

POSITIVITY BOUNDS ON HIGGS-PORTAL DARK MATTER

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[arXiv: 2302.02879 \[hep-ph\]](https://arxiv.org/abs/2302.02879) accepted by JHEP

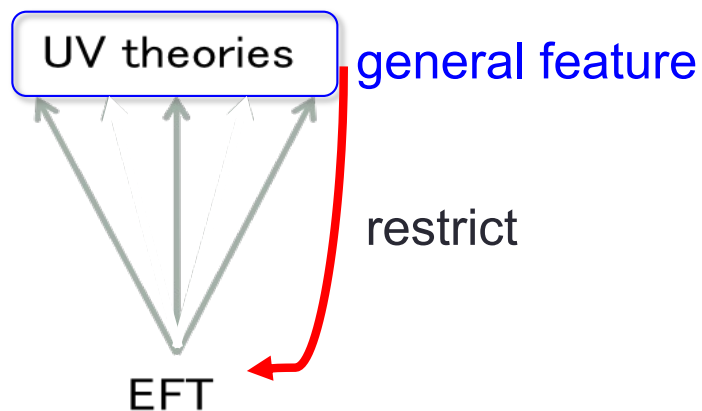
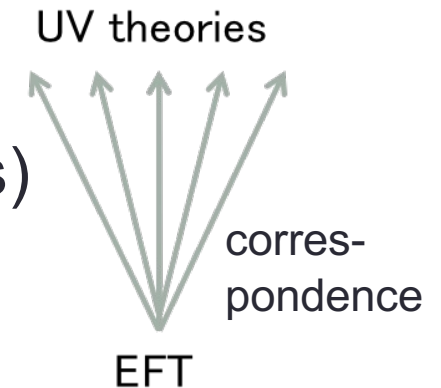
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Positivity Bounds (1/2)

- EFT is for the energy scale $E \ll \Lambda$ (typical energy scale of the UV physics)
- Many UV models correspond with EFT
- From the general feature of UV theory, can we bound on Wilson coefficients of EFT?

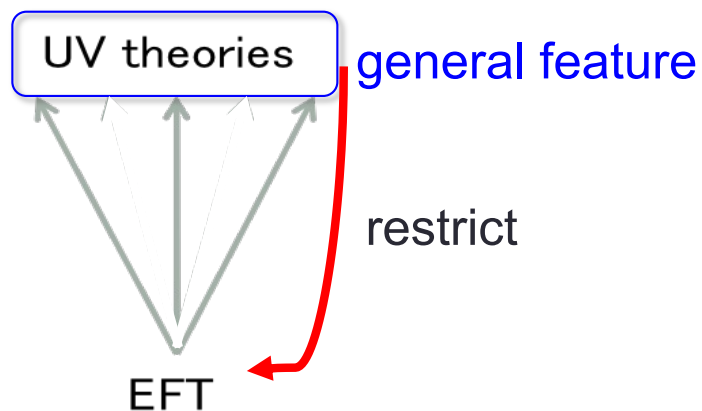
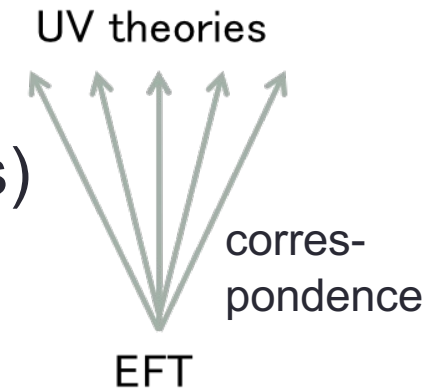


If we base on the local Quantum Field Theory(QFT) for the general feature of UV theory,

1. Special relativity \longrightarrow Lorentz invariance
2. Conservation of probability \longrightarrow Unitarity
3. Causality $- - - \longrightarrow$ Analyticity

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 $E \ll \Lambda$ (typical energy scale of the UV physics)
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→ Positivity Bounds



