



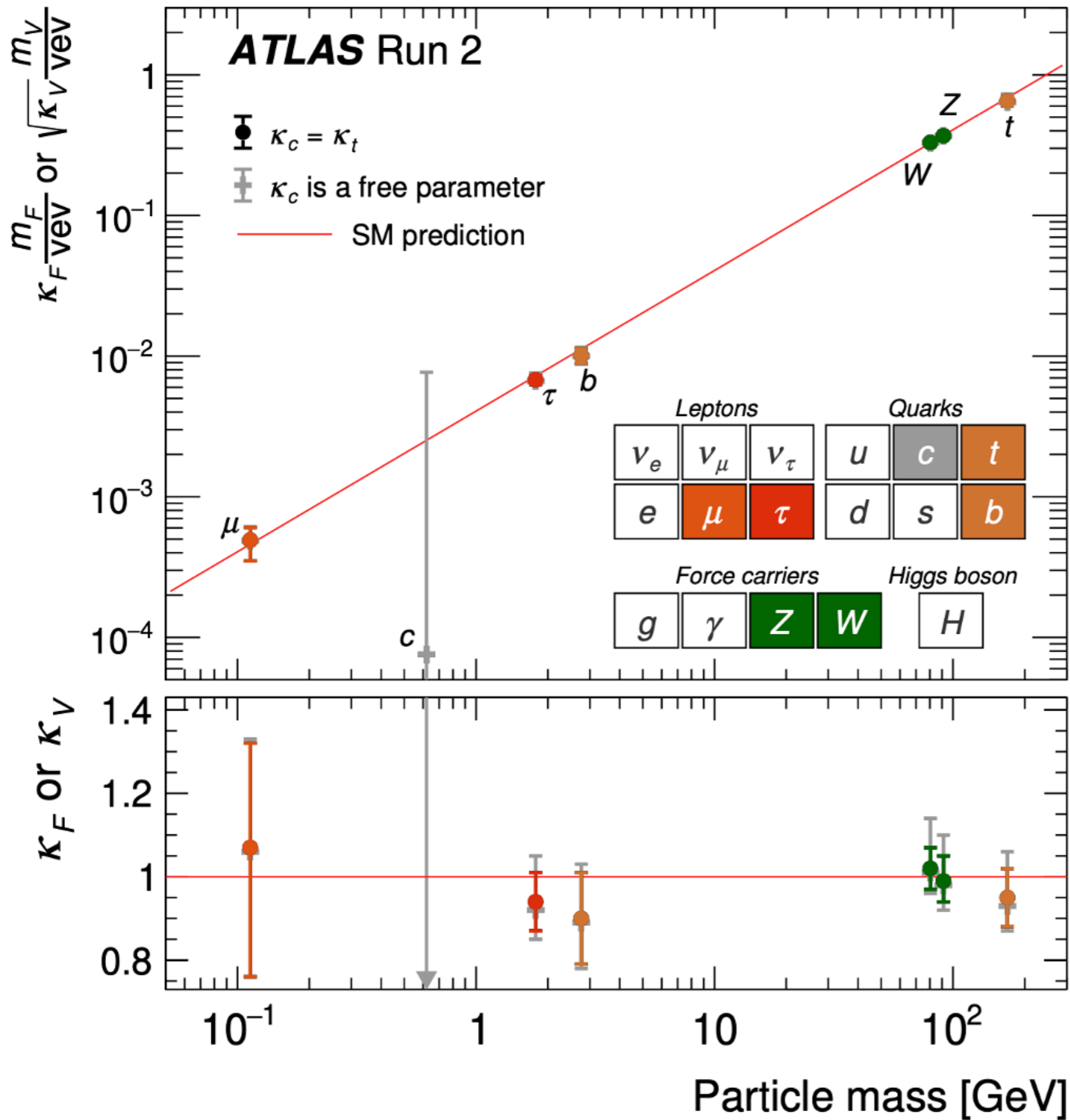
# **Precise predictions for Higgs boson pair production and decay**

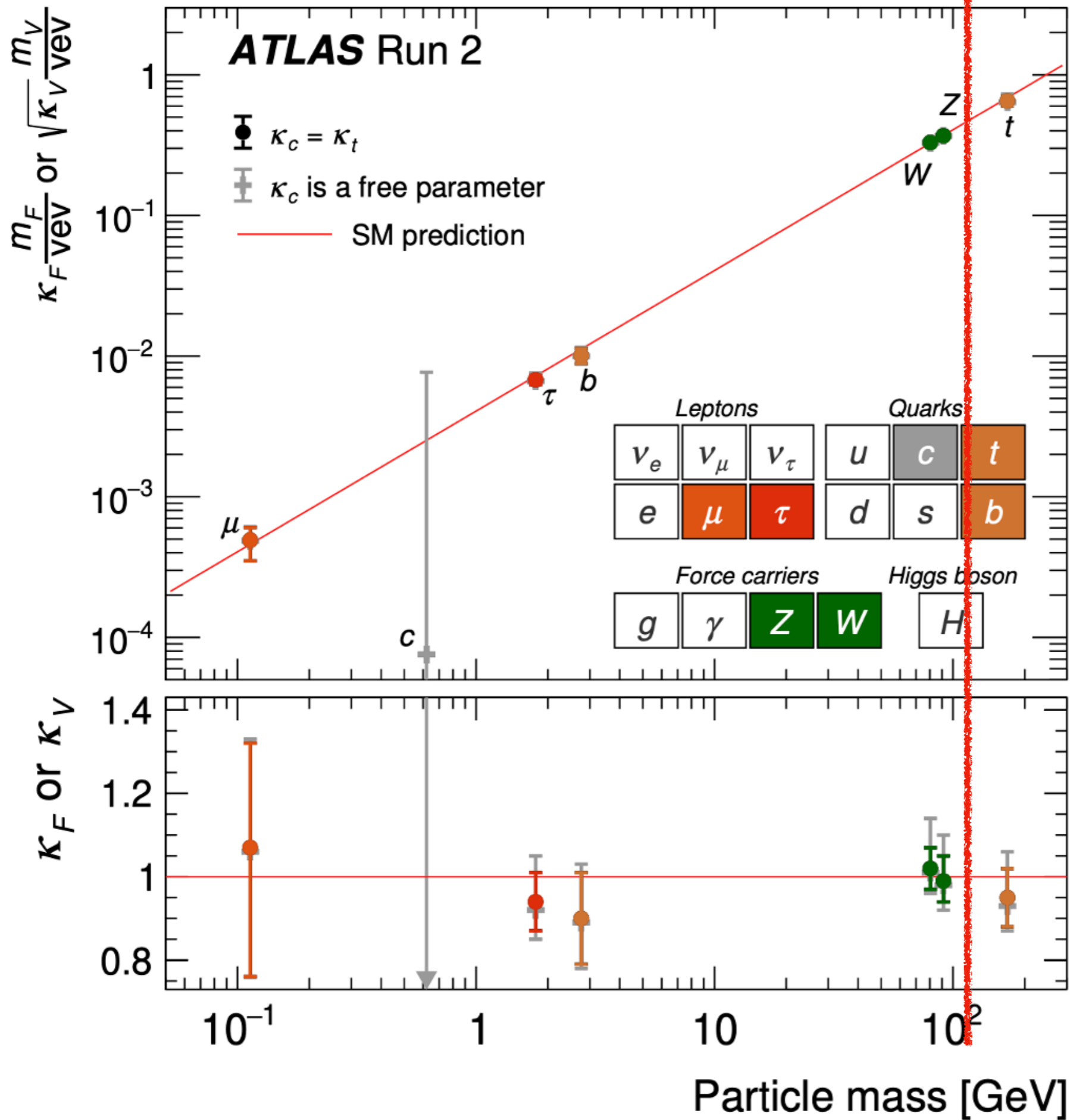
*Jian Wang*

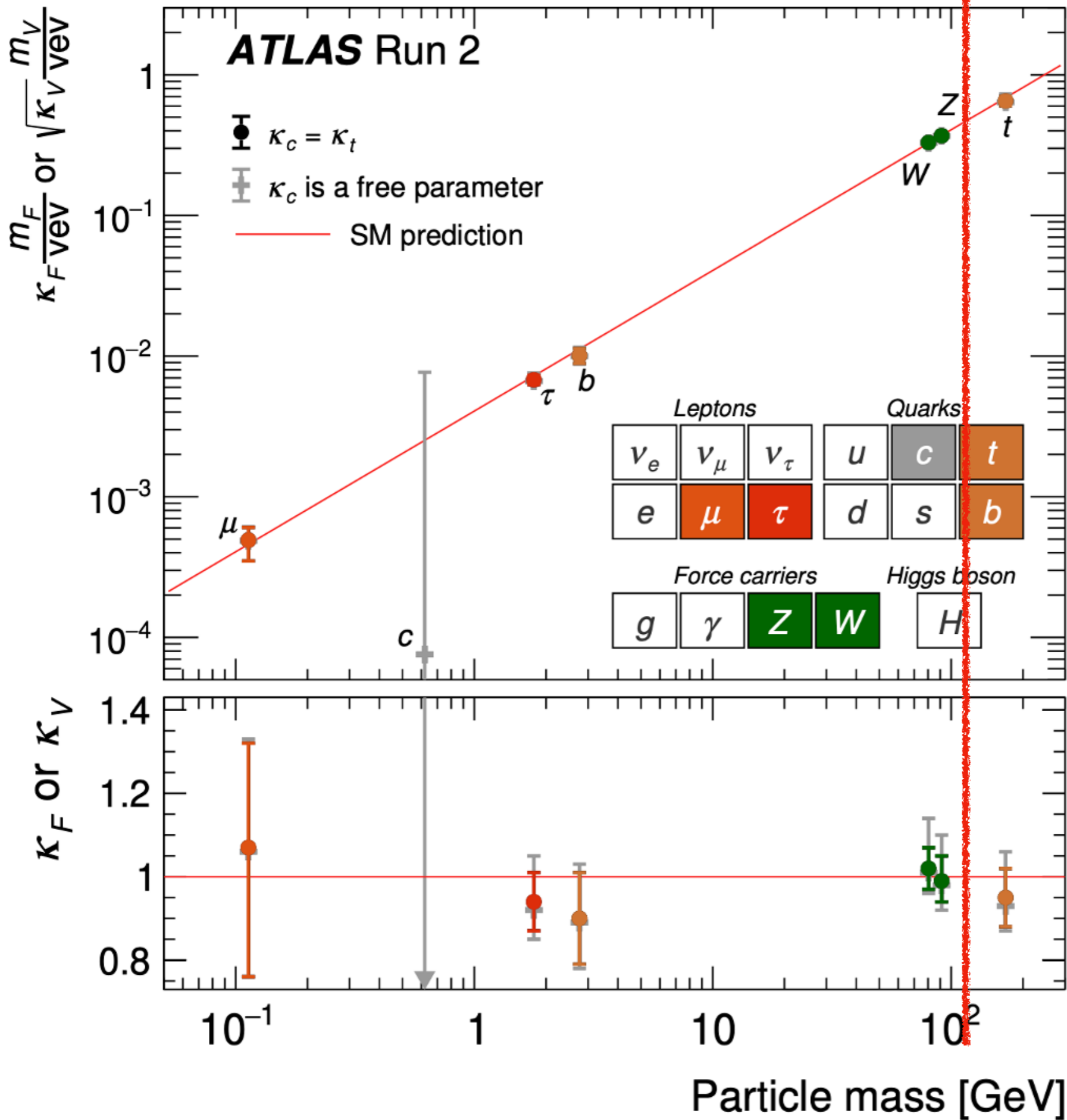
*Shandong University*

*HPNP 2023*

*Osaka University*



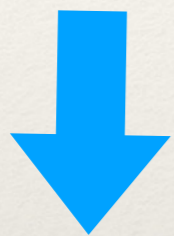




How about the coupling to Higgs boson?

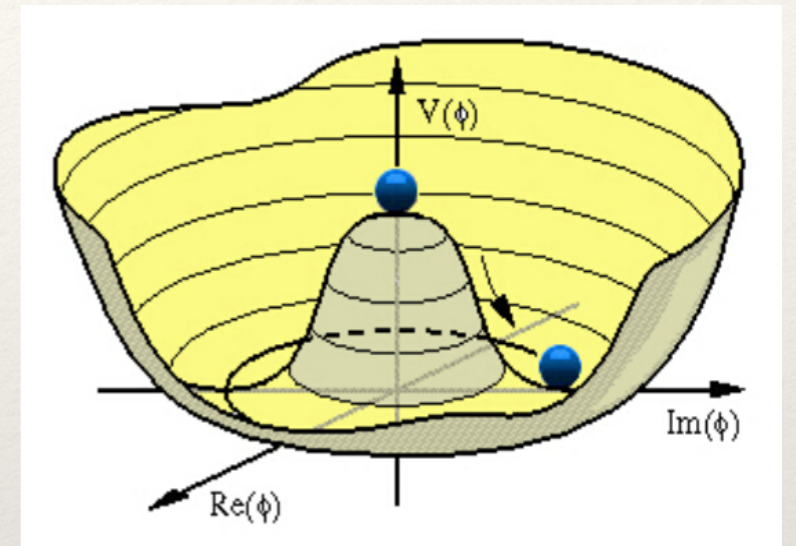
# Higgs self-coupling in the SM

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$



$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$V(h) = \frac{1}{2} m_h^2 h^2 + \sqrt{\frac{\lambda}{2}} m_H h^3 + \frac{1}{4} \lambda h^4$$



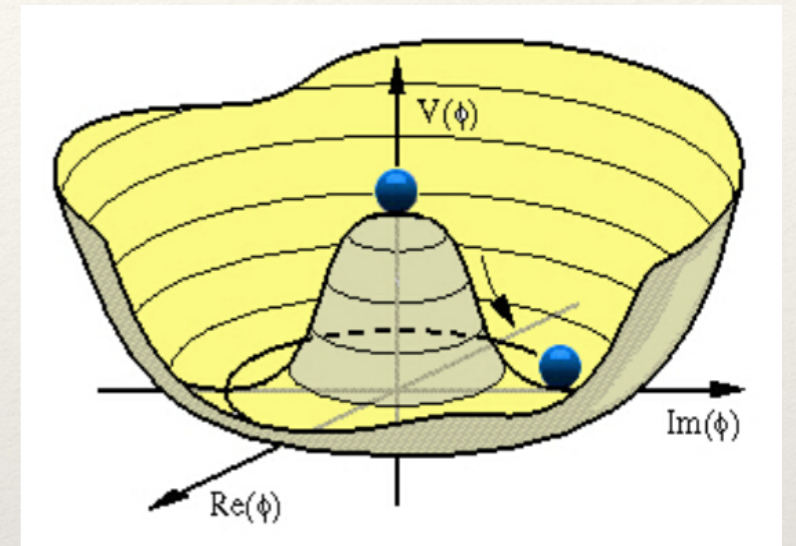
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In some new physics models, the trilinear Higgs self-coupling may change by  $O(100)\%$ , while the couplings with gauge bosons and fermions are still in agreement with SM.

S.Kanemura, et al, PLB558,157

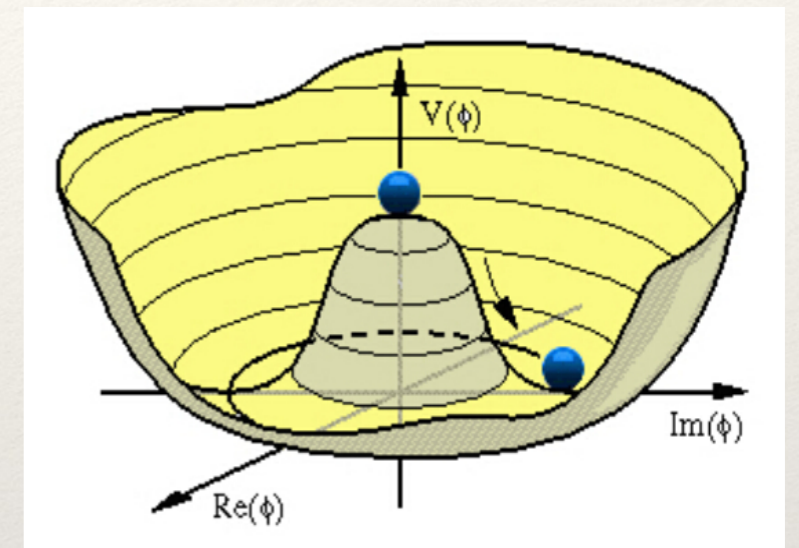
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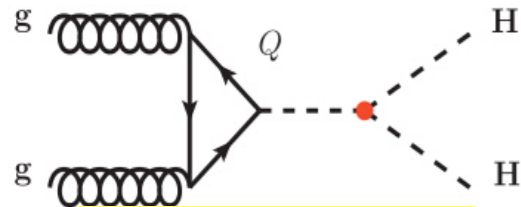
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**We need to measure the trilinear self coupling directly.**

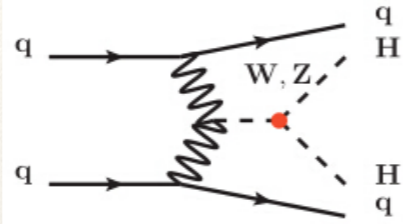
# Higgs pair production

● gluon fusion



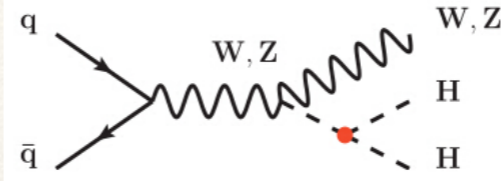
**(NN)NLO QCD**  
~100000 events

● vector boson fusion



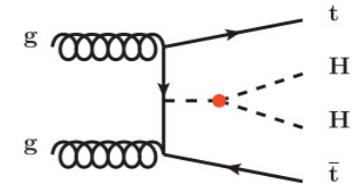
**(N)NNLO QCD**  
~6000 events

● double Higgs-strahlung



**NNLO in QCD**  
~3200 events

● associated production with top quark



**NLO in QCD**  
~3000 events

Borowka, et al, PRL117,012001  
 Baglio, et al, EPJC, 79, 459  
 Chen, Li, Shao, **JW**, PLB 803,135292  
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Ling, Zhang, Ma, Guo, Li, Li,  
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 PRD99,074028

