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Large muon EDM in 2HDM and its extension

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with

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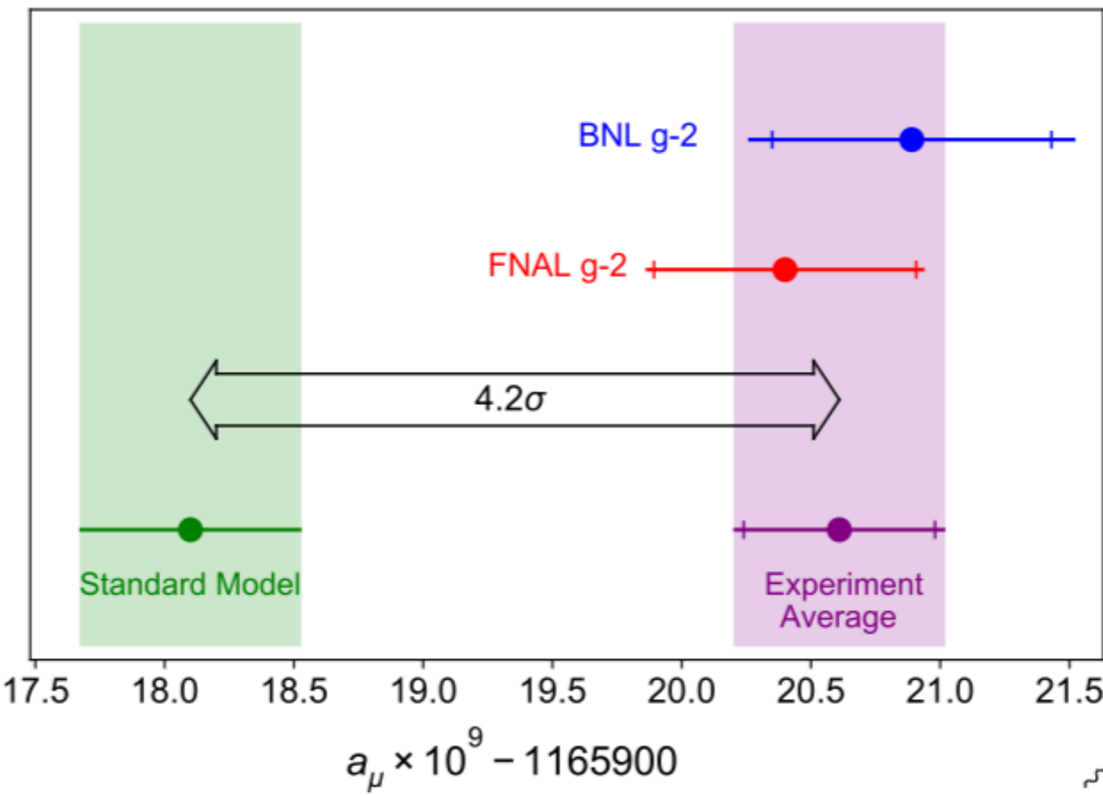
based on PLB831(2022)137194 & JHEP02(2023)234

Higgs as a Probe of New Physics 2023 (HPNP2023), 2023/06/05~09

Introduction

- Muon anomalous magnetic moment (muon *g*-2)

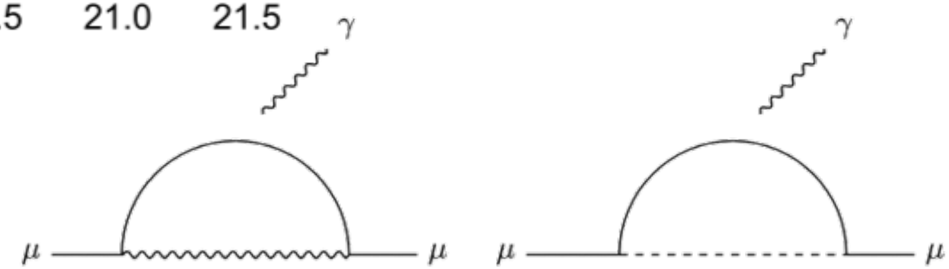
Muon g-2 collab., PRL126(2021)141801



New physics hints

- 2HDM
- MSSM
- U(1) extension
- …

1-loop diagrams



✓ CP conserving process

Introduction

- The same diagrams will have contributions to CP violating processes → electric dipole moment (EDM)

$$\mathcal{H} = -\mu \frac{\mathbf{S}}{|\mathbf{S}|} \cdot \mathbf{B} - d \frac{\mathbf{S}}{|\mathbf{S}|} \cdot \mathbf{E}$$

Relativistic

$$\text{Time reversal}: \ \mathbf{S} \to -\mathbf{S} \quad \mathbf{B} \to -\mathbf{B} \quad \mathbf{E} \to +\mathbf{E}$$

$$\text{Parity}: \ \mathbf{S} \to +\mathbf{S} \quad \mathbf{B} \to +\mathbf{B} \quad \mathbf{E} \to -\mathbf{E}$$

$$\mathcal{L} \supset -a_\mu \frac{e}{4m_\mu} \left(\bar{\mu}\sigma^{\alpha\beta}\mu\right) F_{\alpha\beta} - d_\mu \frac{i}{2} \left(\bar{\mu}\sigma^{\alpha\beta}\gamma_5\mu\right) F_{\alpha\beta}$$

- EDM (d_μ): same type of operator as *g*-2

  if model has CP violating source, EDM is predicted!

- Lepton EDMs constrain models with CPV

Introduction

- Experimental status of EDMs

electron

$$|d_e| < 1.1 \times 10^{-29}\, e\,\mathrm{cm}\ \ (90\%\,\mathrm{C.L.})$$

ACME collab., Nature 562 (2018) 355

$$|d_e| < 4.1 \times 10^{-30}\, e\,\mathrm{cm}\ \ (90\%\,\mathrm{C.L.})$$

JILA, 2212.11841

muon

$$|d_\mu| < 1.8 \times 10^{-19}\, e\,\mathrm{cm}\ \ (95\%\,\mathrm{C.L.})$$

Muon (*g*-2) collab., PRD80(2009)052008

Note: For tau EDM, < $O(10^{-17})$ e cm

Belle collab., PLB551(2003)16

- Indirect bounds

- Minimal flavor violation

NPB292(1987)93, PRL65(1990)2939, PLB500(2001)161, NPB645(2002)155, JHEP08(2014)019

$$|d_\mu| = \left|\frac{m_\mu}{m_e} d_e\right| < 2.3 \times 10^{-27}\, e\,\mathrm{cm}$$

→ Severe constraint for MFV models

- EDM of heavy atoms

Ema, Gao, Pospelov, PRL128(2022)131803, PLB835(2022)137496

$$|d_\mu| < 1.7 \times 10^{-20}\, e\,\mathrm{cm}$$

→ One order smaller than direct bound

Introduction

- Future prospects

	$\lvert d_\mu \rvert$ $[e\,\mathrm{cm}]$	Ref.
Fermilab $(g-2)$ exp.	10^{-21}	[1]
J-PARC	$\mathcal{O}(10^{-21})$	[2]
PSI	6×10^{-23}	[3-5]
J-PARC (dedicated exp.)	10^{-24}	[6]

[1] EPJ Web Conf. 118 (2016) 01005
[2] PTEP2019(2019)5, 053C02
[3] 2102.08838 [hep-ex]
[4] 2201.06561 [hep-ex]
[5] PoS NuFact2021 (2022) 136
[6] PRL93(2004)052001

Muon EDM is also important obs. for NP search!

Note: similar contributions to muon *g*-2 and EDM predict

$$|d_\mu| \sim \frac{e}{2m_\mu}\Delta a_\mu \sim 2.34 \times 10^{-22}\, e\,\mathrm{cm}$$

- What kind of model can predict "large" muon EDM?

Introduction

- Model classification - rough estimate

$\delta_{\rm CPV}$: CPV phase
M : New physics mass scale
λ : Coupling in loop
$y_{\mu\tau}$: LFV coupling
k : Loop level

$$d_\mu \sim \delta_{\rm CPV} \times \begin{cases} \left(\frac{\lambda^2}{16\pi^2}\right)^k \frac{m_\mu}{M^2} & \text{(Spurion)} \\ \frac{y_{\mu\tau}^2}{\lambda^2}\left(\frac{\lambda^2}{16\pi^2}\right)^k \frac{m_\tau}{M^2} & \text{(Flavor changing)} \\ \frac{m_\mu}{M^2} & \text{(Radiative stability)} \\ \lambda\left(\frac{\lambda^2}{16\pi^2}\right)^k \frac{v_H}{M^2} \lesssim \frac{4\pi v}{M^2} & \text{(Tuning, } \lambda \lesssim 4\pi\text{)} \end{cases}$$

NP scale M ⇒

use $\lambda \approx 0.65,\ y_{\mu\tau} \approx 0.3$	1-loop	2-loop
Spurion	300 GeV (75 GeV)	16 GeV (4 GeV)
Flavor changing	580 GeV (140 GeV)	30 GeV (7 GeV)
Radiative stability	5900 GeV (1400 GeV)	
Tuning	1.0×10^6 GeV (2.5×10^5 GeV)	

Large d_μ!

Produce PSI (Fermilab and J-PARC)

Introduction

- We focus on Spurion and radiative stability approaches

Spurion … muon specific two Higgs doublet model (2HDM) for g-2 explanation, JHEP07(2017)012

- ➢ CPV source → scalar potential: $V_\Phi \supset -m_{12}^2 \Phi_1^\dagger \Phi_2 + \frac{\lambda_5}{2}(\Phi_1^\dagger \Phi_2)^2 + \text{h.c.}$
- ➢ Large muon EDM from large tanβ $\sim \mathcal{O}(1000)$

$$\Rightarrow \quad |d_\mu| \sim \mathcal{O}(10^{-23})\, e\,\text{cm}$$

Radiative stability … radiative muon mass model for g-2 explanation, JHEP05(2021)174

- ➢ CPV source → exotic fermion sector: $\mathcal{L} \supset -m_D \bar{\psi}_L \psi_R - \frac{m_{LL}}{2} \bar{\psi}_L \psi_L^c - \frac{m_{RR}}{2} \overline{\psi_R^c} \psi_R + \text{h.c.}$
- ➢ Large muon EDM due to radiative mass

$$d_\mu, m_\mu^{\text{rad}} \propto \frac{y_L y_R}{16\pi^2} \quad \Rightarrow \quad |d_\mu| \sim \mathcal{O}(10^{-22})\, e\,\text{cm}$$

Both models can be explored by future experiments!

