NEW Symmetries of the Two Higgs Doublet Model

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The Two-Higgs Doublet scalar potential

Most general SU(2) × U(1) scalar potential:

$$V = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - \left(m_{12}^2\Phi_1^\dagger\Phi_2 + h.c.\right)$$
$$+ \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2$$
$$+ \left[\frac{1}{2}\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + \lambda_6|\Phi_1|^2\left(\Phi_1^\dagger\Phi_2\right) + \lambda_7|\Phi_2|^2\left(\Phi_1^\dagger\Phi_2\right) + \mathrm{h.c.}\right]$$

m^2_{12}, λ_5, λ_6 and **λ_7 complex - seemingly 14 independent real parameters,**
in fact only 11.

Most frequently studied model: softly broken theory with a Z_2 symmetry,

$\Phi_1 \to -\Phi_1$ and $\Phi_2 \to \Phi_2$, meaning $\lambda_6, \lambda_7 = 0$.

It avoids potentially large flavour-changing neutral currents (FCNC)

Symmetries of the $SU(2)\times U(1)$ 2HDM

The number of free parameters of the potential can be reduced by imposing discrete or continuous global symmetries upon it. The phenomenology of the models possessing those symmetries can be quite interesting.

The simplest example is the already mentioned Z_2 symmetry, where

$\Phi_1 \rightarrow -\Phi_1$ and $\Phi_2 \rightarrow \Phi_2$ $\Rightarrow$ $m^2_{12} = 0, \quad \lambda_6 = \lambda_7 = 0$

Another simple example is the "standard" CP transformation,

$\Phi_1 \rightarrow \Phi_1^*$ and $\Phi_2 \rightarrow \Phi_2^*$ $\Rightarrow$ all parameters real

Another very important symmetry for what's going to follow is a "generalized CP transformation" called CP2,

$\Phi_1 \rightarrow \Phi_2^*$ and $\Phi_2 \rightarrow -\Phi_1^*$ $\Rightarrow$

$$m_{11}^2 = m_{22}^2 \ , \ m_{12}^2 = 0 \ , \ \boxed{\lambda_1 = \lambda_2 \ , \ \lambda_6 = -\lambda_7}$$

Symmetries of the $SU(2)\times U(1)$ 2HDM

- **Symmetries which transform doublets into linear combinations of themselves are called *Higgs Family Symmetries*.**

- **Symmetries which transform doublets into linear combinations of their complex conjugates are called *Generalized CP Symmetries*.**

- **It has been shown, by Igor Ivanov, that there are only *six* of these symmetries in the SU(2)×U(1) invariant potential.**

I.P. Ivanov, Phys. Rev. D75 (2007) 035001
I.P. Ivanov, Phys. Rev. D77 (2008) 015017

- **If one considers the potential invariant under SU(2) only, there are further symmetries, such as *custodial symmetry*. But once taking hypercarge or fermions into account, the relations between couplings resulting from those symmetries will not be stable under renormalization. We will not consider this situation in this work.**

R. A. Battye, G. D. Brawn, and A. Pilaftsis, JHEP 08 (2011) 020
A. Pilaftsis, Phys. Lett. B 706 (2012) 465

Symmetries of the SU(2)×U(1) 2HDM

Higgs-Family Symmetries:

Z_2: $\Phi_1 \to \Phi_1 \quad , \quad \Phi_2 \to -\Phi_2$

U(1): $\Phi_1 \to \Phi_1 \quad , \quad \Phi_2 \to e^{i\theta}\,\Phi_2 \qquad \theta \neq \{0\,,\,\pi\}$

SO(3): $\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \to U \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \qquad \forall_{U \in U(2)}$

Generalized CP Transformations:

CP1: $\Phi_1 \to \Phi_1^* \quad , \quad \Phi_2 \to \Phi_2^*$

CP2: $\Phi_1 \to \Phi_2^* \quad , \quad \Phi_2 \to -\Phi_1^*$

CP3: $\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \Phi_1^* \\ \Phi_2^* \end{pmatrix} \qquad 0 < \theta < \pi/2$

Symmetries of the 2HDM

Each symmetry has a different impact on the scalar potential, originating models with different phenomenologies and a different number *N* of independent parameters:

Symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	N
CP1			real					real	real	real	9
Z_2			0						0	0	7
U(1)			0					0	0	0	6
CP2		m_{11}^2	0		λ_1					$-\lambda_6$	5
CP3		m_{11}^2	0		λ_1			$\lambda_1 - \lambda_3 - \lambda_4$	0	0	4
$SO(3)$		m_{11}^2	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0	3

TABLE 1

Some of these symmetries have phenomenologically viable extensions to the Yukawa sector, thus becoming symmetries of the whole lagrangian, not simply of the potential (Z_2, U(1), CP1, CP2, CP3).

These symmetries may appear differently depending on the *choice of basis* for the 2HDM...

Basis changes

Because both doublets have the same quantum numbers (and are therefore indistinguishable) any linear combination of them will be physically equivalent, and all physical quantities must be independent of this *choice of basis*.

A general change of basis transforms the fields $\{\Phi_1, \Phi_2\}$ in new doublets $\{\Phi'_1, \Phi'_2\}$ related by $\Phi'_a = U_{ab}\, \Phi_b$, where *U* is a generic U(2) matrix, which can be parameterized by three real angles ψ, ξ and χ as

$$U = \begin{pmatrix} e^{i\chi} c_\psi & e^{i(\chi-\xi)} s_\psi \\ -e^{i(\xi-\chi)} s_\psi & e^{-i\chi} c_\psi \end{pmatrix}$$

Renormalization Group Equations

Upon renormalization, each parameter in the potential "runs" with the renormalization scale *μ*, according to their beta-functions. For the most general 2HDM potential at one-loop, they are given by, for the quadratic parameters,

$$\begin{aligned}
\beta_{m_{11}^2} &= 3\lambda_1 m_{11}^2 + (2\lambda_3+\lambda_4)\, m_{22}^2 - 3\,\left(\lambda_6^*\, m_{12}^2 + \text{h.c.}\right) - \frac{1}{4}\,(9g^2+3g'^2)\, m_{11}^2 \\
&\quad + \beta^F_{m_{11}^2}, \\
\beta_{m_{22}^2} &= (2\lambda_3+\lambda_4)\; m_{11}^2 + 3\lambda_2\, m_{22}^2 - 3\,\left(\lambda_7^*\, m_{12}^2 + \text{h.c.}\right) - \frac{1}{4}\,(9g^2+3g'^2)\, m_{22}^2 \\
&\quad + \beta^F_{m_{22}^2}, \\
\beta_{m_{12}^2} &= -3\left(\lambda_6\, m_{11}^2 + \lambda_7\, m_{22}^2\right) + (\lambda_3+2\lambda_4)\; m_{12}^2 + 3\lambda_5\, {m_{12}^2}^* - \frac{1}{4}\,(9g^2+3g'^2) m_{12}^2 \\
&\quad + \beta^F_{m_{12}^2},
\end{aligned}$$

