Electric dipole moment in a complex singlet extension of the Standard Model with degenerate scalars

Based on

Research in progress

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Introduction

One hypothesis to solve baryon asymmetry is **electroweak baryogenesis**

The conditions necessary to generate baryon numbers

Sakharov conditions

1. Baryon number violation
2. C symmetry and CP symmetry violation
3. Departure from thermal equilibrium

- CKM phase cannot fully explain the CP asymmetry
- Strong 1st-order PT gives an upper limit of 70 GeV for the Higgs mass

Model: Complex singlet extension of the SM (CxSM)

- strong 1st order phase transition and gravitational waves
- **CP-violation and electric dipole moment (EDM)**
- feasibility of electroweak baryogenesis

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Electroweak baryogenesis w/ SM

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   → Sphaleron
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   → Chiral gauge interaction, CKM matrix
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・CKM phase cannot fully explain the CP asymmetry

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- Strong 1st-order P the Higgs mass

the SM must be extended

Model: Complex singlet extension of the SM (CxSM)

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- feasibility of electroweak baryogenesis

Model definition

CxSM (Complex singlet extension of the SM)

Barger etal, arXiv:0811.0393

$\cdots$ SM + $SU(2)_L$ singlet complex scalar S

Scalar potential of the standard model Higgs H and the complex scalar S

$$V_0(H,S) = \underbrace{\frac{m^2}{2}H^\dagger H + \frac{\lambda}{4}\left(H^\dagger H\right)^2 + \frac{\delta_2}{2}H^\dagger H|S|^2 + \frac{b_2}{2}|S|^2 + \frac{d_2}{4}|S|^4}_{\text{Global U(1) invariant terms}} + \underbrace{\left(a_1 S + \frac{b_1}{4}S^2 + \text{ H.c. }\right)}_{\text{Soft breaking term}}$$

$$S \to e^{i\phi}S \ (\phi = \text{const.})$$

$m^2, \lambda, \delta_2, b_2, d_2$: real $\qquad$ a_1, b_1 : complex

$$H(x) = \begin{pmatrix} G^+(x) \\ \frac{1}{\sqrt{2}}\left(v + h(x) + iG^0(x)\right) \end{pmatrix}$$

CPV phase derived from singlet scalar

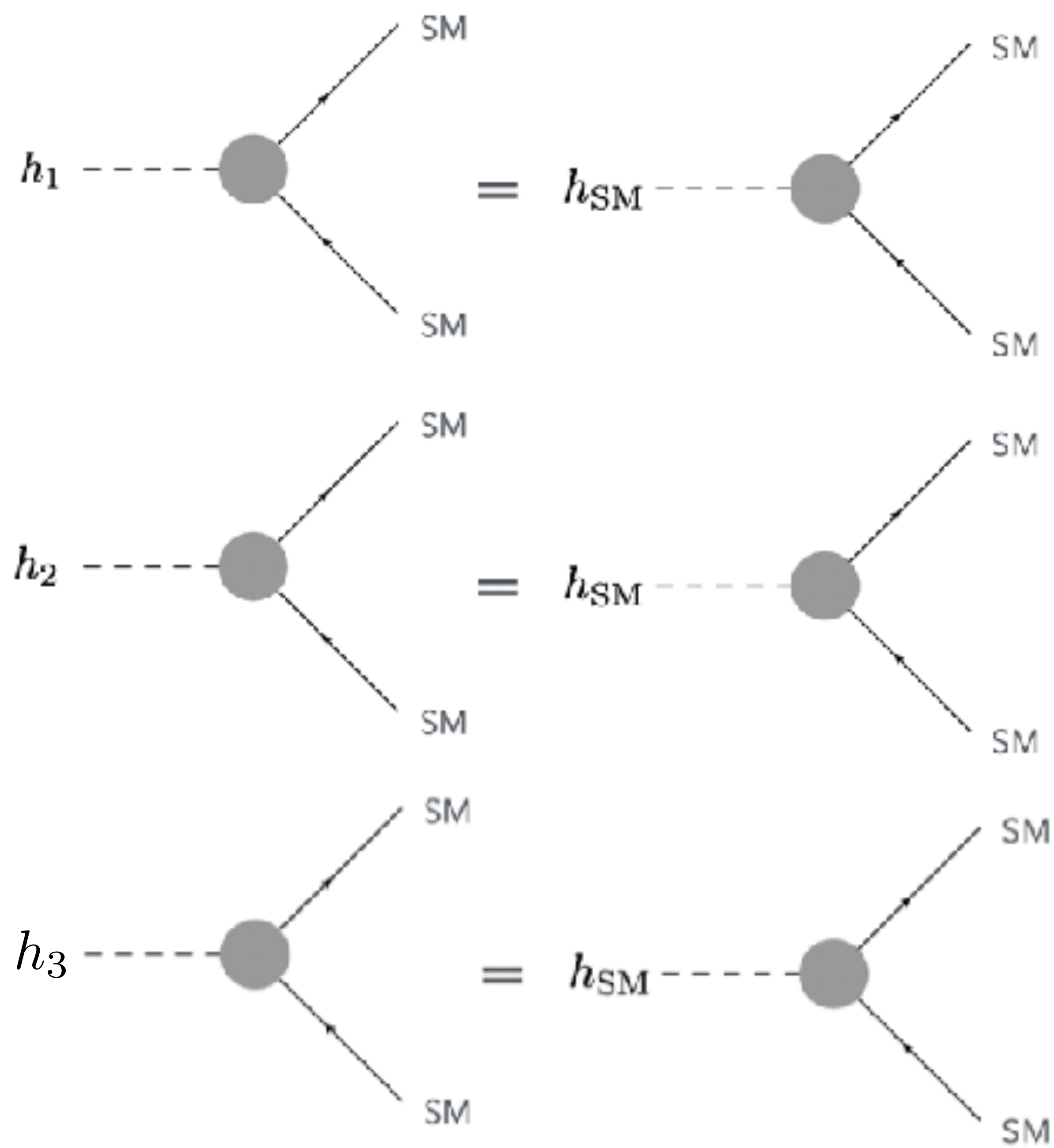
$$S(x) = \frac{1}{\sqrt{2}}\left(v_S^r + iv_S^i + s(x) + i\chi(x)\right) = \frac{1}{\sqrt{2}}\left(|v_S|\,\underline{e^{i\theta_S}} + s(x) + i\chi(x)\right)$$

Degenerate scalar scenario

Cho, CI, Senaha 2022

gauge eigenstates | 3×3 mixing matrix | mass eigenstates

$$(h_{\rm SM}, s, \chi) \xrightarrow{O(\alpha_i)} (h_1, h_2, h_3)$$



$$h_1 = O_{11} h_{\rm SM} + O_{21} s + O_{31} \chi$$
$$h_2 = O_{12} h_{\rm SM} + O_{22} s + O_{32} \chi$$
$$h_3 = O_{13} h_{\rm SM} + O_{22} s + O_{33} \chi$$

Only $h_{\rm SM}$ couples to SM fermions and gauge bosons

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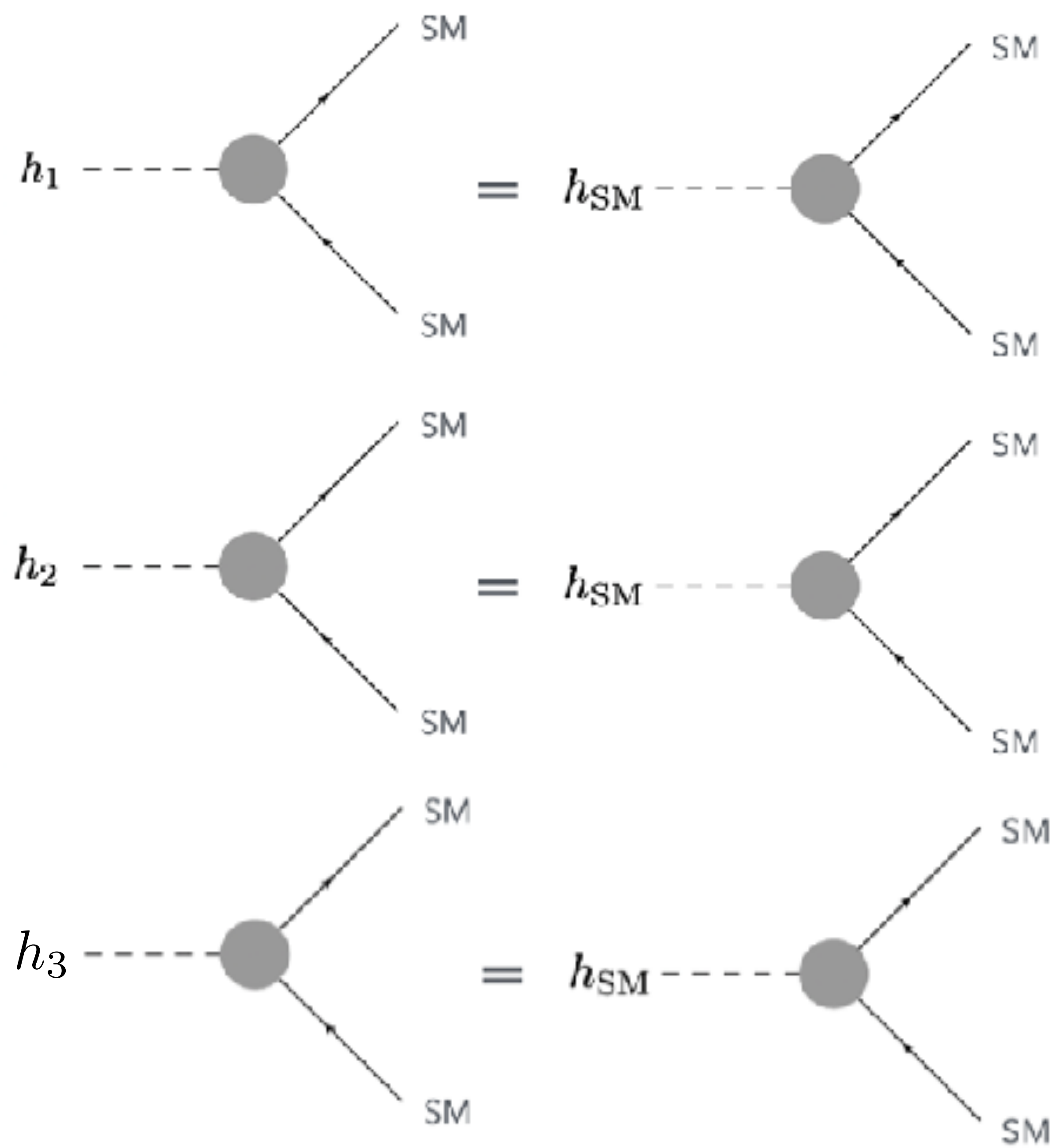
Cho, CI, Senaha 2022

gauge eigenstates $(h_{\rm SM}, s, \chi)$ $\xrightarrow[\quad O(\alpha_i) \quad]{\text{3×3 mixing matrix}}$ mass eigenstates (h_1, h_2, h_3)



$\times O_{11}$

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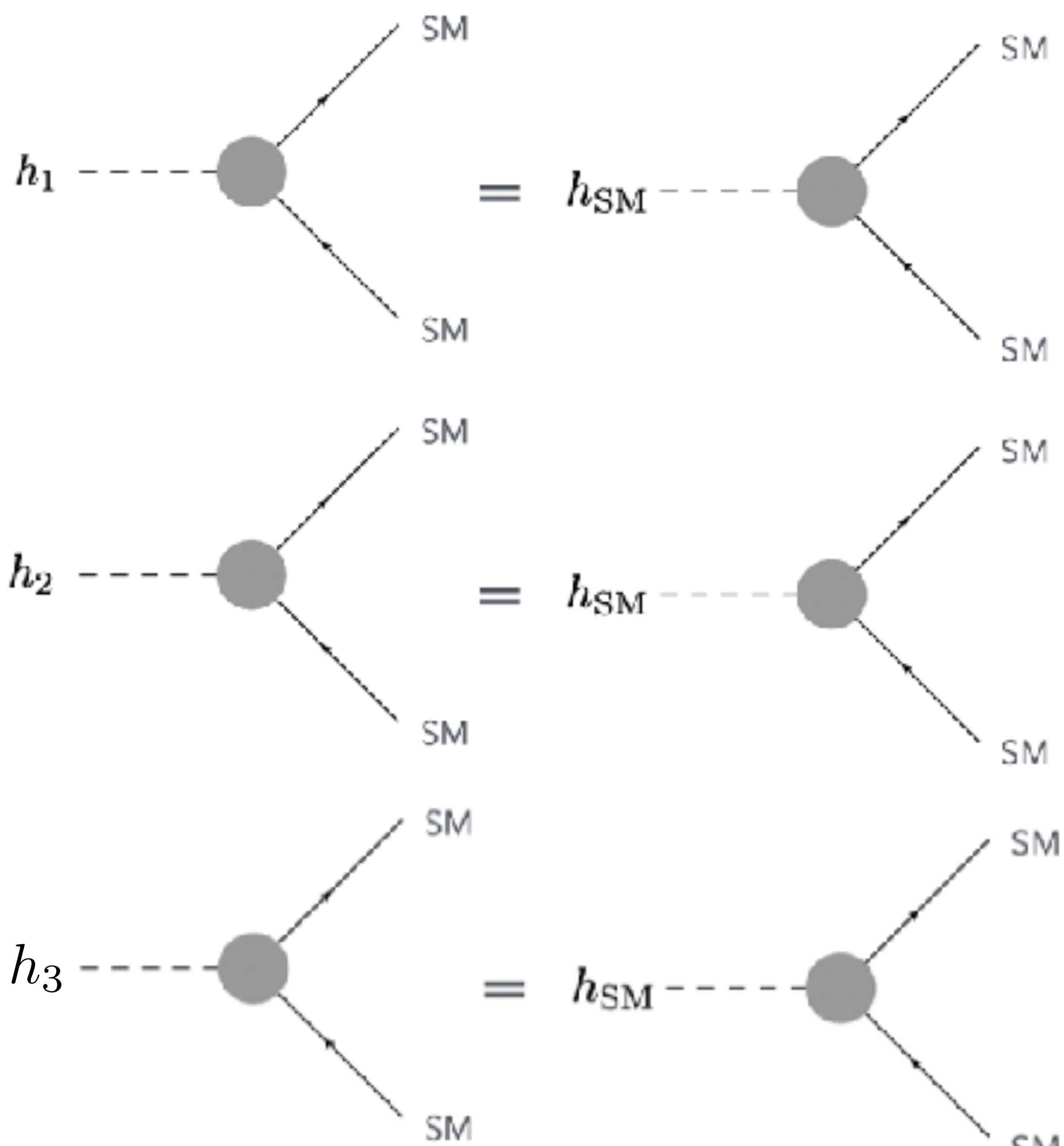

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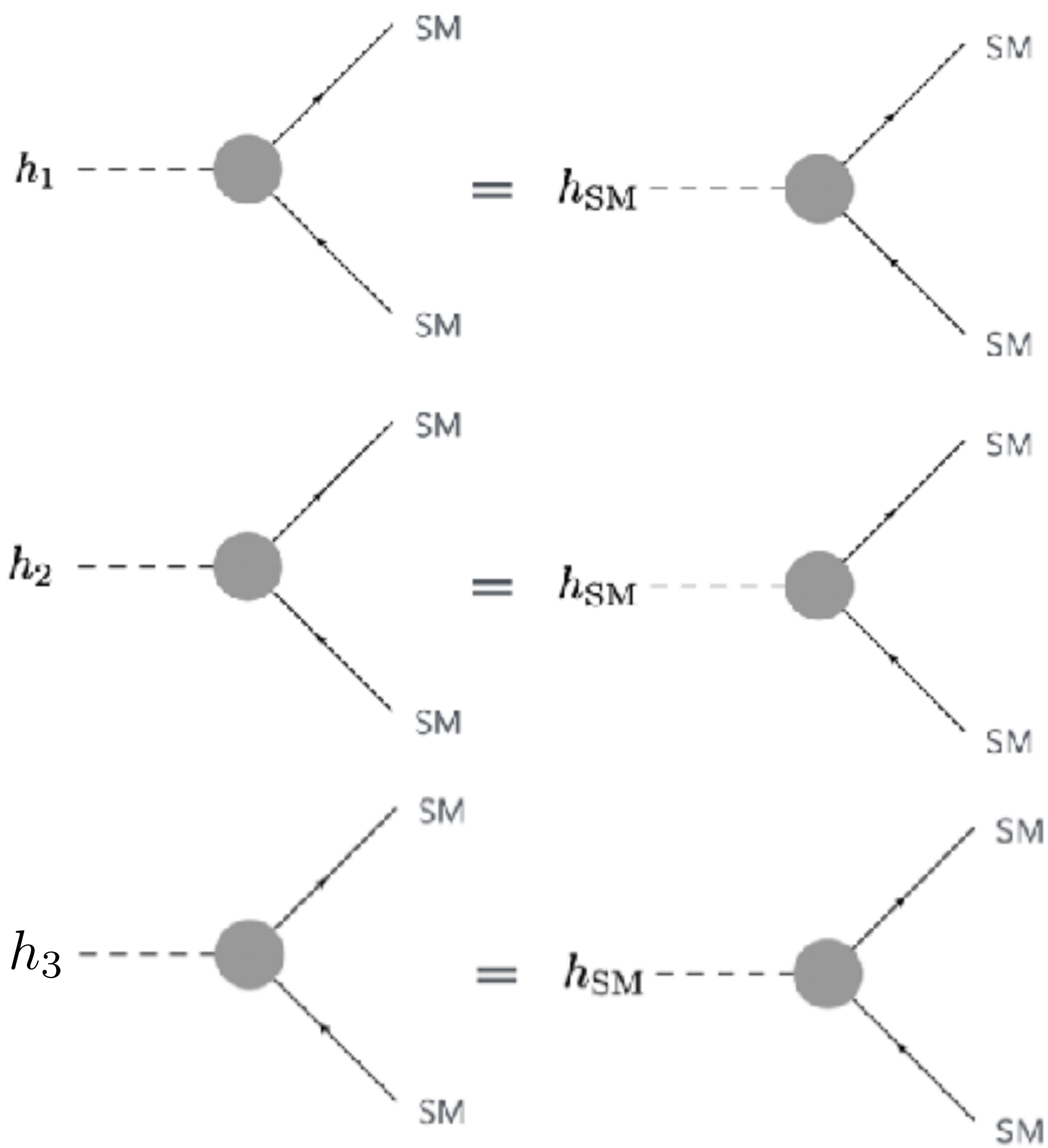

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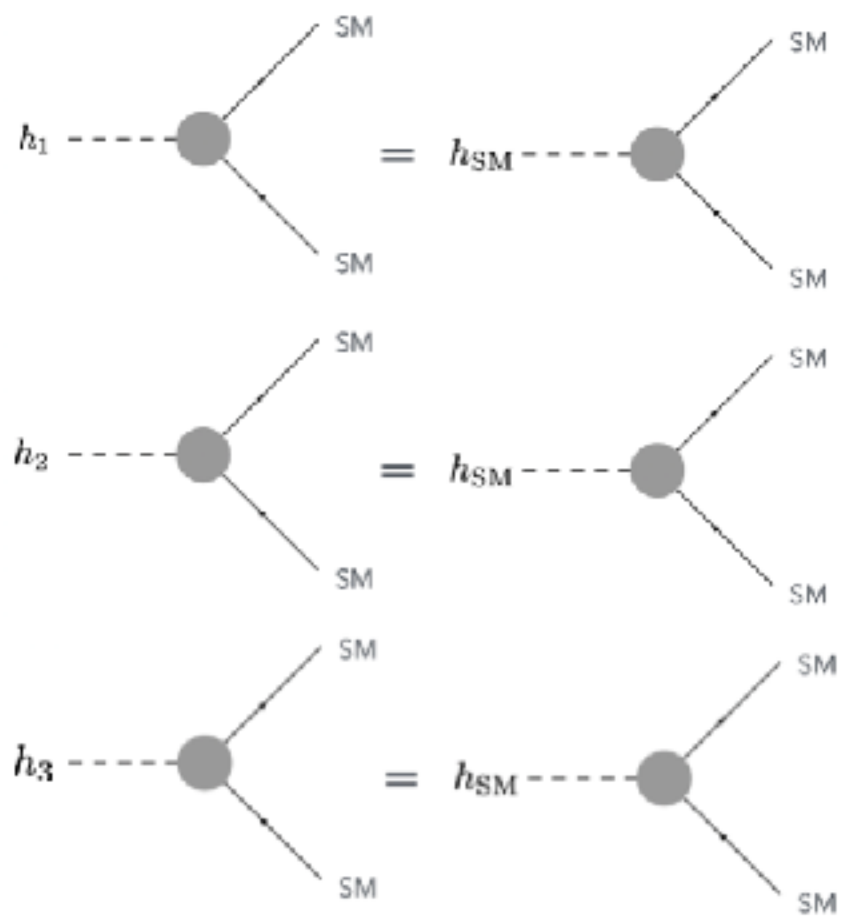

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$$\Gamma\left(h_1 \to \mathrm{SM}\right) = \Gamma\left(h_{\mathrm{SM}} \to \mathrm{SM}\right)\left(m_{h_1}\right) \times O_{11}^2$$
$$\Gamma\left(h_2 \to \mathrm{SM}\right) = \Gamma\left(h_{\mathrm{SM}} \to \mathrm{SM}\right)\left(m_{h_2}\right) \times O_{12}^2$$
$$\Gamma\left(h_3 \to \mathrm{SM}\right) = \Gamma\left(h_{\mathrm{SM}} \to \mathrm{SM}\right)\left(m_{h_3}\right) \times O_{13}^2$$

The orthogonality of the mixing matrix

$$\Sigma_k O_{ik} O_{jk} = \delta_{ij}$$

e.g., $O_{11}^2 + O_{12}^2 + O_{13}^2 = 1$

In Higgs masses degenerate limit

$$\boxed{\Gamma\left(h_1 \to \mathrm{SM}\right) + \Gamma\left(h_2 \to \mathrm{SM}\right) + \Gamma\left(h_3 \to \mathrm{SM}\right) \simeq \Gamma\left(h_{\mathrm{SM}} \to \mathrm{SM}\right) \text{ for } m_{h_1} \simeq m_{h_2} \simeq m_{h_3}}$$

Degenerate scalar scenario

CxSM Extension

S couples to SM fermions only through the mixing angles α

$\Longrightarrow$ pseudoscalar coupling $h_i \bar{f} \gamma_5 f$ does not arise

Even though the complex phases exist in the singlet scalar VEV,
we do not induce CPV in the matter sector in the SM

One possible extension is adding new fermions that couple to S and a new contribution to the Yukawa interaction through a higher dimensional operator.

Scalar extension model + higher dimensional operator

- SM + real singlet scalar
  w/ top Yukawa extension through dimension-5 operator — Espinosa et al. 2011
- SM + complex singlet scalar (no singlet CPV)
  w/ top Yukawa extension through dimension-5 operator — Keus et al. 2017

Higher dimensional operator

We consider the following dimension-5 operators contributing to the top quark and electron Yukawa coupling.

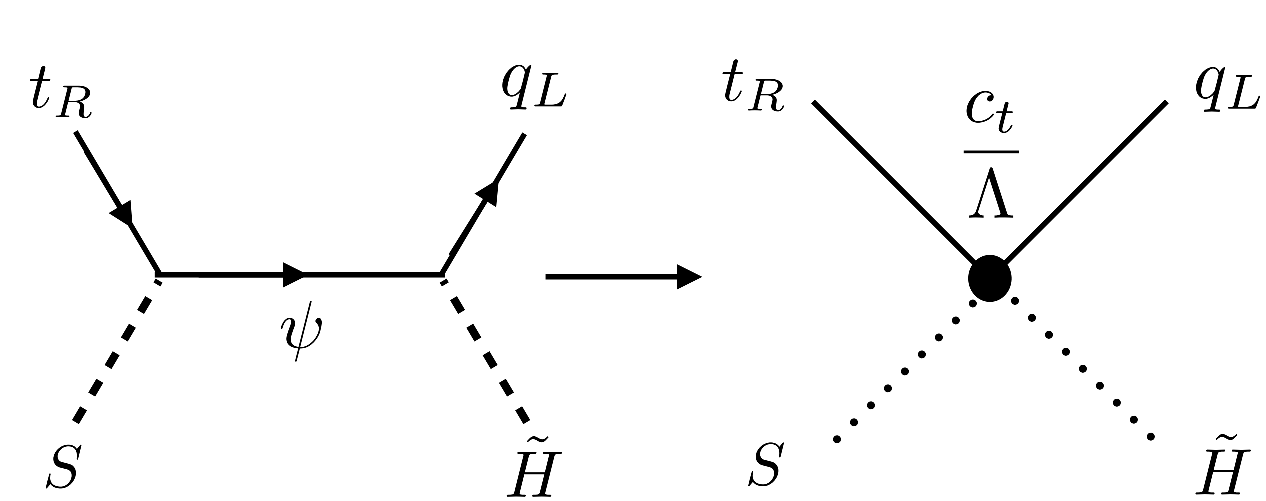
$$\mathcal{L}_t^{\text{Yukawa}} = -\bar{q}_L \tilde{H} \frac{c_t}{\Lambda} S t_R + \text{ H.c},$$

$$\mathcal{L}_e^{\text{Yukawa}} = -\bar{l}_L H \frac{c_e}{\Lambda} S e_R + \text{ H.c},$$

$\tilde{H} = i\tau^2 H^*$ w/ Pauli matrix τ^2

c_i : arbitrary parameters

Λ: the scale of the integrated fermion ψ



(i) If c_t is real, EWBG-related CPV arises only from $\text{Im}S$.

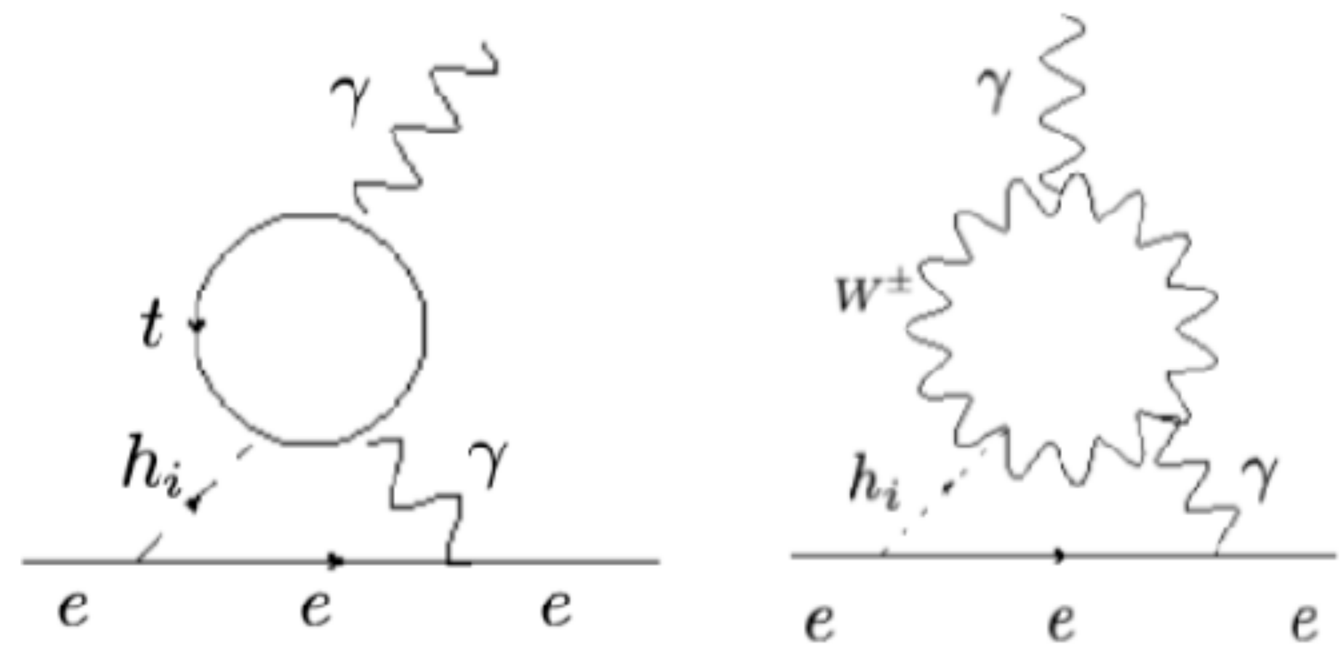
(ii) If c_t is complex, EWBG-related CPV arises from both c_t and S.