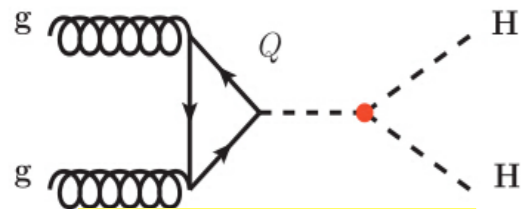
Baglio, et al, JHEP1304,151  
 Li, Li, **JW**, PRD97,074026, PLB765,265

Englert, et al, PLB743,93  
 Liu, Zhang, 1410.1855  
 Frederix, et al, PLB 732,142



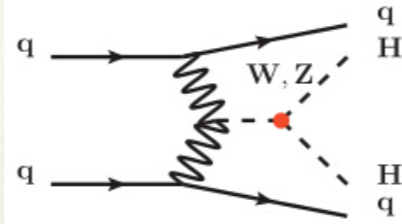
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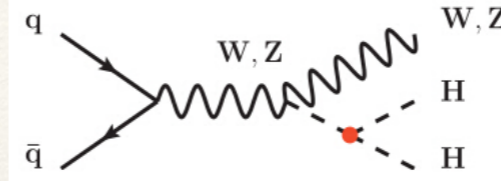
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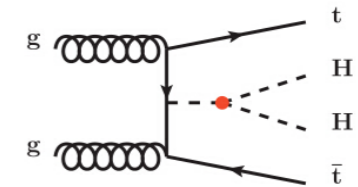
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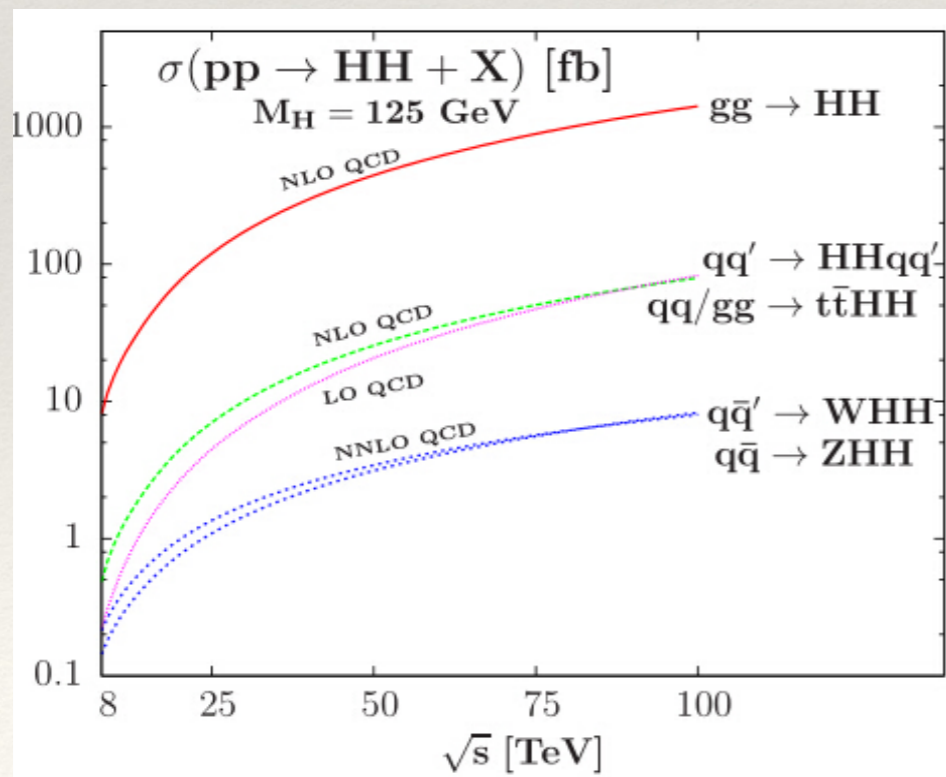
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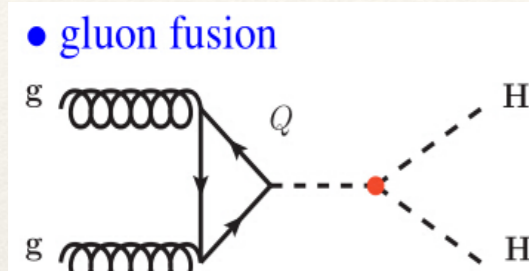
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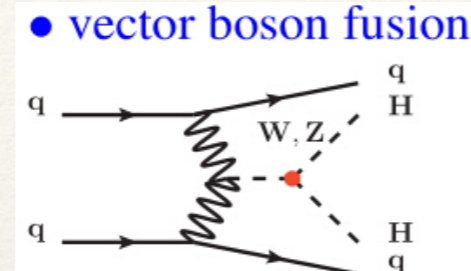
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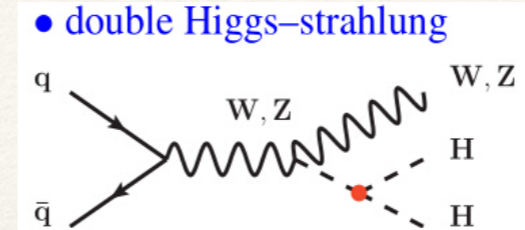
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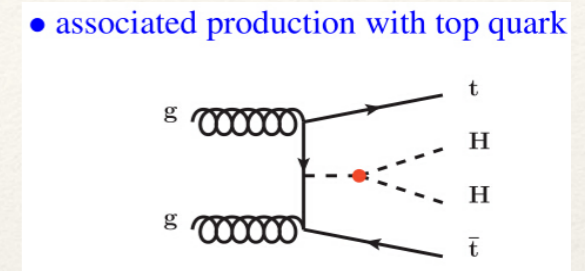
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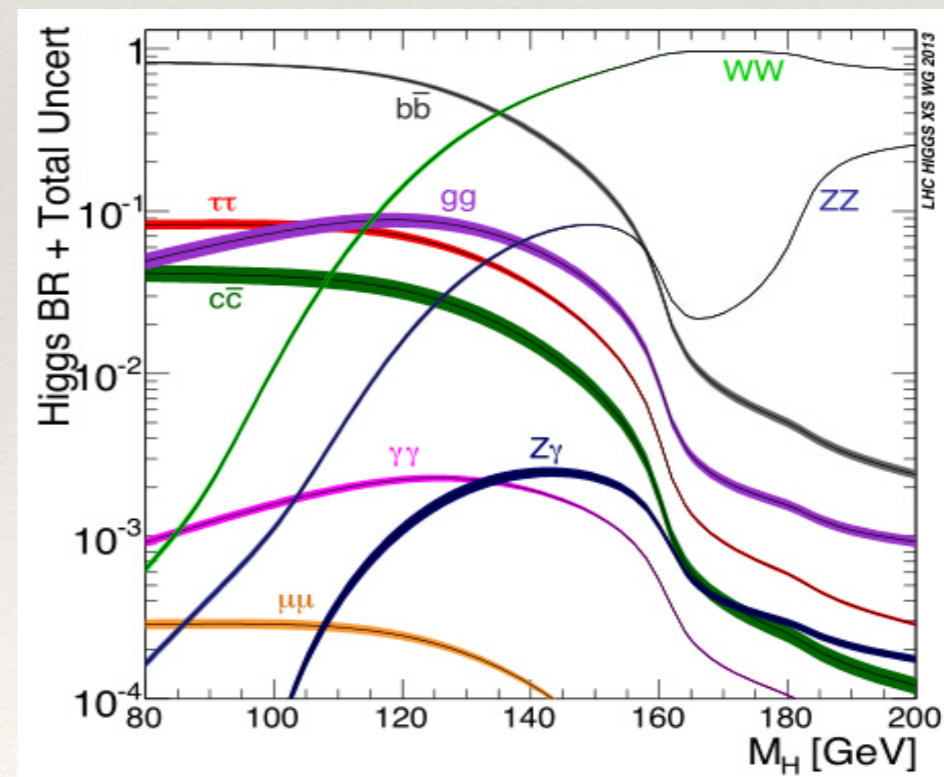
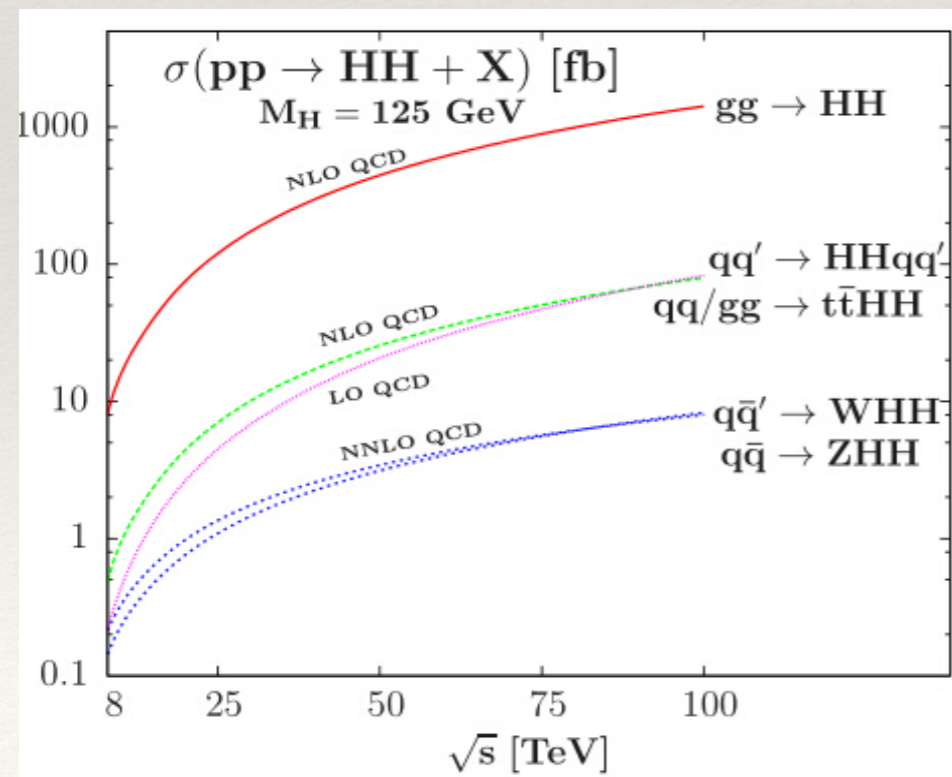
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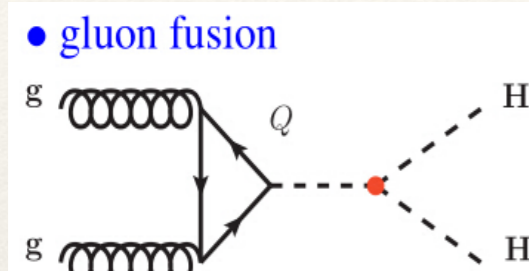
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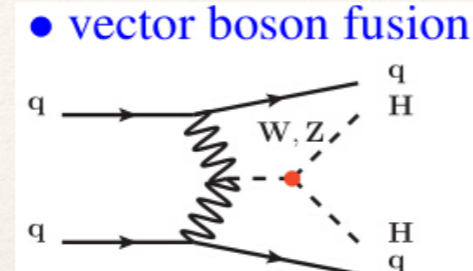
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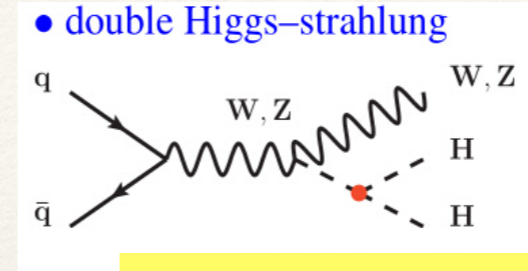
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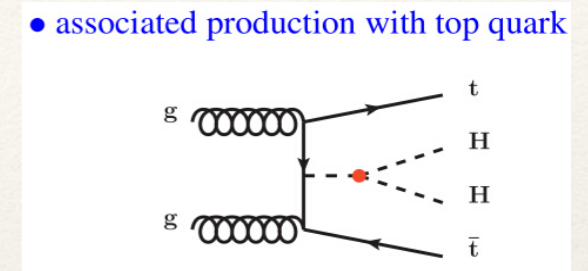
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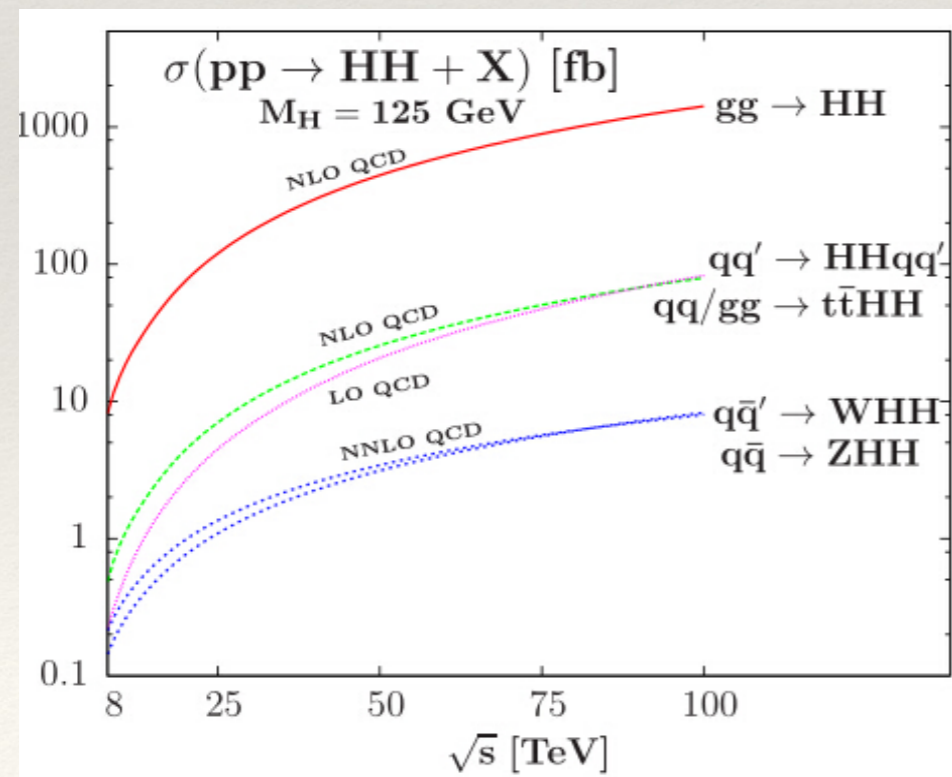
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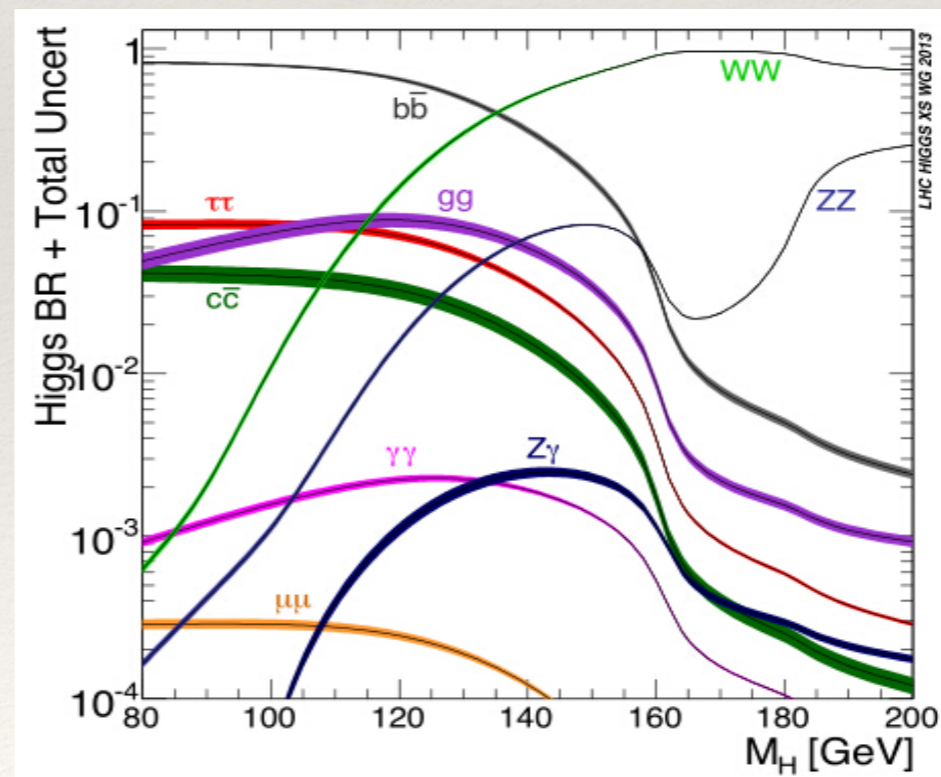
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+



=

$b\bar{b}WW$   
 $b\bar{b}\gamma\gamma$   
 $b\bar{b}\tau\tau$   
 $Wb\bar{b}b\bar{b}$   
 $t\bar{t}b\bar{b}b\bar{b}$   
.....

