Neutron Sources

Neutron Field Theory, moderation and response function

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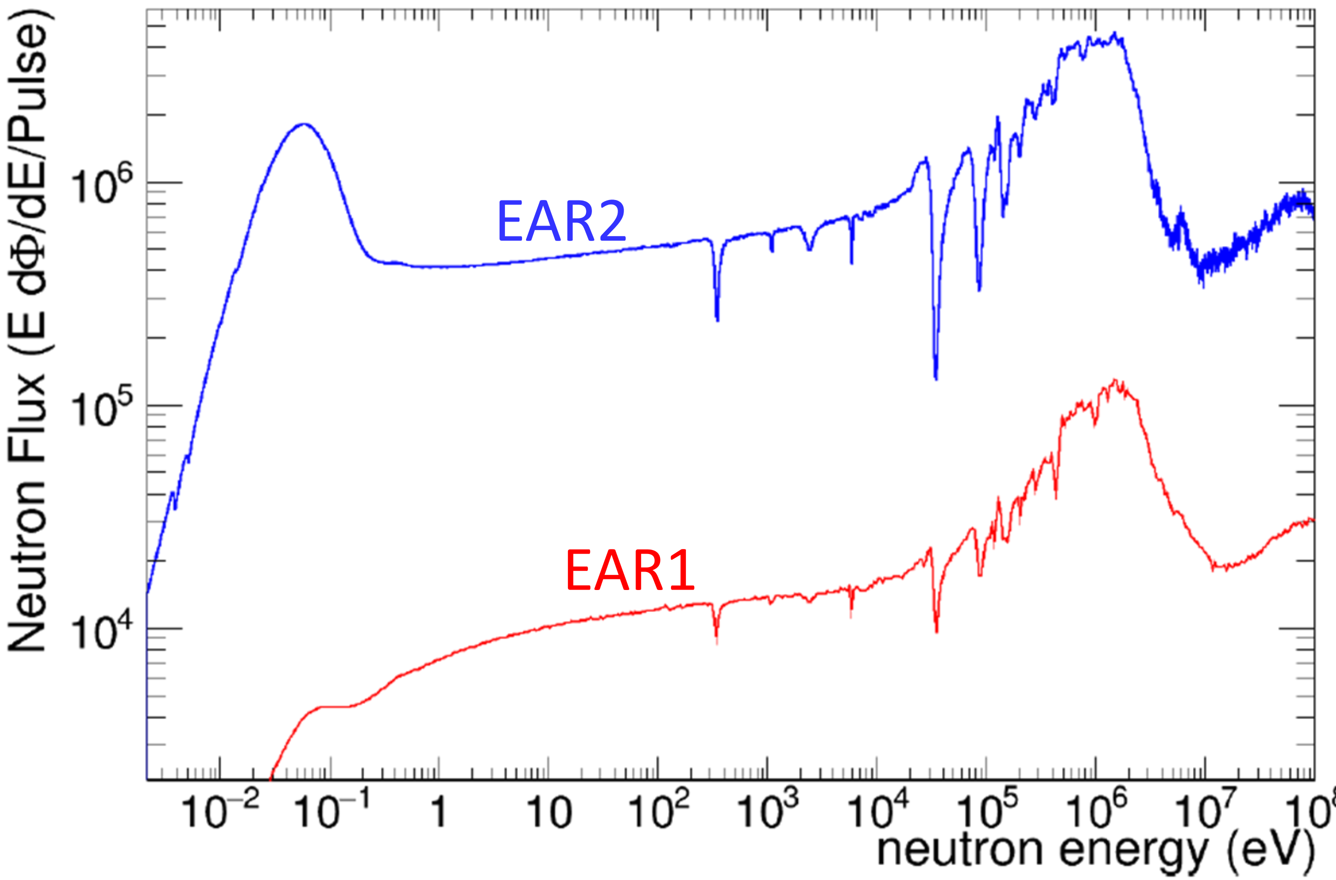
Outline

- Neutron sources
- Slowing down of neutrons
- Neutron Diffusion
- Flux – Fluence – Current
  - Isolethargic
- Resolution function
  - Energy-Time relation
  - Effective Neutron path
- Spallation target optimization



Experimental neutron fluence per 7 $10^{12}p^{+}$

Neutron Sources

Through Reactions:

With low Binding energy of the last neutron (Neutron Separation Energy)

- light nuclei: typically small apart from α-composed atoms
- intermediated: *7-10 MeV*
- heavy: *6-7 MeV*

due to Coulomb barrier, light nuclei play the predominate rule (at low energies)

(α,n)

- Excitation energy ~*10 MeV*, sometimes exothermic, or endothermic.
  → Continuous energy spectrum
  e.g. $^{9}Be + a \rightarrow 12C + n + 5.704\ MeV$

(d,n)

- Due to small binding energy of the deuteron (*2.2 MeV*) and a very highly excited compound nucleus is almost always exothermic
  e.g. $^{7}Li + d \rightarrow {}^{8}Be + n + 15.028\ MeV$

(p,n)

- typically mono-energetic sources
  e.g. $^{7}Li + p \rightarrow 7Be + n\ - 1.646\ MeV$

(γ,n)

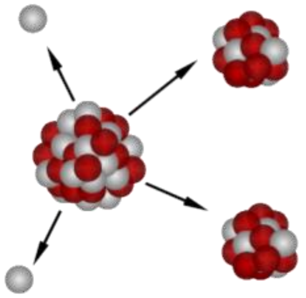
- nearly mono-energetic neutron production
  e.g. $^{9}Be +\ \gamma \rightarrow {}^{8}Be + n\ - 1.666\ MeV$

Nucleus	Binding Energy (Mev)	Nucleus	Binding Energy (Mev)
H^2	2.225	C^{12}	18.720
H^3	6.258	C^{13}	4.937
He^3	7.719	C^{14}	8.176
He^4	20.577	N^{13}	20.326
He^5	–0.956	N^{14}	10.553
Li^6	5.663	N^{15}	10.834
Li^7	7.253	N^{16}	2.500
Li^8	2.033	O^{15}	13.222
Be^8	18.896	O^{16}	15.669
Be^9	1.665	O^{17}	4.142
Be^{10}	6.814	O^{18}	8.047
B^9	18.575	F^{18}	9.141
B^{10}	8.440	F^{19}	10.442
B^{11}	11.456	F^{20}	6.599
C^{11}	13.092		

Binding energy of the last neutron in light nuclei

Neutron Sources

- **Radioactive**
  - (α,n): Ra-^{9}Be, Bi-Be, Pu-Be,...
  - (γ,n): ^{9}Be (*1.66 MeV*), ^{2}H(*2.2 MeV*) (almost mono-energetic)
- **Fission**
  - On average 2.5 $\pm$ 0.1 neutrons



- **Via Bremsstrahlung**
  - Using electron accelerators $E_{e-} \approx$ *50-100 MeV*
  - Heavy target → γ → (γ,n) or photo-fission
- **Spallation**
  - A violent reaction of a high energy particle on a heavy target. Disintegrates the nucleus through intra-nuclear cascade emitting numerous nucleons (protons, neutrons, alpha,...)

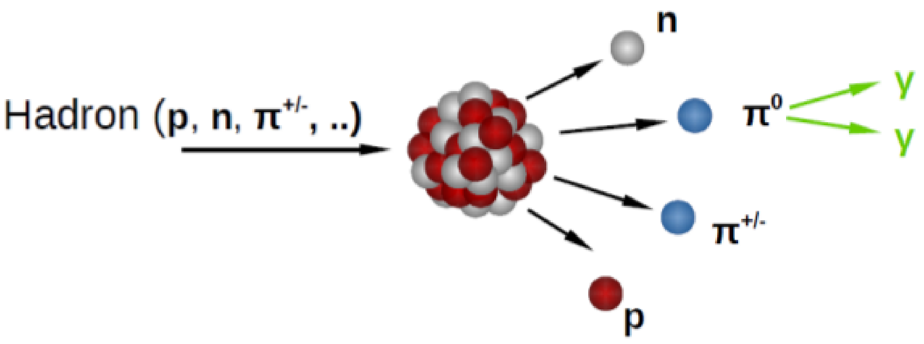
Hadron-Nucleus interactions: basics (simplified)

Fast stage (10^{-22} s)

Hadron interacts with nucleons: particle production possible (mainly **π**)

Intra-nuclear cascade of p, n, π:
→ energetic particles can leave nucleus (→ forward directed)
→ others can deposit energy in nucleus (→ excited state)

e.g. nucleon-nucleon: π production opens at 290 MeV for a free nucleon (somewhat lower for nucleons in nucleus)



fast particle multiplicity ~ log(E)

Hadronic + EM cascades

Slow stage (10^{-16} s)

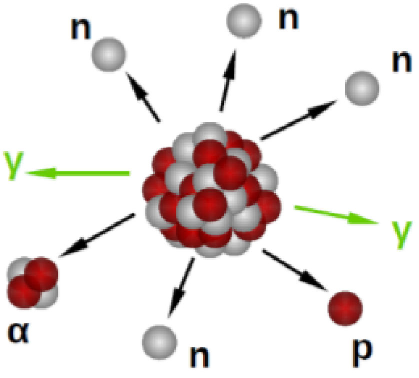
Pre-compound

Equilibrium

Evaporation
(n, light fragments)
γ-deexcitation

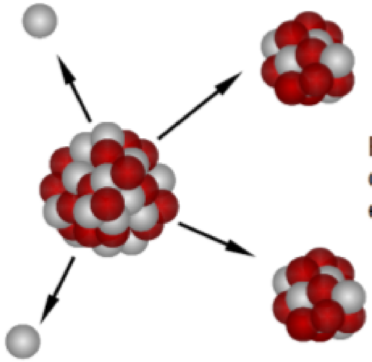
isotropic emission

few MeV



compete

Fission
(heavy elements)

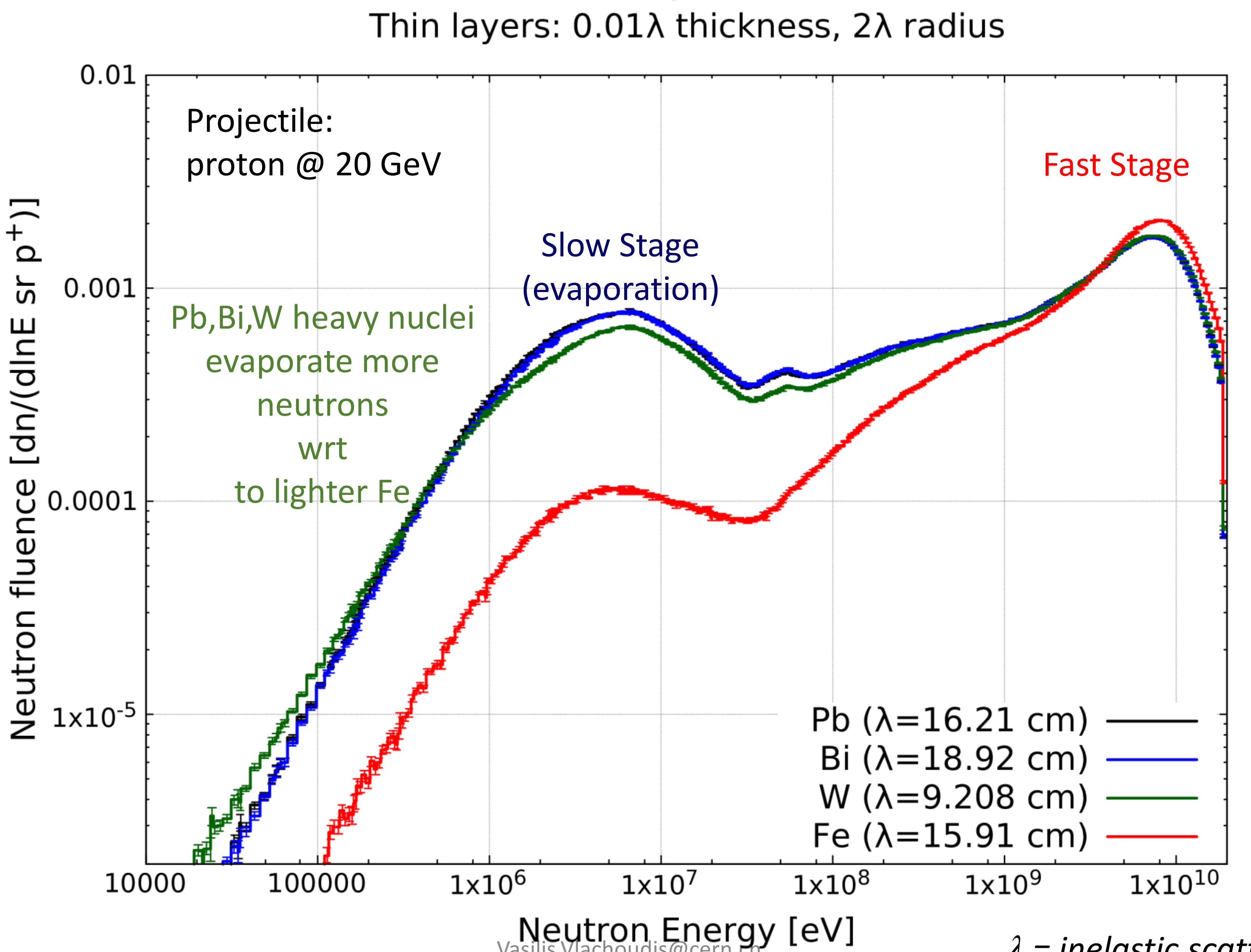


Fission products can also undergo evaporation

→ residuals can be radioactive

- At GeV energies there is no formation of compound nucleus

Hadron-Nucleus: Neutron yield

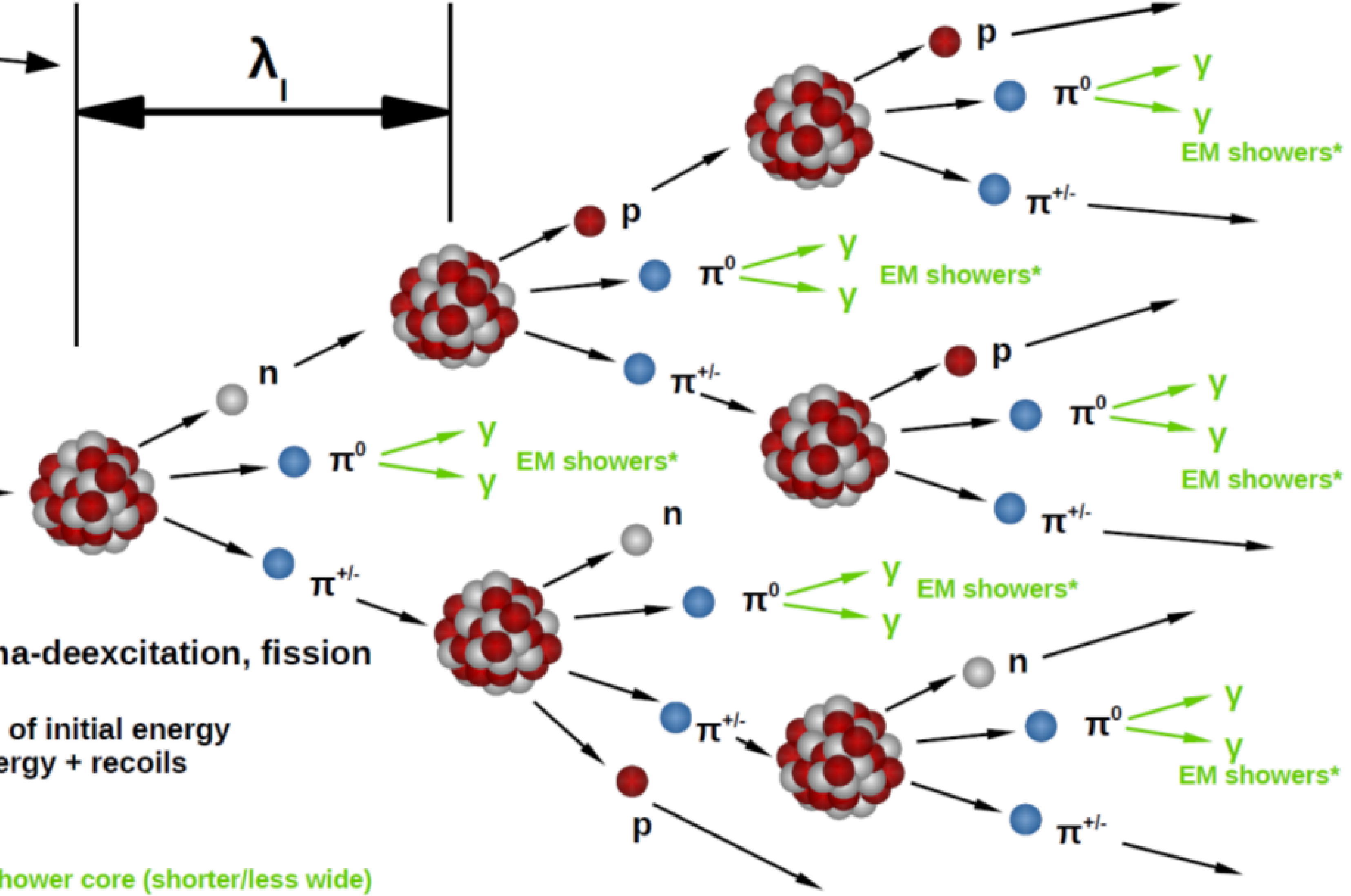


λ = inelastic scattering length

Hadronic shower: basics

Are longer than
EM showers
(X_0 smaller than λ_I)

Hadron (p, n, $\pi^{+/-}$, ..)



+Evaporation, gamma-deexcitation, fission

Non-negligible fraction of initial energy
can go into binding energy + recoils

*EM showers:
→ concentrated along the shower core (shorter/less wide)
→ ~ do not give rise to hadronic showers (photo-nuclear cross section small)
→ not only π^0 but also other particles like η

roughly continues until particle energy
falls below pion production threshold

Hadronic Showers: numbers

Average π^0 fraction:

$$\frac{\pi^0}{all} \approx 0.10\log(E)$$

Average ratio electromagnetic and hadronic particles:

$$\frac{e}{h} \approx 1.1 - 1.35$$

Shower maximum:

$$d_{\max} \approx [0.6\log(E) - 0.2]\lambda$$

Shower depth for 95% longitudinal containment:

$$d_{95\%} \approx d_{max} + 4E^{0.15}\lambda$$

Shower radius for 95% radial containment:

$$R_{95\%} \approx \lambda$$

with: *E* in *GeV*

For *E=20 GeV*
0.3

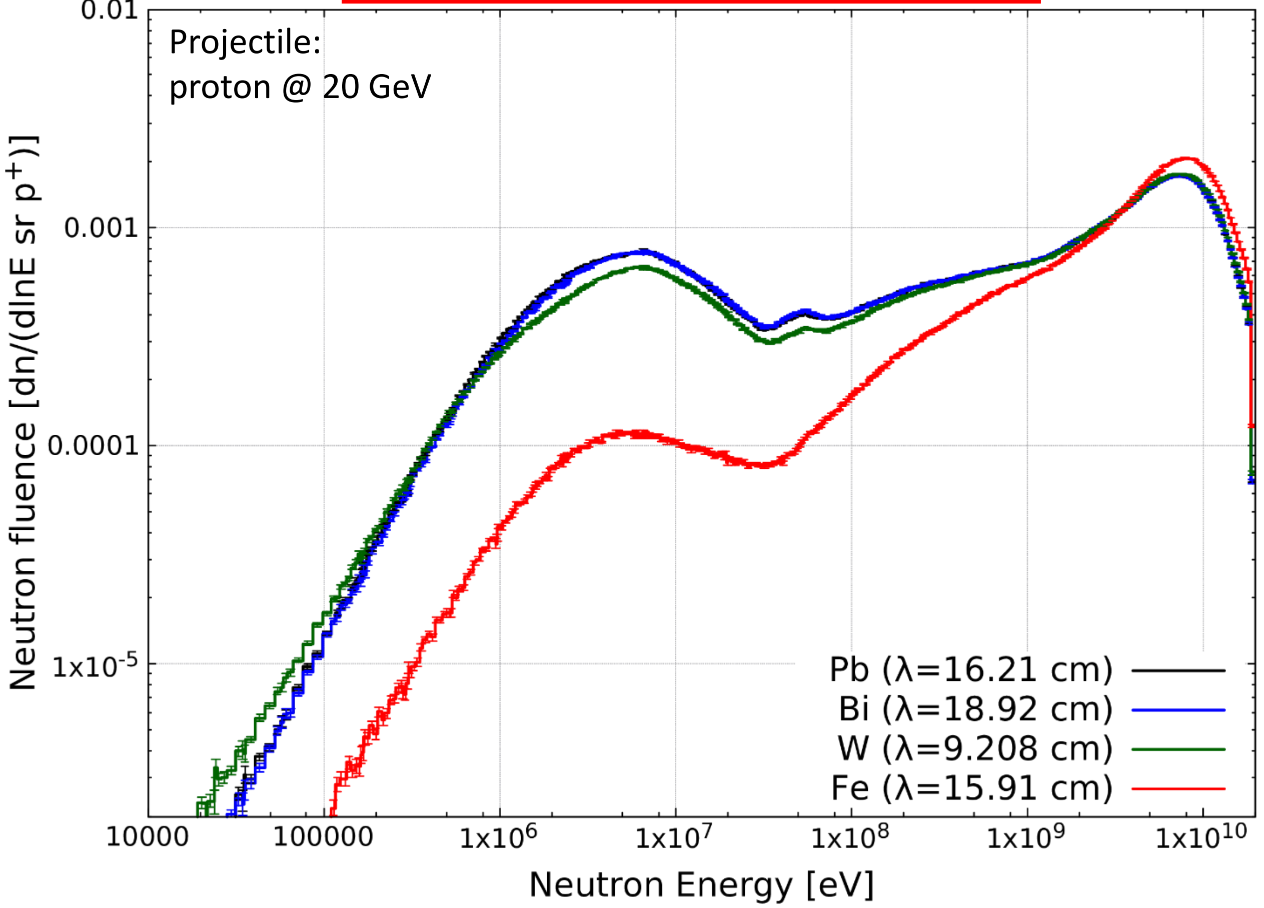
1.6 λ

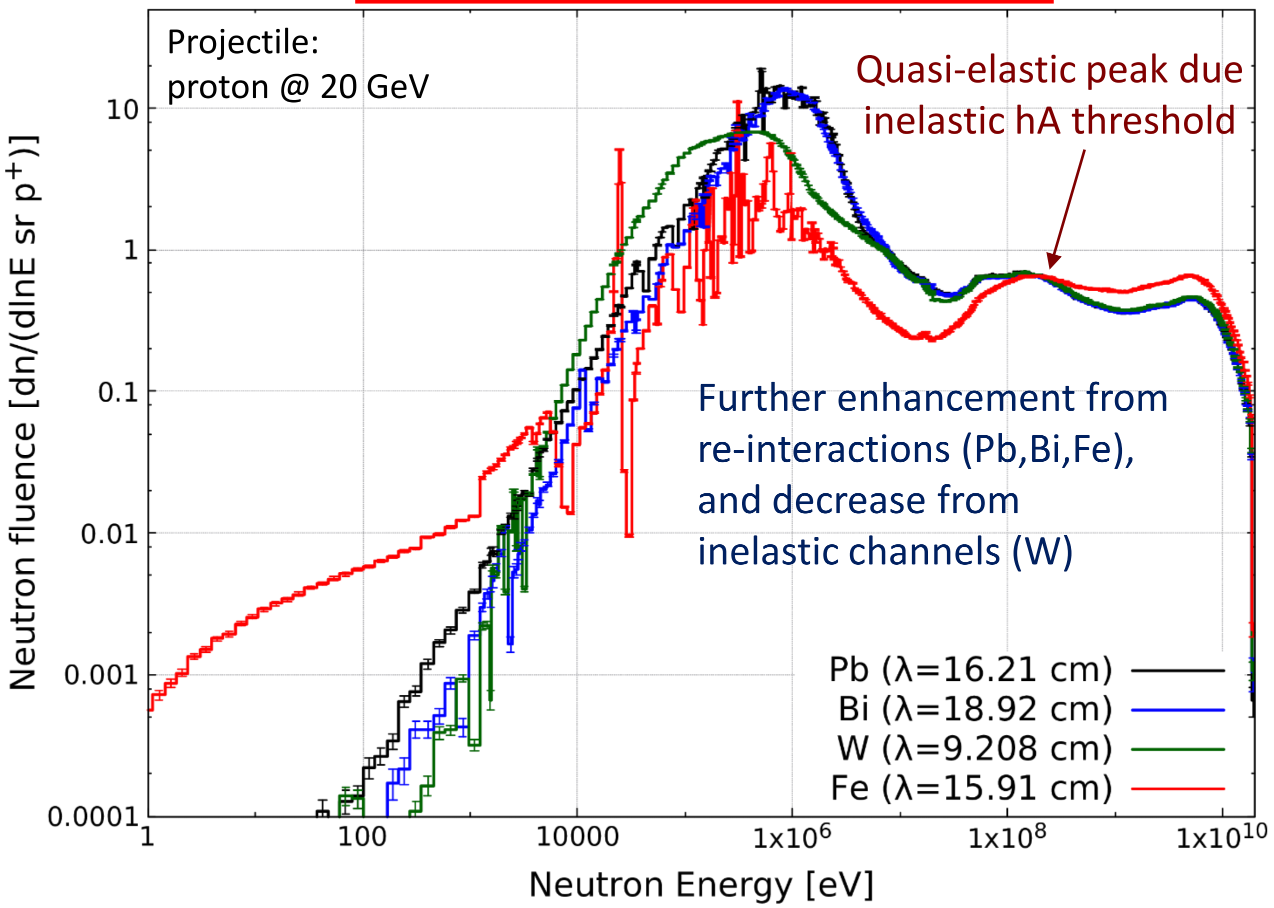
7.9 λ

The Particle Detector BriefBook
ISBN 3-540-64120-3

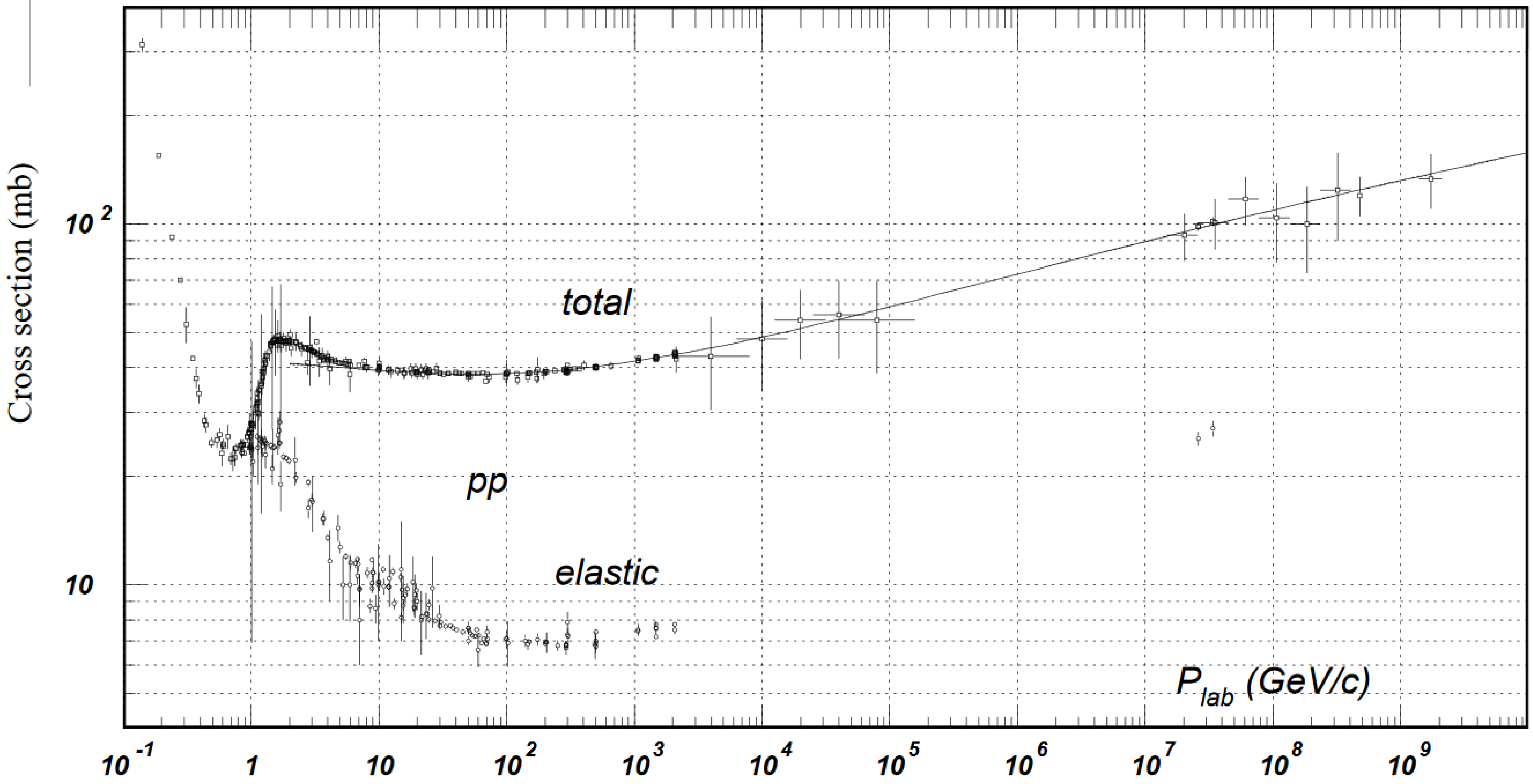
Hadronic shower: Neutron yield

Thin layers: 0.01λ thickness, 2λ radius → Thick layers: 3λ thickness, 2λ radius





Hadron-hadron collision



Neutron production on thick targets

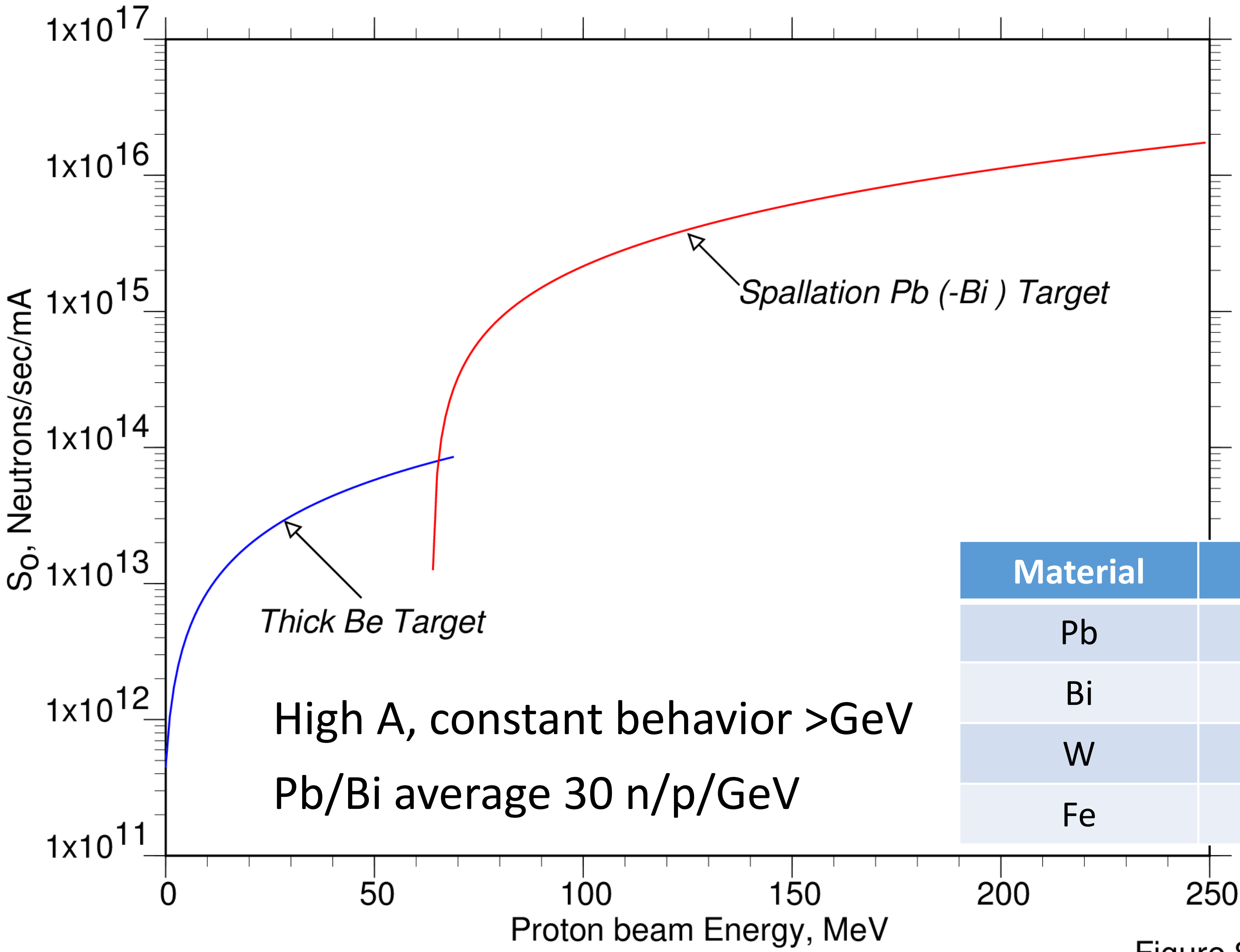


Figure 8

High A, constant behavior >GeV

Pb/Bi average 30 n/p/GeV

Why spallation Sources:

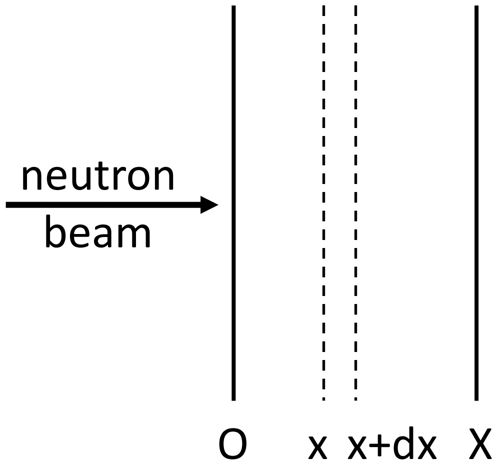
- Spallation x10 more neutrons per heat than fission
- Efficient spallation sources requires proton *E>~100 MeV*
- Pulsed sources allows time-of flight

Material	1%×λ	3×λ	Infinite
Pb	1.6	19	29
Bi	1.6	19	28
W	1.4	17	18
Fe	0.35	6	10

Neutron yield /proton/GeV/(1-$e^{-x/\lambda}$)

Interaction with matter

- Neutron beam traversing a thickness of *dx* suffers a small diminution of dI (from dI to I-dI) in intensity



$$-\frac{dI}{I} = \frac{\sigma N S dx}{S}$$

where: S = surface [cm^2],

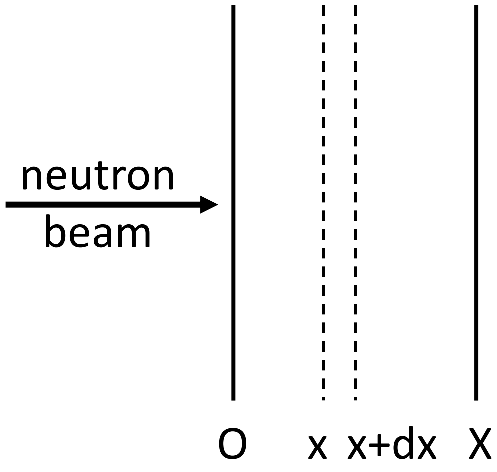
$N = N_A\rho/A$ = atom density [cm^{-3}],

σ = effective target area [$barn = 10^{-24}\ cm^2$]

σ_t = total cross section [*barn*]

$\sigma_t = \sigma_\gamma + \sigma_f + \sigma_s + \ldots$

Interaction with matter



- Integrating gives: $I = I_o e^{-\sigma N X}$
- Probability of loss: $1 - \frac{I}{I_o} = 1 - e^{-\sigma_t N x}$
- Average penetration:

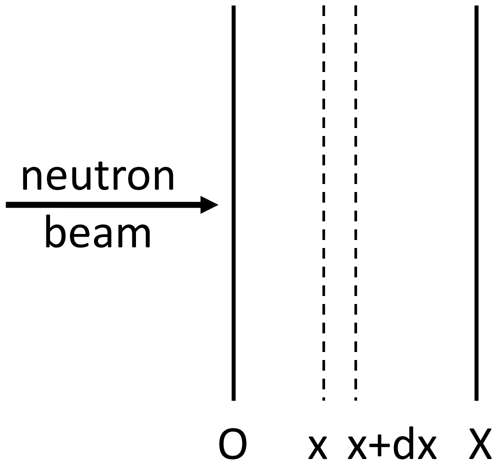
$$\bar{x} = \int_0^\infty x e^{-\sigma_t N_o x} \sigma_t N dx$$

$$= \frac{1}{\sigma_t N} = \frac{1}{\Sigma_t} = \lambda$$

λ = known as *mean free path* [*cm*]
$\Sigma_t = \sigma_t N$ = *macroscopic cross-section* [cm^{-1}]

- σ,λ,Σ: are a function of particle type, material, and energy

Interaction with matter



- Both microscopic and macroscopic cross section are shown to have a similar physical meaning of:

  "probability of interaction per unit length"

  with length measured in different units. Thus, the number of interaction can be obtained from both by multiplying with the corresponding particle track-length.

- Some numbers:

C:	$\sigma_s(1\text{keV}) = 4.7\ b$	$\lambda_s = 2.1\ cm$
Al:	$\sigma_s(1\text{keV}) = 1.42\ b$	$\lambda_s = 11.7\ cm$
Pb:	$\sigma_s(1\text{keV}) = 10.7\ b$	$\lambda_s = 2.8\ \text{cm}$
H_2O:	$\sigma_{sH}(1\text{keV}) = 20.3\ b,$	
	$\sigma_{sO}(1\text{keV}) = 3.85\ b,$	$\lambda_s = 0.67\ \text{cm}$

Flux

- The number of interactions (*reaction rate*) [$cm^{-2}\ s^{-1}$] with a sample in a beam:

$$\frac{n(\boldsymbol{r})\,dx\,\sigma N}{dt} = n(\boldsymbol{r})\frac{dx}{dt}\Sigma = n(\boldsymbol{r})v\Sigma = \frac{n(\boldsymbol{r})v}{\lambda}$$

where: $n(\boldsymbol{r})$ = neutron density [cm^{-3}], having velocity v [cm/s]

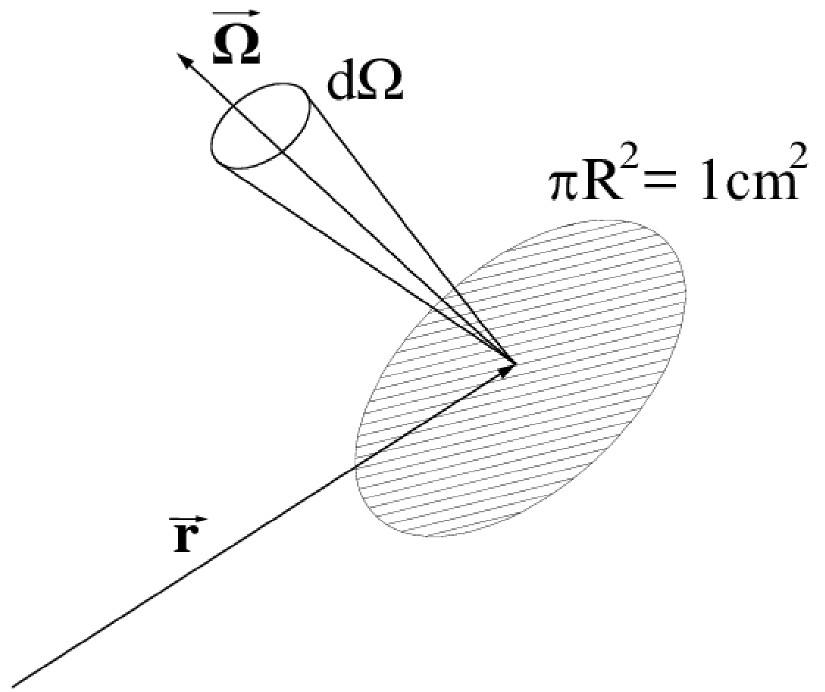
- The quantity $\Phi(E,\boldsymbol{r},\boldsymbol{\Omega},t) = n(\boldsymbol{r},\boldsymbol{\Omega})v$ is known as *differential neutron flux*
- Integrating over all solid angles

$$\Phi(E,\boldsymbol{r},t) = \int_{4\pi}\Phi(E,\boldsymbol{r},\boldsymbol{\Omega},t)d\boldsymbol{\Omega} = n(E,\boldsymbol{r},t)v$$

- we get the *fluence rate* or *flux density*

Flux / Fluence

- has dimensions:
  $[cm^{-3}\ cm\ s^{-1}] = [cm^{-2}\ s^{-1}]$.
- The time integral of the flux density $\Phi(E,\mathbf{r})$ is the fluence $[cm^{-2}]$
- Fluence is measured in *particles per cm^2* but in reality it describes the ***density of particle tracks*** $[cm/cm^3]$***!***
- The number of reactions inside a volume V is given by the formula: (where the product $\Sigma\,\Phi\,V$ is integrated over energy or velocity)



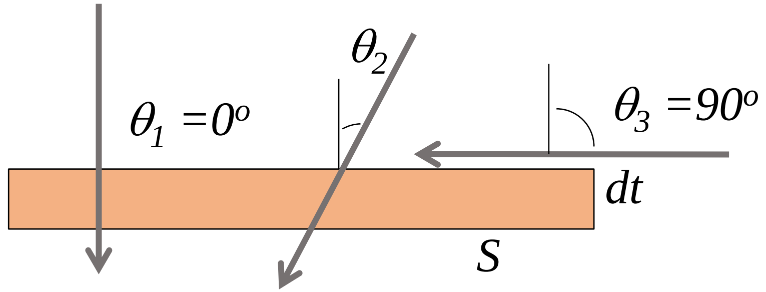
Fluence is equivalent to the particles crossing a surface of $1cm^2$ always perpendicular to the particle direction. Or a sphere with cross section of $1cm^2$

Properties: Isotropic vs Uniform?

