Relation between higher-dimensional gauge theories and gravitational waves from first-order phase transitions

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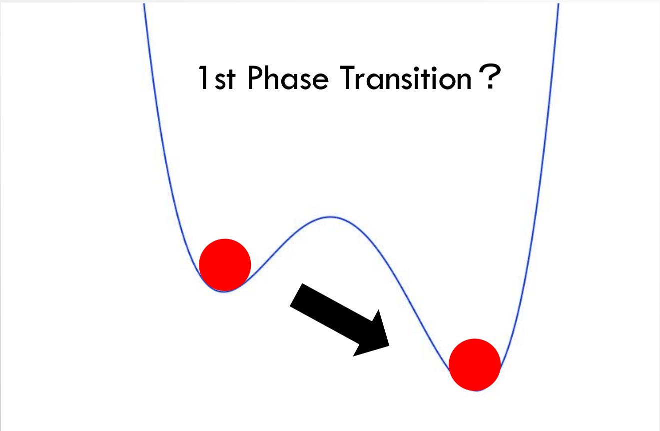
1. Introduction

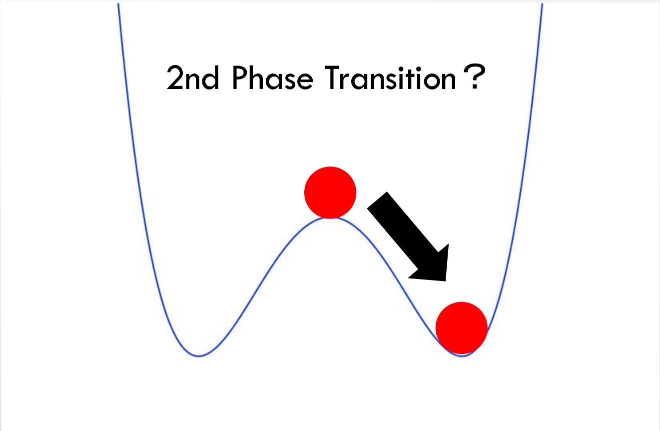
Electroweak Phase Transition (EWPT) occurs

in the universe at EW scale ($\mathcal{O}(10^2)$ GeV)

However

We do not know the type of PT





The types are determined by scalar potential and thermal effect

1. Introduction

If first-order phase transition occurs,

Gravitational Waves are generated !!

[Kosowsky, Turner, Watkins (1992)]

If 1st order PT occurs in Beyond the Standard Model (BSM),

Parameters in BSM and the spectrum of GW

must be related !!

As a candidate for BSM,
we assume a higher-dimensional gauge theory

2. 5D SU(3) gauge theory

5D SU(3) gauge theory [Haba, Hosotani, Kawamura, Yamashita (2004)]

- 4D Minkowski spacetime $\times$ S^1/Z_2
  - Periodic b.c. : $A_M(x^\mu, y+2\pi R) = U A_M(x^\mu, y) U^\dagger$
  - Z_2 parity :

$$A_\mu(x^\mu, -y) = P_0 A_\mu(x^\mu, y) P_0^{-1}$$
$$A_\mu(x^\mu, \pi R - y) = P_1 A_\mu(x^\mu, \pi R + y) P_1^\dagger$$
$$A_y(x^\mu, -y) = -P_0 A_y(x^\mu, y) P_0^{-1}$$
$$A_y(x^\mu, \pi R - y) = -P_1 A_y(x^\mu, \pi R + y) P_1^\dagger$$

4D gauge fields

5th gauge fields

- gauge group : SU(3)
- Extra components of gauge field = Scalar field in 4D

$$A_y \Rightarrow \phi$$

2. 5D SU(3) gauge theory

Effective potential

$$\langle A_y \rangle = \frac{a}{g_4 R}\frac{\lambda^6}{2}$$

$$V_{\rm eff}(a,T) = V_{\rm eff}^{T=0}(a) + V_{\rm eff}^{T\neq 0}(a,T)$$

[Maru, Takenaga (2005)]

- Example of the potential

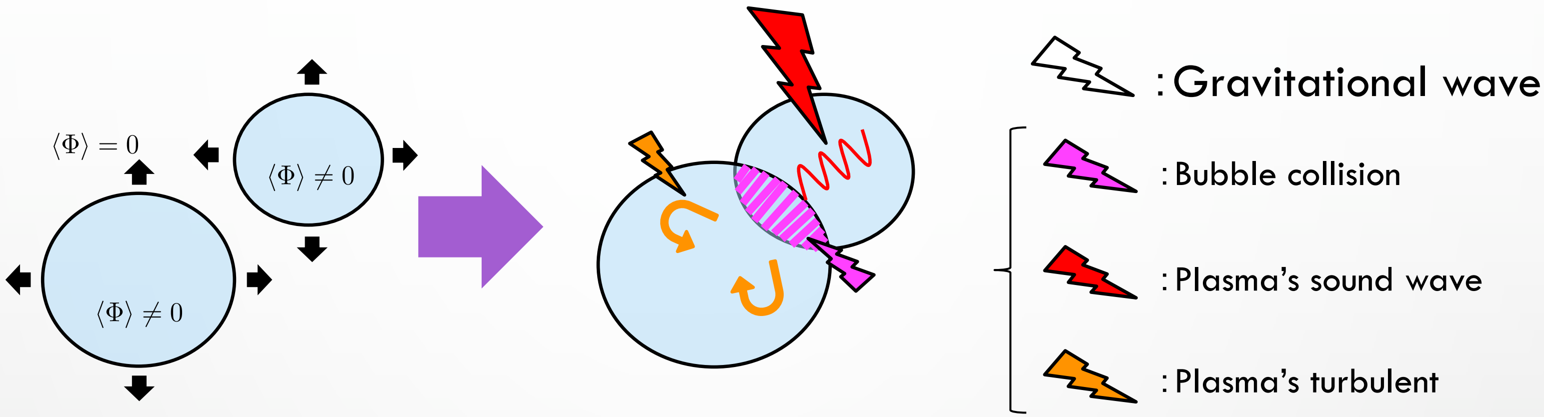
$$V_{\rm eff}^{T=0}(a) = -3C\sum_{n=1}^{\infty}\frac{1}{n^5}\Big(\cos(2\pi na) + 2\cos(\pi na)\Big) + \cdots$$