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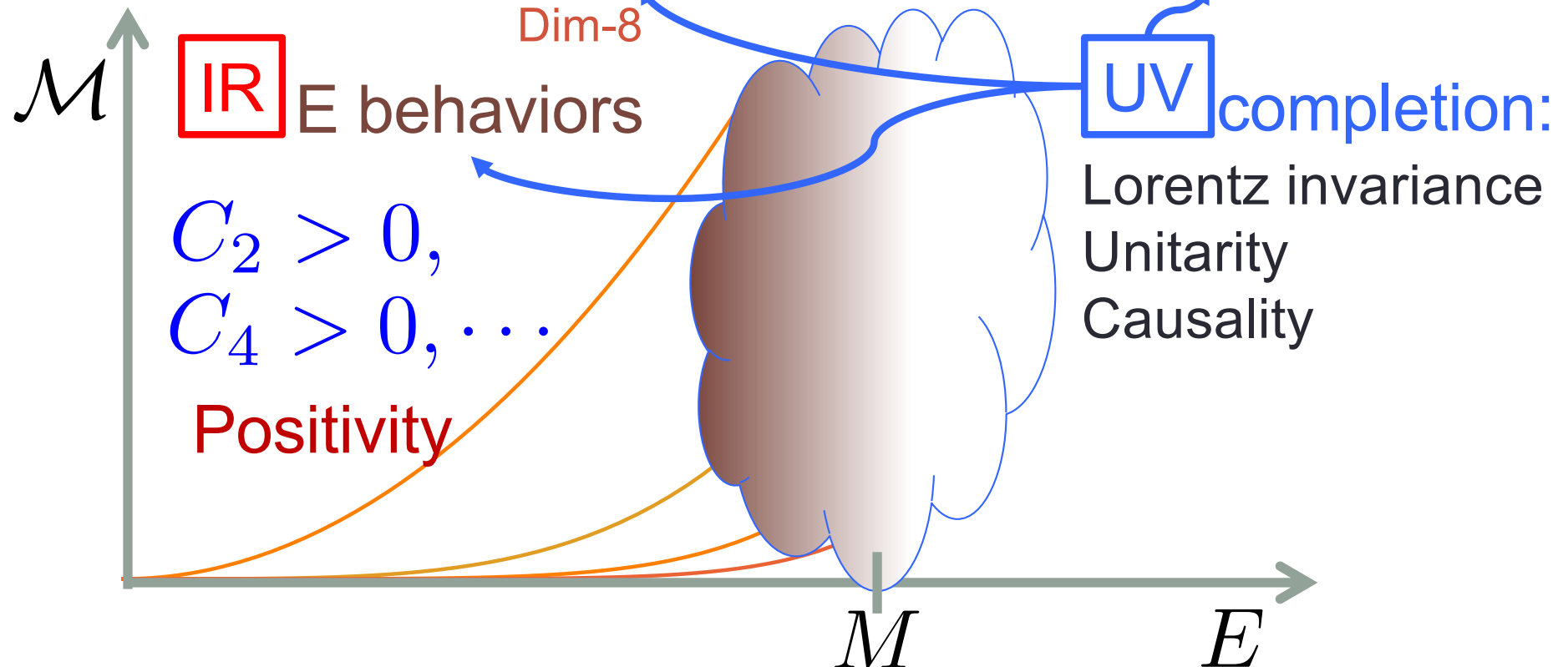
T. N. Pham, T. N. Truong, Phys. Rev. D **31**, 3027 (1985)B. Ananthanarayan, D. Toublan, G. Wanders, Phys. Rev. D **51**, 1093-1100 (1995)A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, R. Rattazzi, JHEP **0610**, 014 (2006)

Positivity Bounds (2/2)

Ref: Slides by [Francesco Riva](#)

- Effective Theory Forward Amplitude (IR):
For $D \geq 8$ Wilson Coefficients

$$\mathcal{M} = C_0 + C_1 \frac{s}{M^2} + \underbrace{C_2}_{\text{Dim-8}} \frac{s^2}{M^4} + C_3 \frac{s^3}{M^6} + \underbrace{C_4}_{\text{Dim-12}} \frac{s^4}{M^8} + \dots$$



Higgs Portal DM operators -positivity side-

- Derivative Coupling for Higgs and Dark Matter Fields

$$O_{H^2\varphi^2}^{(1)} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$

$$O_{H^2\varphi^2}^{(2)} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

- Subject to satisfying **positivity bounds**
- **Spin-2 massive graviton and/or spin-0 radion mediated DM model** is one of the candidates of this scenario as the **partial UV completion**
- Sensitive to **high-energy processes**

Higgs Portal DM operators -positivity side-

- Positivity bounds from the superposed states:

$$O_{H^2\varphi^2}^{(1)} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$

$$O_{H^2\varphi^2}^{(2)} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

$$O_{\varphi^4} = \partial_\mu \varphi \partial^\mu \varphi \partial_\nu \varphi \partial^\nu \varphi$$

$$O_{H^4}^{(1)} = (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$$

$$O_{H^4}^{(2)} = (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$$

$$O_{H^4}^{(3)} = (D_\mu H^\dagger D^\mu H)(D_\nu H^\dagger D^\nu H)$$

Higgs Portal DM operators -positivity side-

- Results:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

Bounds	Channels ($ 1\rangle + 2\rangle \rightarrow 1\rangle + 2\rangle$)
$C_{H^4}^{(1)} + C_{H^4}^{(2)} \geq 0$	$ 1\rangle = \phi_1\rangle, 2\rangle = \phi_3\rangle$
$C_{H^4}^{(1)} + C_{H^4}^{(2)} + C_{H^4}^{(3)} \geq 0$	$ 1\rangle = \phi_1\rangle, 2\rangle = \phi_1\rangle$
$C_{H^4}^{(2)} \geq 0$	$ 1\rangle = \phi_1\rangle, 2\rangle = \phi_2\rangle$
$C_{H^2\varphi^2}^{(1)} \geq 0$	$ 1\rangle = \phi_1\rangle, 2\rangle = \varphi\rangle$
$C_{\varphi^4} \geq 0$	$ 1\rangle = \varphi\rangle, 2\rangle = \varphi\rangle$
$2\sqrt{(C_{H^4}^{(1)} + C_{H^4}^{(2)} + C_{H^4}^{(3)})C_{\varphi^4}} \geq -\left(C_{H^2\varphi^2}^{(1)} + C_{H^2\varphi^2}^{(2)}\right)$	$ 1\rangle = 2\sqrt{C_{\varphi^4}} \phi_1\rangle + \sqrt{-(C_{H^2\varphi^2}^{(1)} + C_{H^2\varphi^2}^{(2)})} \varphi\rangle,$ $ 2\rangle = 1\rangle$ Superposition
$2\sqrt{(C_{H^4}^{(1)} + C_{H^4}^{(2)} + C_{H^4}^{(3)})C_{\varphi^4}} \geq C_{H^2\varphi^2}^{(2)}$	$ 1\rangle = 2\sqrt{C_{\varphi^4}} \phi_1\rangle + \sqrt{C_{H^2\varphi^2}^{(2)}} \varphi\rangle,$ $ 2\rangle = -2\sqrt{C_{\varphi^4}} \phi_1\rangle + \sqrt{C_{H^2\varphi^2}^{(2)}} \varphi\rangle$ Superposition

