

Nonthermal dark matter production from a light thermal scalar

Shao-Ping Li

Institute of High Energy Physics, Chinese Academy of Sciences

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Outline

- Forbidden decay production of freeze-in DM from a light thermal scalar
- Scattering from forbidden scalar decay diagram
- Relative effects of forbidden decay and scattering

Forbidden Decay for Nonthermal DM Production

- In most situations, the nonthermal DM is produced from decay

$$A \rightarrow B + \text{DM} \quad \text{if} \quad m_A > m_B + m_{\text{DM}}$$

- When A is much lighter than DM, the vacuum decay is kinematically forbidden

$$A \nrightarrow B + \text{DM} \quad \text{if} \quad m_A < m_B + m_{\text{DM}}$$

Forbidden Decay for Nonthermal DM Production

- When A equilibrates in a thermal plasma, there exists a temperature regime for *thermally induced decay, or forbidden decay*

$$A \xrightarrow{\text{FD}} B + \text{DM} \quad \text{if} \quad m_A(T > T_c) > m_B + m_{\text{DM}}$$

↪ As a thermal correction to vacuum decay

$$m_A(T = 0) > m_B + m_{\text{DM}}$$

↪ As a purely thermal production of DM

$$m_A(T = 0) < m_B + m_{\text{DM}}$$

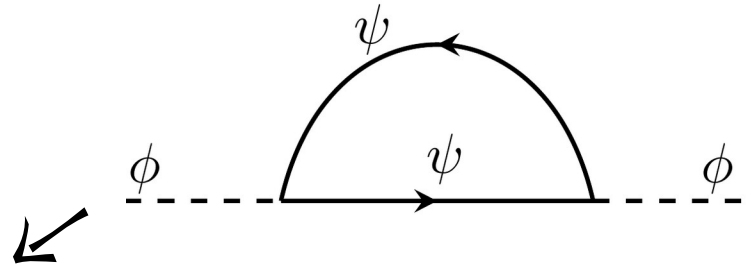
Scattering from Forbidden Decay Diagram

- Usually, scattering is suppressed by higher-order weak couplings and phase-space factors. However, *It is NOT always true in forbidden decay!*
- An example of forbidden scalar decay

$$\mathcal{L} = \underbrace{y_\chi \bar{\chi} \chi \phi}_{\text{DM production}} + \underbrace{y_\psi \bar{\psi} \psi \phi}_{\text{thermalization background}}$$

$$m_\phi(T=0) < 2m_\chi \qquad m_\phi(T > T_c) > 2m_\chi$$

$$\text{Nonthermal DM} \quad m_\chi(T) \ll m_\chi(T=0)$$

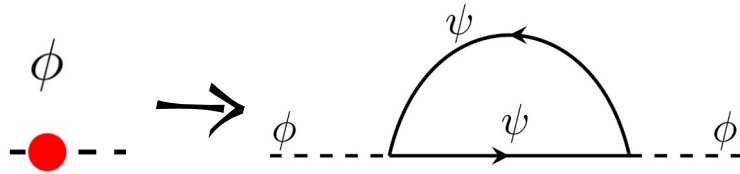


Thermal mass correction at finite T

$$m_\phi \equiv \kappa T$$

Scattering from Forbidden Decay Diagram

$$|\mathcal{M}|_{\phi \rightarrow 2\chi}^2 = \text{[Diagram: } \chi \text{ and } \bar{\chi} \text{ lines meeting at a vertex with } \phi \text{ line, connected by a dashed line with two red dots to another vertex with } \chi \text{ and } \bar{\chi} \text{ lines]} \propto y_\chi^2 \kappa^2$$



$$\hookrightarrow m_\phi^2(T) \sim y_\psi^2 T^2$$

$$\kappa = \mathcal{O}(y_\psi)$$

$$|\mathcal{M}|_{2\psi \rightarrow 2\chi}^2 = \text{[Diagram: } \chi \text{ and } \bar{\chi} \text{ lines meeting at a vertex with } \phi \text{ line, connected by a dashed line with two red dots to a loop of solid lines labeled } \psi \text{ and } \bar{\psi}, \text{ which is then connected to another vertex with } \chi \text{ and } \bar{\chi} \text{ lines]} \propto y_\chi^2 y_\psi^2$$