Factors of *$16\pi^2$* are absorbed into the definition of "*β*", and all fermionic contributions are gathered in the "*F*" terms. Gauge contributions involve the couplings *g* and *g'*. For the quartic couplings, we have

$$
\begin{aligned}
\beta_{\lambda_1} &= 6\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12\,|\lambda_6|^2 \\
&\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) - \frac{3}{2}\lambda_1(3g^2 + g'^2) \,+\, \beta^F_{\lambda_1}, \\
\beta_{\lambda_2} &= 6\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + |\lambda_5|^2 + 12\,|\lambda_7|^2 \\
&\quad + \frac{3}{8}(3g^4 + g'^4 + 2g^2g'^2) - \frac{3}{2}\lambda_2(3g^2 + g'^2) \,+\, \beta^F_{\lambda_2}, \\
\beta_{\lambda_3} &= (\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 2\lambda_3^2 + \lambda_4^2 + |\lambda_5|^2 + 2\left(|\lambda_6|^2 + |\lambda_7|^2\right) + 8\,\mathrm{Re}\,(\lambda_6\lambda_7^*) \\
&\quad + \frac{3}{8}(3g^4 + g'^4 - 2g^2g'^2) - \frac{3}{2}\lambda_3(3g^2 + g'^2) \,+\, \beta^F_{\lambda_3}, \\
\beta_{\lambda_4} &= (\lambda_1 + \lambda_2)\lambda_4 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 4\,|\lambda_5|^2 + 5\left(|\lambda_6|^2 + |\lambda_7|^2\right) + 2\,\mathrm{Re}\,(\lambda_6\lambda_7^*) \\
&\quad + \frac{3}{2}g^2g'^2 - \frac{3}{2}\lambda_4(3g^2 + g'^2) \,+\, \beta^F_{\lambda_4}, \\
\beta_{\lambda_5} &= (\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4)\lambda_5 + 5\left(\lambda_6^2 + \lambda_7^2\right) + 2\lambda_6\lambda_7 \\
&\quad - \frac{3}{2}\lambda_5(3g^2 + g'^2) \,+\, \beta^F_{\lambda_5}, \\
\beta_{\lambda_6} &= (6\lambda_1 + 3\lambda_3 + 4\lambda_4)\lambda_6 + (3\lambda_3 + 2\lambda_4)\lambda_7 + 5\lambda_5\lambda_6^* + \lambda_5\lambda_7^* \\
&\quad - \frac{3}{2}\lambda_6(3g^2 + g'^2) \,+\, \beta^F_{\lambda_6}, \\
\beta_{\lambda_7} &= (6\lambda_2 + 3\lambda_3 + 4\lambda_4)\lambda_7 + (3\lambda_3 + 2\lambda_4)\lambda_6 + 5\lambda_5\lambda_7^* + \lambda_5\lambda_6^* \\
&\quad - \frac{3}{2}\lambda_7(3g^2 + g'^2) \,+\, \beta^F_{\lambda_7},
\end{aligned}
$$

These beta-functions show how symmetry-imposed relations between couplings remain invariant under renormalization. For example, the Z_2 symmetry forces $\lambda_6 = \lambda_7 = 0$, and we see that in that situation we automatically have (let us ignore fermions for now)

$$\beta_{\lambda_6} = \beta_{\lambda_7} = 0$$

So that λ_6 and λ_7 will remain zero, even after loop corrections are taken into account (modulo finite contributions...). This will occur at all orders, due to the presence of a symmetry.

If we look at the λ_5 beta-function for the Z_2 model, it is given by

$$\beta_{\lambda_5} = \left[\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4 - \frac{3}{2}\left(3g^2 + g'^2\right)\right]\lambda_5$$

We see that it possesses a *Fixed Point* – if $\lambda_5 = 0$, then $\beta_{\lambda_5} = 0$ and λ_5 remains zero. This hints at the presence of a larger symmetry than Z_2 which sets this coupling to zero – indeed, that is the effect of the U(1) symmetry mentioned before.

At this point, notice that if $\lambda_1 = \lambda_2$ and $\lambda_6 = -\lambda_7$ (which are the CP2 conditions on the quartic couplings) then another fixed point becomes apparent:

$$\beta_{m_{11}^2+m_{22}^2} = 3(\lambda_1 m_{11}^2 + \lambda_2 m_{22}^2) + (2\lambda_3 + \lambda_4)\,(m_{11}^2 + m_{22}^2) - 3\,\left[(\lambda_6^* + \lambda_7^*) m_{12}^2 + \text{h.c.}\right] - \frac{1}{4}\,(9g^2 + 3g'^2)(m_{11}^2 + m_{22}^2)\,,$$

$$\beta_{\lambda_1-\lambda_2} = 6\,(\lambda_1^2 - \lambda_2^2) + 12\left(|\lambda_6|^2 - |\lambda_7|^2\right) - \frac{3}{2}(\lambda_1 - \lambda_2)\,(3g^2 + g'^2),$$

$$\beta_{\lambda_6+\lambda_7} = 6\,(\lambda_1\lambda_6 + \lambda_2\lambda_7) + (3\lambda_3 + 2\lambda_4)\,(\lambda_6 + \lambda_7) + 6\lambda_5\,(\lambda_6^* + \lambda_7^*) - \frac{3}{2}(\lambda_6 + \lambda_7)\,(3g^2 + g'^2)\,,$$

If the above conditions on the quartic couplings are satisfied, then

$$\boxed{m_{11}^2 + m_{22}^2 = 0}$$

is a fixed point of the one-loop RG equations. This condition on the quadratic parameters is WEIRD and completely different from those shown on Table 1.

Thus, in order to guarantee a softly broken CP3, we demand that there should not exist relations among the physical parameters such that the constraints defining any of those cases of a CP3 invariant potential applies. Then, the conditions expressing a softly broken CP3 become (listing only the RG-stable cases):

Case SOFT-CP3-B:

$$
\begin{aligned}
& M_j = M_k, \quad e_j q_k - e_k q_j = 0, \\
& v^2(e_i q_j - e_j q_i) + e_i e_j (M_j^2 - M_i^2) = 0, \\
& v^2(e_i q_k - e_k q_i) + e_i e_k (M_j^2 - M_i^2) = 0, \\
& 2v^4 q = e_i^2 M_i^2 + (e_j^2 + e_k^2) M_j^2, \\
& 2v^2 M_{H\pm}^2 = e_i^2 M_i^2 + (e_j^2 + e_k^2) M_j^2 + v^2(e_1 q_1 + e_2 q_2 + e_3 q_3),
\end{aligned}
$$

and none of the five cases of CP3 invariant potential found in [18] applies, for any combination of i, j, k.