Electric dipole moment

EDM is good probe to CP-violation

Upper bound on the electron EDM
by JILA experiment [arXiv: 2212.11841]

$$|d_e| < 4.1 \times 10^{-30} e \text{ cm}$$



Two dominant Barr-Zee diagrams
in which electron phase enters

Top-loop

$$(d_e^{h\gamma})_t/e = \frac{1}{3\pi^2}\left(\frac{\alpha_{\rm em} G_F v^2}{\sqrt{2}\pi m_t}\right)\sum_{i=1}^{n}\left[\mathrm{Im}\left(Y_{eeh_i}\right)\mathrm{Re}\left(Y_{tth_i}\right) f\left(\frac{m_t^2}{m_{h_i}^2}\right) + \mathrm{Re}\left(Y_{eeh_i}\right)\mathrm{Im}\left(Y_{tth_i}\right) g\left(\frac{m_t^2}{m_{h_i}^2}\right)\right]$$

W-loop

$$(d_e^{h\gamma})_W/e = -\sum_{i=1}^{n}\frac{\alpha_{\rm em}^2 v}{32\pi^2 s_W^2 m_W^2}\mathrm{Im}\left(Y_{eeh_i}\right) g_{h_iVV}\mathcal{J}_W^\gamma\left(m_{h_i}\right)$$

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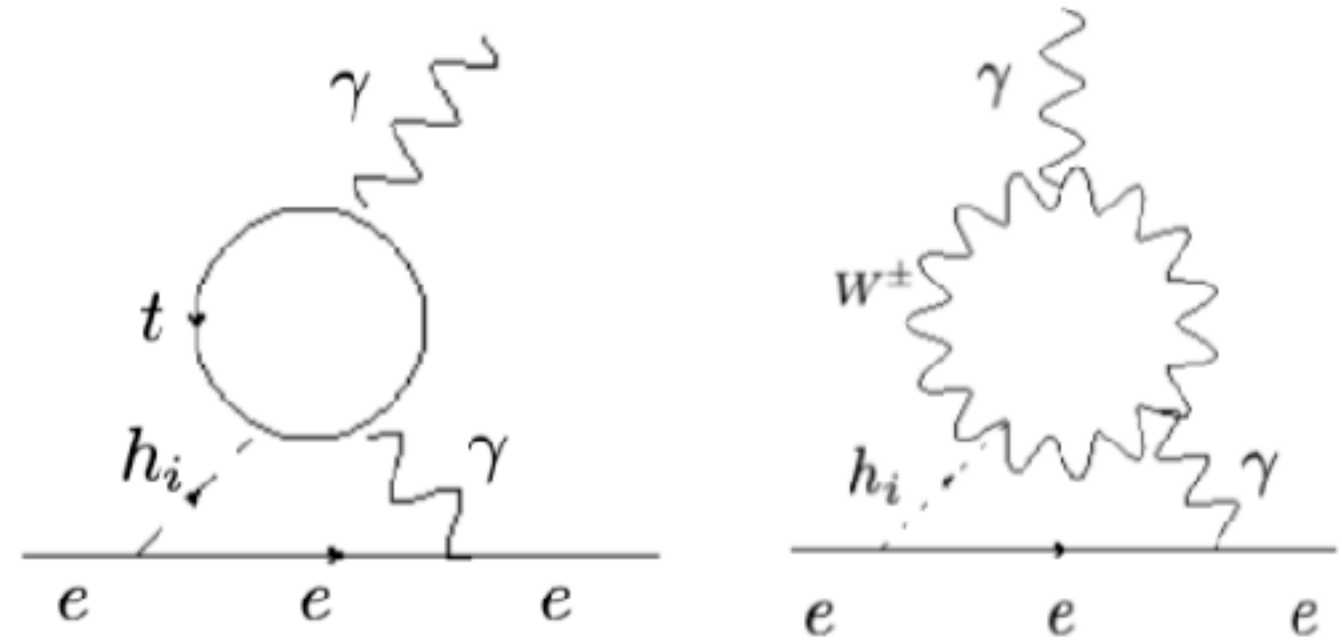

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Electric dipole moment

The couplings of scalar mass eigenstates h_i to top quark and electron

$$Y_{\bar{t}_L t_R h_i} = O_{1i}\frac{m_t}{v} + O_{2i}\frac{1}{2}\frac{c_t v}{\Lambda} + O_{3i}\frac{1}{2}\frac{i c_t v}{\Lambda}$$
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where

$$m_t = \frac{y_t v}{\sqrt{2}} + \frac{c_t v}{2\Lambda}\left(v_S^r + i v_S^i\right),$$
$$m_e = \frac{y_e v}{\sqrt{2}} + \frac{c_e v}{2\Lambda}\left(v_S^r + i v_S^i\right)$$

(i) When CPV arises only from singlet phase (real c_t, c_e)

Top-loop

$$(d_e^{h\gamma})_t/e = \frac{1}{3\pi^2}\left(\frac{\alpha_{\rm em} G_F v^2}{\sqrt{2}\pi m_t}\right)\sum_{i=1}^{3}$$
$$\left\{\underbrace{\left(O_{3i}\frac{1}{2}\frac{c_e v}{\Lambda}\right)}_{\mathrm{Im}\,(Y_{eeh_i})}\underbrace{\left(O_{1i}\frac{m_t}{v} + O_{2i}\frac{1}{2}\frac{c_t v}{\Lambda}\right)}_{\mathrm{Re}\,(Y_{tth_i})} f\left(\frac{m_t^2}{m_{h_i}^2}\right) + \underbrace{\left(O_{1i}\frac{m_e}{v} + O_{2i}\frac{1}{2}\frac{c_e v}{\Lambda}\right)}_{\mathrm{Re}\,(Y_{eeh_i})}\underbrace{\left(O_{3i}\frac{1}{2}\frac{c_t v}{\Lambda}\right)}_{\mathrm{Im}\,(Y_{tth_i})} g\left(\frac{m_t^2}{m_{h_i}^2}\right)\right\}$$

$$\overset{\text{degenerate}}{=} 0 \qquad \because O_{1i}O_{3i} = O_{2i}O_{3i} = 0$$

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(i) When CPV arises only from singlet phase (real c_t, c_e)

Top-loop

Due to the orthogonality of mixing matrix $\Sigma_k O_{ik} O_{jk} = \delta_{ij}$
if the Higgs masses are degenerate i.e., $m_{h_1} = m_{h_2} = m_{h_3}$,
the EDMs are suppressed in each.

$$(d_e^{h\gamma})_t/e = \frac{\cdots}{3\cdots}$$

$$\left\{\underbrace{\left(O_{3i}\frac{1}{2}\frac{c_e v}{\Lambda}\right)}_{\mathrm{Im}\,(Y_{eeh_i})}\underbrace{\left(O_{1i}\frac{m_t}{v} + O_{2i}\frac{1}{2}\frac{c_t v}{\Lambda}\right)}_{\mathrm{Re}\,(Y_{tth_i})} f\left(\frac{m_t^2}{m_{h_i}^2}\right) + \underbrace{\left(O_{1i}\frac{m_e}{v} + O_{2i}\frac{1}{2}\frac{c_e v}{\Lambda}\right)}_{\mathrm{Re}\,(Y_{eeh_i})}\underbrace{\left(O_{3i}\frac{1}{2}\frac{c_t v}{\Lambda}\right)}_{\mathrm{Im}\,(Y_{tth_i})} g\left(\frac{m_t^2}{m_{h_i}^2}\right)\right\}$$

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Electric dipole moment

Benchmark points where strong first-order phase transition occurs

Inputs	v [GeV]	v_S^r [GeV]	v_S^i [GeV]	m_{h_1} [GeV]	m_{h_2} [GeV]	m_{h_3} [GeV]	α_1 [rad]	α_2 [rad]	c_t	c_e	Λ [GeV]
	246.22	0.6	0.5	125.0	124.0	124.5	$\pi/4$	0.0	$y_t^{\rm SM}$	$y_e^{\rm SM}$	1000

almost degenerate

Electric dipole moment

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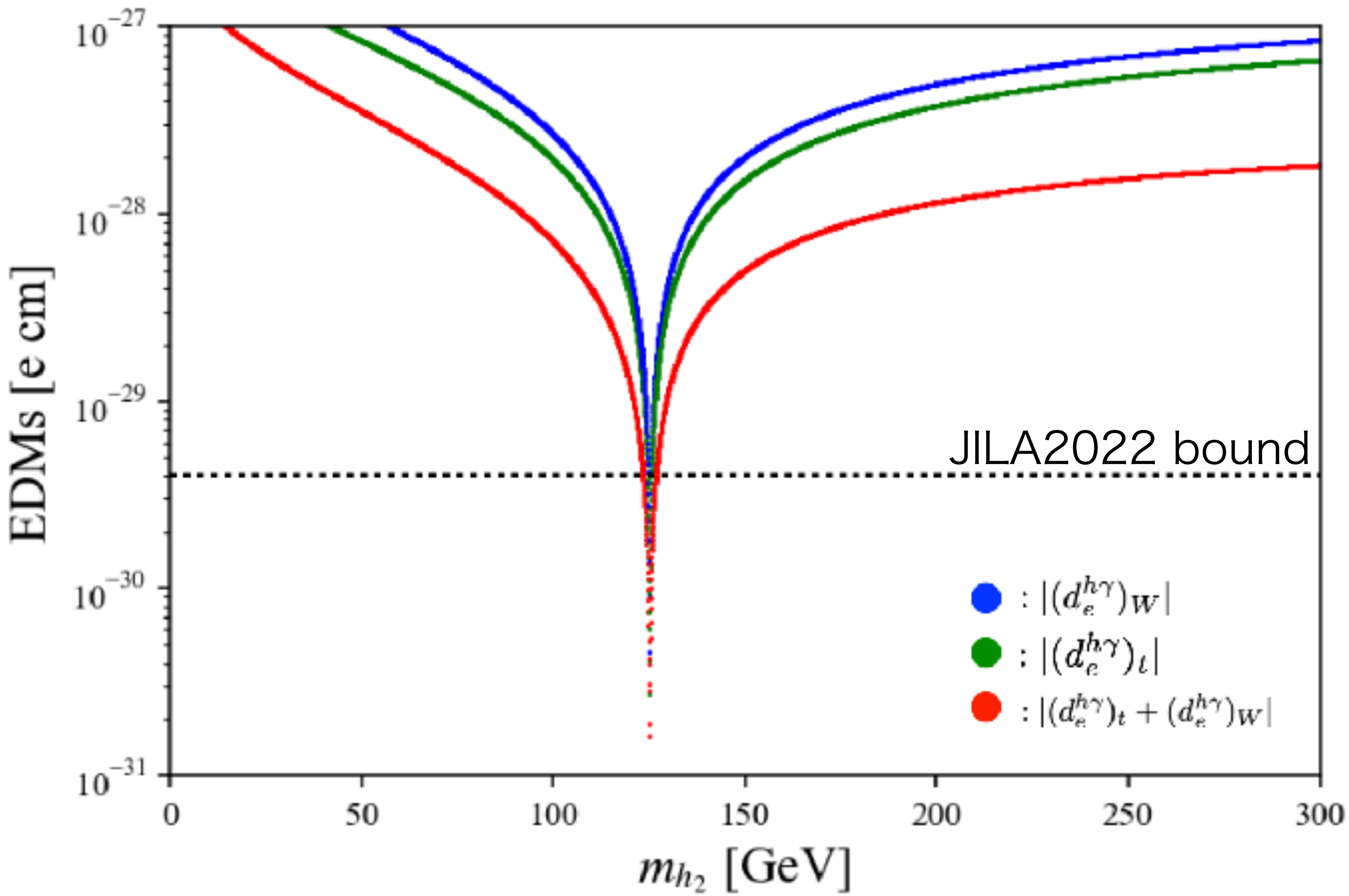

Electric dipole moment

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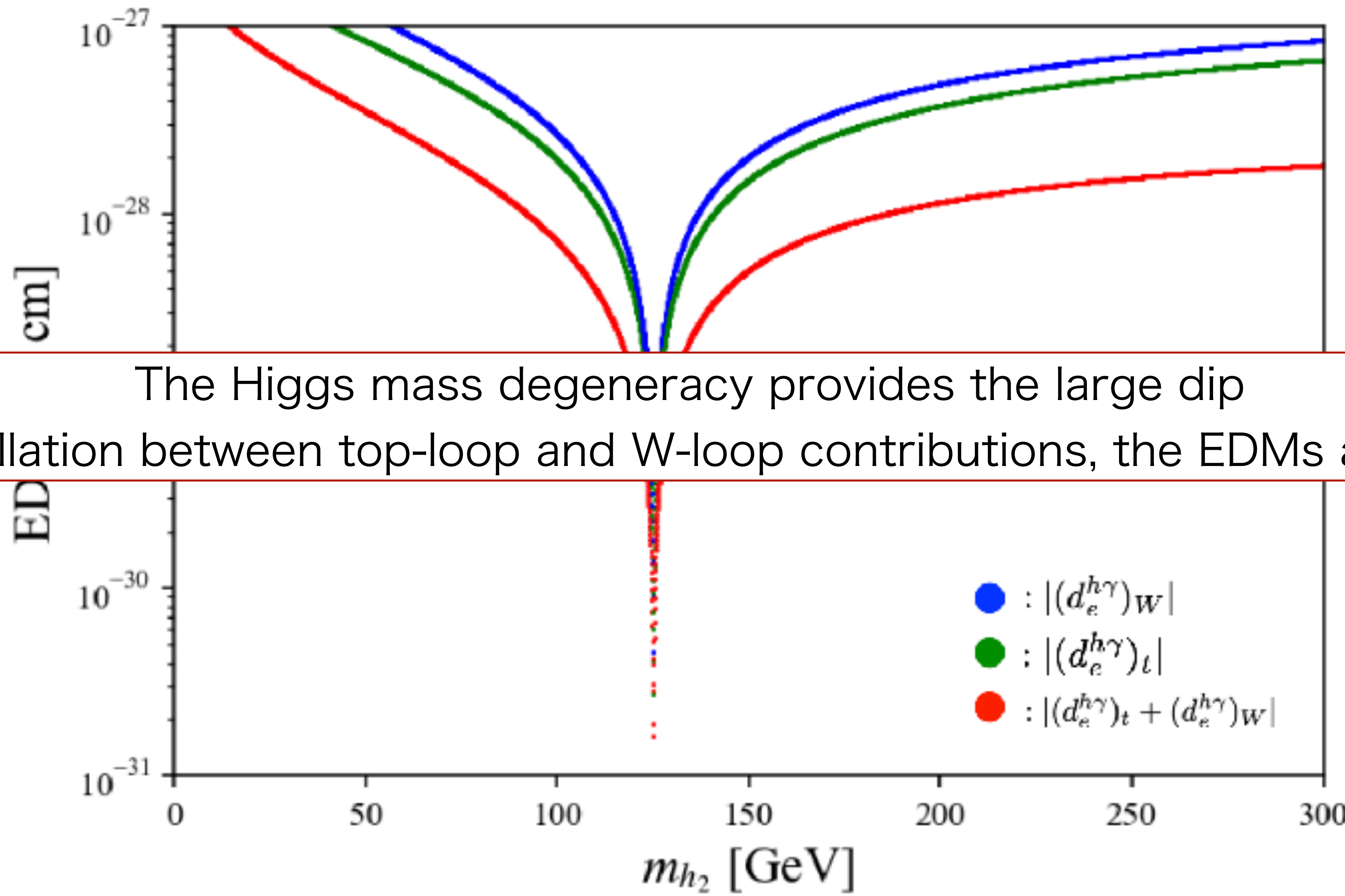

The Higgs mass degeneracy provides the large dip
Also by cancellation between top-loop and W-loop contributions, the EDMs are suppressed.

Electric dipole moment

Cho, CI, Senaha 2022

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almost degenerate

Top-loop contribution $(d_e^{h\gamma})_t = -4.4 \times 10^{-30}$ [e cm]

W-loop contribution $(d_e^{h\gamma})_W = 5.9 \times 10^{-30}$ [e cm]

Sum of two contributions $(d_e^{h\gamma})_t + (d_e^{h\gamma})_W = 1.5 \times 10^{-30}$ [e cm]

$< 4.1 \times 10^{-30}$ [e cm]

(JILA 2022 bound)

Electric dipole moment

(ii) When CPV arises from singlet phase and new Yukawa couplings

$$(c_t = c_t^r + ic_t^i = |c_t|e^{i\theta_t}, c_e = c_e^r + ic_e^i = |c_e|e^{i\theta_e})$$

Top-loop

$$(d_e^{h\gamma})_t/e = \frac{1}{3\pi^2}\left(\frac{\alpha_{\rm em}G_F v^2}{\sqrt{2}\pi m_t}\right)\sum_{i=1}^{3}$$

$$\left\{\underbrace{\left(O_{2i}\frac{1}{2}\frac{c_e^i v}{\Lambda} + O_{3i}\frac{1}{2}\frac{c_e^r v}{\Lambda}\right)}_{\mathrm{Im}\,(Y_{eeh_i})}\underbrace{\left(O_{1i}\frac{m_t}{v} + O_{2i}\frac{1}{2}\frac{c_t^i v}{\Lambda} - O_{3i}\frac{1}{2}\frac{c_t^r v}{\Lambda}\right)}_{\mathrm{Re}\,(Y_{tth_i})} f\left(\frac{m_t^2}{m_{h_i}^2}\right)\right.$$

$$\left. + \underbrace{\left(O_{1i}\frac{m_e}{v} + O_{2i}\frac{1}{2}\frac{c_e^r v}{\Lambda} - O_{3i}\frac{1}{2}\frac{c_e^i v}{\Lambda}\right)}_{\mathrm{Re}\,(Y_{eeh_i})}\underbrace{\left(O_{2i}\frac{1}{2}\frac{c_t^i v}{\Lambda} + O_{3i}\frac{1}{2}\frac{c_t^r v}{\Lambda}\right)}_{\mathrm{Im}\,(Y_{tth_i})} g\left(\frac{m_t^2}{m_{h_i}^2}\right)\right\}$$

$$\overset{\text{degenerate}}{\simeq} \frac{1}{3\pi^2}\left(\frac{\alpha_{\rm em}G_F v^2}{\sqrt{2}\pi m_t}\right)\sum_{i=1}^{3}\frac{v^2}{4\Lambda^2}\left\{\left(O_{2i}^2 c_e^i c_t^r - O_{3i}^2 c_e^r c_t^i\right) f\left(\frac{m_t^2}{m_{h_i}^2}\right) + \left(O_{2i}^2 c_e^r c_t^i - O_{3i}^2 c_e^i c_t^r\right) g\left(\frac{m_t^2}{m_{h_i}^2}\right)\right\}$$

To suppress d_t/e $\quad \dfrac{c_e^i}{c_e^r} = \dfrac{c_t^i}{c_t^r} \Longleftrightarrow \theta_e = \theta_t$ is needed

W-loop

$$(d_e^{h\gamma})_W/e = -\frac{\alpha_{\rm em}}{32\pi^3 v}\sum_{i=1}^{3}\left\{O_{1i}\underbrace{\left(O_{2i}\frac{c_e^i v}{2\Lambda} + O_{3i}\frac{c_e^r v}{2\Lambda}\right)}_{\mathrm{Im}\,(Y_{eeh_i})}\right\}\mathcal{J}_W^\gamma(m_{h_i}) \overset{\text{degenerate}}{=} 0$$

Electric dipole moment

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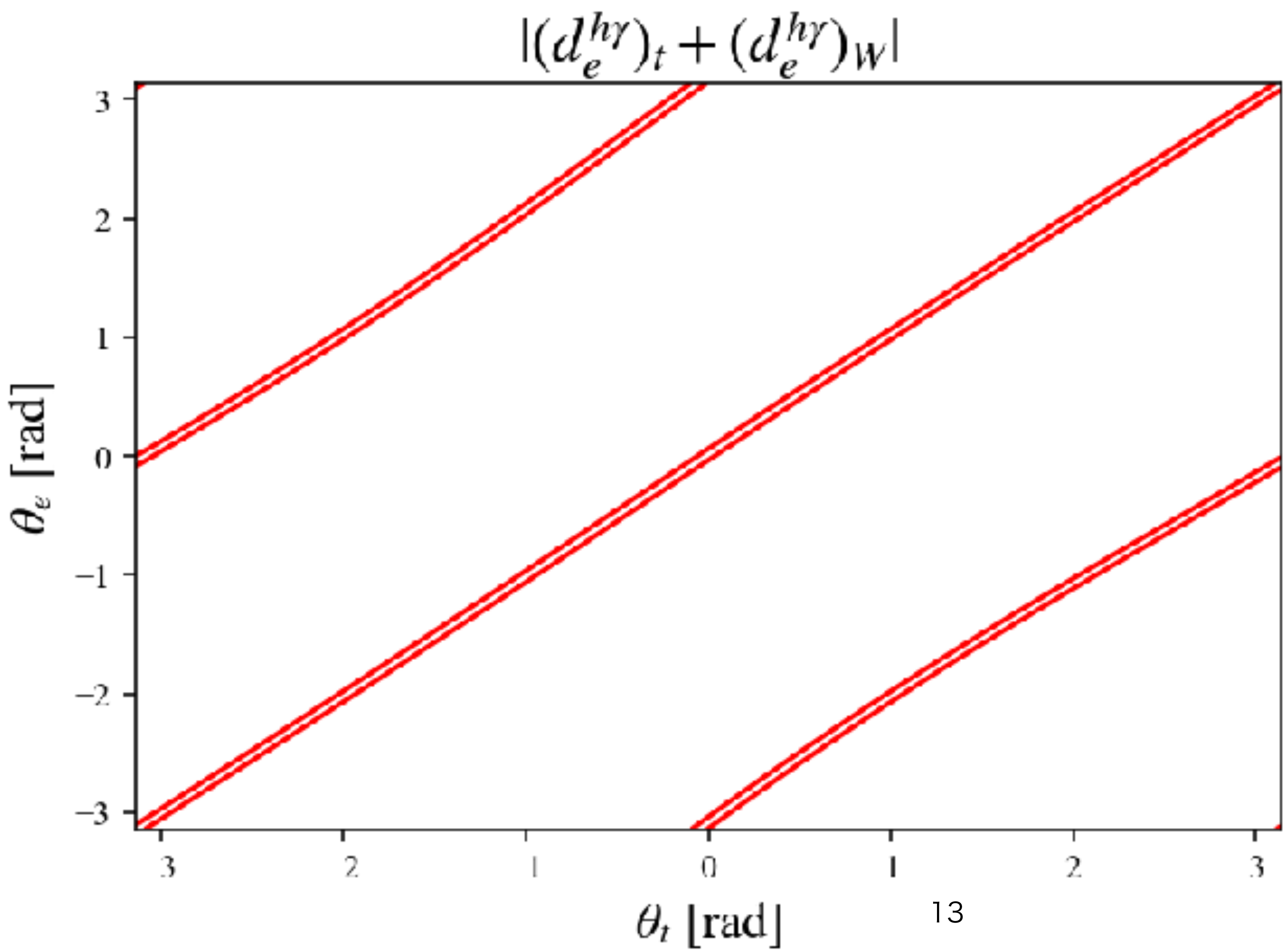


Counters display

$|d_e| = 4.1 \times 10^{-30} [e \cdot \mathrm{cm}]$

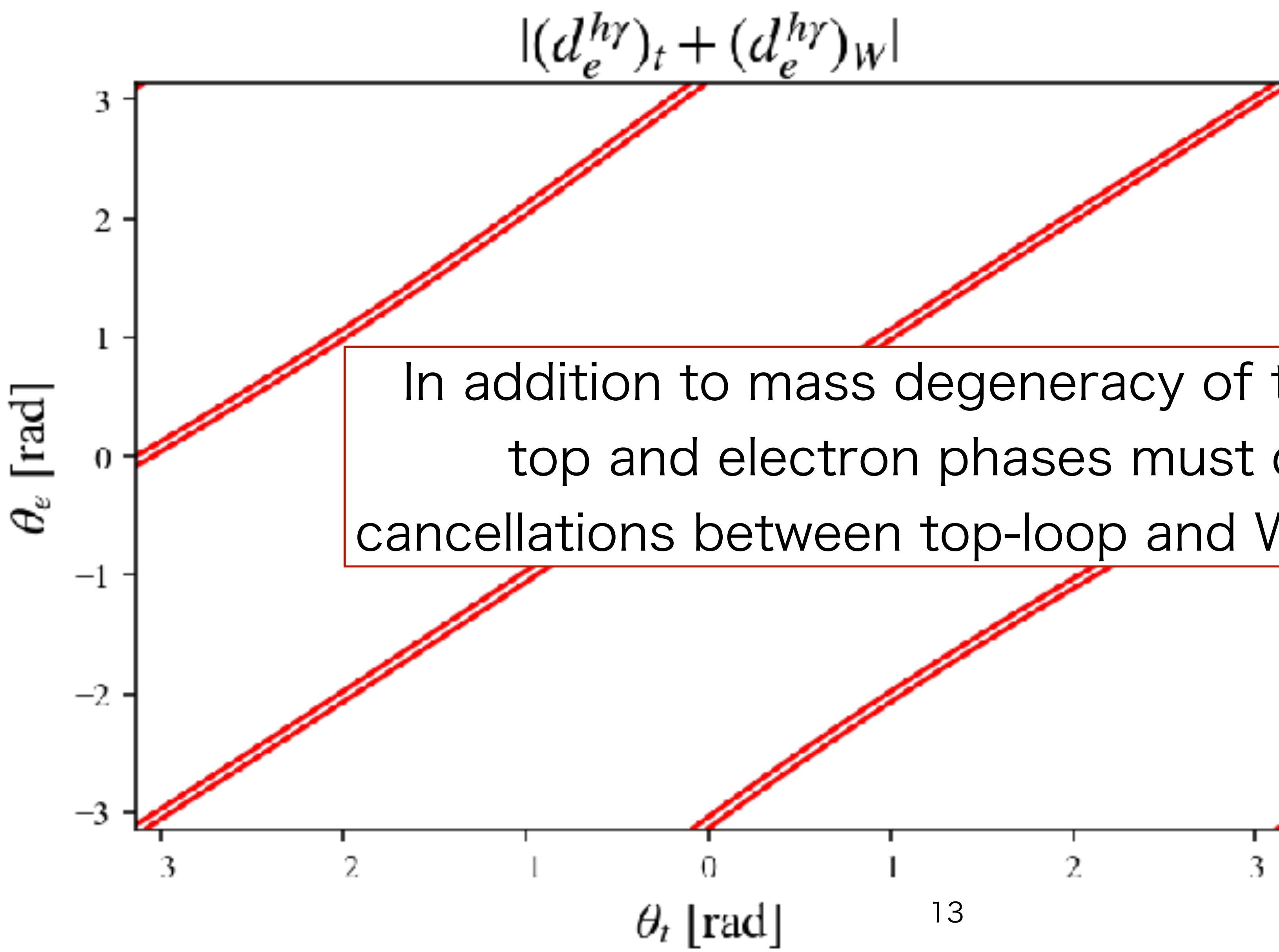
(JILA2022)

Electric dipole moment

Benchmark points where strong first-order phase transition occurs

Inputs	v [GeV]	v_S^r [GeV]	v_S^i [GeV]	m_{h_1} [GeV]	m_{h_2} [GeV]	m_{h_3} [GeV]	α_1 [rad]	α_2 [rad]	$\lvert c_t \rvert$	$\lvert c_e \rvert$	Λ [GeV]
	246.22	0.6	0.5	125.0	124.0	124.5	$\pi/4$	0.0	y_t^{SM}	y_e^{SM}	1000

almost degenerate



Counters display

$|d_e| = 4.1 \times 10^{-30} [e \cdot \mathrm{cm}]$

(JILA2022)

In addition to mass degeneracy of the Higgs boson, top and electron phases must coincide and cancellations between top-loop and W-loop are needed.

Conclusion

- EWPT and EDMs were investigated in the CxSM with dimension-5 operators. The coefficients of the new Yukawa interaction may be real or complex, and in the former case the EWBG-related CPV arises from $\mathrm{Im}S$. We focused on three Higgs degenerate regions that suppress collider signals.

- There are two types of Barr-Zee diagrams including electron phase: top-loop and W-loop.

- electron EDM suppression conditions are as shown in the table below;

	top-loop Barr-Zee diagram	W-loop Barr-Zee diagram
Real $c_t,\ c_e$	Higgs mass degeneracy	Higgs mass degeneracy
Complex $c_t,\ c_e$	Higgs mass degeneracy and $\theta_e = \theta_t$	Higgs mass degeneracy

Depending on the degree of mass degeneracy, the cancellation between the top-loop and the W-loop contributions are necessary.

Backup

electron EDM

$h_i\gamma$-type Barr-Zee diagram

$$(d_e^{h\gamma})_t/e = \frac{1}{3\pi^2}\left(\frac{\alpha_{\rm em}G_F v^2}{\sqrt{2}\pi m_t}\right)\sum_{i=1}^{n}\left[\mathrm{Im}\,(Y_{eeh_i})\,\mathrm{Re}\,(Y_{tth_i})\,f\left(\frac{m_t^2}{m_{h_i}^2}\right)+\mathrm{Re}\,(Y_{eeh_i})\,\mathrm{Im}\,(Y_{tth_i})\,g\left(\frac{m_t^2}{m_{h_i}^2}\right)\right]$$

where

$$f(\tau) = \frac{\tau}{2}\int_0^1 dx\,\frac{1-2x(1-x)}{x(1-x)-\tau}\ln\left(\frac{x(1-x)}{\tau}\right)$$
$$g(\tau) = \frac{\tau}{2}\int_0^1 dx\,\frac{1}{x(1-x)-\tau}\ln\left(\frac{x(1-x)}{\tau}\right).$$

$$(d_e^{h\gamma})_W/e = -\sum_{i=1}^{n}\frac{\alpha_{\rm em}^2 v}{32\pi^2 s_W^2 m_W^2}\,\mathrm{Im}\,(Y_{eeh_i})\,g_{h_iVV}\,\mathcal{J}_W^{\gamma}\,(m_{h_i})$$

where

$$\begin{aligned}\mathcal{J}_W^V\,(m_{h_i}) = {} & \frac{2m_W^2}{m_\phi^2 - m_V^2}\left[-\frac{1}{4}\left\{\left(6-\frac{m_V^2}{m_W^2}\right)+\left(1-\frac{m_V^2}{2m_W^2}\right)\frac{m_{h_i}^2}{m_W^2}\right\}\right. \\ & \times\left(I_1\,(m_W, m_{h_i}) - I_1\,(m_W, m_V)\right) \\ & +\left\{\left(-4+\frac{m_V^2}{m_W^2}\right)+\frac{1}{4}\left(6-\frac{m_V^2}{m_W^2}\right)+\frac{1}{4}\left(1-\frac{m_V^2}{2m_W^2}\right)\frac{m_{h_i}^2}{m_W^2}\right\} \\ & \left.\times\left(I_2\,(m_W, m_{h_i}) - I_2\,(m_W, m_V)\right)\right],\end{aligned}$$

with

$$I_1\,(m_1, m_2) = -2\frac{m_2^2}{m_1^2}f\left(\frac{m_1^2}{m_2^2}\right),$$
$$I_2\,(m_1, m_2) = -2\frac{m_2^2}{m_1^2}g\left(\frac{m_1^2}{m_2^2}\right).$$

General from of W-loop contribution

$$
\begin{aligned}
d_W/e = -\frac{\alpha_{\rm em}}{32\pi^3 v}\Big\{ & O_{11}O_{21}\frac{c_e^i v}{2\Lambda}\mathcal{J}_W^\gamma\left(m_{h_1}\right) + O_{12}O_{22}\frac{c_e^i v}{2\Lambda}\mathcal{J}_W^\gamma\left(m_{h_2}\right) \\
& + O_{13}O_{23}\frac{c_e^i v}{2\Lambda}\mathcal{J}_W^\gamma\left(m_{h_3}\right) + O_{11}O_{31}\frac{c_e^r v}{2\Lambda}\mathcal{J}_W^\gamma\left(m_{h_1}\right) \\
& + O_{12}O_{32}\frac{c_e^r v}{2\Lambda}\mathcal{J}_W^\gamma\left(m_{h_2}\right) + O_{13}O_{33}\frac{c_e^r v}{2\Lambda}\mathcal{J}_W^\gamma\left(m_{h_3}\right)\Big\} \\
= -\frac{\alpha_{\rm em}}{32\pi^3}\frac{1}{2\Lambda}\Big\{ & \left(c_e^i O_{11}O_{21} + c_e^r O_{11}O_{31}\right)\left(\mathcal{J}_W^\gamma\left(m_{h_1}\right) - \mathcal{J}_W^\gamma\left(m_{h_3}\right)\right) \\
& \left(c_e^i O_{12}O_{22} + c_e^r O_{12}O_{32}\right)\left(\mathcal{J}_W^\gamma\left(m_{h_2}\right) - \mathcal{J}_W^\gamma\left(m_{h_3}\right)\right)\Big\} \\
= -\frac{\alpha_{\rm em}}{32\pi^3}\frac{1}{2\Lambda}\Big\{ & \left(c_e^i O_{12}O_{22} + c_e^r O_{12}O_{32}\right)\left(\mathcal{J}_W^\gamma\left(m_{h_2}\right) - \mathcal{J}_W^\gamma\left(m_{h_1}\right)\right) \\
& \underbrace{\left(c_e^i O_{13}O_{23} + c_e^r O_{13}O_{33}\right)\left(\mathcal{J}_W^\gamma\left(m_{h_3}\right) - \mathcal{J}_W^\gamma\left(m_{h_1}\right)\right)\Big\}}_{\text{If } \alpha_2 = 0\text{, this line vanishes}}
\end{aligned}
$$

EDM for thorium monoxide d_{ThO}

The effective EDM for thorium monoxide (ThO) is given by

$$d_{\mathrm{ThO}} = d_e + \alpha_{\mathrm{ThO}} C_S \qquad \alpha_{\mathrm{ThO}} = 1.5 \times 10^{-20}$$

The CPV interactions between the nucleons and electron are defined as

$$\mathcal{L}_{eN}^{\mathrm{NSID}} = -\frac{G_F}{\sqrt{2}} C_S (\bar{N} N)\,(\bar{e} i \gamma_5 e)$$

Furthermore, the CPV 4-fermion interactions between the quarks and electron are represented as

$$\mathcal{L}_{4f}^{\mathrm{CPV}} = \sum_q C_{qe} (\bar{q} q)\,(\bar{e} i \gamma_5 e) \quad \text{where} \quad C_{qe} = \sum_i \frac{\mathrm{Re}\,(Y_{\bar{q}_L q_R h_i})\,\mathrm{Im}\,(Y_{\bar{e}_L e_R h_i})}{2 m_i^2}$$

$$\therefore C_S = -2v^2 \left[6.3\,(C_{ue} + C_{de}) + C_{se} \frac{41\mathrm{MeV}}{m_s} + C_{ce} \frac{79\mathrm{MeV}}{m_c} + 6.2 \times 10^{-2}\mathrm{GeV} \left(\frac{C_{be}}{m_b} + \frac{C_{te}}{m_t} \right) \right]$$

EDM for thorium monoxide d_{ThO}

(i) When CPV arises only from singlet phase (real c_t, c_e)

$$C_{qe}\ (q=u,d,s,c,b) = \sum_i \frac{O_{1i}\frac{m_q}{v}\cdot O_{3i}\frac{1}{2}\frac{c_e v}{\Lambda}}{m_i^2} \overset{\text{degenerate}}{=} 0$$

$$C_{te} = \sum_i \frac{\left(O_{1i}\frac{m_t}{v}+O_{2i}\frac{1}{2}\frac{c_t v}{\Lambda}\right)\left(O_{3i}\frac{1}{2}\frac{c_e v}{\Lambda}\right)}{m_i^2} \overset{\text{degenerate}}{=} 0$$

(ii) When CPV arises from singlet phase and new Yukawa couplings

$$C_{qe}\ (q=u,d,s,c,b) = \sum_i \frac{O_{1i}\frac{m_q}{v}\left(O_{2i}\frac{1}{2}\frac{c_e^i v}{\Lambda}+O_{3i}\frac{1}{2}\frac{c_e^r v}{\Lambda}\right)}{m_i^2} \overset{\text{degenerate}}{=} 0$$

$$C_{te} = \sum_i \frac{\left(O_{1i}\frac{m_t}{v}+O_{2i}\frac{1}{2}\frac{c_t^r v}{\Lambda}-O_{3i}\frac{1}{2}\frac{c_t^i v}{\Lambda}\right)\left(O_{2i}\frac{1}{2}\frac{c_e^i v}{\Lambda}+O_{3i}\frac{1}{2}\frac{c_e^r v}{\Lambda}\right)}{m_i^2}$$

$$\overset{\text{degenerate}}{=} \sum_i \frac{v^2}{4\Lambda}\frac{O_{2i}^2 c_t^r c_e^i - O_{3i}^2 c_t^i c_e^r}{m_i^2} \qquad \therefore \frac{c_e^i}{c_e^r} = \frac{c_t^i}{c_t^r} \leftrightarrow \theta_e = \theta_t$$

Re-phasing invariant CP phases

the re-phasing invariant phase

$$\theta_1 \equiv 2\theta_{a_1} - \theta_{b_1} \to 2\theta'_{a_1} - \theta'_{b_1} = 2\theta_{a_1} - \theta_{b_1}. \longleftarrow \text{case (i)}$$

$$\theta_2 \equiv \theta_{Y^u} - \theta_{c_1^u} + \theta_{a_1} \to \theta'_{Y^u} - \theta'_{c_1^u} + \theta'_{a_1} = \theta_{Y^u} - \theta_{c_1^u} + \theta_{a_1}.$$

$$\theta_3 \equiv \theta_{Y^d} - \theta_{c_1^d} + \theta_{a_1} \to \theta'_{Y^d} - \theta'_{c_1^d} + \theta'_{a_1} = \theta_{Y^d} - \theta_{c_1^d} + \theta_{a_1},$$

$$\theta_4 \equiv \theta_{Y^e} - \theta_{c_1^e} + \theta_{a_1} \to \theta'_{Y^e} - \theta'_{c_1^e} + \theta'_{a_1} = \theta_{Y^e} - \theta_{c_1^e} + \theta_{a_1}.$$

case (ii)

case (ii) $d_e(\theta_2, \theta_4) \overset{\text{degenerate}}{\propto} \sin(\theta_2 - \theta_4)$

To vanish d_e, $\theta_2 = \theta_4$ is needed. $\therefore \theta_{c_1^u} = \theta_{c_1^e}$

Therefore, the number of the rephasing invariant phases is only 4. We may take the convention that $\theta_{Y^u} = \theta_{Y^d} = \theta_{Y^e} = \theta_{b_1} = 0$.