Muon specific 2HDM

Ref: Abe, Sato, Yagyu, JHEP07(2017)012

- Model: muon exclusively couples to 1 scalar doublet

	q_L^a	u_R^a	d_R^a	ℓ_L^e	ℓ_L^τ	ℓ_L^μ	e_R	τ_R	μ_R	Φ_1	Φ_2
$SU(3)_C$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$
$U(1)_Y$	$1/6$	$2/3$	$-1/3$	$-1/2$	$-1/2$	$-1/2$	-1	-1	-1	$1/2$	$1/2$
Z_4	1	1	1	1	1	i	1	1	i	-1	1

Z_4 sym.: only muon couples to Φ_1

2 scalar doublets

- Yukawa couplings and scalar potential

$$\mathcal{L}_Y = -\bar{q}_L \widetilde{\Phi}_2 Y_u u_R - \bar{q}_L \Phi_2 Y_d d_R - \sum_{E=e,\tau} y_E \bar{\ell}_L^E \Phi_2 E_R - y_\mu \bar{\ell}_L^\mu \Phi_1 \mu_R + \text{h.c.}$$

$$V_\Phi = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}\right] + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \left[\frac{\lambda_5}{2}\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right]$$

CP sources, relative phase is physical

Muon specific 2HDM

- Muon magnetic and electric dipole moments

g-2

$$\Delta a_\mu \simeq \frac{m_\mu^4}{8\pi^2 v^2}\frac{\Delta m_H^2}{m_H^4} t_\beta^2 c_{2\theta} \log\left(\frac{m_H^2}{m_\mu^2}\right)$$

EDM

$$d_\mu \simeq -\frac{e\, m_\mu^3}{32\pi^2 v^2}\frac{\Delta m_H^2}{m_H^4} t_\beta^2 s_{2\theta} \log\left(\frac{m_H^2}{m_\mu^2}\right)$$

s_θ: mixing between CP-odd and CP-even heavy Higgses

Both enhance large tanβ and $\Delta m_H^2 = m_{H_2}^2 - m_{H_1}^2$

- Dependence of θ

$$\begin{cases} \theta \to 0 \text{ (or } \pi/2\text{)}: \Delta a_\mu \nearrow,\ d_\mu \searrow \text{ (CP conserving limit)} \\ \theta \to \pi/4: \Delta a_\mu \searrow,\ d_\mu \nearrow \text{ (maximal CP violation)} \end{cases}$$

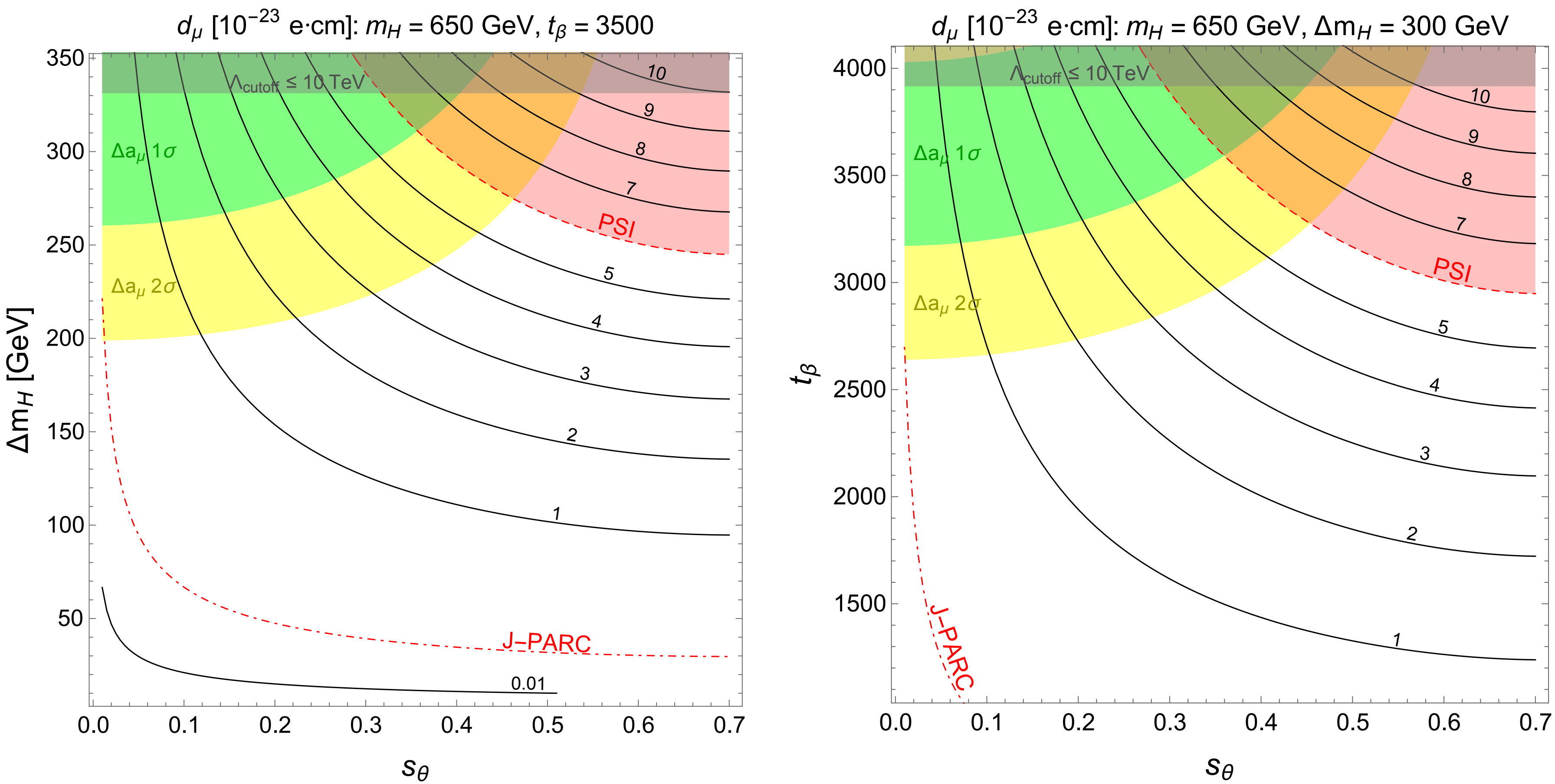
θ → π/8 ($s_\theta \to 0.35$) will be important both for Δa_μ and d_μ

Muon specific 2HDM

Ref: Nakai, Sato, YS, PLB831(2022)137194

Note: $\Delta m_H \equiv \sqrt{m_{H_2}^2 - m_{H_1}^2}$

- Results: tanβ = 3500 (left); Δm_H = 300 GeV (right)



Interesting parameter space: $s_\theta \sim 0.35$

Ref: Nakai, Sato, YS, PLB831(2022)137194

Muon specific 2HDM

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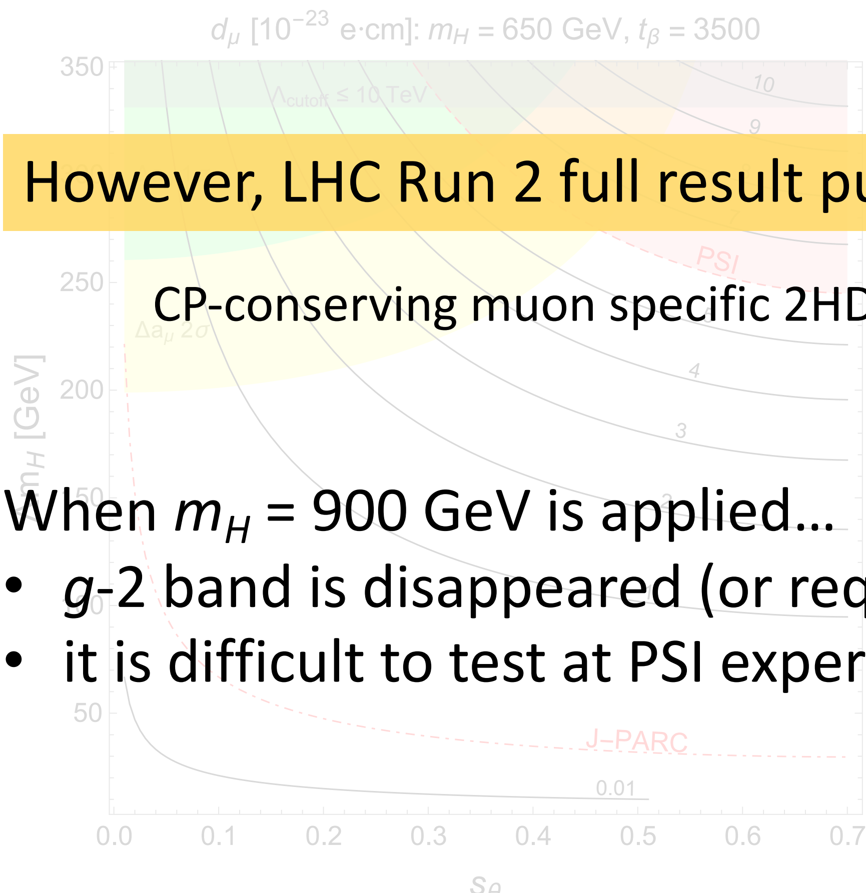
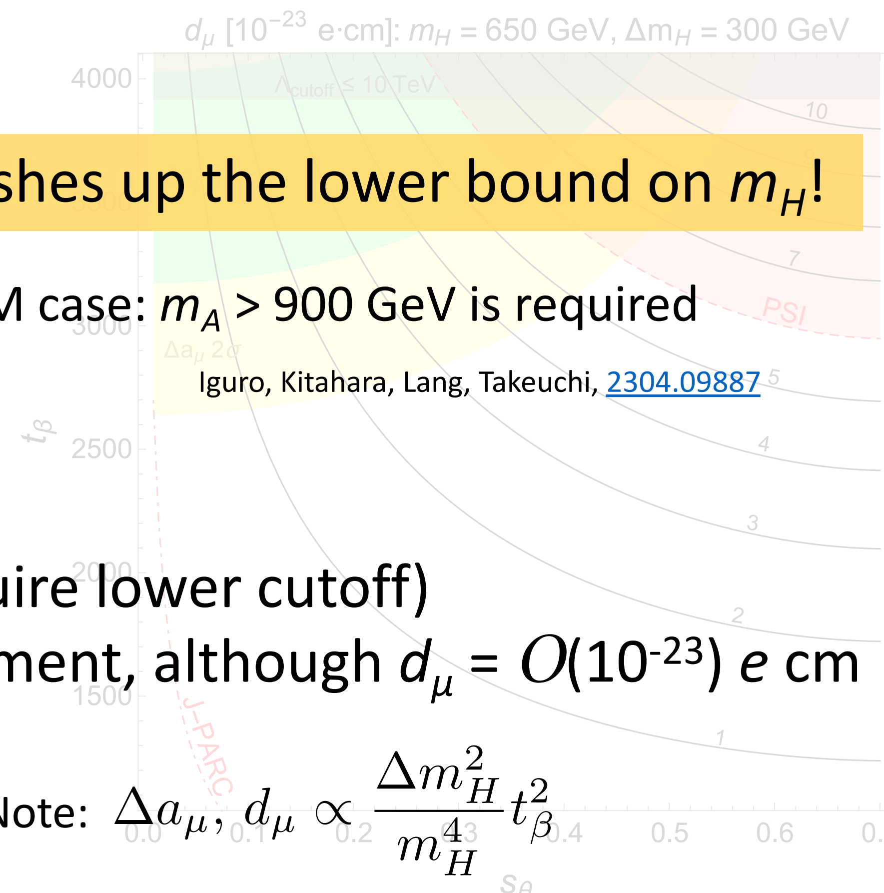




However, LHC Run 2 full result pushes up the lower bound on m_H!