Higgs portal DM $O_{H^2\varphi^2}^{(1)} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$

$$O_{H^2\varphi^2}^{(2)} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

Higgs Portal DM operators - dim4 and dim6 -

- Dim-4 and Dim-6 Higgs Portal DM operators relevant to the phenomenology (relic density, direct and indirect detections):

$$\begin{aligned}
 & -\frac{1}{6\Lambda^4} \left(c_1 m_\varphi^4 \varphi^4 + 4c_2 m_H^4 |H|^4 + 8c'_2 \lambda_H m_H^2 |H|^6 + 4c''_2 \lambda_H^2 |H|^8 \right. \\
 & \quad \left. + 4c_3 m_\varphi^2 m_H^2 \varphi^2 |H|^2 + 4c'_3 \lambda_H m_\varphi^2 \varphi^2 |H|^4 \right) \\
 & + \frac{1}{6\Lambda^4} \left(d_1 m_\varphi^2 \varphi^2 (\partial_\mu \varphi)^2 + 4d_2 m_H^2 |H|^2 |D_\mu H|^2 + 4d'_2 \lambda_H |H|^4 |D_\mu H|^2 \right. \\
 & \quad \left. + 2d_3 m_\varphi^2 \varphi^2 |D_\mu H|^2 + 2d_4 m_H^2 |H|^2 (\partial_\mu \varphi)^2 + 2d'_4 \lambda_H |H|^4 (\partial_\mu \varphi)^2 \right)
 \end{aligned}$$

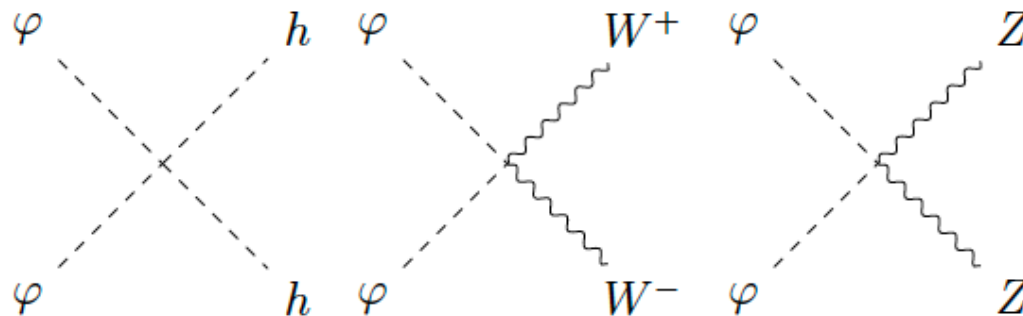
Relic Density

$$\mathcal{L} \supset 4c_3 m_\varphi^2 m_H^2 \varphi^2 |H|^2 + 4c'_3 \lambda_H m_\varphi^2 \varphi^2 |H|^4 \\ + 2d_4 m_H^2 |H|^2 (\partial_\mu \varphi)^2 + 2d'_4 \lambda_H |H|^4 (\partial_\mu \varphi)^2$$

- Higgs-portal interactions **linear** in the **Higgs boson h**

$$\mathcal{L}_{h,\text{linear}} = \frac{1}{3\Lambda^4} h \left[2(c_3 - c'_3) \lambda_H v^3 m_\varphi^2 \varphi^2 - (d_4 - d'_4) \lambda_H v^3 (\partial_\mu \varphi)^2 \right]$$

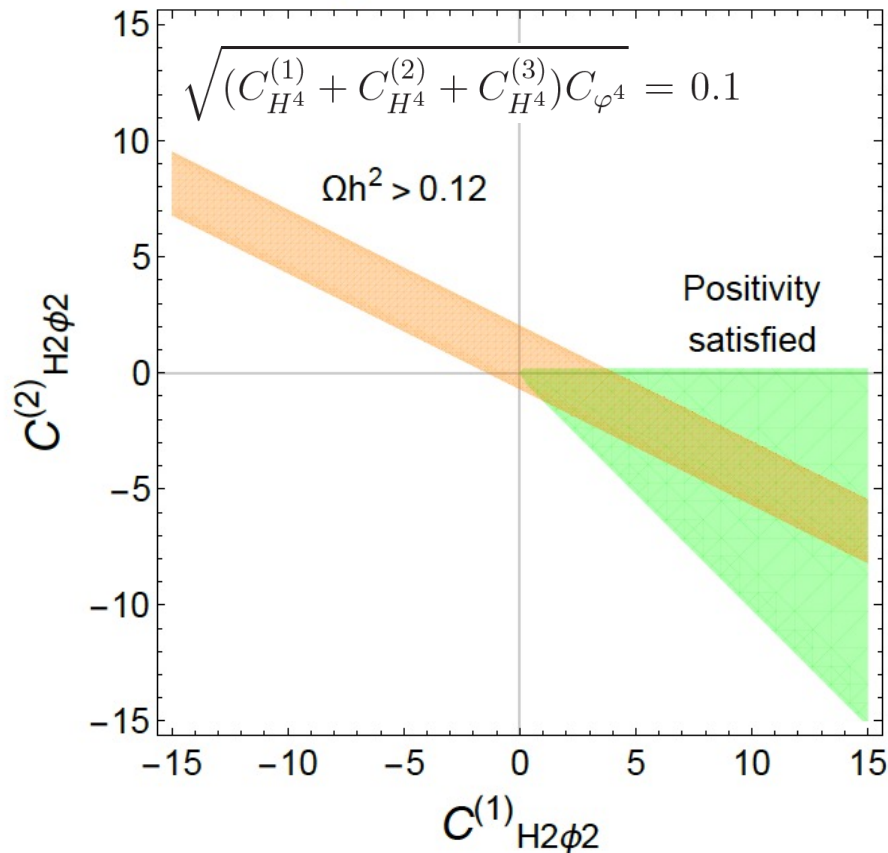
- Feynman diagrams for **DM annihilation processes** when $c'_3=c_3$ and $c'_4=c_4$ ($\varphi\varphi \rightarrow h \rightarrow ff$ are **absent**):