$$V_{\rm eff}^{T\neq 0}(a,T) = 2C\sum_{l=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{[n^2 + l^2/(2\pi RT)^2]^{5/2}}(-3)\Big(\cos(2\pi na) + 2\cos(\pi na)\Big) + \cdots$$

$$C = 3/(64\pi^6 R^4)$$

- R : Radius of compact space S^1
- Tree-level potential vanishes because of gauge symmetry

3. GW from First-Order Phase Transition



- Nucleation temperature T_{nuc}

$$\left.\frac{\Gamma}{H^4}\right|_{T=T_{\rm nuc}} \simeq 1 \quad \Rightarrow \quad \left.\frac{S_3(T)}{T}\right|_{T=T_{\rm nuc}} \simeq 140$$

- Normalized latent heat

$$\alpha = \frac{\epsilon(T_{\rm nuc})}{\rho_{\rm rad}(T_{\rm nuc})}$$

- Duration of PT

$$\tilde{\beta} = \frac{\beta}{H(T_{\rm nuc})} = T\frac{d}{dT}\left(\frac{S_3}{T}\right)\Bigg|_{T=T_{\rm nuc}}$$

GW spectrums depend on these parameters

4. Results

- We investigate two cases

Case 1 : $N_f^{(+)} = 3$, (otherwise) = 0

~~Case 2 : $N_{ad}^{(+)} = 2,\ N_f^{(-)} = 8,\ N_f^{s(+)} = 4,\ N_f^{s(-)} = 2$~~

[Haba, Hosotani, Kawamura, Yamashita (2004)]
[Maru, Takenaga (2005)]

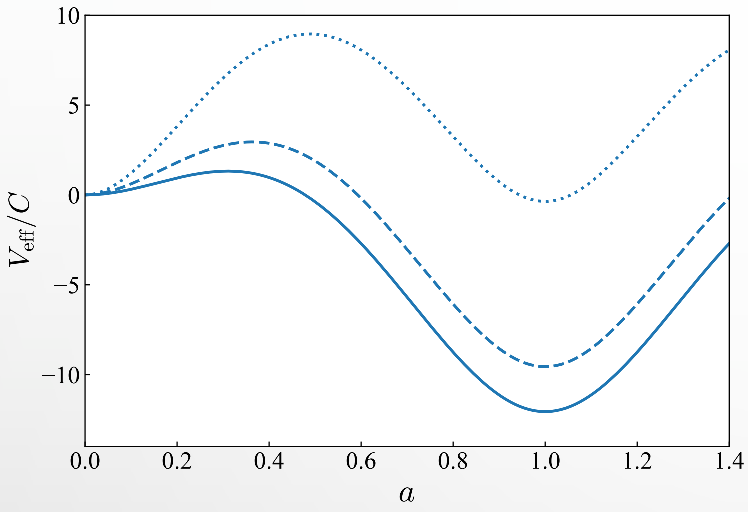
- Use the numerical results of the spectrum of GW

[Caprini et al. (2015)]

- Parameters are compact scale and coupling in 4D

$$\langle A_y \rangle = \frac{a}{g_4 R} \frac{\lambda^6}{2}$$

4. Results



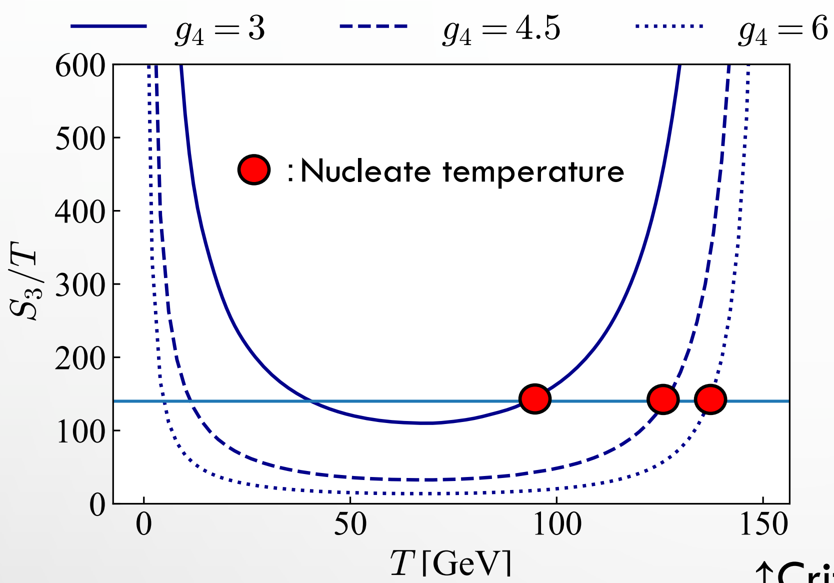
←Potential at critical temperature

←Potential at nucleate temperature

←Potential at zero temperature

- Introduce only fermions in fundamental representation
- A potential barrier even at zero temperature→1st PT
- critical temperature: $T_c = 150$ GeV @ $R = 10^{-3}$