Current vs Fluence

Surface crossing

- Imagine a surface having an infinitesimal thickness dt. A particle incident with an angle θ with respect to the normal of the surface S will travel a segment $dt/\cos\theta$.



- Therefore, we can calculate an average surface fluence by adding $dt/\cos\theta$ for each particle crossing the surface, and dividing by the volume $S\,dt$:

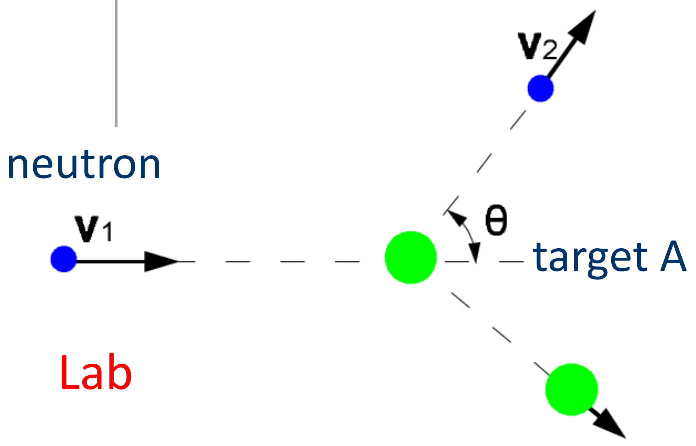
$$\Phi = \lim_{dt\to 0} \frac{\sum_i \frac{dt}{\cos\theta_i}}{S dt}$$

- While the current J counts the number of particles crossing the surface divided by the surface:

$$J = dN/dS$$

The fluence is independent from the orientation of surface *S*,
while the current is NOT!
Q: In an isotropic field can be easily seen that on a flat surface $J = \Phi/2$

Slowing down of neutrons*

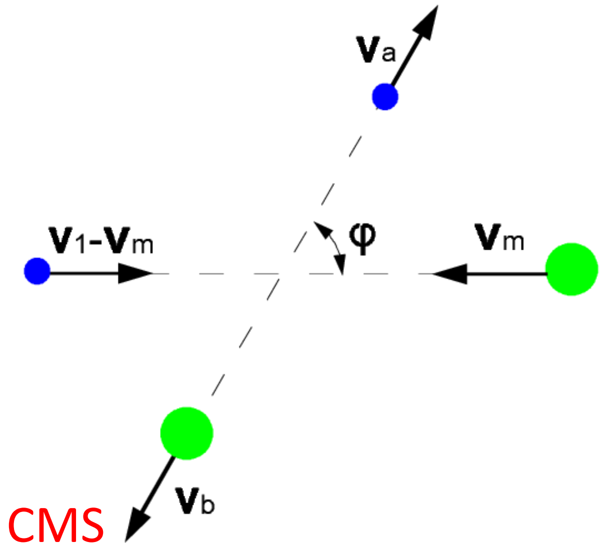


The Center of Mass System CMS is moving with

$$v_m = \frac{v_1}{A+1}$$

From conservation energy & momentum we have

$$|v_a| = |v_1 - v_m| \quad \text{and} \quad |v_m| = |v_b|$$



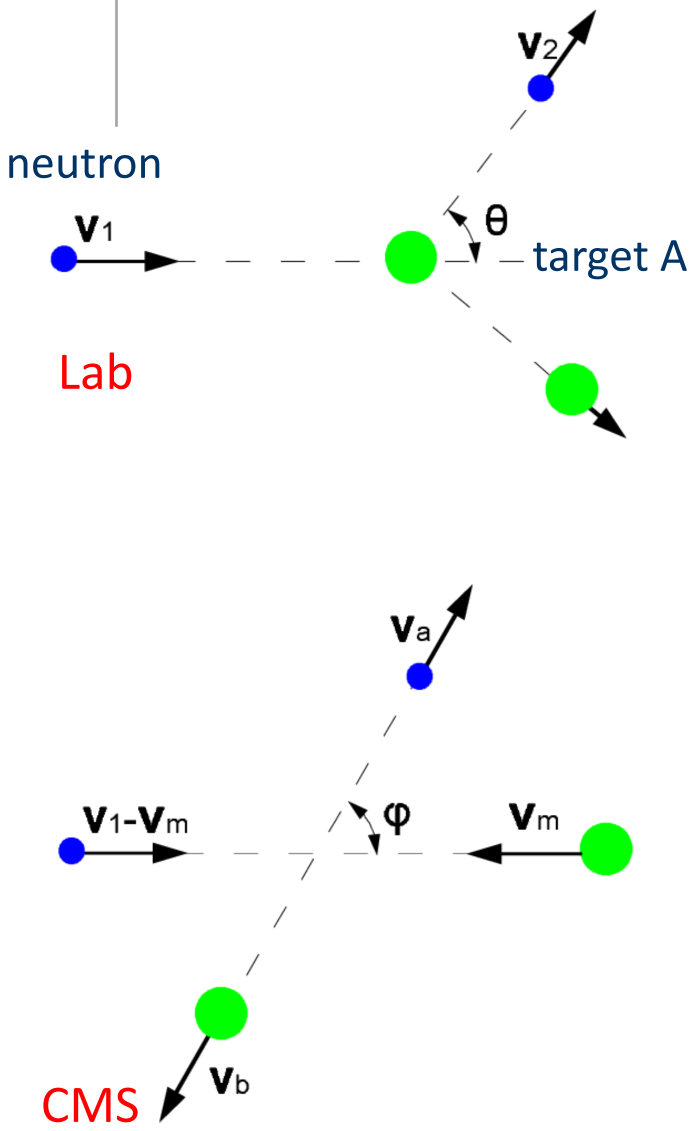
Transforming back to the lab

$$\frac{E_2}{E_1} = \frac{v_2^2}{v_1^2} = \frac{A^2 + 2A\cos\phi + 1}{(A+1)^2}$$

* Assumptions:

i) Non-relativistic kinematics
ii) neutron in epithermal region
iii) target nucleus at stand-still

Slowing down of neutrons



For low energies <*MeV* angle ϕ in C.M. is isotropic
→ flat distribution in $cos\phi$
→ anisotropic in the θ LAB

$$b = \overline{cos\theta} = \frac{2}{3A}$$

The scattering probability to have a final energy in the interval $(E, E+dE)$ is

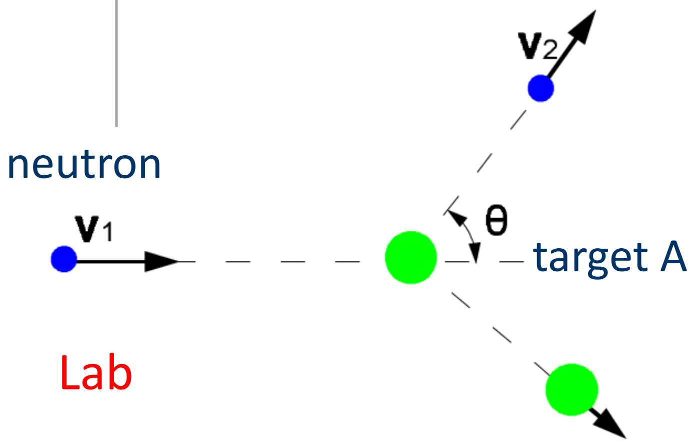
$$F(E)dE = \frac{2\pi\ sin\phi\ d\phi}{4\pi} = -\frac{d(cos\phi)}{2}$$

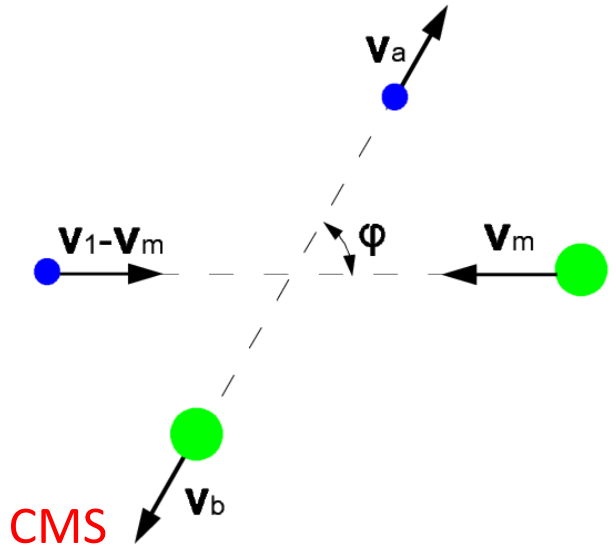
Integrating the $F(E)dE$ we get that

$$\frac{\overline{\Delta E}}{E} = \frac{1-a}{2}$$

with $a = \left(\frac{A-1}{A+1}\right)^2$

Slowing down of neutrons





The quantity *ln(E₁/E₂)avg* is called *lethargy* and represents the *average logarithmic energy* loss per collision:

$$\xi = \overline{ln\frac{E_1}{E_2}} = 1 + \frac{a}{1+a} lna$$

Taylor approximation gives us (for A>1):

$$\xi \approx \frac{2}{A + {}^2/_3}$$

- For A≥10 is a good approximation.
- For A=2 the error of the approximation is 3%

For mixtures:

$$\bar{\xi} = \frac{\sum_i N_i \sigma_s^i \xi_i}{\sum_i N_i \sigma_s^i}$$

Slowing down of neutrons

- lethargy can be used to estimate the number of collisions to moderate from the initial energy E_i to E_f

$$n\xi = ln\frac{E_i}{E_f}$$

- Giving a neutron fluence

$$\Phi(E) = \frac{C}{\bar{\xi}\Sigma_s E}$$

Slowing down power: $\bar{\xi}\Sigma_s$ → Moderation ratio: $\xi\Sigma_s/\Sigma_a$

- larger ξ → faster slow down; larger Σ_s → more often collisions

	H	D	He	Li	Be	C	O	Pb	U
A	1	2	4	7	8	12	16	207	238
a	0	0.111	0.360	0.562	0.640	0.716	0.778	0.981	0.983
ξ	1.0	0.725	0.425	0.268	0.209	0.158	0.120	0.00963	0.00838
n	18	25	43	67	86	114	150	1888	2172

Slow down parameters from 2MeV → 0.025 eV (thermal)

Thermal Neutrons

The velocities of thermal motion of the material nuclei are distributed with a Maxwellian distribution

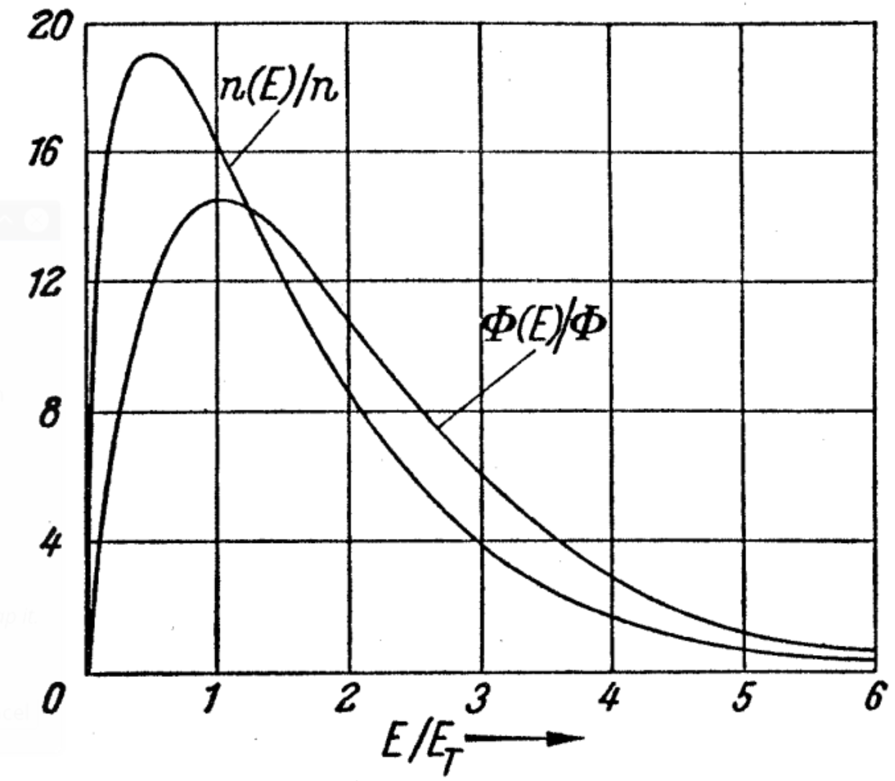
$$n(E)dE = \frac{2\pi n}{(\pi kT)^{3/2}} e^{-E/kT} \sqrt{E}\, dE$$

where n$(E)dE$ is the number of neutrons per cm^3 with energies *[E,E+dE)*, *n* is the total density

The average energy is $\bar{E} = \frac{3}{2} kT$

The most probable energy is $E_T = kT$

At T=20°C=293K, *kT = 0.0253eV, v=2200m/s*

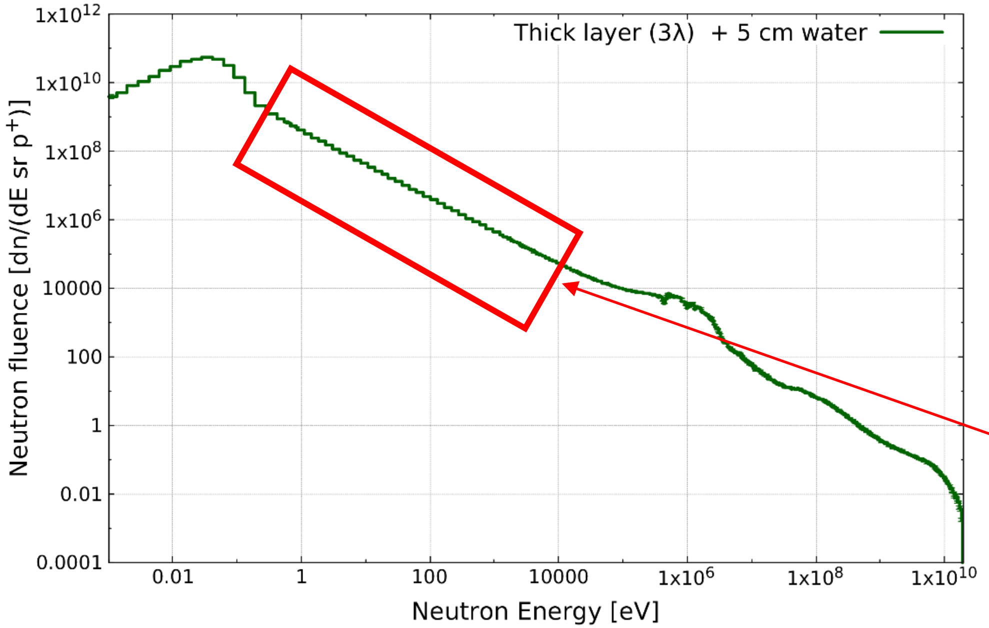


Neutron flux is given

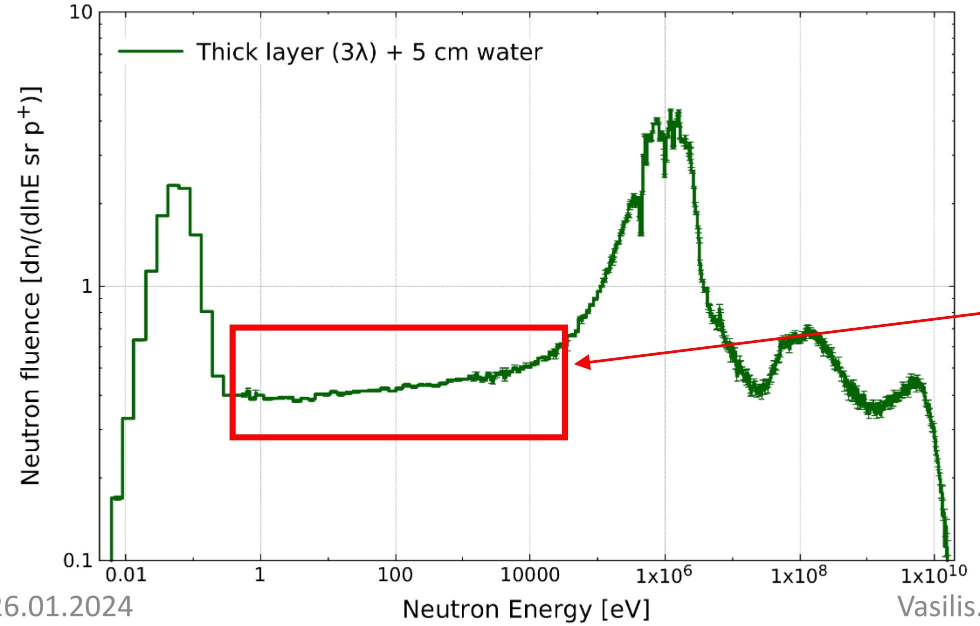
$$\frac{\Phi(E)dE}{\Phi} = e^{-E/E_T} \frac{E}{E_T} \frac{dE}{E_T}$$

Flux/Fluence – Isolethargic

dn/dE



dn/dlnE



- Textbook representation of fluence as $\Phi(E) = dn/dE$ is spanning over several orders of magnitude
  → hides a lot of information
- For energies above thermal the flux is almost: $\Phi(E) \approx \frac{C}{\bar{\xi}\Sigma_s E}$

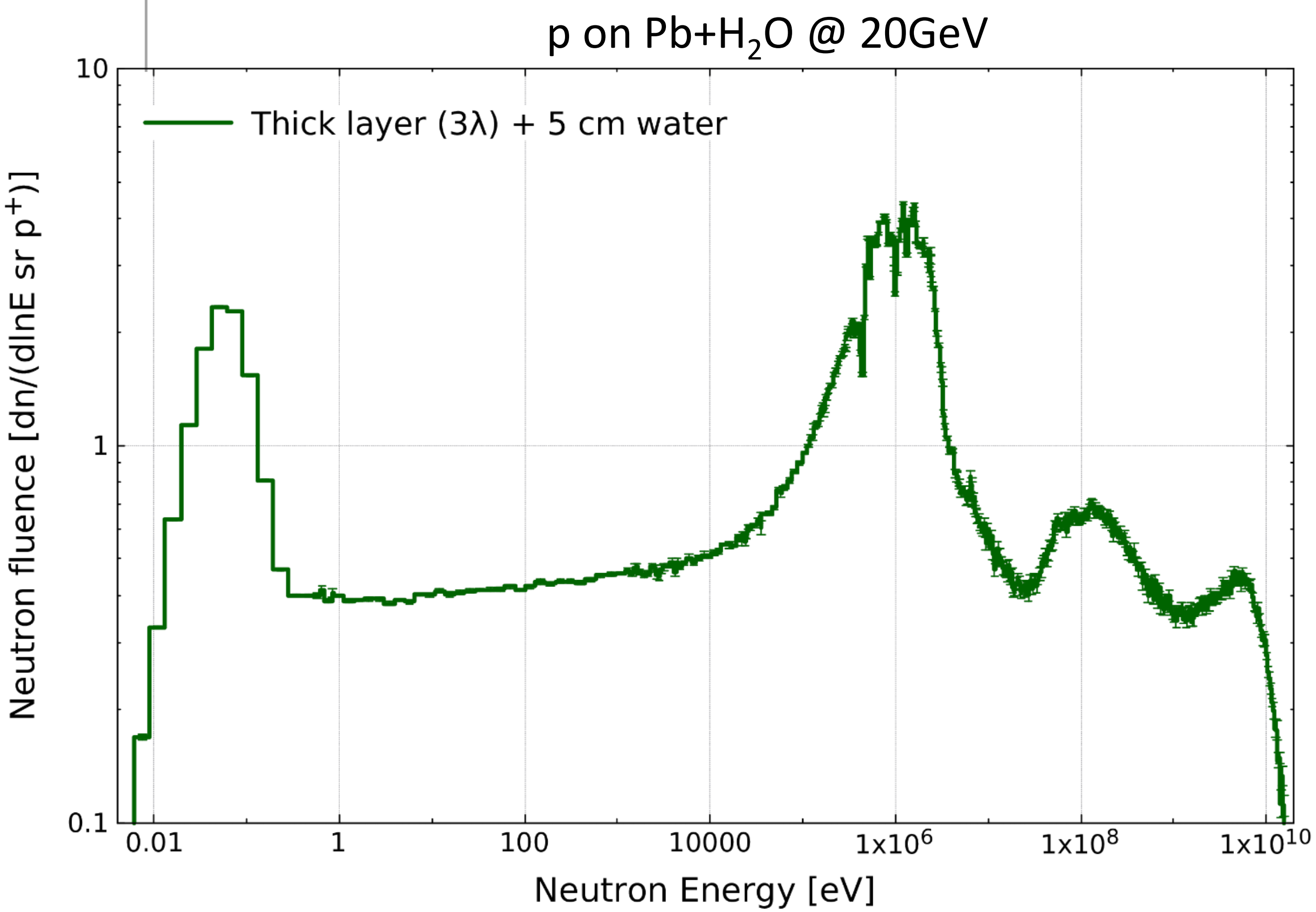
  which can be converted to

$$E\,\Phi(E) = E\frac{dn}{dE} \approx C/\bar{\xi}\Sigma_s$$

$$\Leftrightarrow \frac{dn}{d\ln E} \approx C/\bar{\xi}\Sigma_s = const$$

- Resulting to a histogram “*flat*” in log space in the epithermal region

Isolethargic or Lethargy Plot



Advantages:

- Structures are more visible
- The Y scale is independe on the X unit
  since $\mathrm{d}lnE = \mathrm{d}E/E$= unit less
- When X in log, the areas represent the integral of neutrons

How to read:

- It gives the amount of neutrons at energy *E* for an energy interval *ΔE=E*

Converting histogram to Isolethargic

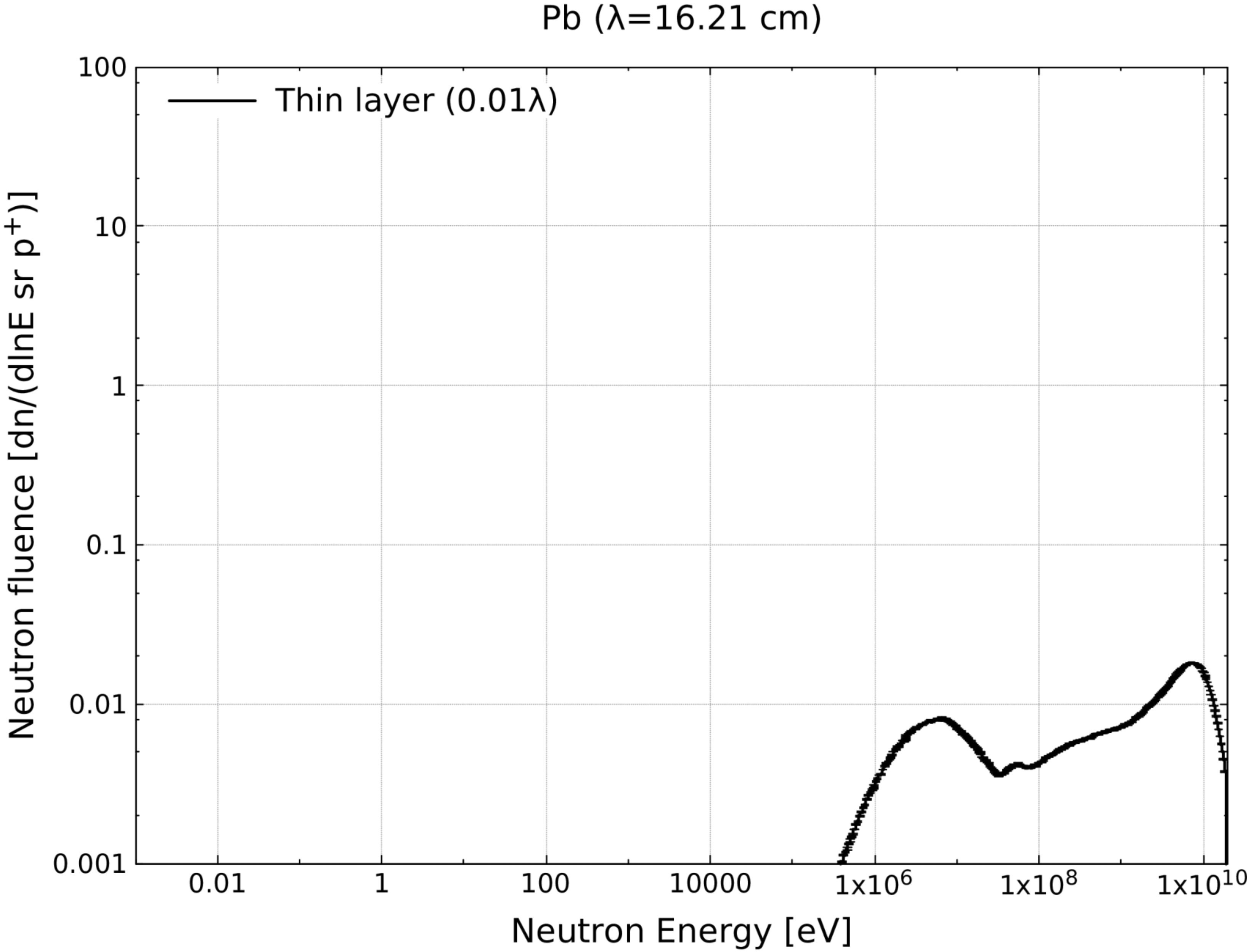
Logarithmic Histogram (base-10):

- Defined as: $X_{min}=log_{10}(E_{min})$, $X_{max}=log_{10}(E_{max})$, N_{bins}
- Log step: $s = (X_{max}-X_{min})/N_{bins}$
- Lower Energy of each bin: $E_i = 10^{X_{min}+i\cdot s}$
- Width of each bin: $\Delta E_i = 10^{X_{min}+(i+1)\cdot s} - 10^{X_{min}+i\cdot s}$
- Geometric Mean of the each bin: $\bar{E}_i = \sqrt{E_i \cdot E_{i+1}} = 10^{X_{min}+(i+\frac{1}{2})\cdot s}$
- To convert to isolethargic can be done by multiplying with:

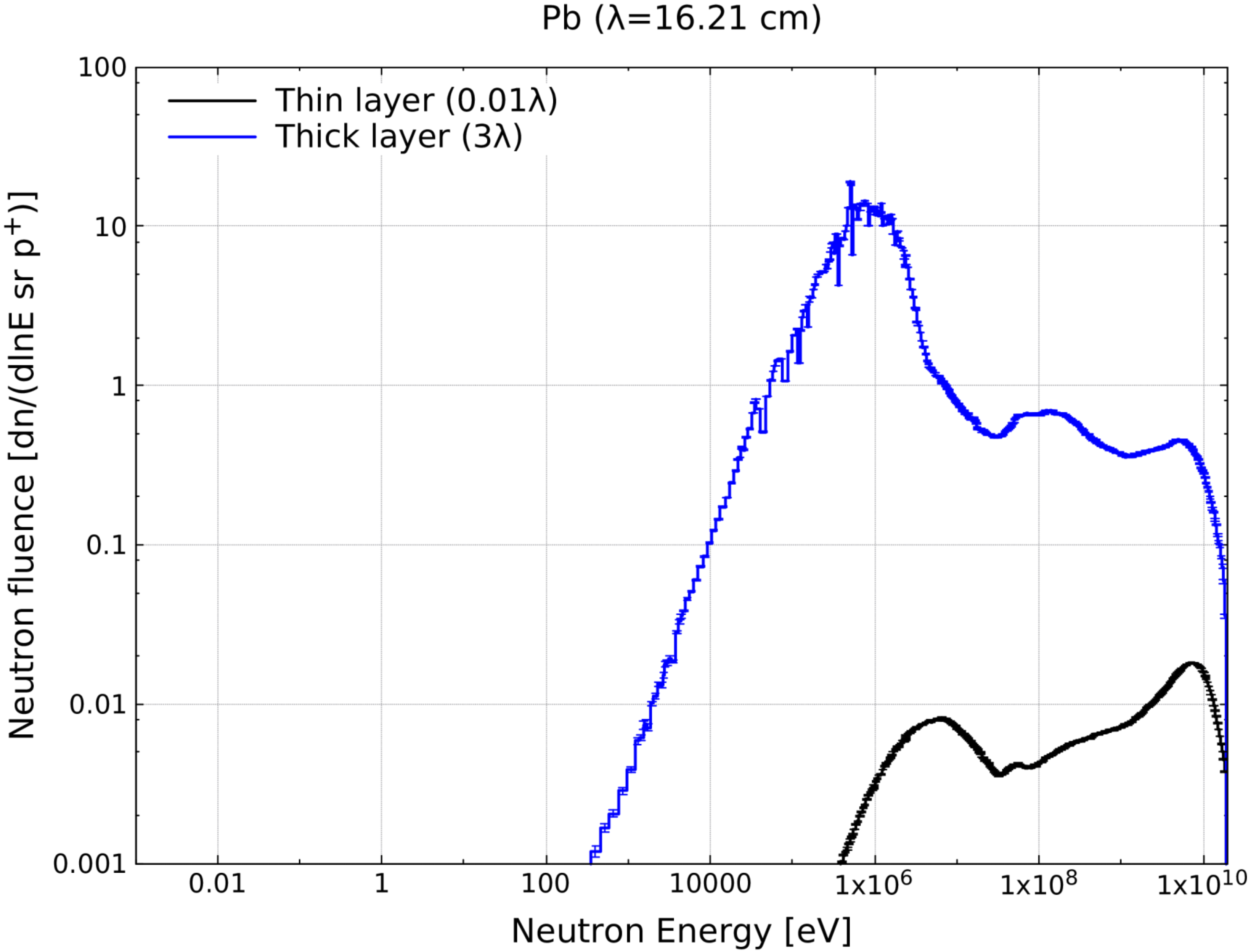
$$f = \frac{\bar{E}}{\Delta E} = \frac{10^{X_{min}+(i+\frac{1}{2})\cdot s}}{10^{X_{min}+(i+1)\cdot s} - 10^{X_{min}+i\cdot s}} = \frac{\sqrt{10^s}}{10^s - 1}$$

Q: difference of geometric mean vs mean for 20 bins per decay?

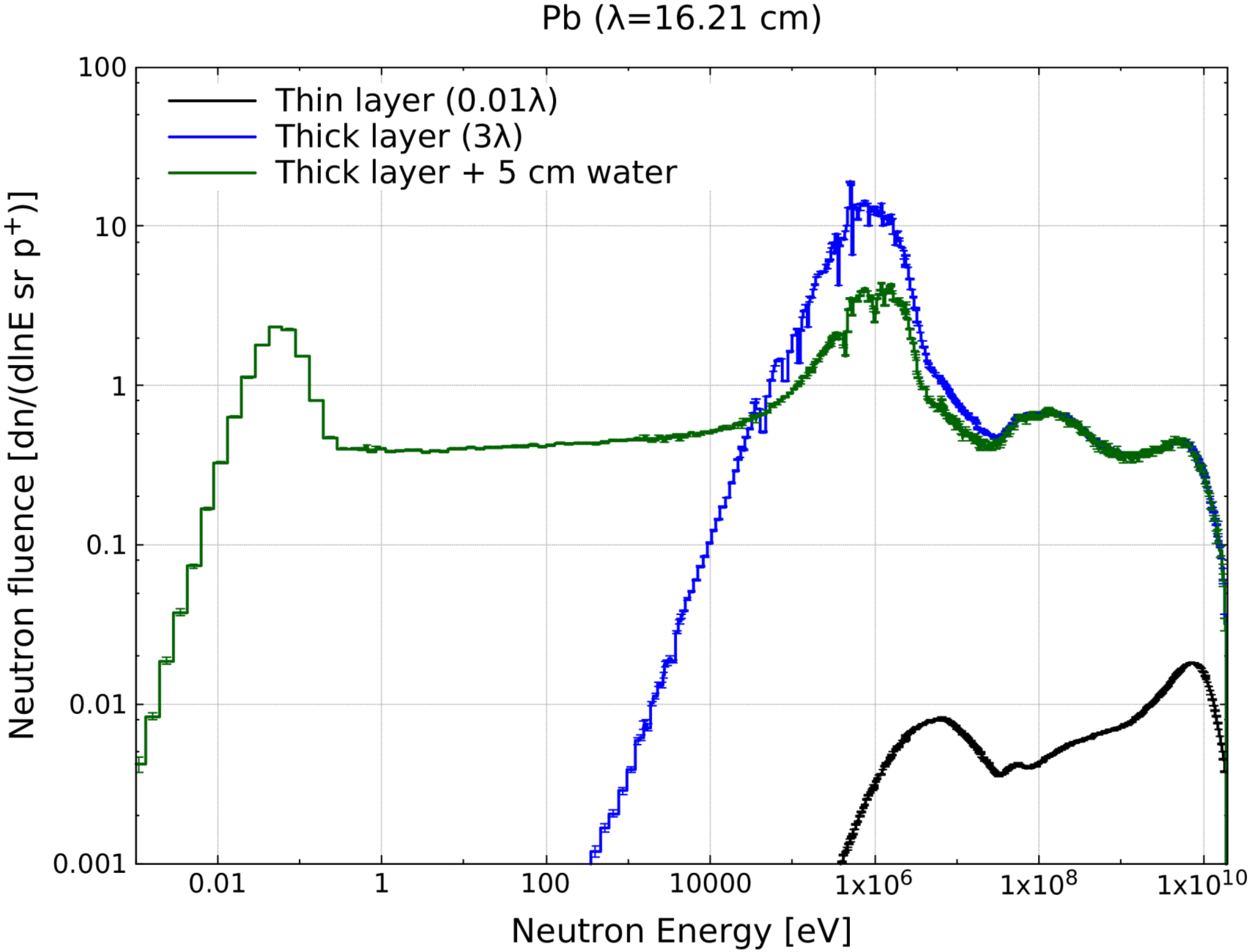
Neutron Yield



Neutron Yield



Neutron Yield

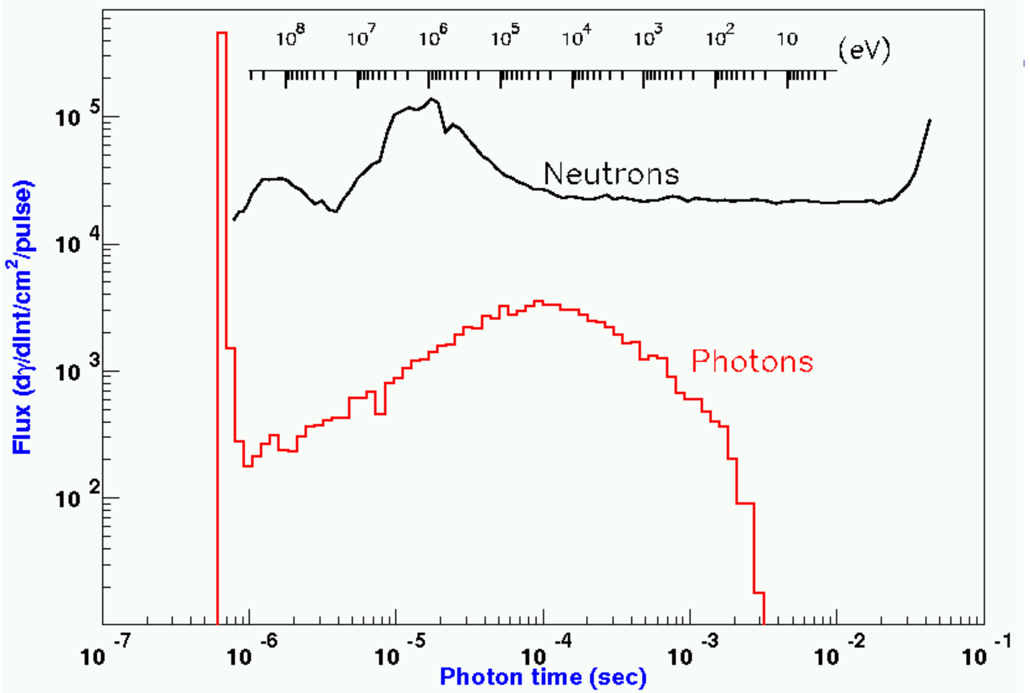


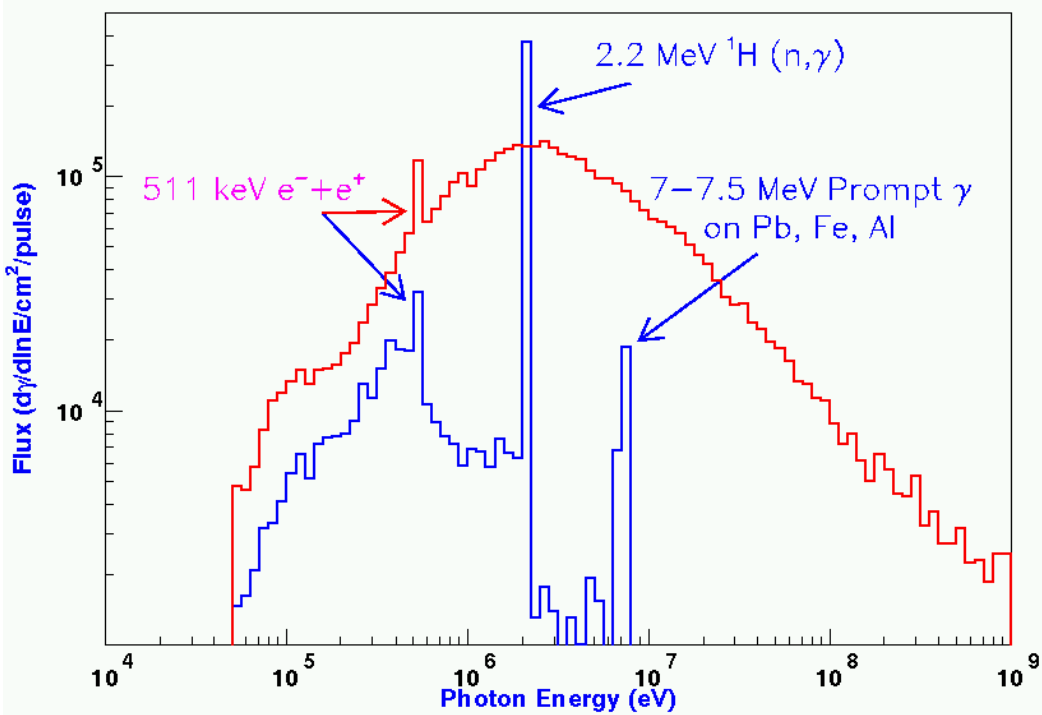
In beam Gamma background

In beam gamma background mostly comes from thermal neutrons radiative capture on ^{1}H

$$^{1}_{1}H + ^{1}_{0}n \rightarrow ^{2}_{1}H + \gamma(2.2 MeV)$$

at n_TOF they arrive to the experimental area EAR1 @185m distance, at times in the range of μs...ms, together with the keV...$100keV$ neutrons