$\kappa_\lambda = \kappa_t = 1$   
 $\kappa_V = \kappa_{2V} = 1$

—●— Observed

----- Median expected

68% expected

95% expected

**bb ZZ**  
 Expected: 40  
 Observed: 32

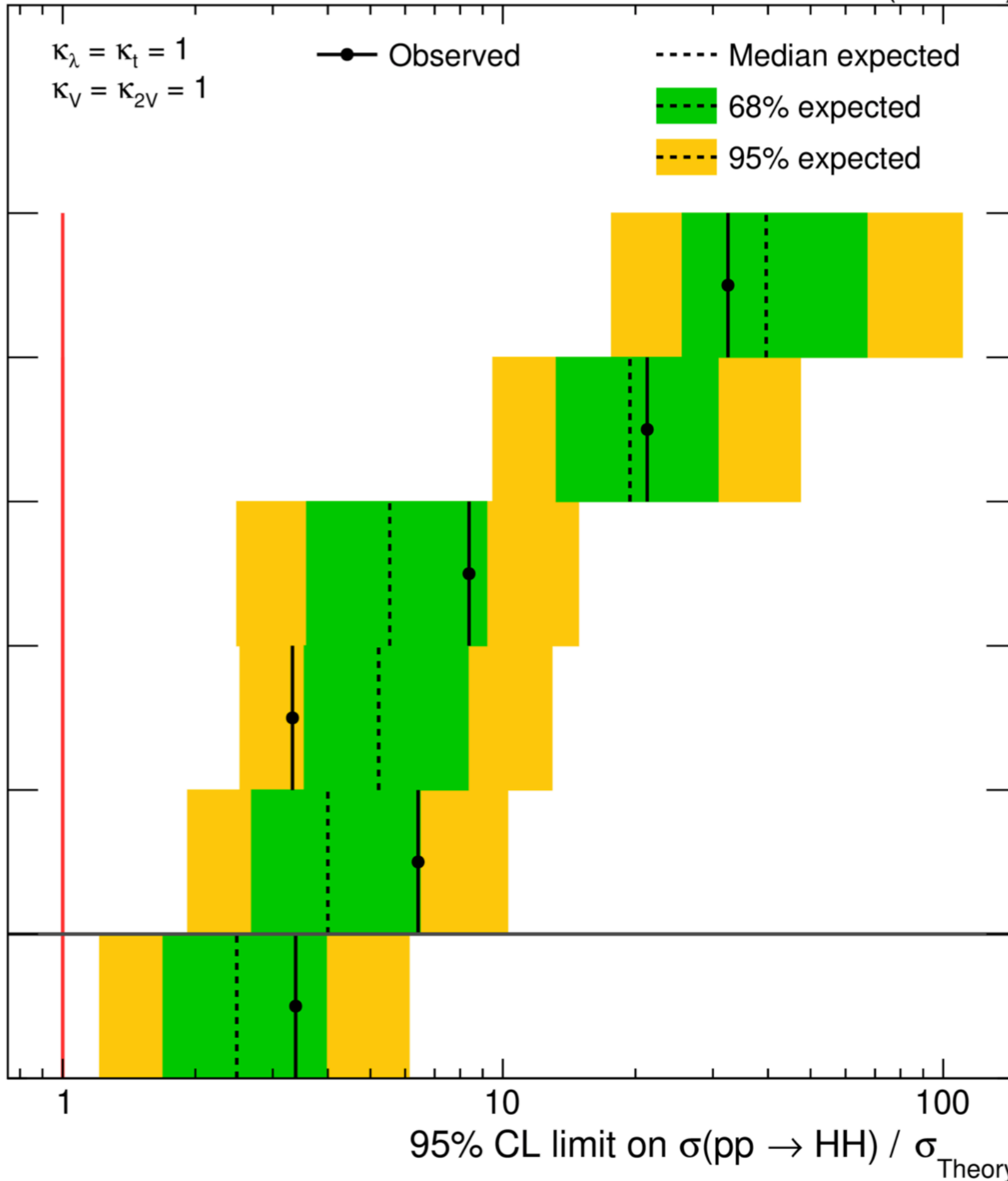
**Multilepton**  
 Expected: 19  
 Observed: 21

**bb  $\gamma\gamma$**   
 Expected: 5.5  
 Observed: 8.4

**bb  $\tau\tau$**   
 Expected: 5.2  
 Observed: 3.3

**bb bb**  
 Expected: 4.0  
 Observed: 6.4

**Combined**  
 Expected: 2.5  
 Observed: 3.4



See the talks  
 by Sanmay  
 and Xiaohu

$$-1.2 < \kappa_\lambda < 6.5$$

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# Why do we need precise prediction?

---

- 1. The measured events numbers do not depend on the theoretical prediction, but the interpretation does.**
- 2. As more data are accumulated, the experimental uncertainties will reduce. Theoretical uncertainties will reduce only after we calculate higher-order corrections.**
- 3. Renormalization and factorization scale uncertainties are sizable, especially for Higgs productions.**

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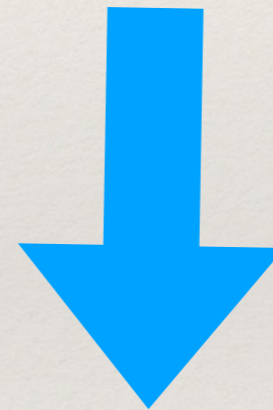
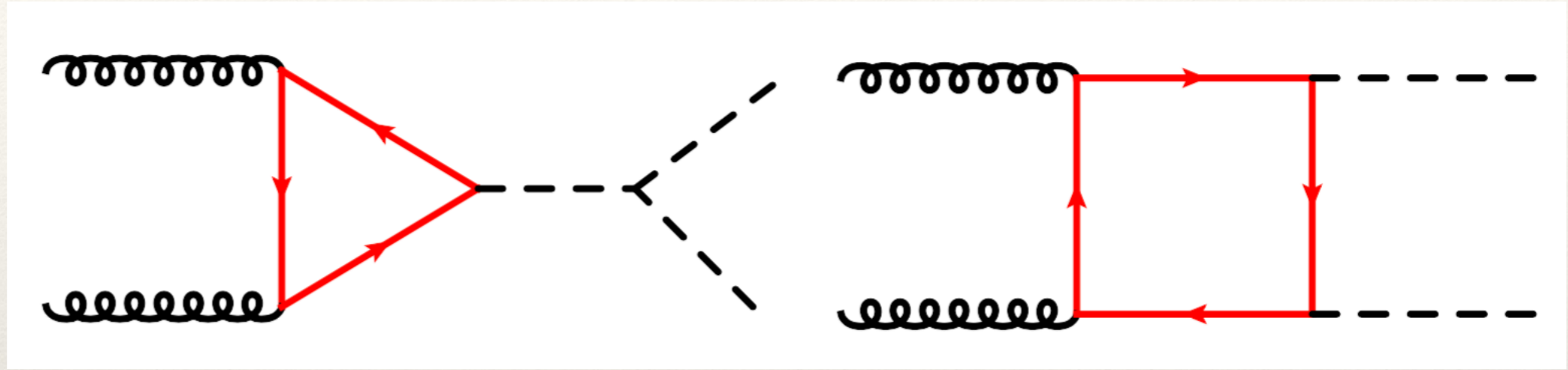
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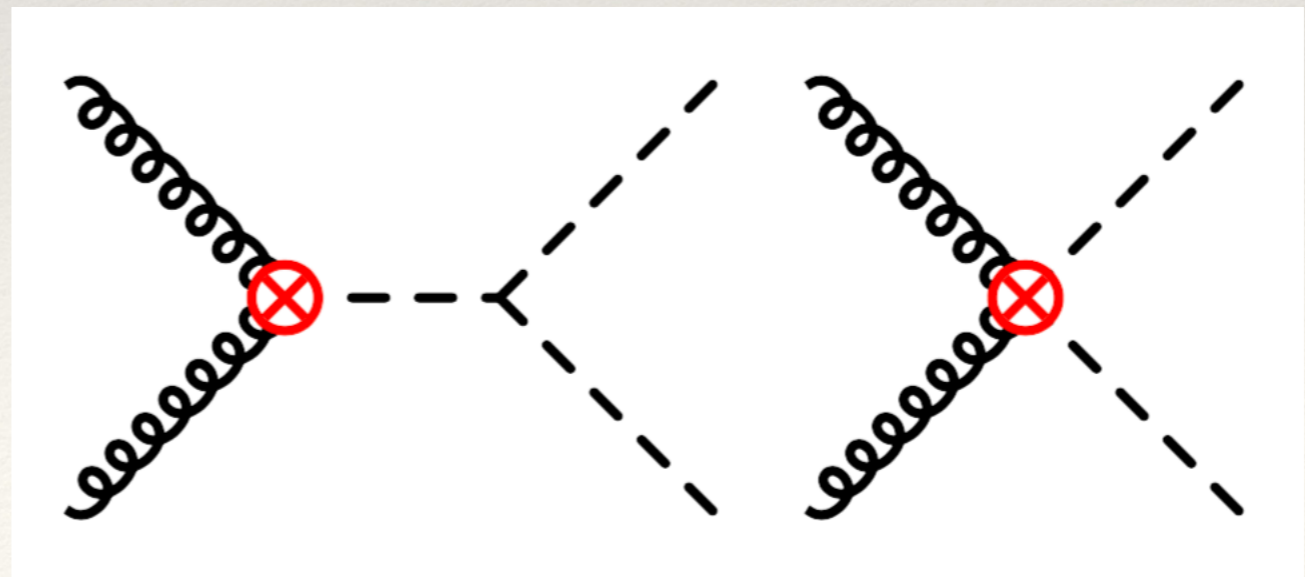
**the more precise the cross section is**

$$gg \rightarrow HH$$

LO in SM



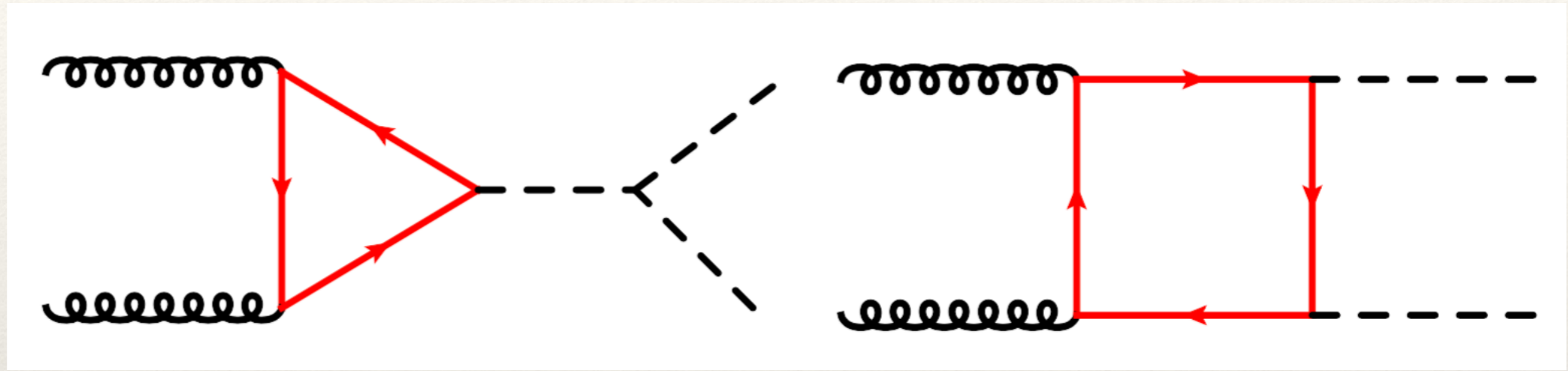
Heavy top quark limit



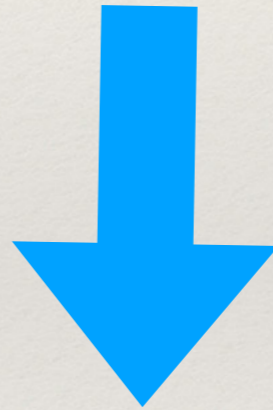


# $gg \rightarrow HH$

LO in SM

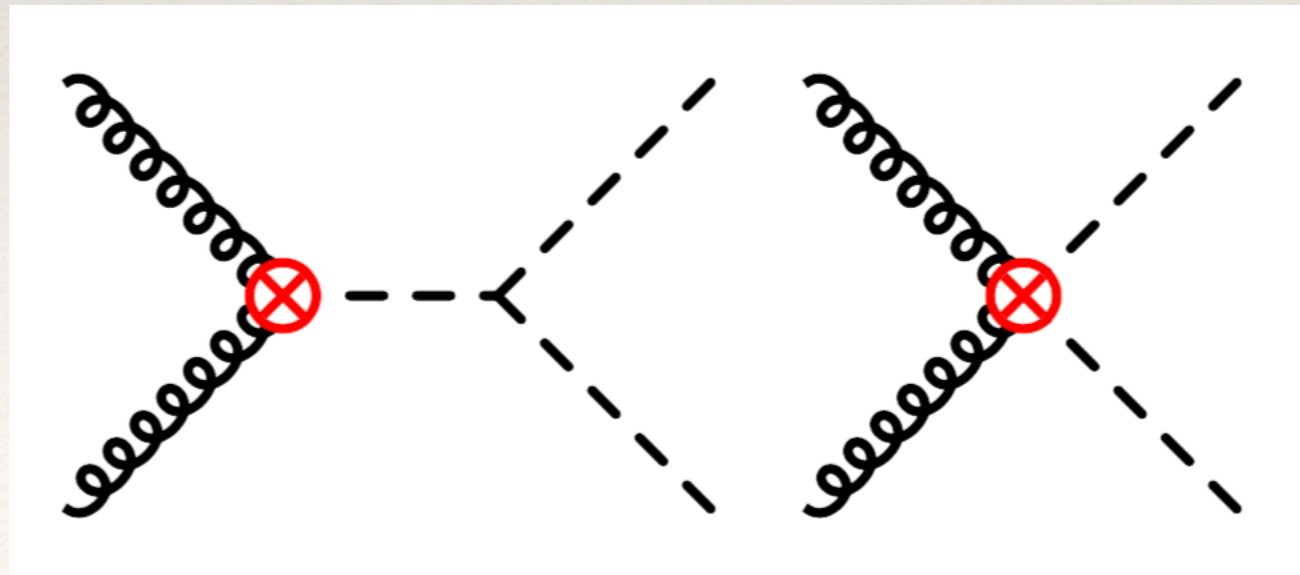


|             |                         |      |
|-------------|-------------------------|------|
| $LO_{m_t}$  | $19.85^{+28\%}_{-21\%}$ | $fb$ |
| $NLO_{m_t}$ | $32.93^{+14\%}_{-13\%}$ | $fb$ |



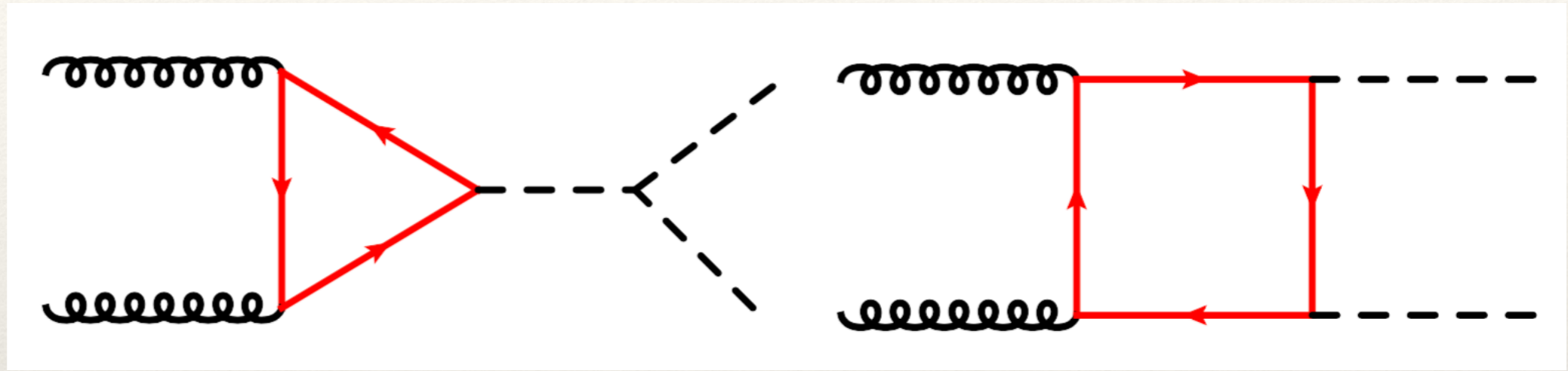
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Borowka, et al, PRL 117, 012001



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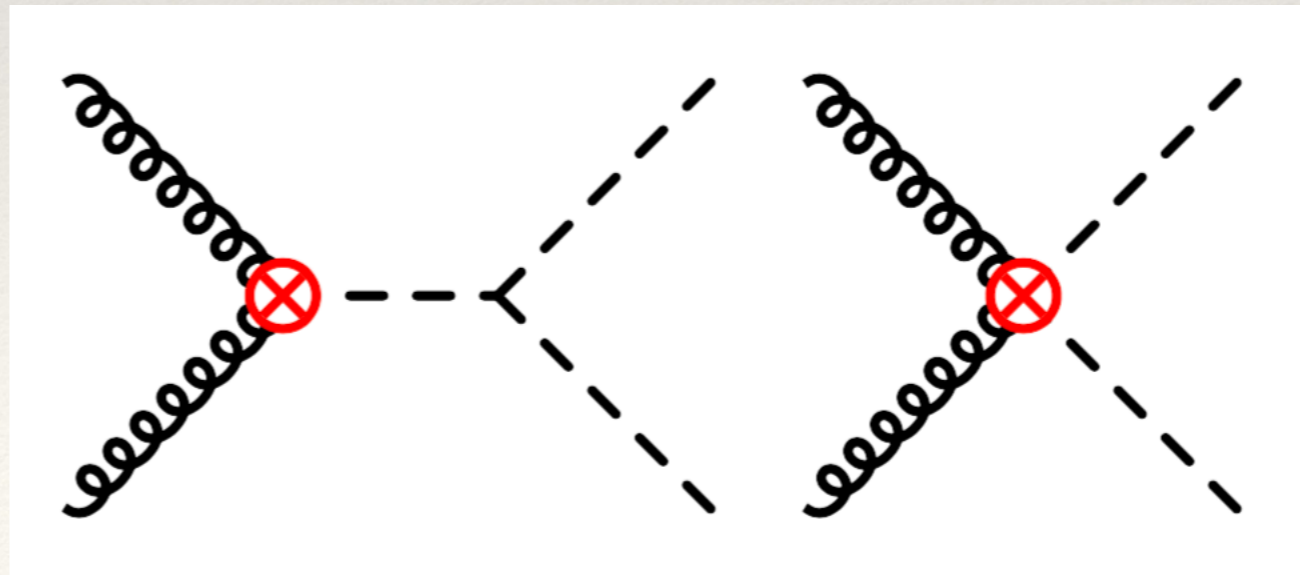


