~~***Boltzmann or Bogoliubov?***~~

~~A Case of Gravitational Higgs Production~~

Is the Higgs Condensate or Fluctuation?

Kunio Kaneta

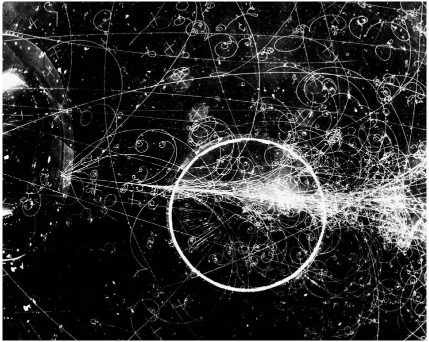
Tokyo Woman's Christian University

+ Kin-ya Oda (TWCU) [2304.12578]

+ Sung Mook Lee (Yonsei U.) and Kin-ya Oda (TWCU) [JCAP09(2022)018; 2206.10929]

Higgs as a Probe of New Physics at Osaka, June 9, 2023

Outline



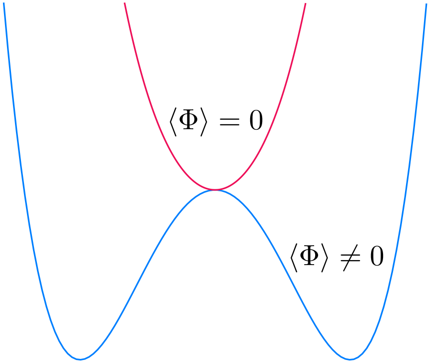
[from FNAL/DOE/NSF]

1. Introduction
2. Condensate or Fluctuation?
3. Phase Space Distribution and Higgs Decay
4. Summary and Outlook

1. Introduction

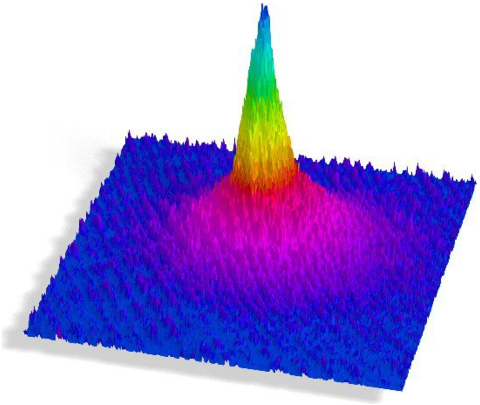
Higgs Phase Transition

Higgs Potential $V(\Phi)$



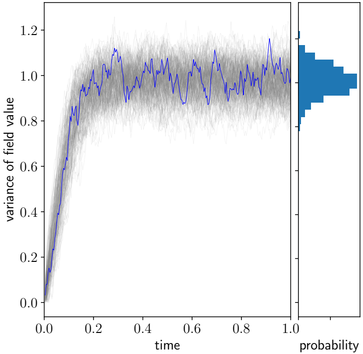
- We live in the broken phase
- Should have been broken since before BBN
- Perhaps symmetric phase above T_{EW}
- $V(\Phi) = V_{T=0}(\Phi) + V_T(\Phi)$ seems good approximation during RD
- But, how about before RD epoch?
- Was the universe in the broken phase or symmetric phase during and after inflation?

EWSB in Two Ways during Inflation



[from Zimmermann group at Toront U.]

- Primordial Higgs condensate
- Exists before/during inflation



- Stochastic Higgs fluctuation
- Excited during inflation

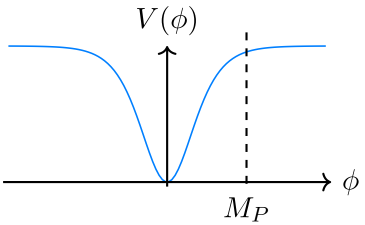
Both characterized by $\langle|\Phi|^2\rangle \neq 0$; **can we distinguish the difference?**

2. Condensate or Fluctuation?

Setup

$$S = \int d^4x\sqrt{-g}\left[\mathcal{L}_{\text{grav}} + \mathcal{L}_{\text{inf}} + \mathcal{L}_{\text{Higgs}}\right], \quad \begin{cases} \mathcal{L}_{\text{inf}} & = & \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) \\ \mathcal{L}_{\text{Higgs}} & = & g^{\mu\nu}(D_\mu\Phi)^\dagger(D_\nu\Phi) - V(\Phi) \end{cases}$$