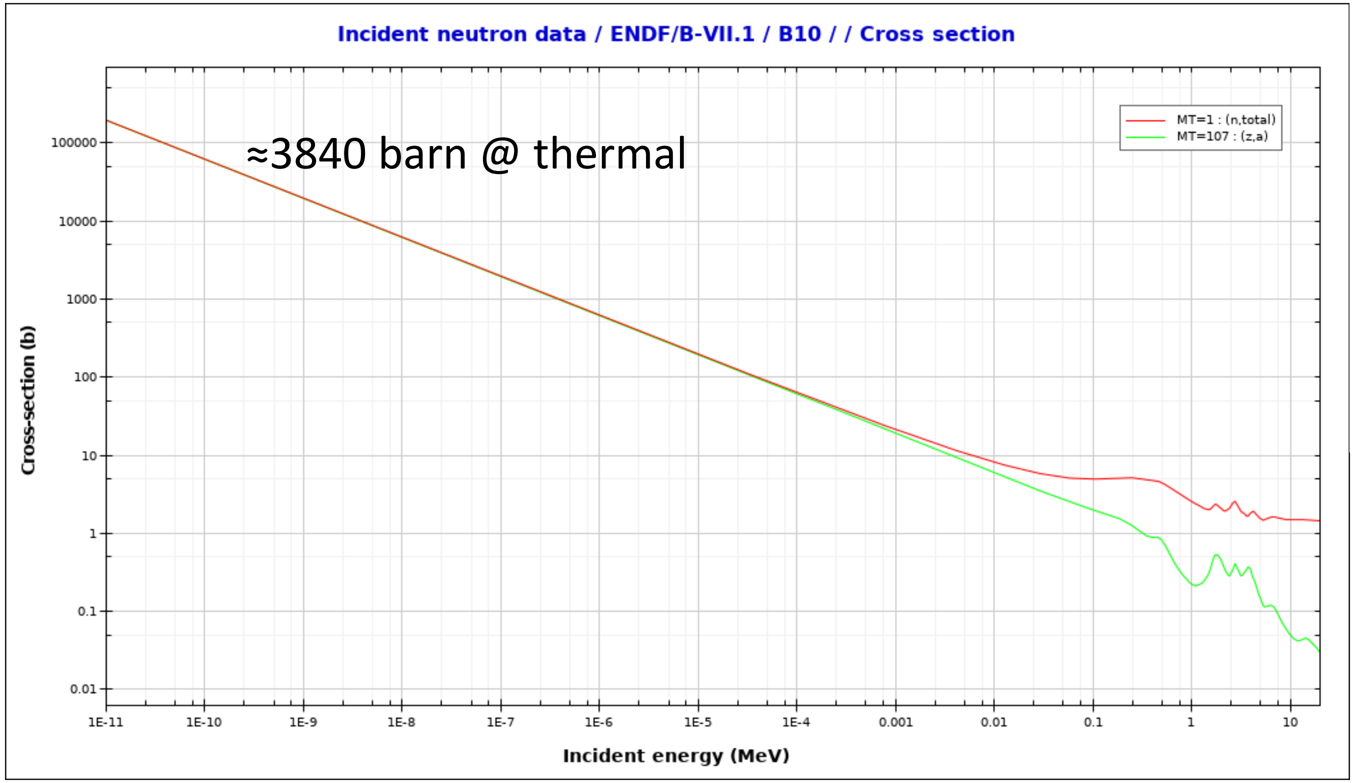


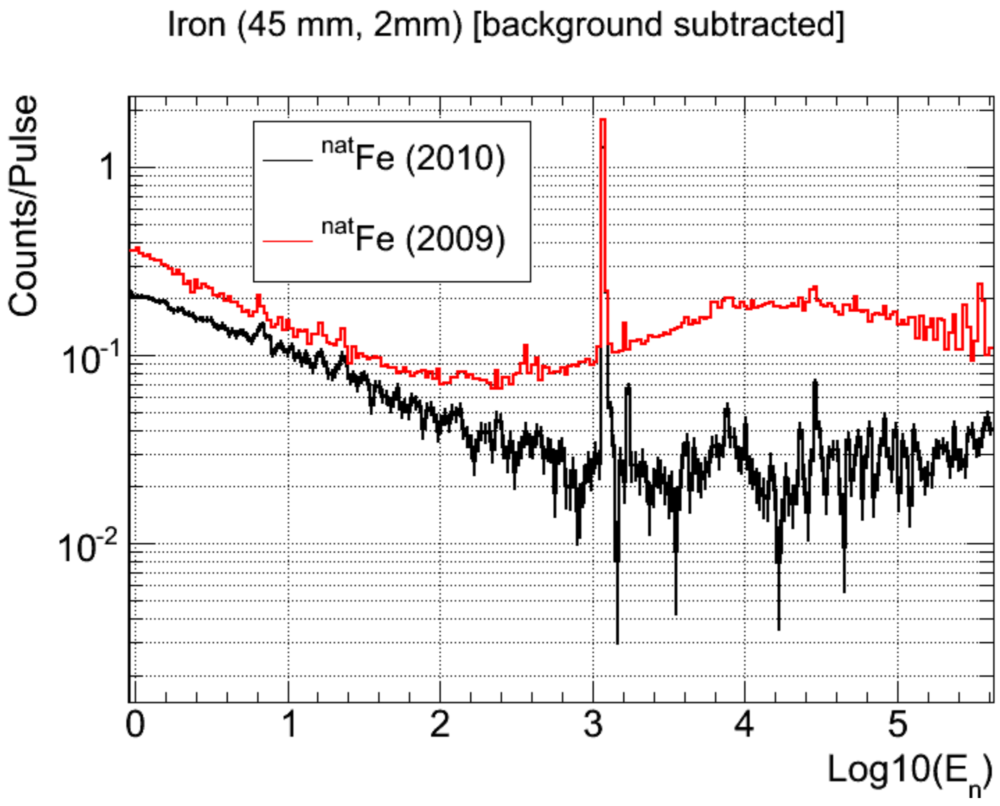


Borated Water effect

One option to suppress this background, was the addition of Boric Acid in the moderator circuit.

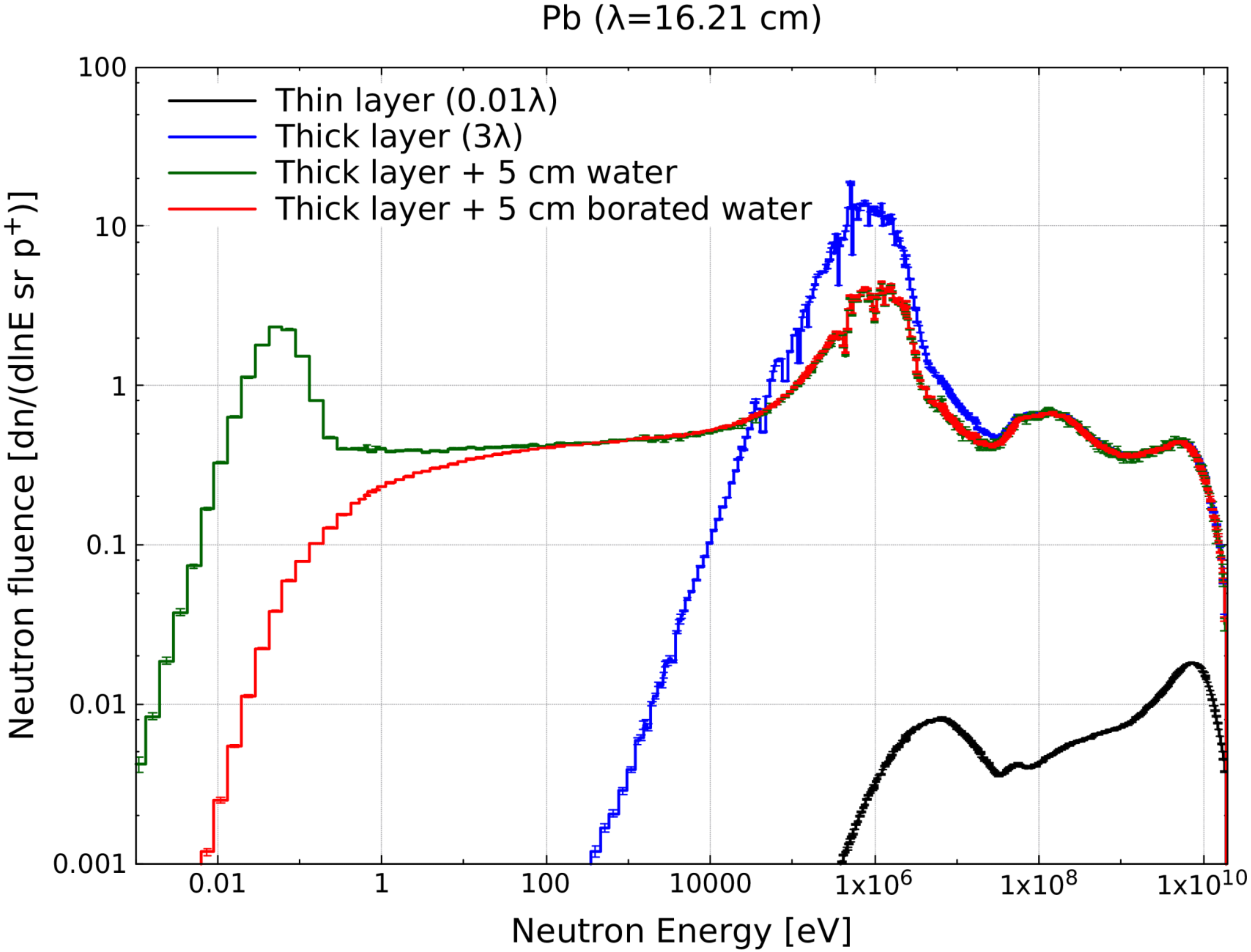
Boric acid: Saturated at 1.28 % enriched with 95% ^{10}B



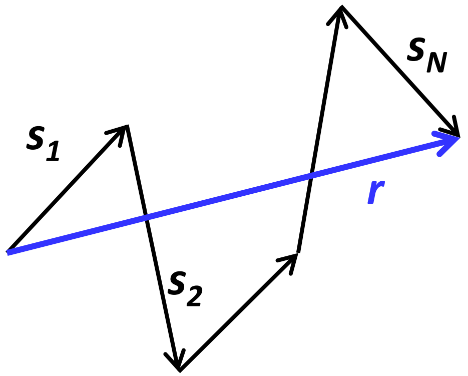


The 2.2 MeV γ is replaced by a *484 keV* (94%) from the $^{10}B(n,\alpha)$ 6% gives *2.79 MeV*

Neutron Yield



Neutron diffusion



Total vector of travel during moderation

$$\boldsymbol{r} = \sum_{i=1}^{N} \boldsymbol{s}_i$$

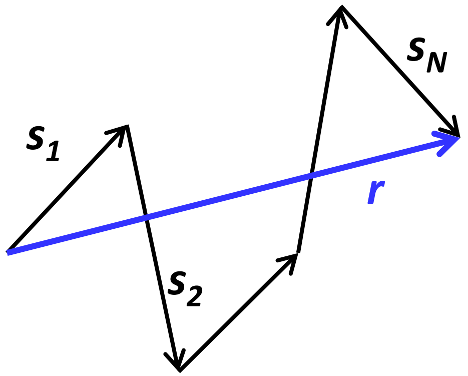
Thus

$$\boldsymbol{r}\cdot\boldsymbol{r} = \boldsymbol{r}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} \boldsymbol{s}_i\,\boldsymbol{s}_j = \sum_{i=1}^{N} \boldsymbol{s}_i^2 + \sum^{\boldsymbol{N}}\sum_{i\neq j} \boldsymbol{s}_i\,\boldsymbol{s}_j$$

If we make the bold assumption we are isotropic in LAB (normally for low energies it is isotropic in CMS not in LAB) → the second term will average to zero

$$\overline{\boldsymbol{r}^2} \cong N\overline{\boldsymbol{s}^2}$$

Neutron diffusion



Combining the probability of a neutron traversing a distance *s* without making interaction and then making a collision in *ds* we have

$$e^{-\frac{s}{\lambda_s}}\frac{ds}{\lambda_s}$$

Assuming that $\Sigma_s \gg \Sigma_a$

$$\overline{\boldsymbol{s}^2} = \int_0^\infty s^2 e^{-\frac{s}{\lambda_s}}\frac{ds}{\lambda_s} = 2\lambda_s^2$$

For n steps

$$\overline{\boldsymbol{r^2}} = 2n\lambda_s^2$$

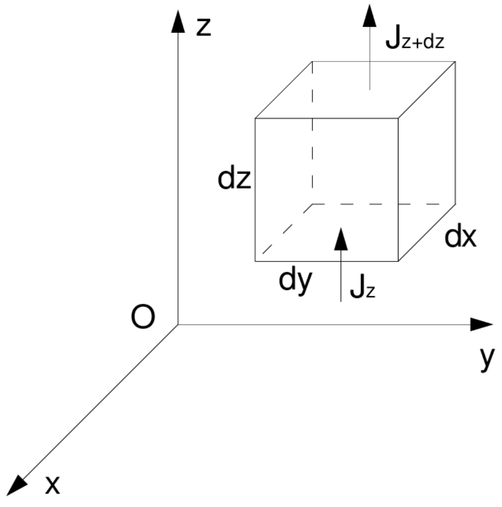
Including the mean cosine per scattering

$$\overline{\boldsymbol{r^2}} = \frac{2n\lambda_s^2}{1-\overline{cos\vartheta}}$$

Finally with the lethargy

$$|\boldsymbol{r}| = \sqrt{\frac{2ln\frac{E_i}{E_f}}{\xi(1-\overline{cos\vartheta})}}\lambda_s$$

Neutron diffusion



Outflow in z direction in a small volume $dV = dxdydz$

$$(J_{z+dz} - J_z)dxdy = -D\left[\left(\frac{\partial\Phi}{\partial z}\right)_{z+dz} - \left(\frac{\partial\Phi}{\partial z}\right)_z\right]dxdy$$

$$= -D\frac{\partial^2\Phi}{\partial z^2}dxdydz$$

where $D = \lambda_s/3(1 - \overline{cos\vartheta})$ diffusion length coefficient

Combining it in 3D it gives $D\nabla^2\Phi(\boldsymbol{r})$

The balance of neutrons per unit volume
→ diffusion equation

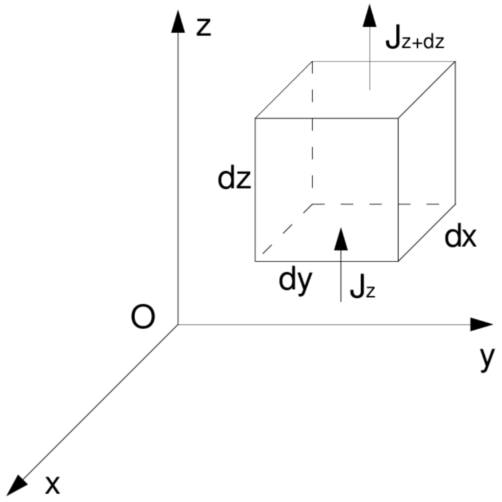
$$\frac{\partial\Phi(\boldsymbol{r})}{\partial t} = S(\boldsymbol{r}) + D\nabla^2\Phi(\boldsymbol{r}) - \Sigma_a\Phi(\boldsymbol{r})$$

production rate — outflow rate — absorption rate

Neutron diffusion

For a point like source in the center (Ficks'law)

$$\frac{S(\boldsymbol{r})}{D} + \nabla^2\Phi(\boldsymbol{r}) - \frac{\Sigma_a}{D}\Phi(\boldsymbol{r}) = 0$$



where $L = \sqrt{D/\Sigma_a} \approx \sqrt{\frac{\lambda_a \lambda_s}{3}}$ = *Diffusion length* [*cm*]

→ specifies the average distance between the place where a neutron is born and the place where it is absorbed.

Notes:

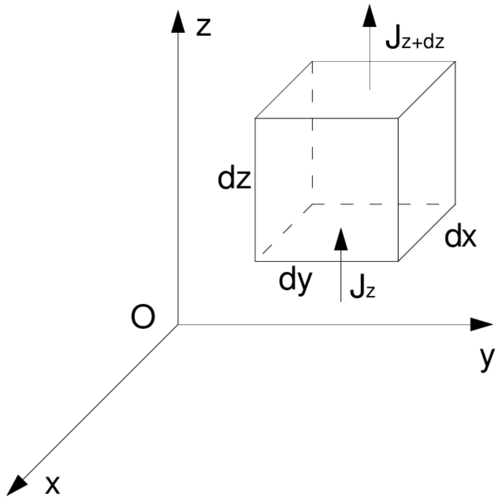
- A neutron reflector should be of the order of a diffusion length
- Infinite medium should be at least one diffusion length

Some numbers:

Carbon: L ≈ 48 *cm*

Lead: L ≈ order of 150 *cm*

Neutron diffusion



- For an infinite medium the solution (Green's function)

$$\Phi(\boldsymbol{r}) \approx So\,\frac{e^{-r/L}}{4\pi D r}$$

where S_o=neutron source rate [*n/s*]
L = diffusion length [cm]
D = diffusion length coefficient

- Flux Enhancement with the use of a moderator:

For a region close to the source r/L<<*1*

$$\Phi(r) \approx S_o/4\pi D r$$

Considerably higher with respect to absence of moderator

$$\Phi_o(r) \approx S_o/4\pi r^2$$

Enhancement $f = \frac{\Phi(r)}{\Phi_o(r)} = r/D$

Carbon: @30cm, *f*=30/(2.1/3) = 42.8

Lead: @30cm, *f*=30/(2.8/3) = 32.1

Energy Time relation

The average increase in lethargy per unit of time during moderation is *u=nξ*

$$\frac{du}{dt} = \xi\Sigma_s \frac{dx}{dt} = \xi\Sigma_s v$$

From the lethargy $\mathrm{u} = n\xi = \ln(\frac{E_i}{E_f})$ we have $v_f = v_i e^{-\frac{u}{2}}$

Therefore: $$\frac{dv}{dt} = -\frac{\xi\Sigma_s}{2} v^2$$

Integrating: $$t = \frac{2}{\xi\Sigma_s}\int_v^{v_o}\frac{dv}{v} = \frac{2}{\xi\Sigma_s}\left(\frac{1}{v_f} - \frac{1}{v_i}\right)$$

Converting to energy: $$E = \frac{2m_n}{(\xi\Sigma_s t)^2} v(t)$$

Energy-Time relation

for A=1 $$\overline{E(t)} = \frac{3m_n}{(\Sigma_s t)^2} = \frac{1.8 eV\mu s^2}{t^2}$$

for A>>1 $$\overline{E(t)} = \frac{m_n}{2}\frac{A(A+2)}{(\Sigma_s t)^2} \approx \frac{A^2}{(\Sigma_s t)^2} 0.522\ eV\mu s^2 cm^{-2}$$

more accurately* $$\overline{E(t)} = \frac{K}{(t+t_o)^2}$$

where $K = \frac{m_n \lambda_s^2 (1-a)^2}{2a^2}$ and $t_o = (1-a)\frac{\lambda_s}{a}\sqrt{\frac{m_n}{2E_o}}$

* R.E.Slovacek et al, Nucl. Sci. and Eng. 62:445-462,1977

Energy-Time relation

Moderation:

$$\overline{E(t)} = \frac{K}{(t + t_o)^2}$$

Flight:

$$E = \frac{m_n L^2}{2\, t^2}$$

Relativistically:

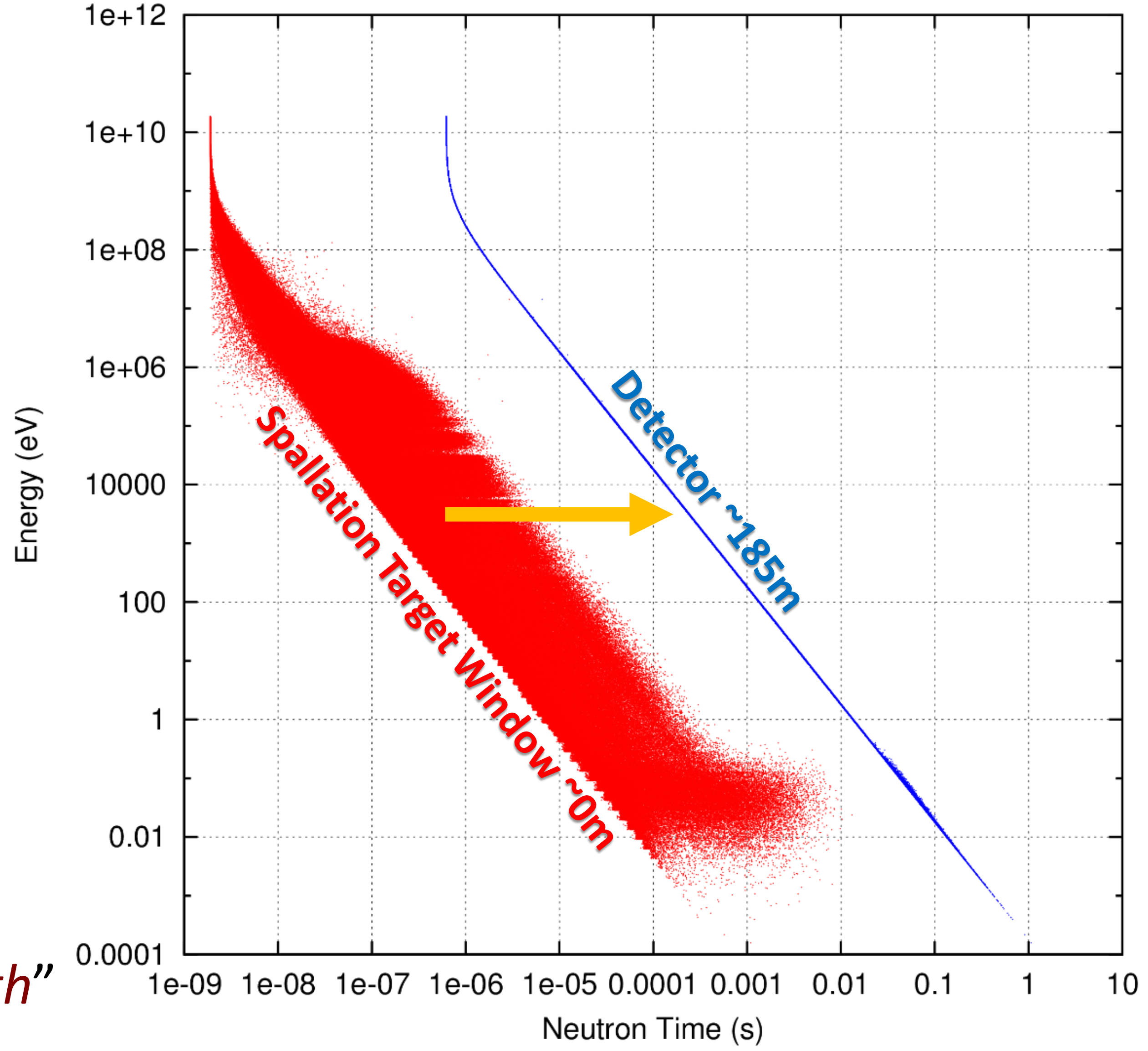
$$\gamma = 1 \Big/ \sqrt{1 - \left(\frac{L}{tc}\right)^2}$$

$$E_{kin} = (\gamma - 1)\, m_n$$

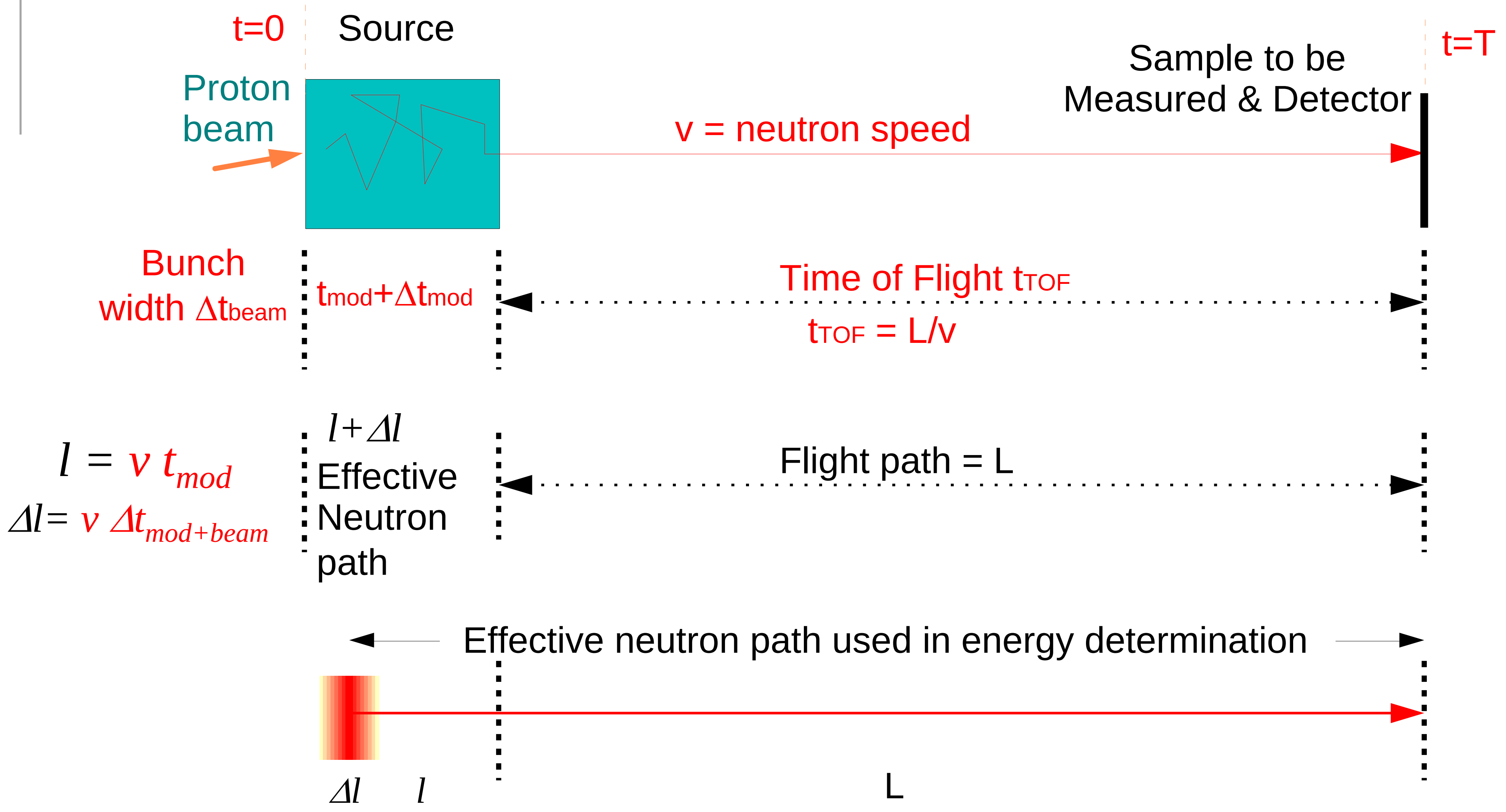
Differentiating we get the uncertainty:

$$\frac{\Delta E}{E} = -2\frac{\Delta t}{t} = 2\frac{\Delta l}{L + l}$$

l = virtual quantity "*effective neutron path*"



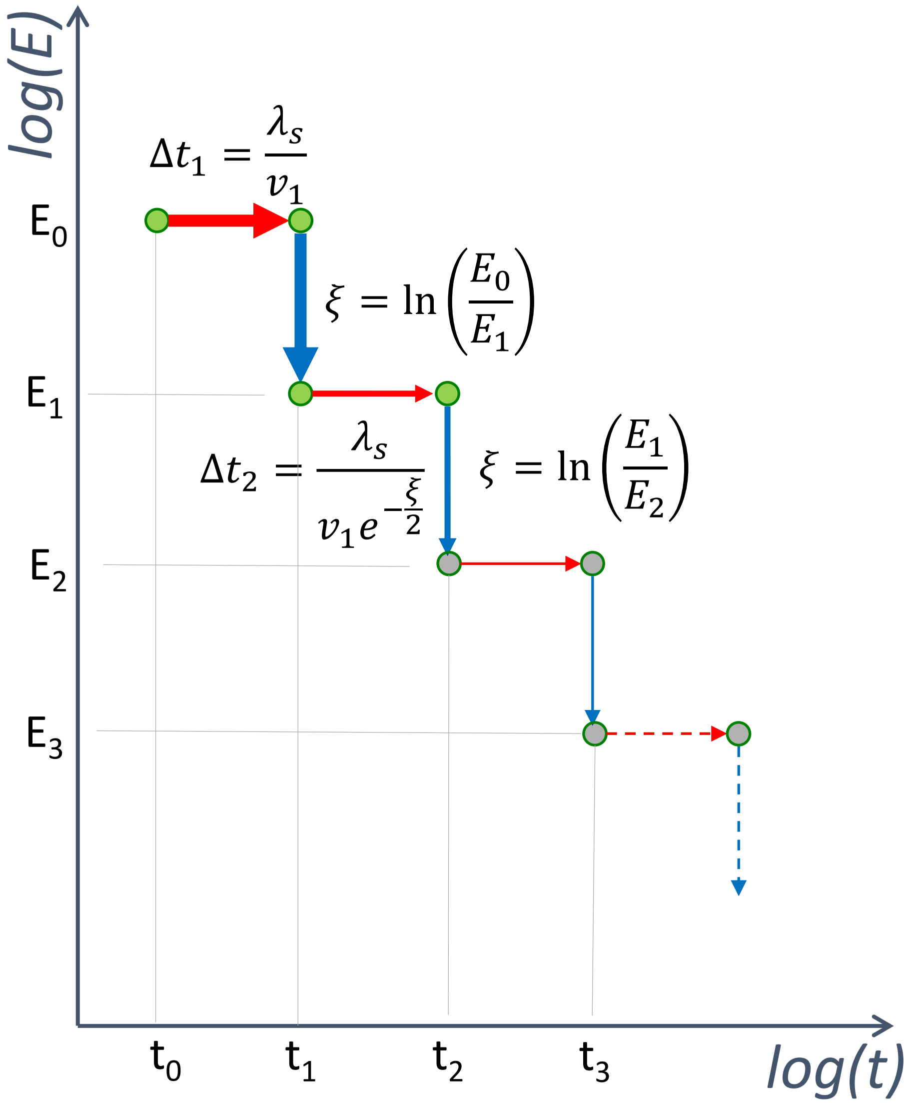
Effective Neutron Path l



The neutron velocity v is derived from the "effective neutron path"
$v=(L+l)/t$ with an uncertainty $\Delta v = \Delta l/t$

WARNING:
Moderation path ≠ effective neutron path

Neutron Moderation



Mean free path

$$\lambda_s = \frac{1}{\Sigma_s} = \frac{1}{\sigma_s \frac{N_A}{A} \rho} \approx const$$

Average energy loss in collision

$$\xi = \ln\left(\frac{E_1}{E_2}\right) \approx \frac{2}{A + 2/3}$$

Velocity after *n* collisions

$$v_n = v_1 \cdot e^{-\frac{n-1}{2}\xi}$$

Time after *n* collisions

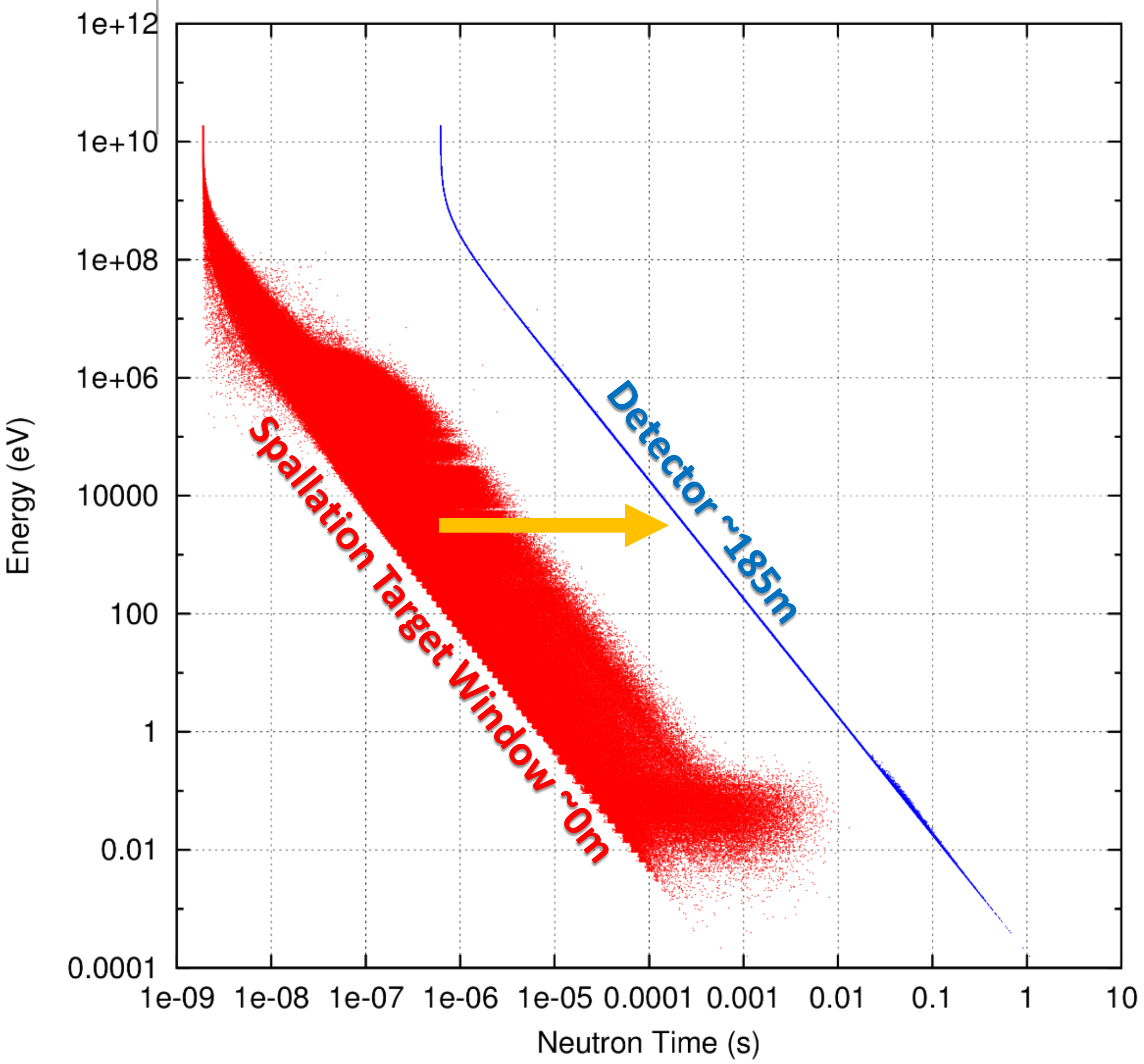
$$t_n = \frac{\lambda_s}{v_1} \sum_{i=1}^{n} e^{\frac{i-1}{2}\xi}$$

Effective length l_n after *n* collisions

$$l_n = v_n \cdot t_n = v_1 \cdot e^{-\frac{n-1}{2}\xi} \cdot \frac{\lambda_s}{v_1} \sum_{i=1}^{n} e^{\frac{i-1}{2}\xi}$$

$$l_\infty \approx 2\frac{\lambda_s}{\xi}$$

Neutron Moderation



Mean free path

$$\lambda_s = \frac{1}{\Sigma_s} = \frac{1}{\sigma_s \frac{N_A}{A} \rho} \approx const$$

Average energy loss in collision

$$\xi = \ln\left(\frac{E_2}{E_1}\right) \approx \frac{2}{A + 2/3}$$

Velocity after *n* collisions

$$v_n = v_1 \cdot e^{-\frac{n-1}{2}\xi}$$

Time after *n* collisions

$$t_n = \frac{\lambda_s}{v_1} \sum_{i=1}^{n} e^{\frac{i-1}{2}\xi}$$

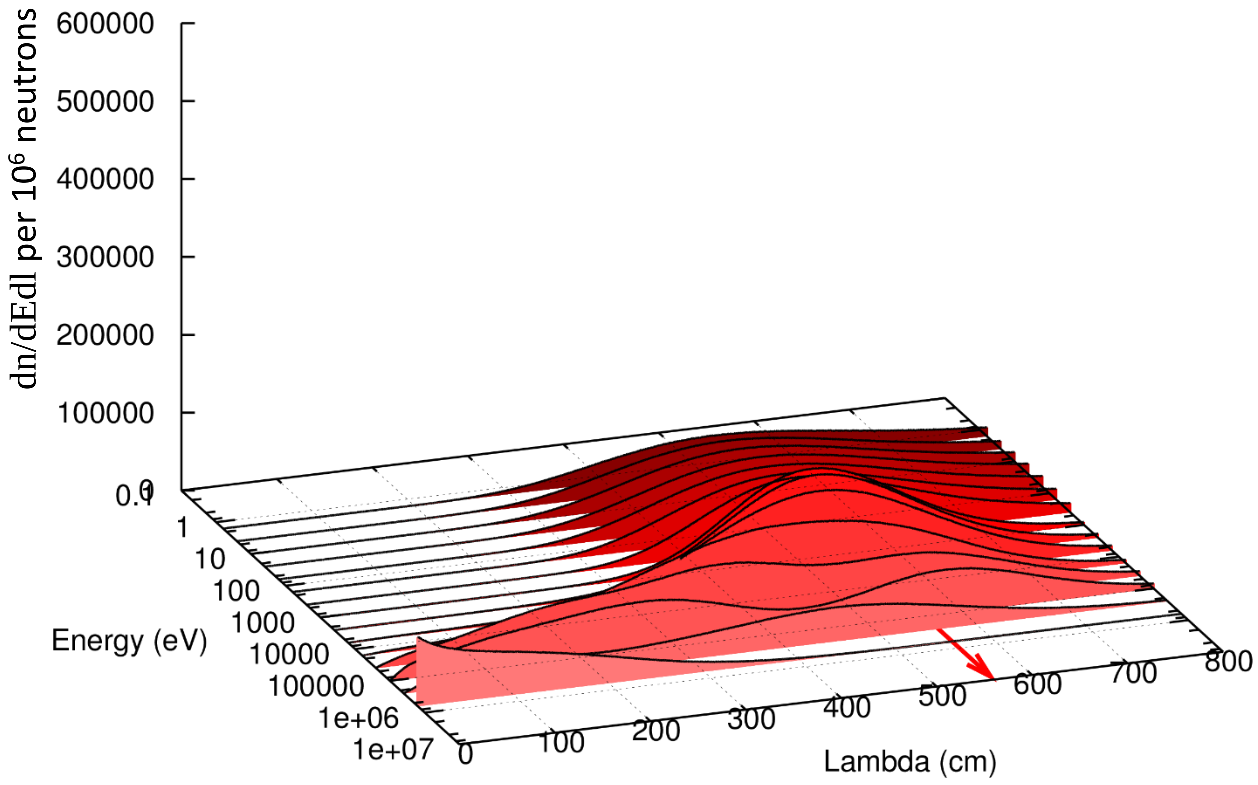
Effective length l_n after *n* collisions

$$l_n = v_n \cdot t_n = v_1 \cdot e^{-\frac{n-1}{2}\xi} \cdot \frac{\lambda_s}{v_1} \sum_{i=1}^{n} e^{\frac{i-1}{2}\xi}$$

$$l_\infty \approx 2 \frac{\lambda_s}{\xi}$$

Effective Neutron Path

- **Lead:**
  $\sigma_s(1\text{keV}) = 11.35$ b
  $\lambda_s = 2.7$ cm
  $\xi = 9.6\times10^{-3}$
  $l \approx 560$ cm