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+66%

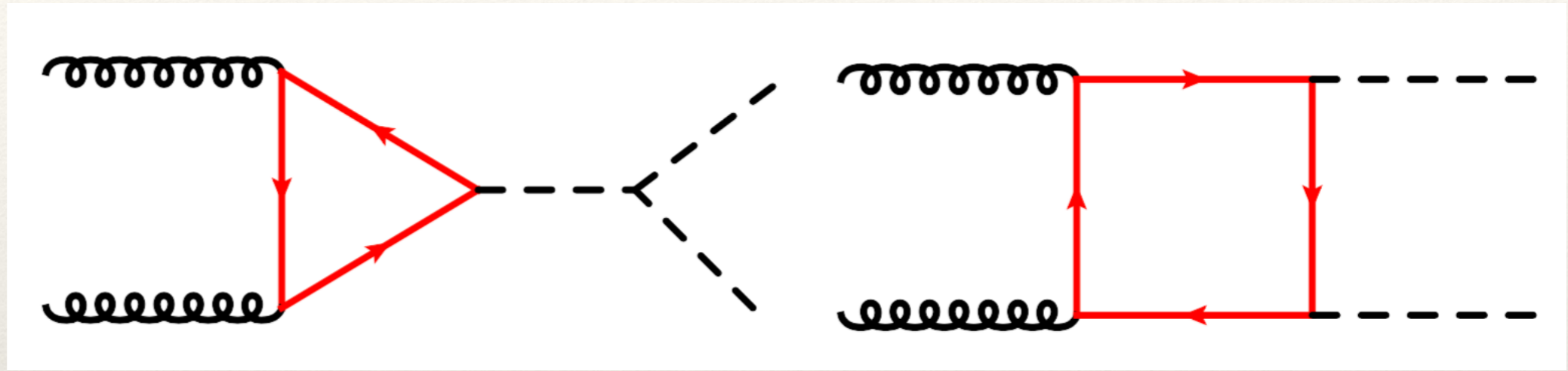
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# gg > HH

LO in SM



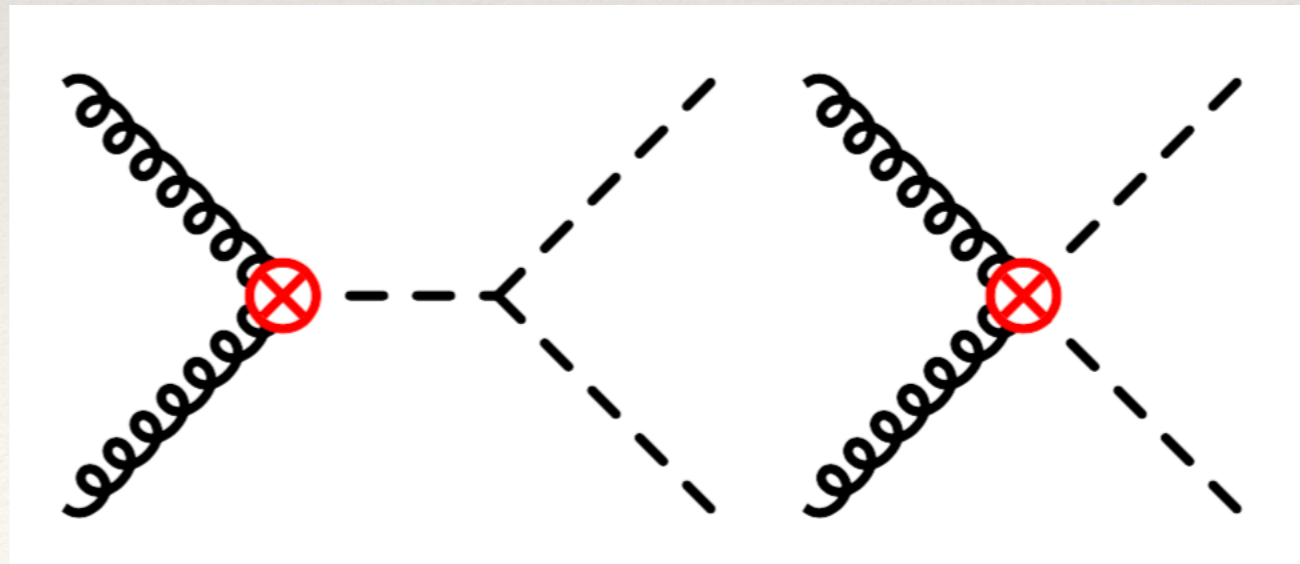
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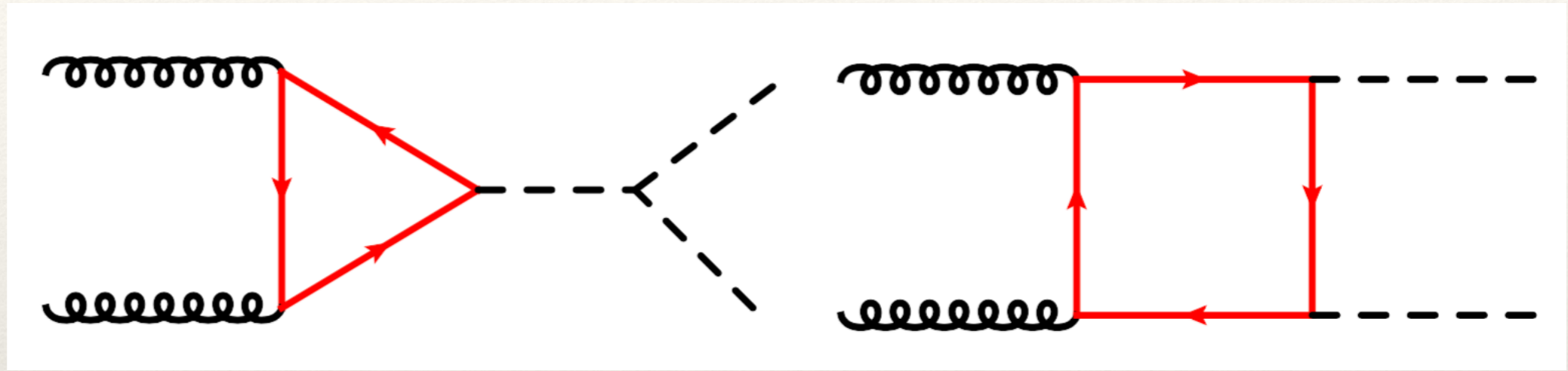
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| $LO_{\infty}$  | $17.07^{+30\%}_{-22\%}$ | $fb$ |
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Dawson, Dittmaier, Spira, PRD58,115012



# gg > HH

LO in SM



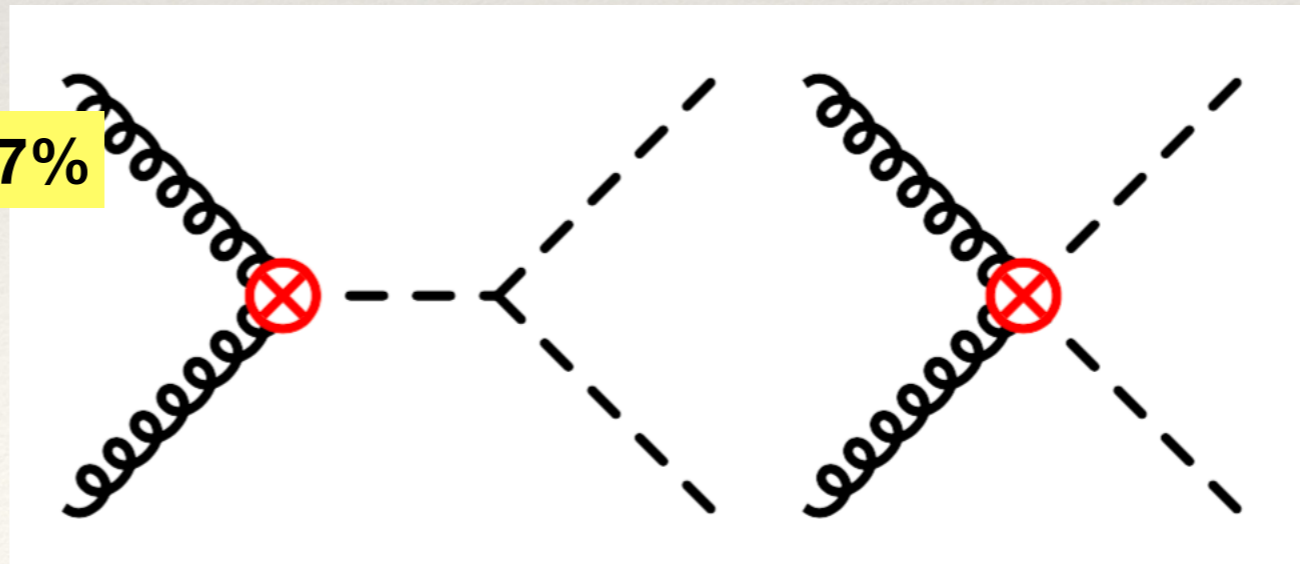
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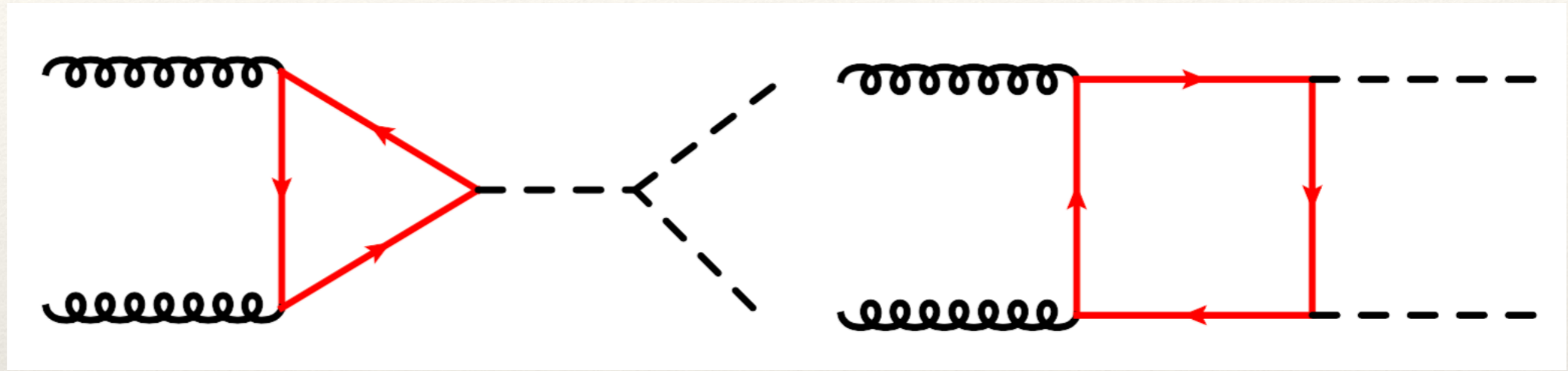
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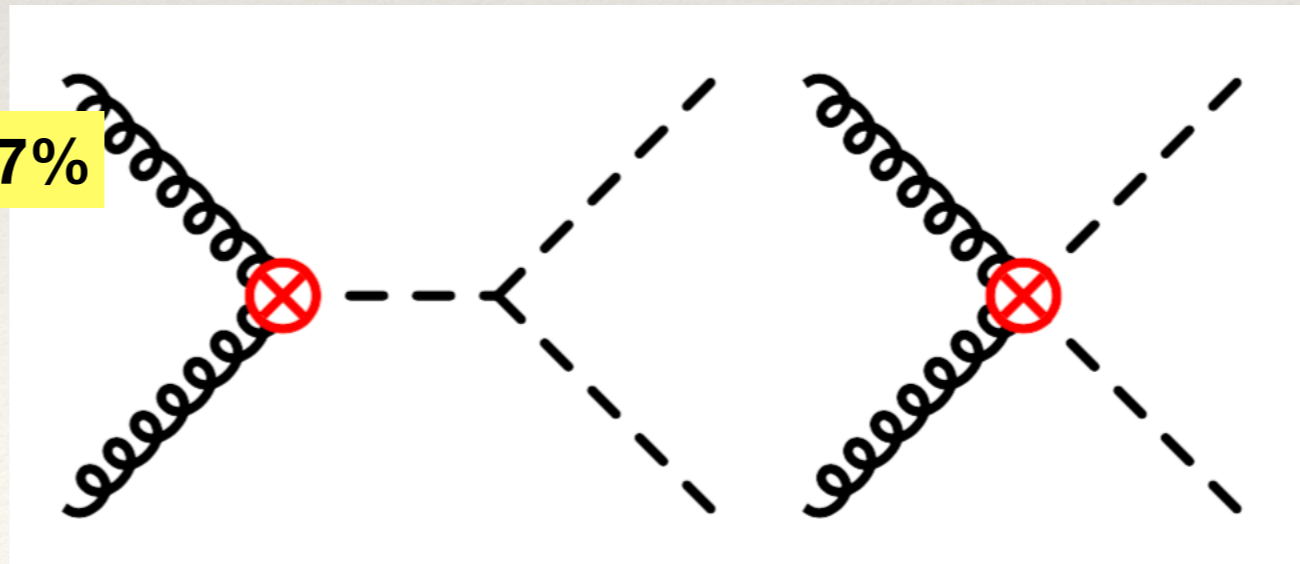
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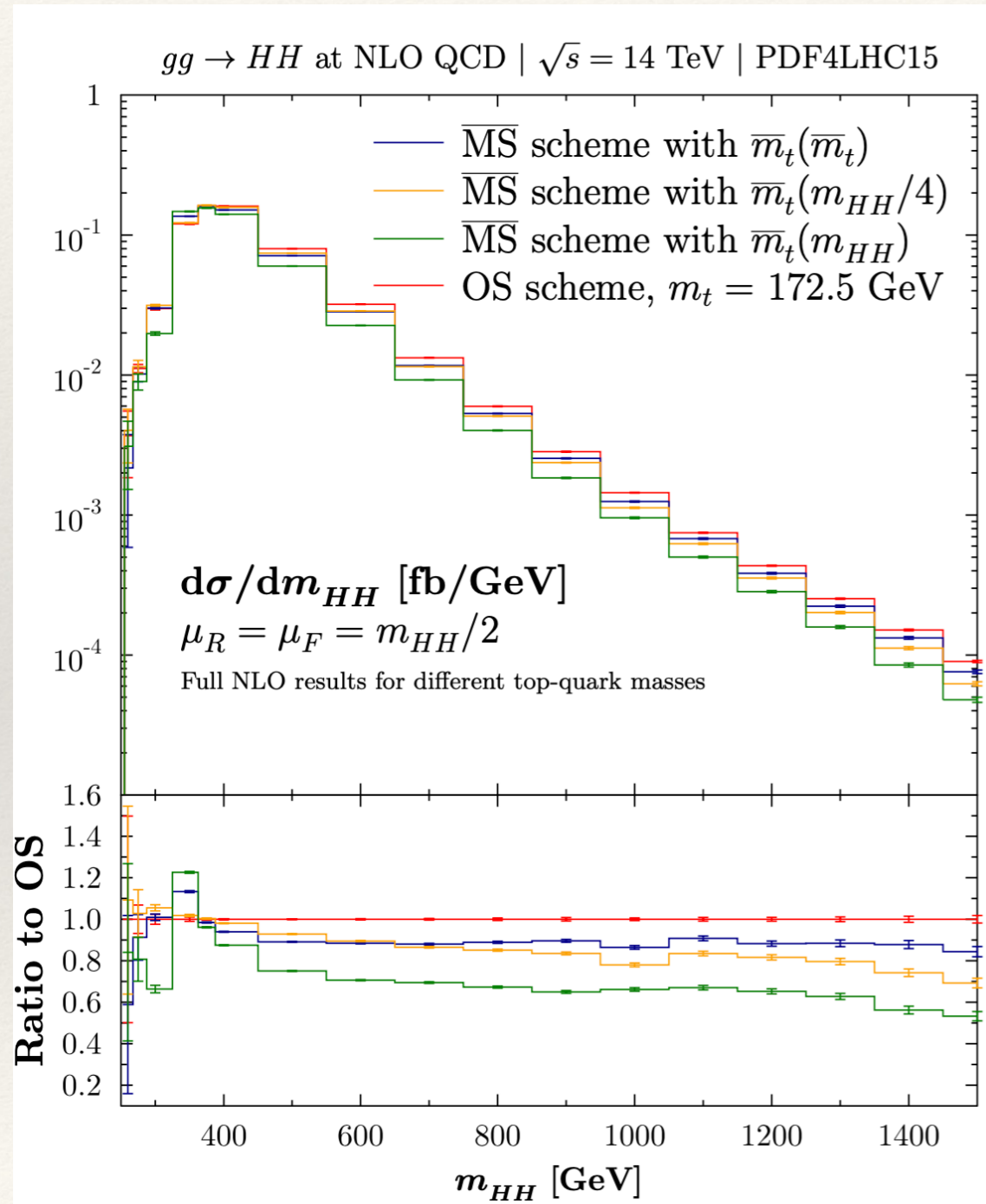
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Dawson, Dittmaier, Spira, PRD58,115012

**difference: 16% at LO to 3% at NLO**



# gg>HH@NLO: Full mt dependence



$$\left. \frac{d\sigma_{NLO}}{dQ} \right|_{Q=300 \text{ GeV}} = 0.02978(7)^{+6\%}_{-34\%} \text{ fb/GeV},$$

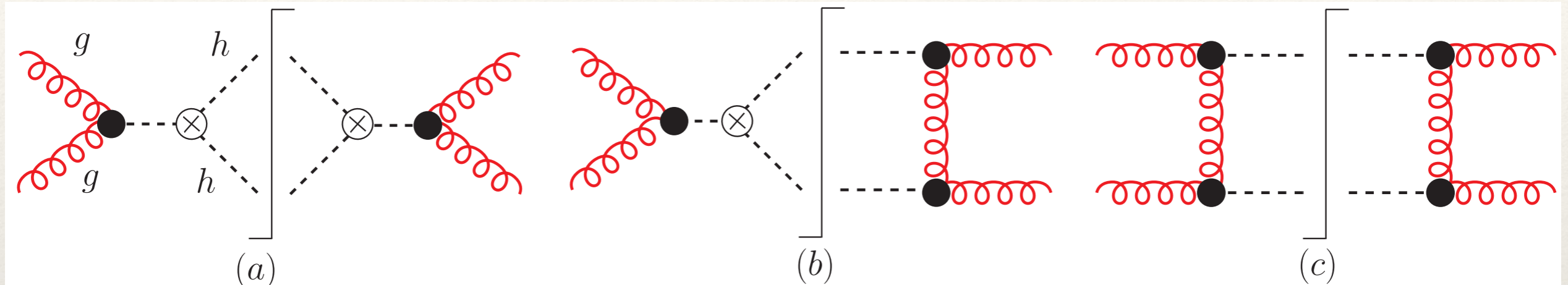
$$\left. \frac{d\sigma_{NLO}}{dQ} \right|_{Q=400 \text{ GeV}} = 0.1609(4)^{+0\%}_{-13\%} \text{ fb/GeV},$$

$$\left. \frac{d\sigma_{NLO}}{dQ} \right|_{Q=600 \text{ GeV}} = 0.03204(9)^{+0\%}_{-30\%} \text{ fb/GeV},$$

$$\left. \frac{d\sigma_{NLO}}{dQ} \right|_{Q=1200 \text{ GeV}} = 0.000435(4)^{+0\%}_{-35\%} \text{ fb/GeV}.$$

$$\begin{aligned} \sqrt{s} = 13 \text{ TeV} : \quad \sigma_{tot} &= 27.73(7)^{+4\%}_{-18\%} \text{ fb}, \\ \sqrt{s} = 14 \text{ TeV} : \quad \sigma_{tot} &= 32.81(7)^{+4\%}_{-18\%} \text{ fb}, \\ \sqrt{s} = 27 \text{ TeV} : \quad \sigma_{tot} &= 127.8(2)^{+4\%}_{-18\%} \text{ fb}, \\ \sqrt{s} = 100 \text{ TeV} : \quad \sigma_{tot} &= 1140(2)^{+3\%}_{-18\%} \text{ fb} \end{aligned}$$

