Coupling Sum Rules and Oblique Corrections in Gauge-Higgs Unification

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GUT inspired GHU in RS space

$$SO(5) \times U(1)_X \times SU(3)_C$$

$$\rightarrow \quad SO(4) \times U(1)_X \times SU(3)_C$$

$$\rightarrow \quad SU(2)_L \times U(1)_Y \times SU(3)_C$$

$$\rightarrow \quad U(1)_{\rm EM} \times SU(3)_C$$

θ_H

AB phase θ_H

$$P e^{i g_A \oint dy \langle A_y \rangle}$$

4D Higgs

Gauge couplings

$$SO(5) \times U(1)_X$$
$$g_A \qquad g_B$$
$$\sim (g_w, \sin\theta_W^0)$$

$$\theta_H = 0.1 \ , \ m_{\rm KK} = 13\,\text{TeV} \ , \ \sin^2\theta_W^0 = 0.230634$$

SM

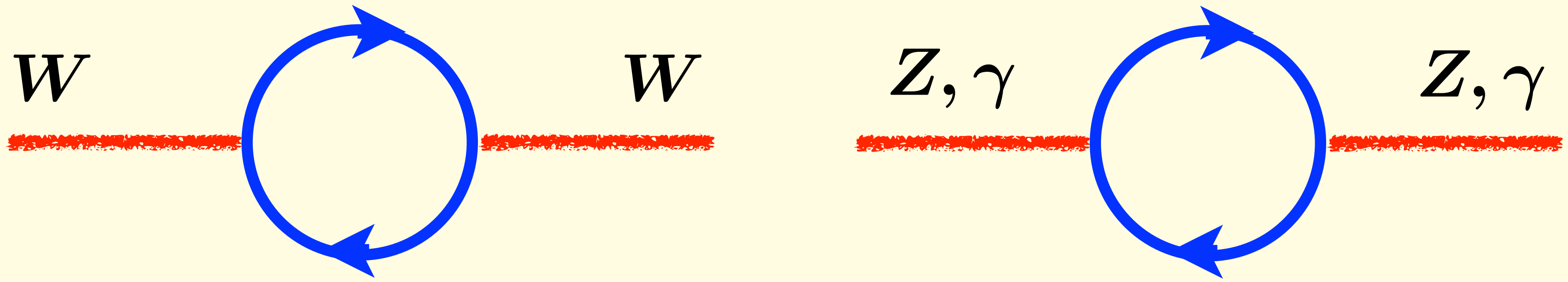
$$\frac{1}{\hat{g}_L^{W\nu_e e}} \begin{pmatrix} \hat{g}_L^{Z\nu_e} \\ \hat{g}_R^{Z\nu_e} \\ \hat{g}_L^{Ze} \\ \hat{g}_R^{Ze} \end{pmatrix} = \begin{pmatrix} 0.500022 \\ 2\times 10^{-32} \\ -0.268800 \\ 0.231198 \end{pmatrix} \qquad \begin{pmatrix} 0.5 \\ 0 \\ -0.2688 \\ 0.2312 \end{pmatrix}$$

$$\frac{1}{\hat{g}_L^{W\nu_e e}} \begin{pmatrix} \hat{g}_L^{Zu} \\ \hat{g}_R^{Zu} \\ \hat{g}_L^{Zd} \\ \hat{g}_R^{Zd} \end{pmatrix} = \begin{pmatrix} 0.346205 \\ -0.153801 \\ -0.422940 \\ 0.077071 \end{pmatrix} \qquad \begin{pmatrix} 0.3459 \\ -0.1541 \\ -0.4229 \\ 0.0771 \end{pmatrix}$$