4. Results



$\leftarrow R = 10^{-3}$

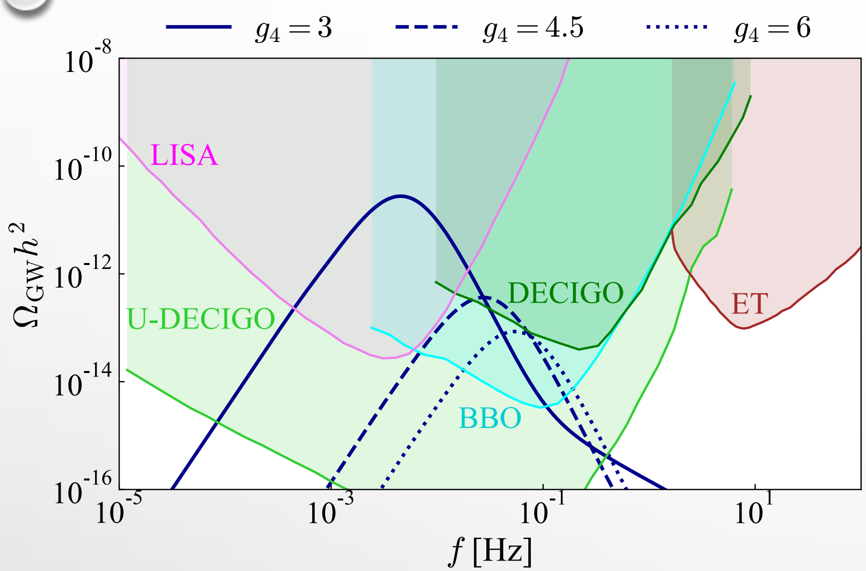
↑Critical temperature

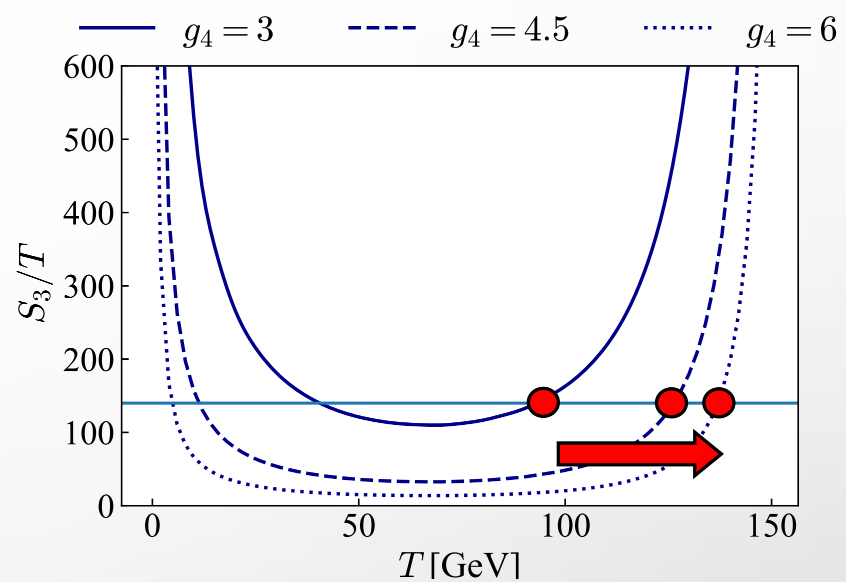
Definition of nucleate temperature

$$\left.\frac{S_3(T)}{T}\right|_{T=T_{\rm nuc}} \simeq 140$$

- Whether 1st PT occurs depends on the coupling
- $g_4 \leq 3 \longrightarrow$ 1st PT does not occur
- Increase $g_4 \longrightarrow T_{nuc}$ approaches T_c