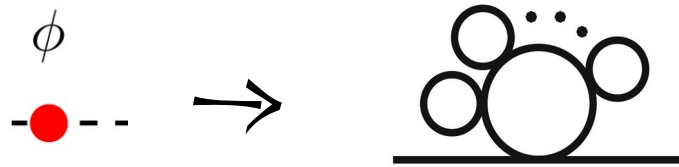
Forbidden decay & scattering coexist, share the same order of couplings

A Special Exception

- If the scalar has only quartic interactions

$$\lambda\phi^4$$

Luc Darmé, *et al*, 1908.05685;
Partha Konar, *et al*, 2110.14411;



One-loop thermal mass correction with daisy rings

$$\curvearrowright m_\phi^2(T) \sim \lambda T^2 \quad \longrightarrow$$

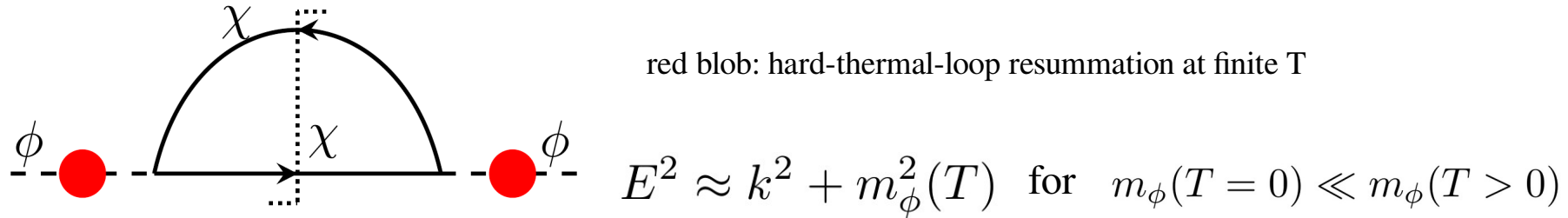
$$|\mathcal{M}|_{\phi \rightarrow 2\chi}^2 \propto m_\phi^2 \sim y_\chi^2 \lambda$$

$$|\mathcal{M}|_{n\phi \rightarrow 2\chi}^2 \sim y_\chi^2 \lambda^2 \quad n=2 \text{ or } 3$$

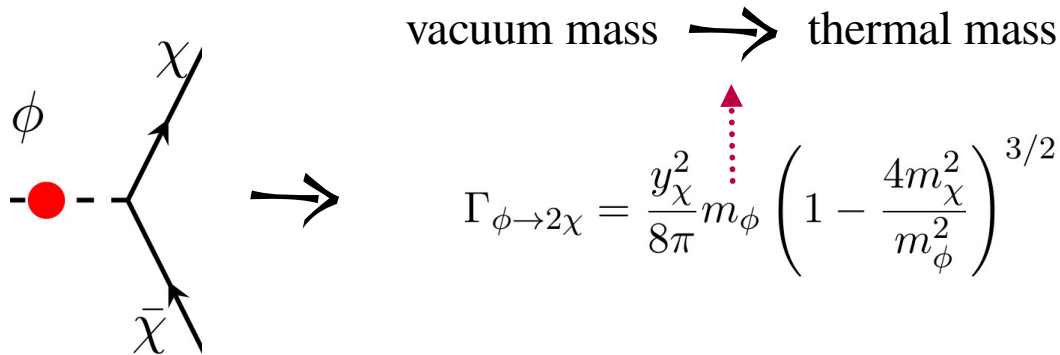
If the quartic coupling is small, the forbidden decay dominates the DM production

Forbidden Scalar Decay Rate

- Method 1: thermal field theory [H.Weldon, PRD, 28 \(1983\) 2007](#)



- Method 2: tree-level amplitude [M. Drewes & J. U. Kang, 1510.05646](#)