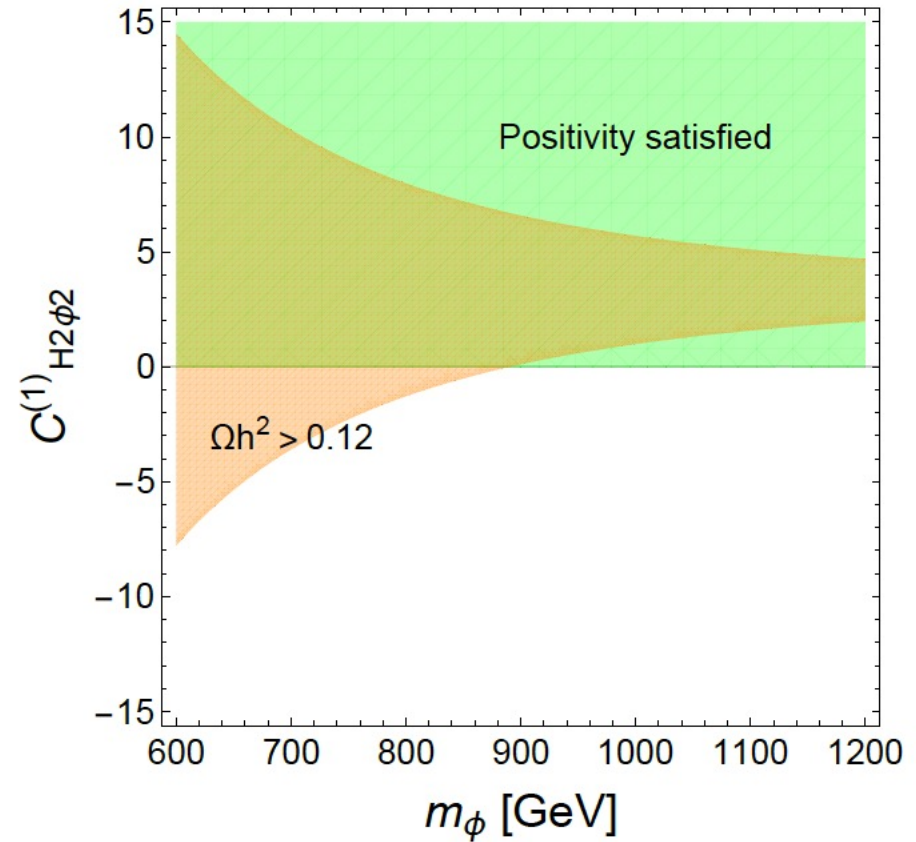
Note that the **tree-level direct detection bounds are absent** in this case

Relic Density

$\Lambda = 2 \text{ TeV}, m_\phi = 950 \text{ GeV}, c_3 = d_3 = c'_3 = d_4 = d'_4 = 2$

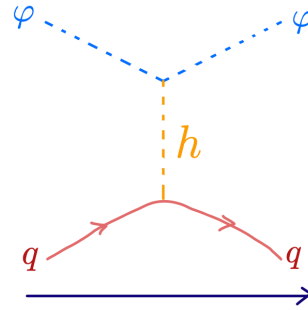


$C_{H^2\phi^2}^{(2)} = -1, \Lambda = 2 \text{ TeV}$
 $c_3 = d_3 = c'_3 = d_4 = d'_4 = 2$



$$O_{H^2\phi^2}^{(1)} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \phi \partial^\nu \phi) \quad O_{H^2\phi^2}^{(2)} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \phi \partial^\nu \phi)$$