# $gg \rightarrow HH @ \text{NNNNLO}$



**NNNLO**

**NNLO**

**NLO**

**Similar to single Higgs**

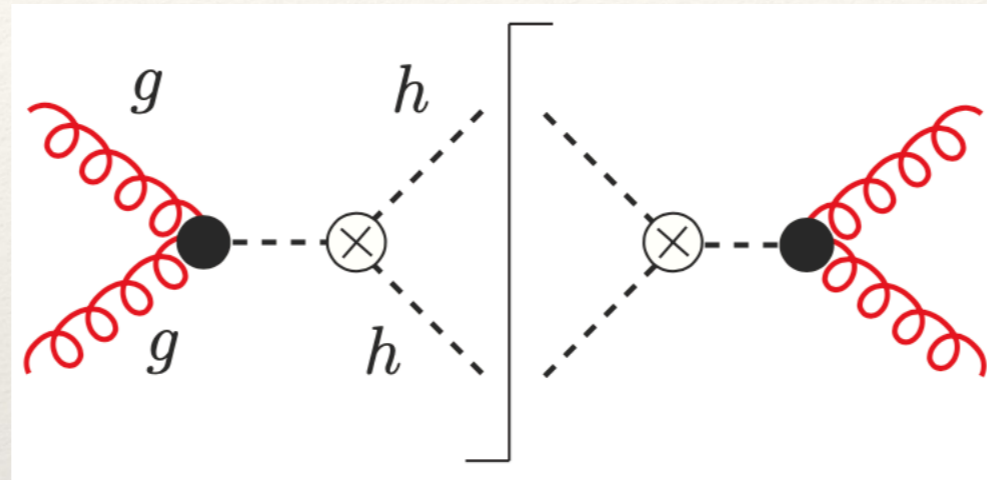
**$q_T$  subtraction**

**Standard methods**

**Many checks:**

- 1. Self consistency (gauge invariance, poles cancellation)**
- 2. Reproduce single Higgs xs up to NNLO**
- 3. Reproduce double Higgs xs up to NNLO**

# Class-(a)



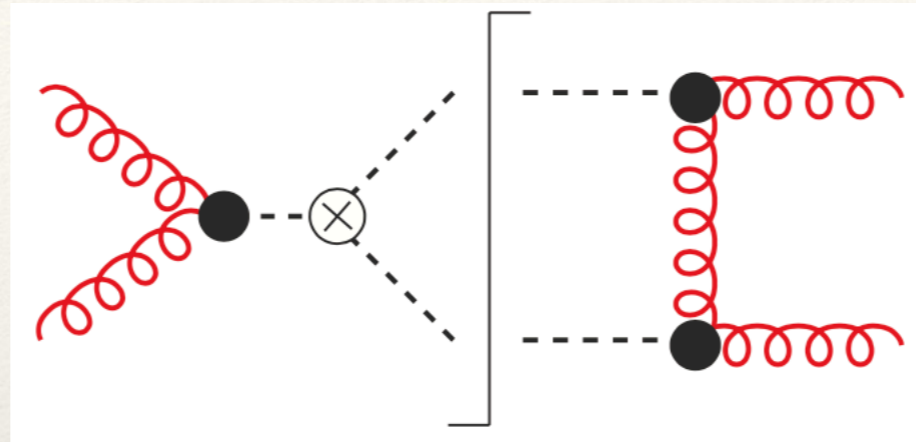
$$\frac{d\sigma_{hh}^a}{dm_{hh}} = f_{h \rightarrow hh} \left( \frac{C_{hh}}{C_h} - \frac{6\lambda_{hhh}v^2}{m_{hh}^2 - m_h^2} \right)^2 \times \left( \sigma_h \Big|_{m_h \rightarrow m_{hh}} \right)$$

$$f_{h \rightarrow hh} = \frac{\sqrt{m_{hh}^2 - 4m_h^2}}{16\pi^2 v^2}$$

Dulat, Lazopoulos, Mistlberger iHixs, 1802.00827



# Class-(b)



$$d\sigma_{hh}^b = d\sigma_{hh}^b \Big|_{p_T^{hh} < p_T^{\text{veto}}} + d\sigma_{hh}^b \Big|_{p_T^{hh} > p_T^{\text{veto}}}$$

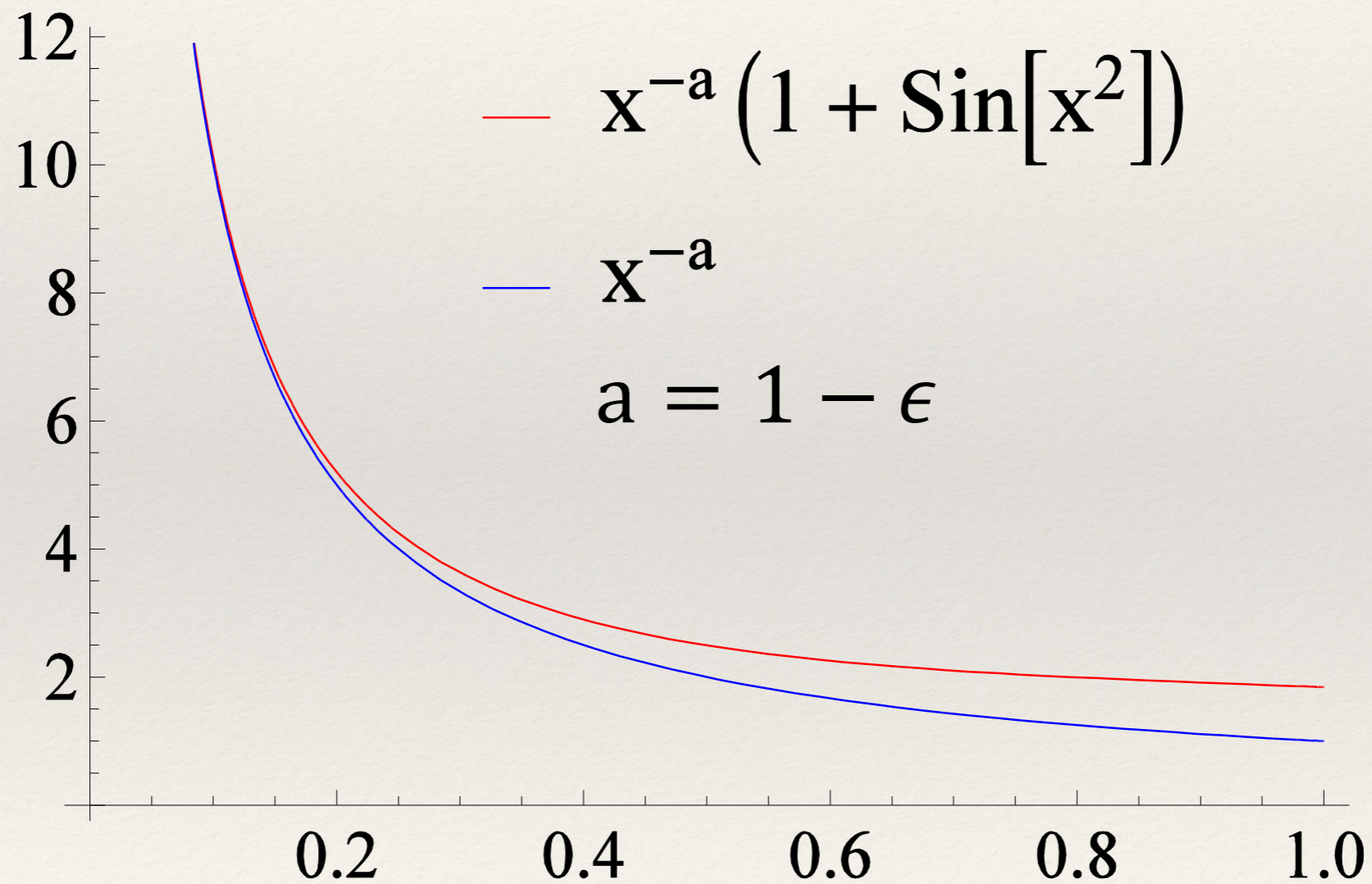
Catani, Grazzini, PRL98,222002

$$\frac{d\sigma_{hh}^b}{dp_T^{hh}} = H^b \otimes B_g \otimes B_g \otimes S \times \left( 1 + \mathcal{O} \left( \frac{(p_T^{hh})^2}{Q^2} \right) \right)$$

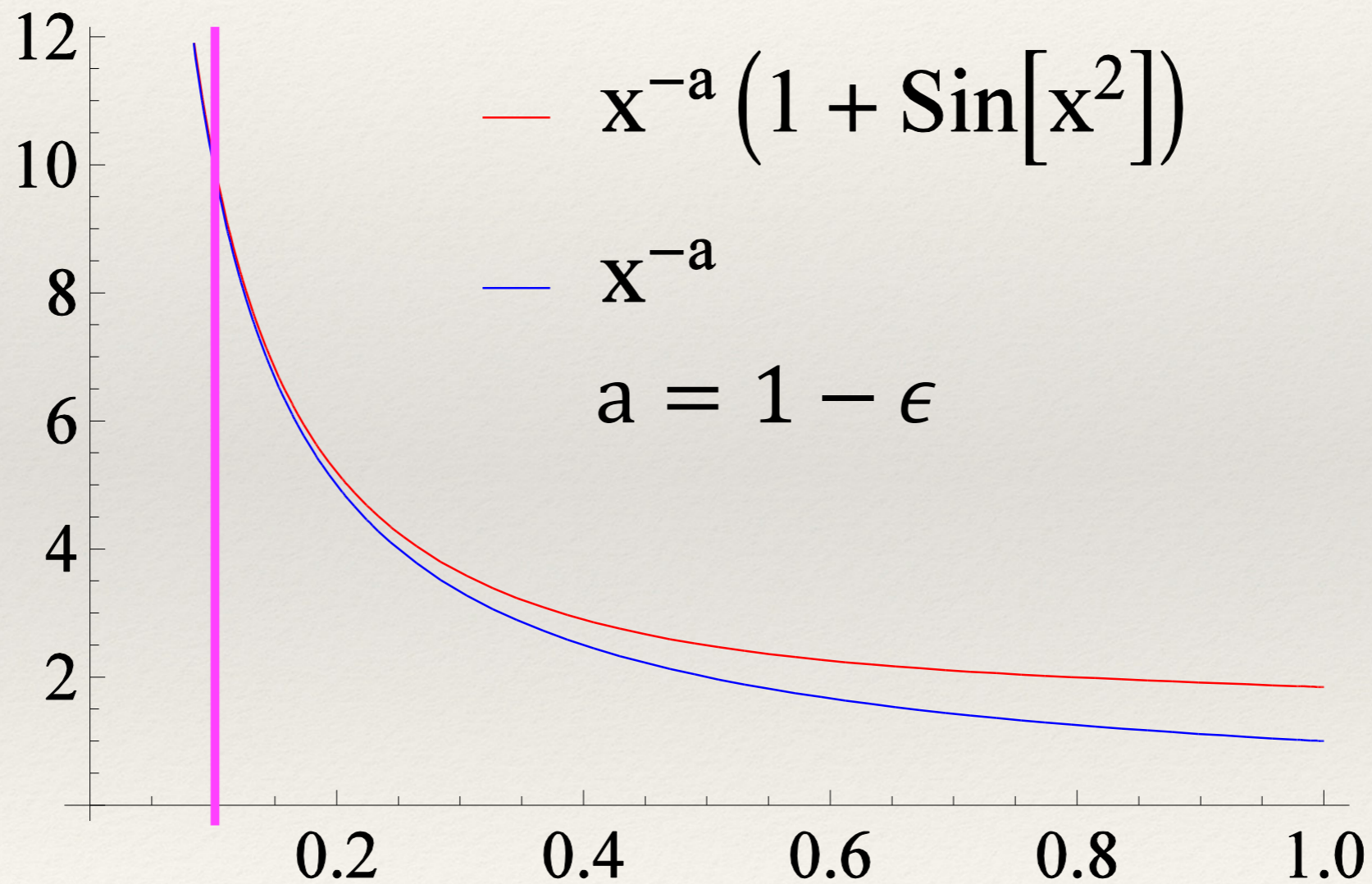
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# The idea of qT subtraction

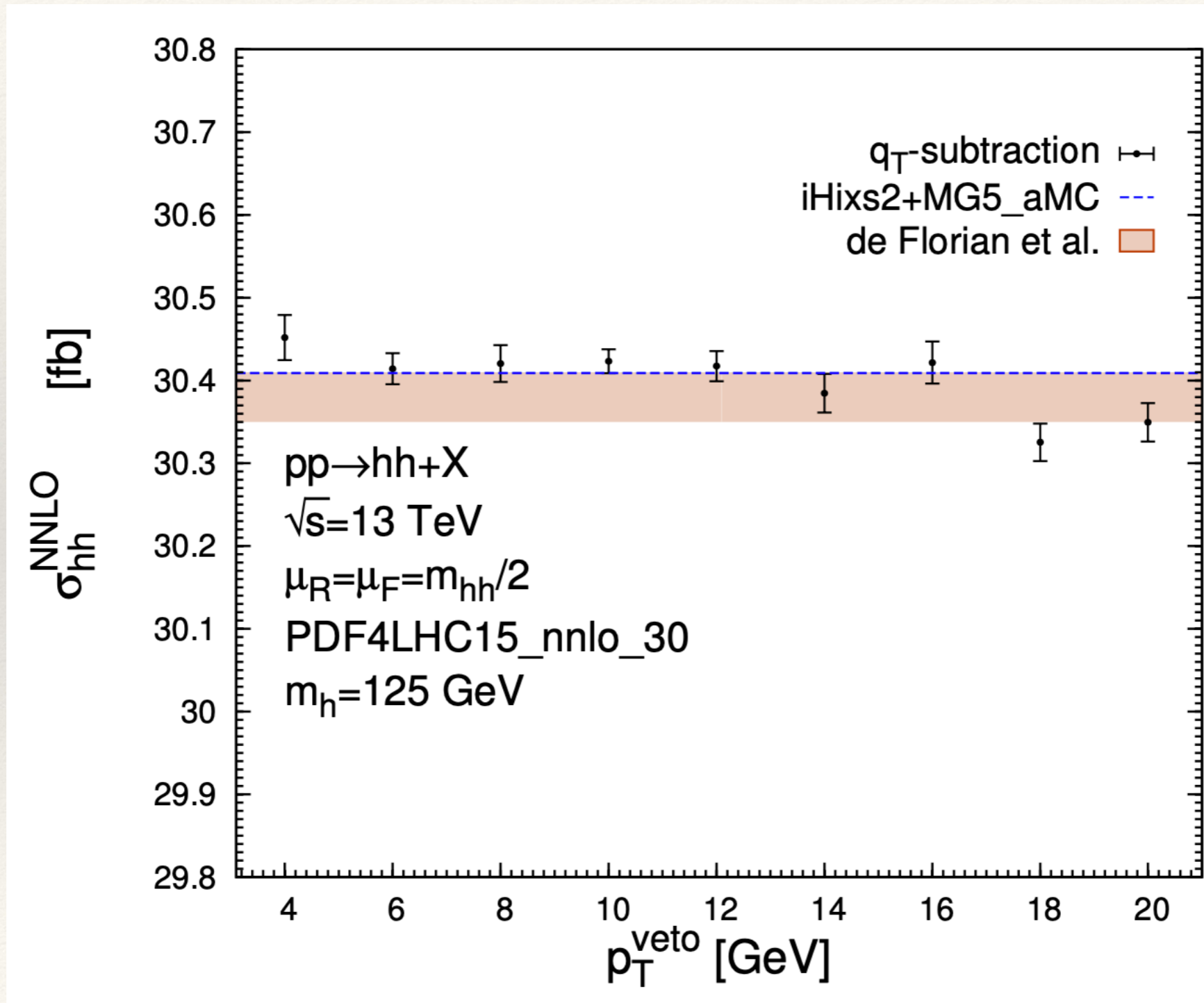
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# The idea of qT subtraction



# Validation of qT subtraction



# How large are NNNLO corrections?

| order \ $\sqrt{s}$ | 13 TeV                                   | 14 TeV                                   | 27 TeV                                   | 100 TeV                                 |
|--------------------|--|--|--|---|
| LO                 | 13.80 <sup>+31%</sup> <sub>-22%</sub>    | 17.06 <sup>+31%</sup> <sub>-22%</sub>    | 98.22 <sup>+26%</sup> <sub>-19%</sub>    | 2015 <sup>+19%</sup> <sub>-15%</sub>    |
| NLO                | 25.81 <sup>+18%</sup> <sub>-15%</sub>    | 31.89 <sup>+18%</sup> <sub>-15%</sub>    | 183.0 <sup>+16%</sup> <sub>-14%</sub>    | 3724 <sup>+13%</sup> <sub>-11%</sub>    |
| NNLO               | 30.41 <sup>+5.3%</sup> <sub>-7.8%</sub>  | 37.55 <sup>+5.2%</sup> <sub>-7.6%</sub>  | 214.2 <sup>+4.8%</sup> <sub>-6.7%</sub>  | 4322 <sup>+4.2%</sup> <sub>-5.3%</sub>  |
| N <sup>3</sup> LO  | 31.31 <sup>+0.66%</sup> <sub>-2.8%</sub> | 38.65 <sup>+0.65%</sup> <sub>-2.7%</sub> | 220.2 <sup>+0.53%</sup> <sub>-2.4%</sub> | 4438 <sup>+0.51%</sup> <sub>-1.8%</sub> |



