Embedded string in $SU(N) \times U(1)$ Higgs model and its application

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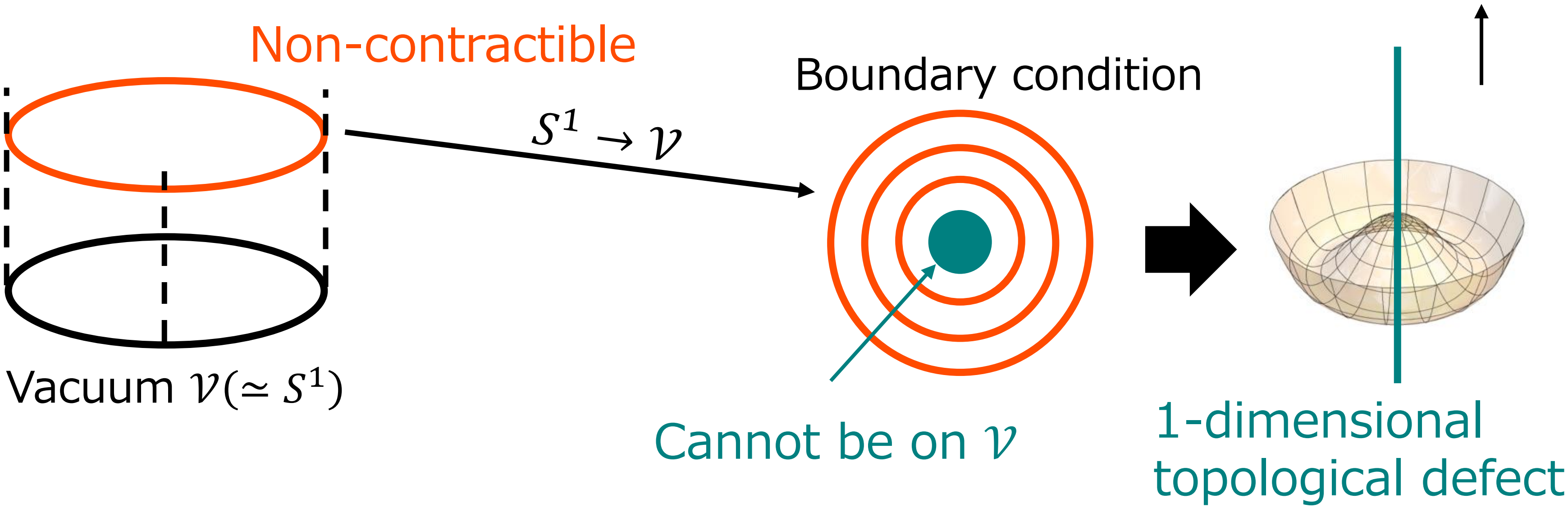
Introduction

Cosmic string is one of powerful tools to probe new physics.

When $U(1) \to\times$, cosmic strings are formed as 1-dimensional topological defects. [Kibble (1976)]



However, cosmic strings are not only topological defects !

= Embedded strings ⟵ Main topics of this talk

Nielsen-Olesen string

In $U(1)$ Higgs model with a potential $V(\phi) = \lambda(|\phi|^2 - v^2)^2$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left|D_\mu\phi\right|^2 - V(\phi) \qquad (D_\mu\phi = (\partial_\mu - igA_\mu)\phi)$$

N-O string solution [Nielsen, Olesen (1973)]

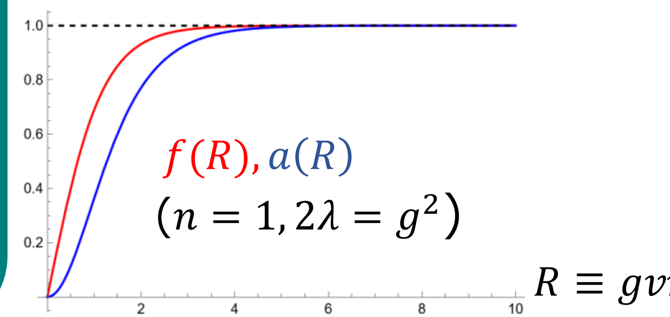
Scalar : $\phi_s(x) = f(r)ve^{in\theta} \qquad (n \in \mathbb{Z})$

Gauge boson : $\vec{A}_s(x) = \frac{na(r)}{gr}\,\vec{e}_\theta,\ A_s^0(x) = 0$

Cylindrical coordinate (r, θ, z)

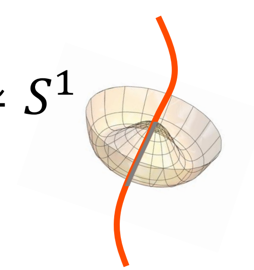
$(f(0) = a(0) = 0, f(\infty) = a(\infty) = 1)$

$f(r)$ and $a(r)$ are derived by solving EoM.



