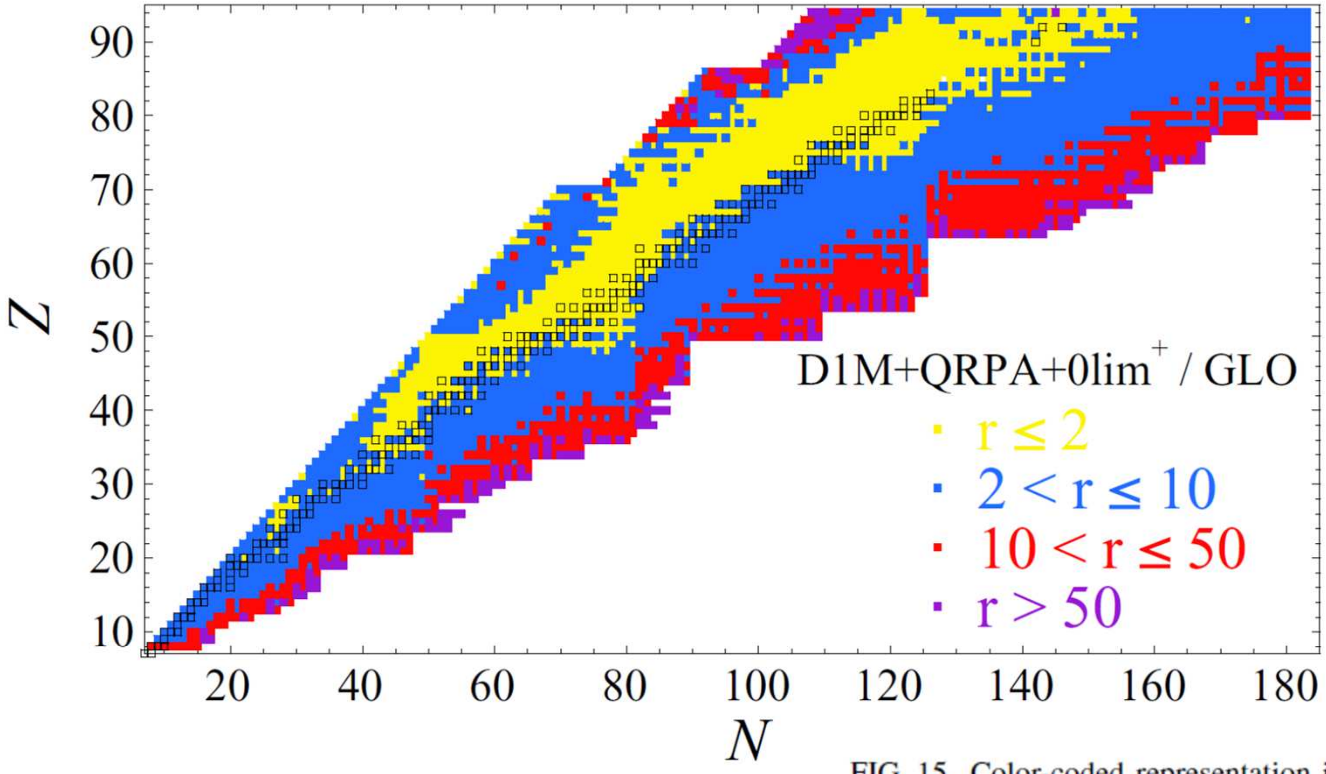
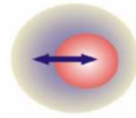
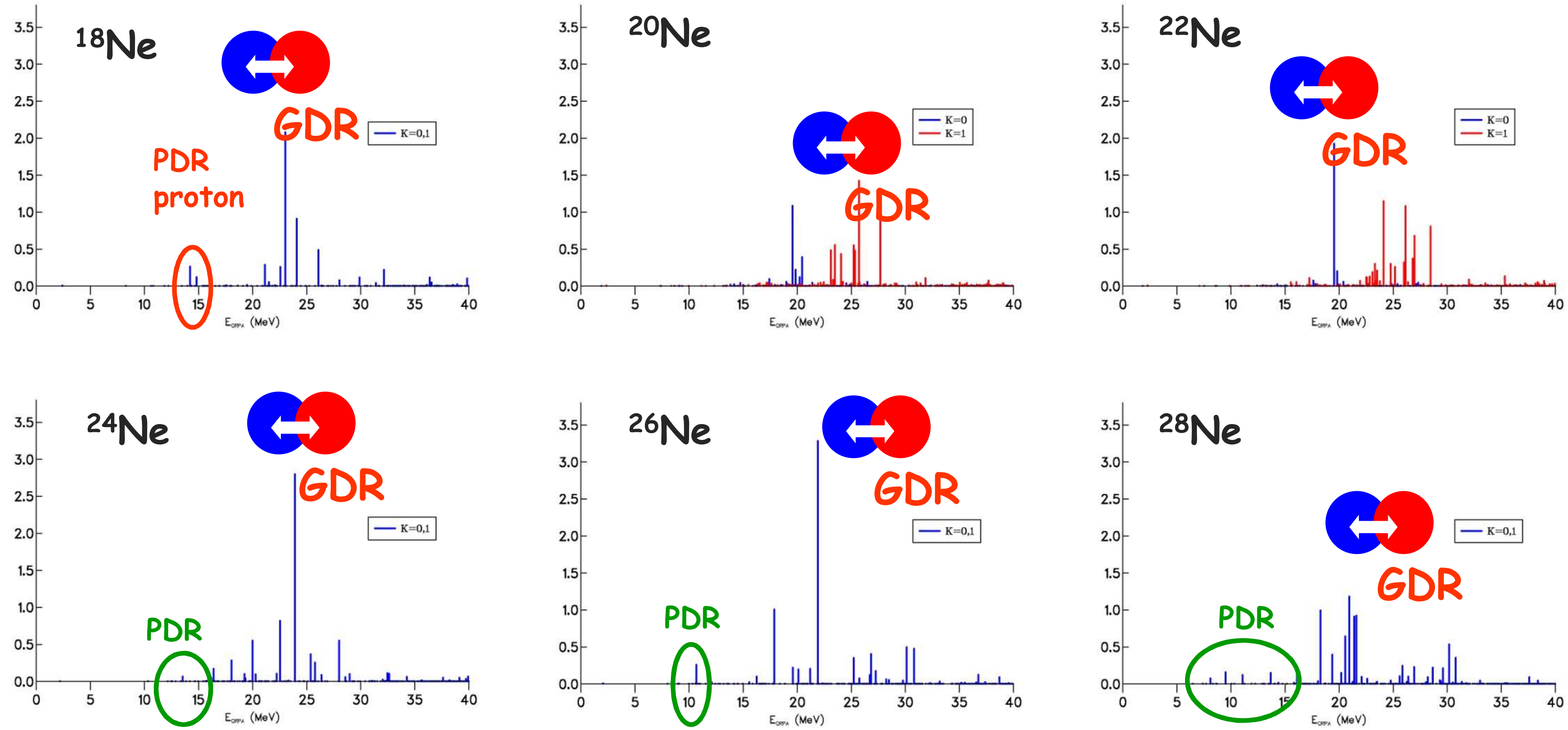


FIG. 15. Color-coded representation in the (N, Z) plane of the ratio of the (n, γ) MACS at $T = 10^9$ K obtained with the present D1M + QRPA + 0lim$^+$ [Eqs. (1) and (2)] to the one obtained with the GLO model [3,17]. Open black squares correspond to the stable nuclei and very long-lived actinides.