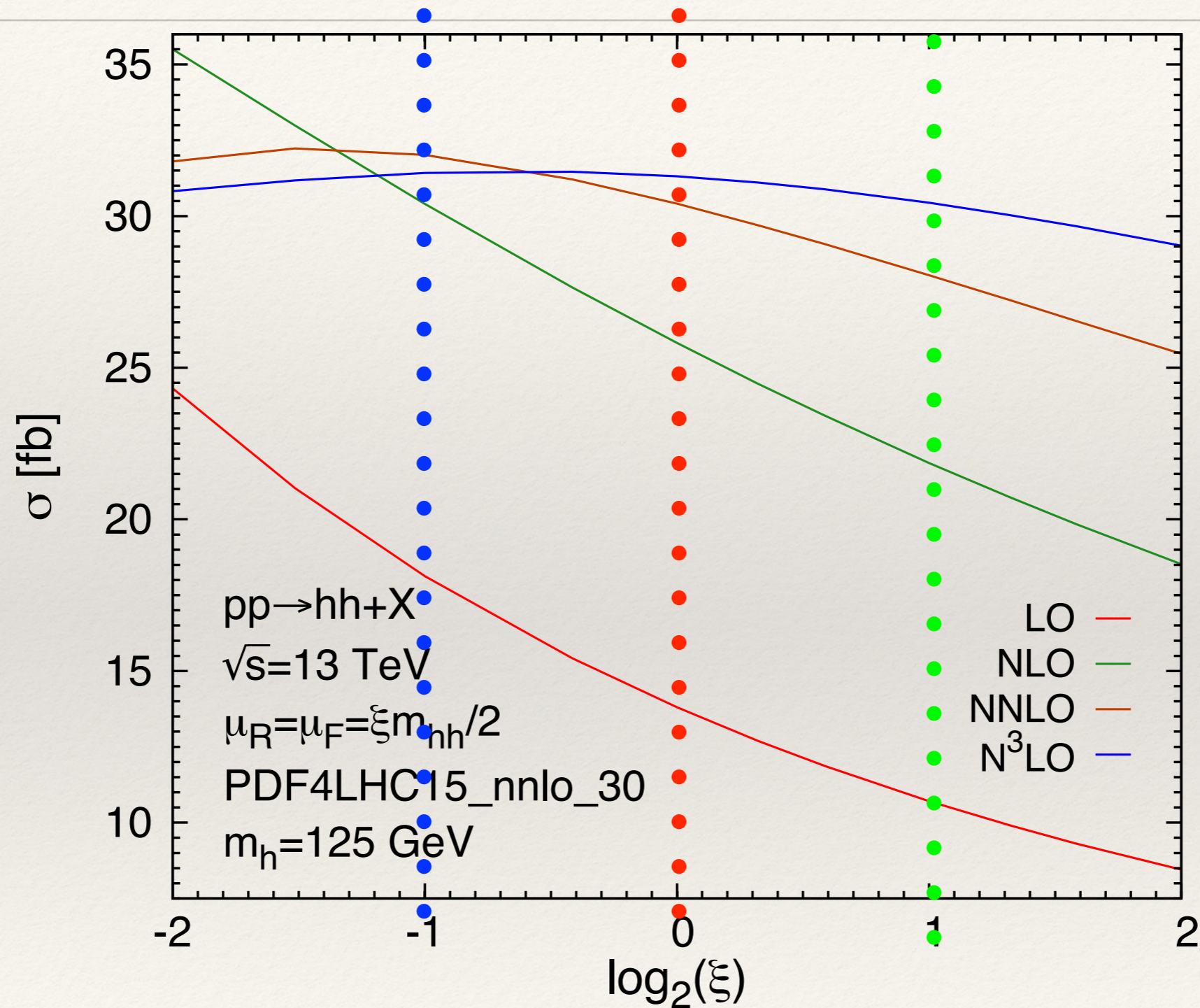
87%

18%

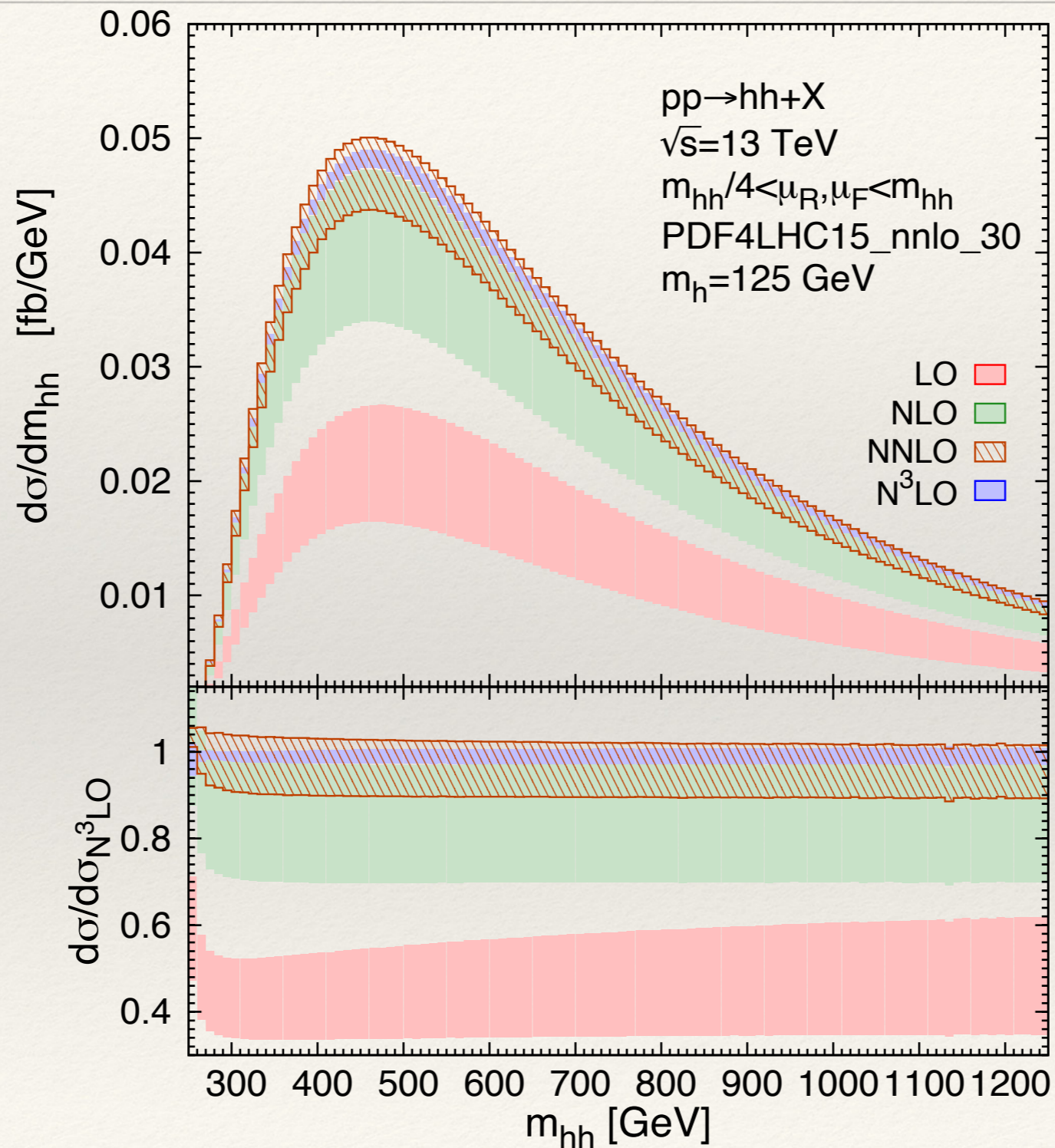
3%

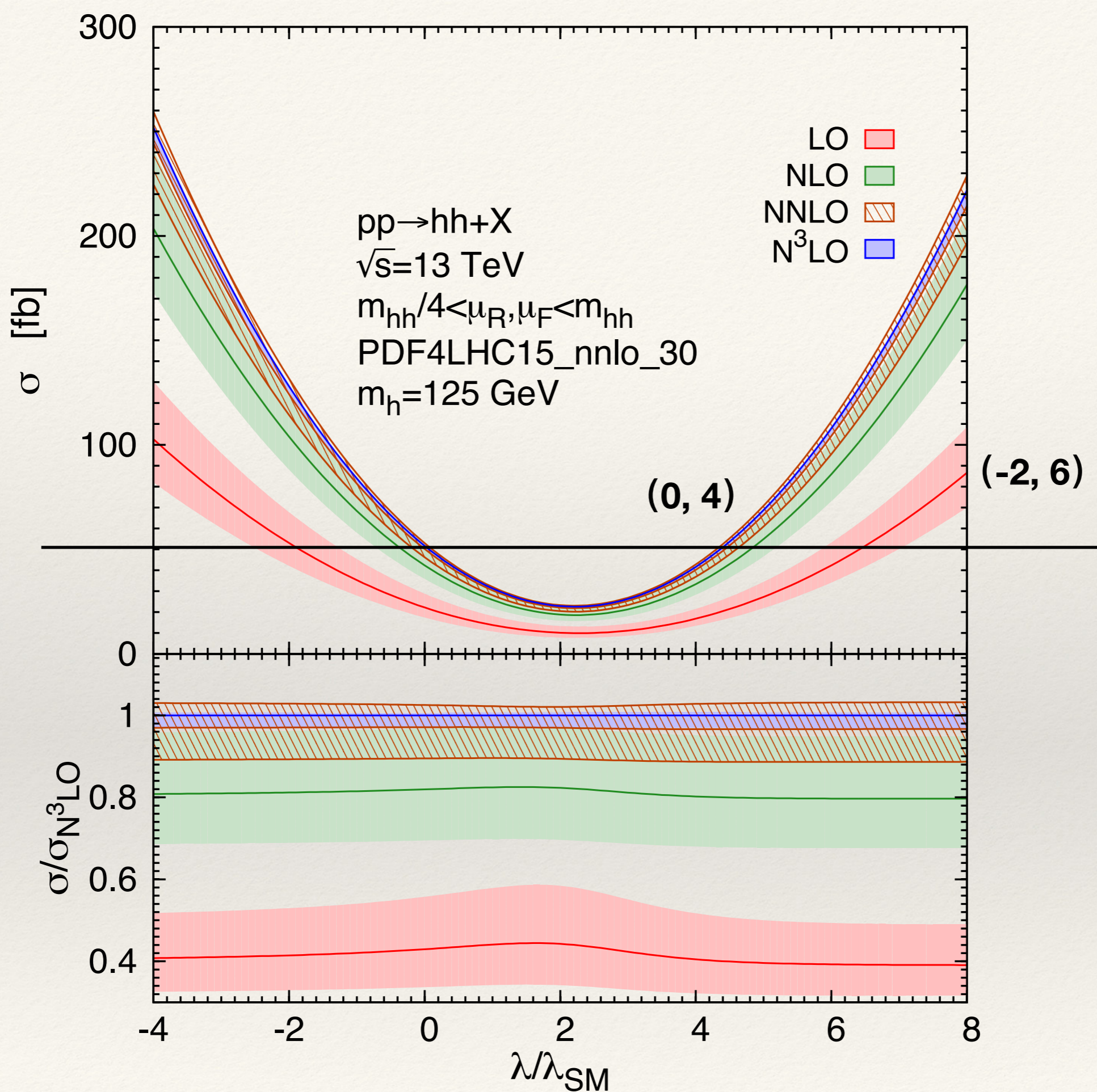
**Scale uncer. less than PDF uncer. 3.3% now !**