CPC CxSM

"Electroweak phase transition in a complex singlet extension of the Standard Model with degenerate scalars", Phys.Lett.B 823 (2021) 136787, **arXiv:2105.11830**

Model definition

Complex singlet extension of the SM（CxSM） Barger etal, arXiv:0811.0393

$$V_0 = \underline{\frac{m^2}{2}|H|^2 + \frac{\lambda}{4}|H|^4 + \frac{\delta_2}{2}|H|^2|S|^2 + \frac{b_2}{2}|S|^2 + \frac{d_2}{4}|S|^4} + \underline{\left(a_1 S + \frac{b_1}{4}S^2 + \text{ c.c. }\right)}$$

Global U(1) and soft breaking terms (minimal set of S.B. operators to realize pNG DM)

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}, \quad S = (v_S + s + i\chi)/\sqrt{2}$$

DM (DM stability ↔ CP sym.)

Mass eigenstates

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

Mass eigenvalues

$$\Lambda^2 \equiv \frac{d_2}{2}v_S^2 - \sqrt{2}\frac{a_1}{2v_S}$$

$\boxed{h_1, h_2}$

$$m^2_{h_1,h_2} = \frac{1}{2}\left(\frac{\lambda}{2}v^2 + \Lambda^2 \mp \sqrt{\left(\frac{\lambda}{2}v^2 - \Lambda^2\right)^2 + 4\left(\frac{\delta_2}{2}vv_S\right)^2}\right)$$

$\boxed{\text{DM}}$

$$m_\chi^2 = -b_1 - \sqrt{2}\frac{a_1}{v_S}$$

Model definition

The general scalar potential

$$V = \frac{m^2}{2}|H|^2 + \frac{\lambda}{4}|H|^4 + \frac{\delta_2}{2}|H|^2|S|^2 + \frac{b_2}{2}|S|^2 + \frac{d_2}{4}|S|^4$$
$$+ \left(a_1 S + \frac{\delta_1}{4}|H|^2 S + \frac{\delta_3}{4}|H|^2 S^2 + \frac{b_1}{4}S^2 + \frac{c_1}{6}S^3 + \frac{c_2}{6}S|S|^2 + \frac{d_1}{8}S^4 + \frac{d_3}{8}S^2|S|^2 + \text{ c.c. } \right)$$

The minimalization condition

$$-m^2 = \frac{\lambda}{2}v^2 + \frac{\delta_2}{2}v_S^2,$$
$$-b_2 = \frac{\delta_2}{2}v^2 + \frac{d_2}{2}v_S^2 + b_1 + 2\sqrt{2}\frac{a_1}{v_S}$$

Mixing angle α

$$\tan 2\alpha = 2\frac{\frac{\delta_2}{2}vv_S}{\frac{\lambda}{2}v^2 - \Lambda^2}, \quad \cos 2\alpha = \frac{\frac{\lambda}{2}v^2 - \Lambda^2}{m_{h_1}^2 - m_{h_2}^2}$$

$$\Lambda^2 \equiv \frac{d_2}{2}v_S^2 - \sqrt{2}\frac{a_1}{2v_S}$$

Mass eigenvalues

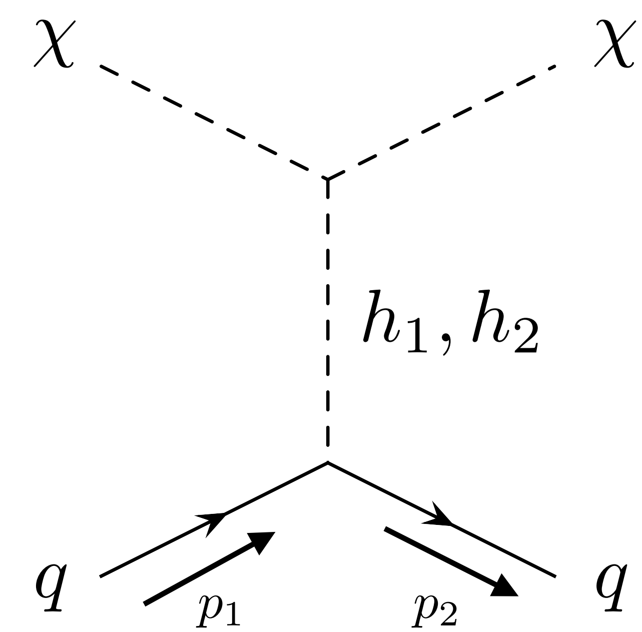
$$m_{h_1,h_2}^2 = \frac{1}{2}\left(\frac{\lambda}{2}v^2 + \Lambda^2 \mp \frac{\frac{\lambda}{2}v^2 - \Lambda^2}{\cos 2\alpha}\right)$$
$$= \frac{1}{2}\left(\frac{\lambda}{2}v^2 + \Lambda^2 \mp \sqrt{\left(\frac{\lambda}{2}v^2 - \Lambda^2\right)^2 + 4\left(\frac{\delta_2}{2}vv_S\right)^2}\right)$$

Degenerate scalar scenario

Abe, Cho, Mawatari arXiv:2101.04887

$$i\mathcal{M}_{h_1} = -i\frac{m_f}{vv_S}\frac{m_{h_1}^2 + \frac{\sqrt{2}a_1}{v_S}}{t - m_{h_1}^2}\sin\alpha\cos\alpha\ \bar{u}(p_3)u(p_1),$$

$$i\mathcal{M}_{h_2} = +i\frac{m_f}{vv_S}\frac{m_{h_2}^2 + \frac{\sqrt{2}a_1}{v_S}}{t - m_{h_2}^2}\sin\alpha\cos\alpha\ \bar{u}(p_3)u(p_1),$$



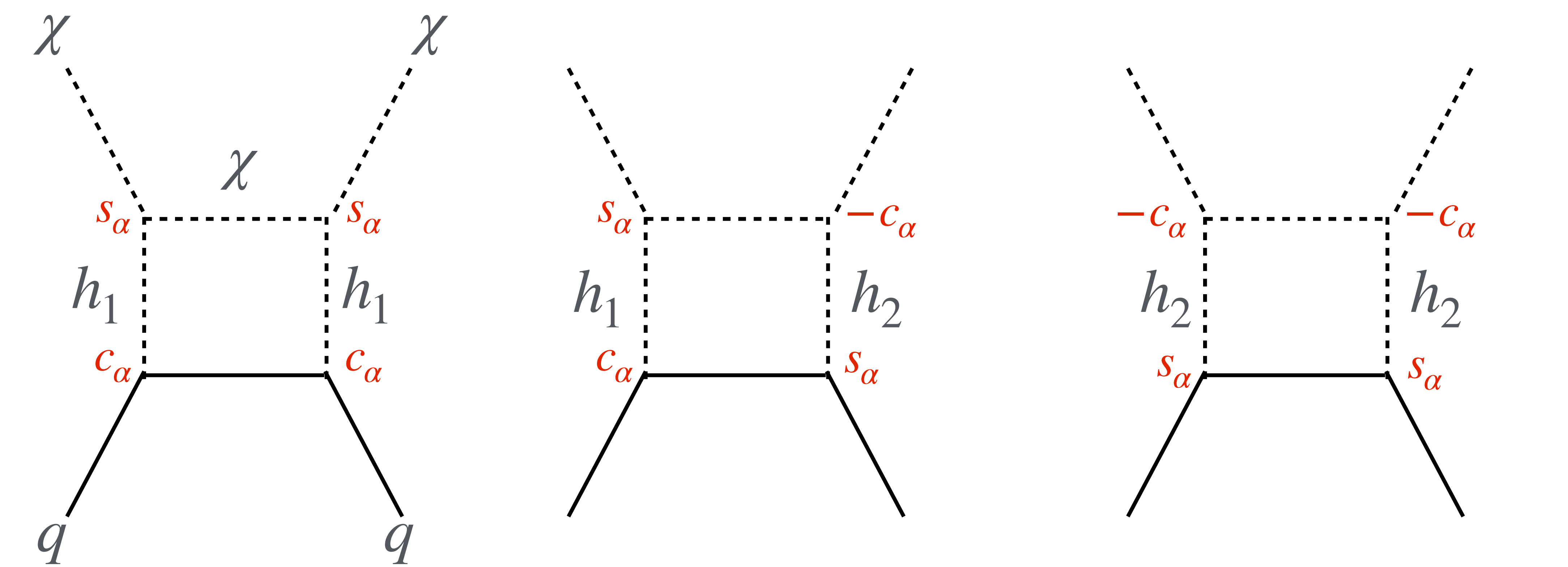
$$\begin{aligned}
i\left(\mathcal{M}_{h_1} + \mathcal{M}_{h_2}\right) =& i\frac{m_f}{vv_S}\left(-\frac{m_{h_1}^2 + \frac{\sqrt{2}a_1}{v_S}}{t - m_{h_1}^2} + \frac{m_{h_2}^2 + \frac{\sqrt{2}a_1}{v_S}}{t - m_{h_2}^2}\right)\sin\alpha\cos\alpha\bar{u}\left(p_3\right)u\left(p_1\right) \\
&\simeq i\frac{m_f}{vv_S}\sin\alpha\cos\alpha\bar{u}\left(p_3\right)u\left(p_1\right) \\
&\times\left\{\left(\frac{\sqrt{2}a_1}{v_S} + t\right)\left(\frac{1}{m_{h_1}^2} - \frac{1}{m_{h_2}^2}\right) + \frac{\sqrt{2}a_1}{v_S}t\left(\frac{1}{m_{h_1}^4} - \frac{1}{m_{h_2}^4}\right)\right\} \text{ @ t}\rightarrow 0
\end{aligned}$$

$$\boxed{\begin{aligned}
&\simeq i\frac{m_f}{vv_S}\sin\alpha\cos\alpha\bar{u}\left(p_3\right)u\left(p_1\right)\frac{\sqrt{2}a_1}{v_S}\left(\frac{1}{m_{h_1}^2} - \frac{1}{m_{h_2}^2}\right) \\
&\simeq 0\ (m_{h1} \sim m_{h2})
\end{aligned}}$$

Degenerate scalar scenario

Degenerate scalar scenario@ one-loop Azevedo etal., 1801.06105

$$\sigma_{\chi N}^{\mathrm{NLO}} = \sin 2\alpha \left(\frac{\mu_{\chi N} f_N m_N}{m_{h_1} m_{h_2}} \right)^2 \frac{\boxed{m_{h_1}^2 - m_{h_2}^2}}{v^3 v_S^3} \times \text{loop func.} \qquad \propto m_{h_1}^2 - m_{h_2}^2$$



$(s_\alpha c_\alpha)^2 f(m_{h_1}, m_{h_1})$ $\quad -2(s_\alpha c_\alpha)^2 f(m_{h_1}, m_{h_2})$ $\quad (-s_\alpha c_\alpha)^2 f(m_{h_2}, m_{h_2})$

Sum = $(s_\alpha c_\alpha)^2 (f(1,1) - f(1,2)) + (s_\alpha c_\alpha)^2 (f(2,2) - f(2,1))$ –> 0 for $m_{h_1} \sim m_{h_2}$

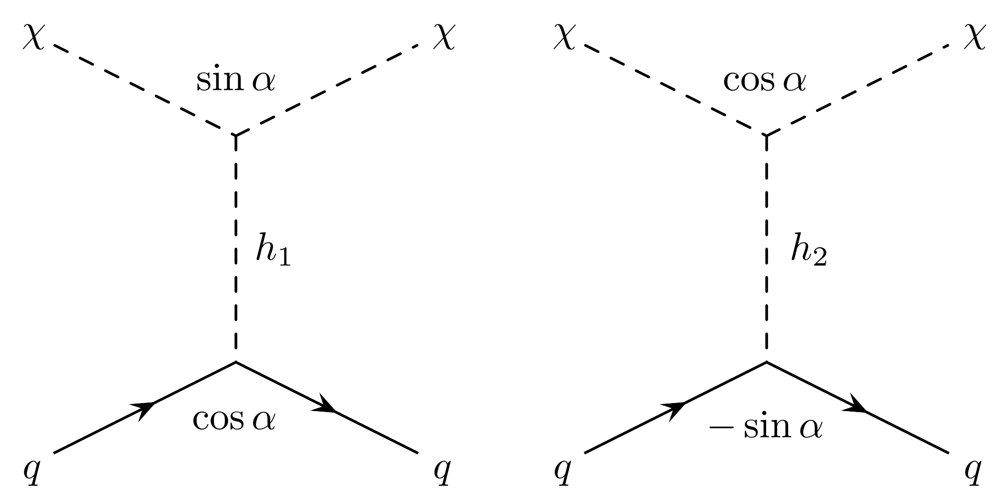
Degenerate scalar scenario

Scalar trilinear interactions

$$\mathcal{L}_S = -\frac{1}{2v_S}\left\{\left(m_{h_1}^2 + \frac{\sqrt{2}a_1}{v_S}\right)\boxed{\sin\alpha}\, h_1\chi^2 + \left(m_{h_2}^2 + \frac{\sqrt{2}a_1}{v_S}\right)\boxed{\cos\alpha}\, h_2\chi^2\right\}$$

Yukawa interactions

$$\mathcal{L}_Y = -\frac{m_f}{v}\bar{f}f\,(h_1\boxed{\cos\alpha} - h_2\boxed{\sin\alpha})$$



$$h_1 = h_{\rm SM}\cos\alpha - s\sin\alpha,\;\; h_2 = -h_{\rm SM}\sin\alpha + s\cos\alpha$$

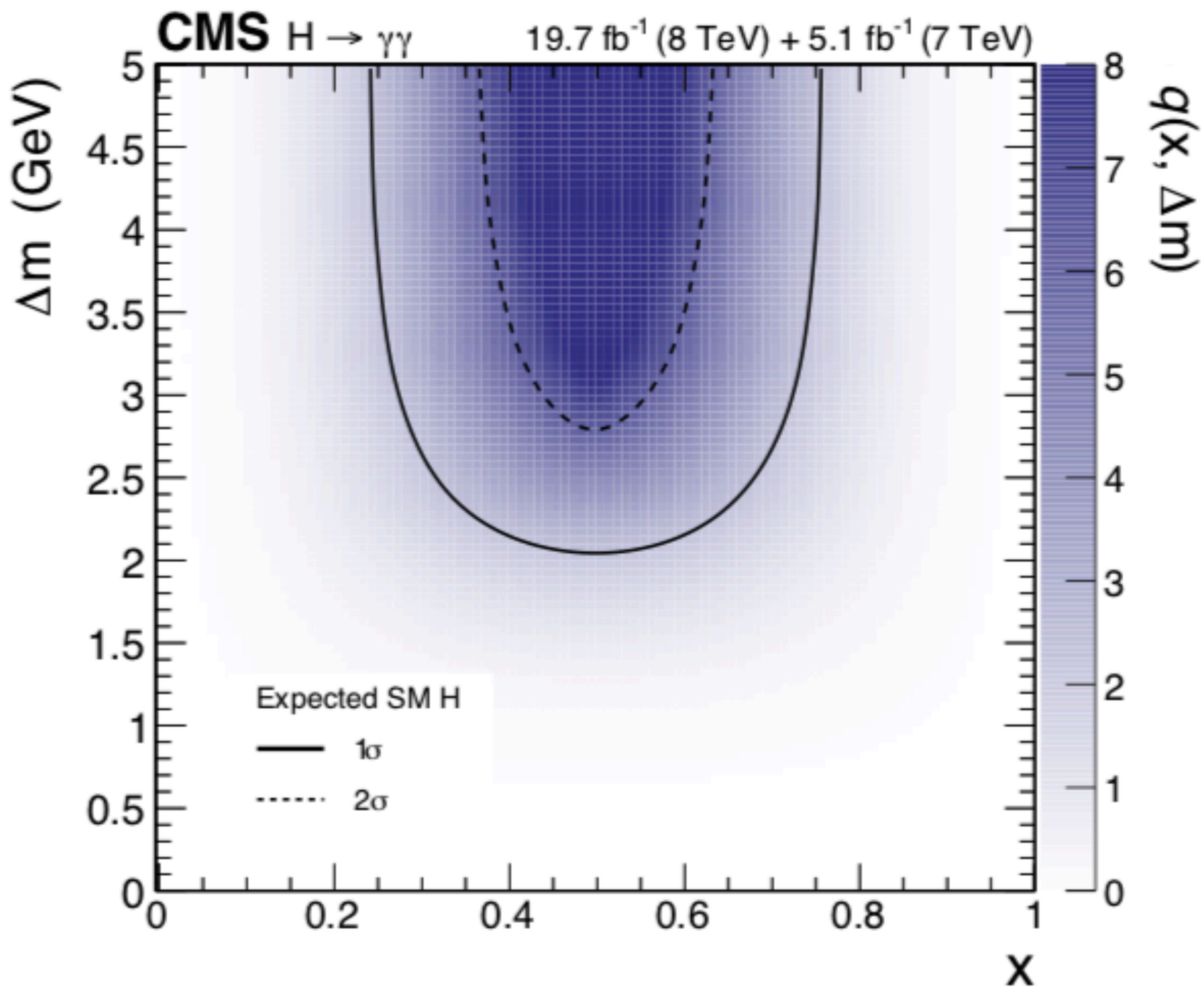
$$\Gamma(h_1 \to \mathrm{SM}) = \Gamma(h_{\rm SM} \to \mathrm{SM})(m_{h_1}) \times \cos^2\alpha$$
$$\Gamma(h_2 \to \mathrm{SM}) = \Gamma(h_{\rm SM} \to \mathrm{SM})(m_{h_2}) \times \sin^2\alpha$$

$$\boxed{\Gamma(h_1 \to \mathrm{SM}) + \Gamma(h_2 \to \mathrm{SM}) \simeq \Gamma(h_{\rm SM} \to \mathrm{SM}) \text{ for } m_{h_1} \simeq m_{h_2}}$$

Degenerate scalar scenario

CMS collaboration, V. Khachatryan et al.,
Eur. Phys. J. C 74 (2014) 3076, [1407.0558].

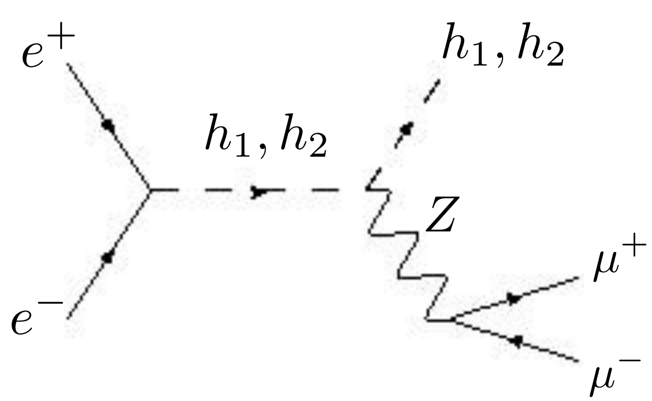
@ LHC

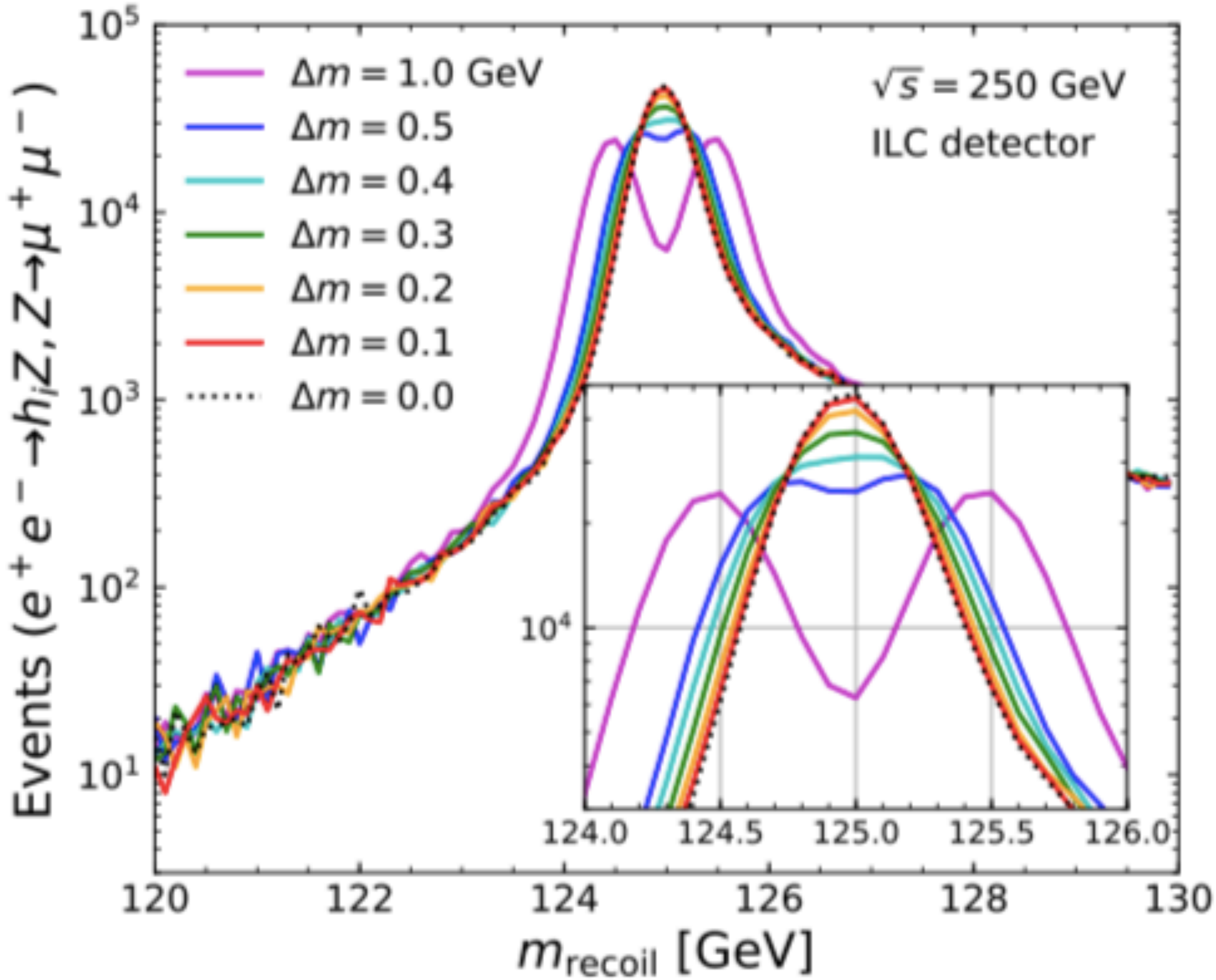


Degenerate scalar scenario

Sachiho Abe , Gi-Chol Cho, Kentarou Mawatari,
arXiv:2101.04887

@ ILC





EWPT

Strong 1st order phase transition (SFOEWPT) → $\dfrac{v_c}{T_c} \gtrsim 1$

T_C : critical temperature
v_C : higgs vev at T_C

[Two calculation schemes on the scalar potential (**gauge independent**)]

HT potential $V^{\mathrm{HT}}(\varphi, \varphi_S; T) = V_0(\varphi, \varphi_S) + \frac{1}{2}\left(\Sigma_H \varphi^2 + \Sigma_S \varphi_S^2\right) T^2$

PRM scheme $\dfrac{\partial V_{\mathrm{eff}}(\varphi, \xi)}{\partial \xi} = -C(\varphi, \xi) \dfrac{\partial V_{\mathrm{eff}}(\varphi, \xi)}{\partial \varphi}$ M. J. Ramsey-Musolf, JHEP 07 (2011), 029.

$$V_0\left(0, v_{S,\,\mathrm{tree}}^{\mathrm{sym}}\right) + V_1\left(0, v_{S,\,\mathrm{tree}}^{\mathrm{sym}}; T\right) = V_0\left(v_{\mathrm{tree}}, v_{S,\,\mathrm{tree}}\right) + V_1\left(v_{\mathrm{tree}}, v_{S,\,\mathrm{tree}}; T\right)$$

v_C, v_{SC} and v_{SC}^{sym} are determined by the use of V^{HT}

[Two resummation methods in evaluating one-loop effective potential (**gauge dependent**)]

$$V_{\mathrm{eff}}(\varphi, \varphi_S; T) = V_0(\varphi, \varphi_S; T) + \sum_i n_i \left[V_{\mathrm{CW}}\left(\bar{m}_i^2\right) + \frac{T^4}{2\pi^2} I_{B,F}\left(\frac{\bar{m}_i^2}{T^2}\right)\right]$$

Parwani scheme Replace $\bar{m}^2$ with thermally corrected field depending masses $\bar{M}^2$

AE scheme $V_{\mathrm{daisy}}(\varphi, \varphi_S; T) = \displaystyle\sum_{\substack{i=h_{1,2},\chi \\ W_L, Z_L, \gamma_L}} -n_i \frac{T}{12\pi}\left[\left(\bar{M}_i^2\right)^{3/2} - \left(\bar{m}_i^2\right)^{3/2}\right]$

EWPT

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$$V_0\left(0, v_{S,\ \mathrm{tree}}^{\mathrm{sym}}\right) + V_1\left(0, v_{S,\ \mathrm{tree}}^{\mathrm{sym}}; T\right) = V_0\left(v_{\mathrm{tree}}, v_{S,\ \mathrm{tree}}\right) + V_1\left(v_{\mathrm{tree}}, v_{S,\ \mathrm{tree}}; T\right)$$

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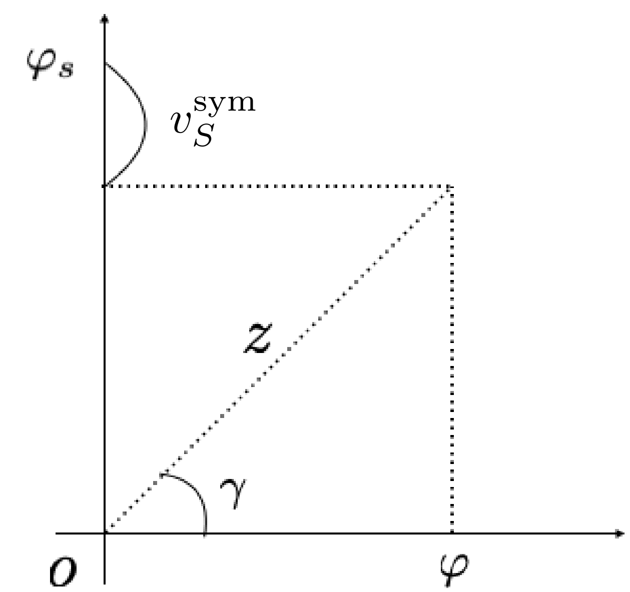
EWPT in the degenerate-scalar scenario

	Gauge independence	Renormalized V_{CW}	One loop contribution
HT potential	○		×
PRM scheme	○	×	○
Parwani scheme	×	○	○
AE scheme	×	○	○

EWPT

Parametrize the two scalar fields using radial coordinates as

$$\varphi = z\cos\gamma, \varphi_S = z\sin\gamma + v_S^{\rm sym}$$



HT potential

$$V^{\rm HT}(\varphi, \varphi_S; T) = V_0(\varphi, \varphi_S) + \frac{1}{2}\left(\Sigma_H\varphi^2 + \Sigma_S\varphi_S^2\right)T^2$$

$$\rightarrow \quad V^{\rm HT}(z, \gamma; T) = c_0 + c_1 z + (c_2 + c_2' T^2)z^2 - c_3 z^3 + c_4 z^4$$