$$\sin^2\theta_W^{\rm SM} = 0.2312$$

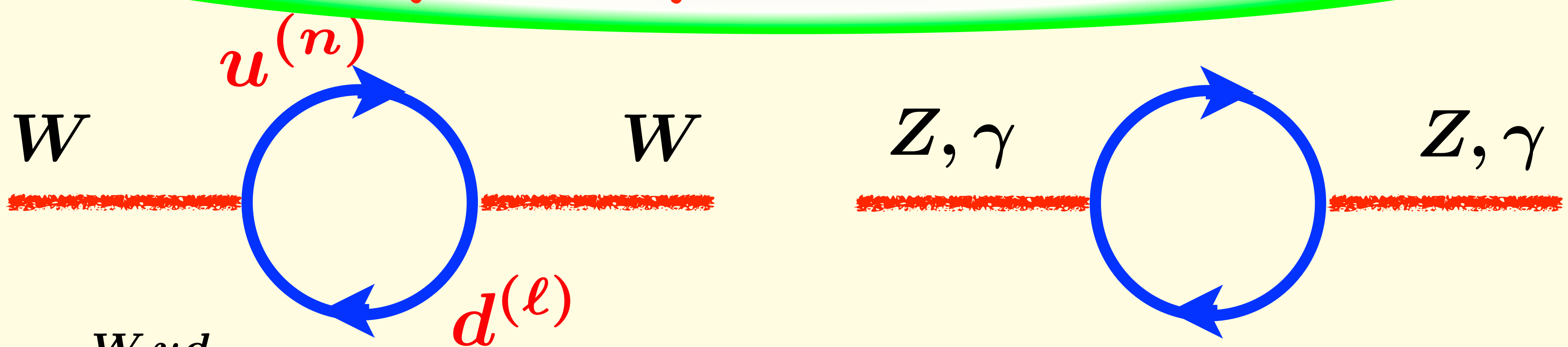
At low energies: Nearly the same as in SM

1 loop oblique corrections ?



In the SM, S, T, U are finite.

1 loop oblique corrections ?



$g^{W,ud}_{L/R,n\ell}$, $m_{u^{(n)}}$, $m_{d^{(\ell)}}$

In the SM, S, T, U are finite.

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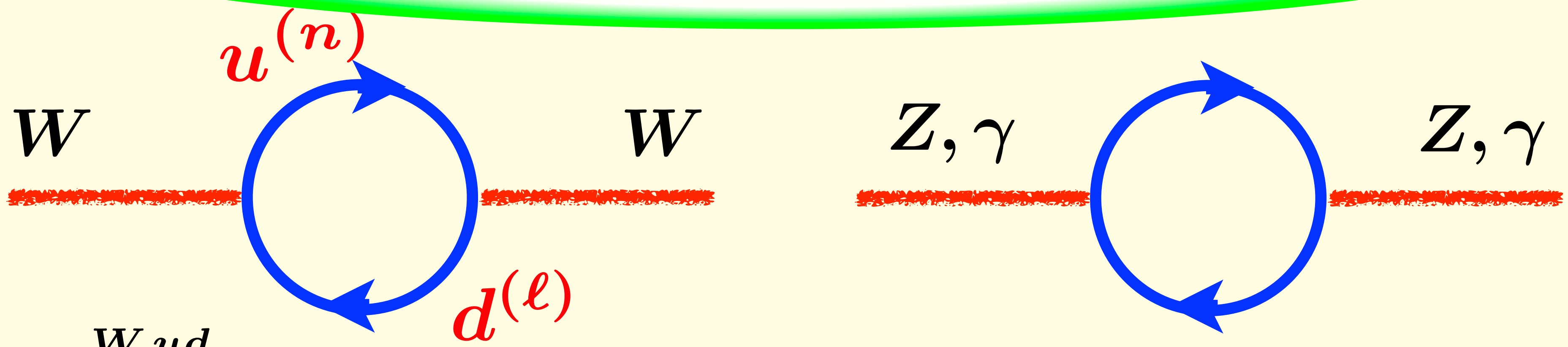


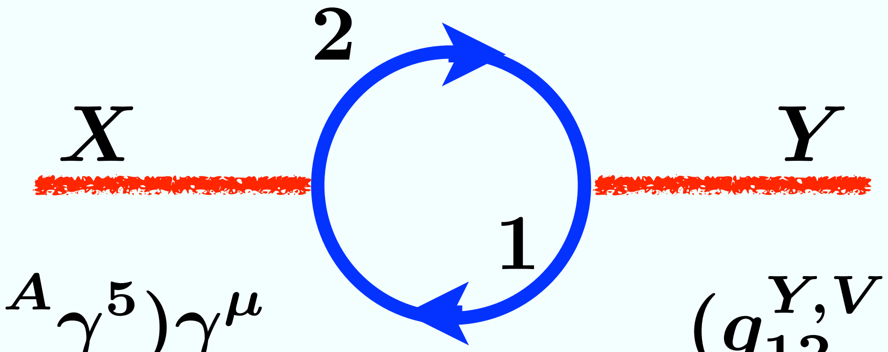
$$g^{W,ud}_{L/R,n\ell} \; , \; m_{u^{(n)}} \; , \; m_{d^{(\ell)}}$$