CP-conserving muon specific 2HDM case: m_A > 900 GeV is required

Iguro, Kitahara, Lang, Takeuchi, 2304.09887

When m_H = 900 GeV is applied…

- g-2 band is disappeared (or require lower cutoff)
- it is difficult to test at PSI experiment, although $d_\mu = \mathcal{O}(10^{-23})$ e cm

Note: $\Delta a_\mu, d_\mu \propto \frac{\Delta m_H^2}{m_H^4} t_\beta^2$

Interesting parameter space: $s_\theta \sim 0.35$

Radiative muon mass (Rad-m_μ) model

Ref: M.J. Baker, P. Cox, R.R. Volkas, JHEP05(2021)174

- Model: singlet fermion + singlet scalar + doublet scalar

	L_L^μ	μ_R	H	ψ_L	ψ_R	ϕ	η
$SU(2)_L$	**2**	**1**	**2**	**1**	**1**	**2**	**1**
Y	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1
L_μ	$-$	$-$	$+$	$+$	$+$	$-$	$-$
X	$+$	$+$	$+$	$-$	$-$	$-$	$-$
S_a	$+$	$-$	$+$	$+$	$+$	$+$	$-$

Assign four Z_2 symmetries:

- L_μ … muon number
  → avoid LFV constraints
- X … exotic number
  → DM stability
- S_a … softly broken
  → forbid Higgs Yukawa

- Lagrangian

$$\mathcal{L} \supset \left(-y_\phi \bar{L}_L \phi^\dagger \psi_R - y_\eta \bar{\psi}_L \eta \mu_R - m_D \bar{\psi}_L \psi_R - \frac{m_{LL}}{2} \bar{\psi}_L \psi_L^c - \frac{m_{RR}}{2} \overline{\psi_R^c} \psi_R + \text{h.c.}\right) - V_{\text{scl}}$$

$$V_{\text{scl}} = \sum_{s=H,\phi,\eta} \left[m_s^2 s^\dagger s + \frac{\lambda_s}{2}(s^\dagger s)^2\right] + \lambda_{H\phi}(H^\dagger H)(\phi^\dagger \phi) + \lambda_{H\eta}(H^\dagger H)(\eta^\dagger \eta) + \lambda_{\phi\eta}(\phi^\dagger \phi)(\eta^\dagger \eta)$$

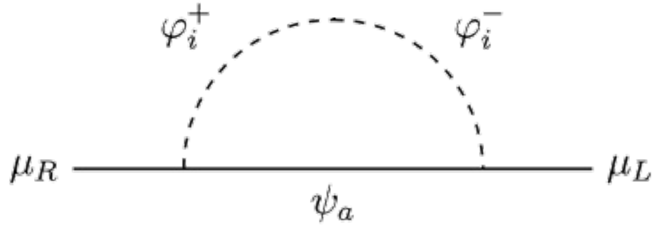
$$+ \lambda'_{H\phi}(H^\dagger \phi)(\phi^\dagger H) + \left(a H \eta^\dagger \phi + \frac{\lambda''_{H\phi}}{2}(H^\dagger \phi)^2 + \text{h.c.}\right)$$

One phase of (m_D, m_{LL}, m_{RR}) cannot removed → CP source of the model

Rad-m_μ model

- Radiative muon mass (with $x_{i,a} \equiv m^2_{\varphi_i^+}/m^2_{\psi_a}$)

$$m_\mu^{\rm rad} = \sum_{i,a} \frac{y_L^{ia} y_R^{ia}}{16\pi^2} m_{\psi_a} \frac{x_{i,a} \ln x_{i,a}}{x_{i,a}-1} \propto \boxed{\frac{y_\phi y_\eta}{16\pi^2} \frac{s_{2\theta} s_{2\alpha}}{4}}$$



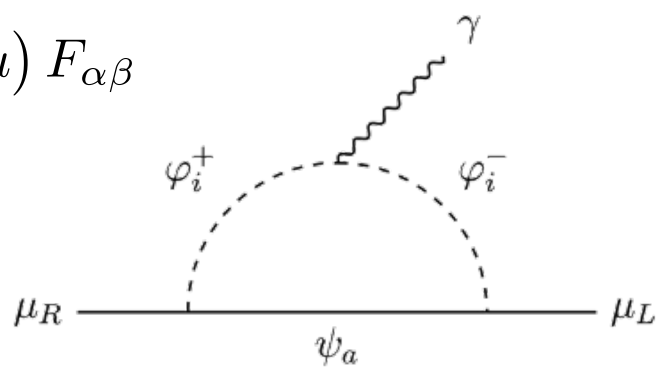
$\psi_{1,2}, \varphi^\pm_{1,2}$: mass eigenstates
α, θ : mixing angles for $\psi_a, \varphi_i^\pm$

- Attach photon → dipole operators:

$$\mathcal{L}_{\rm dipole} = -\frac{e}{4m_\mu} a_\mu \left(\bar{\mu}\sigma^{\alpha\beta}\mu\right) F_{\alpha\beta} - \frac{i}{2} d_\mu \left(\bar{\mu}\sigma^{\alpha\beta}\gamma_5\mu\right) F_{\alpha\beta}$$

⇩

$$a_\mu,\ d_\mu \propto \boxed{\frac{y_\phi y_\eta}{16\pi^2} \frac{s_{2\theta} s_{2\alpha}}{4}}$$



- Loop factor + couplings → m_μ

Enhancement is expected!

Rad-m_μ model

- Constraints on the model

$h \to \mu^+\mu^-$ decay;

$$\kappa_\mu = \sqrt{\frac{\Gamma(h \to \mu^+\mu^-)|_{\rm SM+NP}}{\Gamma(h \to \mu^+\mu^-)|_{\rm SM}}} \simeq 1$$

$Z \to \mu^+\mu^-$ decay

$$\frac{{\rm BR}(Z \to \mu^+\mu^-)}{{\rm BR}(Z \to e^+e^-)} \approx 1$$

⇨ Small NP effects are required

Scalar couplings … unitarity constraints, T-parameter and SM vacuum

trilinear (a) and quartic (λ_i) couplings are constrained

Collider searches for exotic particles

slepton searches can be applied to our model

Dark Matter (DM)

check relic abundance (DM candidate is ψ_1 in our analysis)

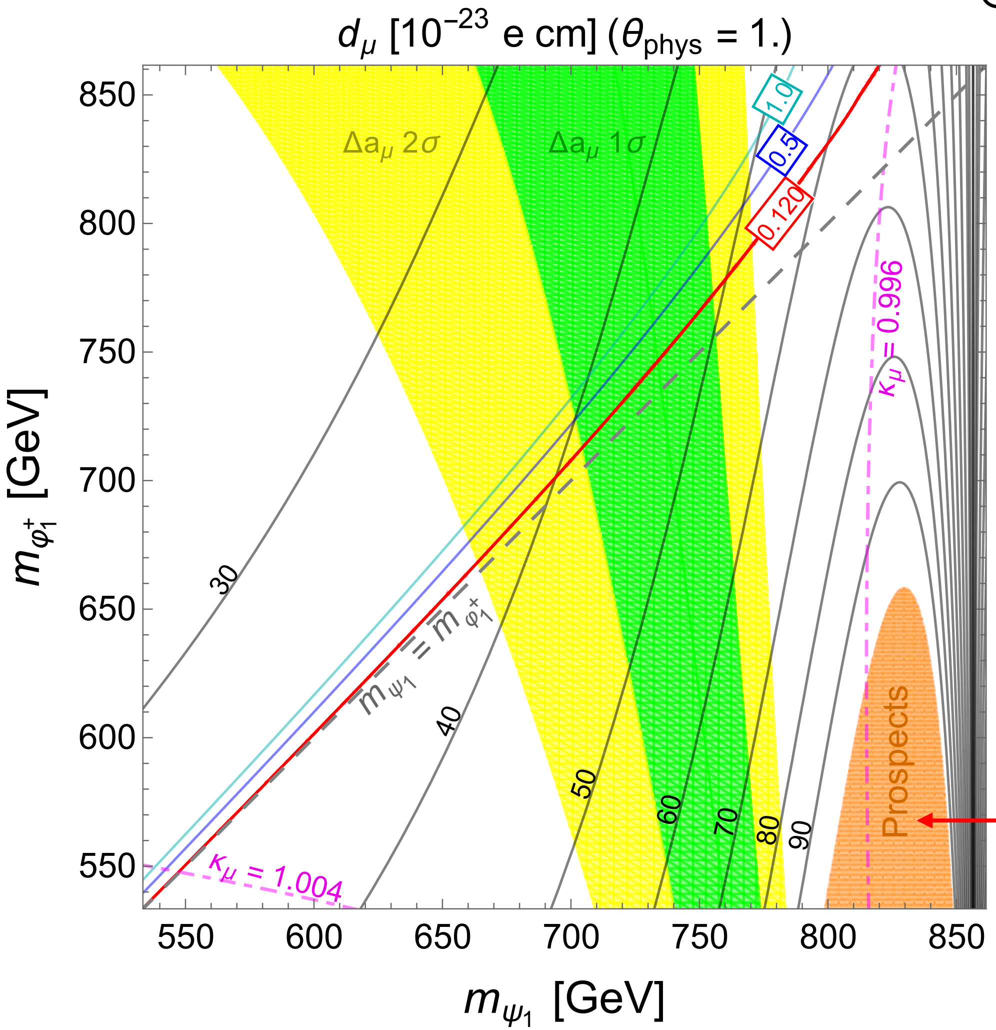
- We explore viable parameter space which is consistent with these constraints and $(g\text{-}2)_\mu$ with large d_μ

Ref: Khaw, Nakai, Sato, YS, Zhang, JHEP02(2023)234

Rad-m_μ model

- Results

Inputs: $\begin{cases} y_\phi = 1.2,\ M_\phi^2 = (1000\,\mathrm{GeV})^2,\ a = 900\,\mathrm{GeV}, \\ m_D = 700\,\mathrm{GeV},\ m_{RR} = 1000\,\mathrm{GeV},\ \theta_{\rm phys} = 1.0 \end{cases}$



- ✓ $d_\mu > 10^{-22}$ e cm!