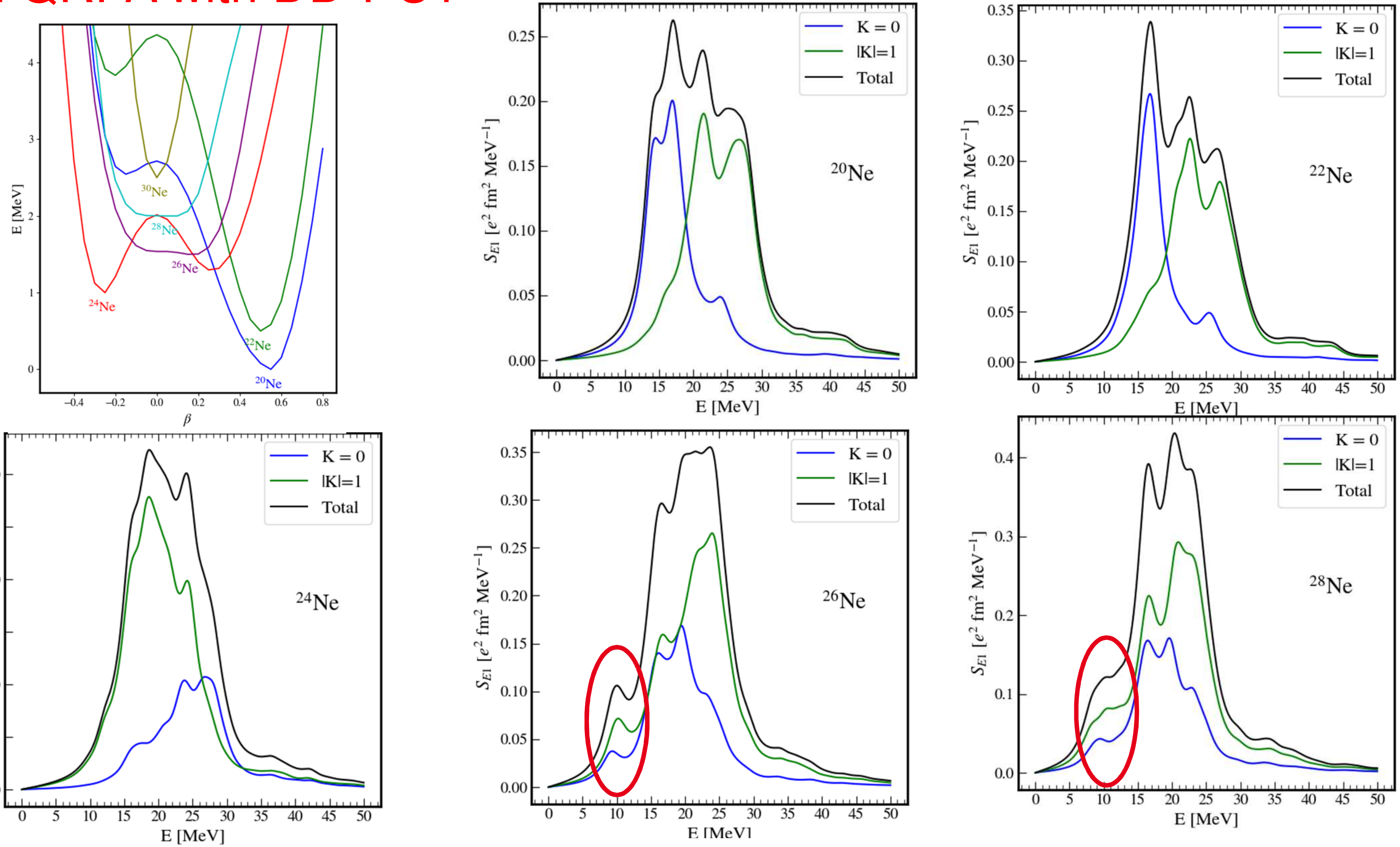
dipole response for Neon isotopes

PDR = 

Increasing neutron number

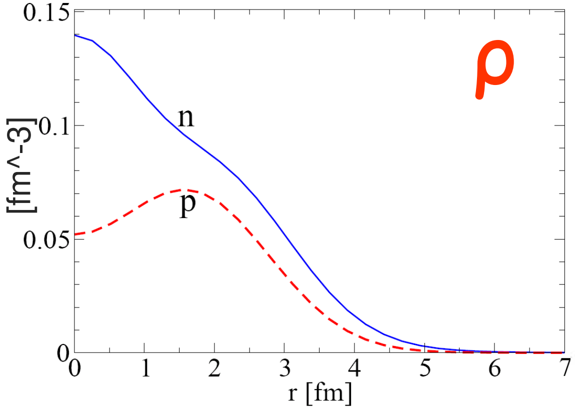


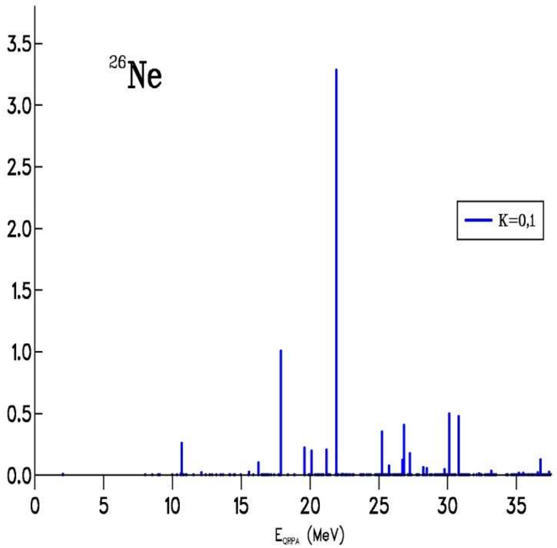
FAM QRPA with DD-PC1

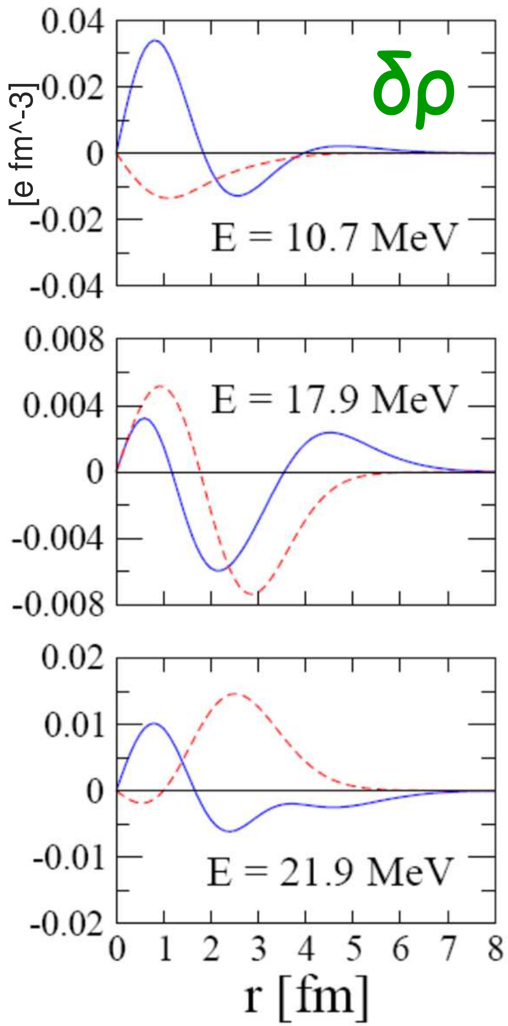


Luis González-Miret thesis, defense scheduled for December 2024

^{26}Ne



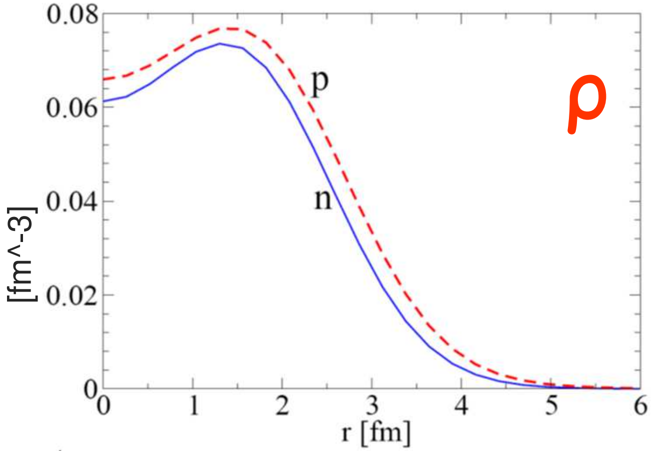


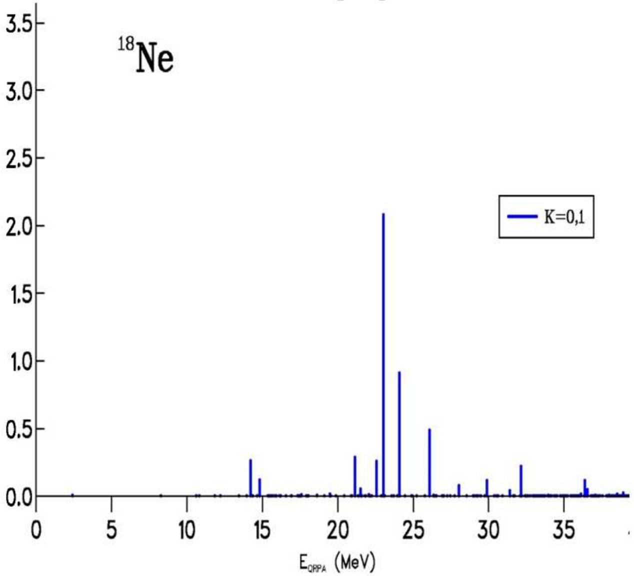


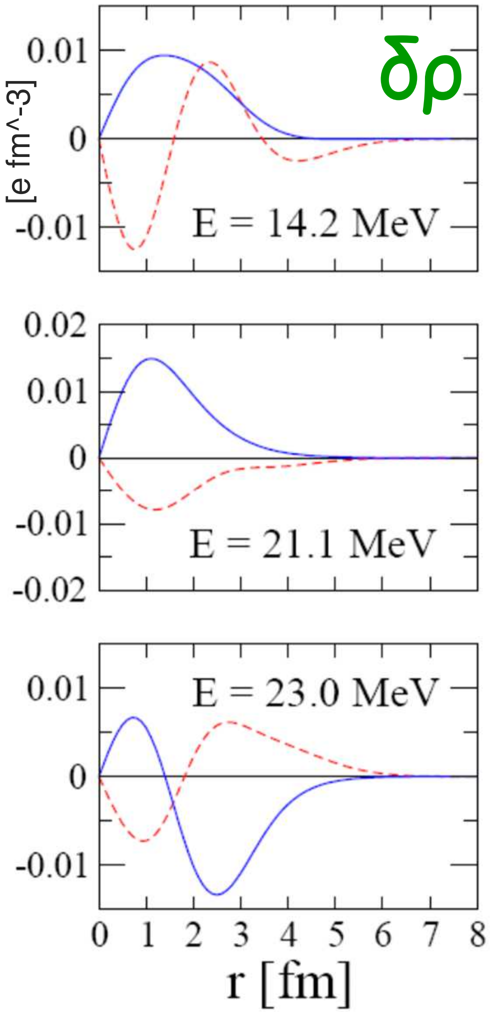
Neutron skin -> neutron PDR

$$\delta\rho \approx \rho_{Ex} - \rho_{GS}$$