In the SM, S, T, U are finite.

$$\hat{g}^{Wud}_{L} = \begin{pmatrix} 0.997645 & -0.024904 & 0.000020 & -0.002827 & 10^{-6} \\ -0.024904 & 0.002498 & 0.028389 & 10^{-7} & 0.000510 \\ 0.000020 & 0.028389 & 0.997618 & -0.024548 & 0.000022 \\ -0.002827 & 10^{-7} & -0.024548 & 0.002498 & 0.027021 \\ 10^{-6} & 0.000510 & 0.000022 & 0.027021 & 0.997620 \end{pmatrix}$$

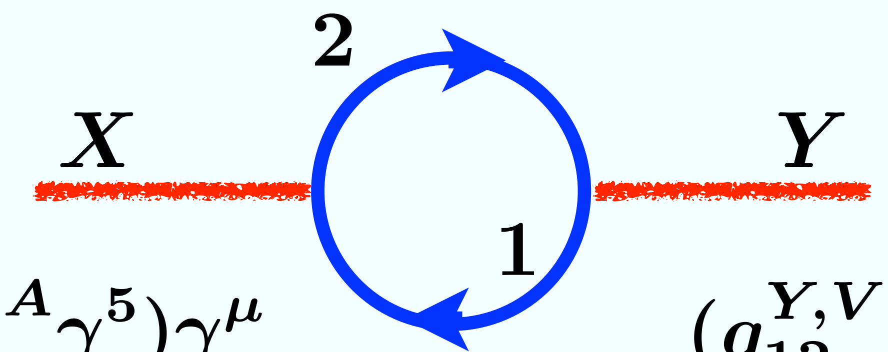
$$\hat{g}^{Wud}_{R} = \begin{pmatrix} 10^{-12} & 10^{-7} & 10^{-7} & 10^{-9} & 10^{-7} \\ 10^{-8} & 0.002498 & 0.024145 & 10^{-8} & 10^{-6} \\ 10^{-7} & 0.024145 & 0.997632 & -0.022564 & 0.000018 \\ 10^{-10} & 10^{-8} & -0.022564 & 0.002498 & 0.025826 \\ 10^{-8} & 10^{-6} & 0.000018 & 0.025826 & 0.997625 \end{pmatrix}$$

$$\Pi^{\mu\nu}_{XY}(p) = \Pi_{XY}(p^2)\eta^{\mu\nu} - \Sigma_{XY}(p^2)p^\mu p^\nu$$



$(g^{X,V}_{21} - g^{X,A}_{21}\gamma^5)\gamma^\mu$ $(g^{Y,V}_{12} - g^{Y,A}_{12}\gamma^5)\gamma^\nu$

$$\Pi^{\mu\nu}_{XY}(p) = \Pi_{XY}(p^2)\eta^{\mu\nu} - \Sigma_{XY}(p^2)p^\mu p^\nu$$



$$(g_{21}^{X,V} - g_{21}^{X,A}\gamma^5)\gamma^\mu \qquad (g_{12}^{Y,V} - g_{12}^{Y,A}\gamma^5)\gamma^\nu$$

$$\Pi^{\rm div}_{XY}(p^2) = \frac{1}{4\pi^2}\Big[(g_{21}^{X,V}g_{12}^{Y,V} + g_{21}^{X,A}g_{12}^{Y,A})\Big\{\frac{1}{3}p^2 - \frac{1}{2}(m_1^2 + m_2^2)\Big\}$$
$$+(g_{21}^{X,V}g_{12}^{Y,V} - g_{21}^{X,A}g_{12}^{Y,A})\, m_1 m_2\Big]\Big(\frac{2}{d-4} + \cdots\Big)$$