In the case of first-order EWPT

$$T_C \simeq \sqrt{\frac{1}{2\Sigma_H}\left(-m^2 - \frac{(v_{SC}^{\rm sym})^2}{2}\delta_2\right)},$$

$$v_C \simeq \sqrt{\frac{2\delta_2 (v_{SC}^{\rm sym})^2}{\lambda}\left(1 - \frac{v_{SC}}{v_{SC}^{\rm sym}}\right)}$$

$$v_C = \lim_{T \nearrow T_C} v(T)$$

$$v_{SC} = \lim_{T \nearrow T_C} v_S(T)$$

$$v_{SC}^{\rm sym} = \lim_{T \searrow T_C} v_S(T)$$

Condition of SFOEWPT

$$\frac{v_c}{T_c} \gtrsim 1$$

EWPT

Tree level potential $V_0=\frac{m^2}{2}|H|^2+\frac{\lambda}{4}|H|^4+\frac{\delta_2}{2}|H|^2|S|^2+\frac{b_2}{2}|S|^2+\frac{d_2}{4}|S|^4+\left(a_1 S+\frac{b_1}{4}S^2+\text{ c.c.}\right)$

$$T_C\simeq\sqrt{\frac{1}{2\Sigma_H}\left(-m^2-\frac{(v_{SC}^{\text{sym}})^2}{2}\delta_2\right)},$$

$$v_C\simeq\sqrt{\frac{2\delta_2\,(v_{SC}^{\text{sym}})^2}{\lambda}\left(1-\frac{v_{SC}}{v_{SC}^{\text{sym}}}\right)}$$

Condition of SFOEWPT

$$\frac{v_c}{T_c}\gtrsim 1$$

About T_C — $T_C\rightarrow$ small, $\delta_2\rightarrow$ positive and sizable

$$\delta_2=\frac{2}{vv_S}\left(m_{h_1}^2-m_{h_2}^2\right)\sin\alpha\cos\alpha$$

$v_S\rightarrow$ small, $\alpha\rightarrow$ the maximal mixing $\dfrac{\pi}{4}$

EWPT

Tree level potential $V_0 = \frac{m^2}{2}|H|^2 + \frac{\lambda}{4}|H|^4 + \frac{\delta_2}{2}|H|^2|S|^2 + \frac{b_2}{2}|S|^2 + \frac{d_2}{4}|S|^4 + \left(a_1 S + \frac{b_1}{4}S^2 + \text{ c.c. }\right)$

$$\boxed{T_C \simeq \sqrt{\frac{1}{2\Sigma_H}\left(-m^2 - \frac{(v_{SC}^{\text{sym}})^2}{2}\delta_2\right)},}$$

$$v_C \simeq \sqrt{\frac{2\delta_2 (v_{SC}^{\text{sym}})^2}{\lambda}\left(1 - \frac{v_{SC}}{v_{SC}^{\text{sym}}}\right)}$$

Condition of SFOEWPT

$$\frac{v_c}{T_c} \gtrsim 1$$

About T_C $\quad T_C \to$ small, $\delta_2 \to$ positive and sizable

$$\delta_2 = \frac{2}{v v_S}(m_{h_1}^2 - m_{h_2}^2)\sin\alpha\cos\alpha$$

$v_S \to$ small, $\alpha \to$ the maximal mixing $\frac{\pi}{4}$

EWPT

$$V_0 = \frac{m^2}{2}|H|^2 + \frac{\lambda}{4}|H|^4 + \frac{\delta_2}{2}|H|^2|S|^2 + \frac{b_2}{2}|S|^2 + \frac{d_2}{4}|S|^4 + \left(a_1 S + \frac{b_1}{4}S^2 + \text{ c.c.}\right)$$

$$T_C \simeq \sqrt{\frac{1}{2\Sigma_H}\left(-m^2 - \frac{\left(v_{SC}^{\rm sym}\right)^2}{2}\delta_2\right)},$$

$$v_C \simeq \sqrt{\frac{2\delta_2\left(v_{SC}^{\rm sym}\right)^2}{\lambda}\left(1 - \frac{v_{SC}}{v_{SC}^{\rm sym}}\right)}$$

Condition of SFOEWPT

$$\frac{v_c}{T_c} \gtrsim 1$$

About v_C

$v_C \rightarrow$ large with an amplification factor $(v_{SC}^{\rm sym})^2\,(1 - v_{SC}/v_{SC}^{\rm sym})$

$$(v_{SC}^{\rm sym})^3 + A v_{SC}^{\rm sym} + B = 0$$

$$A = 2\left(b_1 + b_2 + 2\Sigma_S\right)/d_2$$

$$B = 4\sqrt{2}a_1/d_2$$

$v_{SC}^{\rm sym}$ is scaled by $1/\sqrt{d_2}$

$\therefore d_2 \rightarrow$ small

$$d_2 = \frac{2}{v_S^2}\left[m_{h_1}^2 + \left(m_{h_2}^2 - m_{h_1}^2\right)\cos^2\alpha + \frac{\sqrt{2}a_1}{v_S}\right] \simeq \frac{2}{v_S^2}\left[m_{h_1}^2 + \frac{\sqrt{2}a_1}{v_S}\right]$$

$a_1 < 0$

EWPT

$$V_0 = \frac{m^2}{2}|H|^2 + \frac{\lambda}{4}|H|^4 + \frac{\delta_2}{2}|H|^2|S|^2 + \frac{b_2}{2}|S|^2 + \frac{d_2}{4}|S|^4 + \left(a_1 S + \frac{b_1}{4}S^2 + \text{ c.c.}\right)$$

$$T_C \simeq \sqrt{\frac{1}{2\Sigma_H}\left(-m^2 - \frac{\left(v_{SC}^{\mathrm{sym}}\right)^2}{2}\delta_2\right)},$$

$$\boxed{v_C \simeq \sqrt{\frac{2\delta_2 \boxed{\left(v_{SC}^{\mathrm{sym}}\right)^2}}{\lambda}\boxed{\left(1 - \frac{v_{SC}}{v_{SC}^{\mathrm{sym}}}\right)}}}$$

Condition of SFOEWPT

$$\frac{v_c}{T_c} \gtrsim 1$$

$\boxed{\text{About } v_C}$ $\quad v_C \to$ large with an amplification factor $\boxed{\left(v_{SC}^{\mathrm{sym}}\right)^2\left(1 - v_{SC}/v_{SC}^{\mathrm{sym}}\right)}$

$$\left(v_{SC}^{\mathrm{sym}}\right)^3 + A v_{SC}^{\mathrm{sym}} + B = 0$$

$$A = 2\left(b_1 + b_2 + 2\Sigma_S\right)/d_2$$

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v_{SC}^{sym} is scaled by $1/\sqrt{d_2}$ $\quad \therefore d_2 \to$ small

$$d_2 = \frac{2}{v_S^2}\left[m_{h_1}^2 + \left(m_{h_2}^2 - m_{h_1}^2\right)\cos^2\alpha + \frac{\sqrt{2}a_1}{v_S}\right] \simeq \frac{2}{v_S^2}\left[m_{h_1}^2 + \frac{\sqrt{2}a_1}{v_S}\right] \qquad a_1 < 0$$

EWPT

$$V_0 = \frac{m^2}{2}|H|^2 + \frac{\lambda}{4}|H|^4 + \frac{\delta_2}{2}|H|^2|S|^2 + \frac{b_2}{2}|S|^2 + \frac{d_2}{4}|S|^4 + \left(a_1 S + \frac{b_1}{4}S^2 + \text{ c.c.}\right)$$

$$T_C \simeq \sqrt{\frac{1}{2\Sigma_H}\left(-m^2 - \frac{(v_{SC}^{\text{sym}})^2}{2}\delta_2\right)},$$

$$v_C \simeq \sqrt{\frac{2\delta_2 (v_{SC}^{\text{sym}})^2}{\lambda}\left(1 - \frac{v_{SC}}{v_{SC}^{\text{sym}}}\right)}$$

Condition of SFOEWPT

$$\frac{v_c}{T_c} \gtrsim 1$$

About v_C

$v_C \rightarrow$ large with an amplification factor $(v_{SC}^{\text{sym}})^2\left(1 - v_{SC}/v_{SC}^{\text{sym}}\right)$

$$(v_{SC}^{\text{sym}})^3 + A v_{SC}^{\text{sym}} + B = 0$$

$$A = 2\left(b_1 + b_2 + 2\Sigma_S\right)/d_2$$

$$B = 4\sqrt{2}a_1/d_2$$

v_{SC}^{sym} is scaled by $1/\sqrt{d_2}$

$\therefore d_2 \rightarrow$ small

$$d_2 = \frac{2}{v_S^2}\left[m_{h_1}^2 + (m_{h_2}^2 - m_{h_1}^2)\cos^2\alpha + \frac{\sqrt{2}a_1}{v_S}\right] \simeq \frac{2}{v_S^2}\left[m_{h_1}^2 + \frac{\sqrt{2}a_1}{v_S}\right]$$

$a_1 < 0$

(1) large δ_2 with a positive sign i.e., $|\alpha| \simeq \frac{\pi}{4}$ and $v_S < 1$ GeV

(2) small d_2 i.e., $a_1 < 0$ with its moderate absolute value

Conditions imposed on the parameters

Other conditions imposed on the parameters

- The energy difference between the electroweak vacuum prescribed by (v, v_S) and the local vacuum on the φ_S axis specified by $(0, v_S^{\mathrm{sym}})$

$$
\begin{aligned}
\Delta E =& V_0\left(0, v_S^{\mathrm{sym}}\right) - V_0\left(v, v_S\right) \\
=& \sqrt{2} a_1\left(v_S^{\mathrm{sym}} - v_S\right) + \frac{1}{4}\left(b_1 + b_2\right)\left(\left(v_S^{\mathrm{sym}}\right)^2 - v_S^2\right) + \frac{d_2}{16}\left(\left(v_S^{\mathrm{sym}}\right)^4 - v_S^4\right) \\
& - \frac{m^2}{4} v^2 - \frac{\lambda}{16} v^4 - \frac{\delta_2}{8} v^2 v_S^2
\end{aligned}
$$

ΔE could be negative for $\delta_2 \gg 1$ and $d_2 \ll 1$.

$\Downarrow$

δ_2 and d_2 have the upper and lower bound respectively.

- Bounded from below $\qquad \lambda > 0, d_2 > 0, \lambda d_2 > \delta_2^2$

- Vacuum stability $\qquad \lambda\left(d_2 + \dfrac{2\sqrt{2}\left|a_1\right|}{v_S^3}\right) > \delta_2^2$

- Conditions from perturbation Theory $\qquad \lambda \leqq \dfrac{16}{3}\pi, d_2 \leqq \dfrac{16}{3}\pi$

Benchmark points

Two benchmark points

Inputs	v [GeV]	m_{h_1} [GeV]	m_{h_2} [GeV]	α [rad]	a_1 [GeV^3]	v_S [GeV]	m_χ [GeV]
BP1	246.22	125	124	$\pi/4$	-6576.17	0.6	62.5
BP2	246.22	125	126	$-\pi/4$	-6682.25	0.6	62.5
Outputs	m^2 [GeV^2]	b_1 [GeV^2]	b_2 [GeV^2]	λ	a_1 [GeV^3]	d_2	δ_2
BP1	$-(124.5)^2$	$-(107.7)^2$	$-(178.0)^2$	0.511	-6576.17	1.77	1.69
BP2	$-(125.5)^2$	$-(108.8)^2$	$-(178.4)^2$	0.520	-6682.25	1.70	1.59

Calculate the DM relic density $\Omega_\chi h^2$ and SI cross section with the nucleons $\sigma_{\rm SI}$ in BP1.

(For the moment, m_χ is treated as the varying parameter.)

Benchmark points

Two benchmark points

the varying parameter

Inputs	v [GeV]	m_{h_1} [GeV]	m_{h_2} [GeV]	α [rad]	a_1 [GeV^3]	v_S [GeV]	m_χ [GeV]
BP1	246.22	125	124	$\pi/4$	-6576.17	0.6	variable
BP2	246.22	125	126	$-\pi/4$	-6682.25	0.6	variable
Outputs	m^2 [GeV^2]	b_1 [GeV^2]	b_2 [GeV^2]	λ	a_1 [GeV^3]	d_2	δ_2
BP1	$-(124.5)^2$	$-(107.7)^2$	$-(178.0)^2$	0.511	-6576.17	1.77	1.69
BP2	$-(125.5)^2$	$-(108.8)^2$	$-(178.4)^2$	0.520	-6682.25	1.70	1.59

Calculate the DM relic density $\Omega_\chi h^2$ and SI cross section with the nucleons $\sigma_{\rm SI}$ in BP1.

(For the moment, m_χ is treated as the varying parameter.)

Benchmark points

$$\delta_2 = \frac{2}{v v_S}\left(m_{h_1}^2 - m_{h_2}^2\right) \sin\alpha\cos\alpha$$

Invariant under the transformation $m_{h_1}^2 - m_{h_2}^2 \to -(m_{h_1}^2 - m_{h_2}^2)$ and $\alpha \to -\alpha$

$$d_2 = \frac{2}{v_S^2}\left[m_{h_1}^2 + \left(m_{h_2}^2 - m_{h_1}^2\right)\cos^2\alpha + \frac{\sqrt{2}a_1}{v_S}\right] \simeq \frac{2}{v_S^2}\left[m_{h_1}^2 + \frac{\sqrt{2}a_1}{v_S}\right]$$

The sign of $m_{h_1}^2 - m_{h_2}^2$ cannot be compensated by that of α

DM

We use a public code micrOMEGAs to calculate $\Omega_\chi h^2$ and $\sigma_{\rm SI}$.

The value of $\Omega_\chi h^2$ should not exceed the observed value

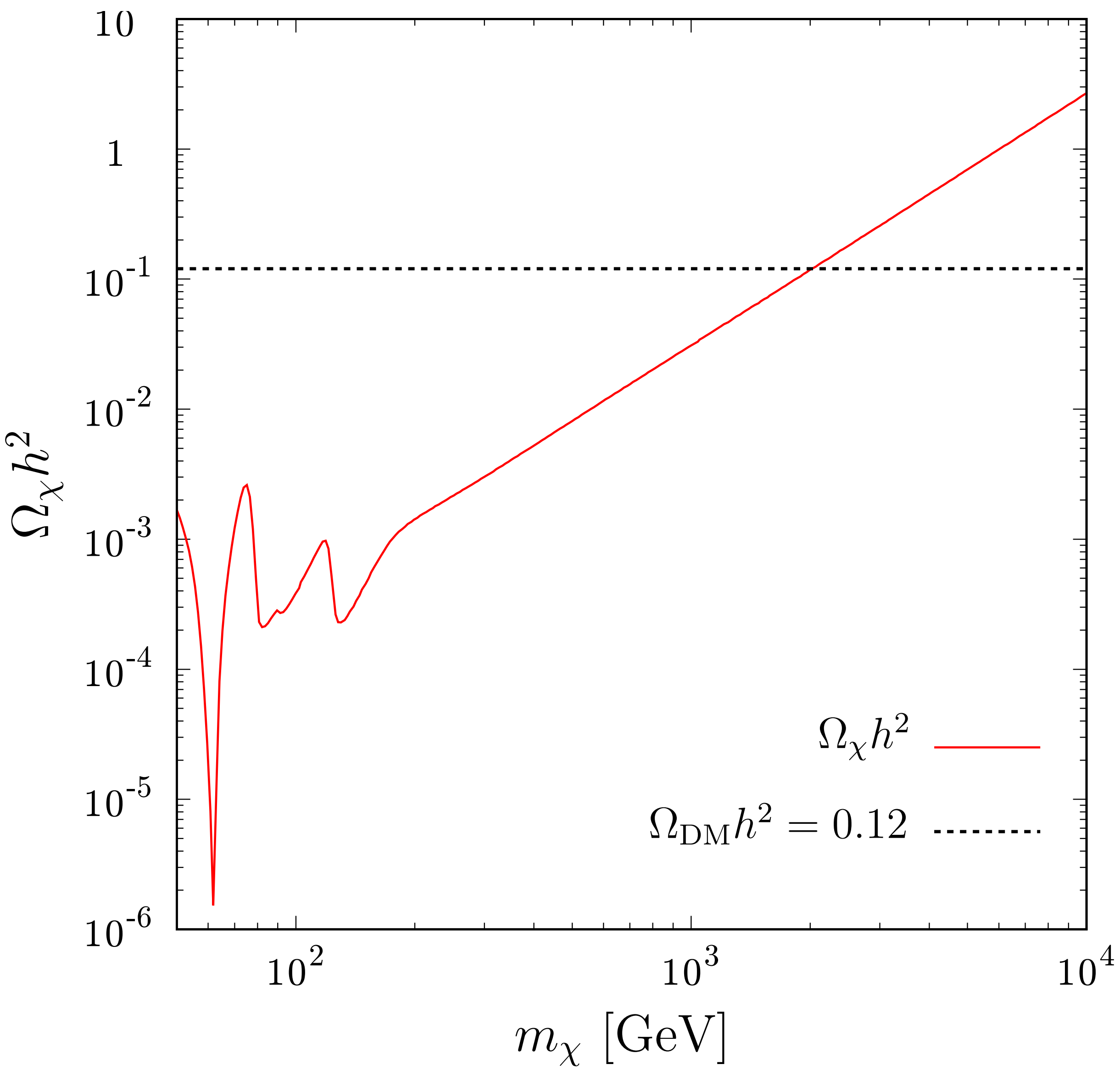
$$\Omega_{\rm DM} h^2 = 0.1200 \pm 0.0012$$

In the case of $m_\chi = 30$ GeV, for instance, the maximum value is $\sigma_{\rm SI} \simeq 4.1 \times 10^{-47}$ cm^2 under the assumption $\Omega_\chi = \Omega_{\rm DM}$.

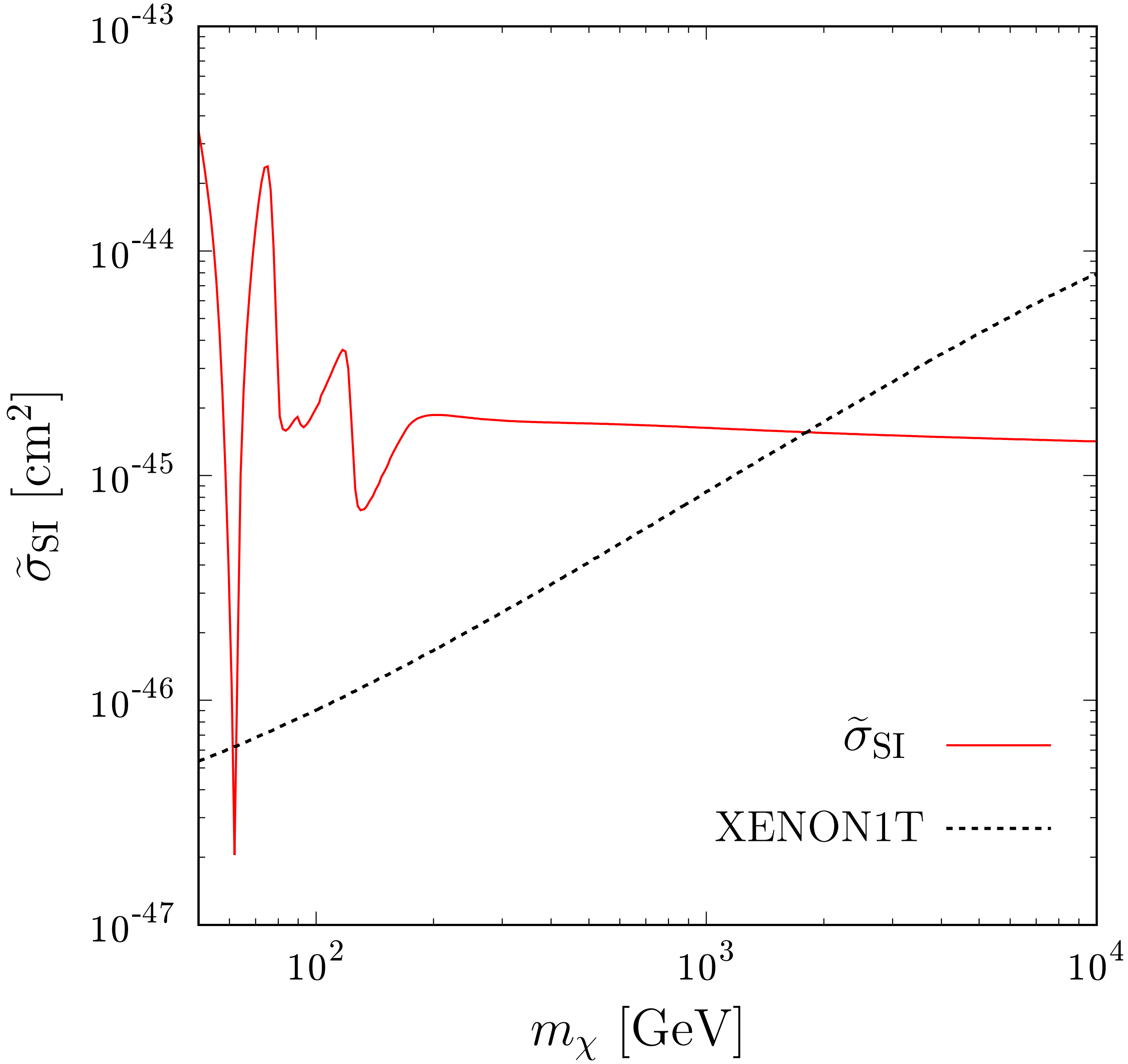
In cases that $\Omega_\chi < \Omega_{\rm DM}$, we scale $\sigma_{\rm SI}$ as

$$\widetilde{\sigma}_{\rm SI} = \left(\frac{\Omega_\chi}{\Omega_{\rm DM}} \right) \sigma_{\rm SI}$$

DM

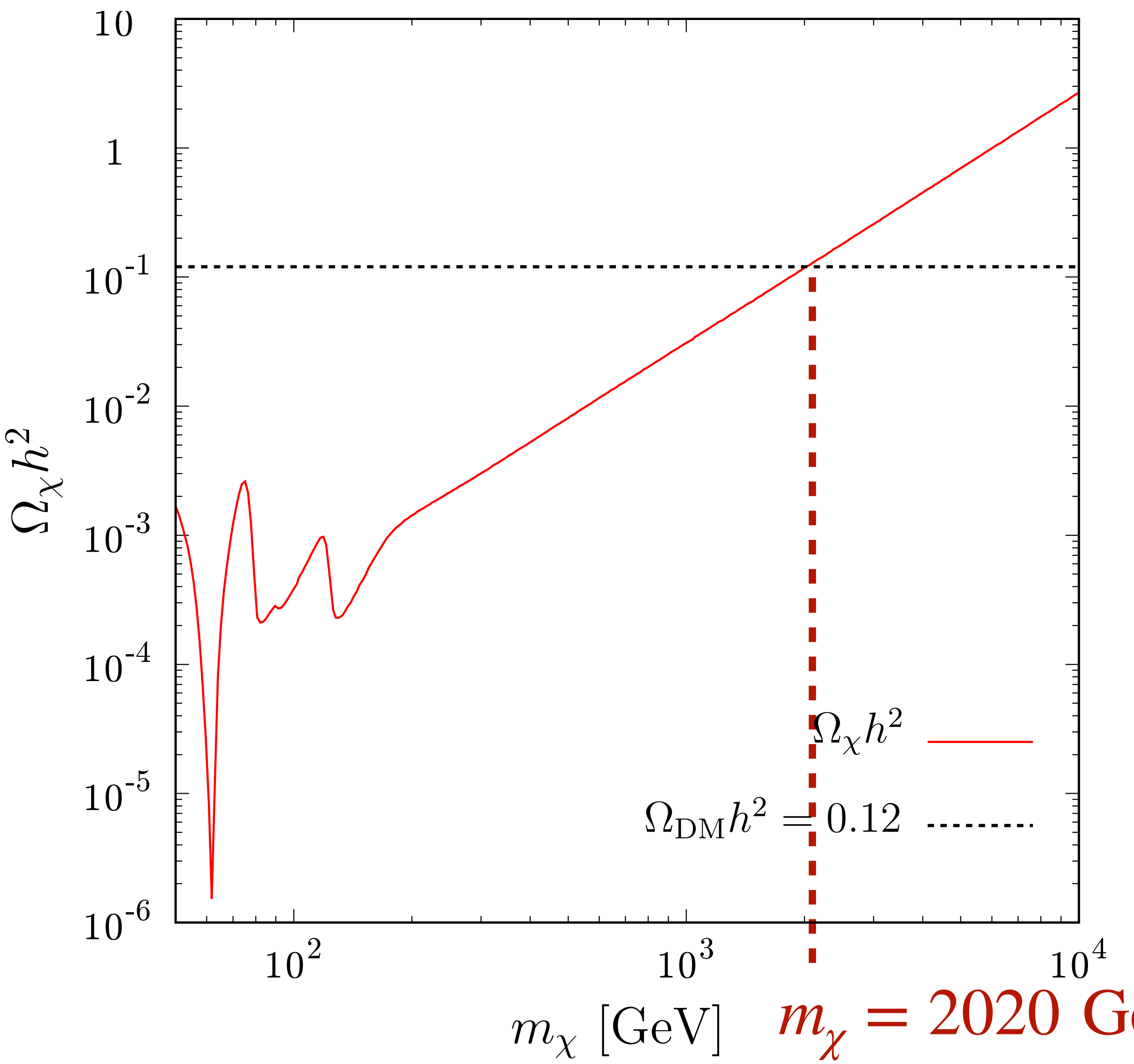


10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$\Omega_\chi h^2$
10^2
10^3
10^4
m_χ [GeV]
$\Omega_\chi h^2$
$\Omega_{\rm DM} h^2 = 0.12$

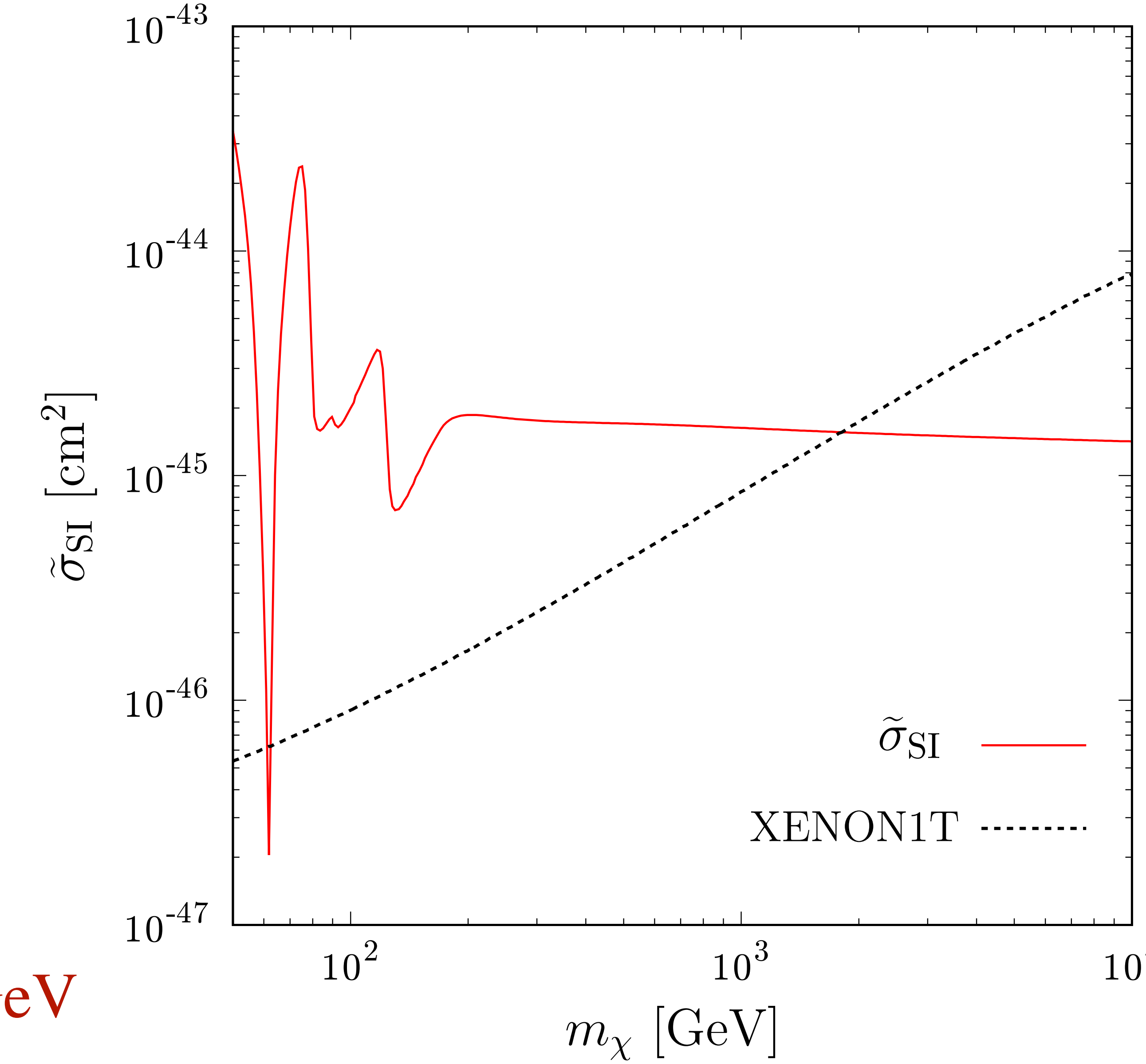


10^{-43}
10^{-44}
10^{-45}
10^{-46}
10^{-47}
$\tilde{\sigma}_{\rm SI}$ [cm^2]
10^2
10^3
10^4
m_χ [GeV]
$\tilde{\sigma}_{\rm SI}$
XENON1T