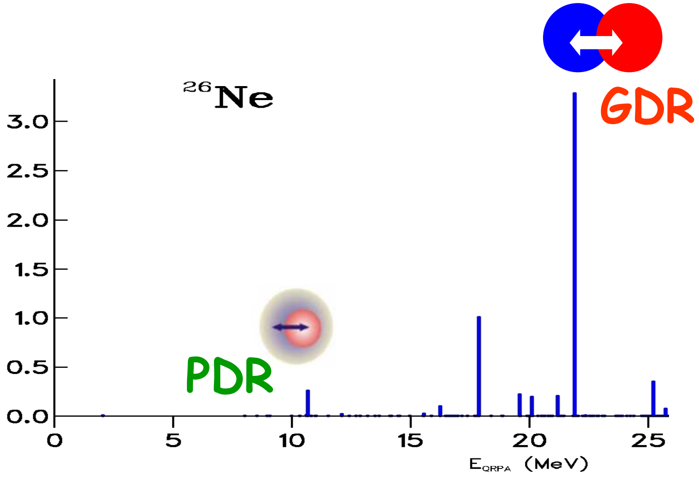
^{18}Ne



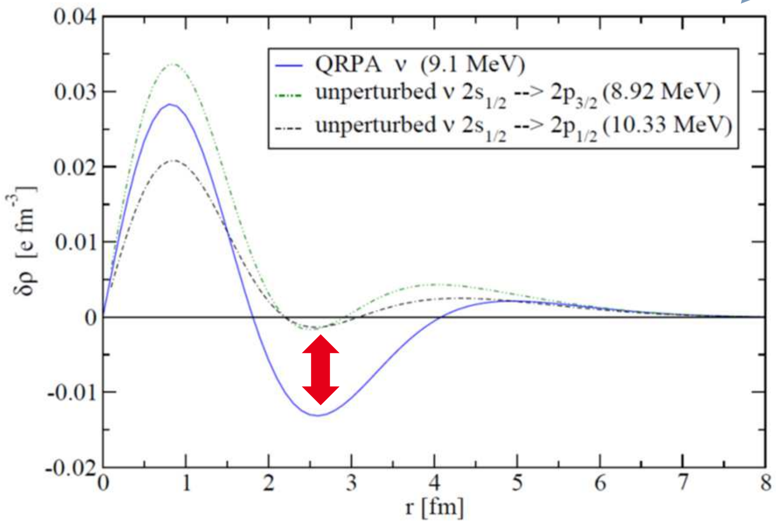




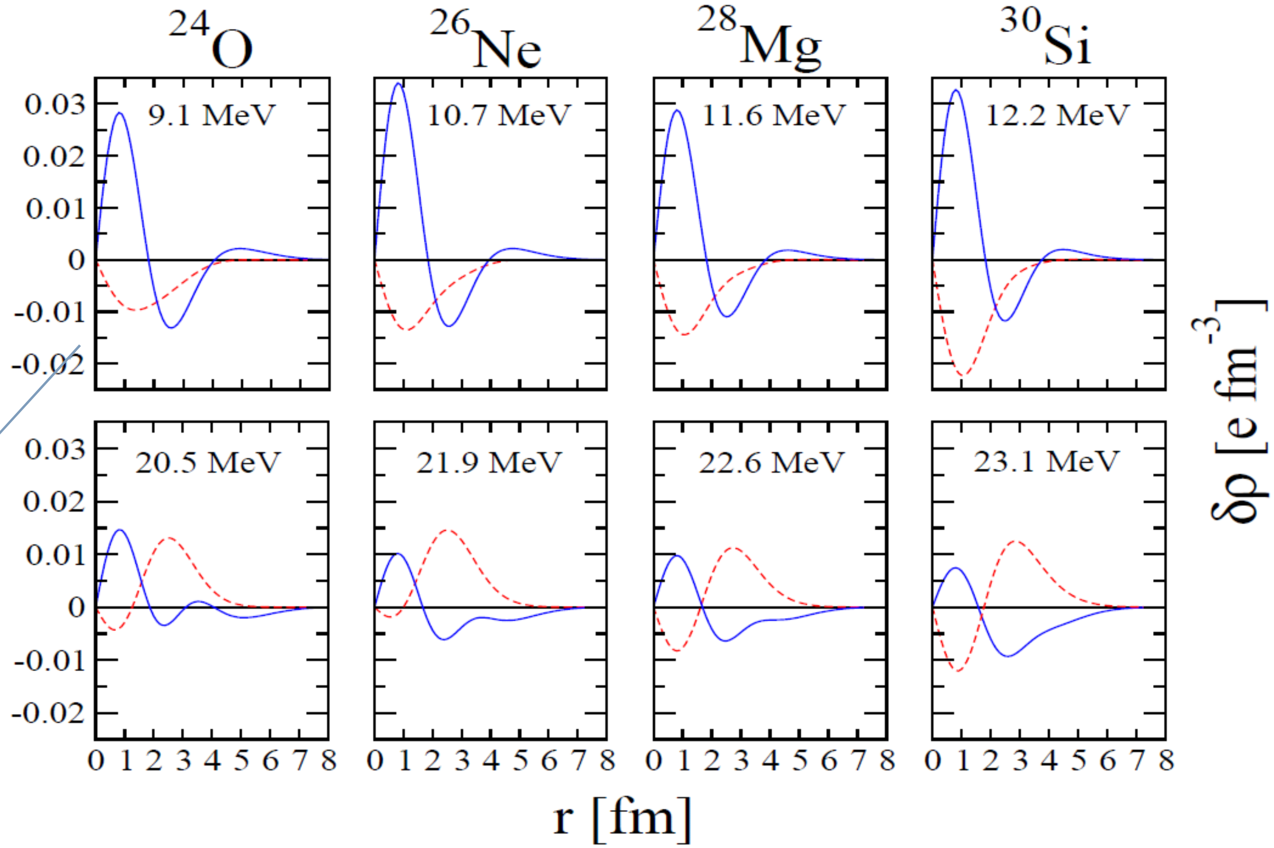
Proton skin -> proton PDR



GDR
PDR
^{26}Ne
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
25
E$_{QRPA}$ (MeV)



QRPA ν (9.1 MeV)
unperturbed ν 2s$_{1/2}$ --> 2p$_{3/2}$ (8.92 MeV)
unperturbed ν 2s$_{1/2}$ --> 2p$_{1/2}$ (10.33 MeV)
δρ [e fm^{-3}]
r [fm]



^{24}O
^{26}Ne
^{28}Mg
^{30}Si
9.1 MeV
10.7 MeV
11.6 MeV
12.2 MeV
20.5 MeV
21.9 MeV
22.6 MeV
23.1 MeV
δρ [e fm^{-3}]
r [fm]