Inflaton sector:



- T-model:
  $V(\phi) = 6\lambda_\phi M_P^4 \tanh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)$
- $\lambda_\phi \simeq 10^{-11} \Rightarrow m_\phi \simeq 10^{13}$ GeV

Higgs sector:

- Consider the simplest (negligible non-minimal coupling)

$$V(\Phi) = -\mu^2|\Phi|^2 + \lambda(\Phi^\dagger\Phi)^2,$$

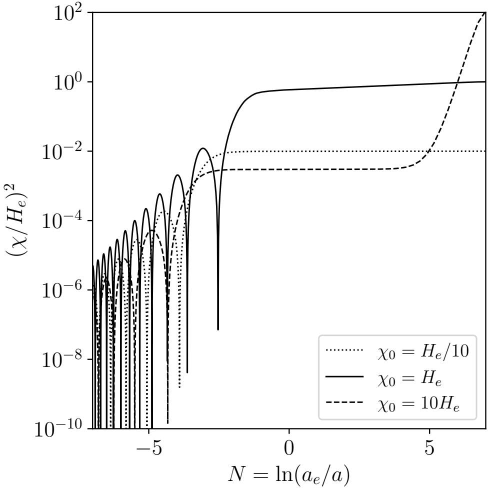
$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\chi_1 + i\chi_2 \\ \chi_3 + i\chi_4\end{pmatrix}$$

- In what follows, we may replace

$$\Phi \to \chi/\sqrt{2}$$

without loss of generality

Primordial Higgs Condensate



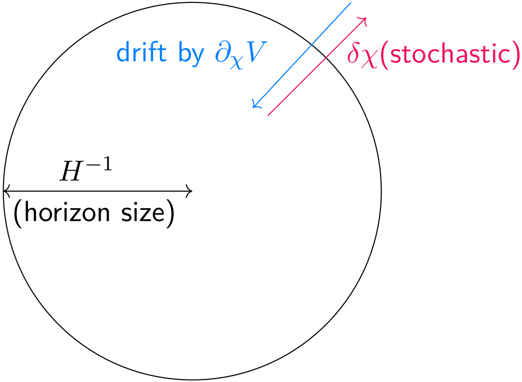
- Suppose Higgs formed a Bose-Einstein condensate before/during inflation
- $\chi(t_{\text{ini}}) = \chi_0$ and zero momentum (no excitations)
- Follows equation of motion:

$$\ddot{\chi} + 3H\dot{\chi} + \partial_\chi V(\chi) = 0$$

- $\langle \chi^2 \rangle_{\text{t-ave}} \neq 0$
- Phase space distribution:

$$f_{\text{cond}}(p,t) = \frac{\rho_{\text{cond}}(t)}{m_{\chi,\text{eff}}}(2\pi)^3\delta^3(\vec{p})$$

Stochastic Higgs Fluctuation



- Suppose $\chi(t_{\text{ini}}) = 0$ (no condensate)
- Decompose $\chi = \overline{\chi}$ (IR) $+ \delta\chi$ (UV)
- EoM becomes the Langevin Eq.:

$$\dot{\overline{\chi}} \simeq -\frac{\partial_{\overline{\chi}} V(\overline{\chi})}{3H} + f^{\delta\chi}$$

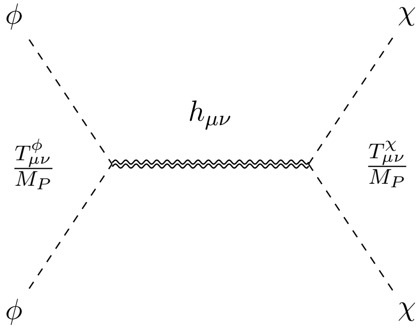
- $\delta\chi$ induces the Gaussian noise $f^{\delta\chi}$:

$$\langle f^{\delta\chi}(t_1) f^{\delta\chi}(t_2)\rangle = \frac{H_e^3}{4\pi^2}\delta(t_1 - t_2)$$