At $r \to \infty$, $\phi = ve^{in\theta}$ ⟷ Non-contractible loops on $U(1) \simeq S^1$

= 1-dimensional topological defect (topological string)



Z string

$$SU(2) \times U(1) \xrightarrow{H:\ (\mathbf{2}, 1/2)} U(1) \quad \text{with } V(H) = \lambda\left(H^\dagger H - v^2\right)^2$$

$$\cup \qquad\qquad\qquad \cup$$

$$U(1) \longrightarrow \times$$

related to the Z boson

Consider Nielsen-Olesen string

$\mathcal{V} \simeq SU(2) \times U(1)/U(1) \simeq S^3$

No non-contractible loops on S^3

$\Rightarrow$ No topological string

Z-string

$$H = \begin{pmatrix} 0 \\ f(r) v e^{i\theta} \end{pmatrix}, \qquad \vec{Z} = -\frac{2z(r)}{\alpha r}\,\vec{e}_\theta, \qquad (\text{others}) = 0$$

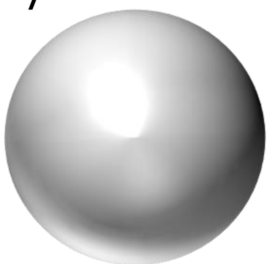
in cylindrical coordinate

$$(f(0) = z(0) = 0, f(\infty) = z(\infty) = 1, \alpha^2 = g_1^2 + g_2^2))$$

Embedded string

When $G \to H$,

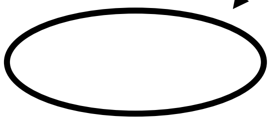
$\mathcal{V} \simeq G/H$



No non-contractible loop...

$\supset$

$\mathcal{V}_{sub} \simeq S^1$



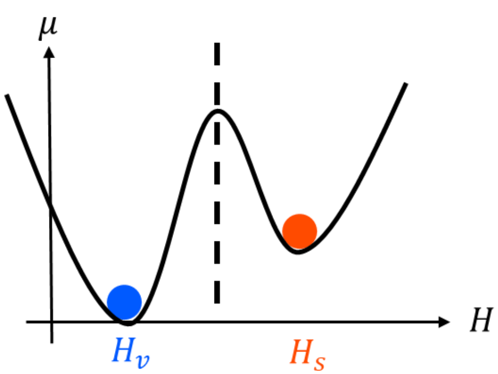
Non contractible loop on $\mathcal{V}_{sub}$

⇩ N-O string of the subsystem

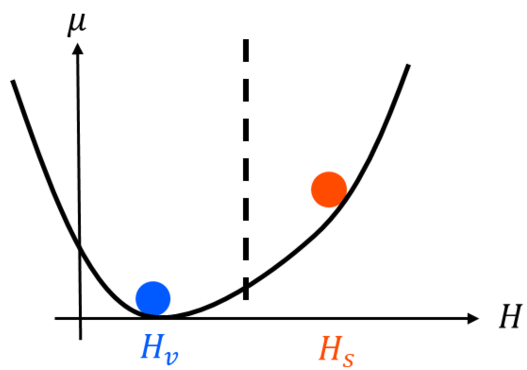
Embedded string

[Vachaspati, Barriola (1992)]
[Vachaspati, Barriola, Bucher (1994)]

However, stability is not topologically guaranteed ...



or



H_v: constant solution
H_s: Z-string solution

Stability of the Z string

[James, Perivolaropoulos, Vachaspati (1993)]

Consider perturbations from the Z-string

$$H = \begin{pmatrix} \phi(x) \\ f(r) v e^{i\theta} + \delta h(x) \end{pmatrix}, \vec{Z} = -\frac{z(r)}{\alpha r}\,\vec{e}_\theta + \delta\vec{Z}(x), \vec{W}^{\pm}(x), \vec{A}(x)$$

↓ Calculate the variations of the energy $\delta\mu$ and find modes decreasing it