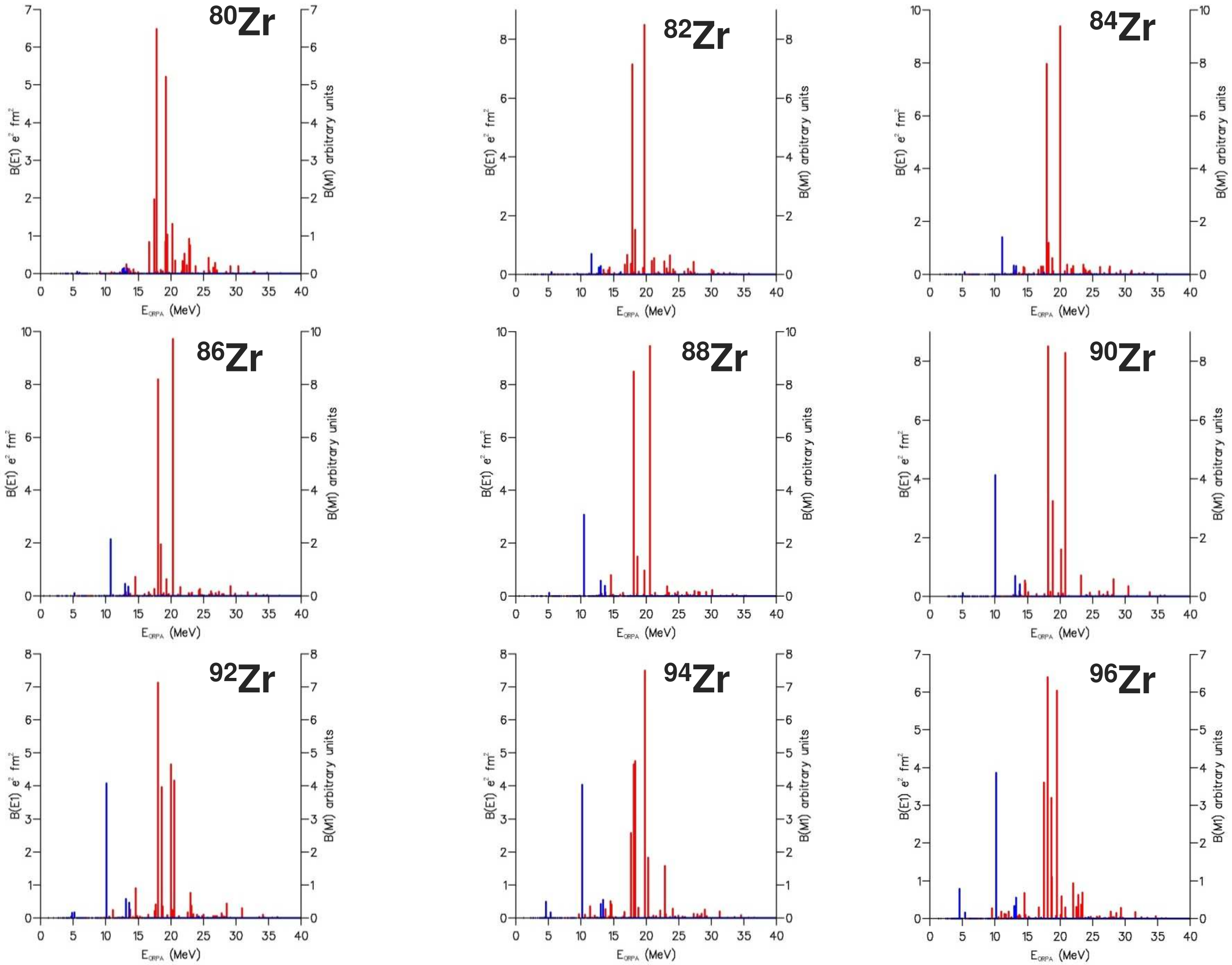
Dipole response for Zr isotopes :

M1 γ strength for ^{92}Zr,
H. Utsunomiya et al,
PRL 100, 162502 (2008)

M1 resonance occur
at similar energy
to the low E1

Low M1 resonances
are associated
to scissor modes

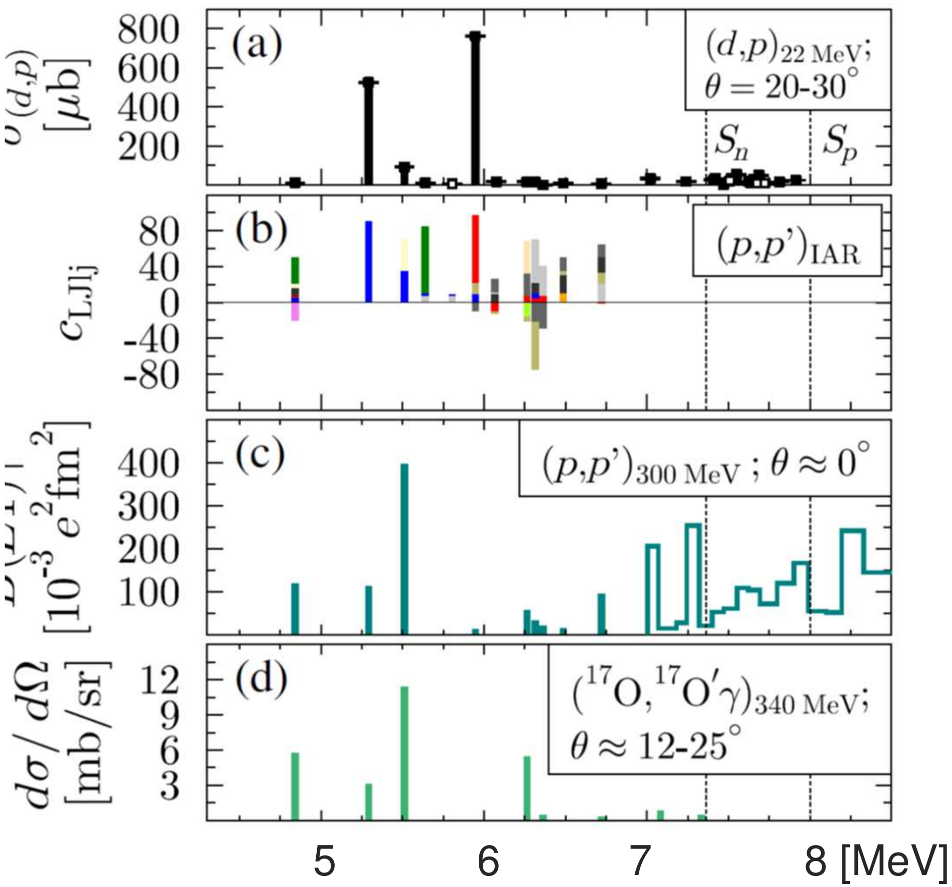
For Zr isotopes,
M1 strength increase with N



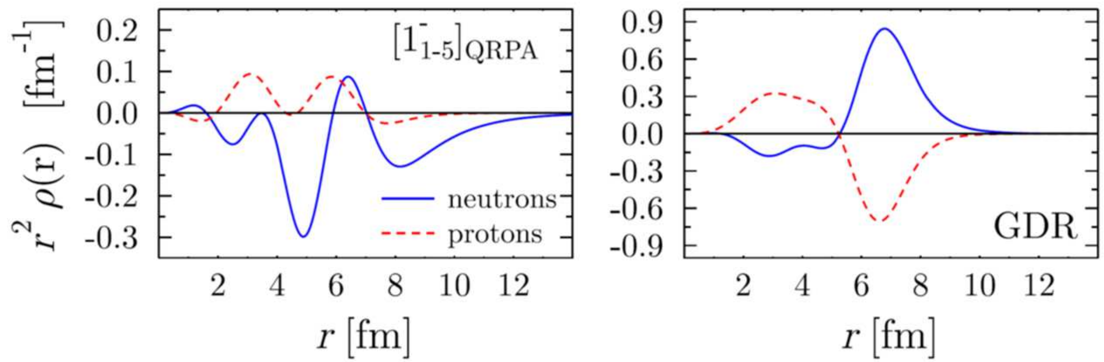
B(E1)
B(M1)

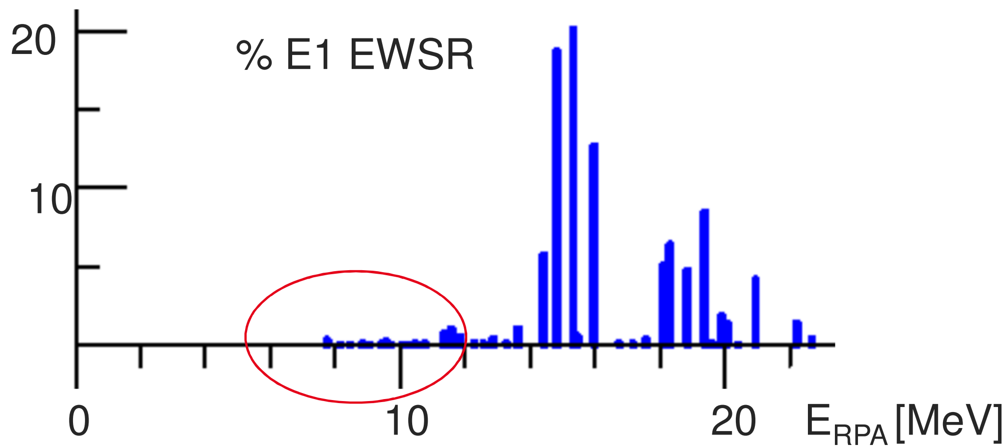
Does the dipole pygmy state occur only for exotic nuclei ?