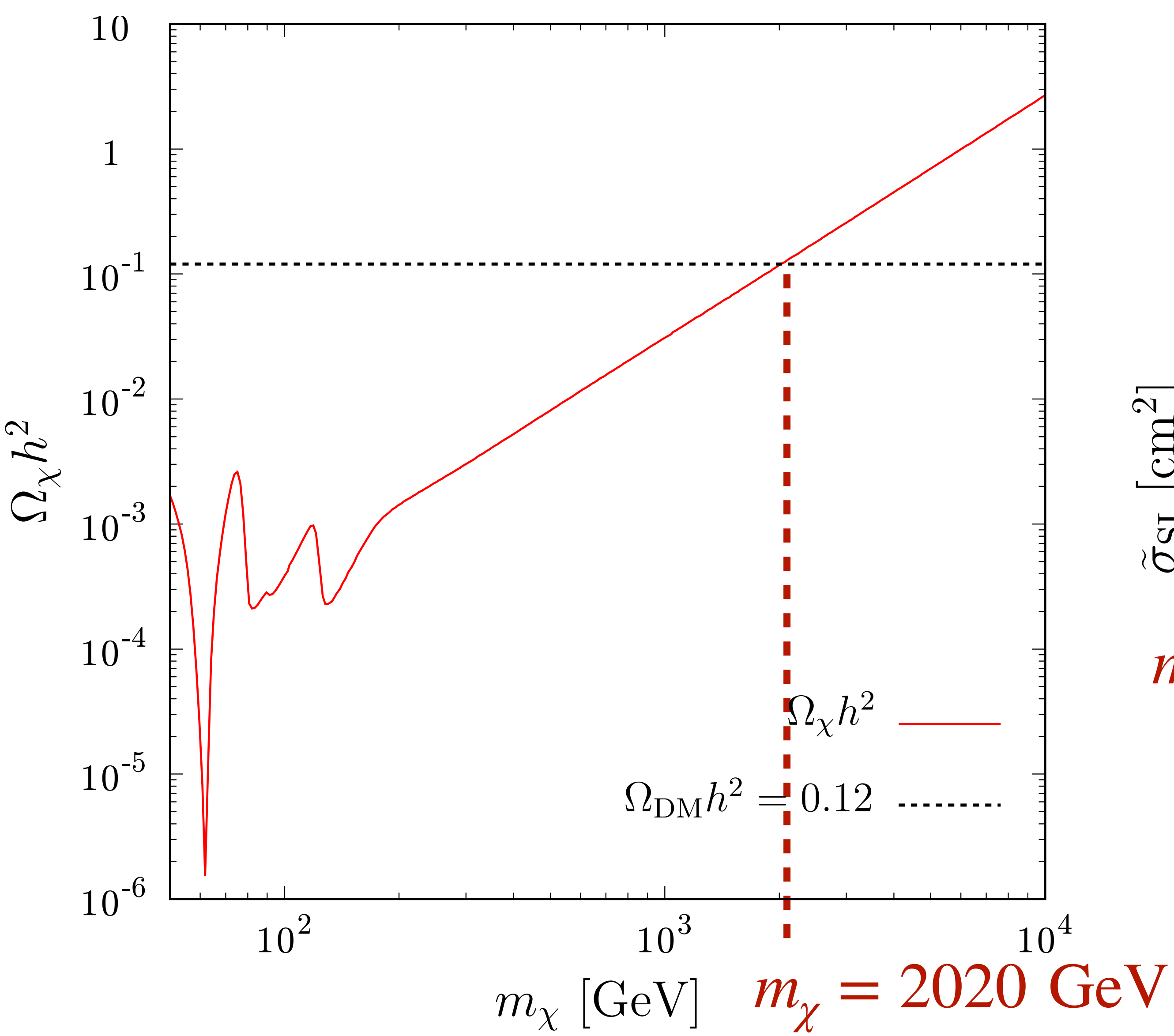
DM



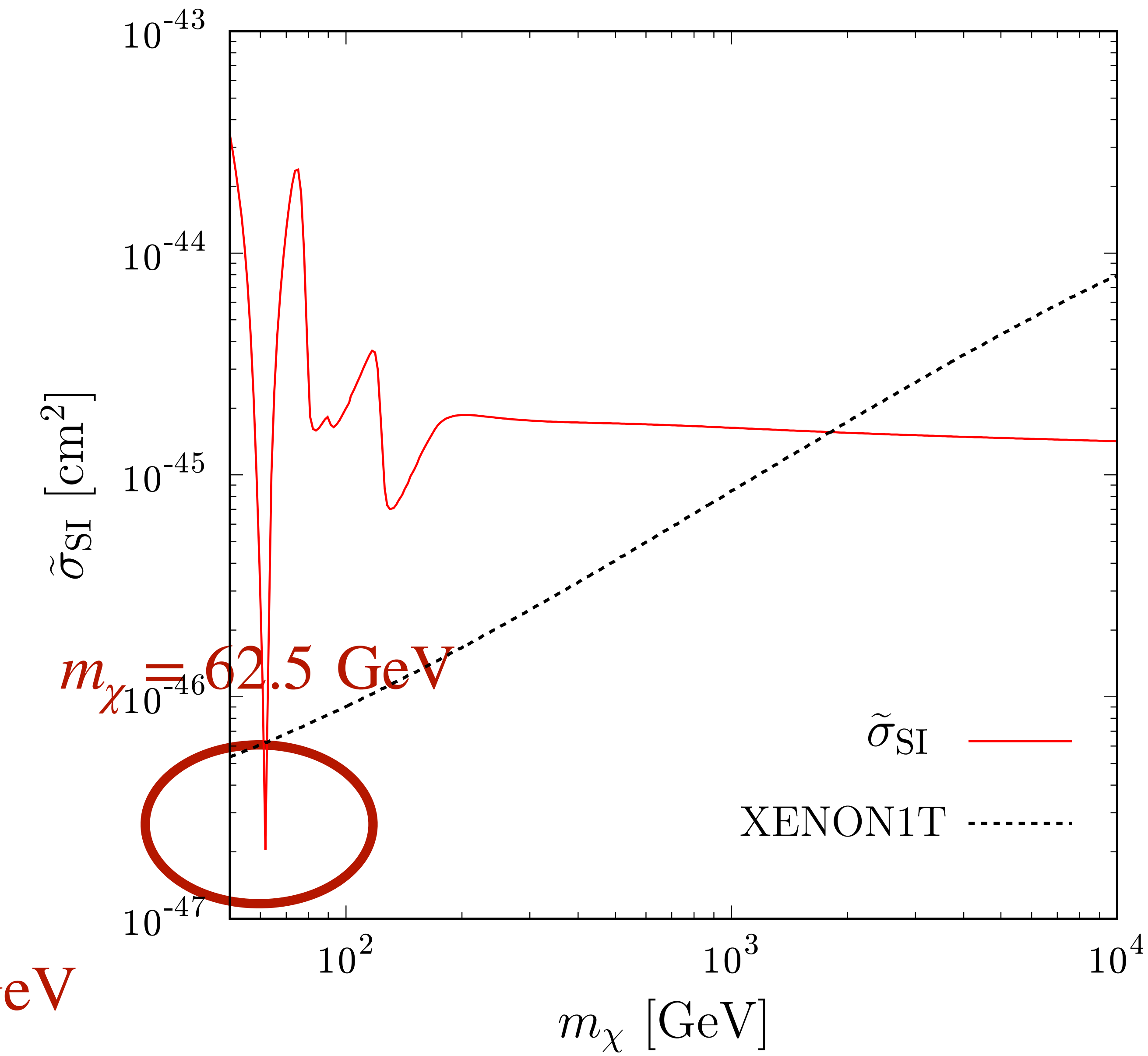
$m_\chi = 2020$ GeV



DM

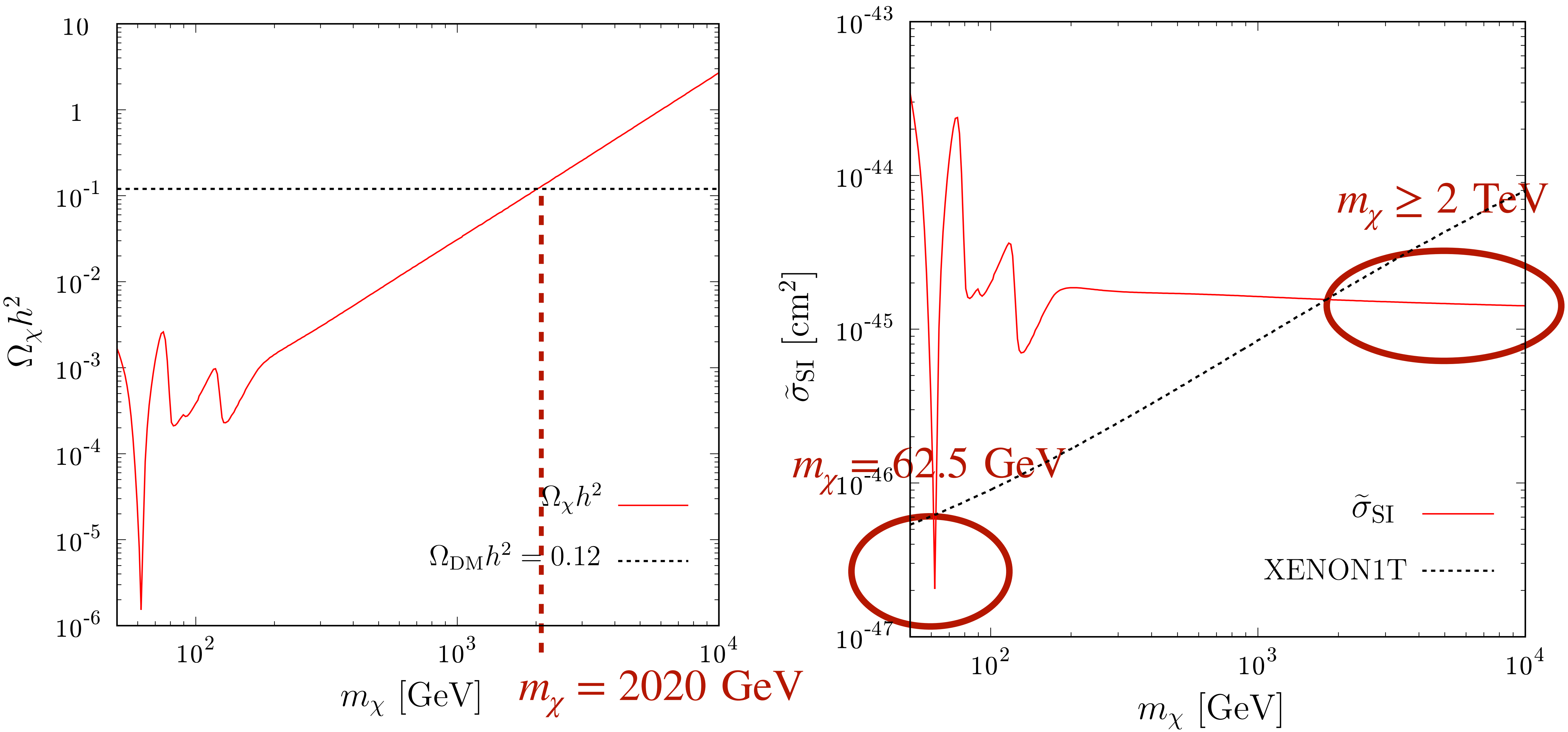


10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$\Omega_\chi h^2$
10^2
10^3
10^4
m_χ [GeV]
$\Omega_\chi h^2$
$\Omega_{\rm DM} h^2 = 0.12$
$m_\chi = 2020$ GeV



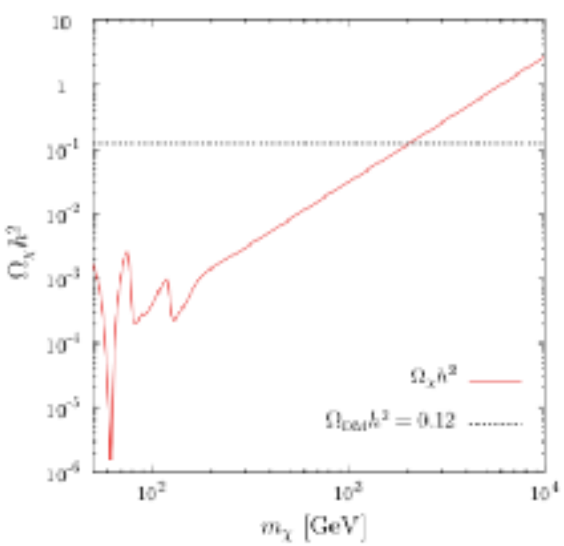
10^{-43}
10^{-44}
10^{-45}
10^{-46}
10^{-47}
$\tilde{\sigma}_{\rm SI}$ [cm^2]
10^2
10^3
10^4
m_χ [GeV]
$\tilde{\sigma}_{\rm SI}$
XENON1T
$m_\chi = 62.5$ GeV

DM

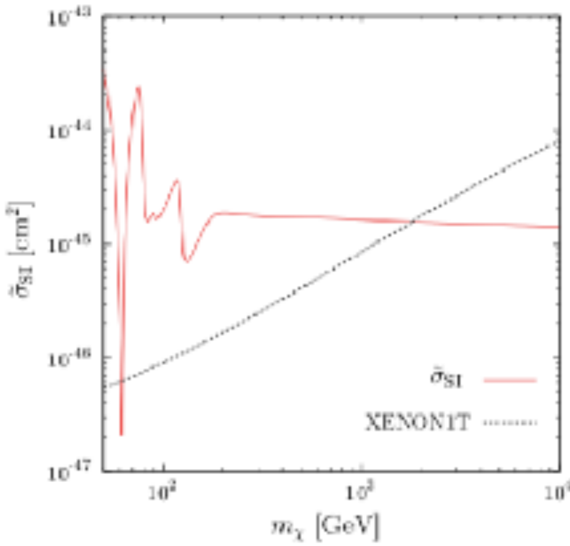


10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
$\Omega_\chi h^2$
$\Omega_{\rm DM} h^2 = 0.12$
10^2
10^3
10^4
m_χ [GeV]
$m_\chi = 2020$ GeV
10^{-43}
10^{-44}
10^{-45}
10^{-46}
10^{-47}
$\tilde{\sigma}_{\rm SI}$ [cm^2]
$m_\chi \geq 2$ TeV
$m_\chi = 62.5$ GeV
$\tilde{\sigma}_{\rm SI}$
XENON1T
10^2
10^3
10^4
m_χ [GeV]

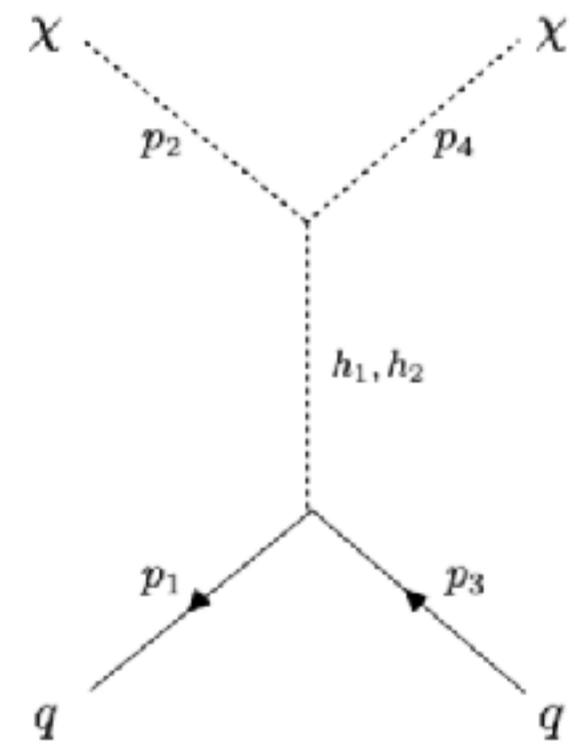
Degenerate scalar scenario and strong 1st-order EWPT



DM relic density $\Omega_\chi h^2$



SI scattering cross section $\sigma_{\rm SI}$



The scattering of dark matter χ and quark q

$$\sigma_{\rm SI} \propto \sin^2\alpha\cos^2\alpha \left(\frac{1}{m_{h_1}^2} - \frac{1}{m_{h_2}^2}\right)^2 \frac{a_1^2}{v_S^4} = \frac{\delta_2^2 v^2}{4m_{h_1}^4 m_{h_2}^4}\frac{a_1^2}{v_S^2}$$

$$\delta_2 = \frac{2}{v v_S}\left(m_{h_1}^2 - m_{h_2}^2\right)\sin\alpha\cos\alpha$$

Strong 1st EWPT

$\delta_2 \to$ large

$v_S \to$ small

(less than 1 GeV)

The core of the cancellation mechanism in the degenerate-scalar scenario:

The suppression of δ_2 owing to $m_{h_1} \simeq m_{h_2}$ with moderate values of v_S.

The conditions for the strong 1st EWPT is incompatible with the suppression mechanism.

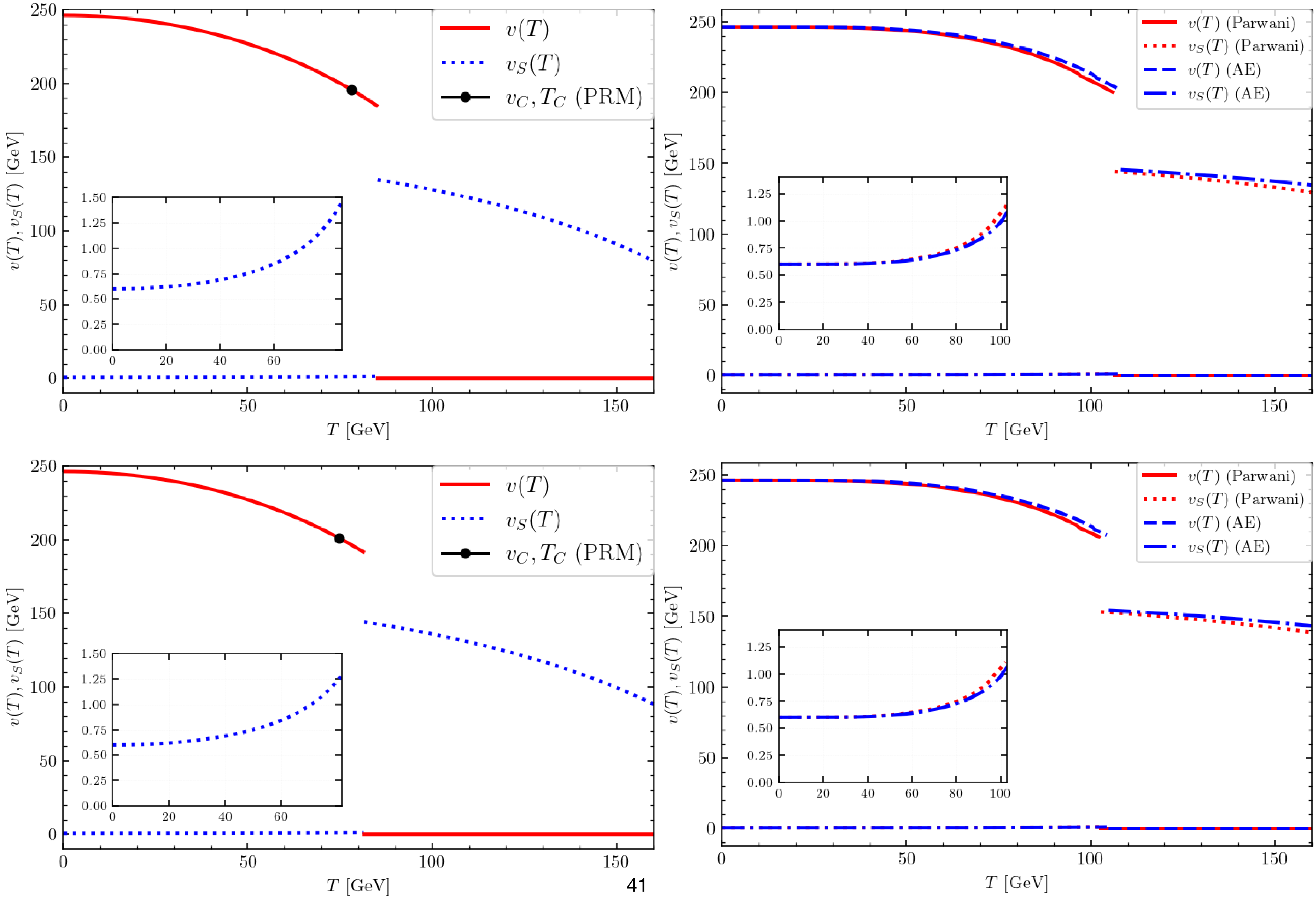
1st-order EWPT

HT/PRM

Parwani/AE

BP1

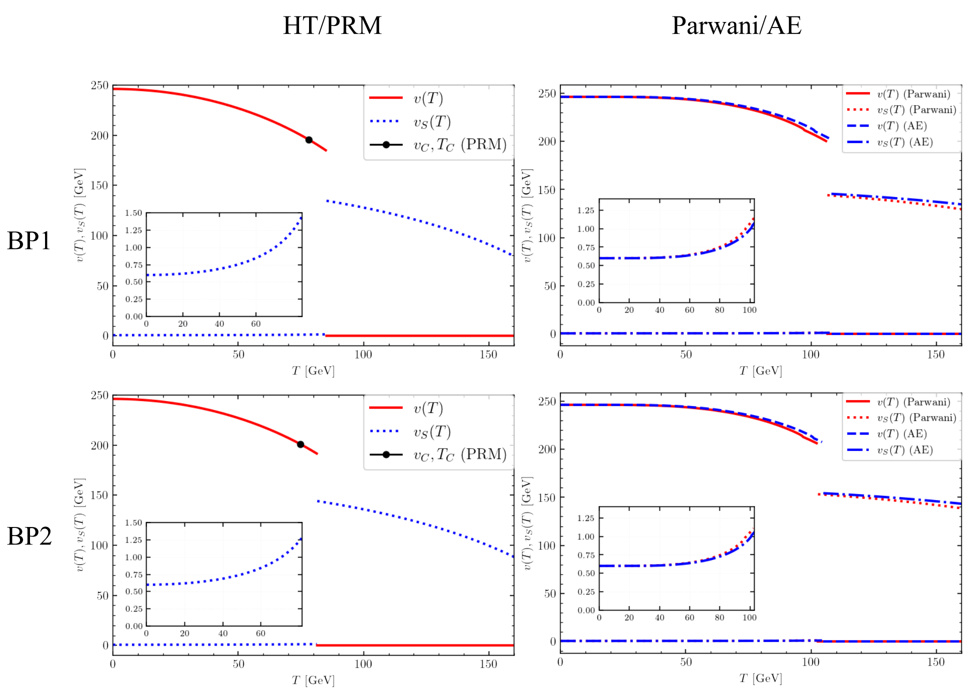
BP2



1st-order EWPT

Condition of SFOEWPT

$$\frac{v_c}{T_c} \gtrsim 1$$



Ex) BP1

	BP1			
Scheme	HT	PRM	Parwani	AE
v_C/T_C	$\frac{184.4}{85.3} = 2.2$	$\frac{195.6}{78.2} = 2.5$	$\frac{201.5}{106.8} = 1.9$	$\frac{202.7}{107.8} = 1.9$
v_{SC} [GeV]	1.5	1.2	1.2	1.2
$v_{SC}^{\rm sym}$ [GeV]	134.6	137.3	144.8	145.3

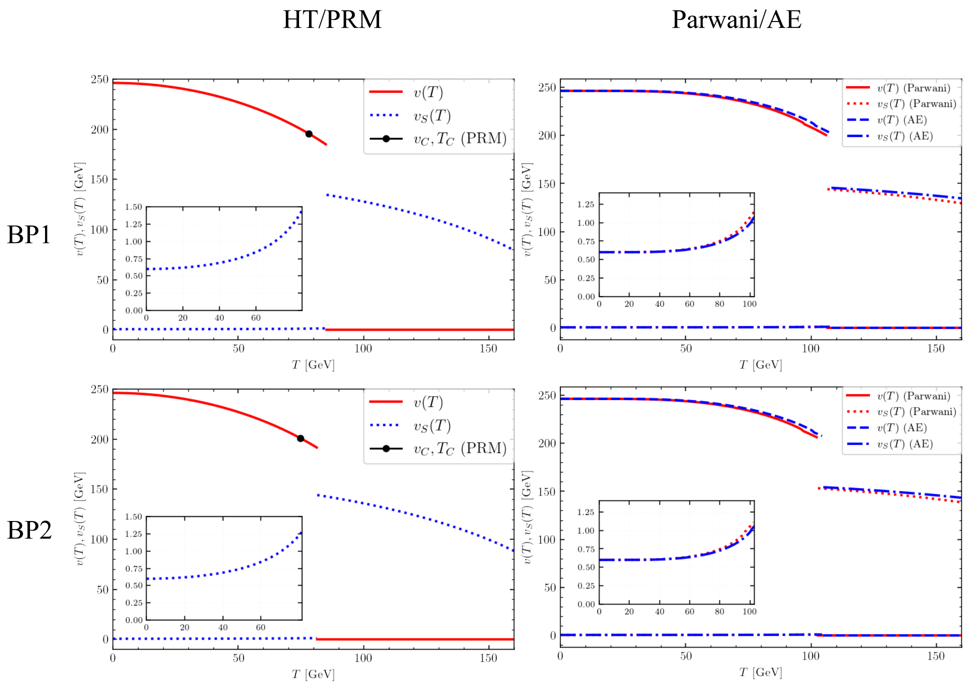
The consequences found in BP1 all apply to BP2 as well.

Strong first-order EWPT in the degenerate-scalar scenario is possible in the both cases $m_{h_1} > m_{h_2}$ and $m_{h_1} < m_{h_2}$.

1st-order EWPT

Condition of SFOEWPT

$$\frac{v_C}{T_C} \gtrsim 1$$



Ex) BP1

	BP1			
Scheme	HT	PRM	Parwani	AE
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v_{SC} [GeV]	1.5	1.2	1.2	1.2
v_{SC}^{sym} [GeV]	134.6	137.3	144.8	145.3

Strong 1st PT !

The consequences found in BP1 all apply to BP2 as well.

Strong first-order EWPT in the degenerate-scalar scenario is possible in the both cases $m_{h_1} > m_{h_2}$ and $m_{h_1} < m_{h_2}$.

1st-order EWPT

The viable DM regions: $m_\chi = 62.5$ GeV, 2 TeV

When $m_\chi = 2$ TeV, one can find the first-order EWPT in the HT, Parwani, and AE schemes while not in the PRM scheme.

$$V_0\left(0, v_{S,\ \mathrm{tree}}^{\mathrm{sym}}\right) + V_1\left(0, v_{S,\ \mathrm{tree}}^{\mathrm{sym}}; T\right) = V_0\left(v_{\mathrm{tree}}, v_{S,\ \mathrm{tree}}\right) + V_1\left(v_{\mathrm{tree}}, v_{S,\ \mathrm{tree}}; T\right)$$

the right-hand side has to be lower than the left-hand side at zero temperature, otherwise, the degeneracy point where T_C is defined would not exist.

Ex) BP1

For $m_\chi \gtrsim 700$ GeV, the right-hand side would exceed the left-hand side.

→ This bound could be relaxed when one includes higher-order corrections.

EWPT

EWPT

Baryon number violation

→ Sphaleron process

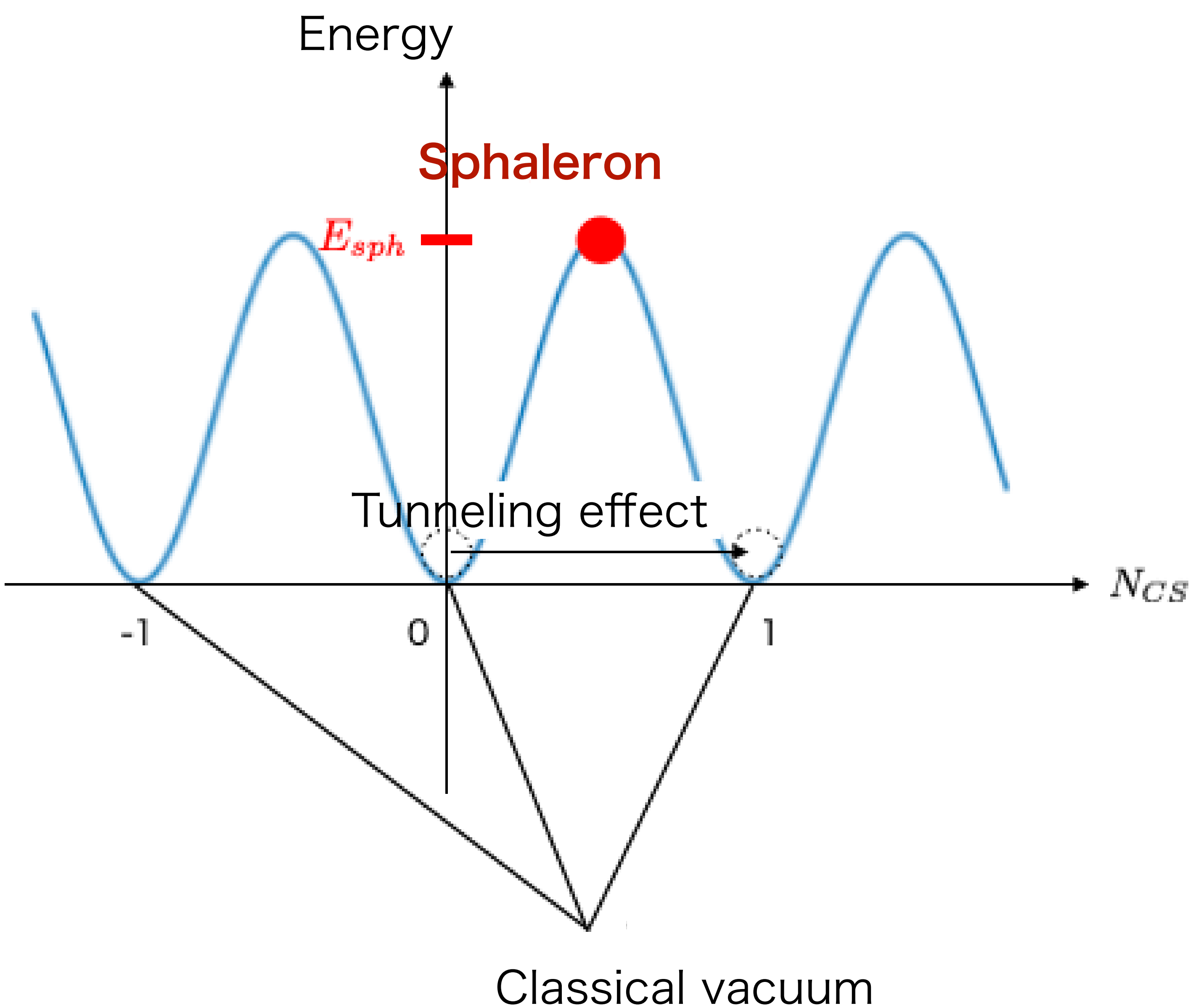
Baryon number

quark : 1/3

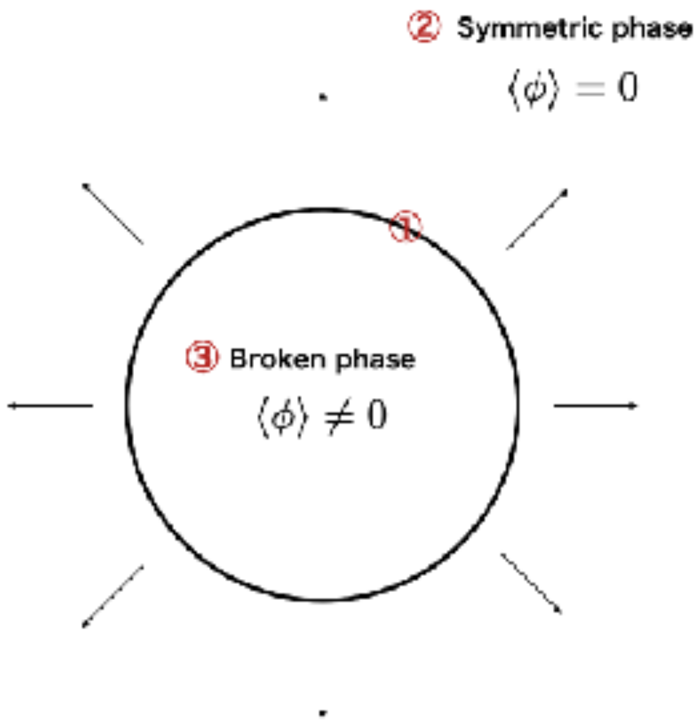
antiquark : -1/3

lepton : 0

boson : 0



EWPT

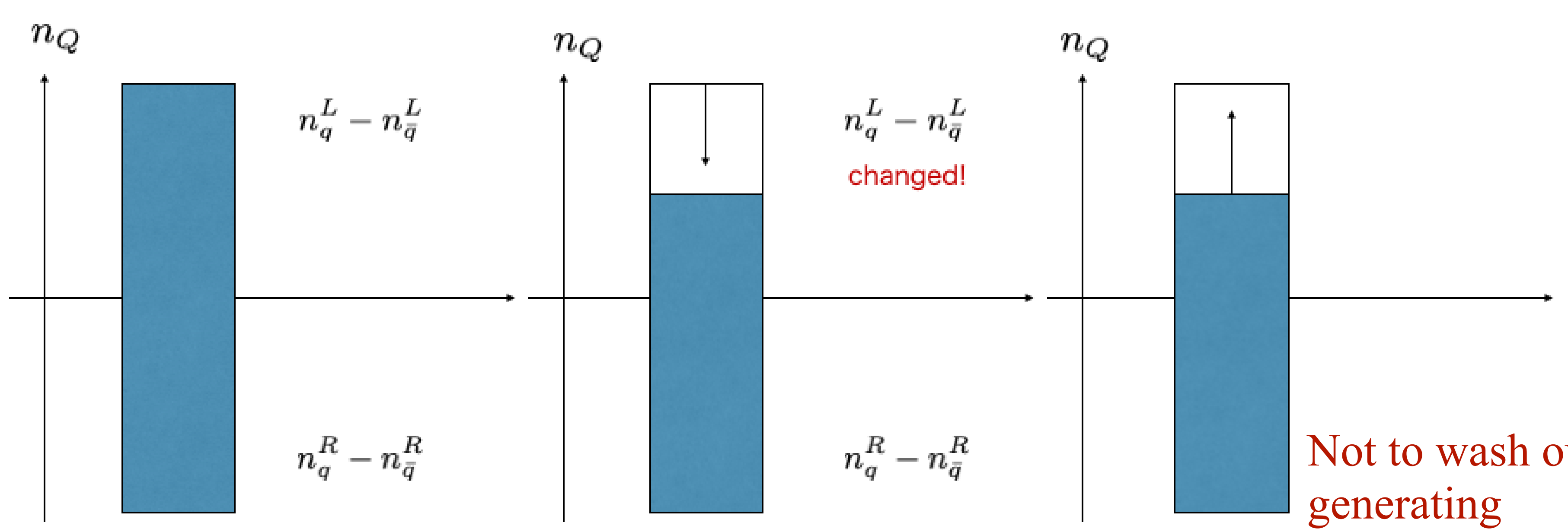


Transmittance, Reflectance

Left-handed quark q^L = Right-handed antiquark $\bar{q}^R$

Left-handed antiquark $\bar{q}^L$ = Right-handed quark q^R

① On the wall ② Symmetric phase ③ Broken phase



Not to wash out generating baryon number

$n_Q \equiv n_q^L - n_{\bar{q}}^L + n_q^R - n_{\bar{q}}^R = 0$

$n_Q \equiv n_q^L - n_{\bar{q}}^L + n_q^R - n_{\bar{q}}^R \neq 0$

baryon number generation

$\boxed{\Gamma_{\rm sph}^{(b)} < H}$ H······Hubble constant

EWPT

The change rate in the baryon number in the broken phase $\Gamma_B^{(b)}(T)$

To generate baryon number

$$\Gamma_B^{(b)}(T)$$ must be small

$$\Gamma_B^{(b)}(T) \simeq (\text{ pre }) \frac{\Gamma_{\rm sph}^{(b)}}{T^3} \simeq (\text{ pre }) e^{-E_{\rm sph}/T}$$