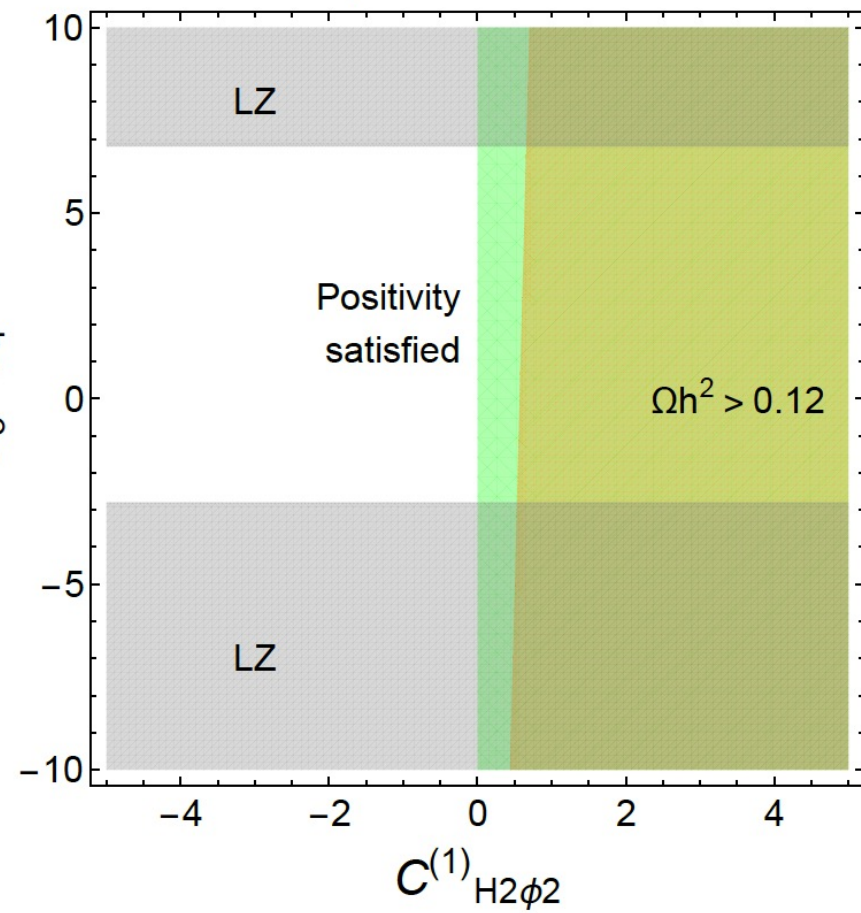
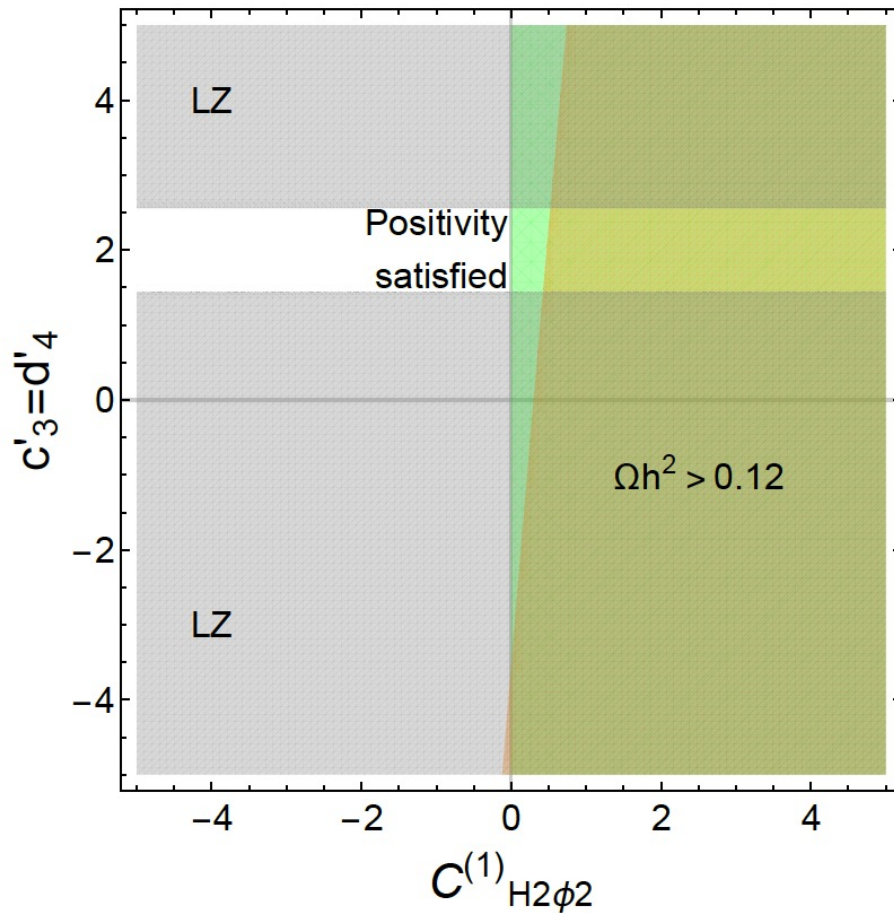
Direct Detection



$$\mathcal{L}_{h,\text{linear}} = \frac{1}{3\Lambda^4} h \left[2(c_3 - c'_3)\lambda_H v^3 m_\phi^2 \varphi^2 - (d_4 - d'_4)\lambda_H v^3 (\partial_\mu \varphi)^2 \right]$$

$\Lambda = 1 \text{ TeV}, m_\phi = 3m_h$
 $C_{H2\phi2}^{(2)} = -1, c_3 = d_3 = d_4 = 2$

$\Lambda = 2 \text{ TeV}, m_\phi = 950 \text{ GeV}$
 $C_{H2\phi2}^{(2)} = -1, c_3 = d_3 = d_4 = 2$



Indirect Detection

$$\mathcal{L} \supset 4c_3 m_\varphi^2 m_H^2 \varphi^2 |H|^2 + 4c'_3 \lambda_H m_\varphi^2 \varphi^2 |H|^4 \\ + 2d_4 m_H^2 |H|^2 (\partial_\mu \varphi)^2 + 2d'_4 \lambda_H |H|^4 (\partial_\mu \varphi)^2$$

Note on some cases:

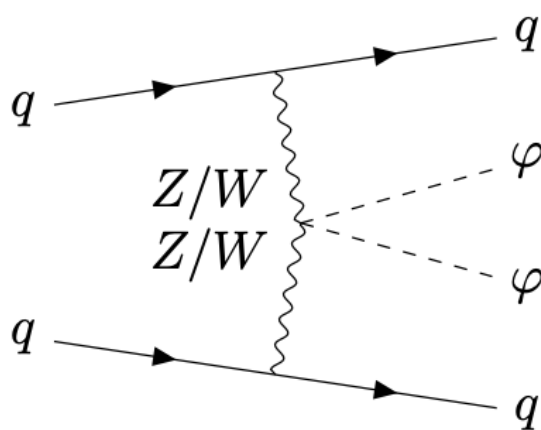
- When $c'_3=c_3$ and $d'_4=d_4$, $\varphi\varphi \rightarrow h \rightarrow ff$ are absent:

$$\mathcal{L}_{h,\text{linear}} = \frac{1}{3\Lambda^4} h \left[2(c_3 - c'_3) \lambda_H v^3 m_\varphi^2 \varphi^2 - (d_4 - d'_4) \lambda_H v^3 (\partial_\mu \varphi)^2 \right]$$