Only one mode can destabilize the Z string

$$\delta\mu = \int R dR\ \zeta(R) \mathcal{O}\left(R; \frac{m_H}{m_Z}, \theta_W\right) \zeta(R)$$

$R \equiv \frac{\alpha v}{2} r$, $\tan\theta_W \equiv g_1/g_2$

m_H, m_Z: mass of scalar and Z boson

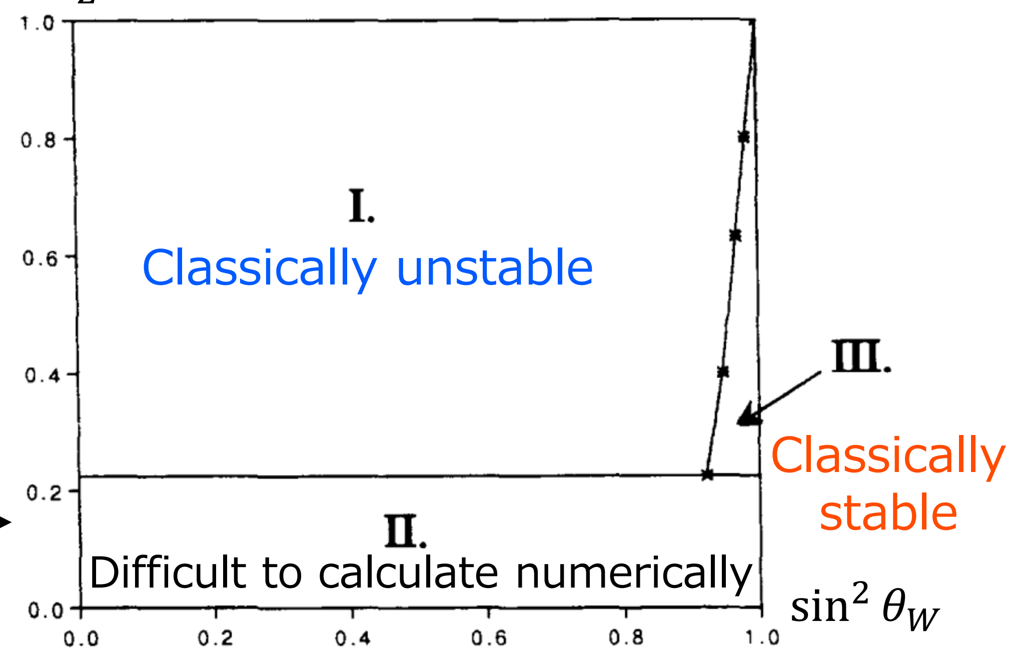
ζ: perturbation mode

→

Standard model

$\frac{m_H}{m_Z}$

Not formed in the SM !



$\sin^2\theta_W$

Motivation of our work

- The region of parameters where embedded strings are formed have been studied only for $SU(2) \times U(1)_X \rightarrow U(1)_Q$
- Nowadays, cosmic strings are important to probe the high energy physics

⇩

How about other symmetry breaking (in BSM)?

Our work

Embedded strings in $SU(N) \times U(1)_X \rightarrow SU(N-1) \times U(1)_Q$

Generalization of the Z string ($N = 2$) "Generalized Z string"

Embedded string in $SU(N)\times U(1)$

We consider $SU(N)\times U(1)_X \xrightarrow{\phi:\left(\boldsymbol{N},\frac{1}{2}\right)} SU(N-1)\times U(1)_Q$

Scalar potential: $V(\phi)=\lambda(|\phi|^2-v^2)^2 \longleftarrow \mathcal{V}\simeq S^{2N-1} \Leftrightarrow$ No non-contractible loop

➡ There is a neutral massive gauge boson $\tilde{Z}_\mu$

$$\tilde{Z}_\mu \equiv \sqrt{\frac{2(N-1)}{N}}\frac{g_N}{\alpha_N}G_\mu^{N^2-1}-\frac{g_1}{\alpha_N}B_\mu$$

G_μ^a, B_μ: $SU(N), U(1)$ gauge bosons

$T^{N^2-1}=\frac{1}{\sqrt{2N(N-1)}}\mathrm{diag}(1,\ldots,1,1-N)$

$\alpha_N^2\equiv\frac{2(N-1)}{N}g_N^2+g_1^2$

⬇ Make an embedded string

Generalized Z-string

$f(0)=z(0)=0, f(\infty)=z(\infty)=1$

$$\phi=\begin{pmatrix}0\\ \vdots\\ 0\\ f(r)ve^{i\theta}\end{pmatrix},\quad \vec{\tilde{Z}}=-\frac{2z(r)}{\alpha_N r}\,\vec{e}_\theta\,,\qquad (\text{others})=0$$