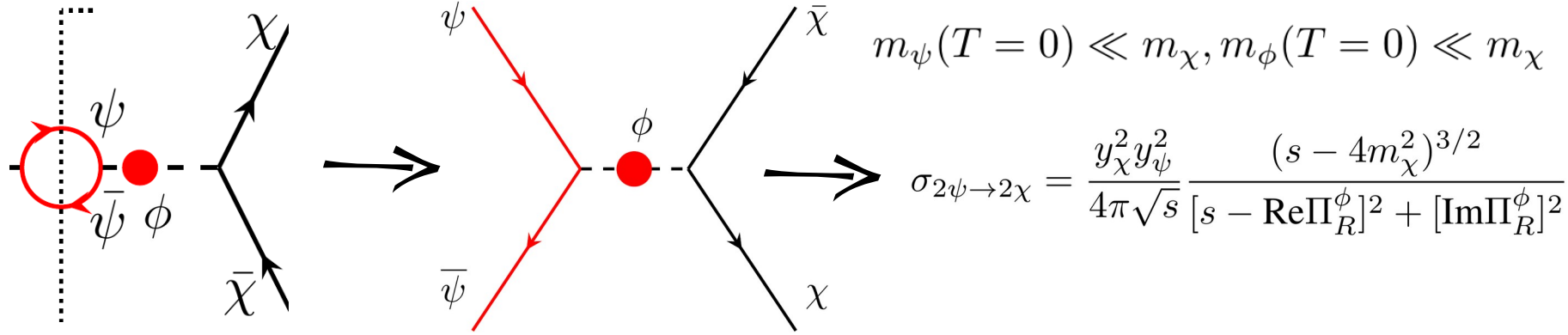


Caution: NOT for fermions!

S.-P. Li, 2301.02835

Scattering Rate

on-shell cut



- If resonance present, include the thermal corrections to off-shell mediator.

Boltzmann Equation for DM Production

$$\dot{n}_\chi + 3Hn_\chi = \underbrace{2\gamma_{\phi \rightarrow 2\chi}}_{\text{forbidden decay}} + \underbrace{2\gamma_{2\psi \rightarrow 2\chi}}_{\text{scattering}}$$

$$\hookrightarrow \gamma_{\phi \rightarrow 2\chi} = \frac{\kappa^3 y_\chi^2 K_1(\kappa)}{16\pi^3} \left(1 - \frac{4m_\chi^2}{\kappa^2 T^2}\right)^{3/2} T^4 \xrightarrow{\text{Vacuum-like dispersion relation}} \propto y_\chi^2 y_\psi^2$$

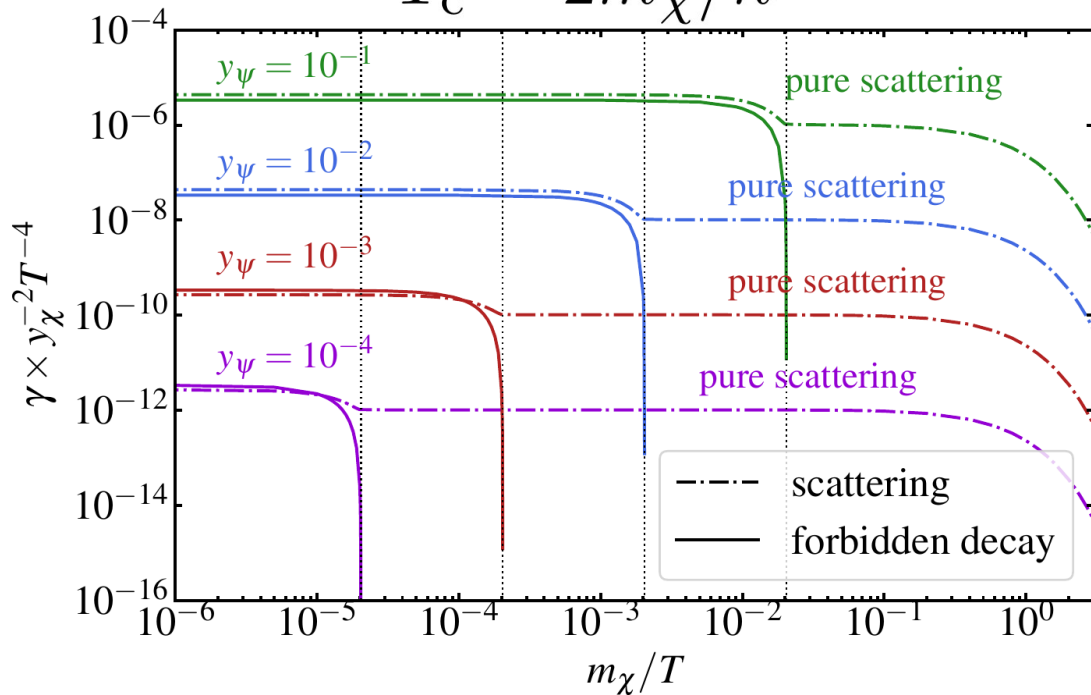
$m_\phi \equiv \kappa T$
 $E^2 - k^2 = \text{Re}_R^\phi, \kappa = y_\psi/6$

$$\hookrightarrow \gamma_{2\psi \rightarrow 2\chi} \approx \frac{T}{32\pi^4} \int_{4m_\chi^2}^{\infty} ds \sigma_{2\psi \rightarrow 2\chi} s^{3/2} K_1(\sqrt{s}/T) \propto y_\chi^2 y_\psi^2$$