- In this case $\varphi\varphi \rightarrow hh$, WW , and ZZ can be constrained by indirect detection
- If we assume that only massive graviton is involved, $\varphi\varphi \rightarrow hh$ also vanish at s-wave, but $\varphi\varphi \rightarrow WW/ZZ$ are s-wave dominant

LHC Search

• High Luminosity LHC (HL-LHC) Search



Amplitude for $W^+W^-/ZZ \rightarrow \varphi\varphi$

- $O_{H^2\varphi^2}^{(2)}$ shows only Mandelstam s and mass dependencies
- $O_{H^2\varphi^2}^{(1)}$ causes t dependency also

Checking **angular distributions**

may help to **distinguish**

between $O_{H^2\varphi^2}^{(1)}$ and $O_{H^2\varphi^2}^{(2)}$

$$O_{H^2\varphi^2}^{(1)} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$

$$O_{H^2\varphi^2}^{(2)} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

X. Li, K. Mimasu, KY, C. Yang,
C. Zhang, S. Y. Zhou, JHEP10(2022)107

Summary and Outlook

- We consider Higgs portal dark matter derivative coupled dim-8 interactions and apply the positivity conditions to them
- We also included dim-4 and dim-6 Higgs portal interactions
- We see constraints from relic density, direct and indirect detections, and the relation to the massive graviton&radion case as an example of the partial UV completion
- For HL-LHC search, utilizing the kinematical distributions may be useful

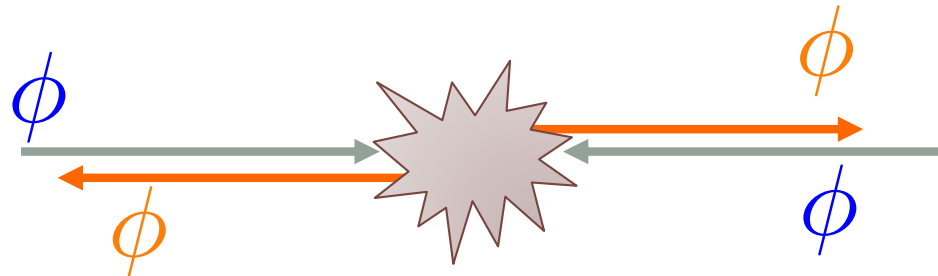
Backup

Positivity Bounds

$$\mathcal{M} = C_0 + C_1 \frac{s}{M^2} + \underbrace{C_2}_{>0} \frac{s^2}{M^4} + C_3 \frac{s^3}{M^6} + C_4 \frac{s^4}{M^8} + \dots$$

massless scalar 2-2 forward elastic scattering:

forward: $t=0$



$|+|| \rightarrow |+||$

elastic

Let us consider the amplitude of this: $\frac{\mathcal{M}(s, 0)}{s^3}$

Positivity Bounds

Forward limit positivity bounds are from:

1. Lorentz Invariance
2. Unitarity \Leftrightarrow Optical theorem:
e.g., elastic case,

$$\text{Im}\mathcal{M}(k_1, k_2 \rightarrow k_1, k_2) = \underline{\underline{s\sigma_{\text{tot}}(k_1, k_2 \rightarrow \text{anything})}}$$

Positive

1. Analyticity* \Leftrightarrow Froissart Bound:

$$|\mathcal{M}(s, \underline{\underline{\cos\theta = 1}})| < \text{Const. } s(\ln s)^2$$

forward Froissart, Martin 1960's
(for real $s \rightarrow \infty$)