  Note: PSI sensitivity ~ 6×10^{-23} e cm
- ✓ Δa_μ: Exotic mass of 700 GeV
- ✓ DM relic → coannihilation is required
- ✓ Enough small contributions to $h \to \mu^+\mu^-$ and $Z \to \mu^+\mu^-$

$d_\mu > 10^{-21}$ e cm is predicted

... FNAL, J-PARC prospects

Summary and Discussion

- We explore the possibilities to predict large muon EDM

  focus on muon specific 2HDM and Rad-m_μ model

  Large tanβ ～ 3500 (muon specific 2HDM) — No loop suppression (Rad-m_μ model)

- Predictions:

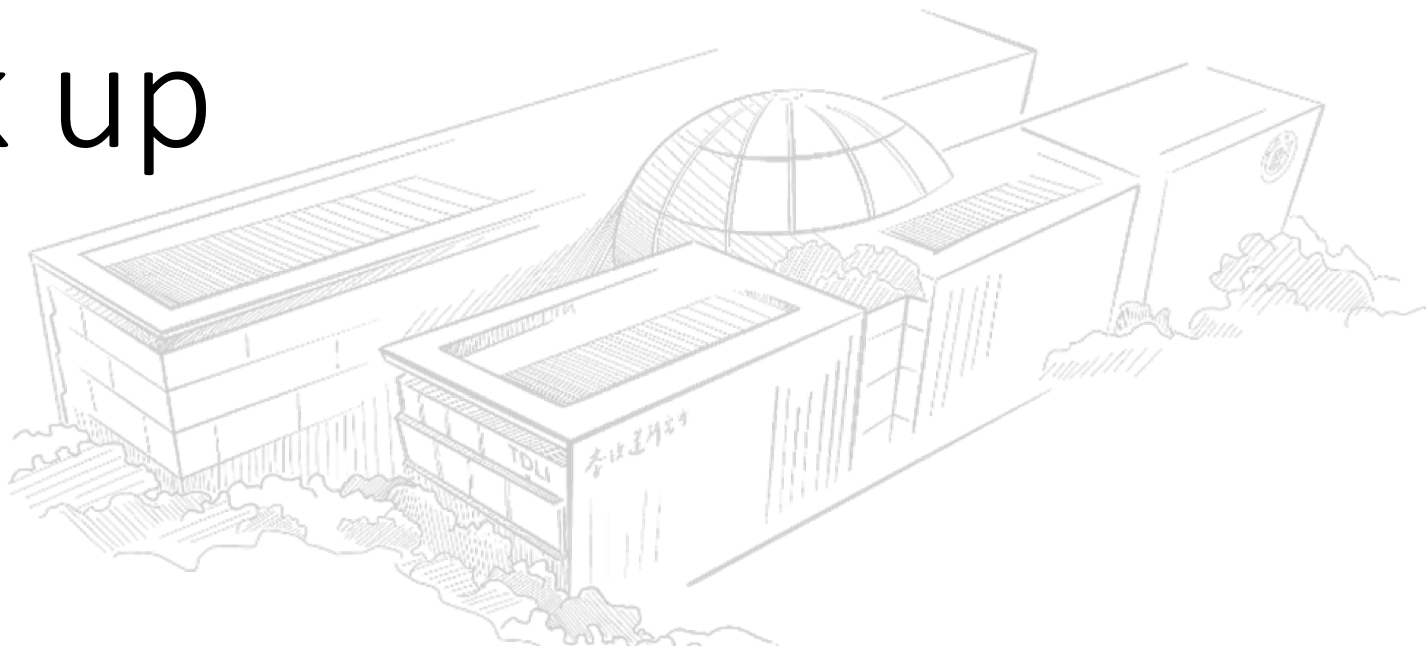
  muon specific 2HDM … $O(10^{-23})$ e cm → constrained by LHC…

  Rad-m_μ model … $O(10^{-22})$ e cm → can be tested by PSI!

- Rad-m_μ model has a parameter space which can be tested by FNAL and J-PRAC experiments ($O(10^{-21})$ e cm)

The muon EDM may have important key for the NP!

Back up

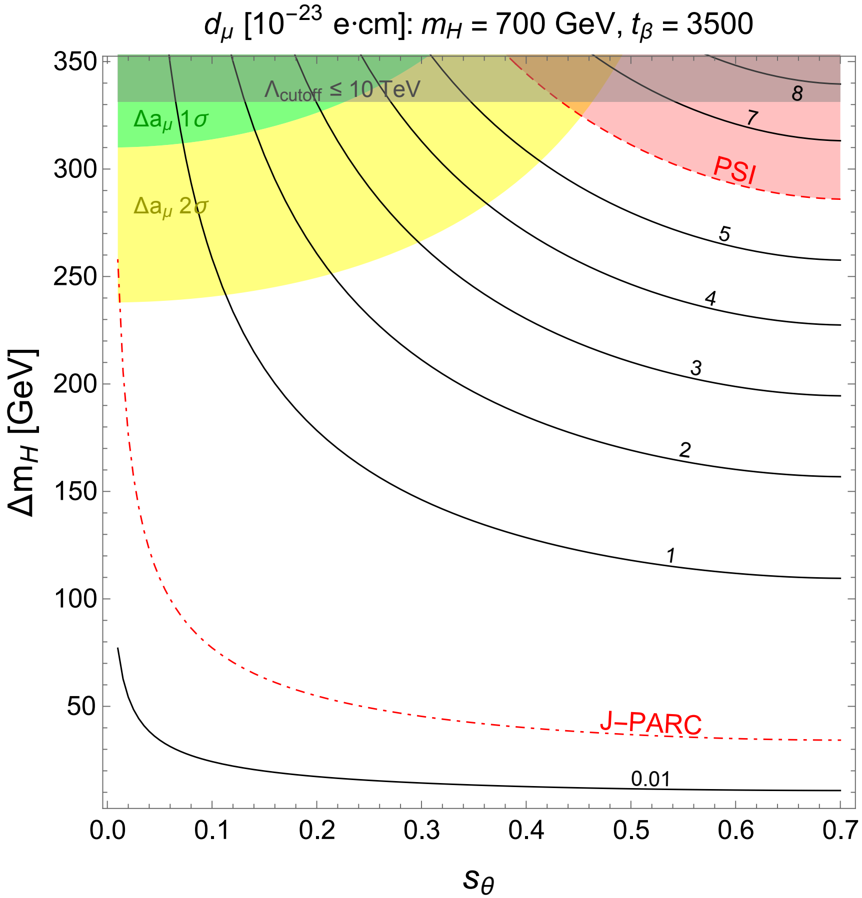


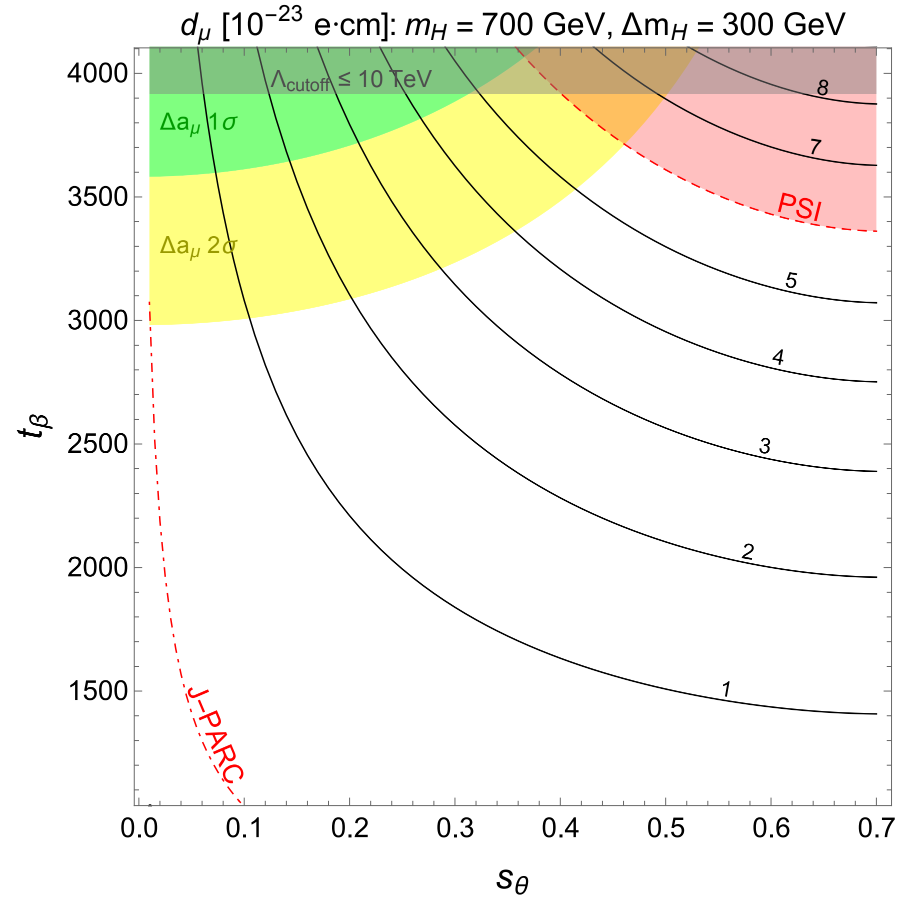
Muon specific 2HDM

Note: $\Delta m_H \equiv \sqrt{m_{H_2}^2 - m_{H_1}^2}$

- Results: tanβ = 3500 (left); Δm_H = 300 GeV (right)

m_H = 700 GeV case





Potential analysis for 2HDM with CPV

$$V_\Phi = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \left[\frac{\lambda_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right]$$

- Mass matrix elements of neutral scalars:

$$\delta_{\tilde{h}\tilde{h}} \equiv \lambda_1 v^2 c_\beta^4 + \lambda_2 v^2 s_\beta^4 + 2\lambda_{345} v^2 s_\beta^2 c_\beta^2 \, ,$$

$$\langle \Phi_\alpha^0 \rangle = v_\alpha, \tan\beta = v_2/v_1, v^2 = v_1^2 + v_2^2$$
$$\lambda_{345} \equiv \lambda_3 + \lambda_4 + \text{Re}\lambda_5$$

$$\delta_{\tilde{h}\tilde{H}} \equiv -(\lambda_1 - \lambda_{345}) v^2 s_\beta c_\beta^3 + (\lambda_2 - \lambda_{345}) v^2 s_\beta^3 c_\beta \, ,$$

$$\delta_{\tilde{H}\tilde{H}} \equiv (\lambda_1 + \lambda_2 - 2\lambda_{345}) v^2 s_\beta^2 c_\beta^2 \, ,$$

$$\delta_{\tilde{h}\tilde{A}} \equiv -\text{Im}\lambda_5 v^2 s_\beta c_\beta \, ,$$

$$\delta_{\tilde{H}\tilde{A}} \equiv \frac{1}{2}(-c_\beta^2 + s_\beta^2) \text{Im}\lambda_5 v^2 \, ,$$

→ 0 for CP conserving models

$\delta_{\tilde{A}\tilde{A}} \equiv -\text{Re}\lambda_5 v^2 \, .$ → CP-odd scalar mass for CP conserving models

Potential analysis for 2HDM with CPV

$$V_\Phi = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \left[\frac{\lambda_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right]$$

- We consider $M^2 > \delta \sim v^2$

mass matrix is approximately diagonalized:

$R^T \widetilde{\mathcal{M}}^2 R = \text{diag}(m_h^2, m_{H_1}^2, m_{H_2}^2, 0)$ → diagonalizing matrix: $R \equiv R_2 R_3$

$$R_2 \simeq \begin{pmatrix} 1 & \delta_{\tilde{h}\tilde{H}}/M^2 & \delta_{\tilde{h}\tilde{A}}/M^2 & 0 \\ -\delta_{\tilde{h}\tilde{H}}/M^2 & 1 & 0 & 0 \\ -\delta_{\tilde{h}\tilde{A}}/M^2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad R_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\theta & s_\theta & 0 \\ 0 & -s_\theta & c_\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Mass eigenvalues:
$$\begin{cases} m_h^2 = \delta_{\tilde{h}\tilde{h}} + \mathcal{O}(1/M^2)\,, \\ m_{H_1}^2 = M^2 + \delta_{\tilde{H}\tilde{H}} c_\theta^2 - 2\delta_{\tilde{H}\tilde{A}} s_\theta c_\theta + \delta_{\tilde{A}\tilde{A}} s_\theta^2 + \mathcal{O}(1/M^2)\,, \\ m_{H_2}^2 = M^2 + \delta_{\tilde{H}\tilde{H}} s_\theta^2 + 2\delta_{\tilde{H}\tilde{A}} s_\theta c_\theta + \delta_{\tilde{A}\tilde{A}} c_\theta^2 + \mathcal{O}(1/M^2)\,. \end{cases}$$