- Scattering & forbidden decay share the same order of couplings

Comparison between Scattering and Decay

$$T_c = 2m_\chi / \kappa$$



- At high T

Thermal correction to mediator included, resonant effect compensates suppression of phase-space factors

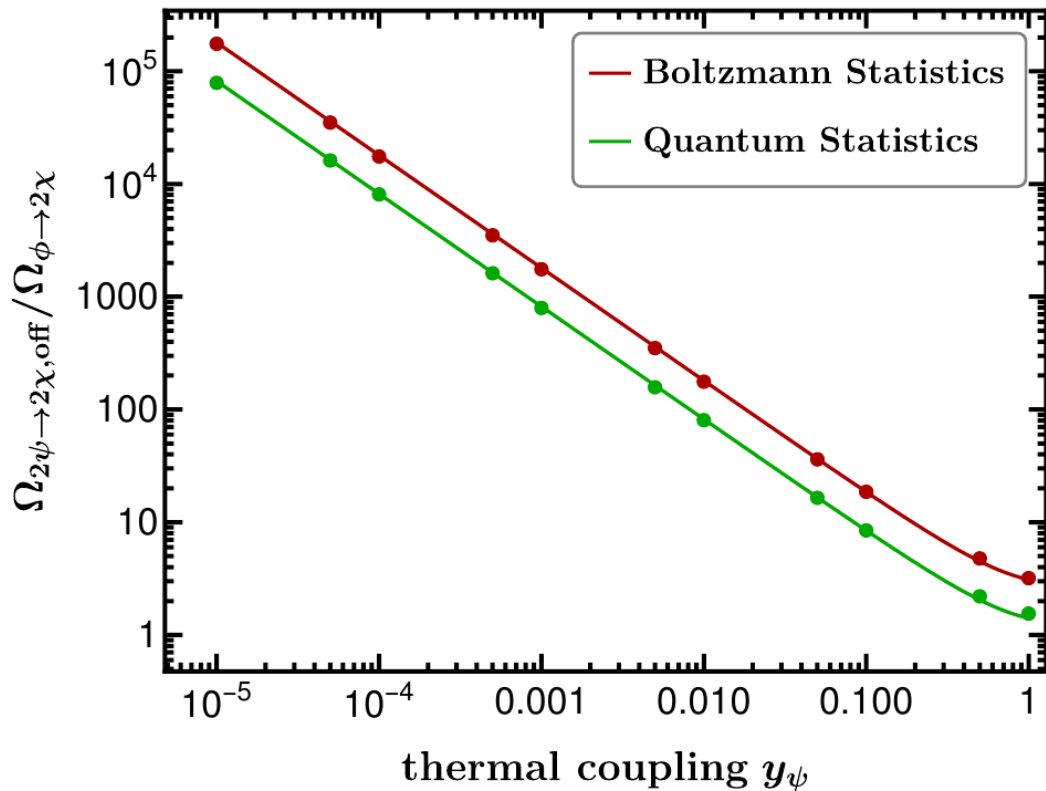
$$\gamma_{2\psi \rightarrow 2\chi} \approx 1.3 \gamma_{\phi \rightarrow \chi}$$

for $y_\psi = 0.1 - 0.01$

- After critical T

Only scattering exists, a longer duration than the forbidden decay

DM Relic Density in Scalar Forbidden Decay



- Fitted formula (Boltzmann)

$$\frac{\Omega_{2\psi \rightarrow 2\chi}}{\Omega_{\phi \rightarrow 2\chi}} \approx 0.8y_\psi + 1.8y_\psi^{-1} + 0.5$$

- For large thermal coupling, both channels are comparable

$$\frac{\Omega_{\text{decay}}}{\Omega_{\text{tot}}} \approx 20\% \quad \text{for} \quad y_\psi = 0.5$$

- For small thermal coupling, the scattering takes over

$$\frac{\Omega_{2\psi \rightarrow 2\chi}}{\Omega_{\phi \rightarrow 2\chi}} \simeq 10^3 \quad \text{for} \quad y_\psi = 10^{-3}$$

Summary

Where to consider forbidden scalar decay?