4. Results



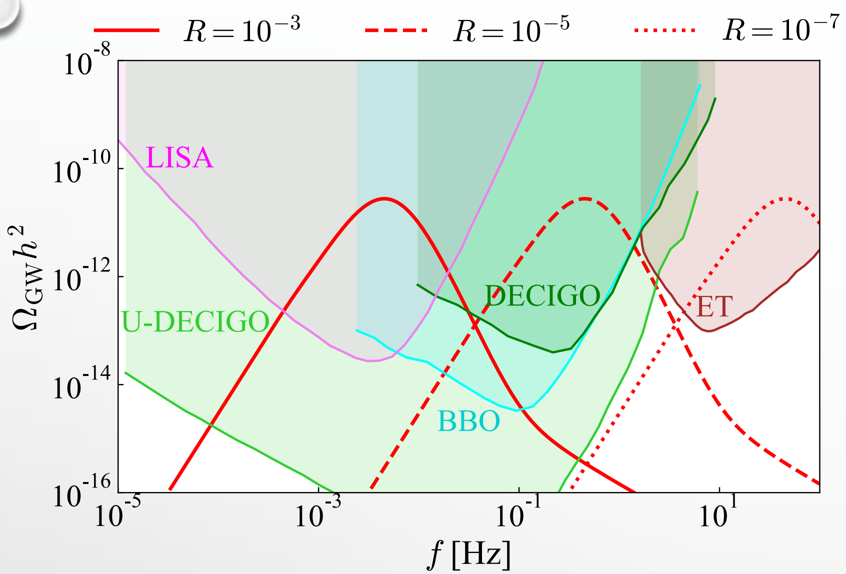


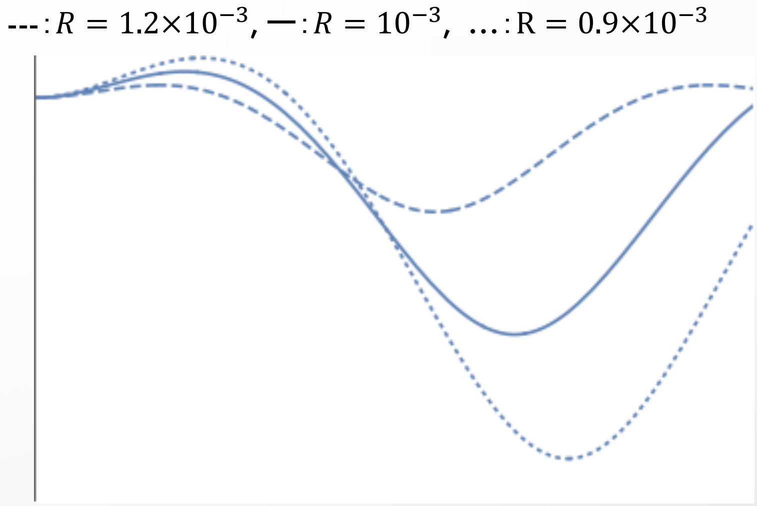
Increase g_4 → Spectrums move to lower right（left panel）

- Increase frequency →Increase T_{nuc}, the tilt of S_3/T（right panel）
- Decrease energy density → decrease latent heat α

$$\alpha = \frac{\epsilon(T_{\rm nuc})}{\rho_{\rm rad}(T_{\rm nuc})}$$

4. Results





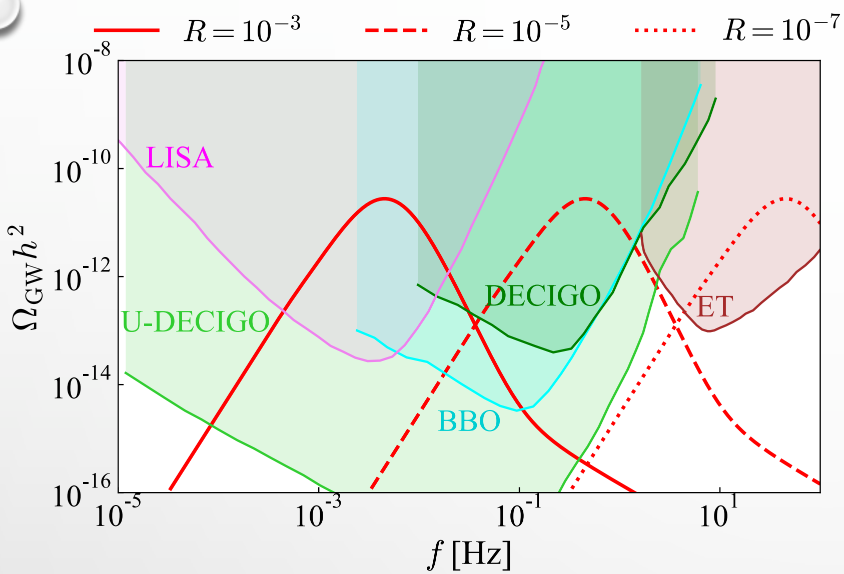
Increase R^{-1} → Spectrums shifts sideways（left panel）