In SM, S, T, U combinations : finite

Coupling Sum Rules

$$[\text{Def}] \quad \hat{g}^{Z,u}_{V,n\ell} = \boxed{\hat{g}^{W^3,u}_{V,n\ell}} - \sin^2\theta^0_W\, Q_u\, \delta_{n\ell} \quad \text{etc}$$

$$A_0^{ud} = \sum_{n,\ell} \left\{ \hat{g}^{W^3u}_{V,n\ell}\hat{g}^{W^3u}_{V,\ell n} + \hat{g}^{Zu}_{A,n\ell}\hat{g}^{Zu}_{A,\ell n} + \hat{g}^{W^3d}_{V,n\ell}\hat{g}^{W^3d}_{V,,\ell n} + \hat{g}^{Zd}_{A,n\ell}\hat{g}^{Zd}_{A,\ell n} \right\}$$

$$A_1^{ud} = \sum_{n,\ell} \left\{ \hat{g}^{W^3u}_{V,n\ell}\hat{g}^{W^3u}_{V,\ell n}(m_{u^{(n)}} - m_{u^{(\ell)}})^2 + \hat{g}^{Zu}_{A,n\ell}\hat{g}^{Zu}_{A,\ell n}(m_{u^{(n)}} + m_{u^{(\ell)}})^2 \right.$$

$$\left. + \hat{g}^{W^3d}_{V,n\ell}\hat{g}^{W^3d}_{V,\ell n}(m_{d^{(n)}} - m_{d^{(\ell)}})^2 + \hat{g}^{Zd}_{A,n\ell}\hat{g}^{Zd}_{A,\ell n}(m_{d^{(n)}} + m_{d^{(\ell)}})^2 \right\}$$

$$B^{ud} = Q_u \sum_n \hat{g}^{W^3u}_{V,nn} + Q_d \sum_n \hat{g}^{W^3d}_{V,nn}$$

$$D_0^{ud} = \sum_{n,\ell} \left\{ \hat{g}^{Wud}_{V,n\ell}\hat{g}^{W^\dagger ud}_{V,\ell n} + \hat{g}^{Wud}_{A,n\ell}\hat{g}^{W^\dagger ud}_{A,\ell n} \right\}$$

$$D_1^{ud} = \sum_{n,\ell} \left\{ \hat{g}^{Wud}_{V,n\ell}\hat{g}^{W^\dagger ud}_{V,\ell n}(m_{u^{(n)}} - m_{d^{(\ell)}})^2 + \hat{g}^{Wud}_{A,n\ell}\hat{g}^{W^\dagger ud}_{A,\ell n}(m_{u^{(n)}} + m_{d^{(\ell)}})^2 \right\}$$

Coupling Sum Rules

$$\begin{cases} A_0^{ud} = h^{ud} B^{ud} \\ D_0^{ud} = 2A_0^{ud} \\ D_1^{ud} = 2A_1^{ud} \end{cases}$$

$$h^{ud} = \hat{g}_{L,00}^{Zu,su2} - \hat{g}_{L,00}^{Zd,su2} = 0.997688$$

Coupling Sum Rules

$$\begin{cases} A_0^{ud} = h^{ud} B^{ud} \\ D_0^{ud} = 2A_0^{ud} \\ D_1^{ud} = 2A_1^{ud} \end{cases}$$

$$h^{ud} = \hat{g}_{L,00}^{Zu,su2} - \hat{g}_{L,00}^{Zd,su2} = 0.997688$$

$$\frac{A_0 - hB}{A_0} \rightarrow \Delta_S \qquad \frac{A_1 - \frac{1}{2}D_1}{A_1} \rightarrow \Delta_T \qquad \frac{A_0 - \frac{1}{2}D_0}{A_0} \rightarrow \Delta_U$$

	A_0	A_1	Δ_S	Δ_T	Δ_U	h
(ν_e, e)	3.24204	44.1410	3.6×10^{-5}	5.3×10^{-5}	-3.4×10^{-7}	0.997690
(ν_μ, μ)	3.24214	43.5089	7.1×10^{-5}	5.3×10^{-5}	-3.4×10^{-7}	0.997687
(ν_τ, τ)	3.24219	43.1535	-1.5×10^{-5}	5.3×10^{-5}	-2.7×10^{-7}	0.997684
(u, d)	3.24210	43.7063	-4.0×10^{-6}	5.5×10^{-5}	-3.4×10^{-7}	0.997688
(c, s)	3.24217	43.2862	2.0×10^{-5}	5.3×10^{-5}	-3.4×10^{-7}	0.997685
(t, b)	3.24262	42.5613	1.0×10^{-4}	4.1×10^{-5}	-2.8×10^{-7}	0.997671