Note that it is the Z-string when $N=2$

Check the stability

Scalar: $\phi(x) = \begin{pmatrix} \phi_1(x) \\ \vdots \\ \phi_{N-1}(x) \\ f(r)e^{in\theta} + \delta\phi(x) \end{pmatrix}$, Gauge boson: $\begin{pmatrix} \vec{G}^a(x) & \vec{G}^+(x) \\ \vec{G}^-(x) & \end{pmatrix}$,

$\vec{\tilde{Z}} = -\frac{z(r)}{\alpha_N r}\vec{e}_\theta + \delta\vec{\tilde{Z}}(x),$

$\vec{\tilde{A}}(x) \equiv \frac{g_1}{\alpha_N}\vec{G}^{N^2-1} + \sqrt{\frac{2(N-1)}{N}}\frac{g_N}{\alpha_N}\vec{B}$

Diagonal part

($:SU(N-1)$ adjoint
$:SU(N-1)$ fundamental
$:SU(N-1)$ singlet)

⇩ Calculate the variations of the energy $\delta\mu$

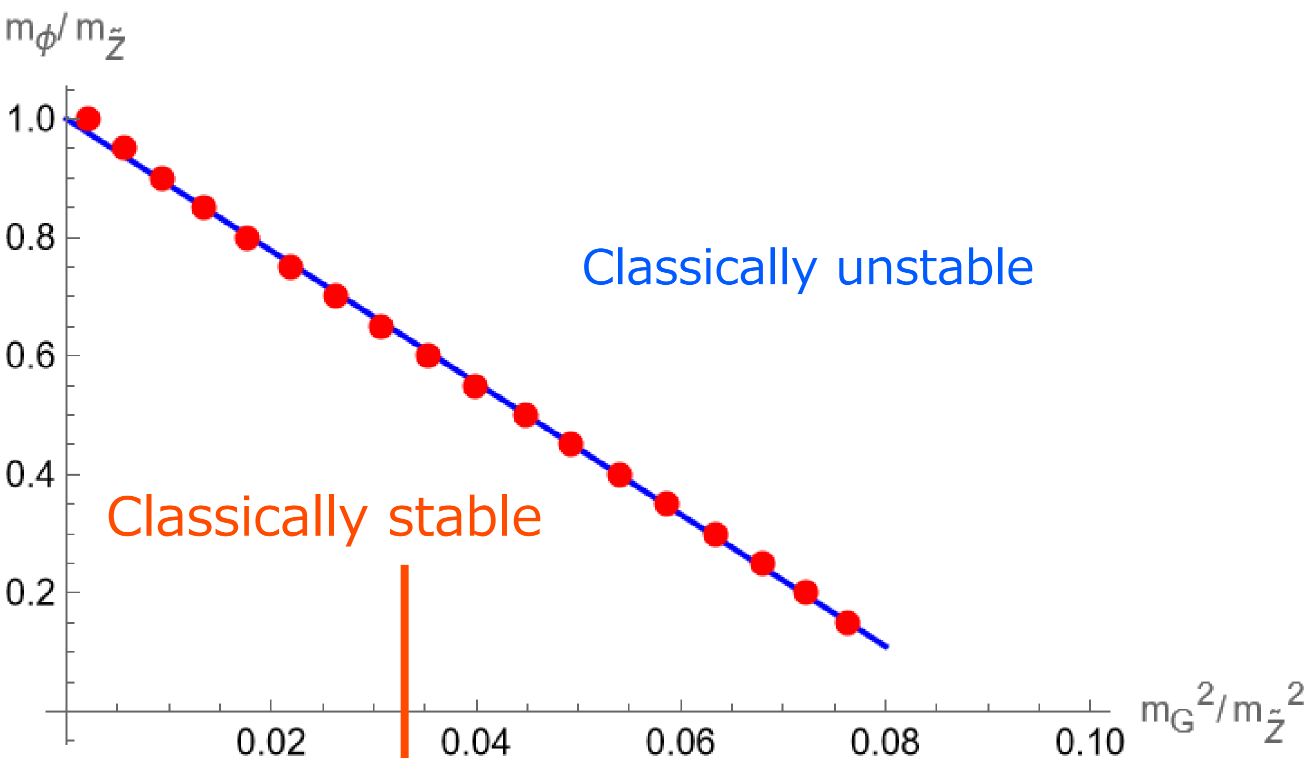
Only fundamental modes can destabilize the generalized Z string.
$\delta\mu$ is divided into $N-1$ parts which are similar to $\delta\mu$ of the Z string.