- ↪ ✓ Heavy DM production from a light scalar mediator

When forbidden scalar decay matters?

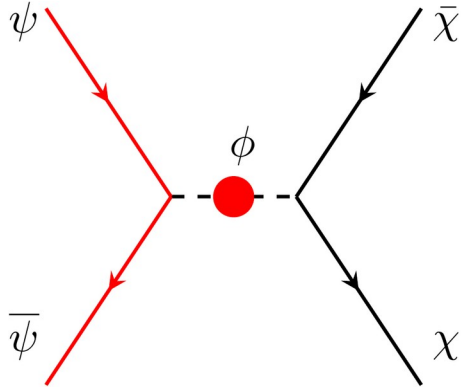
- ↪ ✓ Not-too-weak corrections to the light mediator $\mathcal{O}(0.1 - 1)$
- ✓ Strong quartic self-coupling

How to implement forbidden scalar decay?

- ↪ ✓ Tee-level amplitude with a vacuum-like dispersion (neither for fermions nor possibly for vectors)

Thanks for your attention!

Backup: Scattering Rate w/wo Thermal Propagator



Scalar dispersion relation: $E^2 \approx k^2 + m_\phi^2(T)$ $m_\phi(T) = \frac{y_\psi T}{\sqrt{6}}$

fermion dispersion relation--two modes:

$$\omega_1 \approx k \quad E_\psi^2 - k^2 \approx 2m_\psi^2(T) \quad m_\psi(T) = \frac{y_\psi T}{4}$$

Check if there is an on-shell crossing point $(P_1 + P_2)^2 = s \sim m_\phi^2(T)$

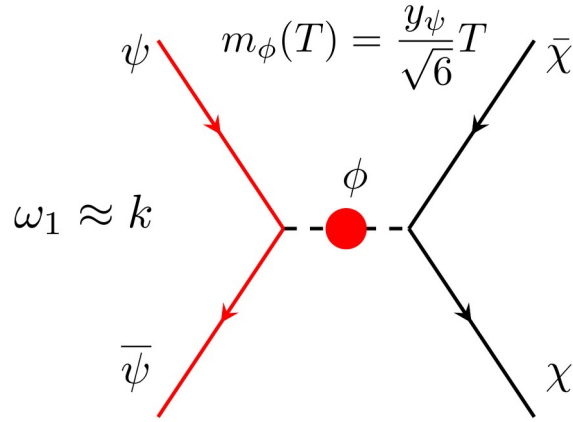


Only the massless mode allows an on-shell crossing in the collinear regime:

$$\theta \equiv \cos^{-1}(\vec{p}_1 \cdot \vec{p}_2 / p_1 p_2) \sim \mathcal{O}(y_\psi)$$

Backup: Scattering Rate w/wo Thermal Propagator

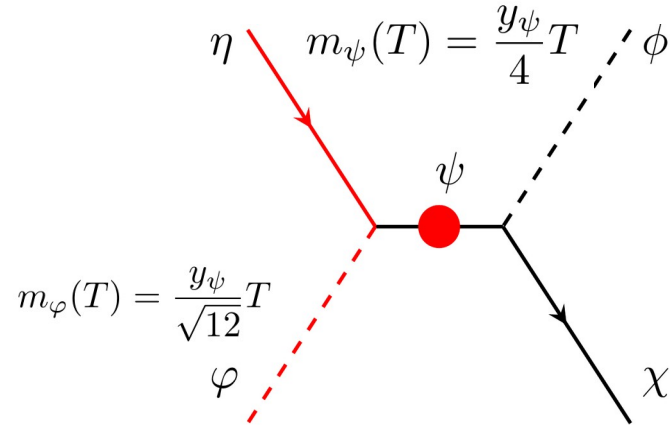
- For scalar mediator



Allow on-shell scalar propagation. Thermal propagator included.

$$\sigma_{2\psi \rightarrow 2\chi} = \frac{y_\chi^2 y_\psi^2}{4\pi\sqrt{s}} \frac{(s - 4m_\chi^2)^{3/2}}{[s - \text{Re}\Pi_R^\phi]^2 + [\text{Im}\Pi_R^\phi]^2}$$

- For fermion mediator



No on-shell fermion propagation. Thermal propagator neglected.

$$\sigma_{\phi\eta \rightarrow \chi\phi} = \frac{y_\chi^2 y_\psi^2}{32\pi s} \left(1 - \frac{m_\phi^2}{s}\right)^2$$