Potential analysis for 2HDM with CPV

$$V_\Phi = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}\right] + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \left[\frac{\lambda_5}{2}\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right]$$

- Expressions for λ_i in $M^2 > \delta$ and large $\tan\beta$ limit

$$\lambda_1 v^2 \simeq m_h^2 + \left(m_H^2 + s_\theta^2 \Delta m_H^2 - M^2\right) t_\beta^2 - 2\delta_{\tilde{h}\tilde{H}} t_\beta \,,$$

$$\lambda_2 v^2 \simeq m_h^2 + \left(m_H^2 + s_\theta^2 \Delta m_H^2 - M^2\right) t_\beta^{-2} + 2\delta_{\tilde{h}\tilde{H}} t_\beta^{-1} \,,$$

$$\lambda_3 v^2 \simeq m_h^2 - m_H^2 - s_\theta^2 \Delta m_H^2 - M^2 + 2m_{H\pm}^2 + \delta_{\tilde{h}\tilde{H}} \left(t_\beta^{-1} - t_\beta\right) \,,$$

$$\lambda_4 v^2 \simeq M^2 + m_H^2 + c_\theta^2 \Delta m_H^2 - 2m_{H\pm}^2 \,,$$

$$\mathrm{Re}\lambda_5 v^2 \simeq M^2 - m_H^2 - c_\theta^2 \Delta m_H^2 \,,$$

$$\mathrm{Im}\lambda_5 v^2 \simeq -\frac{s_{2\theta}}{c_\beta^2 - s_\beta^2} \Delta m_H^2 \,.$$

M^2 is chosen so that λ_1 becomes $O(1)$ in large $\tan\beta$ limit

Potential analysis for 2HDM with CPV

$$V_\Phi = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \left[\frac{\lambda_5}{2} \left(\Phi_1^\dagger \Phi_2 \right)^2 + \text{h.c.} \right]$$

- Theoretical conditions

Vacuum stability

$$\lambda_1 > 0\,, \quad \sqrt{\lambda_1 \lambda_2} + \lambda_3 > 0\,,$$
$$\lambda_2 > 0\,, \quad \sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 - |\lambda_5| > 0\,.$$

Perturbative unitarity

$$\left| \frac{3(\lambda_1 + \lambda_2) \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + 4(2\lambda_3 + \lambda_4)^2}}{2} \right| < 8\pi\,, \quad \left| \lambda_3 + 2\lambda_4 \pm |\lambda_5| \right| < 8\pi\,,$$
$$\left| \frac{(\lambda_1 + \lambda_2) \pm \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_4^2}}{2} \right| < 8\pi\,, \quad \left| \lambda_3 \pm \lambda_4 \right| < 8\pi\,,$$
$$\left| \frac{(\lambda_1 + \lambda_2) \pm \sqrt{(\lambda_1 - \lambda_2)^2 + 4|\lambda_5|^2}}{2} \right| < 8\pi\,, \quad \left| \lambda_3 \pm |\lambda_5| \right| < 8\pi\,.$$

Potential analysis for 2HDM with CPV

- Higgs quartic couplings → related to theoretical conditions

Parameterize: $\frac{\mathrm{Re}\, m_{12}^2}{s_\beta c_\beta} = m_H^2 + s_\theta^2 \Delta m_H^2 - 2\frac{\delta_{\tilde{h}\tilde{H}}}{t_\beta} - v^2 \frac{X}{t_\beta^2}$

$$\lambda_1 v^2 \simeq m_h^2 + X v^2$$

$$\lambda_2 v^2 \simeq m_h^2$$

$$\lambda_3 v^2 \simeq m_h^2 - 2 s_\theta^2 \Delta m_H^2 + 2\Delta m_\pm^2 - \delta_{\tilde{h}\tilde{H}} t_\beta$$

$$\lambda_4 v^2 \simeq \Delta m_H^2 - 2\Delta m_\pm^2$$

$$\mathrm{Re}\lambda_5 v^2 \simeq -c_{2\theta} \Delta m_H^2$$

$$\mathrm{Im}\lambda_5 v^2 \simeq s_{2\theta} \Delta m_H^2$$

θ = 0 → CP conserving limit

Roughly, required to be:

$$X \sim \mathcal{O}(1)$$

$$\delta_{hH} \sim \mathcal{O}(v^2/t_\beta)$$

$$\Delta m_H^2 \sim v^2$$

$$\Delta m_\pm^2 \sim v^2$$

Scalar masses:

$$\begin{cases} m_h^2, m_{H_1}^2, m_{H_2}^2 = m_{H_1}^2 + \Delta m_H^2 \\ m_{H^\pm}^2 = m_{H_1}^2 + \Delta m_\pm^2 \end{cases}$$

Large tanβ is important for Δa_μ, d_μ

Potential analysis for 2HDM with CPV

- 1-loop RGEs for dimensionless couplings

  → relevant couplings: g_i, λ_i, y_t, y_μ

$$\mu \frac{d}{d\mu} c = \frac{1}{16\pi^2} \beta_c$$

$$\beta_{g_1} \simeq +7 g_1^3 , \qquad \beta_{g_2} \simeq -3 g_2^3 , \qquad \beta_{g_3} \simeq -7 g_3^3 ,$$

$$\beta_{\lambda_1} \simeq +\frac{3}{4} g_1^4 + \frac{3}{2} g_1^2 g_2^2 + \frac{9}{4} g_2^4 - 3 g_1^2 \lambda_1 - 9 g_2^2 \lambda_1 + 12 \lambda_1^2 + 4 \lambda_3^2 + 4 \lambda_3 \lambda_4 + 2 \lambda_4^2 + 2 |\lambda_5|^2 + 4 \lambda_1 y_\mu^2 - 4 y_\mu^4 ,$$

$$\beta_{\lambda_2} \simeq +\frac{3}{4} g_1^4 + \frac{3}{2} g_1^2 g_2^2 + \frac{9}{4} g_2^4 - 3 g_1^2 \lambda_2 - 9 g_2^2 \lambda_2 + 12 \lambda_2^2 + 4 \lambda_3^2 + 4 \lambda_3 \lambda_4 + 2 \lambda_4^2 + 2 |\lambda_5|^2 + 12 \lambda_2 y_t^2 - 12 y_t^4 ,$$

$$\beta_{\lambda_3} \simeq +\frac{3}{4} g_1^4 - \frac{3}{2} g_1^2 g_2^2 + \frac{9}{4} g_2^4 + (2 y_\mu^2 + 6 y_t^2 - 3 g_1^2 - 9 g_2^2 + 6 \lambda_1 + 6 \lambda_2 + 4 \lambda_3) \lambda_3 + 2 \lambda_1 \lambda_4 + 2 \lambda_2 \lambda_4 + 2 \lambda_4^2 + 2 |\lambda_5|^2 ,$$

$$\beta_{\lambda_4} \simeq +3 g_1^2 g_2^2 + 8 |\lambda_5|^2 + (2 y_\mu^2 + 6 y_t^2 - 3 g_1^2 - 9 g_2^2 + 2 \lambda_1 + 2 \lambda_2 + 8 \lambda_3 + 4 \lambda_4) \lambda_4 ,$$

$$\beta_{\lambda_5} \simeq +(2 y_\mu^2 + 6 y_t^2 - 3 g_1^2 - 9 g_2^2 + 2 \lambda_1 + 2 \lambda_2 + 8 \lambda_3 + 12 \lambda_4) \lambda_5 ,$$

$$\beta_{y_t} \simeq +\frac{9}{2} y_t^3 + \left(-\frac{17}{12} g_1^2 - \frac{9}{4} g_2^2 - 8 g_3^2 \right) y_t , \qquad \boxed{\beta_{y_\mu} \simeq +\frac{5}{2} y_\mu^3 - \frac{3}{4} \left(5 g_1^2 + 3 g_2^2 \right) y_\mu .}$$

- We have large y_μ : $y_\mu = \dfrac{\sqrt{2} m_\mu}{v} \sqrt{1 + t_{\beta^2}} \simeq 0.6 \times \left(\dfrac{t_\beta}{1000} \right)$

Muon specific 2HDM

- Parameter scan strategy – we have 7 parameters

$$m_H^2,\ \Delta m_H^2,\ \Delta m_\pm^2,\ t_\beta,\ \theta,\ X,\ \delta_{\tilde{h}\tilde{H}}$$

We can optimize these: $\begin{cases} -\frac{m_h^2}{v^2} < X \lesssim 10 \\ |\delta_{\tilde{h}\tilde{H}}| \lesssim 10 \times \frac{v^2}{t_\beta} \end{cases}$

Sub-dominant for Δa_μ, we can fix it as $\Delta m_\pm^2 = \Delta m_H^2/2 \Rightarrow \lambda_4 \simeq 0$

Higher cutoff Λ_{cutoff} can be obtained!

→ satisfy all perturbative unitarity and vacuum stability conditions at this scale

- We focus on $m_H \geqq 650$ GeV to avoid LHC constraint

Abe, Sato, Yagyu, JHEP07(2017)012

- Find parameter space for (s_θ, Δm_H) and (s_θ, tanβ) planes

we assume $\Lambda_{\text{cutoff}} \geqq 10$ TeV is viable parameter space

f couplings in muon specific 2HDM

- Fermion-neutral scalar couplings:

$$\mathcal{L}_Y = -\bar{q}_L \widetilde{\Phi}_2 Y_u u_R - \bar{q}_L \Phi_2 Y_d d_R - \sum_{E=e,\tau} y_E \bar{\ell}_L^E \Phi_2 E_R - \boxed{y_\mu \bar{\ell}_L^\mu \Phi_1 \mu_R} + \text{h.c.}$$

$$\mathcal{L}_{\rm Y}^{\rm int} \supset -\sum_{f\neq\mu} \frac{m_f}{v} \left[\left(R_{1i} + \frac{R_{2i}}{t_\beta} \right) \bar{f} f + i s_f \frac{R_{3i}}{t_\beta} \bar{f}\gamma_5 f \right] \phi_i$$
$$- \frac{m_\mu}{v} \left[(R_{1i} - R_{2i} t_\beta) \bar{\mu}\mu - i R_{3i} t_\beta \bar{\mu}\gamma_5 \mu \right] \phi_i$$

$$s_f = \begin{cases} +1 & \text{for } f = d, e \\ -1 & \text{for } f = u \end{cases}$$