$$\delta\mu = \sum_{k=1}^{N-1} \delta\mu_k(r; m_\phi/m_{\tilde{Z}}, m_G/m_{\tilde{Z}})$$

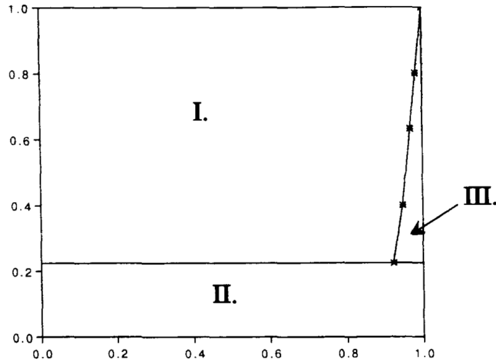
($m_\phi, m_{\tilde{Z}}, m_G$: the mass of scalar, neutral gauge boson, charged gauge boson)

Same as the Z string ! (cf. $m_W/m_Z = \cos\theta_W$)

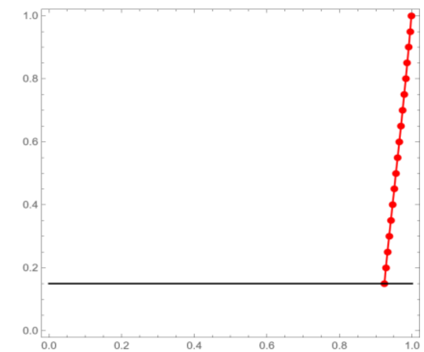
Results



[James, Perivolaropoulos, Vachaspati (1993)]



Ours result



approximately evaluation

$$\frac{m_\phi}{m_{\tilde{Z}}} \leq 1 - 11 \frac{m_G^2}{m_{\tilde{Z}}^2} \Leftrightarrow g_1 \geq \sqrt{\frac{11}{1 - m_\phi / m_{\tilde{Z}}} - \frac{2(N-1)}{N}}\, g_N$$

Application for unification

We consider the case that $SU(N)$ and $U(1)$ have the same origin

$$\phi = (\boldsymbol{N}, q, \boldsymbol{1})\Big|_{g_1{}'} = (\boldsymbol{N}, 1/2, \boldsymbol{1})\Big|_{g_1}$$

$$G \to \cdots \to SU(N) \times U(1) \times H \to SU(N-1) \times U(1) \times H$$

$$g_U = g_N = g_1{}' \xrightarrow{\text{RG running}} g_N = \alpha_{RG}\, g_1{}' = \frac{\alpha_{RG}}{2q} g_1$$

The generalized Z-strings are formed when g_N and g_1 satisfy

$$g_1 \geq \sqrt{\frac{11}{1 - m_\phi/m_{\tilde{Z}}} - \frac{2(N-1)}{N}}\, g_N \quad \Rightarrow \quad q^2 \geq \alpha_{RG}^2 \left[\frac{2.75}{1 - m_\phi/m_{\tilde{Z}}} - \frac{N-1}{2N}\right]$$