M. Spieker et al, PRL125,102503 (2023)



M. Spieker et al, PRL125,102503 (2023)





SP, MM, EPJA 50:88, 2014;
SP, JFB, PFB,EPJA26,25-32,2005

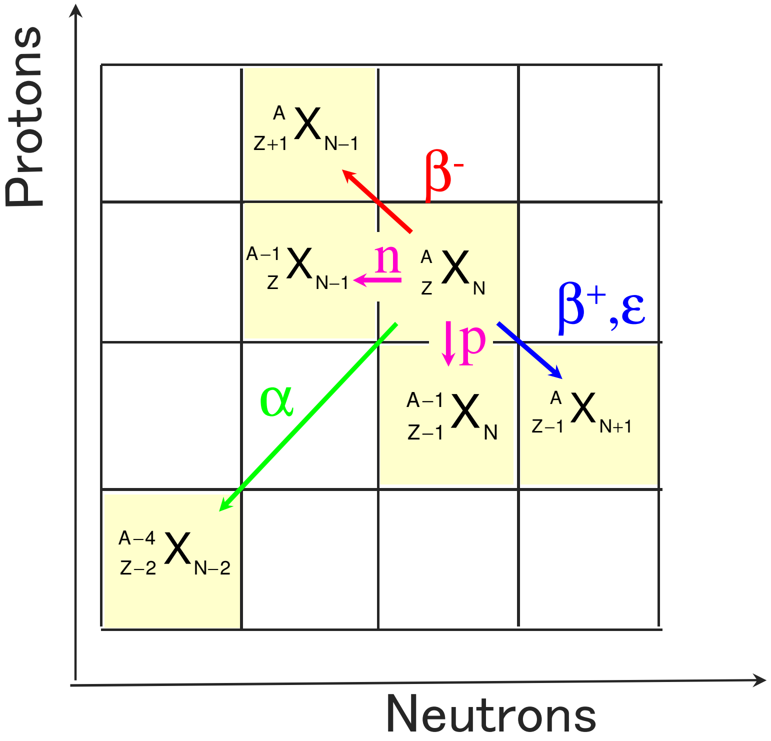
$^{208}_{82}\mathrm{Pb}_{126}$ can be considered as a neutron rich nucleus whatever its “doubly magic “nature

4. Beta decay

Theoretical description within the QRPA framework

Nuclear phenomena: radioactivity

Different types of radioactivity

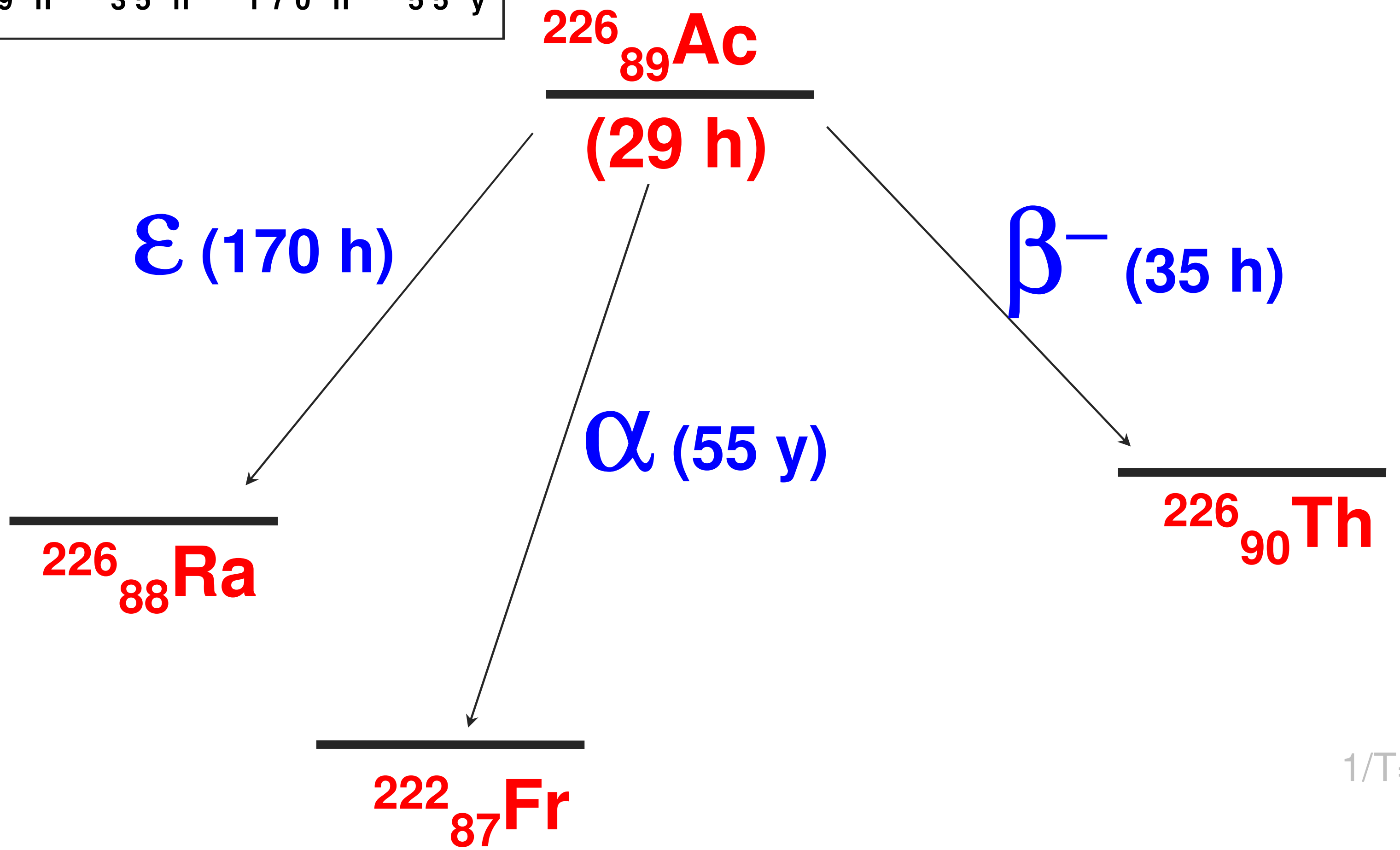


Half–lives cover several orders of magnitude

Nitrogen 16	$T_{1/2}$ = 7.13 s
Oxygen 15	= 2.037 mn
Radium 224	= 3.62 d
Carbon 14	= 5730 y
Molybdenum 100	= 7,3 10^{18} y
Tellurium 128	**= 2.4 10^{24} y**

radioactivity: half-life

$$\frac{1}{29\ \mathrm{h}} = \frac{1}{35\ \mathrm{h}} + \frac{1}{170\ \mathrm{h}} + \frac{1}{55\ \mathrm{y}}$$