- Energy density → Potential changes with similarity（right panel）

  → α, $\tilde{\beta}$ do not depend on R

- Increase frequency → Because $T_{nuc} \sim R^{-1}$ increase

4. Results

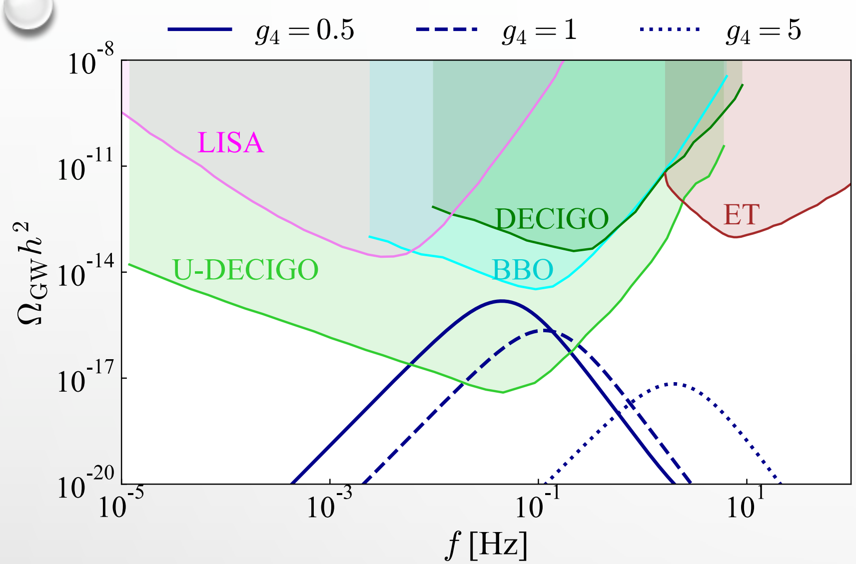


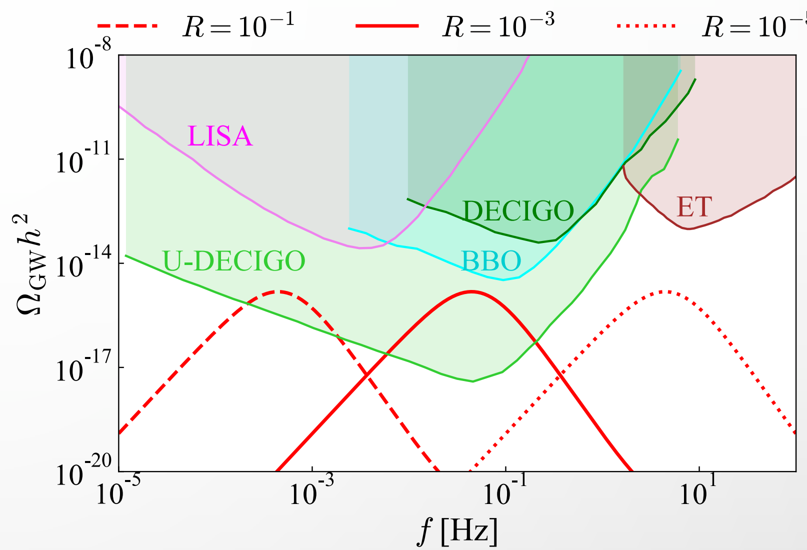
	R [GeV^{-1}]	v [GeV]
LISA	$10^{-4} - 10^{-1}$	$10^{0} - 10^{3}$
DECIGO	$10^{-6} - 10^{-3}$	$10^{2} - 10^{5}$
ET	$10^{-8} - 10^{-6}$	$10^{5} - 10^{7}$

Possibility of verification LISA, DECIGO, Einstein telescope (ET)

Future Work: Extension to electroweak 1st PT

4. Results





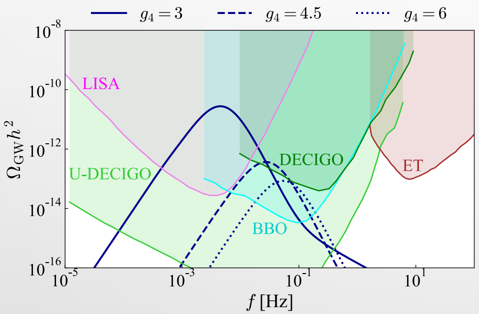
Case 2

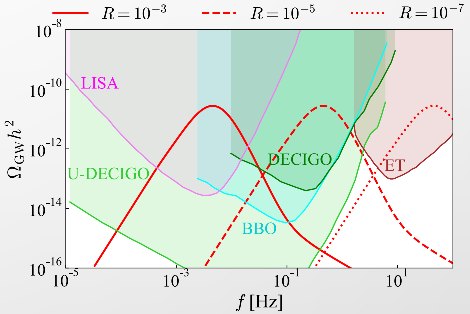
- Energy density is low because potential barrier (latent heat) is small
- In $g_4 \leq 3$, GW is detected by U-DECIGO ?

→At $g_4 = 0.5$, 10^{-4} GeV$\leq R \leq 10^{-1}$ GeV

5. Summary

- We focus on a 5D SU(3) gauge theory and its 1st phase transition
- Relation between higher-dimensional gauge theory and 1st PT mainly depends on compact scale R and coupling in 4D g_4
- Possibility of verification in future experiments (LISA, DECIGO, ET…)



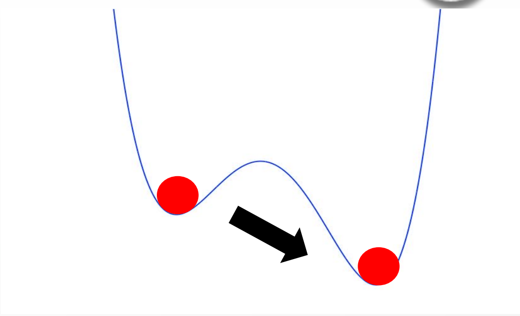


◆Extension to electroweak 1st PT or another physics

- ✓ Application to gauge-Higgs unification
- ✓ Application to grand unified theory

Back up

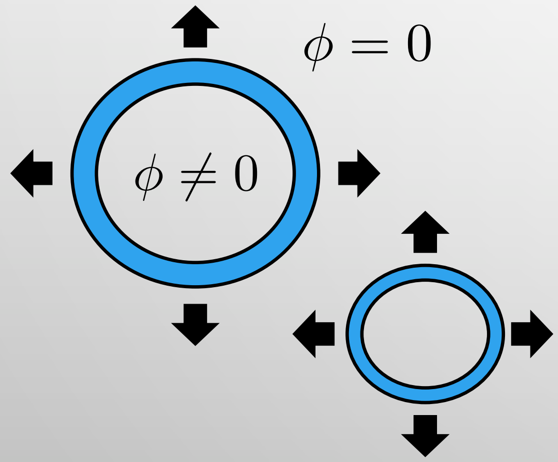
Appendix



First-order Phase Transition

- The potential barrier exists between two minima
- The value of potential becomes discontinuous (the tunneling effect)

in the universe



- Bubbles ($\Phi \neq 0$) appear
- The bubbles expand and collide with other bubbles