$E_{\rm sph}$ …… sphaleron energy

Sphaleron rate/time/volume

$$\Gamma_{\rm sph}^{(b)} \simeq T^4 e^{-E_{\rm sph}/T}$$

$$\boxed{E_{\rm sph} \propto v(T)}$$

Higgs vev must be large

$$\frac{v_c}{T_c} \gtrsim 1$$

EWPT

$$\boxed{\Gamma_B^{(b)}(T) < H} \quad \rightarrow \quad \Gamma_B^{(b)}(T) \simeq (\text{ pre })e^{-E_{\rm sph}/T} < H(T) \simeq 1.66\sqrt{g_*}T^2/m_{\rm P}$$

g_* ……massless dof

$m_{\rm P}$……Plank mass

$$E_{\rm sph} = 4\pi v\mathcal{E}/g_2 \rightarrow$$

g_2 ……SU(2) gauge coupling constant

$$\frac{v}{T} \geq \frac{g_2}{4\pi\mathcal{E}}(42.97 + \text{logcorrections})$$

In the case of the SM

$$m_h = 125\ \text{GeV}, \mathcal{E} = 1.92(T=0) \quad \Rightarrow \quad \frac{v}{T} \geq 1.16$$

EWPT

Effective potential

$$\Gamma\left[\phi_c\right] = -\int d^4x V_{\text{eff}}\left(\phi_c\right)$$

- tree level potential
- zero-temperature one loop potential (the Coleman Weinberg Potential)
- finite-temperature one loop potential

$$V_{\text{eff}}(\varphi,\varphi_S;T) = V_0(\varphi,\varphi_S;T) + \sum_i n_i \left[V_{\text{CW}}\left(\bar{m}_i^2\right) + \frac{T^4}{2\pi^2} I_{B,F}\left(\frac{\bar{m}_i^2}{T^2}\right)\right]$$

$$V_{\text{CW}}\left(\bar{m}_i^2\right) = \frac{\bar{m}_i^4}{64\pi^2}\left(\ln\frac{\bar{m}_i^2}{\bar{\mu}^2} - c_i\right),\ I_{B,F}\left(a^2\right) = \int_0^\infty dx x^2 \ln\left(1 \mp e^{-\sqrt{x^2+a^2}}\right)$$

High temperature expansion

$$I_B\left[m^2\beta^2\right] = \int_0^\infty dx x^2 \log\left[1 - e^{-\sqrt{x^2+\beta^2m^2}}\right] \simeq -\frac{\pi^4}{45} + \frac{\pi^2}{12}\frac{m^2}{T^2} - \frac{\pi}{6}\left(\frac{m^2}{T^2}\right)^{3/2} - \frac{1}{32}\frac{m^4}{T^4}\log\frac{m^2}{a_bT^2}$$

$$I_F\left(m^2\beta^2\right) = \int_0^\infty dx x^2 \log\left[1 + e^{-\sqrt{x^2+\beta^2m^2}}\right] \simeq \frac{7\pi^4}{360} - \frac{\pi^2}{24}\frac{m^2}{T^2} - \frac{1}{32}\frac{m^4}{T^4}\log\frac{m^2}{a_fT^2}$$

EWPT

M. Quiros, [arXiv:hep-ph/9901312 [hep-ph]]

Effective potential of the SM

$$\Gamma\left[\phi_c\right] = -\int d^4 x V_{\text{eff}}\left(\phi_c\right)$$

- tree level potential
- zero-temperature one loop potential (the Coleman Weinberg Potential)
- finite-temperature one loop potential

$$V\left(\phi_c, T\right) = D\left(T^2 - T_o^2\right)\phi_c^2 - ET\phi_c^3 + \frac{\lambda(T)}{4}\phi_c^4$$

$$D = \frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v^2}$$

$$E = \frac{2m_W^3 + m_Z^3}{4\pi v^3}$$

$$T_o^2 = \frac{m_h^2 - 8Bv^2}{4D}$$

$$B = \frac{3}{64\pi^2 v^4}\left(2m_W^4 + m_Z^4 - 4m_t^4\right)$$

$$\lambda(T) = \lambda - \frac{3}{16\pi^2 v^4}\left(2m_W^4 \log\frac{m_W^2}{A_B T^2} + m_Z^4 \log\frac{m_Z^2}{A_B T^2} - 4m_t^4 \log\frac{m_t^2}{A_F T^2}\right)$$

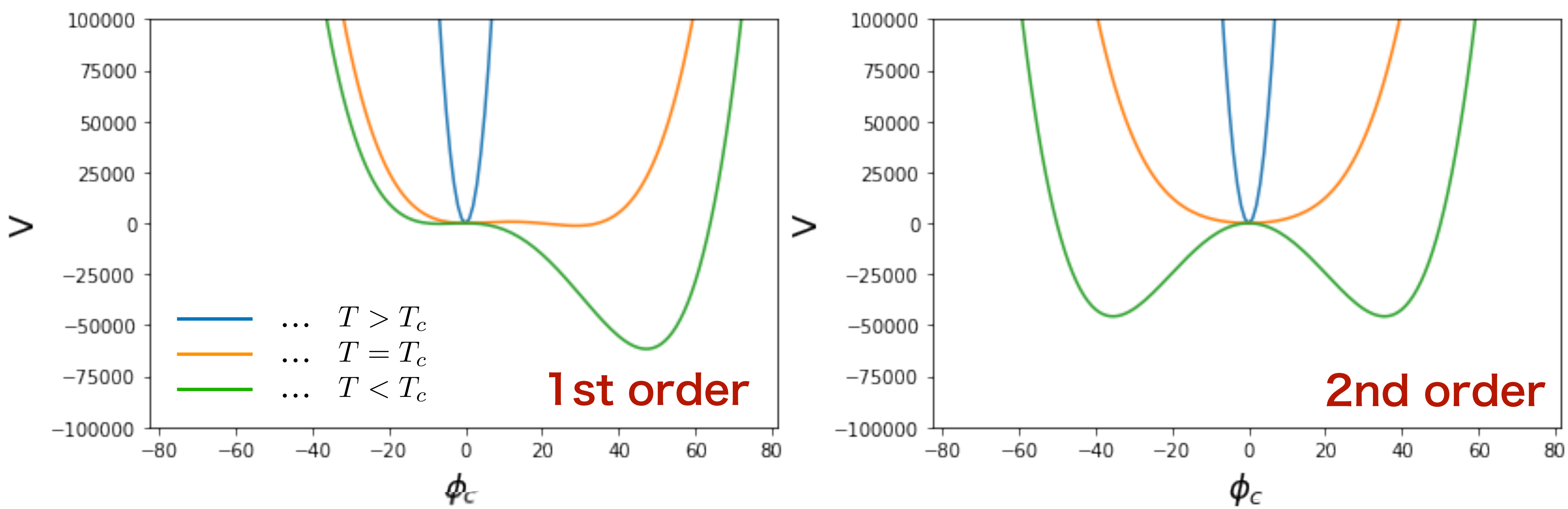
Higgs field

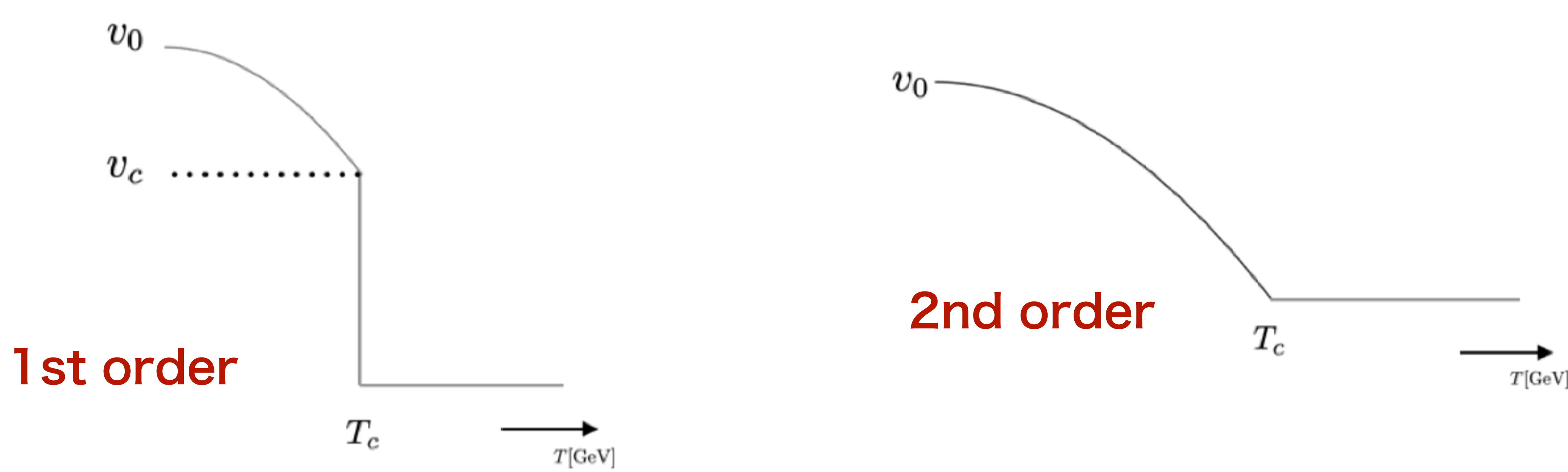
$$H = \begin{pmatrix} \chi_1 + i\chi_2 \\ \frac{\phi_c + h + i\chi_3}{\sqrt{2}} \end{pmatrix}$$

ϕ_c ……real background field

$\chi_a\ (a = 1, 2, 3)$ ……goldstone bosons

EWPT

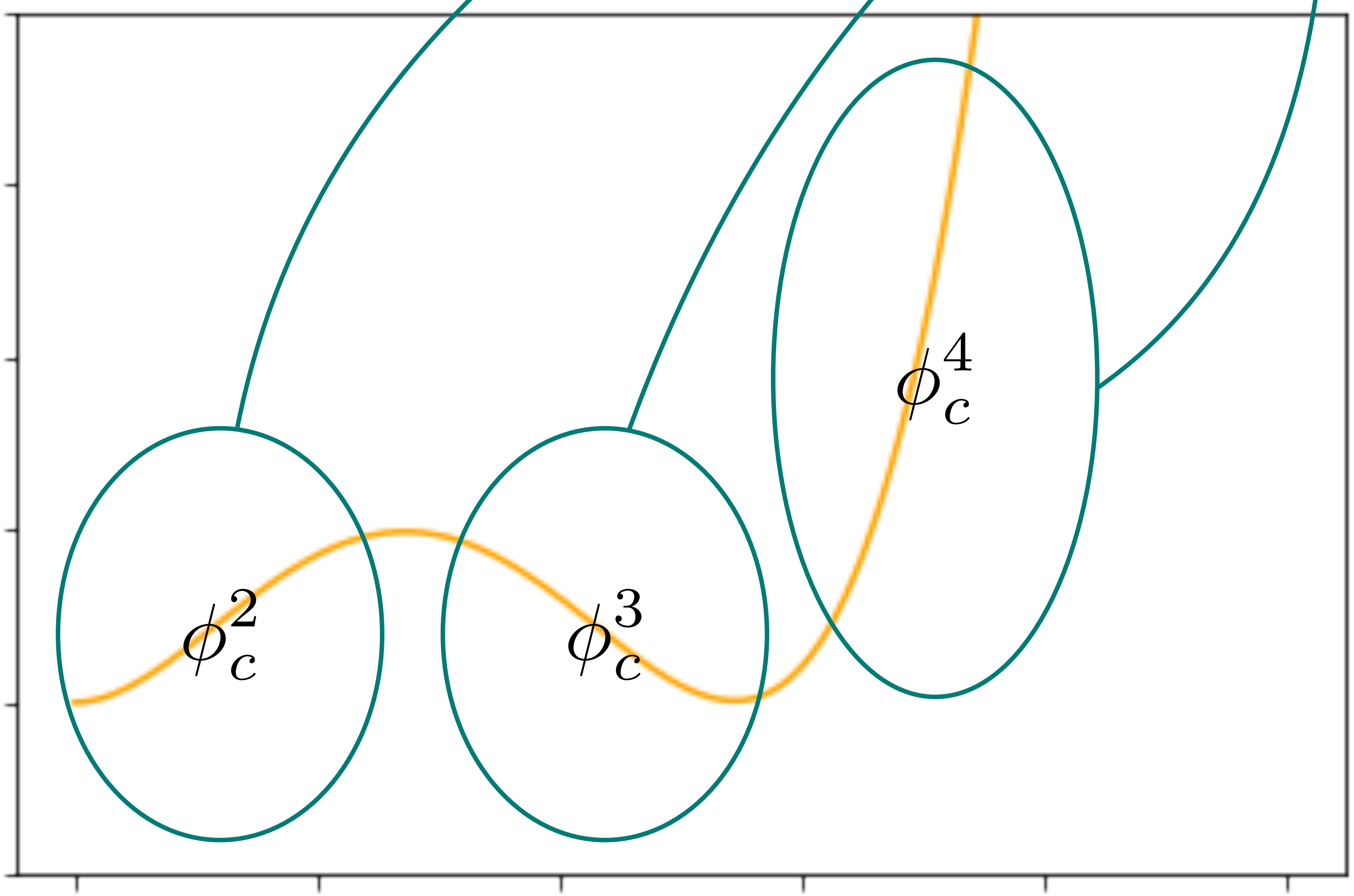




$-ET\phi_c^3$ from finite-temperature boson loop causes a 1st order PT.

EWPT

$$V\left(\phi_c, T\right) = D\left(T^2 - T_o^2\right)\phi_c^2 - ET\phi_c^3 + \frac{\lambda(T)}{4}\phi_c^4$$



$v(T)$ makes discontinuous transition.
(1st order PT)

A barrier is needed between the origin, and $v(T)$

ϕ_c^3 contributes.

EWPT

In the SM, SFOEWPT condition

$$\frac{v_c}{T_c} = \frac{2E}{\lambda(T_c)} \gtrsim 1$$

$$\Downarrow$$

$$m_h \lesssim 64\ \mathrm{GeV}$$

Conflict with observation at LHC → We need to extend the SM!

CPV CxSM

“CP-violating effects on gravitational waves from electroweak phase transition in the CxSM with degenerate scalars”, accepted for PRD, arXiv:2205.12046

Model definition

CPV CxSM

Tadpole condition with respect to h, s, χ

$$\left\langle \frac{\partial V_0}{\partial h} \right\rangle = v\left[\frac{m^2}{2} + \frac{\lambda}{4}v^2 + \frac{\delta_2}{4}\left|v_S\right|^2\right] = 0$$
$$\left\langle \frac{\partial V_0}{\partial s} \right\rangle = v_S^r\left[\frac{b_2}{2} + \frac{\delta_2}{4}v^2 + \frac{d_2}{4}\left|v_S\right|^2 + \frac{b_1^r}{2}\right] + \sqrt{2}a_1^r - \frac{1}{2}b_1^i v_S^i = 0$$
$$\left\langle \frac{\partial V_0}{\partial \chi} \right\rangle = v_S^i\left[\frac{b_2}{2} + \frac{\delta_2}{4}v^2 + \frac{d_2}{4}\left|v_S\right|^2 - \frac{b_1^r}{2}\right] - \sqrt{2}a_1^i - \frac{1}{2}b_1^i v_S^r = 0$$

Mass matrix

$$\mathcal{M}_S^2 = \begin{pmatrix} \frac{\lambda}{2}v^2 & \frac{\delta_2}{2}vv_S^r & \frac{\delta_2}{2}vv_S^i \\ \frac{\delta_2}{2}vv_S^r & \frac{d_2}{2}v_S^{r2} - \frac{\sqrt{2}a_1^r}{v_S^r} + \frac{b_1^i}{2}\frac{v_S^i}{v_S^r} & -\frac{b_1^i}{2} + \frac{d_2}{2}v_S^r v_S^i \\ \frac{\delta_2}{2}vv_S^i & -\frac{b_1^i}{2} + \frac{d_2}{2}v_S^r v_S^i & \frac{d_2}{2}v_S^{i2} + \frac{\sqrt{2}a_1^i}{v_S^i} + \frac{b_1^i}{2}\frac{v_S^r}{v_S^i} \end{pmatrix}$$

Mixing matrix

$$O(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & -s_3 \\ 0 & s_3 & c_3 \end{pmatrix}\begin{pmatrix} c_2 & 0 & -s_2 \\ 0 & 1 & 0 \\ s_2 & 0 & c_2 \end{pmatrix}\begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

CP domain wall

CPV CxSM

Even though $a_1 \neq 0$, one could encounter another domain wall
in the case of spontaneous CPV (called CP domain wall) with $a_1^i = b_1^i = 0$
In this case, V_0 is invariant under Z_2 transformation $\chi \to -\chi$.
Once the Z_2 symmetry is broken spontaneously, the CP domain wall would appear.

$$v_S^i = \pm\sqrt{-{v_S^r}^2 + \frac{2\lambda}{\delta_2^2 - \lambda d_2}\left(-\frac{\delta_2 m^2}{\lambda} + b_2 + \frac{\sqrt{2}a_1^r}{v_S^r}\right)}$$

this vacuum degeneracy is resolved when the explicit CPV is present,
making the CP domain wall unstable.

Parameters

CPV CxSM

Nine degrees of freedom in the scalar potential

$$\left\{m^2, \lambda, \delta_2, b_2, d_2, a_1^r, a_1^i, b_1^r, b_1^i\right\}$$

$\left\{m^2, b_2, b_1^r\right\}$: traded with three scalar VEVs ($\because$ tadpole conditions)

$b_1^i = 0$: a_1^i is absorbs the phase

Five left : traded with $\{m_{h_1}, m_{h_2}, m_{h_3}, \alpha_1, \alpha_2\}$ ($\because$ Mass matrix)

Parameters

CPV CxSM

By using tadpole conditions

$$m^2 = -\frac{\lambda}{2}v^2 - \frac{\delta_2}{2}\left|v_S\right|^2$$

$$b_2 = -\frac{\delta_2}{2}v^2 - \frac{d_2}{2}\left|v_S\right|^2 - \sqrt{2}\left(\frac{a_1^r}{v_S^r} - \frac{a_1^i}{v_S^i}\right)$$

$$b_1^r = -\sqrt{2}\left(\frac{a_1^r}{v_S^r} + \frac{a_1^i}{v_S^i}\right)$$

$$\mathcal{M}_S^2 = \begin{pmatrix} \frac{\lambda}{2}v^2 & \frac{\delta_2}{2}vv_S^r & \frac{\delta_2}{2}vv_S^i \\ \frac{\delta_2}{2}vv_S^r & \frac{d_2}{2}v_S^{r2} - \frac{\sqrt{2}a_1^r}{v_S^r} + \frac{b_1^i}{2}\frac{v_S^i}{v_S^r} & -\frac{b_1^i}{2} + \frac{d_2}{2}v_S^r v_S^i \\ \frac{\delta_2}{2}vv_S^i & -\frac{b_1^i}{2} + \frac{d_2}{2}v_S^r v_S^i & \frac{d_2}{2}v_S^{i2} + \frac{\sqrt{2}a_1^i}{v_S^i} + \frac{b_1^i}{2}\frac{v_S^r}{v_S^i} \end{pmatrix}$$

By using $\left(\mathcal{M}_S^2\right)_{ij} = \sum_k O_{ik}O_{jk}m_{h_k}^2$

$$O(\alpha_i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & -s_3 \\ 0 & s_3 & c_3 \end{pmatrix}\begin{pmatrix} c_2 & 0 & -s_2 \\ 0 & 1 & 0 \\ s_2 & 0 & c_2 \end{pmatrix}\begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\lambda = \frac{2}{v^2}\sum_i O_{1i}^2 m_{h_i}^2$$

$$\delta_2 = \frac{2}{vv_S^r}\sum_i O_{1i}O_{2i}m_{h_i}^2 = \frac{2}{vv_S^i}\sum_i O_{1i}O_{3i}m_{h_i}^2$$

$$d_2 = \frac{2}{v_S^{r2}}\left[\frac{\sqrt{2}a_1^r}{v_S^r} + \sum_i O_{2i}^2 m_{h_i}^2\right] = \frac{2}{v_S^{i2}}\left[-\frac{\sqrt{2}a_1^i}{v_S^i} + \sum_i O_{3i}^2 m_{h_i}^2\right] = \frac{2}{v_S^r v_S^i}\left[\sum_i O_{2i}O_{3i}m_{h_i}^2\right]$$

Parameters
CPV CxSM

By using equations of d_2

$$a_1^r = -\frac{v_S^r}{\sqrt{2}}\left[\sum_i O_{2i}\left(O_{2i} - O_{3i}\frac{v_S^r}{v_S^i}\right)m_{h_i}^2\right]$$

$$a_1^i = \frac{v_S^i}{\sqrt{2}}\left[\sum_i O_{3i}\left(O_{3i} - O_{2i}\frac{v_S^i}{v_S^r}\right)m_{h_i}^2\right]$$

α_3 is not an independent parameter and determined by equations of δ_2

$$\sum_i O_{1i}\left[\frac{O_{2i}}{v_S^r} - \frac{O_{3i}}{v_S^i}\right]m_{h_i}^2 = \frac{\left(\mathcal{M}_S^2\right)_{12}}{v_S^r} - \frac{\left(\mathcal{M}_S^2\right)_{13}}{v_S^i} = 0$$

Degenerate scalar scenario

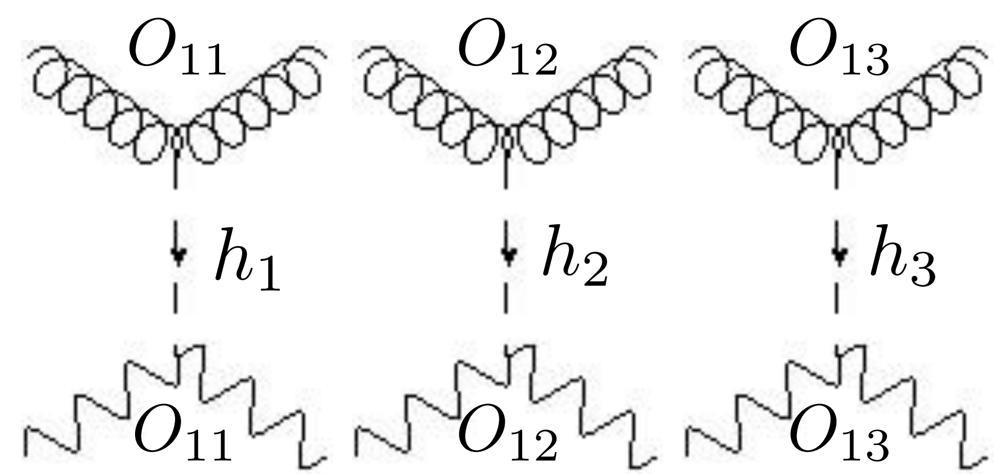
The interactions between Higgs (h) and fermions (f), gauge bosons $(V = W^{\pm}, Z)$

$$\mathcal{L}_{h_i \bar{f} f} = -\frac{m_f}{v} h \bar{f} f = -\frac{m_f}{v} \sum_{i=1-3} \kappa_{if} h_i \bar{f} f$$

$$\mathcal{L}_{h_i VV} = \frac{1}{v} h \left(m_Z^2 Z_\mu Z^\mu + 2 m_W^2 W_\mu^+ W^{-\mu} \right) = \frac{1}{v} \sum_{i=1-3} \kappa_{iV} h_i \left(m_Z^2 Z_\mu Z^\mu + 2 m_W^2 W_\mu^+ W^{-\mu} \right)$$

$$h_1 = O_{11} h_{\mathrm{SM}} + O_{21} s + O_{31} \chi$$
$$h_2 = O_{12} h_{\mathrm{SM}} + O_{22} s + O_{32} \chi \qquad \kappa_i \equiv \kappa_{if} = \kappa_{iV} = O_{1i}$$
$$h_3 = O_{13} h_{\mathrm{SM}} + O_{22} s + O_{33} \chi$$

$$\Gamma(h_1 \to \mathrm{SM}) = \Gamma(h_{\mathrm{SM}} \to \mathrm{SM})(m_{h_1}) \times O_{11}^2$$
$$\Gamma(h_2 \to \mathrm{SM}) = \Gamma(h_{\mathrm{SM}} \to \mathrm{SM})(m_{h_2}) \times O_{12}^2$$
$$\Gamma(h_3 \to \mathrm{SM}) = \Gamma(h_{\mathrm{SM}} \to \mathrm{SM})(m_{h_3}) \times O_{13}^2$$



the orthogonality of the rotation matrix $\Sigma_k O_{ik} O_{jk} = \delta_{ij} \quad \therefore O_{11}^2 + O_{12}^2 + O_{13}^2 = 1$

$$\boxed{\Gamma(h_1 \to \mathrm{SM}) + \Gamma(h_2 \to \mathrm{SM}) + \Gamma(h_3 \to \mathrm{SM}) \simeq \Gamma(h_{\mathrm{SM}} \to \mathrm{SM}) \;\; \text{for } m_{h_1} \simeq m_{h_2} \simeq m_{h_3}}$$

Degenerate scalar scenario

The interactions between Higgs (h) and fermions (f), gauge bosons $(V = W^{\pm}, Z)$

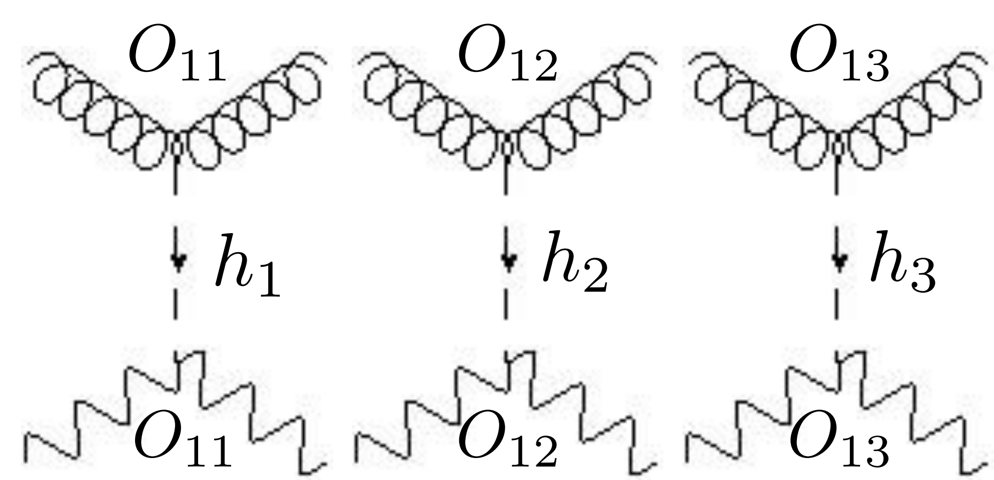
$$\mathcal{L}_{h_i \bar{f} f} = -\frac{m_f}{v} h \bar{f} f = -\frac{m_f}{v} \sum_{i=1-3} \kappa_{if} h_i \bar{f} f$$

$$\mathcal{L}_{h_i VV} = \frac{1}{v} h \left(m_Z^2 Z_\mu Z^\mu + 2 m_W^2 W_\mu^+ W^{-\mu} \right) = \frac{1}{v} \sum_{i=1-3} \kappa_{iV} h_i \left(m_Z^2 Z_\mu Z^\mu + 2 m_W^2 W_\mu^+ W^{-\mu} \right)$$

$$\begin{aligned} h_1 &= O_{11} h_{\mathrm{SM}} + O_{21} s + O_{31} \chi \\ h_2 &= O_{12} h_{\mathrm{SM}} + O_{22} s + O_{32} \chi \\ h_3 &= O_{13} h_{\mathrm{SM}} + O_{22} s + O_{33} \chi \end{aligned}$$

$$\boxed{\kappa_i \equiv \kappa_{if} = \kappa_{iV} = O_{1i}}$$

$$\begin{aligned} \Gamma(h_1 \to \mathrm{SM}) &= \Gamma(h_{\mathrm{SM}} \to \mathrm{SM})(m_{h_1}) \times O_{11}^2 \\ \Gamma(h_2 \to \mathrm{SM}) &= \Gamma(h_{\mathrm{SM}} \to \mathrm{SM})(m_{h_2}) \times O_{12}^2 \\ \Gamma(h_3 \to \mathrm{SM}) &= \Gamma(h_{\mathrm{SM}} \to \mathrm{SM})(m_{h_3}) \times O_{13}^2 \end{aligned}$$



the orthogonality of the rotation matrix $\Sigma_k O_{ik} O_{jk} = \delta_{ij} \quad \therefore O_{11}^2 + O_{12}^2 + O_{13}^2 = 1$

$$\boxed{\Gamma(h_1 \to \mathrm{SM}) + \Gamma(h_2 \to \mathrm{SM}) + \Gamma(h_3 \to \mathrm{SM}) \simeq \Gamma(h_{\mathrm{SM}} \to \mathrm{SM}) \ \text{for } m_{h_1} \simeq m_{h_2} \simeq m_{h_3}}$$