Case SOFT-CP3-C:

$$
\begin{aligned}
& e_k = q_k = 0, \quad v^2(e_i q_j - e_j q_i) + e_i e_j (M_j^2 - M_i^2) = 0, \\
& 2v^4 q = e_i^2 M_i^2 + e_j^2 M_j^2, \\
& (e_j^2 M_i^2 + e_i^2 M_j^2 - v^2 M_k^2) \\
& \times \left[2v^2 M_{H\pm}^2 + e_i^2 M_i^2 + e_j^2 M_j^2 - v^2(2M_k^2 + e_i q_i + e_j q_j) \right] \\
& = 2 e_i^2 e_j^2 (M_j^2 - M_i^2)^2,
\end{aligned}
$$

and none of the five cases of CP3 invariant potential found in [18] applies, for any combination of i, j, k.

Case SOFT-CP3-B contains partial mass degeneracy, which requires $m_{11}^2 + m_{22}^2 = 0$, whereas case SOFT-CP3-C represents the general case of softly broken CP3. Another, RG-unstable case is listed in appendix D for completeness.

Bilinear Formalism

It is very useful for some calculations to express the 2HDM potential in terms of the 4 gauge invariant bilinears one can construct with both doublets:

$$\begin{aligned} r_0 &= \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2\right), \\ r_1 &= \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) = \mathrm{Re}\left(\Phi_1^\dagger\Phi_2\right), \\ r_2 &= -\tfrac{i}{2}\left(\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1\right) = \mathrm{Im}\left(\Phi_1^\dagger\Phi_2\right), \\ r_3 &= \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right). \end{aligned}$$

In terms of these quantities, the potential becomes

$$V = M_\mu\, r^\mu + \Lambda_{\mu\nu}\, r^\mu\, r^\nu$$

$$\begin{aligned} r^\mu &= (r_0\,, r_1\,, r_2\,, r_3) = (r_0\,, \vec{r}), \\ M^\mu &= \left(m_{11}^2 + m_{22}^2\,, 2\mathrm{Re}(m_{12}^2)\,, -2\mathrm{Im}(m_{12}^2)\,, m_{22}^2 - m_{11}^2\right) = (M_0\,, \vec{M})\,, \end{aligned}$$

$$\Lambda^{\mu\nu} = \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\lambda_1+\lambda_2)+\lambda_3 & -\mathrm{Re}\,(\lambda_6+\lambda_7) & \mathrm{Im}\,(\lambda_6+\lambda_7) & \frac{1}{2}(\lambda_2-\lambda_1) \\ -\mathrm{Re}\,(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_5) & \mathrm{Re}\,(\lambda_6-\lambda_7) \\ \mathrm{Im}\,(\lambda_6+\lambda_7) & -\mathrm{Im}\,(\lambda_5) & \lambda_4-\mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_6-\lambda_7) \\ \frac{1}{2}(\lambda_2-\lambda_1) & \mathrm{Re}\,(\lambda_6-\lambda_7) & -\mathrm{Im}\,(\lambda_6-\lambda_7) & \frac{1}{2}(\lambda_1+\lambda_2)-\lambda_3 \end{pmatrix}$$

Basis Transformations in the Bilinear Formalism

One of the reasons the bilinear formalism is so useful is just how simple basis transformations look using bilinears. A generic basis transformation (see slide 10) via a generic 2×2 unitary matrix *U* corresponds to an *O(3) rotation matrix* ***R*** in the bilinear space, with entries given by

$$R_{ij}(U) = \mathrm{Tr}\left(U^\dagger \sigma_i U \sigma_j\right)/2$$

where σ_i are the Pauli matrices. Then:

- The quantities r_0, $M_0 = m_{11}^2 + m_{22}^2$ and $\Lambda_{00} = \frac{1}{2}(\lambda_1 + \lambda_2) + \lambda_3$ are basis-invariant.
- The quantities $\vec{r}$, $\vec{M}$ and $\vec{\Lambda}$ transform as vectors under basis transformations – they are simply "rotated" by the matrix *R*.

Basis Transformations

$$\vec{r} = (r_1\,, r_2\,, r_3)$$

$$\vec{M} = \left(2\mathrm{Re}(m_{12}^2)\,, -2\mathrm{Im}(m_{12}^2)\,, m_{22}^2 - m_{11}^2\right)$$

$$\vec{\Lambda} = \left(-\mathrm{Re}\,(\lambda_6 + \lambda_7)\;,\;\mathrm{Im}\,(\lambda_6 + \lambda_7)\;,\;\frac{1}{2}(\lambda_2 - \lambda_1)\right)$$

$\Rightarrow$

$$\vec{r}\,' = R\,\vec{r}\,,$$

$$\vec{M}' = R\,\vec{M}\,,$$

$$\vec{\Lambda}' = R\,\vec{\Lambda}\,,$$

- **Finally, the matrix Λ,**

$$\Lambda = \begin{pmatrix} \lambda_4 + \mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_5) & \mathrm{Re}\,(\lambda_6 - \lambda_7) \\ -\mathrm{Im}\,(\lambda_5) & \lambda_4 - \mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_6 - \lambda_7) \\ \mathrm{Re}\,(\lambda_6 - \lambda_7) & -\mathrm{Im}\,(\lambda_6 - \lambda_7) & \frac{1}{2}(\lambda_1 + \lambda_2) - \lambda_3 \end{pmatrix}$$

transforms as a matrix under rotations with *R*,

$$\Lambda' = R\,\Lambda\,R^T$$

Structure of beta-functions in the Bilinear Formalism

Bednyakov computed the tree-loop 2HDM beta-functions using the bilinear formalism. We are going to use his methods to prove that the set of conditions

$$\{m_{11}^2 + m_{22}^2 = 0 \ , \ \ \lambda_1 = \lambda_2 \ \ , \ \ \lambda_6 = -\lambda_7\}$$

is RG invariant *to all orders of perturbation theory*.

First, consider that the most general set of basis invariant quantities one can build with the quartic couplings of the potential is given by

$$I_{1,1} = \Lambda_{00}\,, \qquad I_{1,2} = \mathrm{tr}\Lambda$$

$$I_{2,1} = \vec{\Lambda}\cdot\vec{\Lambda}\,, \qquad I_{2,2} = \mathrm{tr}\Lambda^2$$

$$I_{3,1} = \vec{\Lambda}\cdot\Lambda\vec{\Lambda}\,, \qquad I_{3,2} = \mathrm{tr}\Lambda^3$$

$$I_{4,1} = \vec{\Lambda}\cdot\Lambda^2\vec{\Lambda}\,,$$

I.P. Ivanov, Phys. Lett. B 632 (2006) 360
A. V. Bednyakov. JHEP 11 (2018) 154

These are all "scalars" for basis transformations – if a vector appears, it's in a scalar product with another vector. And there are only 3 independent vectors we can build with dimensionless couplings:

- $\vec{\Lambda}$ **is a vector for basis transformations.**
- $\Lambda\,\vec{\Lambda}$ **is a vector for basis transformations.**
- $\Lambda^2\,\vec{\Lambda}$ **is a vector for basis transformations.**

No higher powers of the matrix Λ need be considered, because this matrix obeys the following equation (Cayley-Hamilton theorem):

$$\Lambda^3 = (\mathrm{tr}\Lambda)\Lambda^2 - \frac{1}{2}\left[(\mathrm{tr}\Lambda)^2 - \mathrm{tr}\Lambda^2\right]\Lambda + \frac{1}{6}\left[(\mathrm{tr}\Lambda)^3 - 3\mathrm{tr}\Lambda\,\mathrm{tr}\Lambda^2 + 2\mathrm{tr}\Lambda^3\right]\mathbb{1}_{3\times 3}$$

This severely limits the number of independent basis-invariant dimensionless couplings to the seven quantities I_{jk} shown in the previous slide.