- Only muon coupling is enhanced by tanβ

Note: $R_{11} \approx 1, R_{1i} \ll 1$

1-loop integrals for Δa_μ and d_μ

- Neutral scalar contribution

Δa_μ:

$$I_S(r) \equiv \int_0^1 dx \frac{x^2(2-x)}{rx^2 - x + 1} \quad \text{(CP-even)}$$

$$I_P(r) \equiv \int_0^1 dx \frac{-x^3}{rx^2 - x + 1} \quad \text{(CP-odd)}$$

d_μ:

$$f_0(r) \equiv \int_0^1 dx \frac{x^2}{rx^2 - x + 1}$$

- Charged scalar contribution

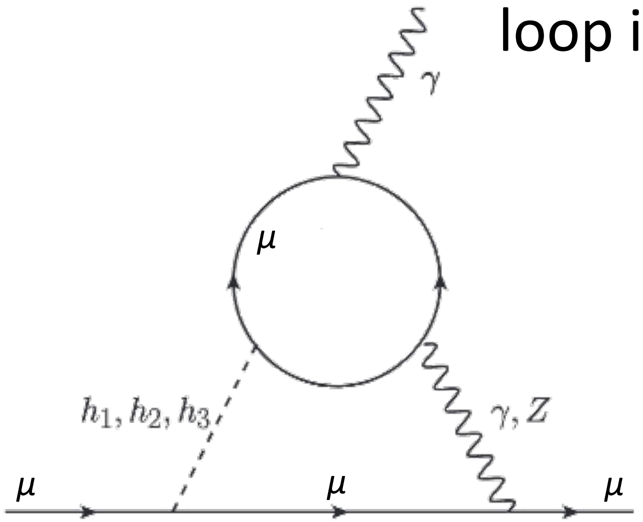
Δa_μ:

$$I_C(r) \equiv \int_0^1 dx \frac{-x(1-x)}{rx + 1 - r}$$

2-loop contribution to d_μ

- Dominant one: Barr-Zee type diagram with muon loop

$$d_\mu^{i,\phi_i\text{-}\gamma\text{-}\gamma} = -\frac{e\,m_\mu}{(4\pi)^2 v^2} r_i \left(R_{1i} - R_{2i} t_\beta\right) R_{3i} t_\beta \frac{2\alpha}{\pi} I_\mu(r_i)$$



loop integral: $I_\mu(r) = \int_0^1 dx \frac{1 - x(1-x)}{r - x(1-x)} \ln\left[\frac{x(1-x)}{r}\right]$

JHEP01(2014)106, JHEP08(2017)031, JHEP12(2019)068

✓ 1- and 2-loop contributions are

$$d_\mu^{i,\phi_i\text{-}\gamma\text{-}\gamma} = d_\mu^i \times \frac{2\alpha}{\pi} \frac{I_\mu(r_i)}{f_0(r_i)}$$

- Facts: $\begin{cases} 2\alpha/\pi \sim 4.6 \times 10^{-3} \\ I_\mu(r) \sim -(15\text{-}19) f_0(r) \end{cases}$

2-loop contribution is important!

T-parameter

Peskin and Takeuchi, PRL65(1990)964; PRD46(1992)381

- ρ-parameter: $\rho = \dfrac{m_W^2}{m_Z^2 \cos\theta_W^2}$ (= 1 in the SM)
- NP contributions deviate it from 1: $\Delta\rho = \alpha T$

  experimental status (PDG): $|T| \lesssim 0.2$
- In 2HDM, mass differences among $H_{1,2}$ and H^+ are crucial

Grimus, Lavoura, Ogreid, Osland, J.Phys.G35(2008)075001; NPB801(2008)81

- For muon specific 2HDM, Δm_H is enough small to satisfy

  e.g.) m_H = 650 GeV with Δm_H = 320 GeV and s_θ = 0.35 → T = -0.03
- Radiative m_μ model also predicts small $T \sim$ 0.002

$h \rightarrow \mu^+\mu^-$ decay in μ2HDM

- Yukawa interaction in mass basis for scalars:

$$\mathcal{L}_Y^{\rm int} = -\sum_{f\neq\mu}\frac{m_f}{v}\left[\left(R_{1i}+\frac{R_{2i}}{t_\beta}\right)\bar{f}f + is_f\frac{R_{3i}}{t_\beta}\bar{f}\gamma_5 f\right]\phi_i - \frac{m_\mu}{v}\left[(R_{1i}-R_{2i}t_\beta)\bar{\mu}\mu - iR_{3i}t_\beta\bar{\mu}\gamma_5\mu\right]\phi_i$$

$$+\left\{-\frac{\sqrt{2}}{vt_\beta}\sum_{a=1}^{3}\bar{u}^a\left(m_{d^a}P_R - m_{u^a}P_L\right)d^aH^+\left[\frac{\sqrt{2}}{v}t_\beta m_\mu\bar{\nu}_\mu P_R\mu - \frac{\sqrt{2}}{vt_\beta}\sum_{\ell\neq\mu}m_\ell\bar{\nu}_\ell P_R\ell\right]H^+ + {\rm h.c.}\right\}$$

- Large tan limit, h-μ-μ coupling may be large

current experimental status (95% C.L.):

$$|\kappa_\mu| < 1.47 \quad ({\rm ATLAS})$$

$$0.61 < |\kappa_\mu| < 1.44 \quad ({\rm CMS})$$

ATLAS collab., PLB812(2021)135980
CMS collab., JHEP01(2021)148

μ2HDM

$$\Leftrightarrow \qquad \kappa_\mu = R_{11} - R_{21}t_\beta \simeq 1 + \frac{\delta_{\tilde{h}\tilde{H}}}{M^2}t_\beta \sim 1 + \frac{\mathcal{O}(v^2)}{m_H^2}$$

Mass spectrum in radiative m_μ model

- Fermion sector

$$-\frac{1}{2}\begin{pmatrix}\bar{\psi}_L & \overline{\psi^c_R}\end{pmatrix}\begin{pmatrix}m_{LL} & m_D \\ m_D & m_{RR}\end{pmatrix}\begin{pmatrix}\psi^c_L \\ \psi_R\end{pmatrix} + \text{h.c.}$$

$$\mathcal{M}_{\psi,\text{diag}} = U_\psi^\dagger \mathcal{M}_\psi U_\psi^*\,, \quad U_\psi = \begin{pmatrix} c_\alpha & s_\alpha e^{-i\tau} \\ -s_\alpha e^{i\tau} & c_\alpha \end{pmatrix}$$

Eigenvalues:
$$\begin{cases} m^2_{\psi_1} = \frac{1}{2}\left(m^2_{LL} + m^2_{RR} + 2\,|m_D|^2 - \Delta m^2_\psi\right) \\ m^2_{\psi_2} = \frac{1}{2}\left(m^2_{LL} + m^2_{RR} + 2\,|m_D|^2 + \Delta m^2_\psi\right) \end{cases}$$

$$\Delta m^2_\psi = \sqrt{\left(m^2_{LL} - m^2_{RR}\right)^2 + 4\,|m_D|^2\left|m_{LL}e^{-i\theta_\text{phys}} + m_{RR}e^{i\theta_\text{phys}}\right|^2}$$

Mixing angle and phase:

$$\sin 2\alpha = \frac{2|m_D|}{\Delta m^2_\psi}\left|m_{LL}e^{-i\theta_\text{phys}} + m_{RR}e^{i\theta_\text{phys}}\right|; \quad \tan\tau = -\frac{m_{LL} - m_{RR}}{m_{LL} + m_{RR}}\tan\theta_\text{phys}$$

Physical phase of model: $\theta_\text{phys} = \frac{1}{2}\left(\theta_L + \theta_R - 2\theta_D\right)$

$\tau = 0$ when $\theta_\text{phys} = 0$ or $m_{LL} = m_{RR}$

Diagonalization of M_ψ

- M_ψ is complex symmetric matrix

  Real eigenvalues can be obtained by $M_\psi^\dagger M_\psi$

$$\mathcal{M}_\psi^\dagger \mathcal{M}_\psi = \begin{pmatrix} m_{LL}^2 + |m_D|^2 & |m_D| \left(m_{LL} e^{-i\theta_{\rm phys}} + m_{RR} e^{i\theta_{\rm phys}} \right) \\ |m_D| \left(m_{LL} e^{i\theta_{\rm phys}} + m_{RR} e^{-i\theta_{\rm phys}} \right) & m_{RR}^2 + |m_D|^2 \end{pmatrix}$$

  Comparing with diagonal matrix, $M_{\psi,\mathrm{diag}}^\dagger M_{\psi,\mathrm{diag}}$ = diag($m_{\psi 1}^2$, $m_{\psi 2}^2$)

$$U_\psi^* \mathcal{M}_{\psi,\rm diag}^\dagger \mathcal{M}_{\psi,\rm diag} U_\psi^T = \begin{pmatrix} m_{\psi_1}^2 c_\alpha^2 + m_{\psi_2}^2 s_\alpha^2 & (m_{\psi_2}^2 - m_{\psi_1}^2) s_\alpha c_\alpha e^{i\tau} \\ (m_{\psi_2}^2 - m_{\psi_1}^2) s_\alpha c_\alpha e^{-i\tau} & m_{\psi_1}^2 s_\alpha^2 + m_{\psi_2}^2 c_\alpha^2 \end{pmatrix}$$

- Each off-diagonal element gives sin2α and tanτ:

$$|m_D| \left(m_{LL} e^{-i\theta_{\rm phys}} + m_{RR} e^{i\theta_{\rm phys}} \right) = (m_{\psi_2}^2 - m_{\psi_1}^2) s_\alpha c_\alpha e^{i\tau}$$

Physical phase of model: $\theta_{\rm phys} = \frac{1}{2} \left(\theta_L + \theta_R - 2\theta_D \right)$

Mass spectrum in radiative m_μ model

- Scalar sector

Parameterize exotic scalars:

$$\phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(\sigma_\phi + i a_\phi) \end{pmatrix}, \quad \eta = \eta^+$$

neutral scalar (basis (σ_ϕ, a_ϕ))

$$\mathcal{M}_0^2 = \begin{pmatrix} M_\phi^2 + \frac{\lambda_{H\phi}^+}{2} v_H^2 & 0 \\ 0 & M_\phi^2 + \frac{\lambda_{H\phi}^-}{2} v_H^2 \end{pmatrix} \equiv \begin{pmatrix} m_{\sigma_\phi}^2 & 0 \\ 0 & m_{a_\phi}^2 \end{pmatrix}$$

charged scalar (basis (ϕ^+, η^+))

$$\mathcal{M}_\pm^2 = \begin{pmatrix} M_\phi^2 & \frac{a v_H}{\sqrt{2}} \\ \frac{a v_H}{\sqrt{2}} & M_\eta^2 \end{pmatrix}, \quad M_{\phi,\eta}^2 = m_{\phi,\eta}^2 + \frac{\lambda_{H\phi,H\eta}}{2} v_H^2$$

⇨ Diagonalizing matrix: $U_s \equiv \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}$ $\quad \sin 2\theta = \frac{\sqrt{2} a v_H}{m_{\varphi_2^+}^2 - m_{\varphi_1^+}^2}$

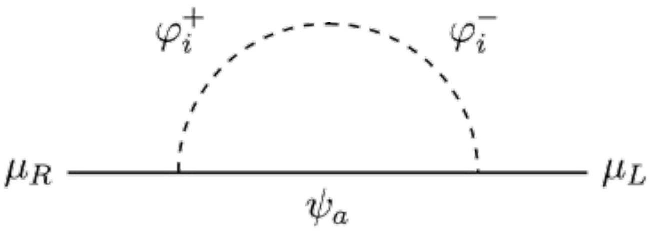
Eigenvalues:

$$\begin{cases} m_{\varphi_1^+}^2 = \frac{1}{2}\left[M_\phi^2 + M_\eta^2 - \sqrt{(M_\phi^2 - M_\eta^2)^2 + 2a^2 v_H^2}\right] \\ m_{\varphi_2^+}^2 = \frac{1}{2}\left[M_\phi^2 + M_\eta^2 + \sqrt{(M_\phi^2 - M_\eta^2)^2 + 2a^2 v_H^2}\right] \end{cases}$$