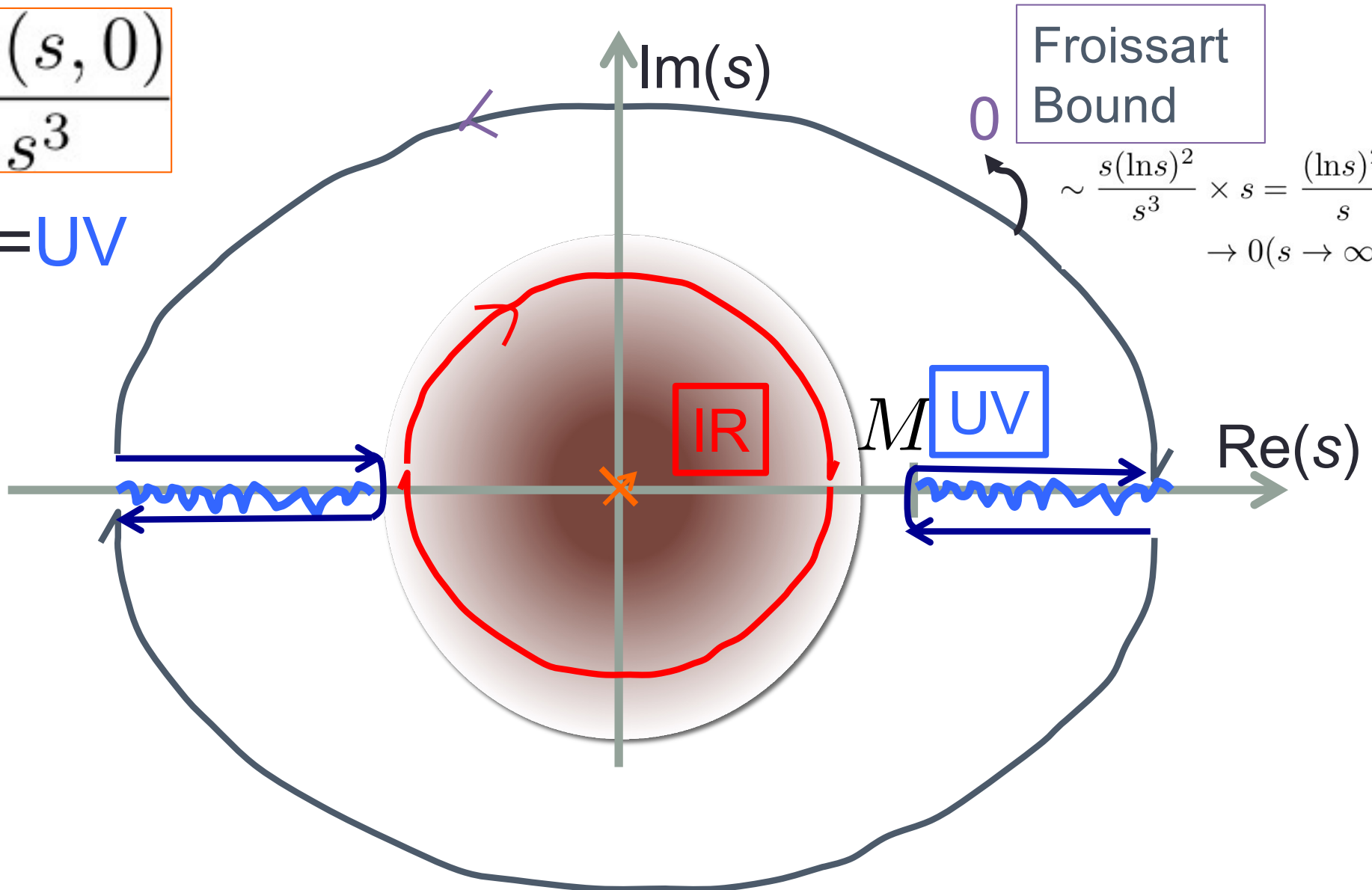
*Analyticity of the amplitude besides poles and branch cuts on real axis

Positivity Bounds

massless scalar 2-2 forward elastic scattering amplitude:

$$\frac{\mathcal{M}(s, 0)}{s^3}$$

IR=UV



Higgs Portal DM operators

- Massive Graviton and Radion case-

- Higgs/DM and Graviton Interaction:

$$-\frac{c_H}{M} G^{\mu\nu} T_{\mu\nu}^H - \frac{c_\varphi}{M} G^{\mu\nu} T_{\mu\nu}^\varphi$$

- Higgs/DM and Radion Interaction:

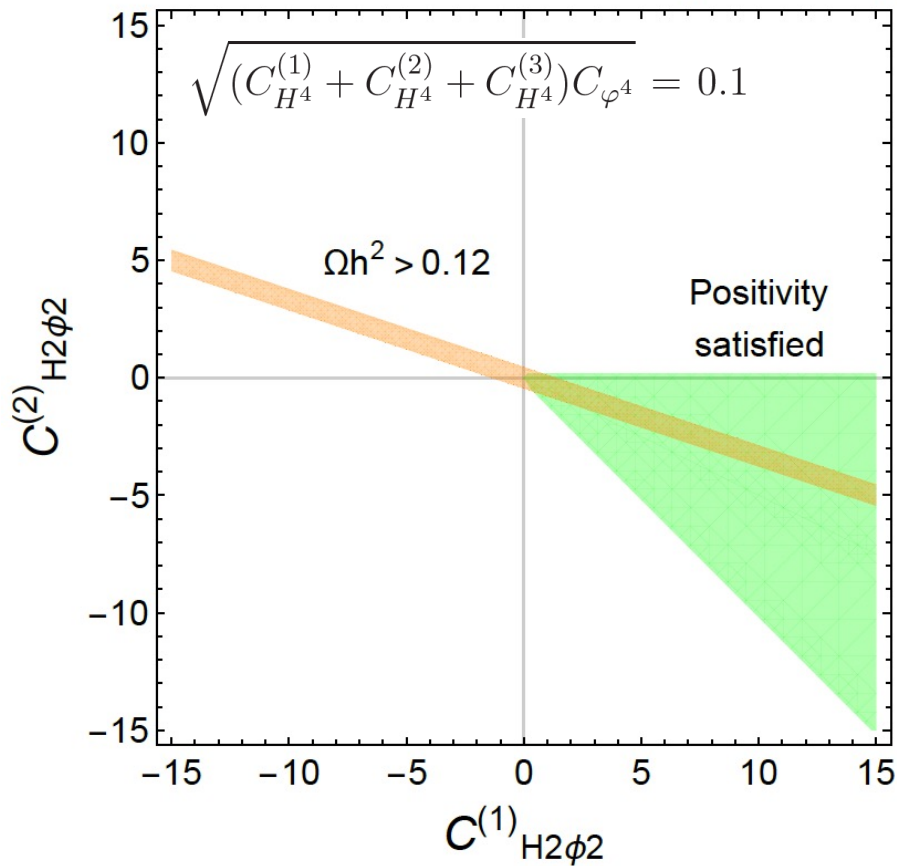
$$\mathcal{L}_r = \frac{c_H^r}{\sqrt{6}M} r T^H + \frac{c_\varphi^r}{\sqrt{6}M} r T^\varphi$$

- After Integrating out Massive Graviton/Radion, we can identify coefficients of dim-4, 6, and 8 operators as an example
- We found that **they satisfied the positivity conditions** as far as $c_H c_\varphi \geq 0$, (attractive force for the graviton)