With basis invariance dictating the shape of the all-order beta function for $\vec{\Lambda}$,

$$\beta_{\vec{\Lambda}} = a_0 \, \vec{\Lambda} + a_1 \, \Lambda \, \vec{\Lambda} + a_2 \, \Lambda^2 \, \vec{\Lambda}$$

we obtain a remarkable result:

- $\vec{\Lambda} = \vec{0}$ *is a fixed point of the RG equation for this quantity, to all orders of perturbation theory.*

Remembering the definition of $\vec{\Lambda}$,

$$\vec{\Lambda} = \left(-\mathrm{Re}\,(\lambda_6 + \lambda_7) \, , \, \mathrm{Im}\,(\lambda_6 + \lambda_7) \, , \, \frac{1}{2}(\lambda_2 - \lambda_1) \right)$$

we see that that $\vec{\Lambda} = \vec{0}$ implies $\lambda_1 = \lambda_2$ and $\lambda_6 = - \lambda_7$ - again, the CP2 conditions on the quartic couplings.

Things become *really* interesting when we look at the beta-functions for the quadratic couplings...

All-order beta-functions for quadratic parameters

Remember that the quadratic parameters are organized in a basis invariant quantity M_0 and a vector $\vec{M}$,

$$M_0 = m_{11}^2 + m_{22}^2 \qquad \vec{M} = \left(2\mathrm{Re}(m_{12}^2)\,,\, -2\mathrm{Im}(m_{12}^2)\,,\, m_{22}^2 - m_{11}^2\right)$$

For the quadratic parameters, the "geometry" arising from basis invariance and dimensional analysis gives

$$\underbrace{\beta_{M_0}}_{\text{Must be a scalar}} = \underbrace{b_0\, M_0 + b_1\, \vec{\Lambda}\cdot\vec{M} + b_2\, \vec{\Lambda}\cdot\left(\Lambda\vec{M}\right) + b_3\, \vec{\Lambda}\cdot\left(\Lambda^2\vec{M}\right)}_{\text{Combination of all scalars one can build with correct mass dimensions}}$$

Must be a scalar

Combination of all scalars one can build with correct mass dimensions

The b_i are basis-invariant quantities containing the I_{jk} and also gauge couplings.

Likewise,

$$\beta_{\vec{M}} = c_0 \vec{M} + c_1 \Lambda \vec{M} + c_2 \Lambda^2 \vec{M} + c_3 I_M \vec{\Lambda} + c_4 I_M \Lambda \vec{\Lambda} + c_5 I_M \Lambda^2 \vec{\Lambda}$$

Must be a vector

Combination of all vectors one can build with correct mass dimensions

The c_i are basis-invariant quantities containing the I_{jk} and gauge couplings. I_M stands for linear combinations of the basis-invariant combinations with mass dimension in the previous equation.

With these expressions, we discover TWO fixed points of the RG equations to all orders of perturbation theory.

$$\beta_{\vec{\Lambda}} = a_0\,\vec{\Lambda} + a_1\,\Lambda\,\vec{\Lambda} + a_2\,\Lambda^2\,\vec{\Lambda}$$

$$\beta_{\vec{M}} = c_0\,\vec{M} + c_1\,\Lambda\,\vec{M} + c_2\,\Lambda^2\,\vec{M} + c_3\,I_M\,\vec{\Lambda} + c_4\,I_M\,\Lambda\,\vec{\Lambda} + c_5\,I_M\,\Lambda^2\,\vec{\Lambda}$$

$$\vec{\Lambda} = \left(-\mathrm{Re}\,(\lambda_6+\lambda_7)\ ,\ \mathrm{Im}\,(\lambda_6+\lambda_7)\ ,\ \frac{1}{2}(\lambda_2-\lambda_1)\right)$$

$$\vec{M} = \left(2\mathrm{Re}(m_{12}^2)\,,\ -2\mathrm{Im}(m_{12}^2)\,,\ m_{22}^2 - m_{11}^2\right)$$

- **If $\vec{\Lambda} = \vec{0}$, then $\vec{M} = \vec{0}$ is a fixed point of the RG equations to all orders of perturbation theory.**

This means that the conditions $\{\vec{M} = \vec{0}\,,\ \vec{\Lambda} = \vec{0}\}$ **, which are equivalent to**

$$m_{11}^2 = m_{22}^2\ ,\quad m_{12}^2 = 0\quad ,\quad \lambda_1 = \lambda_2\ ,\quad \lambda_6 = -\lambda_7$$

are preserved under renormalization to all orders of perturbation theory – ***this is the CP2 symmetry*****.**

But the other fixed point brings something new...

$$\beta_{\vec{\Lambda}} = a_0 \vec{\Lambda} + a_1 \Lambda \vec{\Lambda} + a_2 \Lambda^2 \vec{\Lambda}$$

$$\beta_{M_0} = b_0 M_0 + b_1 \vec{\Lambda} \cdot \vec{M} + b_2 \vec{\Lambda} \cdot \left(\Lambda \vec{M}\right) + b_3 \vec{\Lambda} \cdot \left(\Lambda^2 \vec{M}\right)$$

$$\vec{\Lambda} = \left(-\mathrm{Re}\,(\lambda_6 + \lambda_7)\ ,\ \mathrm{Im}\,(\lambda_6 + \lambda_7)\ ,\ \frac{1}{2}(\lambda_2 - \lambda_1)\right)$$

$$M_0 = m_{11}^2 + m_{22}^2$$

- **If $\vec{\Lambda} = \vec{0}$, then $M_0 = 0$ is a fixed point of the RG equations to all orders of perturbation theory.**

This means that the conditions $\{M_0 = 0\,,\ \vec{\Lambda} = \vec{0}\}$ **, which are equivalent to**

$$\left\{m_{11}^2 + m_{22}^2 = 0\ ,\ \ \lambda_1 = \lambda_2\ ,\ \ \lambda_6 = -\lambda_7\right\}$$

are preserved under renormalization to all orders of perturbation theory – *this is NOT the CP2 symmetry*. This is a new set of conditions on the 2HDM parameters, which none of the symmetries from Table 1 can reproduce.

The set of conditions

$$\{m_{11}^2 + m_{22}^2 = 0 \ , \ \ \lambda_1 = \lambda_2 \ \ , \ \ \lambda_6 = -\lambda_7\}$$

is therefore preserved under renormalization to all orders of perturbation theory – it must be the result of some kind of symmetry...? The relations between quartic couplings are identical to those of the CP2 model, but the relation between quadratic parameters is completely different. NO FERMIONS YET!