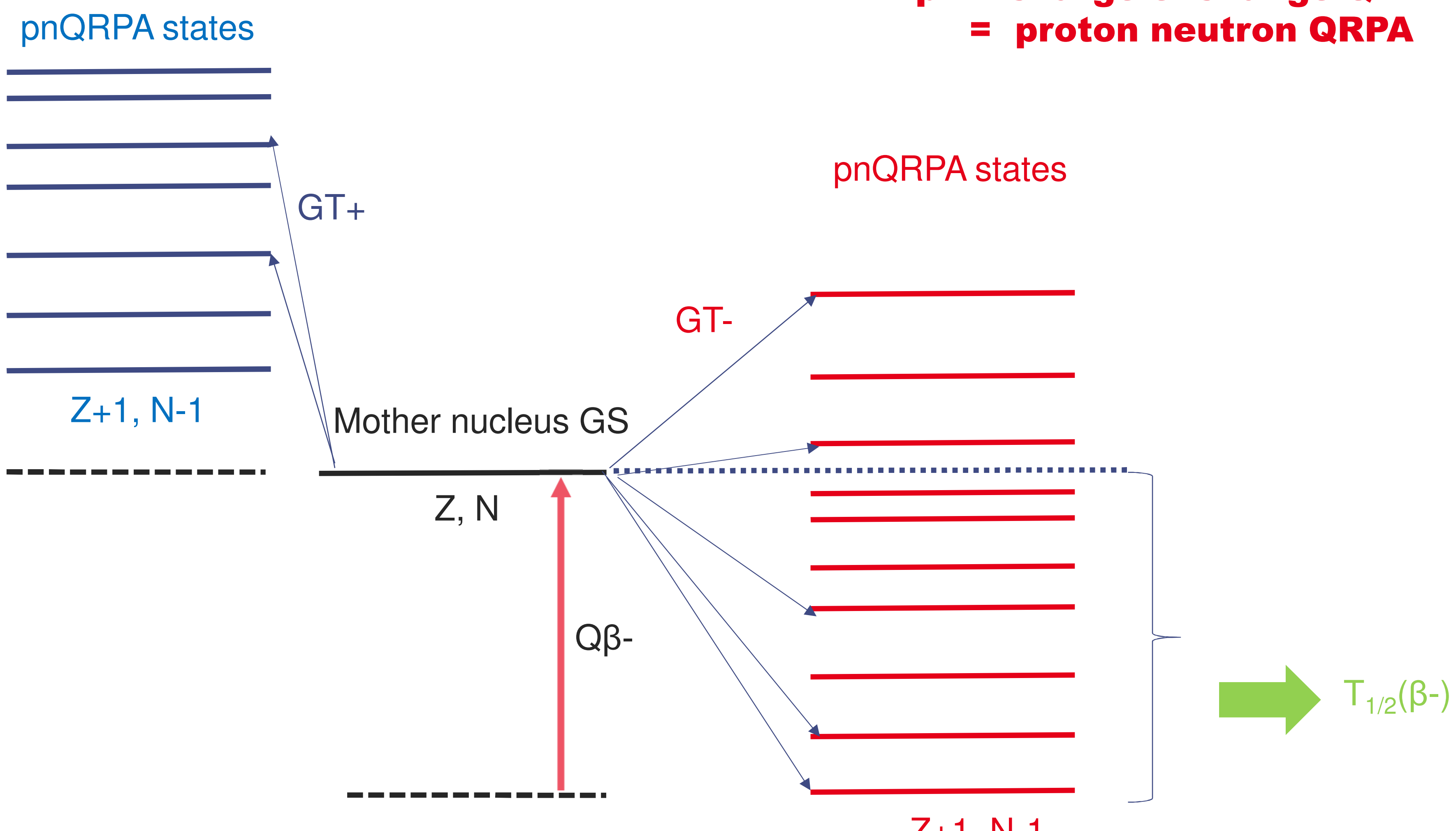


$\lambda = \lambda_\varepsilon + \lambda_\alpha + \lambda_\beta$

$1/T = 1/T_\varepsilon + 1/T_\alpha + 1/T_\beta$

Microscopic approach of beta decay

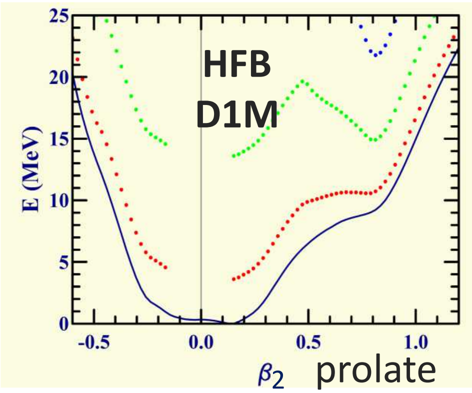
Xpn = charge exchange QRPA
= proton neutron QRPA



Sophie Péru, CEA,DAM, DIF, sophie.peru-desenfants@cea.fr

Charge exchange QRPA : ^{76}Ge a deformed nucleus

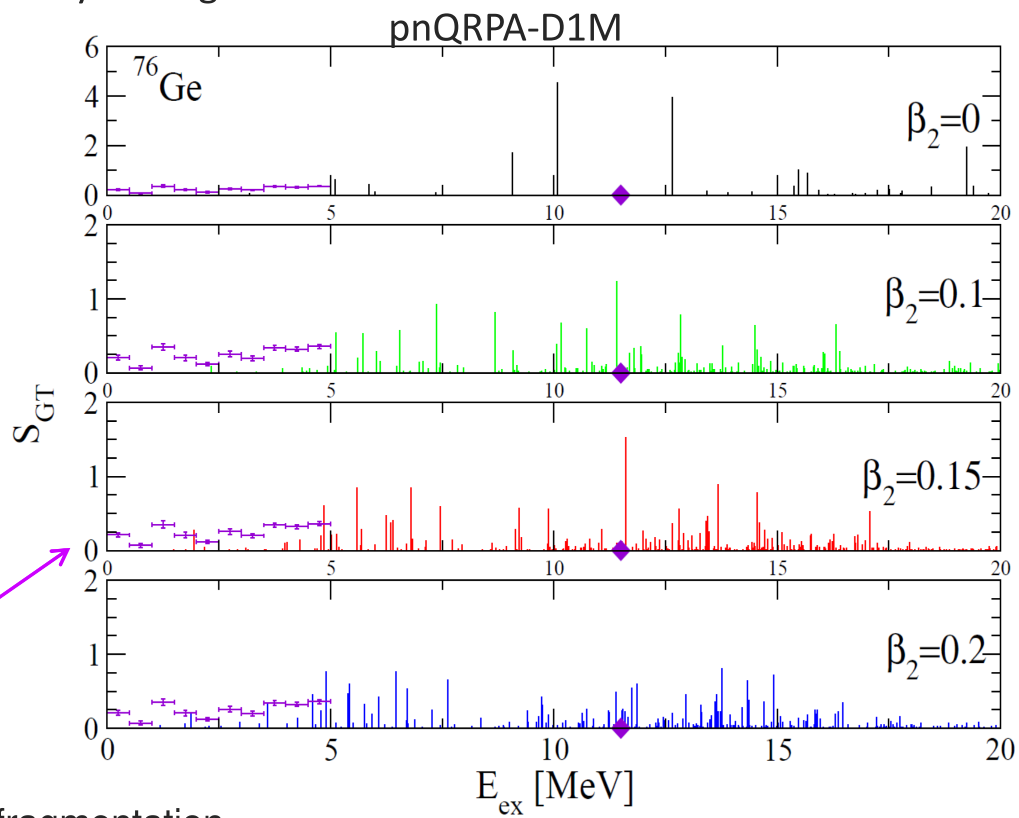
GT $J^{\pi}=1^{+}$ distributions obtained by adding twice the $K^{\pi}=1^{+}$ result to the $K^{\pi}=0^{+}$ one