# How to choose a scale?



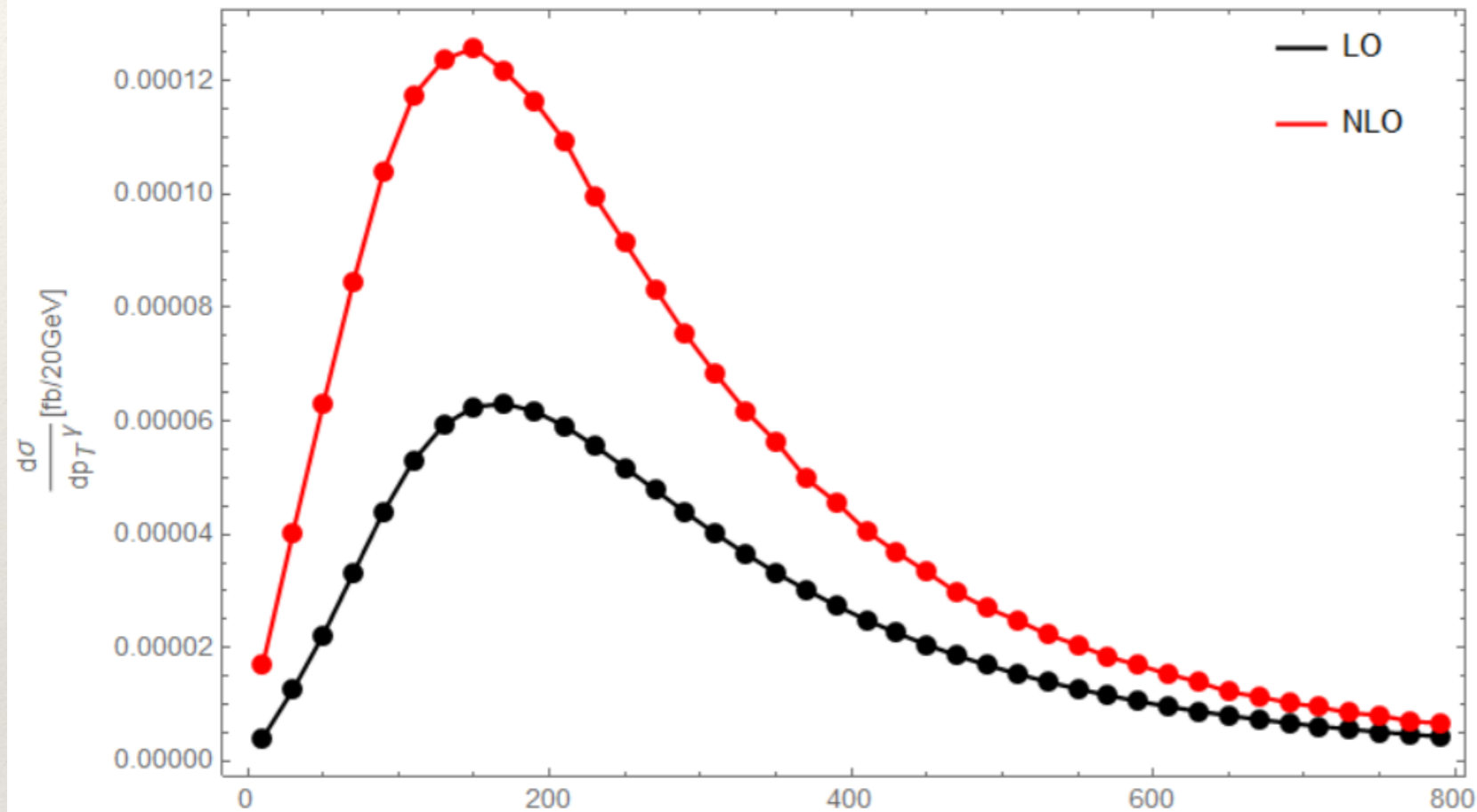
# Invariant mass of Higgs pair





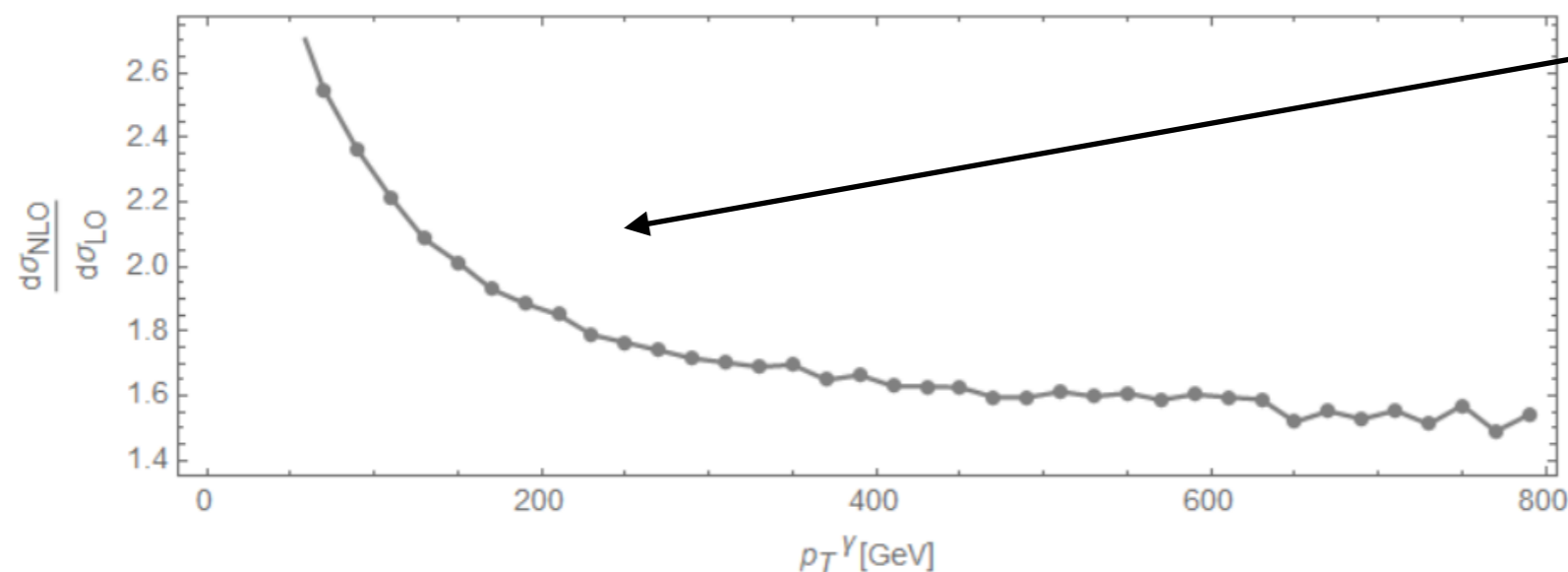


# $gg \rightarrow HH$ including decay

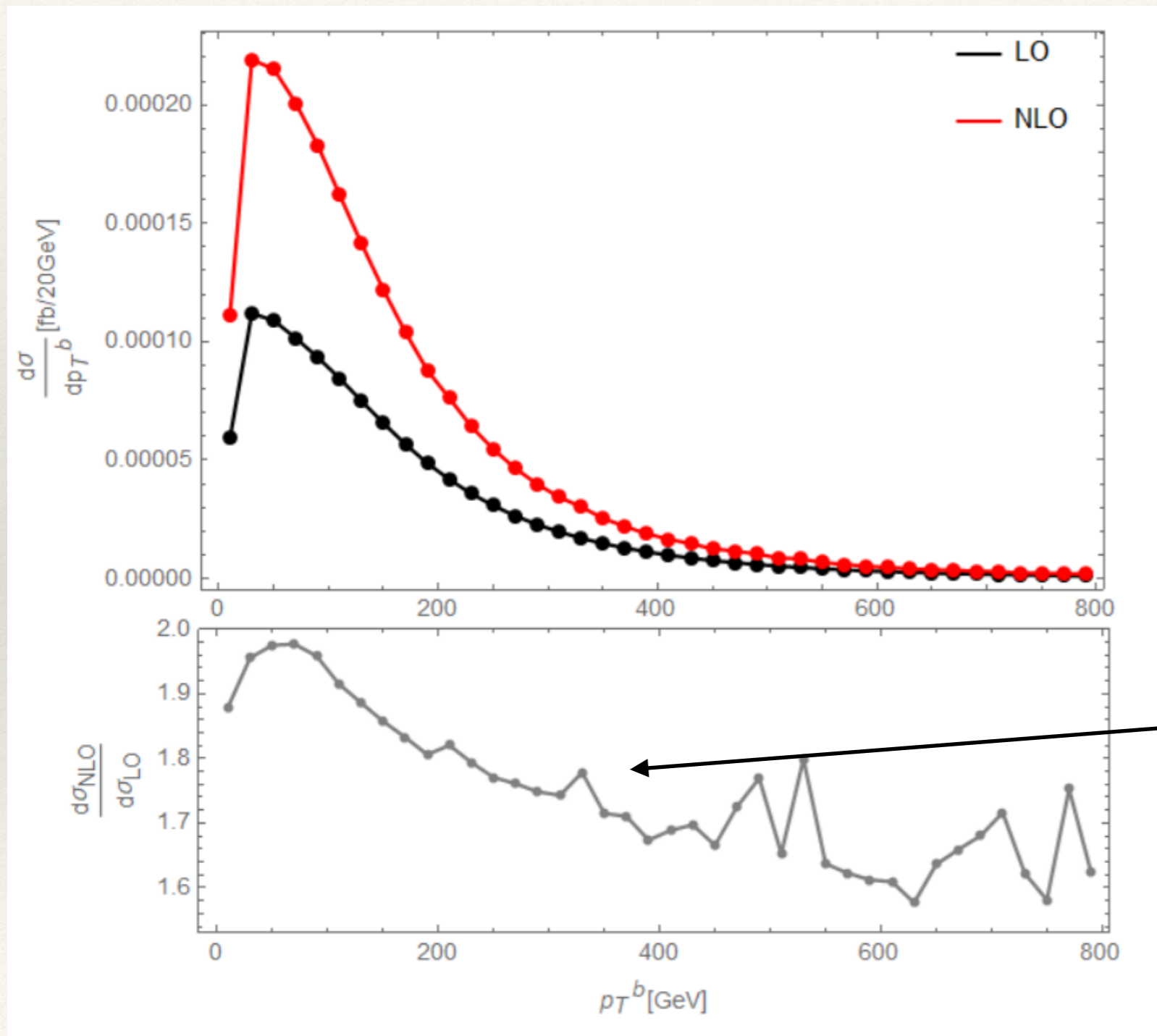


LO prod. & LO decay vs.  
NLO prod. & LO decay

Non-constant enhancement



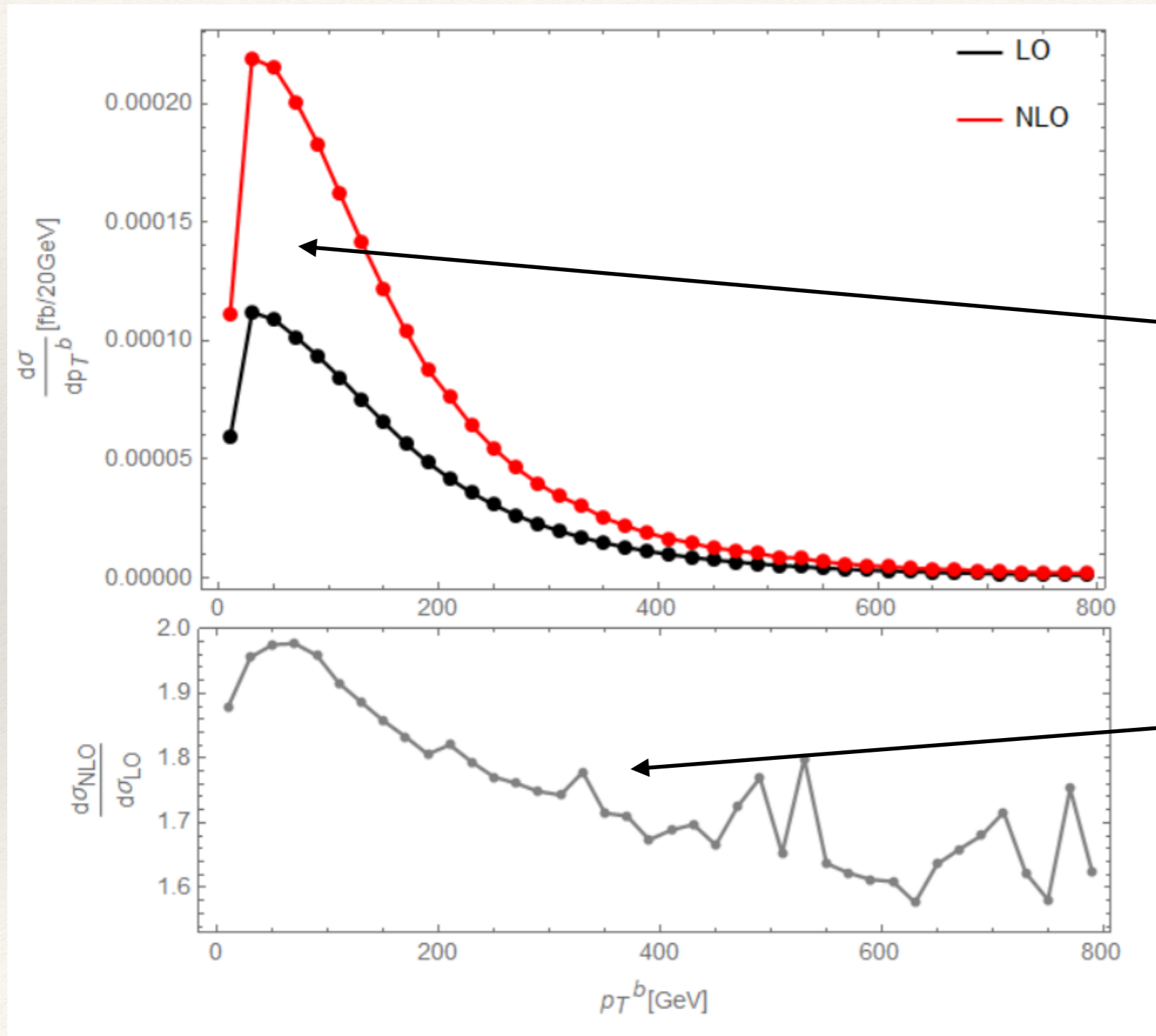
# $gg \rightarrow HH$ including decay



LO prod. & LO decay vs.  
NLO prod. & LO decay

Non-constant enhancement

# $gg \rightarrow HH$ including decay

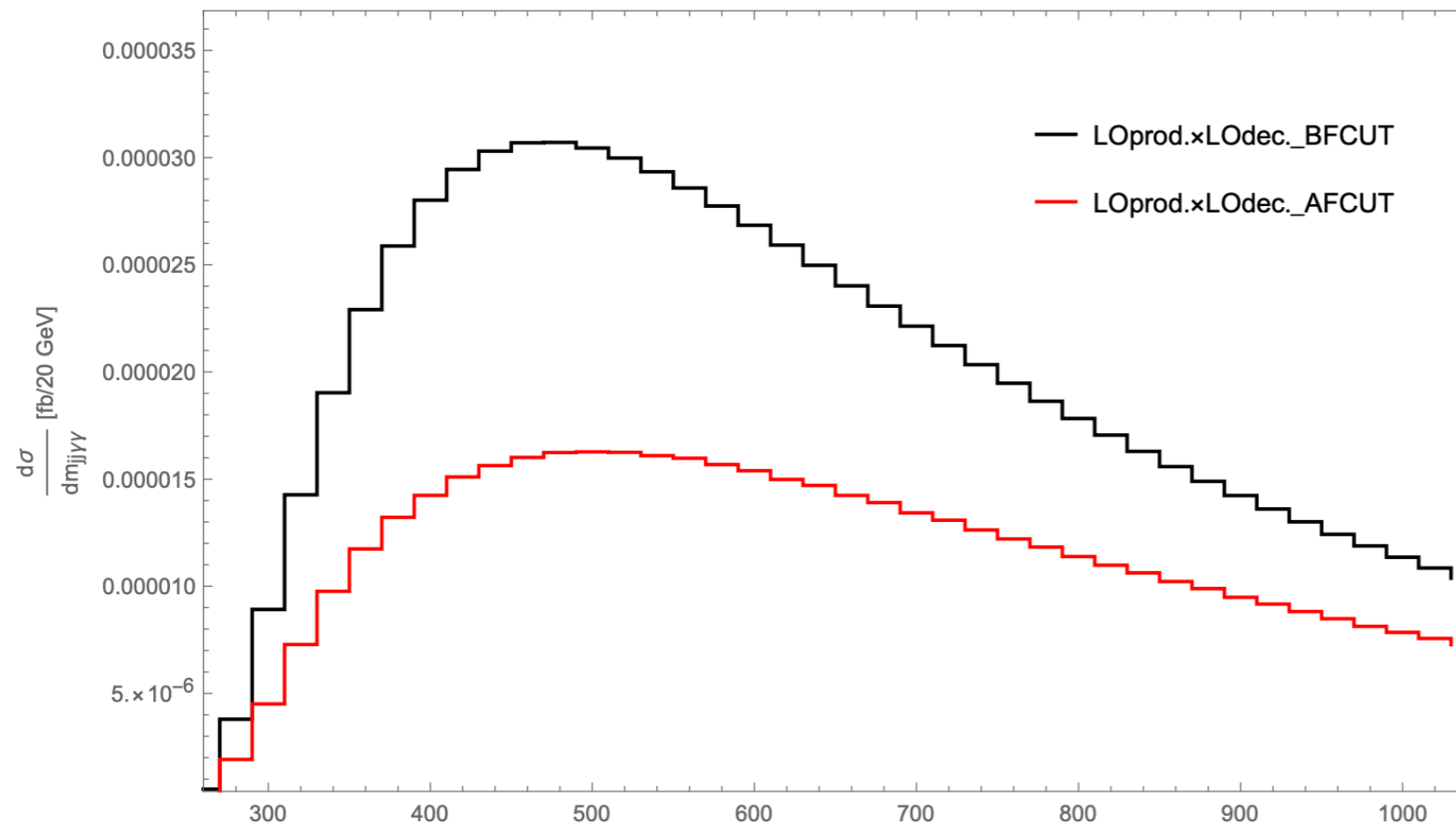


LO prod. & LO decay vs.  
NLO prod. & LO decay

Peak at  $\sim 30$  GeV

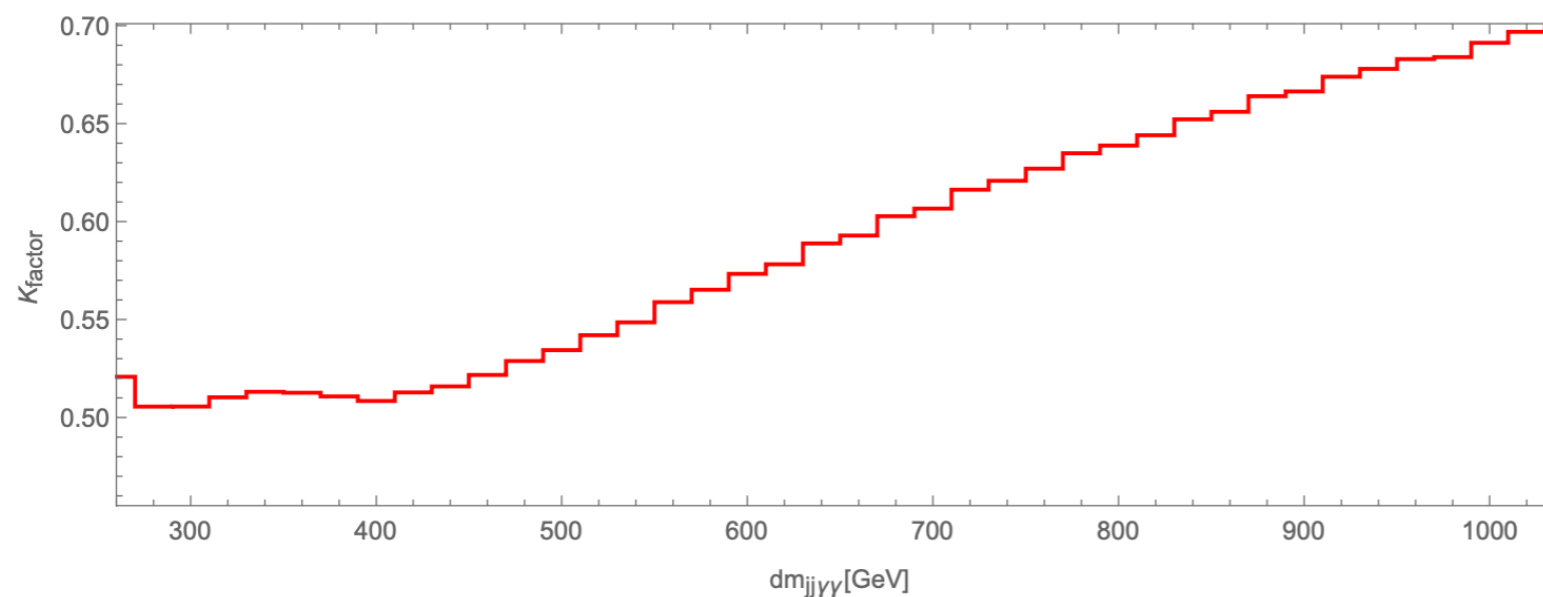
Non-constant enhancement

# $gg \rightarrow HH$ including decay

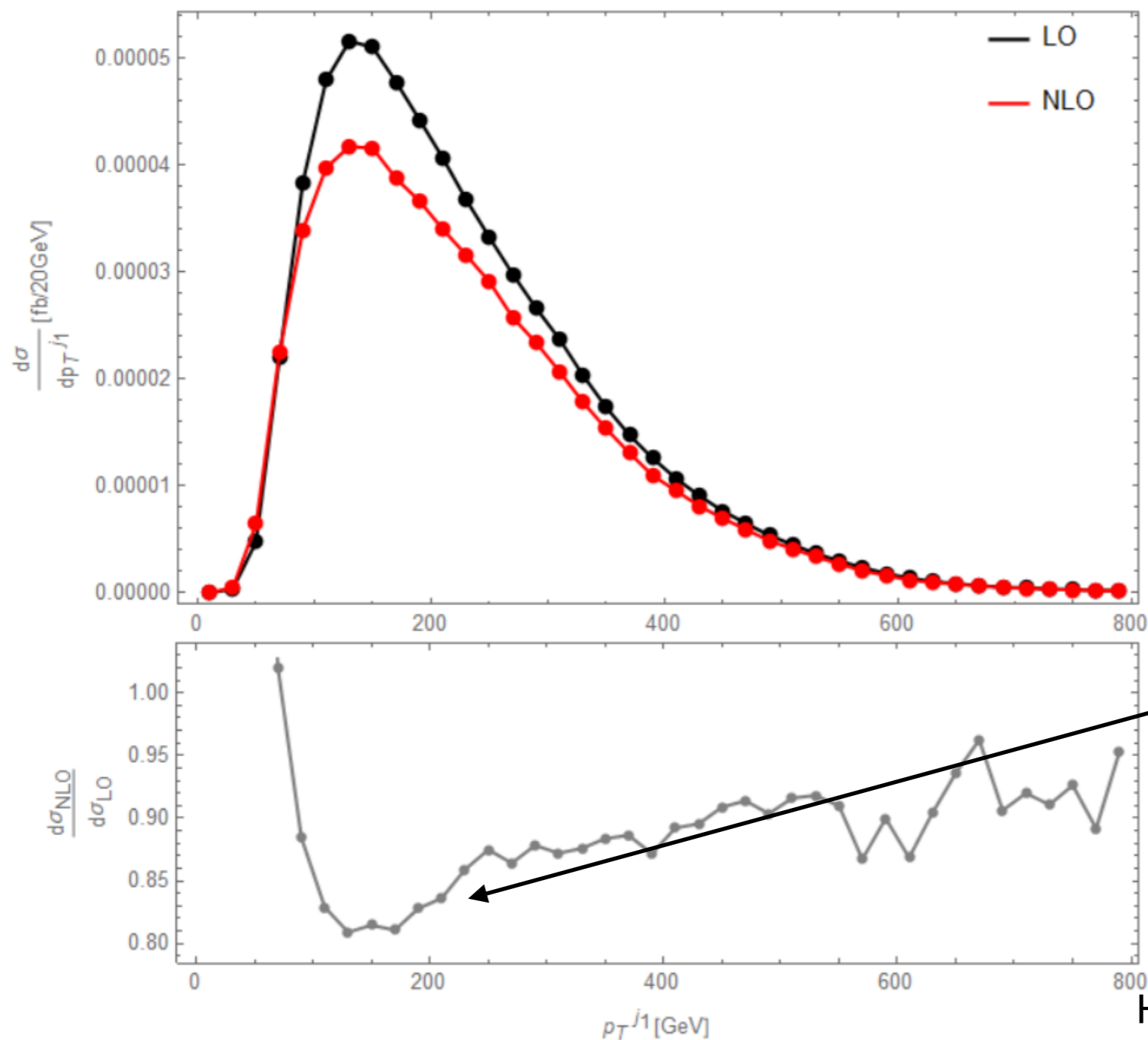


$p_j^T, p_\gamma^T > 25 \text{ GeV}$   
 $|\eta_j|, |\eta_\gamma| < 2.5$   
 $\Delta R_{jj,\gamma j,\gamma\gamma} > 0.4$   
 $90 \text{ GeV} < m_{jj} < 190 \text{ GeV}$