$$\theta_H = 0.1 \ , \ m_{\rm KK} = 13\,{\rm TeV} \ , \ n, \ell = 0 \sim 12$$

$A_0 = h\,B$

$$h^{ud} = \hat{g}^{Zu,su2}_{L,00} - \hat{g}^{Zd,su2}_{L,00} = 0.997688$$

			u_L	d_L	u_R	d_R
$SU(2)_L$	$\hat{g}^{Z,a}$	:	0.498844	−0.498844	0.0012459	−0.000026196
$SU(2)_R$	$\hat{g}^{Z,b}$	:	10^{-23}	10^{-26}	0.0012459	−0.000026196
$SO(5)/SO(4)$	$\hat{g}^{Z,c}$	:	10^{-12}	10^{-13}	−0.0024918	0.000052392
$SU(2)_{\rm eff}$	$\hat{g}^{Z,su2}$	:	0.498844	−0.498844	10^{-11}	10^{-13}

$$W : \quad \frac{g_w\, \hat{g}^{W\beta}_{L,00}}{\sqrt{2}} \left(T^1_{\rm eff} + i\, T^2_{\rm eff}\right) ,$$

$$Z : \quad \frac{g_w\, h^\beta}{\cos\theta^0_W} \left(T^3_{\rm eff} - \frac{\sin^2\theta^0_W}{h^\beta}\, Q_{\rm EM}\right)$$

Oblique corrections

$$\alpha_* S^\beta = \frac{\sin^2 2\theta_W^0}{m_Z^2}\left\{\Pi_{ZZ}^\beta(m_Z^2) - \Pi_{ZZ}^\beta(0) - \frac{\cos 2\theta_W^0 + h^\beta - 1}{\sin\theta_W^0 \cos\theta_W^0}\Pi_{Z\gamma}^\beta(m_Z^2) - \left(1 + \frac{h^\beta - 1}{\cos^2\theta_W^0}\right)\Pi_{\gamma\gamma}^\beta(m_Z^2)\right\}$$

$$\alpha_* T^\beta = \frac{1}{m_Z^2\cos^2\theta_W^0}\Pi_{WW}^\beta(0) - \frac{1}{m_Z^2}\Pi_{ZZ}^\beta(0)$$

Note : $\boldsymbol{m_Z \cos\theta_W^0 \neq m_W^{\mathbf{tree}}}$

Oblique corrections

$$\alpha_* S^\beta = \frac{\sin^2 2\theta_W^0}{m_Z^2}\left\{\Pi_{ZZ}^\beta(m_Z^2) - \Pi_{ZZ}^\beta(0) - \frac{\cos 2\theta_W^0 + h^\beta - 1}{\sin\theta_W^0 \cos\theta_W^0}\Pi_{Z\gamma}^\beta(m_Z^2) - \left(1 + \frac{h^\beta - 1}{\cos^2\theta_W^0}\right)\Pi_{\gamma\gamma}^\beta(m_Z^2)\right\}$$

$$\alpha_* T^\beta = \frac{1}{m_Z^2\cos^2\theta_W^0}\Pi_{WW}^\beta(0) - \frac{1}{m_Z^2}\Pi_{ZZ}^\beta(0)$$

Note : $m_Z \cos\theta_W^0 \neq m_W^{\rm tree}$

β	S^β	T^β	U^β
(ν_e, e)	0.0010	0.0126	3.7×10^{-6}
(ν_μ, μ)	0.0009	0.0122	3.7×10^{-6}
(ν_τ, τ)	0.0016	0.0129	3.7×10^{-6}
(u, d)	0.0028	0.0382	1.1×10^{-5}
(c, s)	0.0026	0.0360	1.1×10^{-5}
(t, b)	0.0013	0.0058	7.9×10^{-6}
total	0.010	0.12	0.00004
per level	8.4×10^{-4}	9.7×10^{-3}	3.4×10^{-6}