β_2(min. HFB) = 0.15 γ(min.HFB) = 0°

β_2 (0^{+}_{1}:5DCH) = 0.26 γ(0^{+}_{1}:5DCH) = 26°

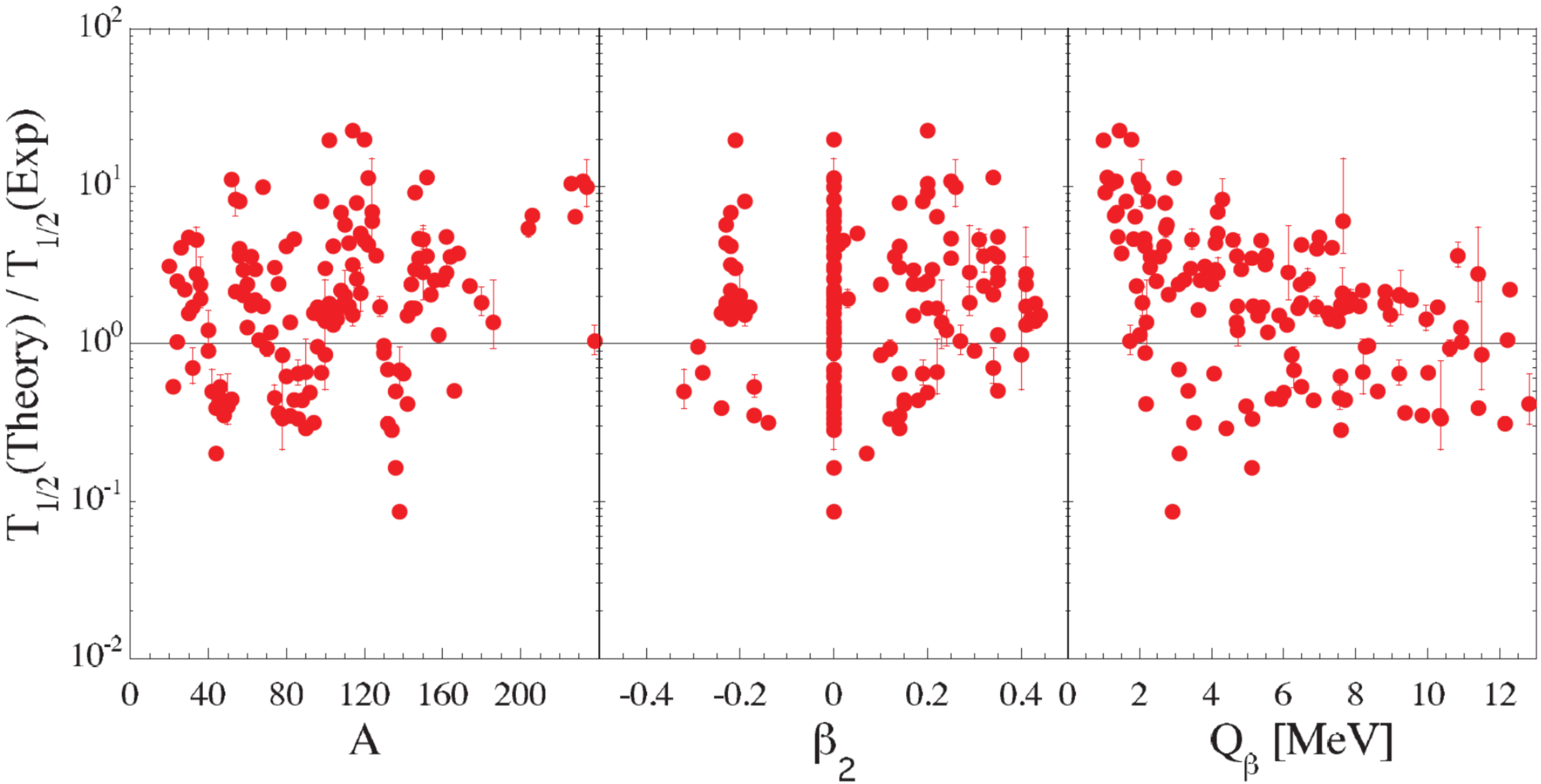
Experiment
Thies et al., Phys. Rev. C 86, 014304 (2012)



M. Martini, S. Péru and S. Goriely, Phys. Rev. C **89**, 044306 (2014)

- The deformation tends to increase the fragmentation
- Displacements of the peaks
- Deformation influences the low energy strength hence β decay half-lives are expected to be affected

β⁻ decay half-life $T_{1/2}$ Comparison experimental data



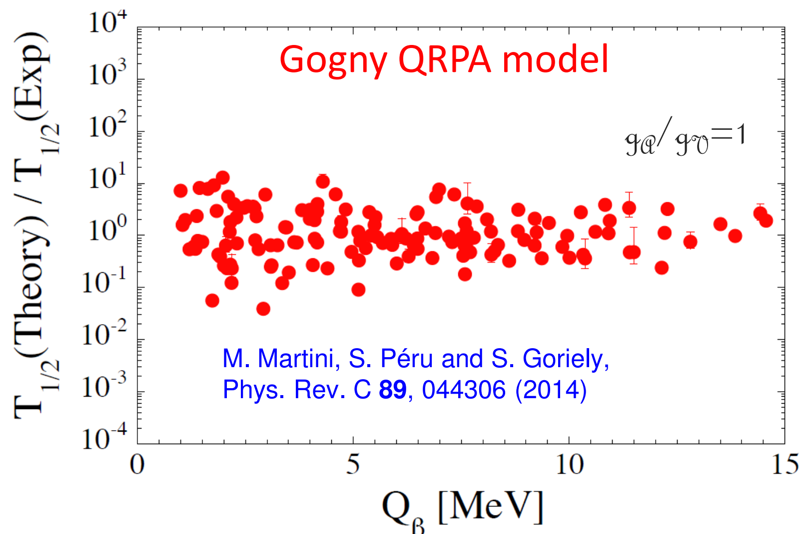
M. Martini, S. Péru and S. Goriely, Phys. Rev. C **89**, 044306 (2014)

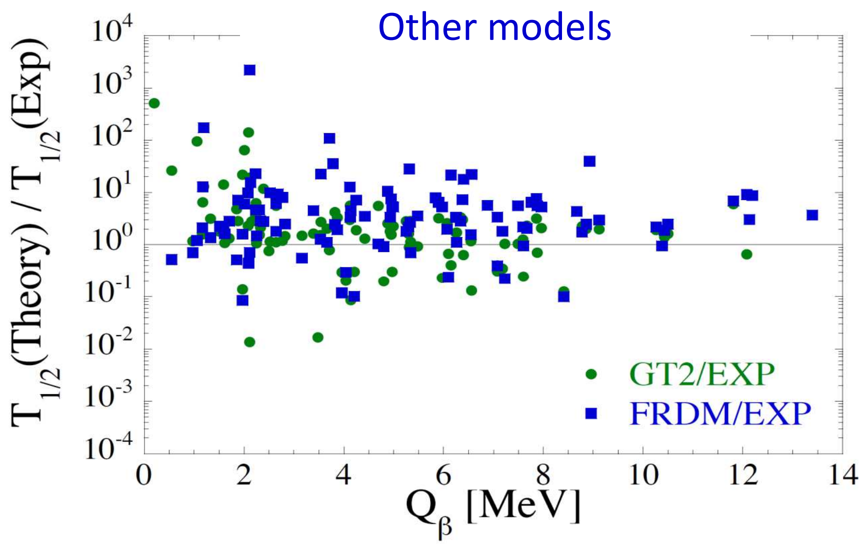
β^- decay half-life $T_{1/2}$ Comparison with other models and experimental data

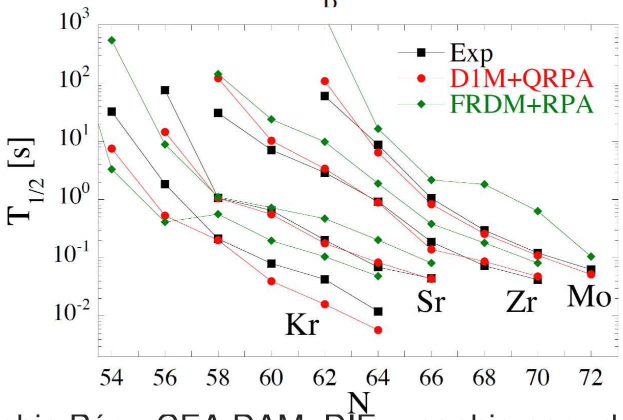
$$\frac{\ln 2}{T_{1/2}} = \frac{(g_A/g_V)^2_{\rm eff}}{D} \sum_{E_{ex}=0}^{Q_\beta} f_0(Z, A, Q_\beta - E_{ex}) S_{GT}(E_{ex})$$

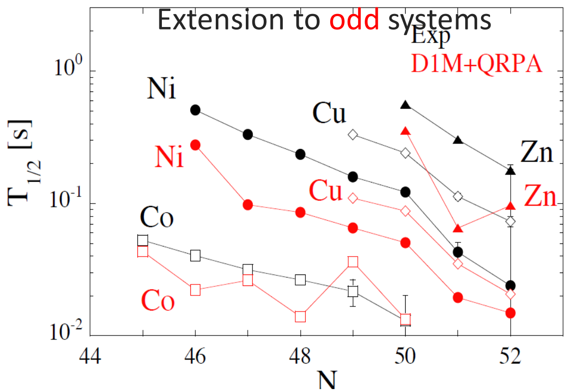
FRDM: Moller et al., ADNDT, 66,131 (1997)

GT2: Tachibana et al. Prog. Theor. Phys., 84, 641 (1990)