These are also *basis-invariant* conditions. This ensures that they are NOT a basis change from any of the six usual symmetries.

But none of the Higgs Family symmetries (unitary transformations on the doublets) or Generalized CP symmetries (anti-unitary transformations on the doublets) can produce these relations between couplings.

The bilinear formalism provides a formal way to at least have an idea of how this symmetry comes about...

Bilinear Formalism "Interpretation"

Remember that the potential can be written in terms of bilinears as

$$V = M_0\, r_0 + \Lambda_{00}\, r_0^2 - \vec{M}\,.\,\vec{r} - 2\left(\vec{\Lambda}\,.\,\vec{r}\right) r_0 + \vec{r}\,.\,(\Lambda\,\vec{r})$$

- **For this potential to be invariant under** $\vec{r} \rightarrow -\vec{r}$ **it would be necessary that**

$$\{\vec{M} = \vec{0}\,,\, \vec{\Lambda} = \vec{0}\}$$

This is the bilinear interpretation of CP2.

- **But there is a second way to obtain** $\vec{\Lambda} = \vec{0}$ **: suppose we require that the above potential be invariant under** $r_0 \rightarrow -r_0$. **This can only happen if**

$\{M_0 = 0\,,\, \vec{\Lambda} = \vec{0}\}$ **, that is,** $\{m_{11}^2 + m_{22}^2 = 0\ ,\ \lambda_1 = \lambda_2\ ,\ \lambda_6 = -\lambda_7\}$

- **These are exactly the relations we found to be RG-invariant to all orders.**

The r_0-symmetry

- **Invariance of the scalar potential under the transformation** $r_0 \to -r_0$ **formally reproduces the all-order RG invariant conditions we have found...**

- **... but there is a problem: since**

$$r_0 \;=\; \frac{1}{2}\left(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2\right) \;,$$

this quantity is guaranteed to be positive, whatever we do to the doublets!

- **But even if we were to analytically extend the bilinears by redefining** r_0 **to be**

$$r_0 \;=\; \pm\frac{1}{2}\left(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2\right) \;,$$

the scalar kinetic terms would *not* be invariant under $r_0 \to -r_0$ **!**

The scalar kinetic terms are given by

$$
\begin{aligned}
T &= (D_\mu \Phi_i)^\dagger \, D^\mu \Phi_i \\
&= K_1^\mu \left\{ \left(\partial_\alpha \Phi_i^\dagger\right) (\sigma_\mu)_{ij} \, (\partial^\alpha \Phi_j) + \frac{ig'}{2} \left[\left(\partial_\alpha \Phi_i^\dagger\right) (\sigma_\mu)_{ij} \, \Phi_j - \Phi_i^\dagger \, (\sigma_\mu)_{ij} \, (\partial_\alpha \Phi_j) \right] B^\alpha \right. \\
&\quad \left. + \frac{ig}{2} \left[\left(\partial_\alpha \Phi_i^\dagger\right) (\sigma_\mu)_{ij} \, \sigma^a \Phi_j - \Phi_i^\dagger \, (\sigma_\mu)_{ij} \, \sigma^a \, (\partial^\alpha \Phi_j) \right] W^{a\alpha} + \frac{g \, g'}{2} \Phi_i^\dagger \, (\sigma_\mu)_{ij} \, \sigma^a \, \Phi_j \, W_\alpha^a B^\alpha \right\} \\
&\quad + \frac{1}{2} K_2^\mu \left(g'^2 \, B_\alpha B^\alpha + g^2 \, W_\alpha^a W^{a\alpha} \right) r_\mu \, ,
\end{aligned}
$$

with $K_1^\mu = K_2^\mu = (1\,,\,0\,,\,0\,,\,0)$ **and** $\sigma^\mu = (\mathbb{1}\,,\,\sigma_i)$ **, where σ_i are the Pauli matrices. The transformation $r_0 \to -r_0$ only affects the last term, it is not clear how all terms with derivatives or gauge boson fields would transform to leave the kinetic terms invariant.**

So the transformation $r_0 \to -r_0$ is at least a useful mnemonic to reproduce the parameter relations of the r_0-symmetry, but there may be something deeper behind it...

New 2HDM symmetries

Combining the new relations between couplings ("r_0-symmetry")

$$\{m_{11}^2 + m_{22}^2 = 0 \ , \ \lambda_1 = \lambda_2 \ , \ \lambda_6 = -\lambda_7\}$$

with the other six symmetries, we obtain new 2HDM models, with new coupling relationships which are RG invariant to all orders.

We will designate the new symmetries with the prefix "0", so for instance, "0CP1" will refer to the application of the r_0 and CP1 symmetries, and "$0Z_2$" refers to the application of r_0 and Z_2.

Symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
r_0		$-m_{11}^2$			λ_1					$-\lambda_6$
0CP1		$-m_{11}^2$	real		λ_1			real	real	$-\lambda_6$
$0Z_2$		$-m_{11}^2$	0		λ_1				0	0
0U(1)		$-m_{11}^2$	0		λ_1			0	0	0
0CP2	0	0	0		λ_1					$-\lambda_6$
0CP3	0	0	0		λ_1			$\lambda_1 - \lambda_3 - \lambda_4$	0	0
$0SO(3)$	0	0	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

TABLE 2

- **Three of the new models – 0CP2, 0CP3 and 0SO(3) – have vanishing quadratic terms, so no electroweak symmetry breaking can occur in them.**

- **However, it is always possible and interesting to study models where we introduce *soft breaking terms*. And we can do so by keeping the new relation,**

$$m_{11}^2 = -m_{22}^2$$

  as it will be preserved under renormalization to all orders, given the restrictions existing on the quartic sector.

- **All of these models – with or without soft breaking terms, but with the above relation maintained – have very curious phenomenological aspects.**

- **For instance, for all of them, a Decoupling Limit is not possible, provided the above relation between quadratic parameters holds.**

- **In other words, the r_0-symmetry requires light extra scalars, and can therefore be tested – and disproven – at LHC.**

- **These models have some very interesting phenomenological aspects...**

Can the r_0-symmetry be extended to the whole lagrangian to include fermions?

- **The r_0-symmetry induces CP2-like relations in the scalar quartic couplings – or even more restrictive ones, like CP3, when combined with other symmetries.**

- **Therefore we need Yukawa couplings which preserve the CP2 (CP3 is another possibility) symmetry among quartic couplings.**

- **The most general Yukawa lagrangian is given by (forget neutrinos)**

$$-\mathcal{L}_Y = \bar{q}_L(\Gamma_1\Phi_1 + \Gamma_2\Phi_2)n_R + \bar{q}_L(\Delta_1\tilde{\Phi}_1 + \Delta_2\tilde{\Phi}_2)p_R + \bar{l}_L(\Pi_1\Phi_1 + \Pi_2\Phi_2)l_R + \text{H.c.}$$

with generic 3×3 complex matrices Γ_i for the down quarks, Δ_i for the up quarks and Π_i for the charged leptons.