$\theta_H = 0.1$, $m_{\rm KK} = 13\,{\rm TeV}$, $n, \ell = 0 \sim 12$

Corrections : small

PDG: RPP 2022

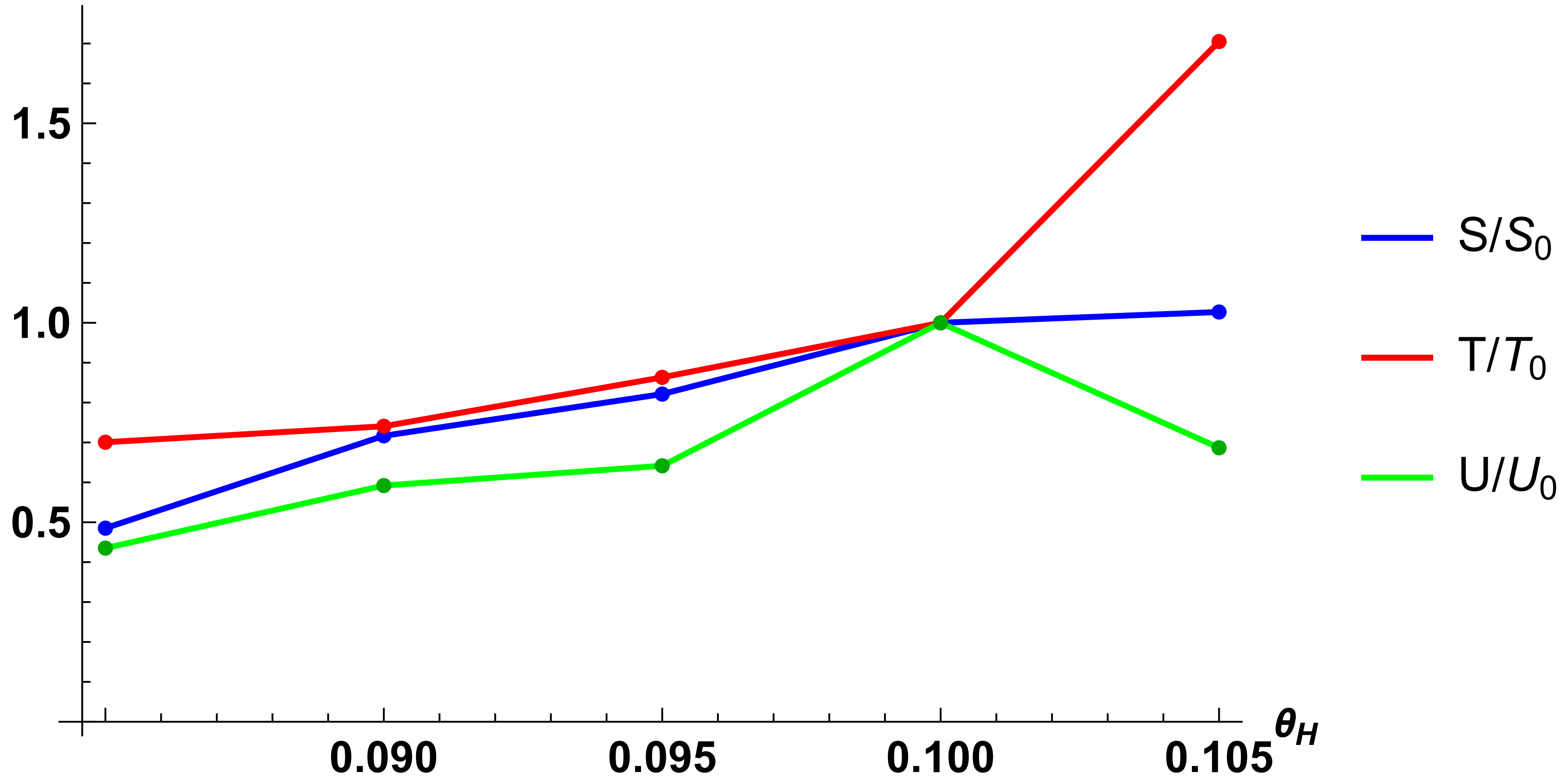
$$S^{RPP} = -0.02 \pm 0.10$$
$$T^{RPP} = 0.03 \pm 0.12$$
$$U^{RPP} = 0.01 \pm 0.11$$

Need to include

$S^{(n)}, T^{(n)}, U^{(n)}$

for $W^{(n)}, Z^{(n)}, \gamma^{(n)}$

θ_H -dependence



Summary

GUT inspired GHU in RS space

Coupling sum rules

$$A_0 = hB \ , \quad D_0 = 2A_0 \ , \quad D_1 = 2A_1$$

Oblique corrections : small

$$S = 0.01 \ , \quad T = 0.12 \ , \quad U = 0.00004$$

$$(n_{\max} = 12)$$

(Need to include $W^{(n)}, Z^{(n)}, \gamma^{(n)}$)