Appendix

orbifold

$$S^1/Z_2$$

Periodic boundary condition

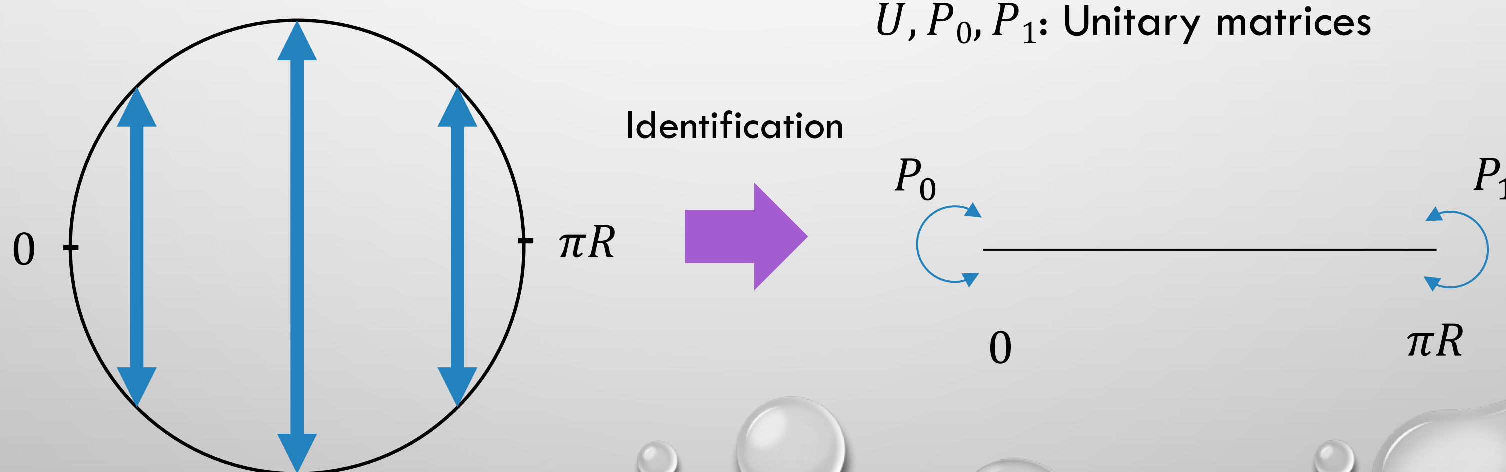
$$A_M(x^\mu, y+2\pi R) = U A_M(x^\mu, y) U^\dagger$$

Z_2 Parity

$$A_\mu(x^\mu, -y) = P_0 A_\mu(x^\mu, y) P_0^{-1}$$

$$A_\mu(x^\mu, \pi R - y) = P_1 A_\mu(x^\mu, \pi R + y) P_1^\dagger$$

U, P_0, P_1: Unitary matrices



Appendix

Symmetry Breaking Pattern

$P_0 = P_1 = \mathrm{diag}(1, 1, -1)$

1. Explicitly breaking by P_0, P_1

$$SU(3) \rightarrow SU(2)\times U(1)$$

2. Spontaneously breaking by the VEV of 5th gauge field

[Hosotani (1983, 1989)]

$$SU(2)\times U(1) \rightarrow U'(1)\times U(1) \text{ or } U(1)$$

SU(2)

$$A_\mu = \left(\begin{array}{cc|c} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{array}\right)$$

U(1)

Scalar doublet

$$A_y = \left(\begin{array}{cc|c} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ \hline (+,+) & (+,+) & (-,-) \end{array}\right)$$

VEV

$(P_0, P_1) = (+,+)$: zero modes of massless fields

Appendix

$$W = P\exp\left(ig\oint_{S^1} dy A_y\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi a) & i\sin(\pi a) \\ 0 & i\sin(\pi a) & \cos(\pi a) \end{pmatrix}$$

$SU(3)$ $\xrightarrow{\text{orbifold}}$ $SU(2)\times U(1)$ $\xrightarrow{\text{spontaneous}}$ change by a

- $a = 0$: $SU(2)\times U(1)$
- $a = 1$: $SU(2)\times U(1) \rightarrow U'(1)\times U(1)$
- $a \neq 0,1$: $SU(2)\times U(1) \rightarrow U(1)$

Appendix

Periodic boundary condition & Z_2 parity

➡ Kaluza-Klein (KK) expansion

ex.)

$$A_\mu\left(x^\mu, y\right)_{(+,+)} = \frac{1}{\sqrt{2\pi R}} A_\mu^{(0)}(x^\mu)_{(+,+)} + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} A_\mu^{(n)}(x^\mu)_{(+,+)} \cos\left(\frac{ny}{R}\right)$$

We find KK mass from E.O.M

$$(\Box + \partial_y \partial_y) A_\mu = 0$$

$$m_{KK}^2 = \frac{n^2}{R^2}, \ \frac{(n \pm a)^2}{R^2}, \cdots$$

n : KK mode (integer)