Higgs coupling
CPV CxSM

The interactions between Higgs (h) and fermions (f), gauge bosons $(V = W^{\pm}, Z)$

$$\mathcal{L}_{h_i \bar{f} f} = -\frac{m_f}{v} h \bar{f} f = -\frac{m_f}{v} \sum_{i=1-3} \kappa_{if} h_i \bar{f} f$$

$$\mathcal{L}_{h_i VV} = \frac{1}{v} h \left(m_Z^2 Z_\mu Z^\mu + 2 m_W^2 W_\mu^+ W^{-\mu} \right) = \frac{1}{v} \sum_{i=1-3} \kappa_{iV} h_i \left(m_Z^2 Z_\mu Z^\mu + 2 m_W^2 W_\mu^+ W^{-\mu} \right)$$

$$\kappa_{if} = O_{1i}, \quad \kappa_{iV} = O_{1i}$$

In SM limit, $\kappa_{1f} = \kappa_{1V} = 1$ and $\kappa_{2,3f} = \kappa_{2,3V} = 0$

Degenerate scalar scenario

CPV CxSM

For example, the cross section of the process $gg \to h_i \to VV^*$ would be cast into the following form

amplitude

$$\mathcal{M}_{gg\to h_i \to VV^*} = \sum_{i=1}^{3} \mathcal{M}^{\mathrm{SM}}_{gg\to h} \kappa_{if} \frac{1}{s - m_{h_i}^2 + i m_{h_i}\Gamma_{h_i}} \kappa_{iV} \mathcal{M}^{\mathrm{SM}}_{h\to VV^*}$$

Squared amplitude

$$\left|\mathcal{M}_{gg\to h_i \to VV^*}\right|^2 = \left|\mathcal{M}^{\mathrm{SM}}_{gg\to h}\right|^2 \left|\mathcal{M}^{\mathrm{SM}}_{h\to VV^*}\right|^2 \left[\sum_{i=1}^{3} \frac{\kappa_{if}^2 \kappa_{iV}^2}{\left(s - m_{h_i}^2\right)^2 + m_{h_i}^2 \Gamma_{h_i}^2} \right.$$
$$\left. + 2\,\mathrm{Re} \sum_{i<j} \frac{\kappa_{if}\kappa_{jf}\kappa_{iV}\kappa_{jV}}{\left(s - m_{h_i}^2 + i m_{h_i}\Gamma_{h_i}\right)\left(s - m_{h_j}^2 - i m_{h_j}\Gamma_{h_j}\right)}\right]$$

Degenerate scalar scenario

CPV CxSM

Due to $\Gamma_{h_i} < \Gamma_h^{\mathrm{SM}}(\simeq 4.1\mathrm{MeV}) \ll m_{h_i}(\simeq 125\mathrm{GeV})$, we can use NWA

$$\pi\delta\left(s-m^2\right)=\lim_{\Gamma\to 0}\frac{m\Gamma}{\left(s-m^2\right)^2+m^2\Gamma^2} \quad \to \quad \int_{-\infty}^{\infty}\frac{ds}{2\pi}\frac{1}{\left(s-m^2\right)^2+m^2\Gamma^2}=\frac{1}{2m\Gamma}$$

If $i=1$, $j=2$, interference term cast into the following form

$$\begin{aligned}
I &= \int_{-\infty}^{\infty}\frac{ds}{2\pi}\,\mathrm{Re}\left[\frac{1}{\left(s-m_1^2+im_1\Gamma_1\right)\left(s-m_2^2-im_2\Gamma_2\right)}\right] \\
&= \int_{-\infty}^{\infty}\frac{ds}{2\pi}\frac{\left(s-m_1^2\right)\left(s-m_2^2\right)+m_1m_2\Gamma_1\Gamma_2}{\left\{\left(s-m_1^2\right)\left(s-m_2^2\right)+m_1m_2\Gamma_1\Gamma_2\right\}^2+\left\{m_1\Gamma_1\left(s-m_2^2\right)-m_2\Gamma_2\left(s-m_1^2\right)\right\}^2} \\
&= \frac{m_1\Gamma_1+m_2\Gamma_2}{\left(m_1^2-m_2^2\right)^2+\left(m_1\Gamma_1+m_2\Gamma_2\right)^2}
\end{aligned}$$

When $m_1=m_2=m$ $\qquad I=\dfrac{1}{m\left(\Gamma_1+\Gamma_2\right)}$

Degenerate scalar scenario

CPV CxSM

$$I = \frac{m_1\Gamma_1 + m_2\Gamma_2}{\left(m_1^2 - m_2^2\right)^2 + \left(m_1\Gamma_1 + m_2\Gamma_2\right)^2}$$

When $\left(m_1^2 - m_2^2\right)^2 < m_1\Gamma_1 + m_2\Gamma_2 \quad \rightarrow \quad \left|m_{h_1} - m_{h_2}\right| \lesssim \Gamma_{h_1} + \Gamma_{h_2}$

interference term is important

In our benchmark points the smallest mass differences is 500 MeV and the sum of the total decay widths are at most $\Gamma_h^{\mathrm{SM}} = 4.1\mathrm{MeV}$.

Experimental constraints on Higgs total decay width

$$\Gamma_h^{\mathrm{exp}} < 14.4\mathrm{MeV}\ (\mathrm{ATLAS}\ )\ \mathrm{and}\ \Gamma_h^{\mathrm{exp}} = 3.2^{+2.4}_{-1.7}\mathrm{MeV}(\mathrm{CMS})$$

which are not precise enough to constrain Γ_{h_i} in our scenario at this moment.

Degenerate scalar scenario

CPV CxSM

Cross section

$$\sigma_{gg\to h_i\to VV^*} \simeq \sigma^{\rm SM}_{gg\to h}\left[\sum_i \frac{\kappa_{if}^2\kappa_{iV}^2}{\Gamma_{h_i}}\right]\Gamma^{\rm SM}_{h\to VV^*}$$

$$\sigma_{gg\to h_i\to VV^*} \simeq \sigma^{\rm SM}_{gg\to h}\cdot {\rm Br}^{\rm SM}_{h\to VV^*} \qquad \because \Gamma_{h_i}\simeq \kappa_i^2\Gamma_h^{\rm SM}\ ,\sum_i \kappa_i^2 = 1$$

Degenerate scalar scenario

CPV CxSM

$$\left(\mathcal{M}_S^2\right)_{ij} \simeq \sum_k O_{ik} O_{jk} m_h^2 = \delta_{ij} m_h^2$$

$$\mathcal{M}_S^2 = \begin{pmatrix} \frac{\lambda}{2} v^2 & \frac{\delta_2}{2} v v_S^r & \frac{\delta_2}{2} v v_S^i \\ \frac{\delta_2}{2} v v_S^r & \frac{d_2}{2} v_S^{r2} - \frac{\sqrt{2} a_1^r}{v_S^r} + \frac{b_1^i}{2} \frac{v_S^i}{v_S^r} & -\frac{b_1^i}{2} + \frac{d_2}{2} v_S^r v_S^i \\ \frac{\delta_2}{2} v v_S^i & -\frac{b_1^i}{2} + \frac{d_2}{2} v_S^r v_S^i & \frac{d_2}{2} v_S^{i2} + \frac{\sqrt{2} a_1^i}{v_S^i} + \frac{b_1^i}{2} \frac{v_S^r}{v_S^i} \end{pmatrix} \quad O(\alpha_i) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & -s_3 \\ 0 & s_3 & c_3 \end{pmatrix} \begin{pmatrix} c_2 & 0 & -s_2 \\ 0 & 1 & 0 \\ s_2 & 0 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\delta_2 = \frac{2}{v v_S^r} \sum_i O_{1i} O_{2i} m_{h_i}^2 = \frac{2}{v v_S^i} \sum_i O_{1i} O_{3i} m_{h_i}^2$$

One may consider the cases $|\delta_2| \ll 1$ and $|d_2| \ll 1$ to satisfy the orthogonality of the rotation matrix. However, the size of δ_2 is closely related to the strong First-order EWPT.

Thus, we have to take $v_S^{r_i^i}/v \ll 1$ while maintaining $\delta_2 = \mathcal{O}(1)$ and $d_2 = \mathcal{O}(1)$

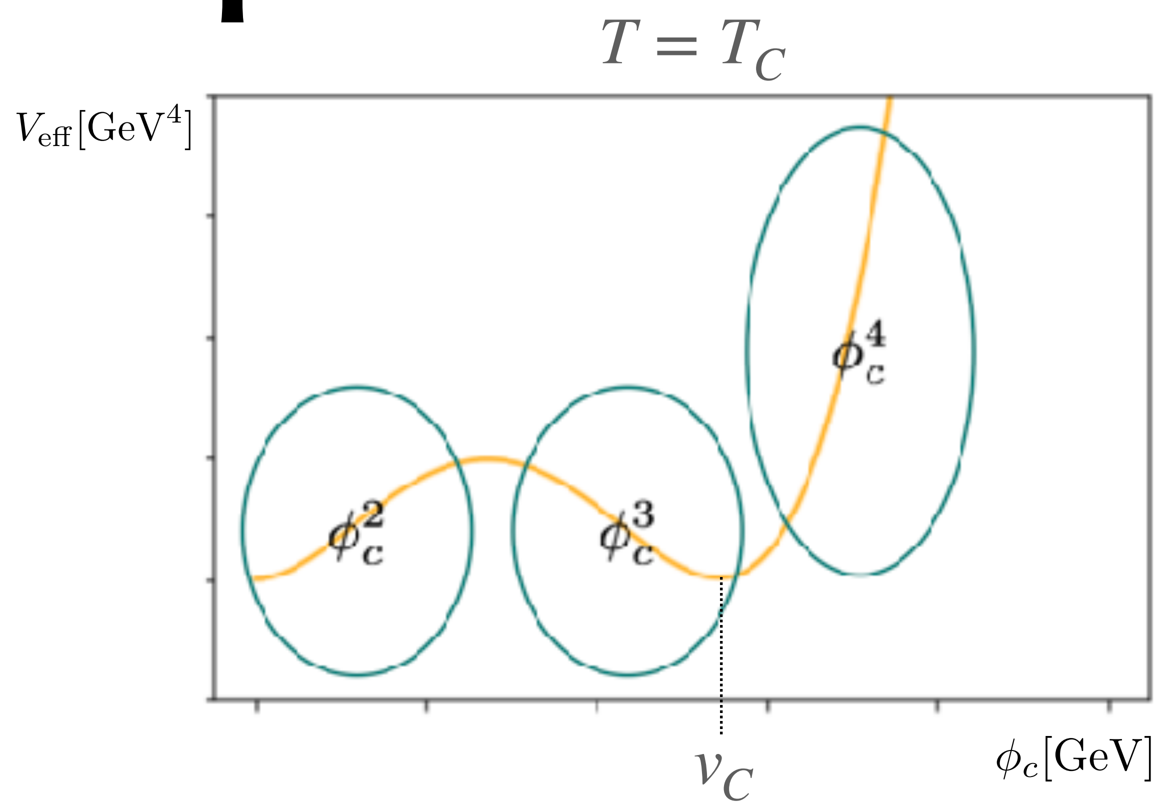
Electroweak phase transition

Strong 1st order EWPT

$$\frac{v_c}{T_c} \gtrsim 1$$

T_C : critical temperature

v_C : Higgs vev at T_C



Ordinary PT : The cubic terms of field derived from boson-loop at finite temperature is important

CxSM PT : **structure of tree level potential** is most important

$$V^{\rm HT}\left(\varphi, \varphi_S^r, \varphi_S^i; T\right) = V_0\left(\varphi, \varphi_S^r, \varphi_S^i\right) + \frac{T^2}{2}\left[\Sigma_H \varphi^2 + \Sigma_S \varphi_S^{r2} + \Sigma_S \varphi_S^{i2}\right]$$

radial coordinate $\varphi = z\cos\gamma, \quad \varphi_S^r = z\sin\gamma\cos\theta + \tilde{v}_S^r, \quad \varphi_S^i = z\sin\gamma\sin\theta + \tilde{v}_S^i$

$$\rightarrow \quad V^{\rm HT}(z,\gamma;T) = c_0 + c_1 z + (c_2 + c_2' T^2) z^2 \underline{- c_3 z^3} + c_4 z^4$$

$$c_3 = -\frac{s_{\gamma_C} c_{\gamma_C}^2}{4}\left(c_{\theta_C}\tilde{v}_S^r + s_{\theta_C}\tilde{v}_S^i\right)\left(\delta_2 + d_2 t_{\gamma_C}^2\right)$$

Qualitative discussion

CPV CxSM

HT potential

$\varphi, \varphi_S^r, \varphi_S^i \cdots$ Back ground fields of $H, \mathrm{Re}S, \mathrm{Im}S$

$$
\begin{aligned}
V^{\mathrm{HT}}\left(\varphi, \varphi_S^r, \varphi_S^i; T\right) =& V_0\left(\varphi, \varphi_S^r, \varphi_S^i\right) + \frac{T^2}{2}\left[\Sigma_H \varphi^2 + \Sigma_S \varphi_S^{r2} + \Sigma_S \varphi_S^{i2}\right] \\
=& \frac{m^2}{4}\varphi^2 + \frac{\lambda}{16}\varphi^4 + \frac{\delta_2}{8}\varphi^2\left(\varphi_S^{r2} + \varphi_S^{i2}\right) + \frac{d_2}{16}\left(\varphi_S^{r2} + \varphi_S^{i2}\right)^2 \\
&+ \sqrt{2}\left(a_1^r \varphi_S^r - a_1^i \varphi_S^i\right) + \frac{1}{4} b_1^r\left(\varphi_S^{r2} - \varphi_S^{i2}\right) + \frac{b_2}{4}\left(\varphi_S^{r2} + \varphi_S^{i2}\right) \\
&+ \frac{T^2}{2}\left[\Sigma_H \varphi^2 + \Sigma_S \varphi_S^{r2} + \Sigma_S \varphi_S^{i2}\right]
\end{aligned}
$$

Three scalar fields are reexpressed in terms of the polar coordinate

$$
\varphi = z\cos\gamma, \quad \varphi_S^r = z\sin\gamma\cos\theta + \tilde{v}_S^r, \quad \varphi_S^i = z\sin\gamma\sin\theta + \tilde{v}_S^i
$$

Qualitative discussion

CPV CxSM

Potential at T_C

$$V\left(z_C,\gamma_C,\theta_C;T_C\right)=c_4 z^2\left(z-z_C\right)^2,\quad z_C=\frac{c_3}{2c_4}$$

$$V^{\mathrm{HT}}\ni -c_3 z^3+c_4 z^4$$

$$c_3=-\frac{s_{\gamma_C}c_{\gamma_C}^2}{4}\left(c_{\theta_C}\tilde{v}_S^r+s_{\theta_C}\tilde{v}_S^i\right)\left(\delta_2+d_2 t_{\gamma_C}^2\right)$$

$$c_4=\frac{c_{\gamma_C}^4}{16}\left(\lambda+2\delta_2 t_{\gamma_C}^2+d_2 t_{\gamma_C}^4\right)$$

$$t_{\gamma_C}=\frac{v_{SC}^r-\tilde{v}_{SC}^r}{v_C c_{\theta_C}}=\frac{v_{SC}^i-\tilde{v}_{SC}^i}{v_C s_{\theta_C}}.$$

When $|t_{\gamma_C}|\ll 1$

$$v_C\simeq\sqrt{\frac{2\delta_2}{\lambda}\left(\left|\tilde{v}_{SC}\right|^2-\tilde{v}_{SC}^i\left(\tilde{v}_{SC}^i-t_{\theta_C}\tilde{v}_{SC}^r\right)\right)\left(1-\frac{v_{SC}^r}{\tilde{v}_{SC}^r}\right)},$$

$$T_C\simeq\sqrt{\frac{1}{2\Sigma_H}\left[-m^2-\frac{\delta_2}{2}\left|\tilde{v}_{SC}\right|^2\right]},$$

Qualitative discussion

CPV CxSM

When 1st-order phase transition occur

$$v_C \simeq \sqrt{\frac{2\delta_2}{\lambda}\left(|\tilde{v}_{SC}|^2 - \tilde{v}^i_{SC}\left(\tilde{v}^i_{SC} - t_{\theta_C}\tilde{v}^r_{SC}\right)\right)\left(1 - \frac{v^r_{SC}}{\tilde{v}^r_{SC}}\right)},$$

$$T_C \simeq \sqrt{\frac{1}{2\Sigma_H}\left[-m^2 - \frac{\delta_2}{2}|\tilde{v}_{SC}|^2\right]},$$

Strong 1st-order PT

$$\frac{v_c}{T_c} \gtrsim 1$$

By exchanging the indices r and i we have equivalent expression of v_C

large δ_2, small $\tilde{v_S}^r, \tilde{v_S}^i$ are preferred

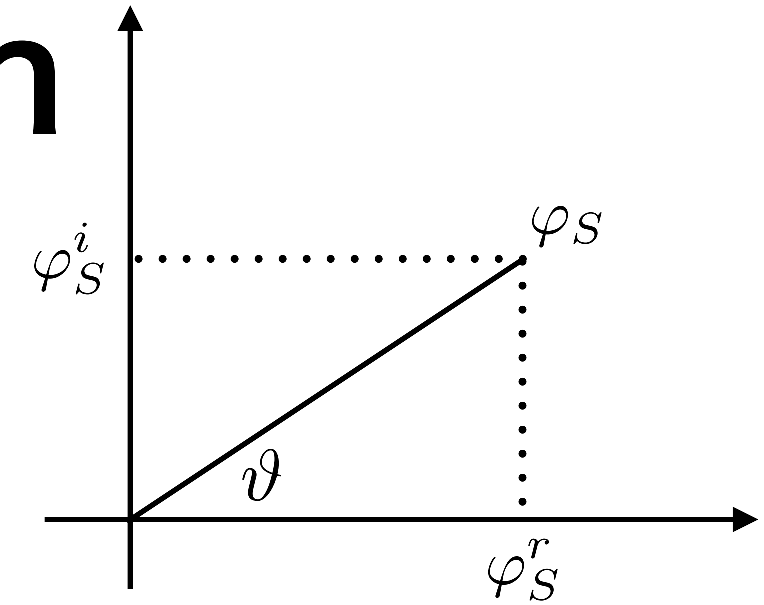
$$\delta_2 = \frac{2}{vv^r_S}\sum O_{1i}O_{2i}m^2_{h_i} = \frac{2}{vv^i_S}\sum O_{1i}O_{3i}m^2_{h_i}$$

There is a contribution to the CPV CxSM specific PT

small ${v_S}^r, {v_S}^i$ are preferred

Qualitative discussion

CPV CxSM



Phase dependent part of HT potential

$$V^{\mathrm{HT}}(\vartheta_S) = \sqrt{2}\left(a_1^r\varphi_S^r - a_1^i\varphi_S^i\right) + \frac{1}{4}b_1^r\left(\varphi_S^{r2} - \varphi_S^{i2}\right)$$
$$= \sqrt{2}\varphi_S\left(a_1^r\cos\vartheta_S - a_1^i\sin\vartheta_S\right) + \frac{1}{4}b_1^r\varphi_S^2\left(\cos^2\vartheta_S - \sin^2\vartheta_S\right),$$

$$\varphi_S^r = \varphi_S\cos\vartheta_S \quad \varphi_S^i = \varphi_S\sin\vartheta_S$$

Temperature dependent structure

$T^2\Sigma_S\left(\varphi_S^{r2} + \varphi_S^{i2}\right) = T^2\Sigma_S\varphi_S^2$ $\rightarrow$ phase is time- independent

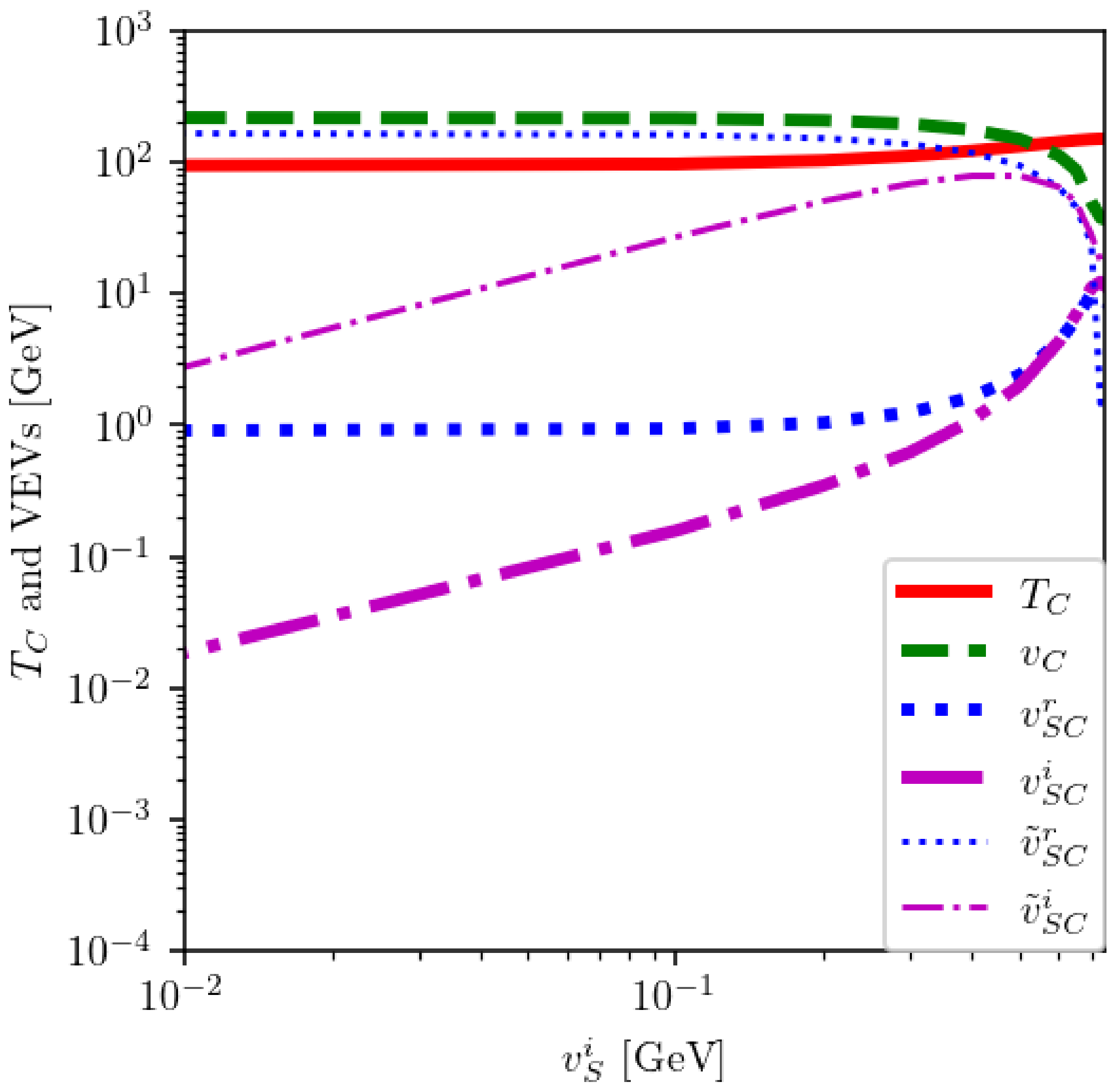
$\langle\vartheta_S(T)\rangle = \theta_S(T)$ stays on its zero temperature value $\theta_S(T=0) = \tan^{-1}(v_S^i/v_S^r)$.

Electroweak phase transition

$(0, \tilde{v}_S^r, \tilde{v}_S^i)$ @symmetric phase

$\downarrow$

(v, v_S^r, v_S^i) @broken phase



	CPV			CPC	
	$v_S^i = 0.3$ GeV	$v_S^i = 0.4$ GeV	$v_S^i = 0.5$ GeV	$m_\chi = 62.5$ GeV	$m_\chi = 2$ TeV
v_C/T_C	$\frac{196.1}{112.3} = 1.7$	$\frac{177.2}{122.5} = 1.4$	$\frac{150.9}{132.8} = 1.1$	$\frac{200.1}{106.1} = 1.9$	$\frac{205.3}{108.7} = 1.9$
v_{SC}^r [GeV]	1.249	1.634	2.403	1.250	1.171
v_{SC}^i [GeV]	0.624	1.089	2.003	—	—
$\tilde{v}_{SC}^r$ [GeV]	137.9	118.5	94.82	144.2	146.2
$\tilde{v}_{SC}^i$ [GeV]	68.97	79.01	79.01	—	—

$\frac{v_C}{T_C} \lesssim 1$ for $v_S^i \gtrsim 0.5$

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v_N/T_N	$\frac{239.0}{66.85} = 3.6$	$\frac{211.7}{102.0} = 2.1$	$\frac{177.2}{123.1} = 1.4$	$\frac{241.8}{57.20} = 4.2$	$\frac{242.4}{57.99} = 4.2$
v_{SN}^r [GeV]	0.657	0.921	1.446	0.636	0.634
v_{SN}^i [GeV]	0.328	0.614	1.205	—	—
$\tilde{v}_{SN}^r$ [GeV]	143.7	122.3	97.26	150.1	150.2
$\tilde{v}_{SN}^i$ [GeV]	71.83	81.55	81.05	—	—
Δ	40.5%	16.7%	7.3%	46.0%	46.7%

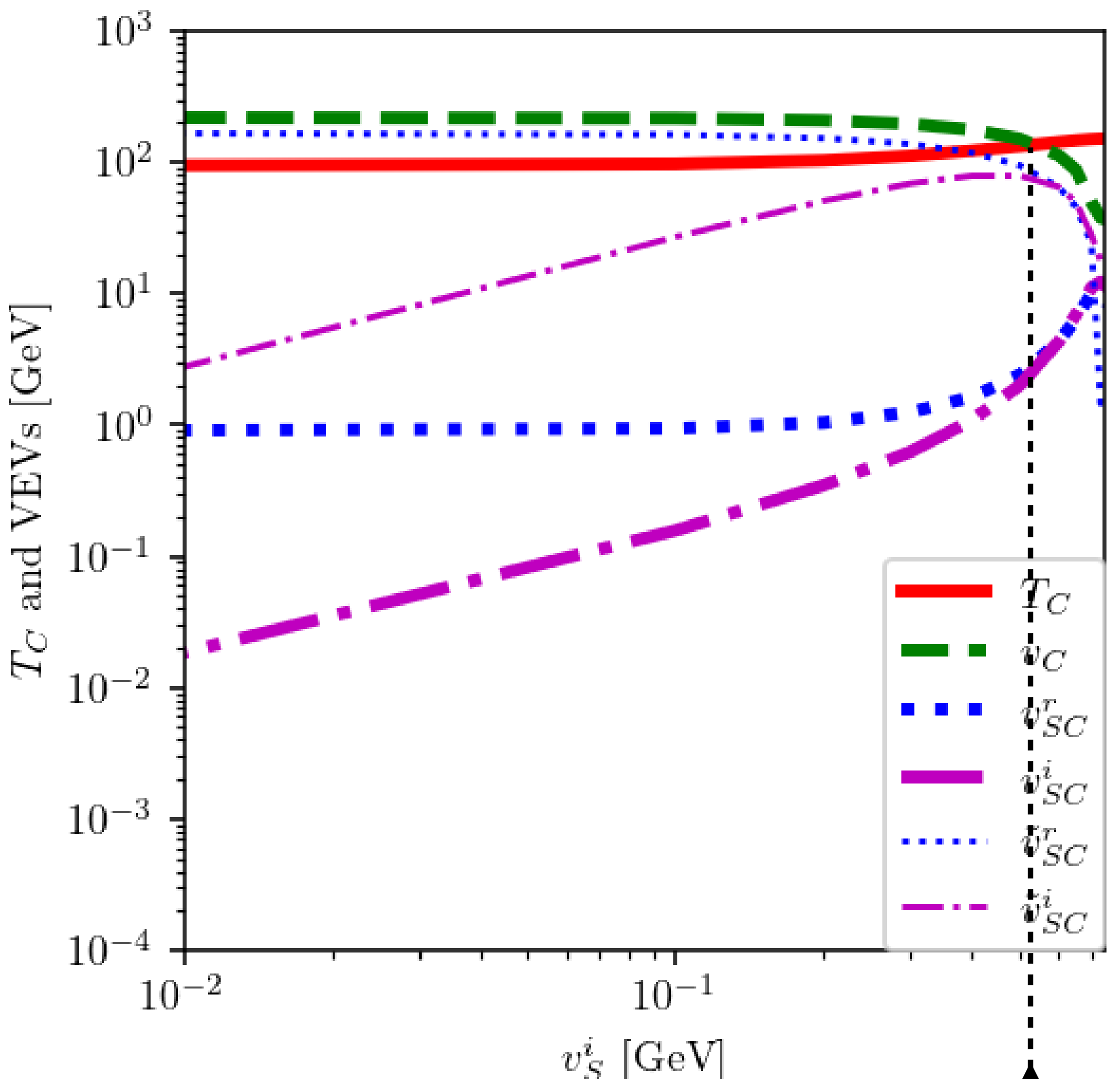
Bubbles do not nucleate $v_S^i \lesssim 0.3$

Electroweak phase transition

$(0, \tilde{v}_S^r, \tilde{v}_S^i)$ @symmetric phase

$\downarrow$

(v, v_S^r, v_S^i) @broken phase



$v_S^i = 0.5$

	CPV			CPC	
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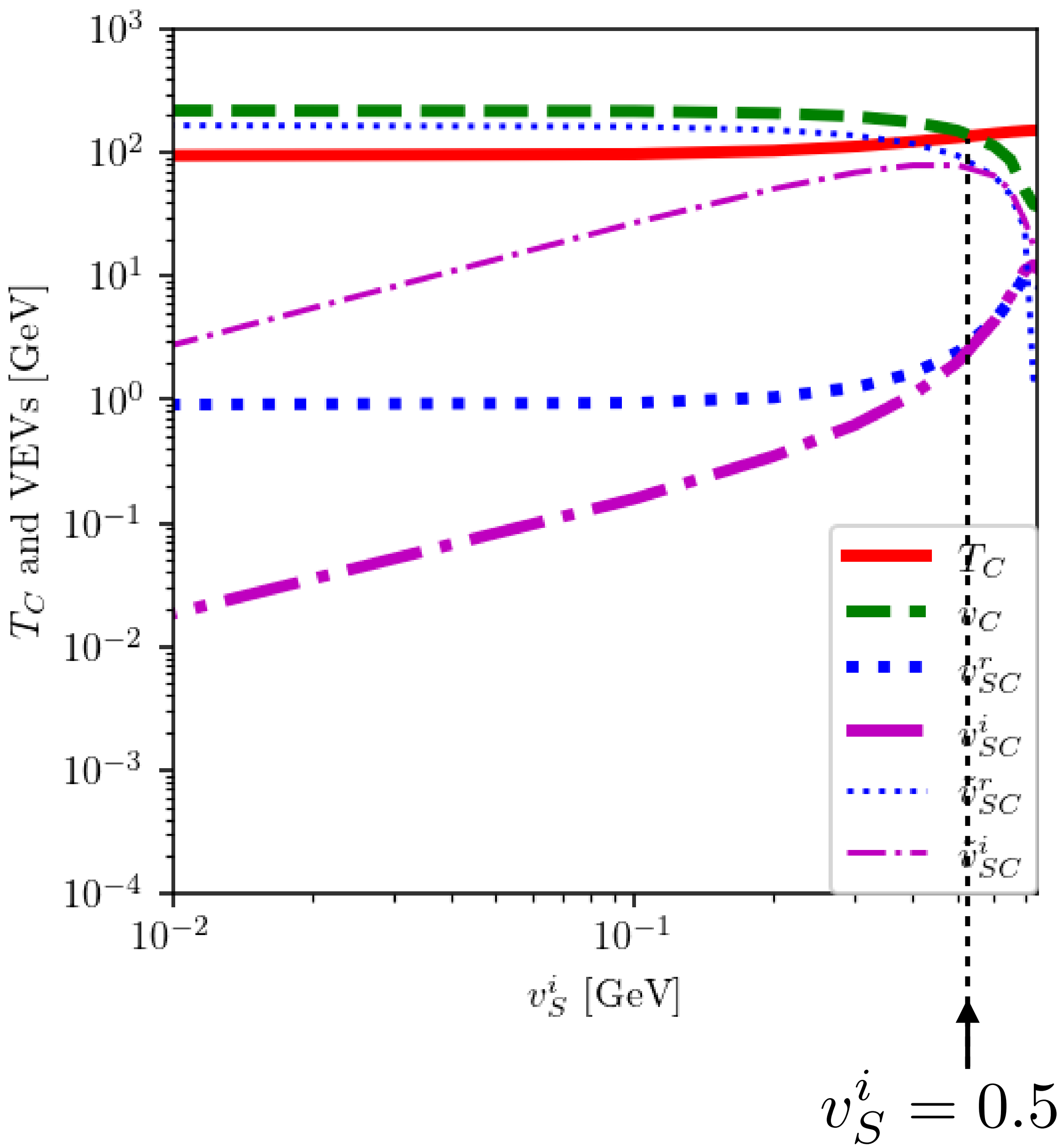
$(0, \tilde{v}_S^r, \tilde{v}_S^i)$ @symmetric phase