- **The form of these matrices which preserves CP2 and CP3 is known.**

P. Ferreira and J.P. Silva, Eur. Phys. J. C 69 (2010) 45

- **For CP2,**

$$\Gamma_1 = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{12} & -a_{11} & 0 \\ 0 & 0 & 0 \end{pmatrix} \;,\; \Gamma_2 = \begin{pmatrix} -a_{12}^* & a_{11}^* & 0 \\ a_{11}^* & a_{12}^* & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Analogous expressions (with different couplings) for the up and leptonic Yukawa matrices. CP2 implies a massless down, up and charged lepton.

- **For CP3,**

$$\Gamma_1 = \begin{pmatrix} i\,a_{11} & i\,a_{12} & a_{13} \\ i\,a_{12} & -i\,a_{11} & a_{23} \\ a_{31} & a_{32} & 0 \end{pmatrix} \;,\; \Gamma_2 = \begin{pmatrix} i\,a_{12} & -i\,a_{11} & -a_{23} \\ -i\,a_{11} & -i\,a_{12} & a_{13} \\ -a_{32} & a_{31} & 0 \end{pmatrix}$$

All a_{ij} real. Analogous expressions (with different couplings) for the up and leptonic Yukawa matrices. CP3 reproduces well fermion masses and magnitudes of the CKM matrix, but cannot seem to fit the value of the Jarlskog invariant.

- **We have shown that the relation $m_{11}^2 + m_{22}^2 = 0$ is RG-preserved for the scalar and gauge couplings. What if one includes the fermion contributions in the beta-functions?**

The Yukawa-only contributions to the relevant one-loop beta-functions are

$$\begin{aligned}
\beta^{F,1L}_{m_{11}^2} &= \left[3\,\mathrm{Tr}(\Delta_1\Delta_1^\dagger) + 3\,\mathrm{Tr}(\Gamma_1\Gamma_1^\dagger) + \mathrm{Tr}(\Pi_1\Pi_1^\dagger)\right] m_{11}^2 \\
&\quad - \left\{\left[3\,\mathrm{Tr}(\Delta_1^\dagger\Delta_2) + 3\,\mathrm{Tr}(\Gamma_1^\dagger\Gamma_2) + \mathrm{Tr}(\Pi_1^\dagger\Pi_2)\right] m_{12}^2 + \mathrm{h.c.}\right\} \\
\beta^{F,1L}_{m_{22}^2} &= \left[3\mathrm{Tr}(\Delta_2\Delta_2^\dagger) + 3\mathrm{Tr}(\Gamma_2\Gamma_2^\dagger) + \mathrm{Tr}(\Pi_2\Pi_2^\dagger)\right] m_{22}^2 \\
&\quad - \left\{\left[3\,\mathrm{Tr}(\Delta_1^\dagger\Delta_2) + 3\,\mathrm{Tr}(\Gamma_1^\dagger\Gamma_2) + \mathrm{Tr}(\Pi_1^\dagger\Pi_2)\right] m_{12}^2 + \mathrm{h.c.}\right\}
\end{aligned}$$

Something amazing now happens. For both the CP2 and CP3 Yukawa textures,

$$\mathrm{Tr}(\Delta_1\Delta_1^\dagger) = \mathrm{Tr}(\Delta_2\Delta_2^\dagger)\ ,\ \ \mathrm{Tr}(\Gamma_1\Gamma_1^\dagger) = \mathrm{Tr}(\Gamma_2\Gamma_2^\dagger)\ ,\ \ \mathrm{Tr}(\Pi_1\Pi_1^\dagger) = \mathrm{Tr}(\Pi_2\Pi_2^\dagger)$$

as well as

$$\mathrm{Tr}(\Delta_1\Delta_2^\dagger) = \mathrm{Tr}(\Gamma_1\Gamma_2^\dagger) = \mathrm{Tr}(\Pi_1\Pi_2^\dagger) = 0$$

Therefore we obtain

$$\beta^{F,1L}_{m_{11}^2+m_{22}^2} = \left[3\,\mathrm{Tr}(\Delta_1\Delta_1^\dagger) + 3\,\mathrm{Tr}(\Gamma_1\Gamma_1^\dagger) + \mathrm{Tr}(\Pi_1\Pi_1^\dagger)\right] \left(m_{11}^2 + m_{22}^2\right)$$

- **At one-loop, since**

$$\beta^{F,1L}_{m_{11}^2+m_{22}^2} = \left[3\,\mathrm{Tr}(\Delta_1\Delta_1^\dagger) + 3\,\mathrm{Tr}(\Gamma_1\Gamma_1^\dagger) + \mathrm{Tr}(\Pi_1\Pi_1^\dagger)\right]\,(m_{11}^2 + m_{22}^2)$$

the relation $m_{11}^2 + m_{22}^2 = 0$ **is a fixed point of the RG equations and is preserved at one-loop!**

- **The same conclusion holds at two-loops! Using SARAH or PyR@TE, we obtain**

$$\beta^{2L}_{m_{11}^2+m_{22}^2} = X\,(m_{11}^2 + m_{22}^2)$$

where "X" is a complicated expression of couplings. Again we see that $m_{11}^2 + m_{22}^2 = 0$ **is a fixed point of the RG equations.**

- **This seems too incredible to be a coincidence. But an all-order demonstration with the Yukawa sector has not (yet...) been possible.**

(careful – there is a known SARAH bug that affects this calculation, a patch is available)

F. Staub, arXiv:0806.0538
F. Staub, Comput. Phys. Commun. 181 (2010) 1077
F. Staub, Comput.Phys. Commun. 182 (2011) 808
F. Lyonnet and I. Schienbein, Comput. Phys.Commun. 213 (2017) 181

Phenomenological consequences of the new symmetries

The r_0-model

This is the model resulting from only applying the r_0-symmetry $r_0 \to -r_0$. The scalar potential is given by

$$V = m_{11}^2 \left[\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right] - \left[m_{12}^2 \Phi_1^\dagger\Phi_2 + \text{h.c.}\right] + \tfrac{1}{2}\lambda_1 \left[(\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2\right] + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$