Appendix

Effective potential (zero temperature)

$$V_{\text{eff}}^{T=0}(a) \equiv \underline{V_{\text{eff}}^{g}(a)} + \underline{V_{\text{eff}}^{m}(a)}$$

gauge fields　matter fields

$$V_{\text{eff}}^{g}(a) = -3C \sum_{n=1}^{\infty} \frac{1}{n^5} \Big(\cos(2\pi n a) + 2\cos(\pi n a) \Big)$$

$$\begin{aligned} V_{\text{eff}}^{m}(a) = &(4N_f^{(+)} - 2N_f^{s(+)})C \sum_{n=1}^{\infty} \frac{1}{n^5} \cos(\pi n a) + (4N_f^{(-)} - 2N_f^{s(-)})C \sum_{n=1}^{\infty} \frac{1}{n^5} \cos(\pi n(a-1)) \\ &+ 4N_{\text{ad}}^{(+)} C \sum_{n=1}^{\infty} \frac{1}{n^5} \Big(\cos(2\pi n a) + 2\cos(\pi n a) \Big) \\ &+ 4N_{\text{ad}}^{(-)} C \sum_{n=1}^{\infty} \frac{1}{n^5} \Big(\cos(2\pi n(a-1/2)) + 2\cos(\pi n(a-1)) \Big) \end{aligned}$$

$N_f^{(+)}$など：degree of freedom

$C = 3/(64\pi^6 R^4)$

Appendix

Effective potential (finite temperature) [Maru, Takenaga (2005)]

Compactify S^1 in the direction of time

$\rightarrow$Introduce T as its radius.

$$
\begin{aligned}
V_{\rm eff}^{T\neq 0}(a,T) = 2C\sum_{l=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{[n^2+l^2/(2\pi R T)^2]^{5/2}}\Big[\Big(-3+4(-1)^l N_{\rm ad}^{(+)}\Big)\Big(\cos(2\pi n a)+2\cos(\pi n a)\Big) \\
+4(-1)^l N_{\rm ad}^{(-)}\Big(\cos(2\pi n(a-1/2))+2\cos(\pi n(a-1))\Big) \\
+\Big(4(-1)^l N_f^{(+)}-2N_f^{s(+)}\Big)\cos(\pi n a) \\
+\Big(4(-1)^l N_f^{(-)}-2N_f^{s(-)}\Big)\cos(\pi n(a-1))\Big]
\end{aligned}
$$

Appendix

Introduce matter fields [Haba, Hosotani, Kawamura, Yamashita (2004)]

$$\phi(x,-y) = \eta P_0 \phi(x,y) \quad , \quad \phi(x,\pi R - y) = \eta' P_1 \phi(x,\pi R + y)\,,$$

$$\psi(x,-y) = \eta P_0 \gamma^5 \psi(x,y) \quad , \quad \psi(x,\pi R - y) = \eta' P_1 \gamma^5 \psi(x,\pi R + y)\,,$$

$$\psi^a(x,-y) = \eta P_0 \gamma^5 \psi^a(x,y) P_0^\dagger \quad , \quad \psi^a(x,\pi R - y) = \eta' P_1 \gamma^5 \psi^a(x,\pi R + y) P_1^\dagger$$

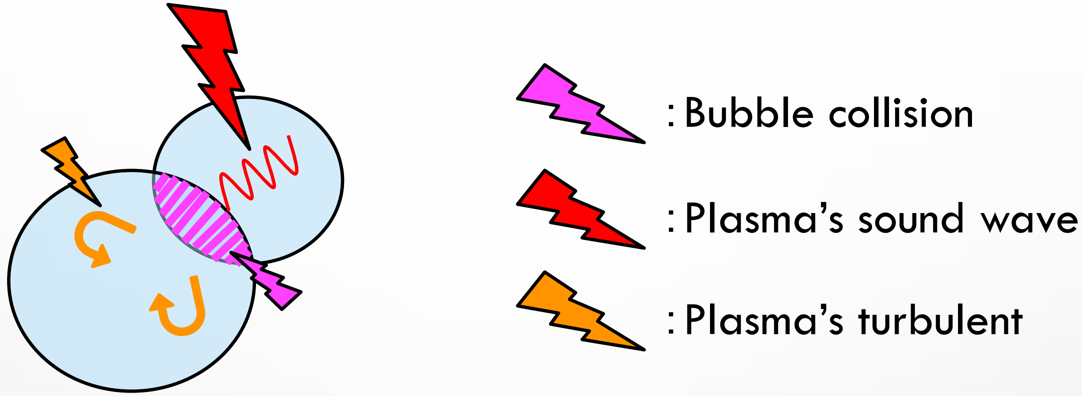
$(\eta, \eta' = \pm)$

ϕ : Scalar in fundamental rep.

ψ : Fermion in fundamental rep.

ψ^a : Fermion in adjoint rep.

3. GW from First Phase Transition



- Contributions to GW is the sum of above three sources
- To computing the spectrum of GW, need numerical calculation
- Approximate calculation has known [Caprini et al. (2015)]

※Plasma's sound wave (main contribution)

$$\Omega_{\rm sw} h^2 = 2.65 \times 10^{-6} \tilde{\beta}^{-1} \left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{\frac{1}{3}} v_w (f/f_{\rm sw})^3 \left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{\frac{7}{2}}$$

$$f_{\rm sw} = 1.9 \times 10^{-5}{\rm Hz}\ v_w^{-1} \tilde{\beta} \left(\frac{T_{\rm nuc}}{100\ {\rm GeV}}\right) \left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

Appendix

Detail for approximate calculation [Caprini et al. (2015)]