Radiative muon mass model

- Radiative muon mass (with $x_{i,a} \equiv m^2_{\varphi_i^+}/m^2_{\psi_a}$)

$$m_\mu^{\rm rad} = \sum_{i,a} \frac{y_L^{ia} y_R^{ia}}{16\pi^2} m_{\psi_a} \frac{x_{i,a}\ln x_{i,a}}{x_{i,a}-1} \propto \frac{y_\phi y_\eta}{16\pi^2}\frac{s_{2\theta}s_{2\alpha}}{4}$$



$\psi_{1,2}, \varphi^\pm_{1,2}$: mass eigenstates

- Similar diagram (attach SM Higgs) gives effective Yukawa:

$$y_\mu^{\rm eff}(p_h^2) = -\sum_{i,j,a} \frac{y_L^{ia} y_R^{ja} A_{ij}}{16\pi^2} m_{\psi_a} C_0(m_\mu^2, m_\mu^2, p_h^2, m^2_{\varphi_i^+}, m^2_{\psi_a}, m^2_{\varphi_j^+})$$

⇨ $|m_\mu^{\rm rad}| \neq \frac{|y_\mu^{\rm eff}|}{\sqrt{2}} v_H$:constraint will come from $h \to \mu^+\mu^-$ decay

Relevant couplings:

(i,a)	y_L^{ia}	y_R^{ia}	(i,j)	A_{ij}
$(1,1)$	$-y_\phi c_\theta s_\alpha e^{-i\tau}$	$-y_\eta s_\theta c_\alpha$	$(1,1)$	$-a s_{2\theta} + \sqrt{2} v_H (\lambda_{H\phi} c_\theta^2 + \lambda_{H\eta} s_\theta^2)$
$(1,2)$	$y_\phi c_\theta c_\alpha$	$-y_\eta s_\theta s_\alpha e^{i\tau}$	$(1,2)$	$a c_{2\theta} + \sqrt{2} v_H (\lambda_{H\phi} - \lambda_{H\eta}) s_\theta c_\theta$
$(2,1)$	$-y_\phi s_\theta s_\alpha e^{-i\tau}$	$y_\eta c_\theta c_\alpha$	$(2,1)$	$a c_{2\theta} + \sqrt{2} v_H (\lambda_{H\phi} - \lambda_{H\eta}) s_\theta c_\theta$
$(2,2)$	$y_\phi s_\theta c_\alpha$	$y_\eta c_\theta s_\alpha e^{i\tau}$	$(2,2)$	$a s_{2\theta} + \sqrt{2} v_H (\lambda_{H\phi} s_\theta^2 + \lambda_{H\eta} c_\theta^2)$

Diagonalizing matrices:

$$U_\psi = \begin{pmatrix} c_\alpha & s_\alpha e^{-i\tau} \\ -s_\alpha e^{-i\tau} & c_\alpha \end{pmatrix} \text{ for } \psi_{L,R}$$

$$U_s = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \text{ for } \phi^+, \eta^+$$

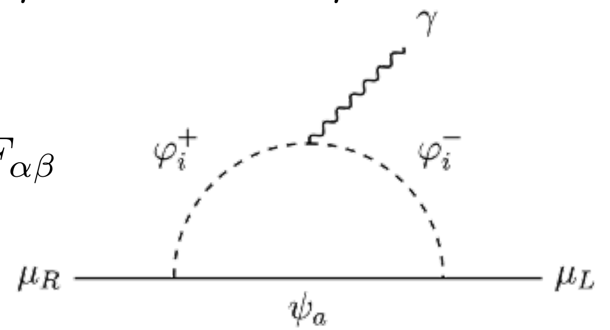
Radiative muon mass model

- Point: m_μ^{rad} has phase $m_\mu^{\text{rad}} = m_\mu e^{i\theta_\mu}$

Need chiral rotation

$$\mu \to e^{-i\theta_\mu \gamma_5/2}\mu$$

- This affects dipole operators



$$\mathcal{L}_{\text{dipole}} = -\frac{e}{2} C_T(q^2) \left(\bar{\mu}\sigma^{\alpha\beta}\mu\right) F_{\alpha\beta} - \frac{e}{2} C_{T'}(q^2) \left(\bar{\mu} i \sigma^{\alpha\beta}\gamma_5 \mu\right) F_{\alpha\beta}$$

$$= -\frac{e}{4m_\mu} a_\mu \left(\bar{\mu}\sigma^{\alpha\beta}\mu\right) F_{\alpha\beta} - \frac{i}{2} d_\mu \left(\bar{\mu}\sigma^{\alpha\beta}\gamma_5\mu\right) F_{\alpha\beta}$$

$$\Rightarrow \quad g-2 : a_\mu = 2m_\mu \left(C_T(0)\cos\theta_\mu + C_{T'}(0)\sin\theta_\mu\right)$$

$$\text{EDM} : d_\mu = e\left(C_{T'}(0)\cos\theta_\mu - C_T(0)\sin\theta_\mu\right)$$

$$C_T(0) = \sum_{i,a} \frac{\text{Re}[y_L^{ia} y_R^{ia}]}{16\pi^2} \frac{x_{i,a}^2 - 1 - 2x_{i,a}\ln(x_{i,a})}{2m_{\psi,a}(1-x_{i,a})^3}, \quad C_{T'}(0) = \sum_{i,a} \frac{\text{Im}[y_L^{ia} y_R^{ia}]}{16\pi^2} \frac{x_{i,a}^2 - 1 - 2x_{i,a}\ln(x_{i,a})}{2m_{\psi,a}(1-x_{i,a})^3}$$

- Loop factor and some coupling can be replaced by m_μ

$$m_\mu^{\text{rad}}, C_T(0), C_{T'}(0) \propto \frac{y_\phi y_\eta}{16\pi^2}$$

Enhancement is expected!

Full forms for $y_\mu^{\rm eff}(p_h^2), C_T(q^2), C_{T'}(q^2)$

$$y_\mu^{\rm eff}(p_h^2) = \sum_{i,j,a}\Bigg\{-\frac{y_L^{ia}y_R^{ja}A_{ij}}{16\pi^2}m_{\psi_a}C_0(m_\mu^2, m_\mu^2, p_h^2, m_{\varphi_i^+}^2, m_{\psi_a}^2, m_{\varphi_j^+}^2)$$
$$+\frac{A_{ij}}{16\pi^2}m_\mu^{\rm rad}\Big[y_R^{ia\,*}y_R^{ja}C_1(m_\mu^2, p_h^2, m_\mu^2, m_{\psi_a}^2, m_{\varphi_i^+}^2, m_{\varphi_j^+}^2)$$
$$+y_L^{ia}y_L^{ja\,*}C_2(m_\mu^2, p_h^2, m_\mu^2, m_{\psi_a}^2, m_{\varphi_i^+}^2, m_{\varphi_j^+}^2)\Big]\Bigg\}$$

$$C_T(q^2) = \sum_{i,a}\Bigg\{\frac{{\rm Re}[y_L^{ia}y_R^{ia}]}{16\pi^2}m_{\psi_a}\Big[Q_S\Big(C_0(q^2, m_{\psi_a}^2, m_{\varphi_i^+}^2) + 2C_1(q^2, m_{\psi_a}^2, m_{\varphi_i^+}^2)\Big) - 2Y_\psi C_1(q^2, m_{\varphi_i^+}^2, m_{\psi_a}^2)\Big]$$
$$-\frac{|y_L^{ia}|^2 + |y_R^{ia}|^2}{16\pi^2}{\rm Re}[m_\mu^{\rm eff}]\Big[Q_S C_{\rm sub}(q^2, m_{\psi_a}^2, m_{\varphi_i^+}^2) + Y_\psi C_{\rm sub}(q^2, m_{\varphi_i^+}^2, m_{\psi_a}^2)\Big]\Bigg\}$$

$$C_{T'}(q^2) = \sum_{i,a}\Bigg\{\frac{{\rm Im}[y_L^{ia}y_R^{ia}]}{16\pi^2}m_{\psi_a}\Big[Q_S\Big(C_0(q^2, m_{\psi_a}^2, m_{\varphi_i^+}^2) + 2C_1(q^2, m_{\psi_a}^2, m_{\varphi_i^+}^2)\Big) - 2Y_\psi C_1(q^2, m_{\varphi_i^+}^2, m_{\psi_a}^2)\Big]$$
$$-\frac{|y_L^{ia}|^2 + |y_R^{ia}|^2}{16\pi^2}{\rm Im}[m_\mu^{\rm eff}]\Big[Q_S C_{\rm sub}(q^2, m_{\psi_a}^2, m_{\varphi_i^+}^2) + Y_\psi C_{\rm sub}(q^2, m_{\varphi_i^+}^2, m_{\psi_a}^2)\Big]\Bigg\}$$

with
$$\begin{cases} C_0(q^2, m_{\psi_a}^2, m_{\varphi_i^+}^2) \equiv C_0(m_\mu^2, m_\mu^2, q^2, m_{\varphi_i^+}^2, m_{\psi_a}^2, m_{\varphi_i^+}^2) \\ C_N(q^2, m_A^2, m_B^2) \equiv C_N(m_\mu^2, q^2, m_\mu^2, m_A^2, m_B^2, m_B^2) \quad (N = 1, 11, 12) \\ C_{\rm sub}(q^2, m_A^2, m_B^2) \equiv C_1(q^2, m_A^2, m_B^2) + C_{11}(q^2, m_A^2, m_B^2) + C_{12}(q^2, m_A^2, m_B^2) \end{cases}$$

Muon couplings to bosons

- Higgs coupling: $\mathcal{L}_{\text{eff}} \supset -\frac{y_\mu^{\text{eff}}}{\sqrt{2}} \bar{\mu}_L \mu_R h^0 + \text{h.c.}$

  Decay width in our model

$$\Gamma_{h \to \mu^+ \mu^-} = \frac{m_h}{16\pi} \sqrt{1 - \frac{4m_\mu^2}{m_h^2}} \left[\left(1 - \frac{4m_\mu^2}{m_h^2}\right) \left(\text{Re}\, y_\mu^{\text{eff}}\right)^2 + \left(\text{Im}\, y_\mu^{\text{eff}}\right)^2 \right]$$

$$\Rightarrow \quad |\kappa_\mu| = \frac{1}{\sqrt{2}} \frac{v_H}{m_\mu} \sqrt{\left(\text{Re}\, y_\mu^{\text{eff}}\right)^2 + \left(1 - \frac{4m_\mu^2}{m_h^2}\right)^{-1} \left(\text{Im}\, y_\mu^{\text{eff}}\right)^2}$$