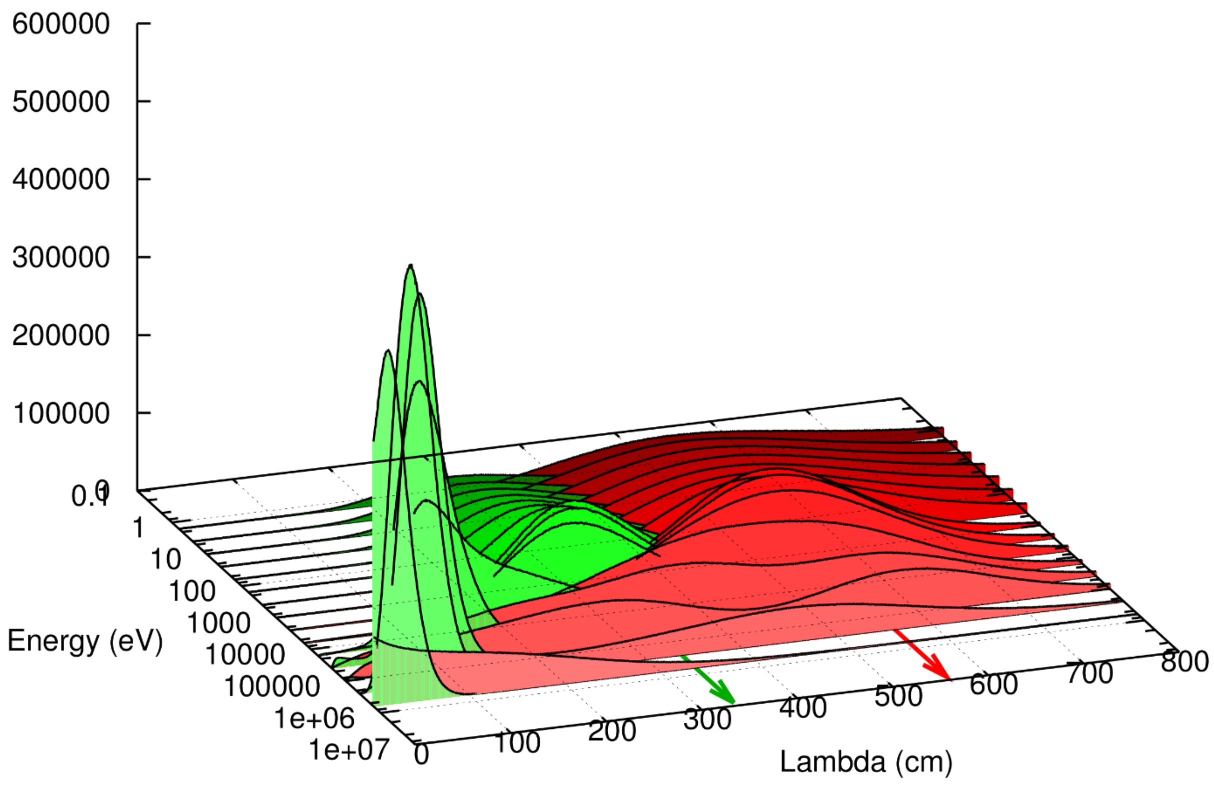


Effective Neutron path in infinite block with E_0=1MeV

Effective Neutron Path

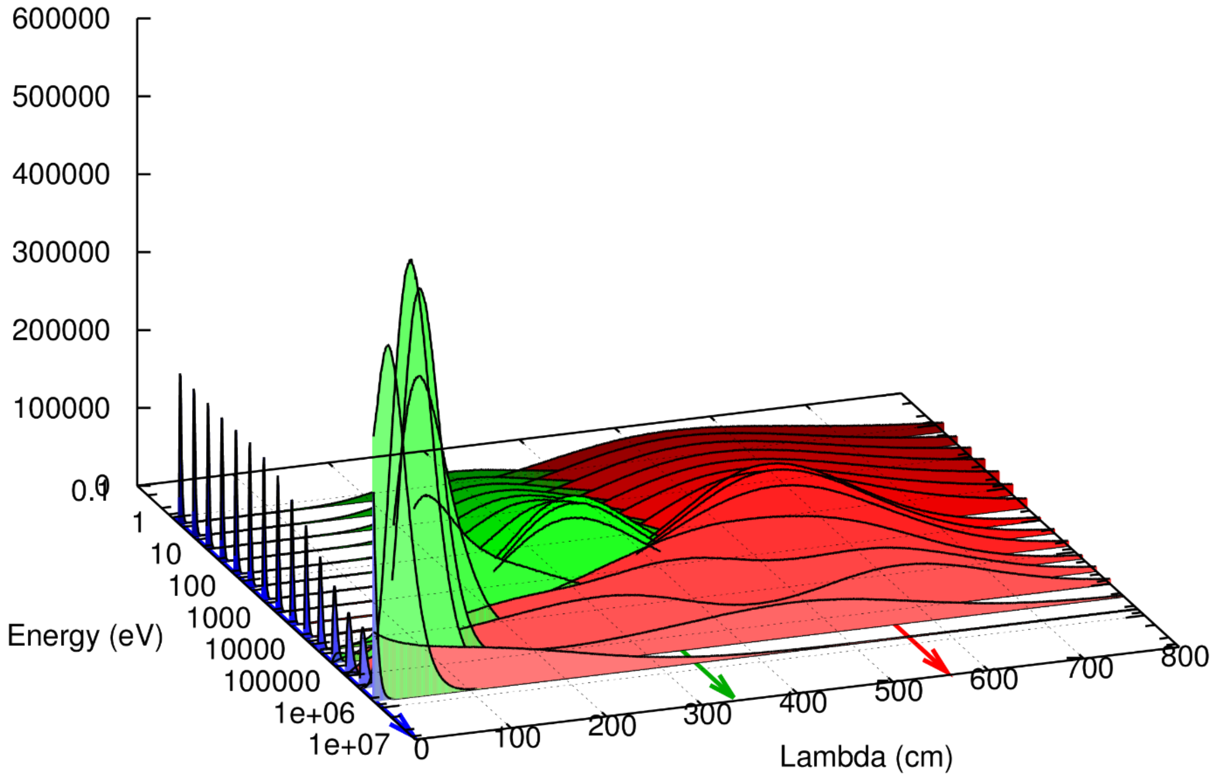
- **Lead:**
  σ_s(1keV) = 11.35 b
  λ_s = 2.7 cm
  $\xi = 9.6\times10^{-3}$
  l ≈ 560 cm

- **Aluminum:**
  σ_s(1keV) = 1.42 b
  λ_s = 11.7 cm
  $\xi = 7.2\times10^{-2}$
  l ≈ 320cm



Effective Neutron Path

- **Lead:**
  $\sigma_s(1\text{keV}) = 11.35$ b
  $\lambda_s = 2.7$ cm
  $\xi = 9.6\times10^{-3}$
  $l \approx 560$ cm
- **Aluminum:**
  $\sigma_s(1\text{keV}) = 1.42$ b
  $\lambda_s = 11.7$ cm
  $\xi = 7.2\times10^{-2}$
  $l \approx 320$cm
- **Water:**
  $\sigma H_s(1\text{keV}) = 20.3$ b
  $\sigma O_s(1\text{keV}) = 3.85$ b
  $\lambda_s = 0.67$ cm
  $\xi_H = 1,\ \xi_O = 0.12$
  $l_H \approx 1.5$ cm



Effective Neutron Path Spread

The lambda spread is determined by the spread of energy loss after collision

Maximum (θ=0°)

$$E_{2max} = E_1$$

Minimum (θ=180°)

$$E_{2\min} = \left[\frac{A-1}{A+1}\right]^2 E_1$$

and a mean scattering angle

$$b = \overline{cos\theta} = \frac{2}{3A}$$

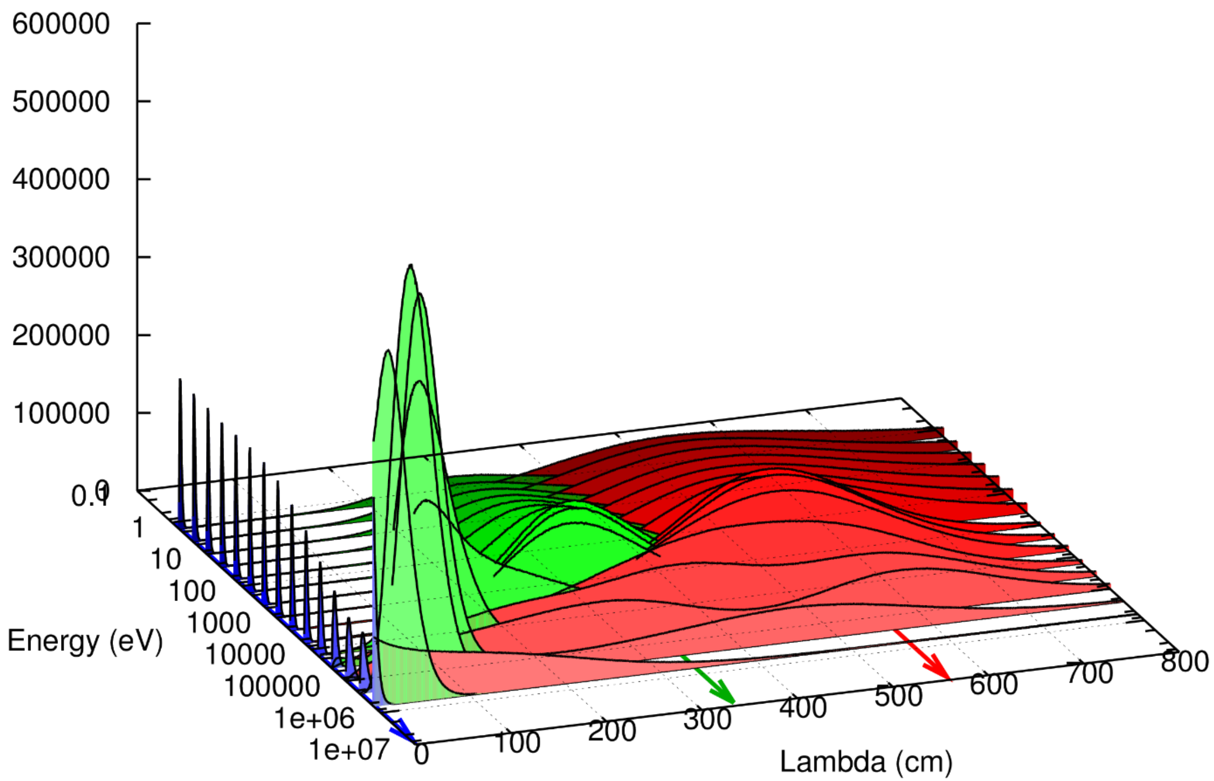
using the central limiting theorem to sum up all contributions:

For (A>>1)

$$\overline{\left(\frac{\Delta E}{E}\right)^2} \approx \frac{8}{3A}$$

For H (A=1)

$$\overline{\left(\frac{\Delta E}{E}\right)^2} = \frac{7}{3}$$



Effective Neutron Path Spread

- **Lead:**

$$\sqrt{(\Delta E/_E)^2} = 11.4\%$$

$$\Delta l \approx 32\text{cm}$$

- **Aluminum:**

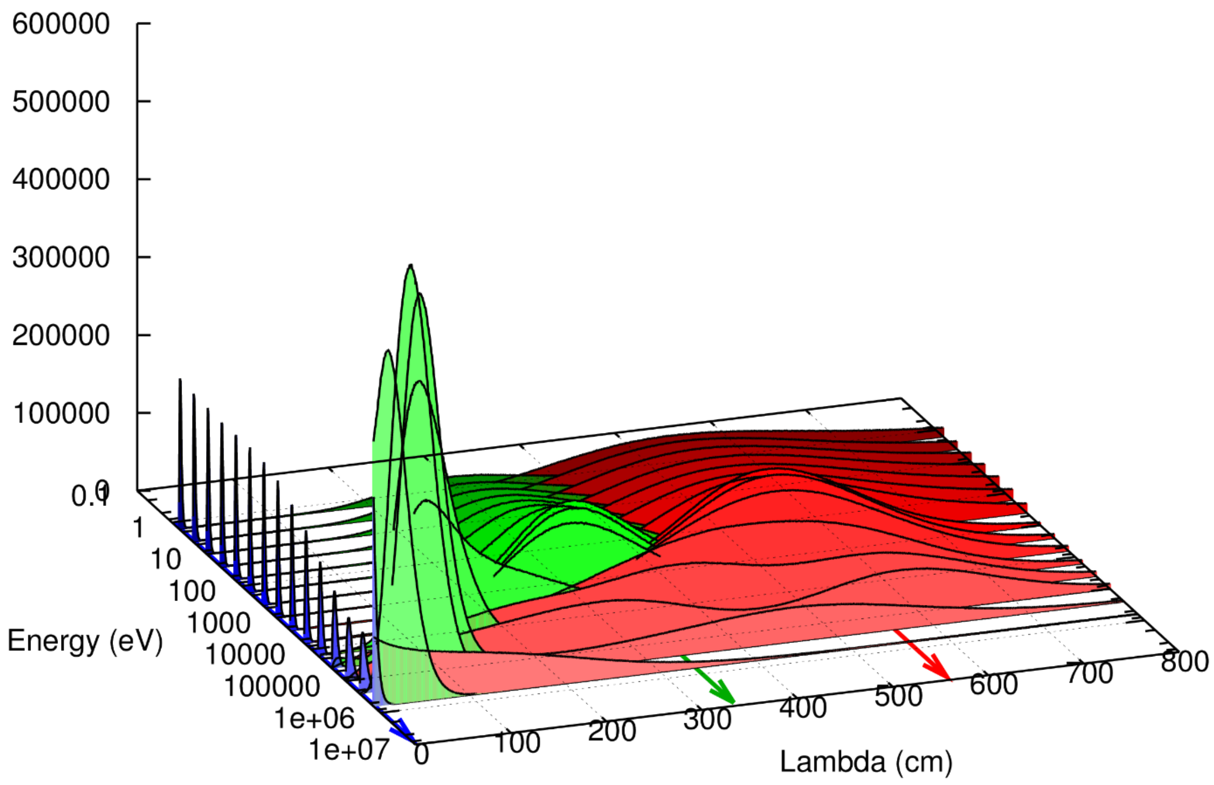
$$\sqrt{(\Delta E/_E)^2} = 31.4\%$$

$$\Delta l \approx 53\text{cm}$$

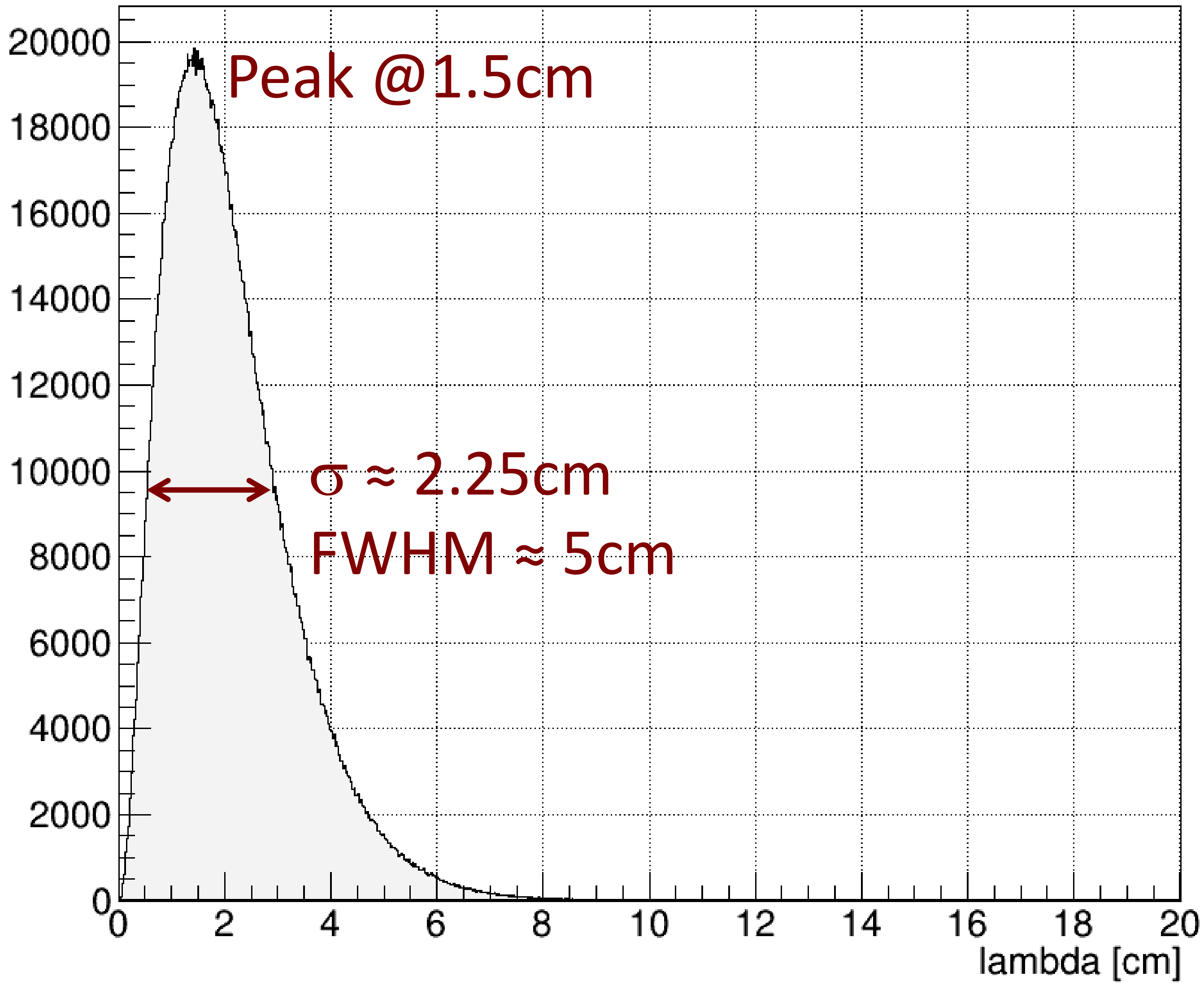
- **Water:**

$$\sqrt{(\Delta E/_E)^2} = 150\%$$

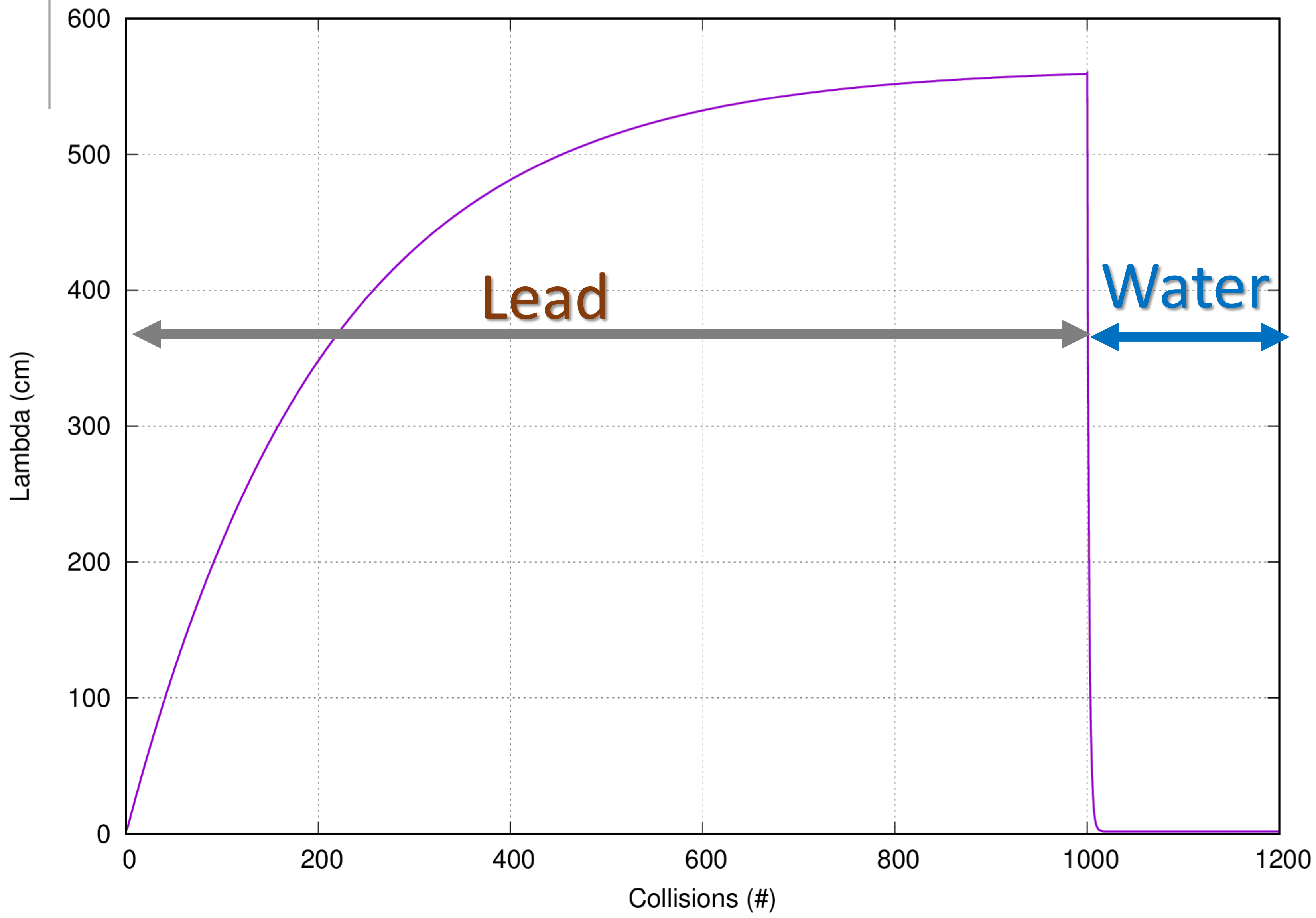
$$\Delta l \approx 2.25\text{cm}$$



Water Energy resolution @1eV



Lambda vs # neutron collisions

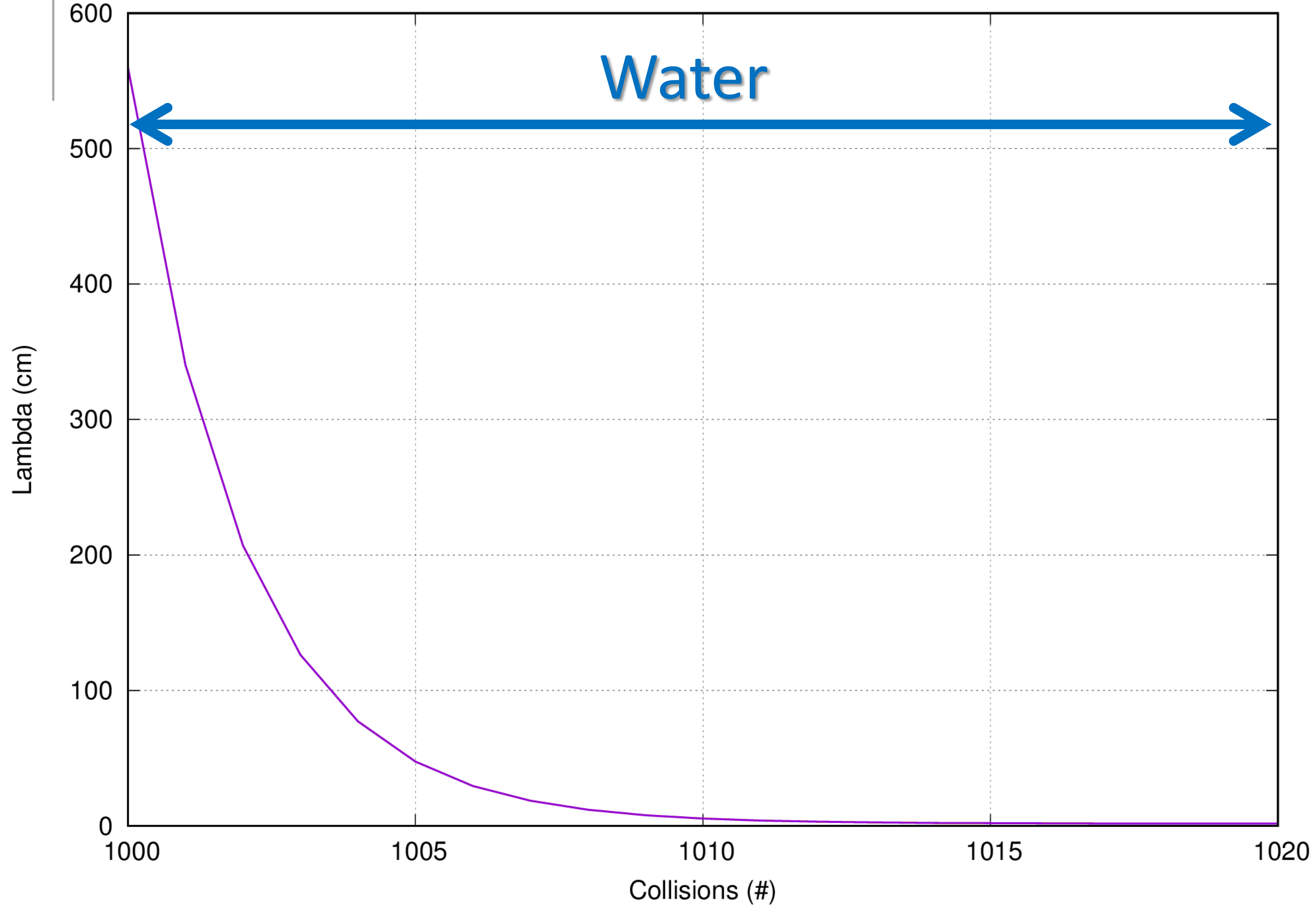


Note:

- to thermalize 2MeV in Pb we need ~1900 collisions
- *n*=1000 col ≈ keV energies
- Average distance traveled (moderation path)

$$r = \sqrt{2n\lambda_s^2} \approx \sqrt{2 \cdot 1000 \cdot 2.5^2} = 111\ cm$$

Lambda vs # collisions

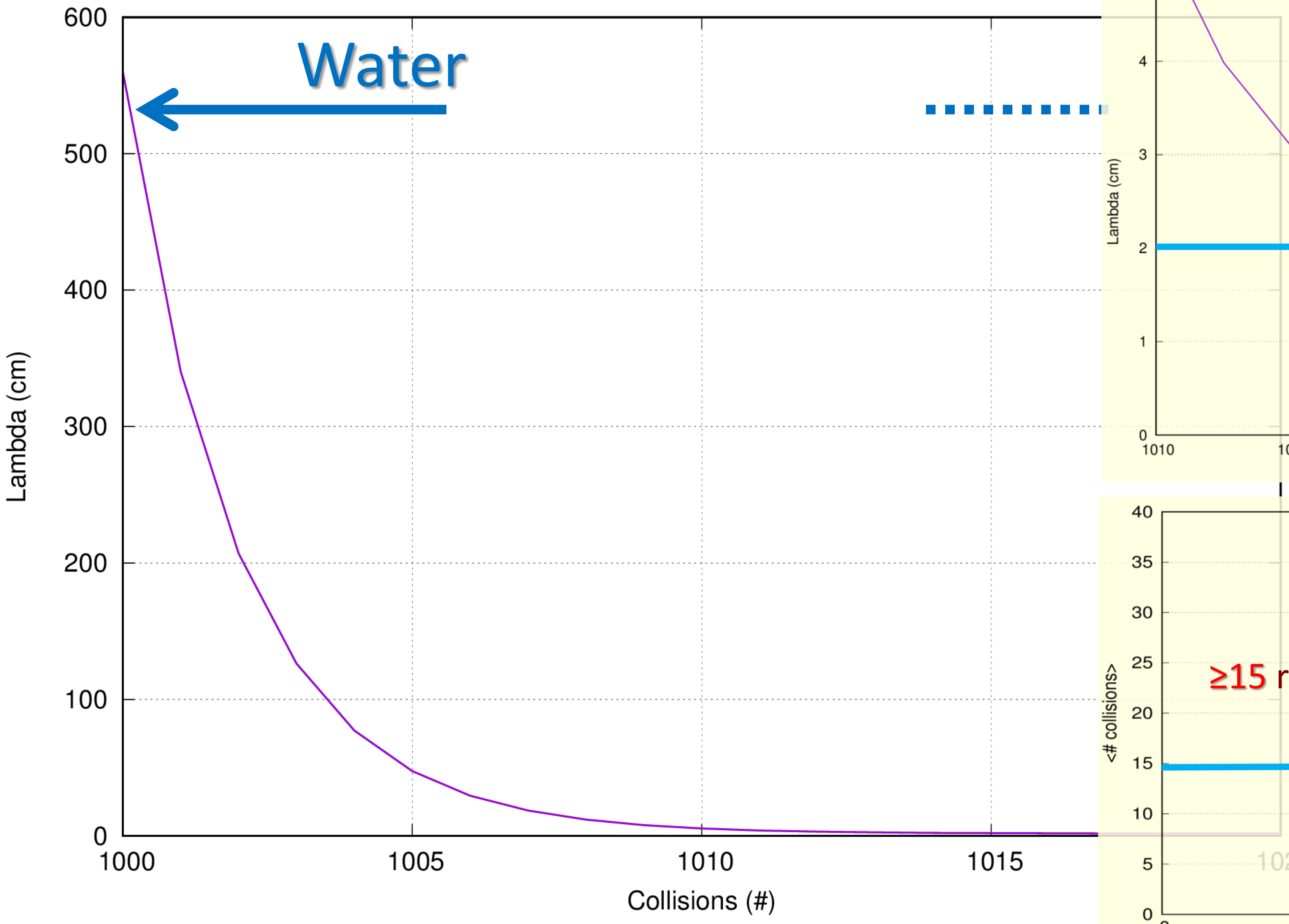


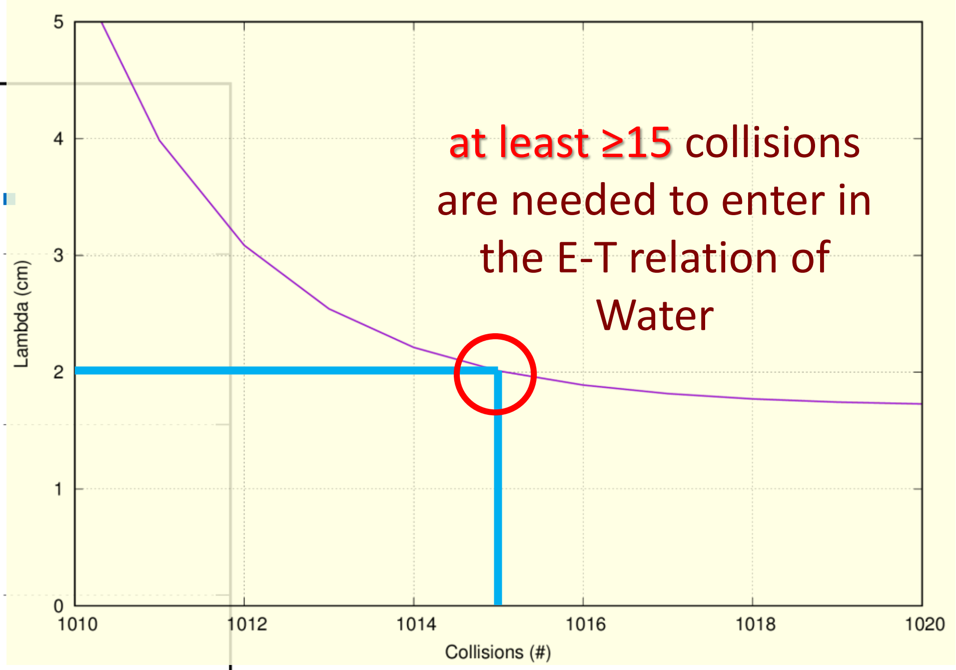
Note:

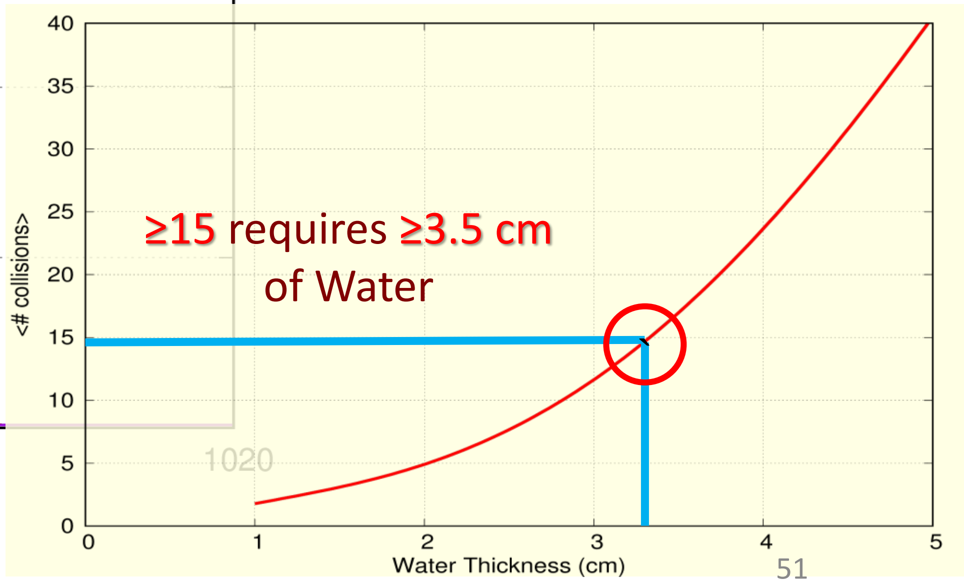
- to thermalize 2MeV in Water (due to H) we need ~18 collisions
- *n*=10 col ≈ keV energies
- Average distance traveled (moderation path)

$$r = \sqrt{2N\lambda_s^2} \approx \sqrt{2 \cdot 10 \cdot 0.67^2} = 3.0\ cm$$

Lambda vs # collisions







Optimization: Figure of Merit

Distance dependence

- Neutron Fluence:

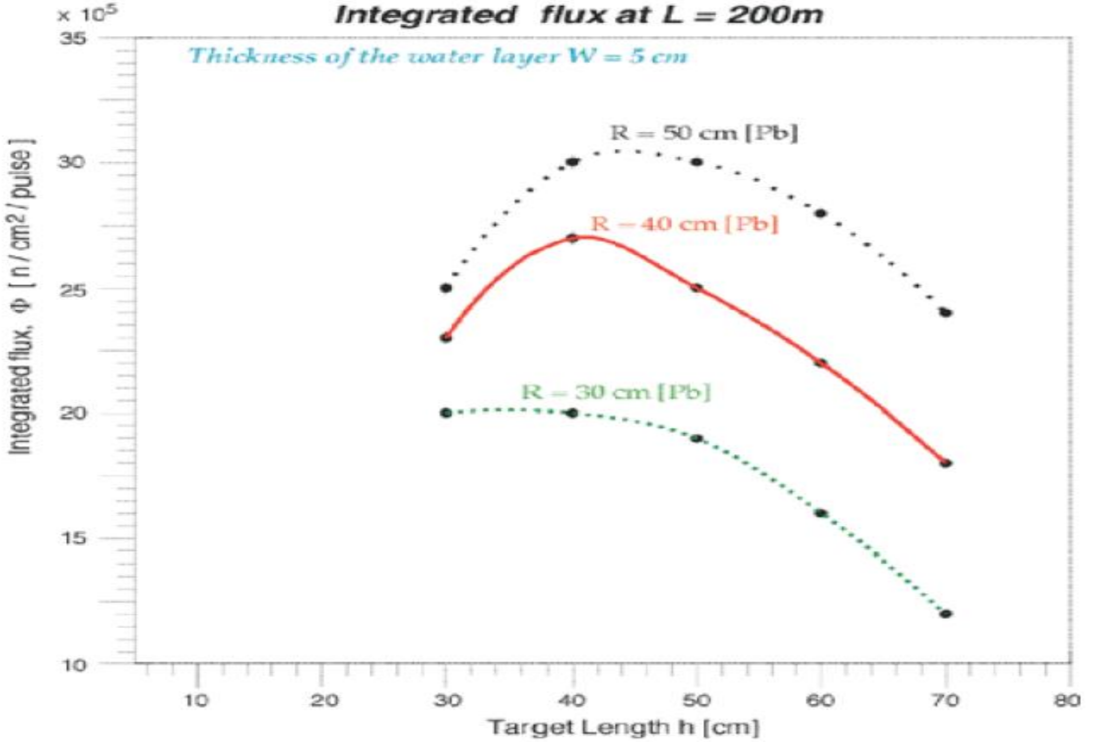
$$\Phi = \frac{\Phi_o}{L^2}$$

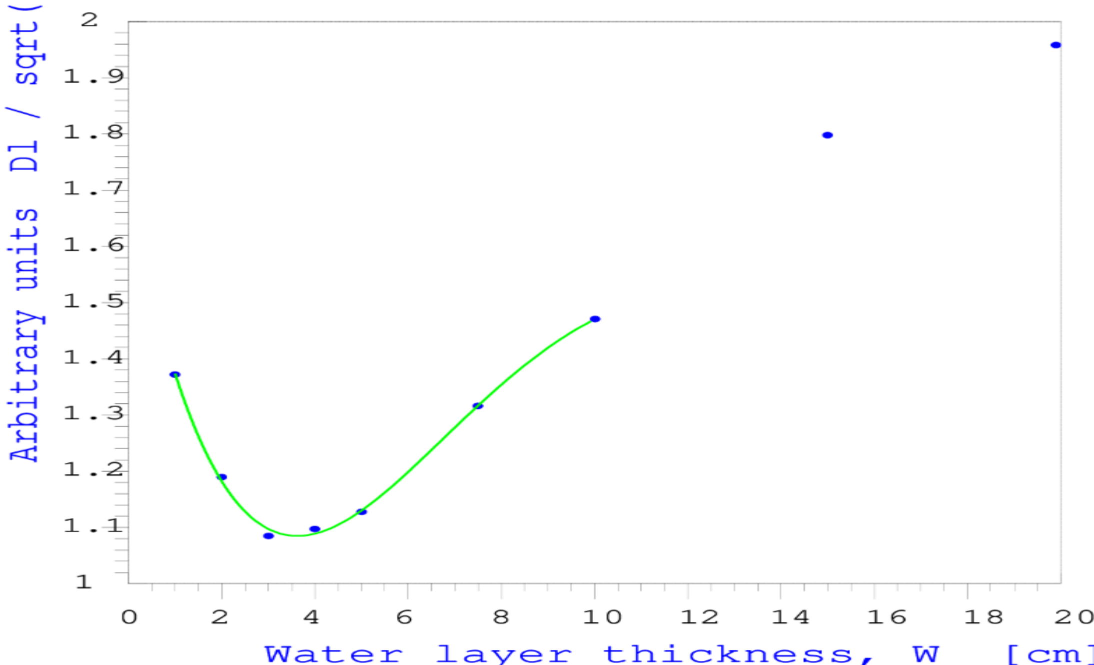
- Resolution:

$$\frac{\Delta E}{E} = 2\frac{\Delta l}{L+l} \approx 2\frac{\Delta l}{L}$$

- Figure of Merit

$$\frac{\Delta E/_E}{\sqrt{\Phi}} \approx \frac{2\Delta l}{\sqrt{\Phi_o}}$$





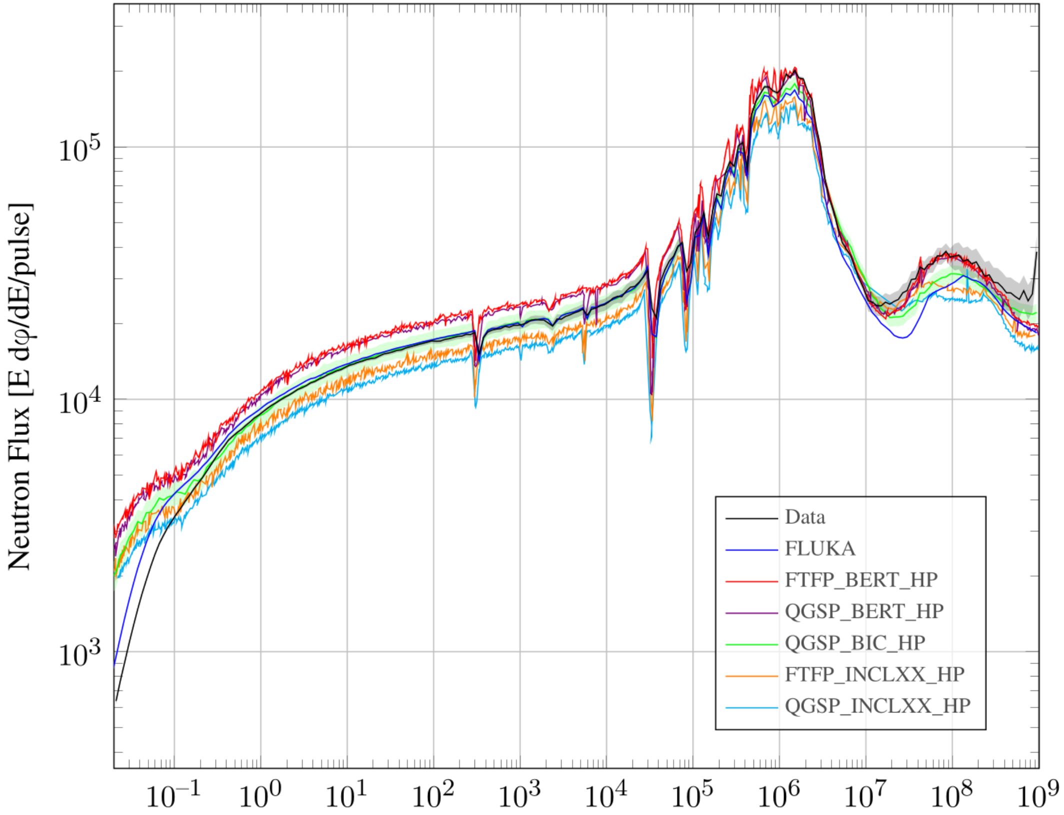
Neutron source optimization recipe

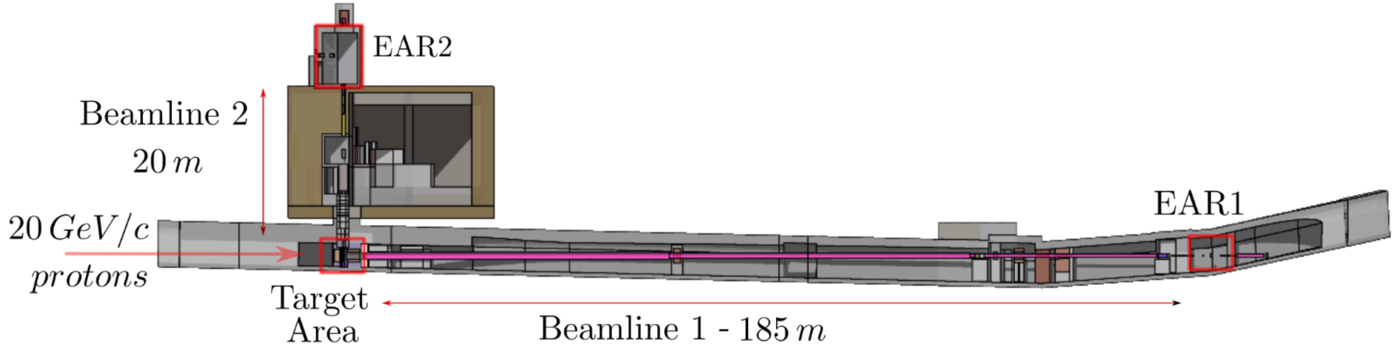
Ingredients:

- A fast accelerator
- Freshly heavy A target (with low XS)
- Room temperature water
- 2 collimators
- 1 sweeping magnet
- ^{10}B optional

Instructions:

1. If you have a fast accelerator lying around, Spallation is your best friend.
2. Cut the heavy A target to dimensions (to contain 95% of the shower):
   - Length 3-4 λ_{inel}
   - Radius >1 λ_{inel}
3. Shoot fast protons on it
4. Soak the produced spallation neutrons in room temperature water, ~4cm in length
   - Optional: to spice it up add a jest of ^{10}B
5. let them fly in vacuum
6. collimate
7. use a sweeping magnet to remove charged particles
8. collimate again
9. enjoy your neutron spectrum





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