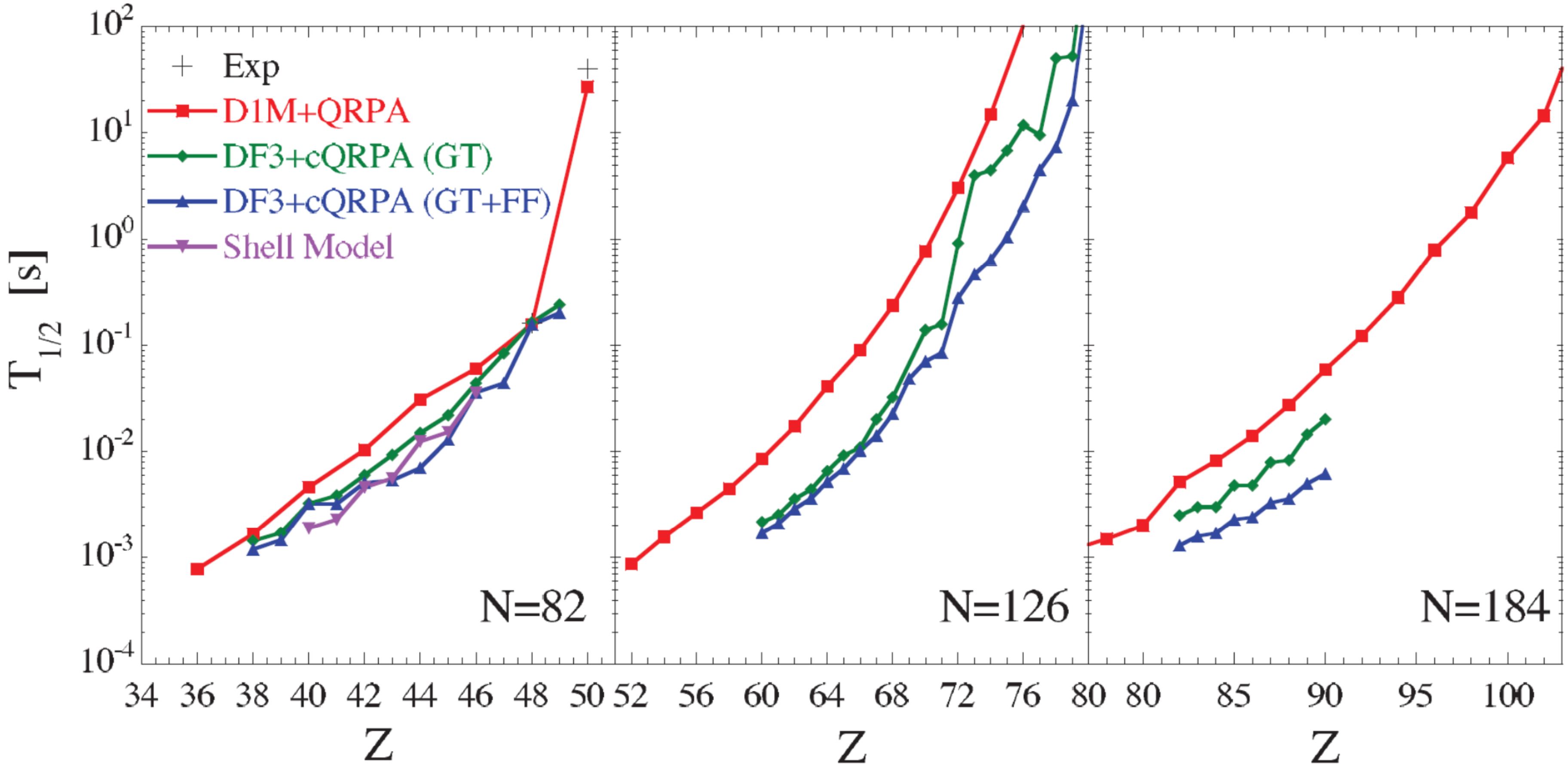


Experimental results

Z.Y. Xu et al, PRL 113, 032505 (2014)

β-decay Half lives of ^{76,77}Co, ^{79,80}Ni and ^{81}Cu : Experimental indication of a Doubly Magic ^{78}Ni

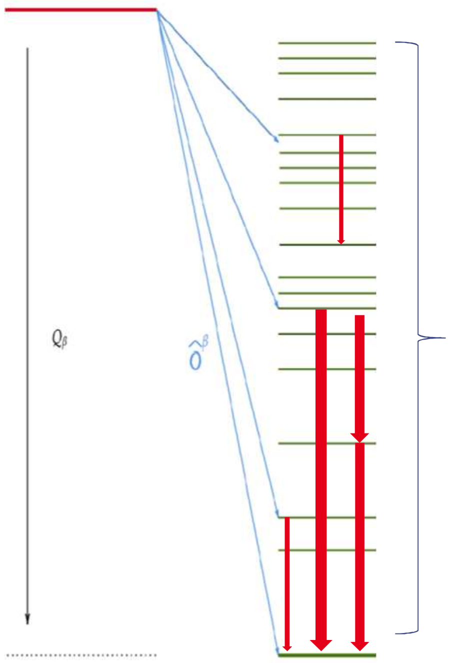
β^- decay half-life $T_{1/2}$ Comparison with other models and experimental data

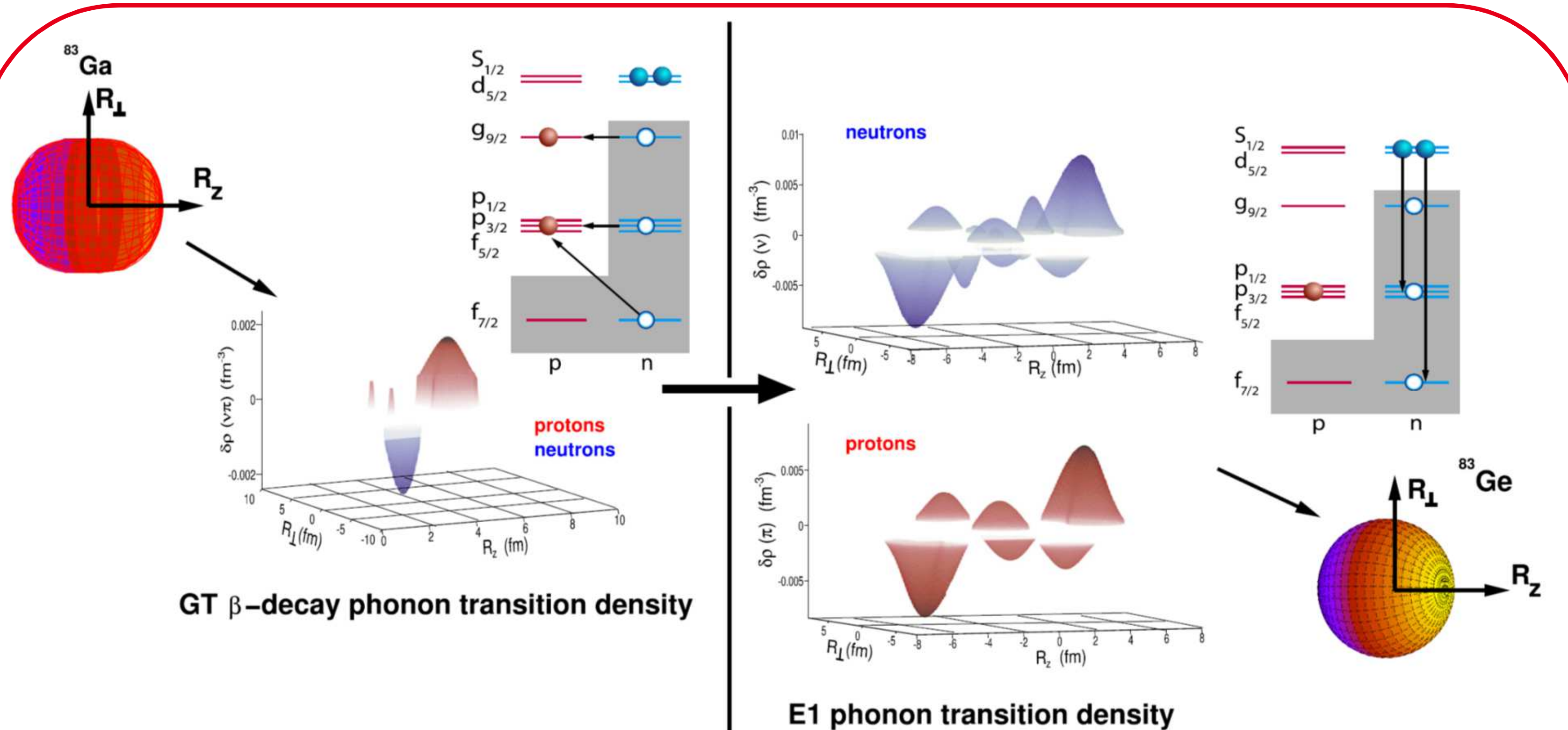


M. Martini, S. Péru and S. Goriely, Phys. Rev. C **89**, 044306 (2014)

Unique QRPA framework for beta decay and pygmy analysis

Unexpected high-energy gamma emission from decaying exotic nuclei.





A. Gottardo et al, PLB772, 359-362 (2017)

Little recap

- Exotic nuclei cover most of the nuclear chart
- All of them are important for astrophysical studies
- Transition probabilities and half-lives are challenging observables for models
  - ✓ This last point implies that an accurate description of both ground and excited nuclear states is needed