- Peak of energy density

$$\Omega_{\rm env}h^2 = 1.67\times10^{-5}\tilde{\beta}^{-2}\left(\frac{\kappa_\varphi\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)\frac{3.8(f/f_\varphi)^{2.8}}{1+2.8(f/f_\varphi)^{3.8}},$$

$$\Omega_{\rm sw}h^2 = 2.65\times10^{-6}\tilde{\beta}^{-1}\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}v_w(f/f_{\rm sw})^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{\frac{7}{2}},$$

$$\Omega_{\rm turb}h^2 = 3.35\times10^{-4}\tilde{\beta}^{-1}\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}v_w\frac{(f/f_{\rm turb})^3}{[1+(f/f_{\rm turb})]^{\frac{11}{3}}(1+8\pi f/h_n)}$$

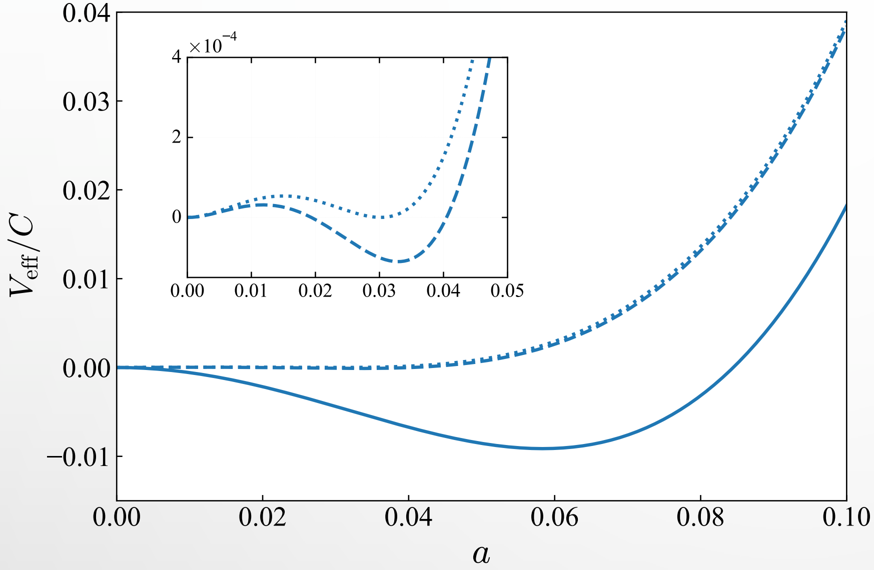
- Peak of gravitational frequency

$$f_{\rm env} = 1.65\times10^{-5}{\rm Hz}\;\tilde{\beta}\left(\frac{0.62}{1.8-0.1v_w+v_w^2}\right)\left(\frac{T_{\rm nuc}}{100\ {\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}},$$

$$f_{\rm sw} = 1.9\times10^{-5}{\rm Hz}\;v_w^{-1}\tilde{\beta}\left(\frac{T_{\rm nuc}}{100\ {\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}},$$

$$f_{\rm turb} = 2.7\times10^{-5}{\rm Hz}\;v_w^{-1}\tilde{\beta}\left(\frac{T_{\rm nuc}}{100\ {\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

Appendix



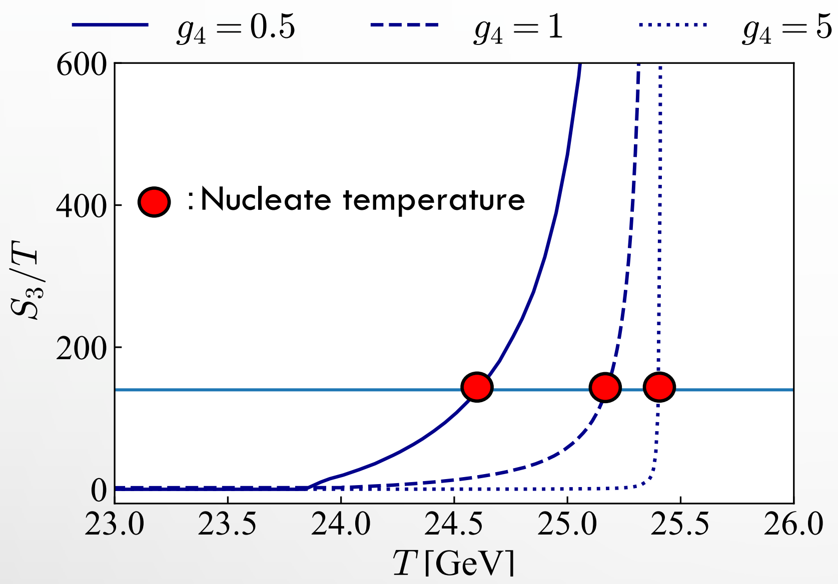
… : potential at critical temperature

--- : potential at nucleate temperature

— : potential at zero temperature

- To realize Higgs mass, introduce several fields
- No potential barrier at zero temperature
- critical temperature : $T_c = 25$ GeV @ $R = 10^{-3}$

Appendix



$\leftarrow R = 10^{-3}$

Definition of nucleate temperature

$$\left.\frac{S_3(T)}{T}\right|_{T=T_{nuc}} \simeq 140$$

- Increase $g_4 \rightarrow T_{nuc}$ approaches T_c
- T_{nuc} slightly change $\rightarrow$ Because of small potential barrier
- Decrease temperature $\rightarrow$ approach to potential at zero temperatute

$\rightarrow S_3/T$ is always less than 140