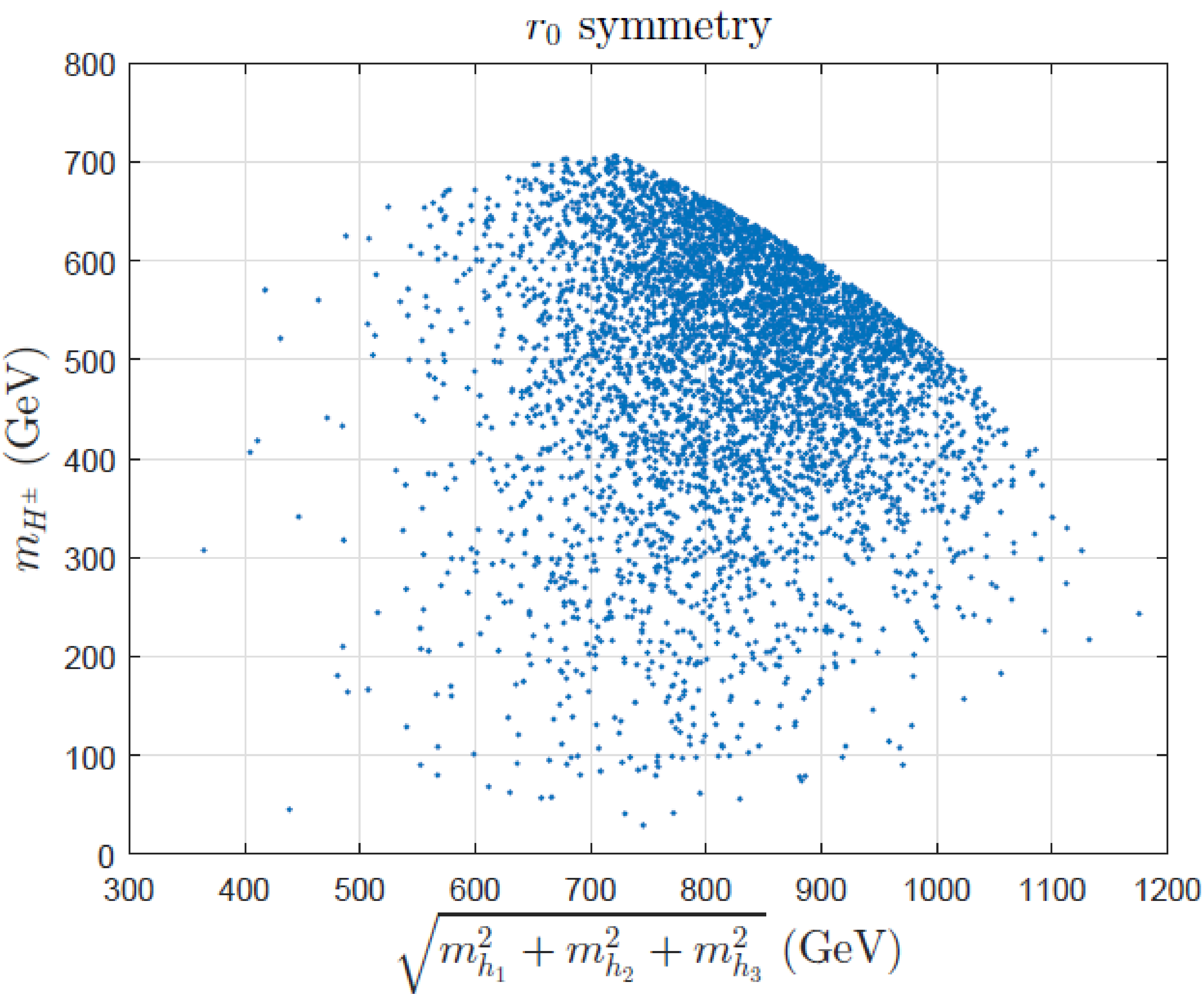
$$+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_6\left[(\Phi_1^\dagger\Phi_1) - (\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\} ,$$

and without loss of generality we can rotate to a basis where λ_5 is real and $\lambda_6 = \lambda_7 = 0$.

$$V = m_{11}^2 \left[\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right] - \left[m_{12}^2 \Phi_1^\dagger\Phi_2 + \text{h.c.}\right] + \tfrac{1}{2}\lambda_1 \left[(\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2\right] + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$

$$+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right] .$$

The presence of λ_5 and a complex m^2_{12} means that this model has *explicit CP violation*. This may be verified using basis-invariant methods.



r_0 symmetry
800
700
600
500
400
300
200
100
0
$m_{H^\pm}$ (GeV)
300
400
500
600
700
800
900
1000
1100
1200
$\sqrt{m_{h_1}^2 + m_{h_2}^2 + m_{h_3}^2}$ (GeV)

The 0CP1 model

In the simplified base where $\lambda_6 = \lambda_7 = 0$ and all couplings are real, the scalar potential is written as

$$V = m_{11}^2 \left[\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right] - m_{12}^2 \left[\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right] + \frac{1}{2}\lambda_1 \left[(\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2\right]$$
$$+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right] ,$$

In the CP1 model, for some regions of parameter space CP-conserving vacua are possible; in others, CP-violating vacua can be found.

The r_0-symmetry changes that!

The 0CP1 model

- **CP-CONSERVING VACUUM:** $\langle\Phi_1\rangle = \frac{v_1}{\sqrt{2}} \quad , \quad \langle\Phi_2\rangle = \frac{v_2}{\sqrt{2}}$

A vacuum with real vevs is possible, and it originates two CP-even scalars (h and H), a pseudoscalar A and a charged scalar:

$$M_H^2 = \frac{1}{2}v^2\left(\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5\right), \quad M_h^2 = \lambda_1 v^2,$$
$$M_A^2 = \frac{1}{2}v^2\left(\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5\right), \quad M_{H^\pm} = \frac{1}{2}\left(\lambda_1 + \lambda_3\right)v^2$$

Again, the dependence on the quadratic terms vanishes because of the r_0 symmetry condition $m_{11}^2 = -m_{22}^2$ and *no decoupling limit is possible*. A quick scan requiring unitarity and boundedness from below for the quartic couplings yields

$$M_{H^\pm} \leq 711 \text{ GeV}$$
$$M_A \leq 708 \text{ GeV}$$
$$M_H \leq 961 \text{ GeV}$$

The 0CP1 model

- **SPONTANEOUS CP VIOLATION:** $\langle \Phi_1 \rangle = \frac{v_1}{\sqrt{2}} \quad , \quad \langle \Phi_2 \rangle = \frac{v_2 \, e^{i\theta}}{\sqrt{2}}$

If one tries to find a vacuum with complex vevs, the minimisation conditions give

$$(\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5) v_2 (v_1^2 + v_2^2) \cos\delta \ = \ 0$$

$$(\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5) v_2 (v_1^2 + v_2^2) \sin\delta \ = \ 0$$

The only solution with non-zero vevs and phase would require

$$\lambda_5 \ = \ \lambda_1 + \lambda_3 + \lambda_4$$

However, this relation between the couplings is RG-unstable, therefore the *tree-level minimisation conditions do not allow for spontaneous breaking of CP* – if it can occur, it must arise from loop corrections to the potential.

The $0Z_2$ model

The scalar potential is written as

$$V = m_{11}^2 \left[\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right] + \tfrac{1}{2}\lambda_1 \left[(\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2\right] + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)$$

$$+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right] ,$$

Two vacua should be possible, for different choices of parameters: one that preserves the Z_2 symmetry (inert model) with one of the vevs equal to zero; and another for which the Z_2 symmetry is spontaneously broken, where both doublets have non-zero vevs.

Again, the r_0-symmetry changes that, preventing one of them - at tree-level, at least.