- For a sufficiently long time, $\overline{\chi}$ reaches

$$\langle\overline{\chi}^2\rangle \sim \frac{H_e^2}{\sqrt{\lambda}} \quad \Rightarrow \quad m^2_{\chi,\text{eff}} \simeq 3\lambda\langle\overline{\chi}^2\rangle$$

Gravitational Particle Production from Inflaton Scattering



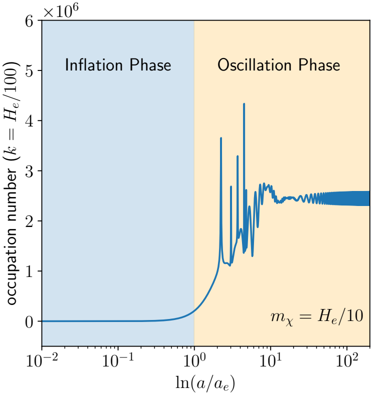
- Gravitational χ production during oscillation phase
- Phase space distribution:

$$f_{\chi}(k) \simeq \frac{9\pi}{64}\left(\frac{H_e}{m_\phi}\right)^3\left(\frac{m_\phi}{k}\right)^{9/2}$$

$$\begin{cases} k: \text{comoving momentum} \\ p: \text{physical momentum} \\ p = k/a(t) \end{cases}$$

- Energy conservation:
  $p = m_\phi \longleftrightarrow k = a m_\phi > m_\phi$

Gravitational Particle Production from Phase Transition



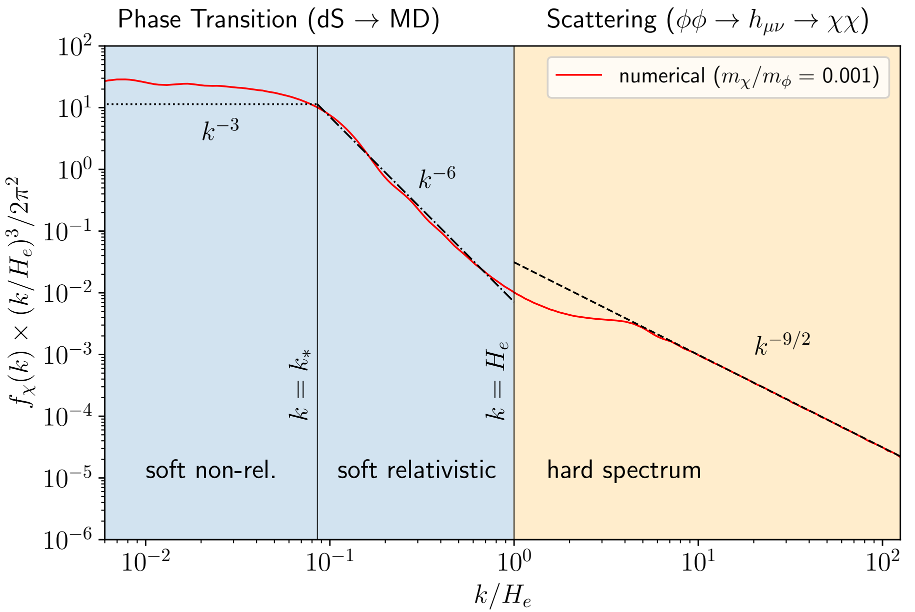
- Particle production through abrupt phase transition (dS $\rightarrow$ MD)
- Initial condition: Bunch-Davies vacuum (no particle)
- $\omega_\chi^{(\mathrm{dS})} \neq \omega_\chi^{(\mathrm{MD})}$, producing particles
- Phase space distribution:

$$f_\chi(k) \simeq \begin{cases} \frac{9}{64}\left(\frac{H_e}{k}\right)^6 & (k > k_*) \\ \frac{9}{32}\left(\frac{H_e}{m_{\chi,\mathrm{eff}}}\right)\left(\frac{H_e}{k}\right)^3 & (k < k_*) \end{cases}$$

$$k_* = \left(\frac{H_e^2 m_{\chi,\mathrm{eff}}}{2}\right)^{1/6}$$

- Only $k < H_e$ may be excited

Two Sources of Gravitational Particle Production



3. Phase Space Distribution and Higgs Decay

Phase Space Distribution

$\langle|\Phi|^2\rangle \neq 0$ in both cases, but $f_h(k)$ is very different:

Condensate:

$$f_h(p,t) = \frac{\rho_{\mathsf{cond}}(t)}{m_{h,\mathrm{eff}}}(2\pi)^3\delta^3(\vec{p})$$

$$m_{h,\mathrm{eff}} \approx \sqrt{\lambda|\Phi_0|^2}, \quad \rho_{\mathsf{cond}}(t) \approx \begin{cases} \rho_{h,0} & (H > m_{h,\mathrm{eff}}) \\ \rho_{h,0}(a/a_{\mathrm{osc}})^{-4} & (H < m_{h,\mathrm{eff}}) \end{cases}$$

Fluctuation:

$$f_h(p,t) = f_{\mathsf{hard}}(p,t) + f_{\mathsf{sfot,R}}(p,t) + f_{\mathsf{soft,NR}}(p,t)$$

$$\approx \begin{cases} \frac{9\pi}{64}\left(\frac{H_e}{m_\phi}\right)^3\left(\frac{m_\phi}{k}\right)^{9/2} & (k > m_\phi \sim H_e) \\ \frac{9}{64}\left(\frac{H_e}{k}\right)^6 & (k > k_*) \\ \frac{9}{32}\left(\frac{H_e}{m_{\chi,\mathrm{eff}}}\right)\left(\frac{H_e}{k}\right)^3 & (k < k_*) \end{cases}$$

Do they have any phenomenological implication?

Higgs Decay

- Suppose λ is a free parameter $\Rightarrow$ When $m_{h,\text{eff}} > T_{\text{RH}}$, Higgs is very non-thermal
- If $\sqrt{\lambda} \gtrsim g, y_t$, $h \to WW, ZZ, t\bar{t} \Rightarrow$ Take Γ_h as a free parameter

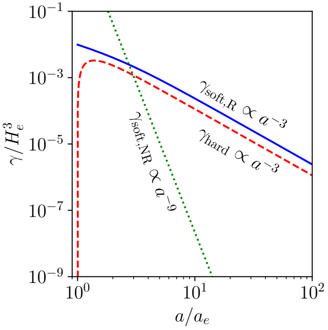
Condensate:

- $\dot{\rho}_h + 4H\rho_h = -\Gamma_h \rho_h \quad \Rightarrow \quad \rho_h(t) = \rho_{h,0} \left(\frac{a}{a_{\text{osc}}}\right)^{-4} e^{-\Gamma_h t}$
- Decays away at $t_{\text{dec}} = \Gamma_h^{-1} \quad \Rightarrow \quad \frac{a_{\text{dec}}}{a_e} \simeq \left(\frac{3H_e}{2\Gamma_h}\right)^{2/3}$

Higgs Decay

- Suppose λ is a free parameter $\Rightarrow$ When $m_{h,\mathrm{eff}} > T_{\mathrm{RH}}$, Higgs is very non-thermal
- If $\sqrt{\lambda} \gtrsim g, y_t$, $h \to WW, ZZ, t\bar{t} \Rightarrow$ Take Γ_h as a free parameter

Fluctuation:



- $\dot{n}_h + 3Hn_h = -\gamma\Gamma_h$

$$\gamma = \int \frac{d^3p}{(2\pi)^3} \frac{m_{h,\mathrm{eff}}}{\sqrt{m_{h,\mathrm{eff}}^2 + p^2}} f_h(p,t)$$

- $\gamma_{\mathrm{soft,R}}$ is dominant in decaying
- $n_h(a_{\mathrm{dec}}) = 0 \quad \Rightarrow \quad \dfrac{a_{\mathrm{dec}}}{a_e} \simeq \left(\dfrac{6H_e}{\Gamma_h}\right)^{2/3}$

4. Summary

Summary and Outlook

- EWSB by either the primordial Higgs condensate or the stochastic fluctuation
- They may be distinguished by **phase space distribution**
- Higgs is non-thermal when $T_{\rm RH} < m_{h,\rm eff} \sim \sqrt{\lambda} H_e$
- Demonstrated the perturbative Higgs decay as a possible application
- Could be many implications (inf. models, thermal effect, DM, BAU, GWs, etc.)