$\downarrow$

(v, v_S^r, v_S^i) @broken phase



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Δ	40.5%	16.7%	7.3%	46.0%	46.7%

Bubbles do not nucleate $v_S^i \lesssim 0.3$

Benchmark points

CPV CxSM and CPC CxSM

Phys.Lett.B 823 (2021) 136787,
arXiv:2105.11830

CPV

Inputs	v [GeV]	v_S^r [GeV]	v_S^i [GeV]	m_{h_1} [GeV]	m_{h_2} [GeV]	m_{h_3} [GeV]	α_1 [rad]	α_2 [rad]
BP1	246.22	0.6	0.3	125.0	124.0	124.5	$\pi/4$	0.0
BP2	246.22	0.6	0.4	125.0	124.0	124.5	$\pi/4$	0.0
BP3	246.22	0.6	0.5	125.0	124.0	124.5	$\pi/4$	0.0
Outputs	m^2	b_2 [GeV^2]	b_1^r [GeV^2]	λ	δ_2	d_2	a_1^r [GeV^3]	a_1^i [GeV^3]
BP1	$-(124.5)^2$	$-(121.2)^2$	-7.717×10^{-12}	0.511	1.51	1.111	$-(18.735)^3$	$(14.870)^3$
BP2	$-(124.5)^2$	$-(107.3)^2$	5.145×10^{-12}	0.511	1.40	0.962	$-(18.735)^3$	$(16.367)^3$
BP3	$-(124.5)^2$	$-(90.82)^2$	0.0000	0.511	1.29	0.820	$-(18.735)^3$	$(17.630)^3$

CPC

Inputs	v [GeV]	m_{h_1} [GeV]	m_{h_2} [GeV]	α [rad]	a_1 [GeV^3]	v_S [GeV]	m_χ [GeV]
BP4	246.22	125	124	$\pi/4$	-6576.17	0.6	62.5
BP5	246.22	125	124	$\pi/4$	-6576.17	0.6	2000
Outputs	m^2 [GeV^2]	b_1 [GeV^2]	b_2 [GeV^2]	λ	a_1 [GeV^3]	d_2	δ_2
BP4	$-(124.5)^2$	$(107.7)^2$	$(178.0)^2$	0.511	-6576.17	1.77	1.69
BP5	$-(124.5)^2$	$-(1996)^2$	$(1991)^2$	0.511	-6576.17	1.77	1.69

Benchmark points

CPV CxSM and CPC CxSM

Phys.Lett.B 823 (2021) 136787,
arXiv:2105.11830

CPV

Inputs	v [GeV]	v_S^r [GeV]	v_S^i [GeV]	m_{h_1} [G	degenerate	[GeV]	α_1 [rad]	α_2 [rad]
BP1	246.22	0.6	0.3	125.0	124.0	124.5	$\pi/4$	0.0
BP2	246.22	0.6	0.4	125.0	124.0	124.5	$\pi/4$	0.0
BP3	246.22	0.6	0.5	125.0	124.0	124.5	$\pi/4$	0.0
Outputs	m^2	b_2 [GeV^2]	b_1^r [GeV^2]	λ	δ_2	d_2	a_1^r [GeV^3]	a_1^i [GeV^3]
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Benchmark points

CPV CxSM and CPC CxSM

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CPV

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Benchmark points

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Outputs	m^2	b_2 [GeV^2]	b_1^r [GeV^2]	λ	δ_2	d_2	a_1^r [GeV^3]	a_1^i [GeV^3]
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CPC

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Benchmark points

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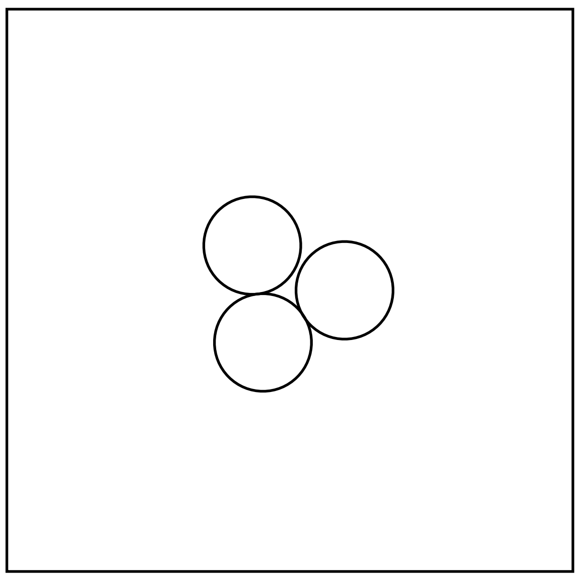
Bubble nucleation

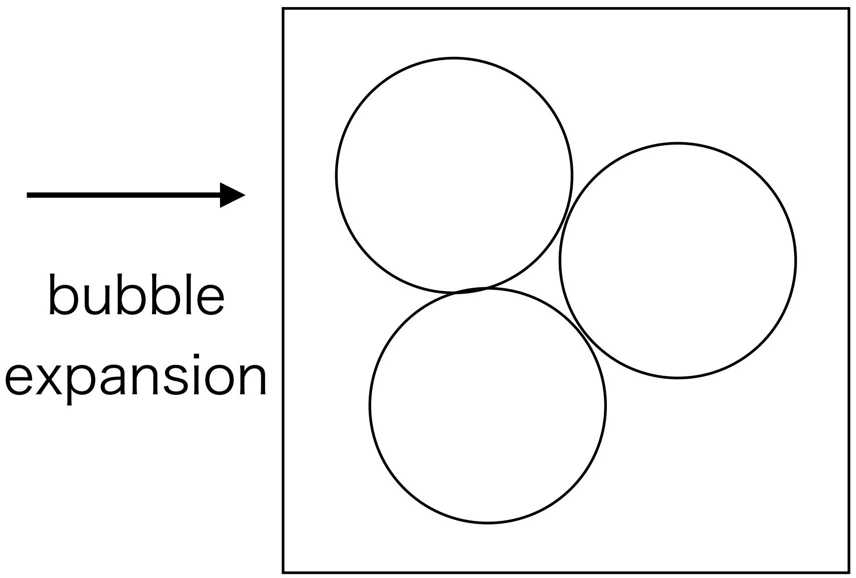
$T = T_N$



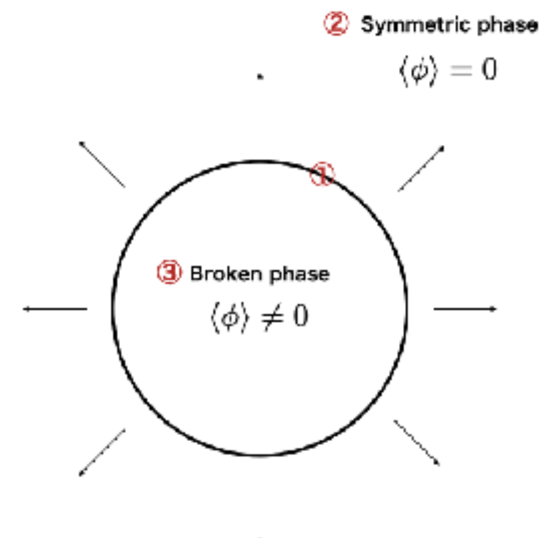
nucleation

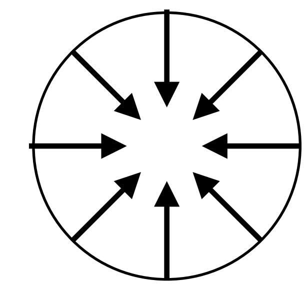
bubble expansion

$T = T_N + \Delta T$



collision
→ completion of PT





Not all bubbles will expand
→ the radius of the bubble must be above a certain value (critical bubble)

T_N and <u>bubble wall profile</u> are derived from the energy of critical bubble

Parameters important for BAU realization

Bubble nucleation and bubble wall profile

nucleation rate per time per volume

$$\Gamma_N(T) \simeq T^4 \left(\frac{S_3(T)}{2\pi T}\right)^{3/2} e^{-S_3(T)/T}$$

$S_3(T)$: critical bubble energy at T

Definition of nucleation temperature T_N

$$\frac{\Gamma_N\left(T_N\right)}{H^3\left(T_N\right)} = H\left(T_N\right) \simeq 1.66\sqrt{g_*\left(T_N\right)}\frac{T_N^2}{m_{\mathrm{P}}}$$

$H(T_N)$: Hubble parameter at T_N

$$\frac{S_3\left(T_N\right)}{T_N} - \frac{3}{2}\ln\left(\frac{S_3\left(T_N\right)}{T_N}\right) = 143.4 - 2\ln\left(\frac{g_*\left(T_N\right)}{100}\right) - 4\ln\left(\frac{T_N}{100\mathrm{GeV}}\right)$$

$S_3(T_N)/T_N \lesssim 140$ are needed for EWPT

Bubble nucleation and bubble wall profile

Gauge-Higgs system

$$\mathcal{L}_{\text{gauge-Higgs}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H)^\dagger D^\mu H + \partial_\mu S^* \partial^\mu S - V(H,S)$$

$$D_\mu = \partial_\mu + ig_2\frac{\tau^a}{2}A^a_\mu + ig_1\frac{1}{2}B_\mu$$

Energy functional when $A_0 = 0$

$$E[H,S;T] = \int d^3\boldsymbol{x}\left[\frac{1}{4}F^a_{ij}F^a_{ij} + \frac{1}{4}B_{ij}B_{ij} + (D_iH)^\dagger D_iH + \partial_i S^*\partial_i S + V(H,S;T)\right],$$

Pure gauge configuration

$$ig_2\frac{\tau^a}{2}A^a_\mu = (\partial_\mu U_2)\,U_2^{-1}, \quad i\frac{g_1}{2}B_\mu = (\partial_\mu U_1)\,U_1^{-1}$$

Then,

$$E = \int d^3\boldsymbol{x}\left[(\partial_i H)^\dagger \partial_i H + \partial_i S^* \partial_i S + V(H,S;T)\right]$$

Bubble nucleation and bubble wall profile

Energy functional

$$S_3(T) = \int d^3x \left[(\partial_i H)^\dagger \partial_i H + \partial_i S^* \partial_i S + V_{\text{eff}}(H, S; T) \right]$$

$$\langle H(x) \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho(x) \end{pmatrix}, \quad \langle S(x) \rangle = \frac{1}{\sqrt{2}} \left(\rho_S^r(x) + i \rho_S^i(x) \right)$$

$$S_3(T) = 4\pi \int_0^\infty dr r^2 \left[\frac{1}{2} \left(\frac{d\rho}{dr} \right)^2 + \frac{1}{2} \left(\frac{d\rho_S^r}{dr} \right)^2 + \frac{1}{2} \left(\frac{d\rho_S^i}{dr} \right)^2 + \bar{V}_{\text{eff}} \left(\rho, \rho_S^r, \rho_S^i; T \right) \right]$$

EOMs

$$\frac{d^2\rho}{dr^2} + \frac{2}{r} \frac{d\rho}{dr} - \frac{\partial \bar{V}}{\partial \rho} = 0$$

$$\frac{d^2\rho_S^r}{dr^2} + \frac{2}{r} \frac{d\rho_S^r}{dr} - \frac{\partial \bar{V}}{\partial \rho_S^r} = 0$$

$$\frac{d^2\rho_S^i}{dr^2} + \frac{2}{r} \frac{d\rho_S^i}{dr} - \frac{\partial \bar{V}}{\partial \rho_S^i} = 0$$

w/ boundary conditions

$$\lim_{r\to\infty} \rho(r) = 0, \quad \lim_{r\to\infty} \rho_S^r(r) = \tilde{v}_S^r, \quad \lim_{r\to\infty} \rho_S^i(r) = \tilde{v}_S^i,$$

$$\left. \frac{d\rho(r)}{dr} \right|_{r=0} = 0, \quad \left. \frac{d\rho_S^r(r)}{dr} \right|_{r=0} = 0, \quad \left. \frac{d\rho_S^i(r)}{dr} \right|_{r=0} = 0.$$

Bubble nucleation and bubble wall profile

Thick wall regime

The bubble wall thickness L_ω is larger than the typical interaction length.

At temperature T, the interaction length is expressed as $l \simeq 1/T$

$$\therefore L_\omega > 1/T$$

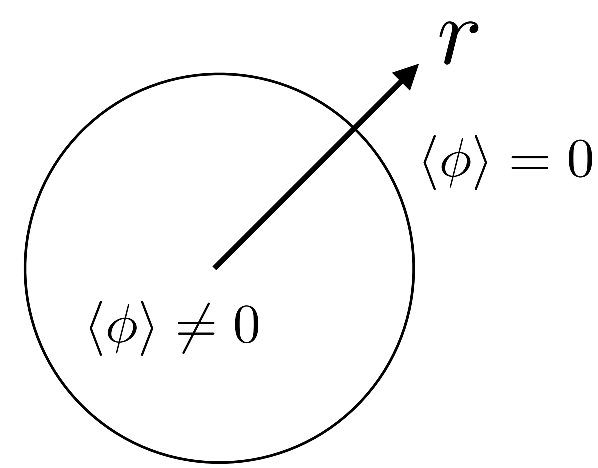
In BP1 $\quad L_w \simeq 0.1\mathrm{GeV}^{-1}$

$$1/T_N \simeq 0.01\mathrm{GeV}^{-1}$$

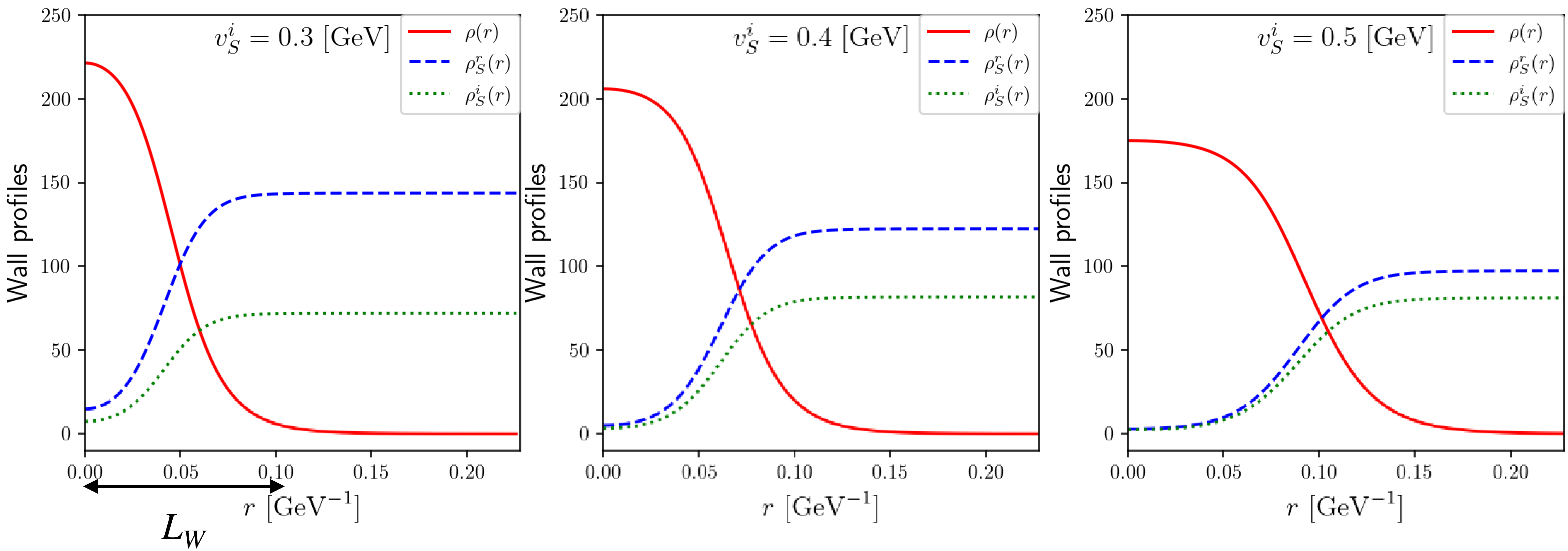
$\rightarrow$ thick wall

Bubble nucleation

$$\langle H(r)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \rho(r) \end{pmatrix}, \quad \langle S(r)\rangle = \frac{1}{\sqrt{2}}\left(\rho_S^r(r) + i\rho_S^i(r)\right)$$



*The energy of the bubble is smallest when the configuration is spherically symmetric
→ the scalar fields depend only on the radial direction



The wall is thinner when a strong first-order phase transition occurs

Bubble nucleation

Before evaluating the CPV phase related to EWBG

S couples to fermions only through the mixing angles α

pseudoscalar coupling $h_i \bar{f} \gamma_5 f$ does not arise

$\rightarrow$ Even though the complex phases exist in the scalar potential and the singlet scalar VEV, we do not induce CPV in the matter sector in the SM

One possible extension is adding new fermions that couple to S.
We consider the following higher dimensional operators contributing to the top Yukawa coupling.

$$\mathcal{L} = -y_t \bar{q}_L \tilde{H} \left(1 + \frac{c_1}{\Lambda} S + \frac{c_2}{\Lambda^2} |S|^2 + \frac{c_3}{\Lambda^2} S^2 + \cdots \right) t_R + \text{ H.c}$$

q_L : left-handed doublet fermion

$\tilde{H} = i\tau^2 H^*$ w/ Pauli matrix τ^2

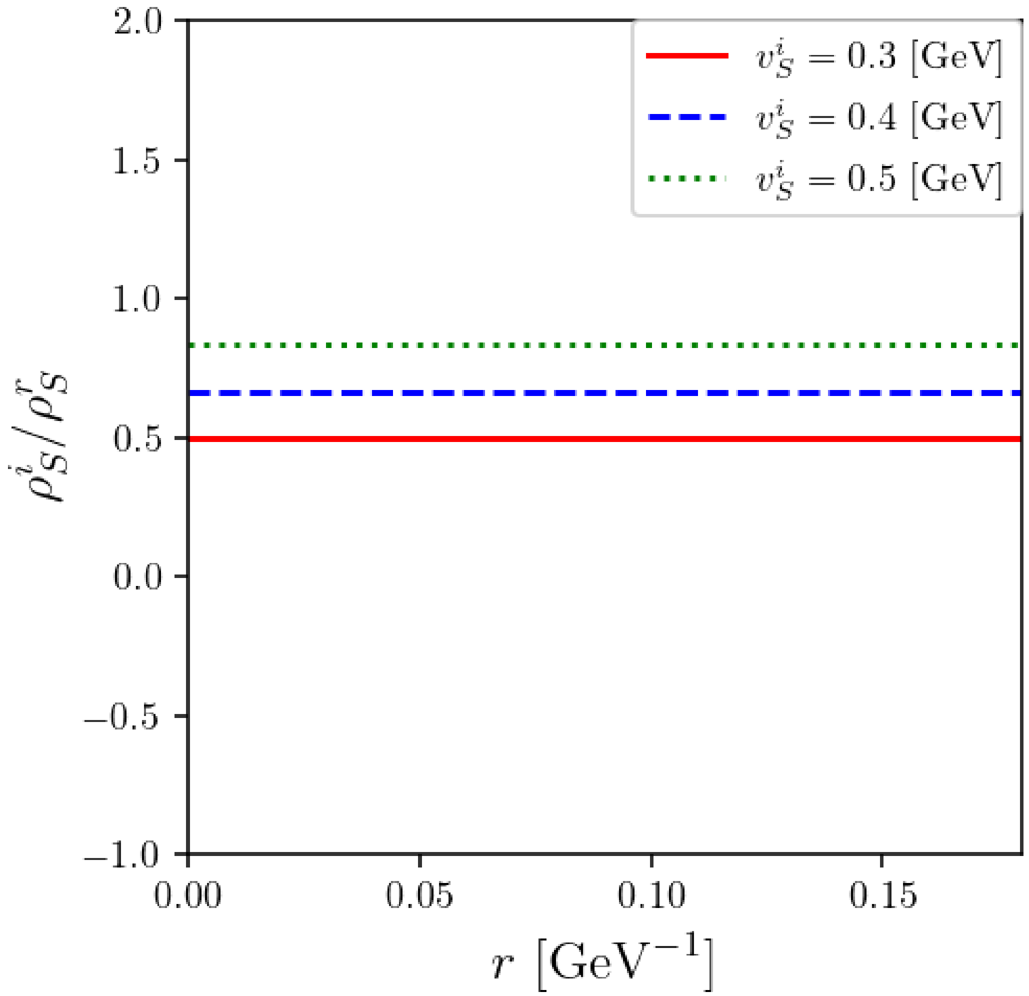
c_i : arbitrary parameters

Λ: the scale of the integrated fermion

Bubble nucleation

CPV phase related to EWBG $\theta(r) \equiv \tan^{-1}\left(\frac{\rho_S^i(r)}{\rho_S^r(r)}\right)$

the baryon number arises from the r derivative of θ



$$\mathcal{L} = -y_t \bar{q}_L \tilde{H}\left(1 + \boxed{\frac{c_1}{\Lambda}S} + \frac{c_2}{\Lambda^2}|S|^2 + \frac{c_3}{\Lambda^2}S^2 + \cdots\right) t_R + \text{ H.c}$$

top mass during PT

$$m_t(r) = \frac{y_t \rho(r)}{\sqrt{2}}\left(1 + \frac{c_1}{\sqrt{2}\Lambda}\left(\rho_S^r(r) + i\rho_S^i(r)\right)\right) \equiv |m_t(r)|\, e^{i\theta_t(r)}$$

$$\theta_t(r) = \tan^{-1}\left(\frac{\rho_S^i(r)}{\sqrt{2}\Lambda/c_1 + \rho_S^r(r)}\right)$$

the addition of this term allows θ to vary with r

→ Baryon number would generate!

The change with respect to r is constant for all benchmark points
→ baryon number generation is not possible?

Gravitational waves

GWs spectrum

$$\Omega_{\mathrm{GW}}(f)h^2 = \Omega_{\mathrm{col}}(f)h^2 + \Omega_{\mathrm{sw}}(f)h^2 + \Omega_{\mathrm{turb}}(f)h^2$$

$$\Omega_{\mathrm{col}}h^2 = 1.67\times 10^{-5}\left(\frac{\beta}{H_*}\right)^{-2}\left(\frac{\kappa_{\mathrm{col}}\alpha}{1+\alpha}\right)^{2}\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{0.11v_w^3}{0.42+v_w^2}\right)\frac{3.8\,(f/f_{\mathrm{col}})^{2.8}}{1+2.8\,(f/f_{\mathrm{col}})^{3.8}}$$

$$\Omega_{\mathrm{sw}}h^2 = 2.65\times 10^{-6}\left(\frac{\beta}{H_*}\right)^{-1}\left(\frac{\kappa_{v}\alpha}{1+\alpha}\right)^{2}\left(\frac{100}{g_*}\right)^{1/3}v_w\left(\frac{f}{f_{\mathrm{sw}}}\right)^{3}\left(\frac{7}{4+3\,(f/f_{\mathrm{sw}})^2}\right)^{7/2}$$

$$\Omega_{\mathrm{turb}}h^2 = 3.35\times 10^{-4}\left(\frac{\beta}{H_*}\right)^{-1}\left(\frac{\kappa_{\mathrm{turb}}\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3}v_w\frac{(f/f_{\mathrm{turb}})^3}{[1+(f/f_{\mathrm{turb}})]^{\frac{11}{3}}\,(1+8\pi f/h_*)}$$

v_w : wall velocity

$$h_* = 1.65\times 10^{-5}\left(\frac{T_*}{100\mathrm{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}\ \mathrm{Hz}$$

Gravitational waves

For $v_w \approx 1$

$$\kappa_{\rm col} \simeq \frac{1}{1+0.715\alpha}\left(0.715\alpha + \frac{4}{27}\sqrt{\frac{3\alpha}{2}}\right)$$

$$\kappa_v \simeq \frac{\alpha}{0.73+0.083\sqrt{\alpha}+\alpha}$$

$$\kappa_{\rm turb} \simeq (0.05-0.1)\kappa_v$$

Peak frequency

$$f_{\rm col} = 16.5\times 10^{-6}\left(\frac{\beta}{H_*}\right)\left(\frac{0.62}{1.8-0.1v_w+v_w^2}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}\ {\rm Hz}$$

$$f_{\rm sw} = 1.9\times 10^{-5}\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}\ {\rm Hz}$$

$$f_{\rm turb} = 2.7\times 10^{-5}v_w^{-1}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}\ {\rm Hz}$$

Gravitational wave

As a phenomenological consequence, **gravitational waves** from the first-order EWPT are also evaluated

The amplitudes and frequencies of GWs would be modulated according to the amount of latent heat (α) and/or duration of the phase transition (β).

$$\alpha \equiv \frac{\epsilon\left(T_*\right)}{\rho_{\mathrm{rad}}\left(T_*\right)}, \quad \beta \equiv H_* T_* \frac{d}{dT}\left(\frac{S_3(T)}{T}\right)\bigg|_{T=T_*}$$

$$\epsilon(T) = \Delta V_{\mathrm{eff}} - T\frac{\partial \Delta V_{\mathrm{eff}}}{\partial T}, \quad \rho_{\mathrm{rad}}(T) = \frac{\pi^2}{30} g_*(T) T^4 \qquad T_* = T_N$$

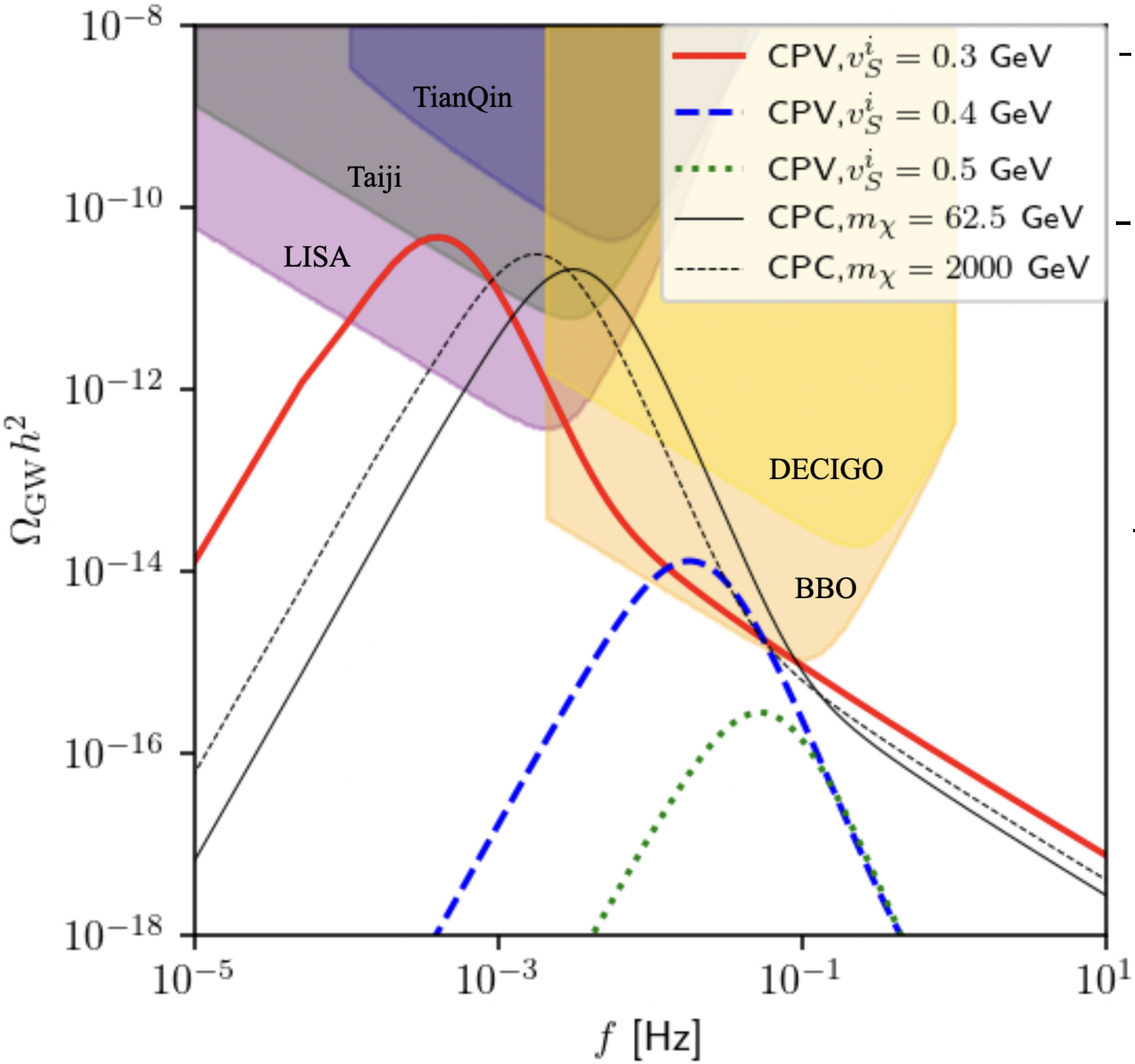
Sources of GWs

$$\Omega_{\mathrm{GW}}(f)h^2 = \underbrace{\Omega_{\mathrm{col}}(f)h^2}_{\text{bubble collision}} + \underbrace{\Omega_{\mathrm{sw}}(f)h^2}_{\text{sound wave}} + \underbrace{\Omega_{\mathrm{turb}}(f)h^2}_{\text{turbulence}}$$

The GWs spectrum $\Omega_{\mathrm{GW}}(f)h^2$ is amplified by a larger α and smaller β

realized by a stronger first-order phase transition.

Gravitational wave



- overlay the sensitivity curves of the future GW experiments
- The case of BP1 and CPC has a large spectrum, which could be verified by some experiments
- If $\theta = \tan^{-1}\left(v_S^i/v_S^r\right) \lesssim 0.7$, GWs are verifiable.