- **INERT $0Z_2$ MODEL:** $\langle \Phi_1 \rangle = \frac{v}{\sqrt{2}} \quad , \quad \langle \Phi_2 \rangle = 0$

- **Dark matter candidates (H or A) found, but no decoupling possible.**

- **All extra scalar masses should be inferior to roughly 710 GeV due to unitarity bounds on the quartic couplings.**

- **Masses have identical expressions to those found for 0CP1, but now neither H nor A couple to electroweak gauge bosons!**

$$M_H^2 = \frac{1}{2} v^2 \left(\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5\right), \quad M_h^2 = \lambda_1 v^2,$$
$$M_A^2 = \frac{1}{2} v^2 \left(\lambda_1 + \lambda_3 + \lambda_4 - \lambda_5\right), \quad M_{H^\pm} = \frac{1}{2} \left(\lambda_1 + \lambda_3\right) v^2$$

- **Analogous situation occurs for the inert 0U(1) model – for that case, H and A are actually degenerate in mass.**

- **SPONTANEOUS Z_2 BREAKING IN $0Z_2$:** $\langle\Phi_1\rangle = \frac{v_1}{\sqrt{2}}$, $\langle\Phi_2\rangle = \frac{v_2}{\sqrt{2}}$

The minimisation conditions for this vacuum now give

$$m_{11}^2 = -\frac{1}{2}\left[\lambda_1 v_1^2 + (\lambda_3+\lambda_4+\lambda_5) v_2^2\right] ;$$

$$0 = (\lambda_1+\lambda_3+\lambda_4+\lambda_5)\, v_2\, (v_1^2+v_2^2)$$

The second of these conditions can only be satisfied, for non-zero vevs, if

$$\lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 = 0$$

This condition is not RG-stable!

Therefore, analogous to the 0CP1 case, the *tree-level minimisation conditions do not allow for spontaneous breaking of Z_2* – if it can occur, it must arise from loop corrections to the potential. An analogous situation occurs for spontaneous breaking of U(1) in the 0U(1) model.

The softly-broken 0U(1) model

Without loss of generality we can choose a real soft-breaking term,

$$
\begin{aligned}
V \;=\;& m_{11}^2\left[\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right] - m_{12}^2\left[\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right] + \tfrac{1}{2}\lambda_1\left[(\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2\right] \\
& +\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1).
\end{aligned}
$$

A vacuum with two real vevs is possible, and again no decoupling limit is possible. But a very interesting thing occurs:

$$
M_h^2 = \lambda_1 v^2, \quad M_H^2 = M_A^2 = \frac{1}{2}\left(\lambda_1 + \lambda_3 + \lambda_4\right) v^2, \quad M_{H\pm} = \frac{1}{2}\left(\lambda_1 + \lambda_3\right) v^2
$$

There is a mass degeneracy between H and A – but H is CP-even, and couples to electroweak gauge bosons, and A does not, since it is a pseudoscalar.

Therefore, we can expect that this degeneracy is lifted by loop corrections – but the mass splitting between the H and A should be small, *since it arises from radiative corrections*!

CONCLUSIONS

- **We have identified relations between 2HDM scalar couplings which are invariant under RG equations to all orders in scalar and gauge couplings; and at least to two-loop orders if fermions are included.**

- **This proves/strongly implies an underlying symmetry of the model. A formal way of obtaining the relations between scalar couplings is provided by the bilinear formalism – $r_0 \rightarrow -r_0$ – but there are no transformations on the fields which give this, neither does this transformation preserve the scalar kinetic terms *(but we have a really crazy idea!)*.**

- **We identified several new symmetries, with new relations between couplings invariant under renormalization and analysed the phenomenology of such models.**

- **The r_0 symmetry eliminates all dependence of quadratic parameters in the scalar masses and prevents these models from having a *decoupling limit*.**

- **Explicit CP violation is possible (r_0 model) but spontaneous CP violation not possible at tree-level, at least (0CP1 model).**

- **Spontaneous breaking of Z_2 or U(1) symmetries not possible at tree-level, at least. A softly broken 0U(1) model gives tree-level mass degenerate A and H, but that degeneracy is expected to be lifted by radiative corrections – *this model predicts therefore a* *mass between these two particles*.**

- **Given the names of the authors – Ferreira, Grzadko Osland and– these might be called *FGOO Symmetrie***

- **... but they are so WEIRD that a better name might be *GOOFy Symmetries !***

A modest and crazy proposal to explain the r_0 symmetry...

- **Write doublets in terms of their real component fields as**

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix}$$

- **Transform these component fields as**

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix}$$

- **This transformation acts on the doublets *and their hermitian conjugates* in an "*inconsistent*" manner:**

$$\begin{aligned} \Phi_1 &\rightarrow -\Phi_2^* \\ \Phi_1^\dagger &\rightarrow \Phi_2^T, \\ \Phi_2 &\rightarrow \Phi_1^*, \\ \Phi_2^\dagger &\rightarrow -\Phi_1^T. \end{aligned}$$

This is not the "†" of the previous!

- **However weirdly, this transformation on the component fields of the doublets yields exactly the bilinear r_0-symmetry: $r_0 \rightarrow - r_0$, $r_i \rightarrow r_i$**

- **What about the kinetic terms? These remain invariant if we assume the following transformation laws for the spacetime derivatives and gauge fields:**

$$\begin{aligned} \partial_\mu &\rightarrow -i\partial_\mu, \\ B_\mu &\rightarrow iB_\mu, \\ W_{1\mu} &\rightarrow iW_{1\mu}, \\ W_{2\mu} &\rightarrow -iW_{2\mu}, \\ W_{3\mu} &\rightarrow iW_{3\mu}. \end{aligned}$$

Imaginary Spacetime!

$$x_\mu \rightarrow ix_\mu$$

- **The effect of these transformations on the scalar covariant derivatives is**

$$D^\mu\Phi_1 \to i\,(D^\mu\Phi_2)^*\,, \qquad (D^\mu\Phi_1)^\dagger \to -i\,(D^\mu\Phi_2)^T\,,$$
$$D^\mu\Phi_2 \to -i\,(D^\mu\Phi_1)^*\,, \quad (D^\mu\Phi_2)^\dagger \to i\,(D^\mu\Phi_1)^T$$

which renders the scalar kinetic terms invariant.

- **Likewise for the gauge kinetic terms:** $\mathcal{L}^B \;=\; -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{i\mu\nu}W_i^{\mu\nu}$

With $B^{\mu\nu} = \partial^\nu B^\mu - \partial^\mu B^\nu$ **and** $W_i^{\mu\nu} = \partial^\nu W_i^\mu - \partial^\mu W_i^\nu + g\epsilon_{ijk}W_j^\mu W_k^\nu$ **, these tensors transform as**

$$\begin{aligned} B^{\mu\nu} &\to B^{\mu\nu}, \\ W_1^{\mu\nu} &\to W_1^{\mu\nu}, \\ W_2^{\mu\nu} &\to -W_2^{\mu\nu}, \\ W_3^{\mu\nu} &\to W_3^{\mu\nu}, \end{aligned}$$

and the kinetic terms are found to be invariant! Still working on fermions...