**Very loose cuts decrease the x-section significantly!**



# $gg \rightarrow HH$ including NLO decay

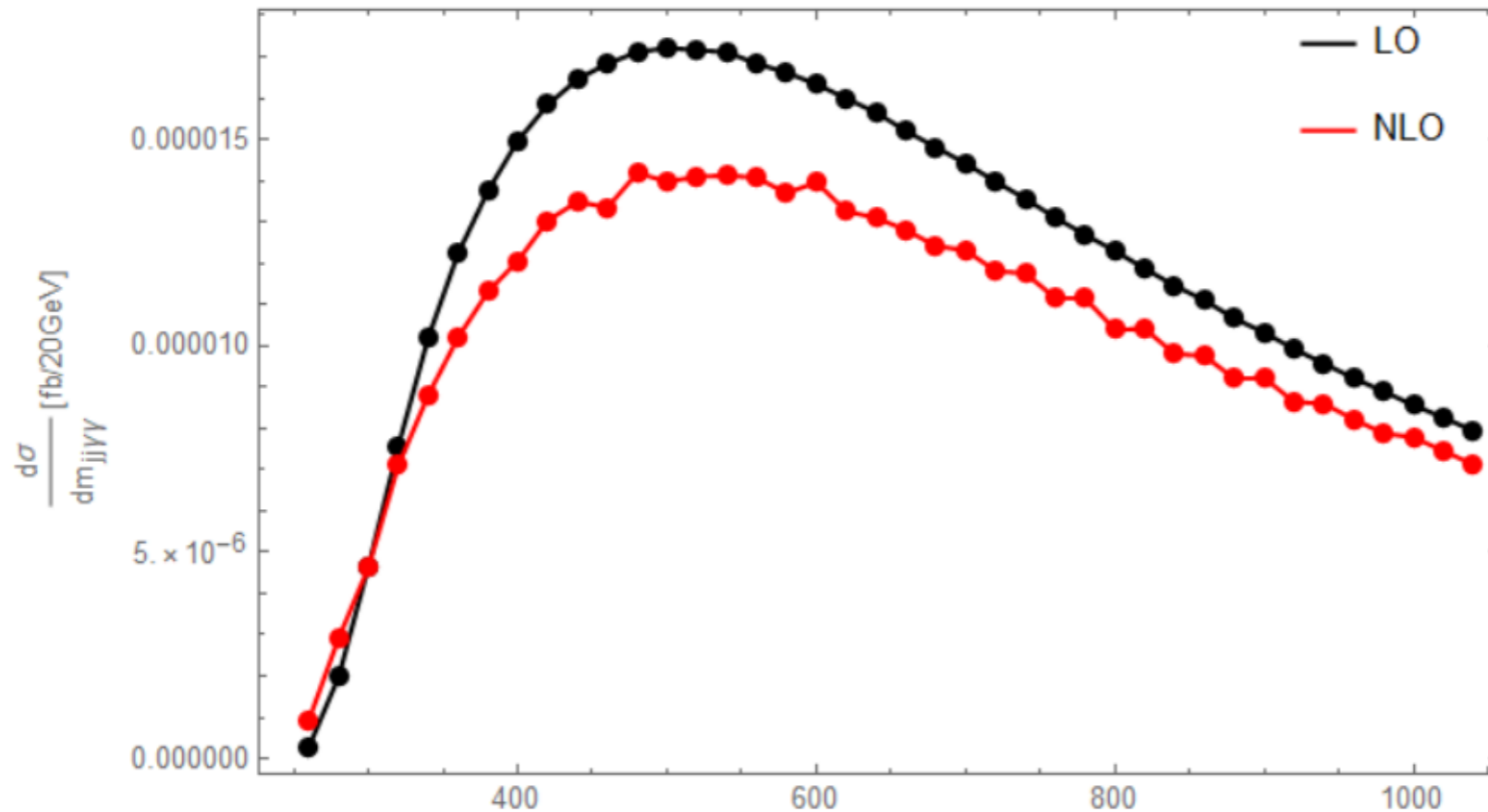


**LO prod. & LO decay vs.  
LO prod. & NLO decay**

$p_j^T, p_\gamma^T > 25$  GeV  
 $|\eta_j|, |\eta_\gamma| < 2.5$   
 $\Delta R_{jj, \gamma j, \gamma\gamma} > 0.4$   
 $90\text{GeV} < m_{jj} < 190\text{GeV}$

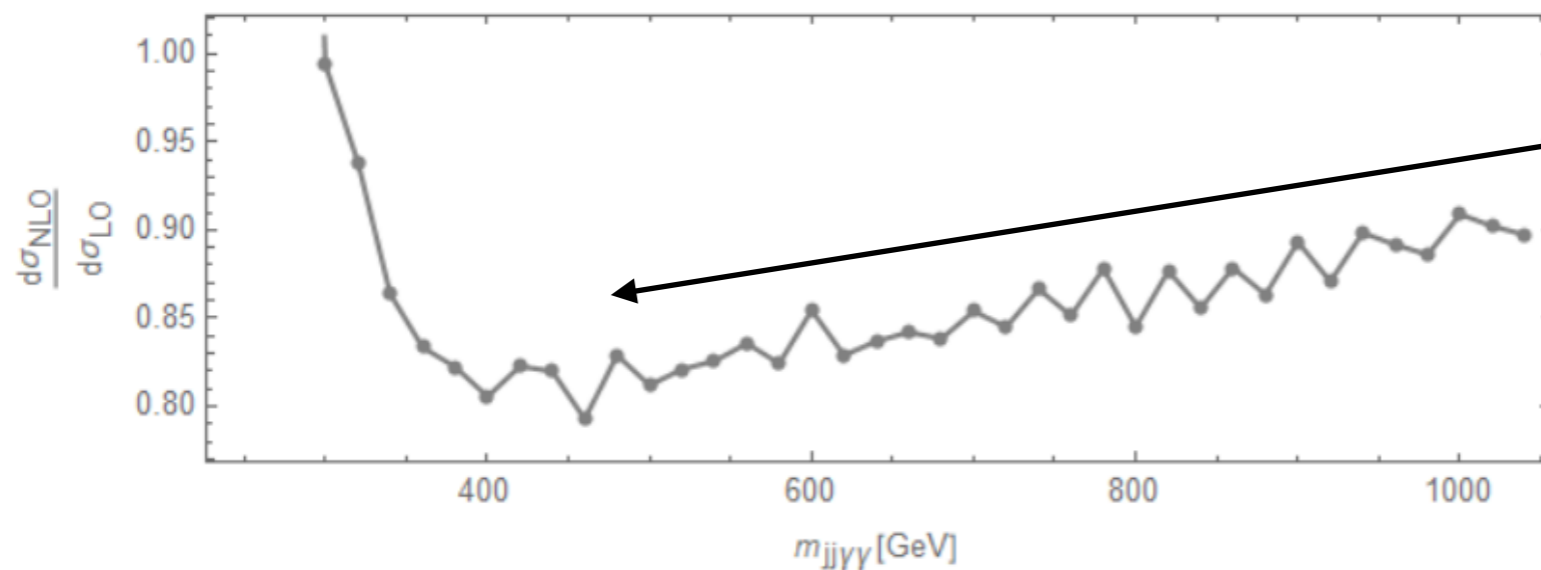
**Up to 20% reduction**

# $gg \rightarrow HH$ including NLO decay



**LO prod. & LO decay vs.  
LO prod. & NLO decay**

$p_j^T, p_\gamma^T > 25 \text{ GeV}$   
 $|\eta_j|, |\eta_\gamma| < 2.5$   
 $\Delta R_{jj,\gamma j,\gamma\gamma} > 0.4$   
 $90\text{GeV} < m_{jj} < 190\text{GeV}$



**Up to 20% reduction**

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# Conclusion

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- Measuring Higgs self-couplings is of great importance in the future.
- Precise theoretical prediction is needed to properly interpret the data.
- The dominant channel  $gg \rightarrow HH$  has been calculated up to NLO/NNLO in the finite/infinite  $m_t$  scheme.
- The cut effects and higher order corrections in decay are also significant, and thus should be considered for a detailed study.



*Thank you!*



# **Back-up slides**

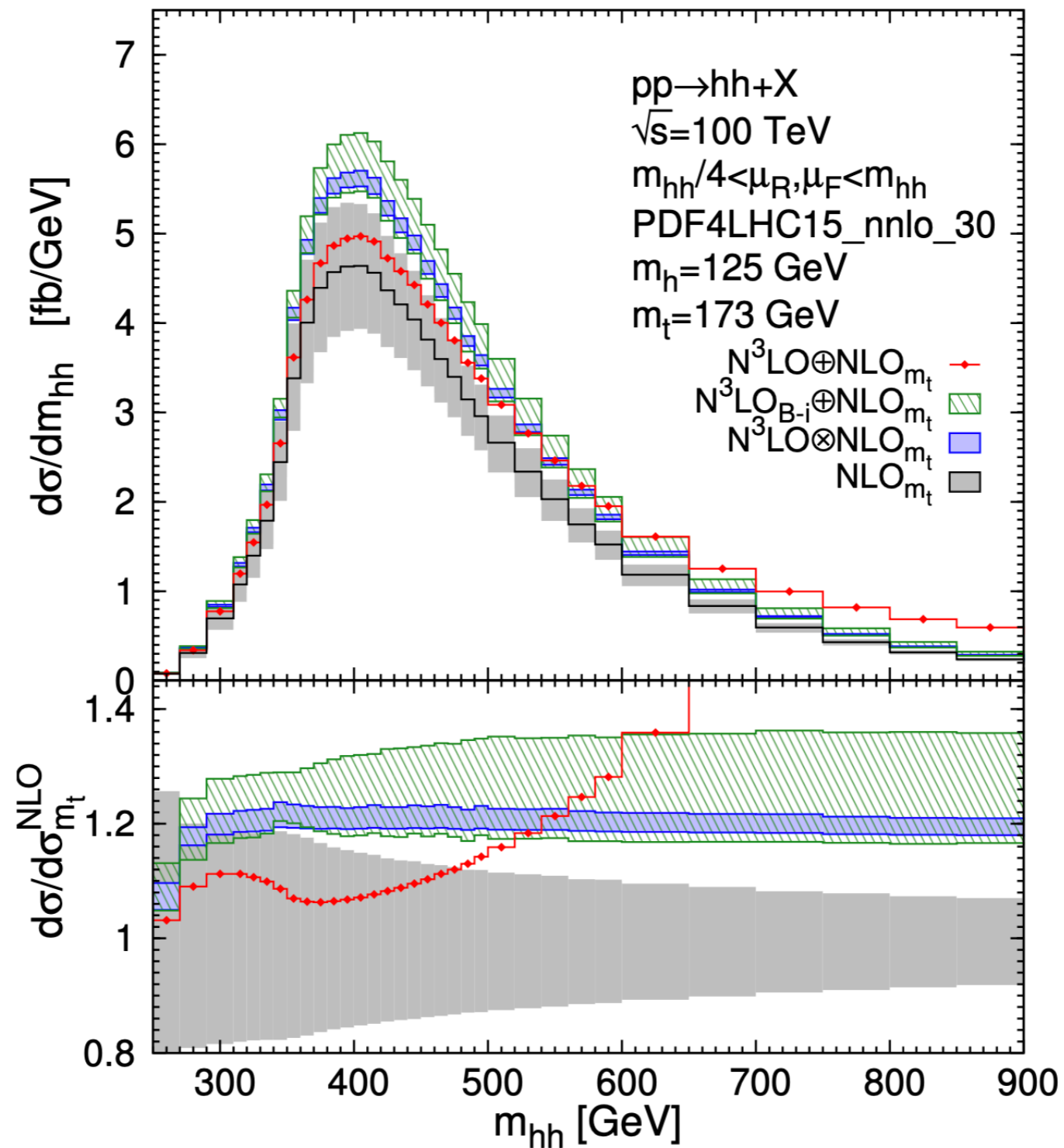
# Top quark mass effects

$$d\sigma^{\mathbf{N}^k \mathbf{LO} \oplus \mathbf{N}^l \mathbf{LO}}_{m_t} = d\sigma^{\mathbf{N}^l \mathbf{LO}}_{m_t} + \Delta\sigma_{m_t \rightarrow \infty}^{k,l}$$

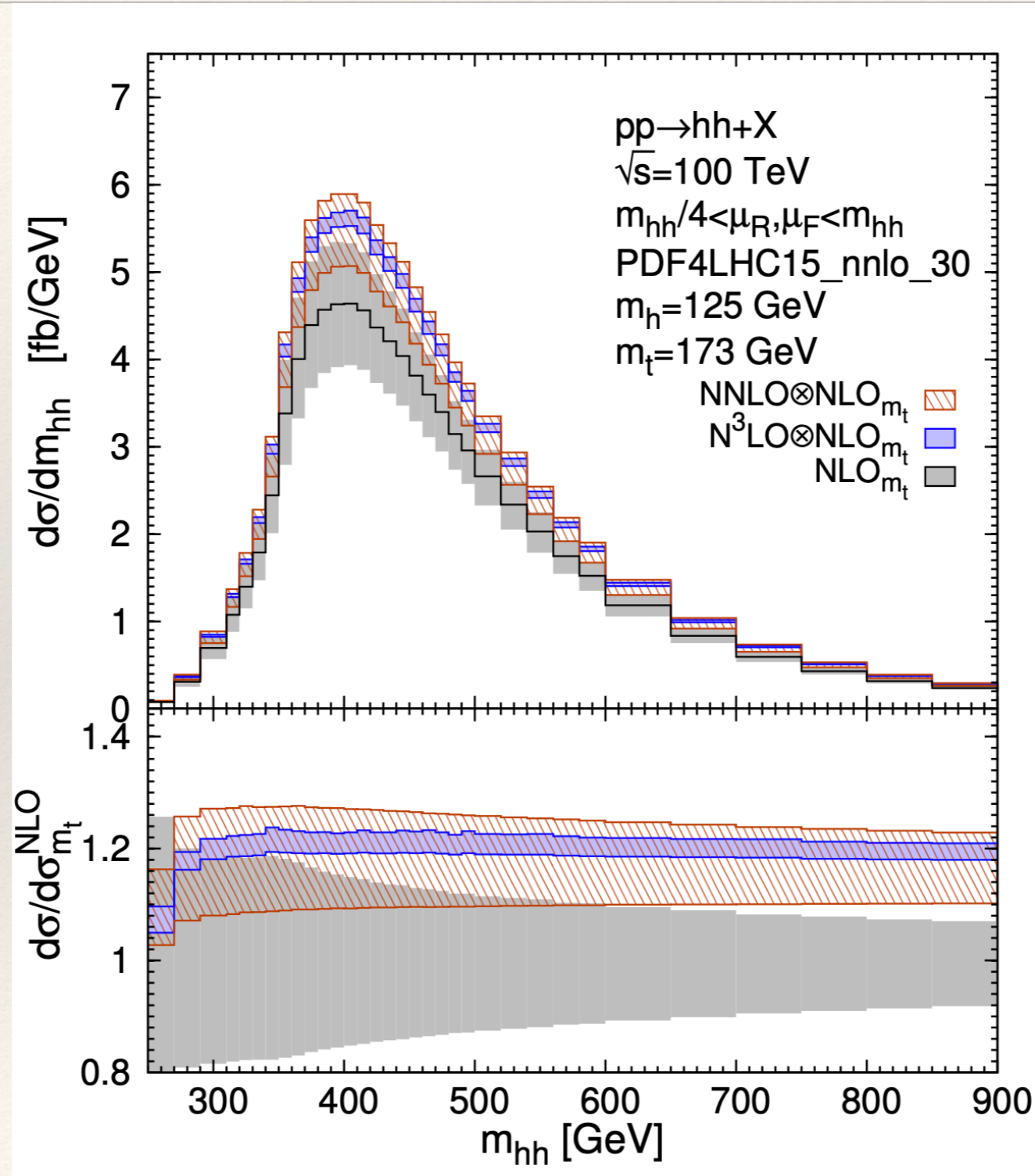
$$d\sigma^{\mathbf{N}^k \mathbf{LO}_{B-i} \oplus \mathbf{N}^l \mathbf{LO}}_{m_t} = d\sigma^{\mathbf{N}^l \mathbf{LO}}_{m_t} + \Delta\sigma_{m_t \rightarrow \infty}^{k,l} \frac{d\sigma^{\mathbf{LO}}_{m_t}}{d\sigma^{\mathbf{LO}}_{m_t \rightarrow \infty}}$$

$$d\sigma^{\mathbf{N}^k \mathbf{LO} \otimes \mathbf{N}^l \mathbf{LO}}_{m_t} = d\sigma^{\mathbf{N}^l \mathbf{LO}}_{m_t} \frac{d\sigma^{\mathbf{N}^k \mathbf{LO}}_{m_t \rightarrow \infty}}{d\sigma^{\mathbf{N}^l \mathbf{LO}}_{m_t \rightarrow \infty}} = d\sigma^{\mathbf{N}^l \mathbf{LO}}_{m_t} + \Delta\sigma_{m_t \rightarrow \infty}^{k,l} \frac{d\sigma^{\mathbf{N}^l \mathbf{LO}}_{m_t}}{d\sigma^{\mathbf{N}^l \mathbf{LO}}_{m_t \rightarrow \infty}}$$

# $gg \rightarrow HH @ \text{NNNLO}$



# $gg \rightarrow HH @ \text{NNNNLO}$



# $gg \rightarrow HH @ \text{NNNNLO}$