Constraint for the rep. of ϕ in G

We apply it for

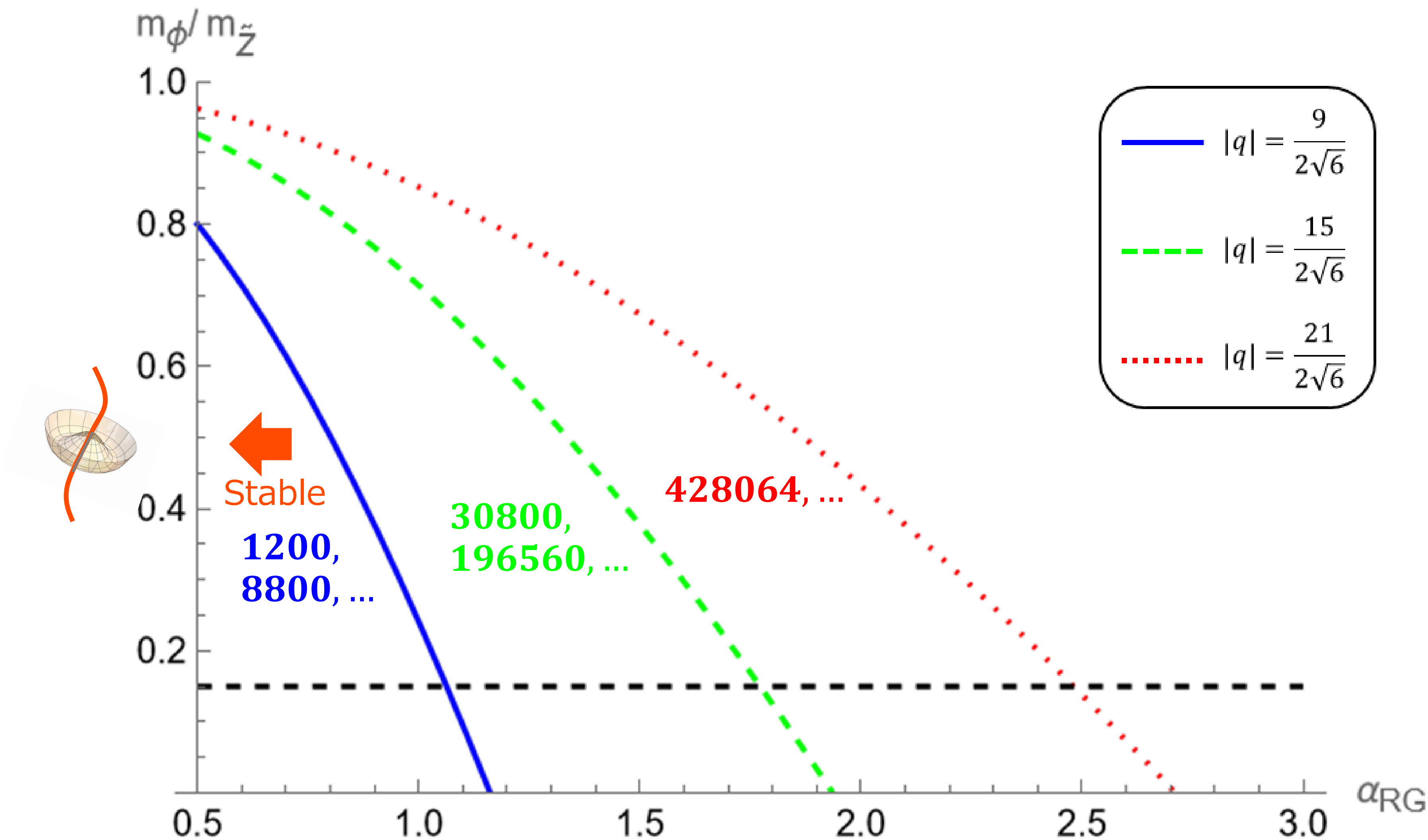
$$\boldsymbol{SO(10) \to SU(3)_C \times SU(2)_L \times \underline{SU(2)_R \times U(1)_X} \to SU(3)_C \times SU(2)_L \times \underline{U(1)_Y}}$$

$$\phi = (\boldsymbol{1}, \boldsymbol{1}, \boldsymbol{2}, q)$$

$$SO(10) \to SU(3)_C \times SU(2)_L \times \underline{SU(2)_R \times U(1)_X} \to SU(3)_C \times SU(2)_L \times \underline{U(1)_Y}$$

($\alpha_{RG} = g_{2R}/g_{1X}$ at the breaking scale)

$\phi = (\mathbf{1}, \mathbf{1}, \mathbf{2}, q)$



Summary

- Embedded strings are not topological defects, but the classical solutions having 1-dimensional excited region (= cosmic string)

- Whether embedded strings are formed or not depends on not only the broken symmetry, but also values of the parameters in models. The condition of the formation have been well-studied only for the $SU(2) \times U(1)$ Higgs model (Z string).

- We have generalized the Z string for the $SU(N) \times U(1)$ Higgs model and found that its stability can be determined the ratios of the masses $(m_\phi / m_{\tilde{Z}}, m_G / m_{\tilde{Z}})$. It is consistent with the results of the Z string.

- We have applied the formation condition to the case that $SU(N)$ and $U(1)$ have the same origin, and found that a higher dimensional scalar is needed for the generalized Z string formation