Relic Density -Graviton and Radion case-

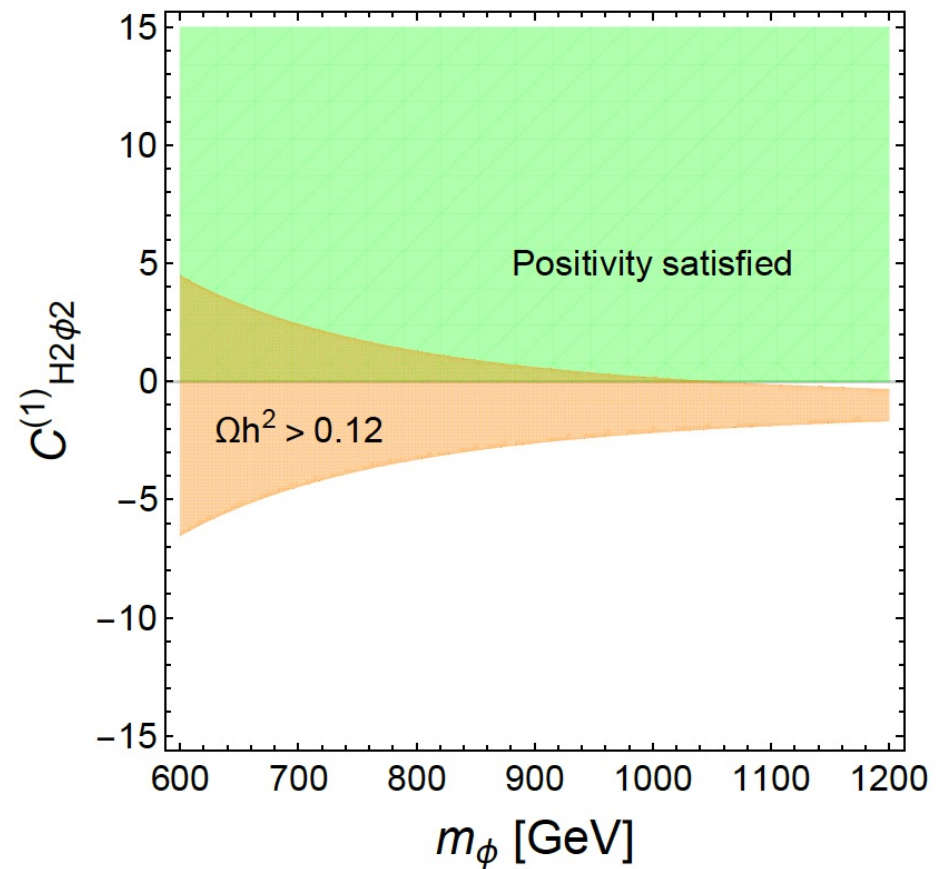
$\Lambda = 2 \text{ TeV}, m_\phi = 950 \text{ GeV}$

$$c_3 = d_3 = c'_3 = d'_4 = -1.5C^{(1)}_{H^2\phi^2} - 6C^{(2)}_{H^2\phi^2}$$



$C^{(2)}_{H^2\phi^2} = -1, \Lambda = 2 \text{ TeV}$

$$c_3 = d_3 = c'_3 = d'_4 = -1.5C^{(1)}_{H^2\phi^2} - 6C^{(2)}_{H^2\phi^2}$$



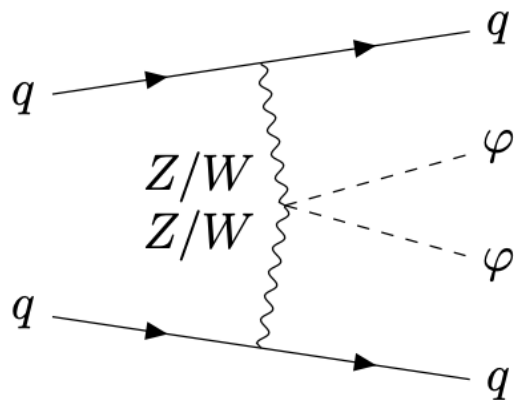
$$O_{H^2\phi^2}^{(1)} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi) \quad O_{H^2\phi^2}^{(2)} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$

LHC Search

ATLAS measurement with 139/fb at the 13 TeV LHC

- 95% upper limits: **0.11 pb** G. Aad *et al.* [ATLAS], JHEP **08**, 104 (2022)

$\sqrt{s} = 13 \text{ TeV LHC}, L_{\text{int}} = 139 \text{ fb}^{-1}$	$\sigma^{\text{VBF}} \times B_{\text{inv}} = \mathbf{0.11 \text{ pb}}$ ($m_H = 1 \text{ TeV}$)
$\Lambda = 1 \text{ TeV}, m_\varphi = 375 \text{ GeV}$	cross section from EFT operators
$(C_{H^2\varphi^2}^{(1)}, C_{H^2\varphi^2}^{(2)}) = (40, 40)$	0.28 pb Excluded
$(C_{H^2\varphi^2}^{(1)}, C_{H^2\varphi^2}^{(2)}) = (32, 32)$	0.11 pb Excluded
$(C_{H^2\varphi^2}^{(1)}, C_{H^2\varphi^2}^{(2)}) = (40, 0)$	0.012 pb
$(C_{H^2\varphi^2}^{(1)}, C_{H^2\varphi^2}^{(2)}) = (0, 40)$	0.097 pb



$$O_{H^2\varphi^2}^{(1)} = (D_\mu H^\dagger D_\nu H)(\partial^\mu \varphi \partial^\nu \varphi)$$

$$O_{H^2\varphi^2}^{(2)} = (D_\mu H^\dagger D^\mu H)(\partial_\nu \varphi \partial^\nu \varphi)$$