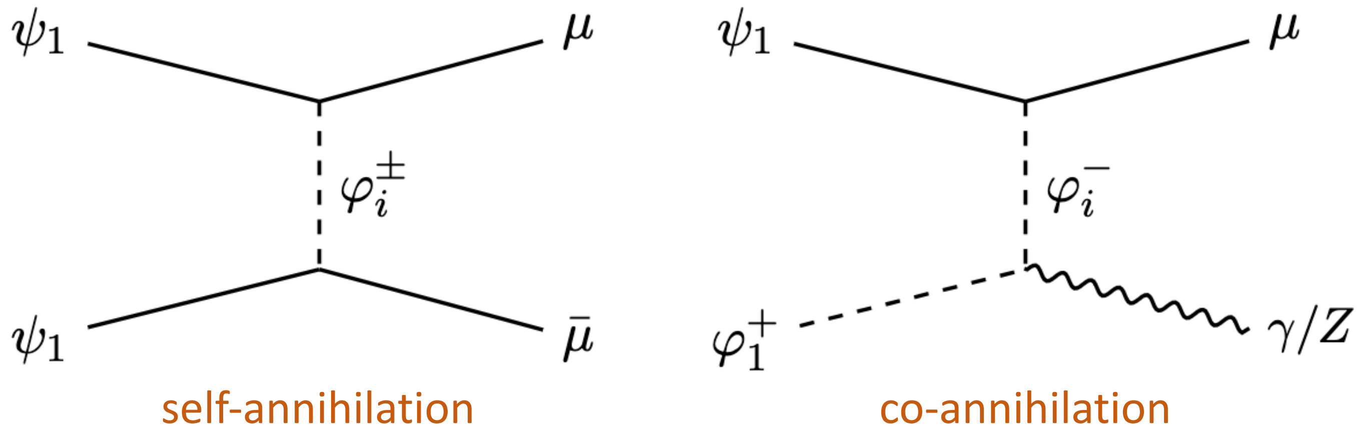
- Z boson coupling: $\mathcal{L}_Z \supset \frac{g}{\cos\theta_W} \bar{\mu} \gamma^\alpha \left[\left(g_L^\mu + \delta g_L^\mu\right) P_L + \left(g_R^\mu + \delta g_R^\mu\right) P_R \right] \mu Z_\alpha$

$$\frac{\Gamma(Z \to \mu^+ \mu^-)}{\Gamma(Z \to e^+ e^-)} \simeq 1 + \frac{2 g_L^e \text{Re}\left(\delta g_L^\mu\right) + 2 g_R^e \text{Re}\left(\delta g_R^\mu\right)}{(g_L^e)^2 + (g_R^e)^2}$$

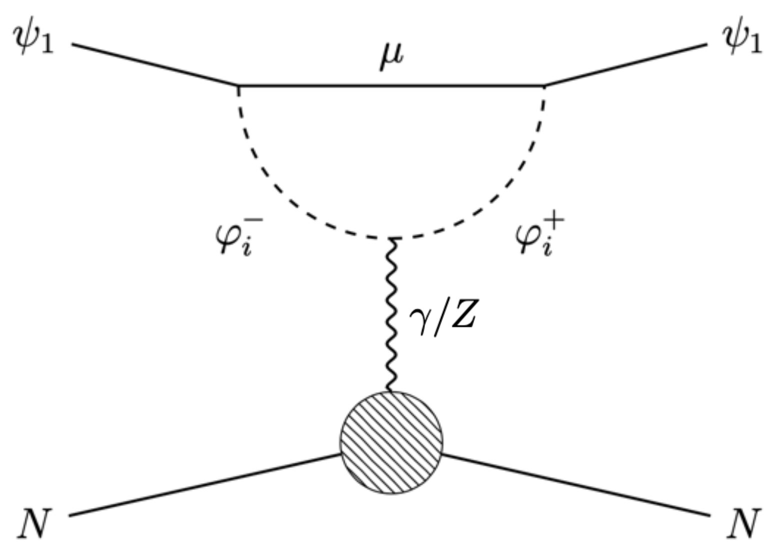
$\delta g_{L,R}^\mu$ in our model can be found in Ref. JHEP04(2021)151

Dark matter in Rad-m_μ model (ψ_1)

- Main annihilation mode: muon in the final state



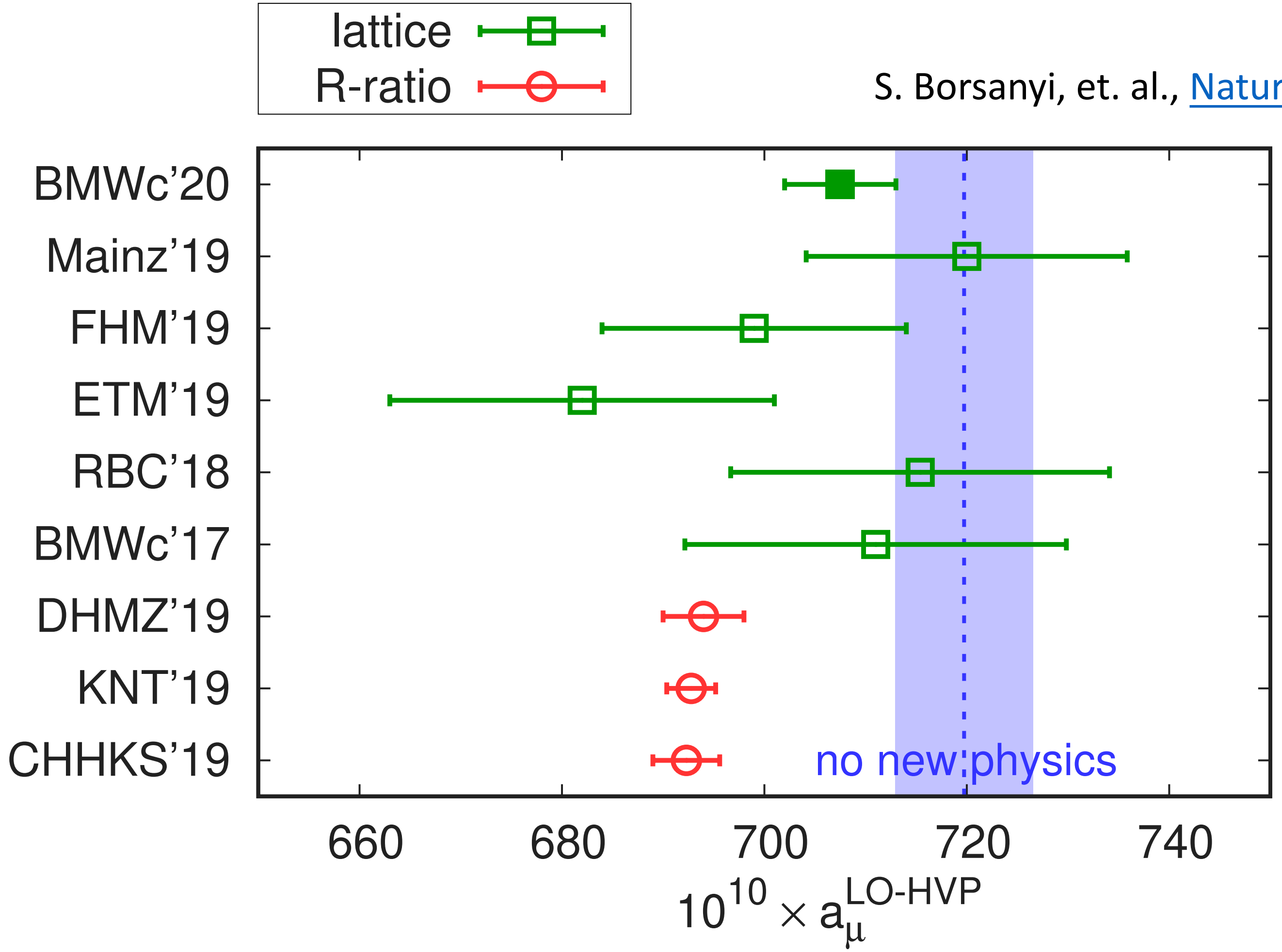
- Direct detection:



Lattice results on HVP

- Result: $a_\mu^{\text{LO-HVP}} = 707.5(2.3)(5.0) \times 10^{-10}$

S. Borsanyi, et. al., [Nature 593 (2021) 51]



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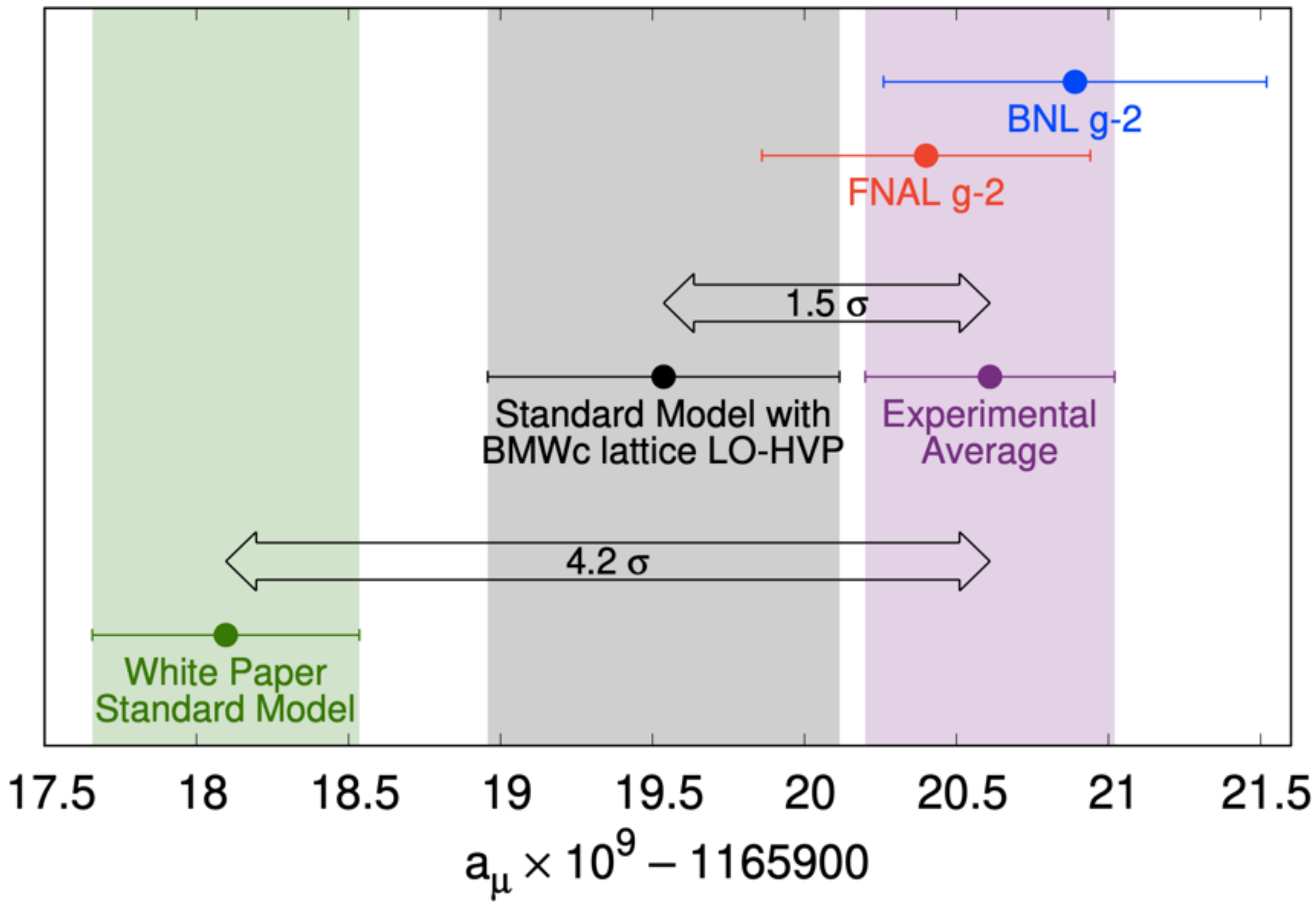

Figure from talk of L. Lellouch (Wits ICPP iThemba Labs seminar)