A model with two scalars in the view of effective theory

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Outline

Contents

Research Background

(1) Higgs Boson

- It has been discovered in 2012 which confirms its mass of around 125 GeV
- A scalar boson with spin 0 and charge 0
- Responsible for fermions (quark and lepton) and gauge boson mass generations.

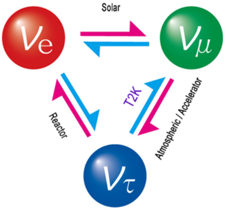
Problems

- Experiment results of Higgs do not tell us clearly how Spontaneous symmetry breaking occurs and how many Higgs particles are included in the process.
- Is there only one Higgs Boson?
  $\rightarrow$ BSM with multiple Higgs is possible.
- Is there a heavier mass than the energy range of the current experiment?
  $\rightarrow$ The other (second) Higgs boson has not been observed yet.

Research Background...

(2) Neutrino

- Three flavors exist and oscillate with each other. (electron neutrino ν_e, muon neutrino ν_μ, tau neutrino ν_τ)
- Charge is zero with a very tiny mass.
- Only interacting weakly with matter.



Neutrino oscillation between three generations

https://www.secretsofuniverse.in/8-interesting-facts-about-neutrinos/

Problems

- Mass Hierarchy problem (Normal or Inverted hierarchies), Either is Majorana or Dirac particles, etc.
- In order to explain the small mass of the neutrinos with the Yukawa couplings of neutrinos, we must assume a quite small of Yukawa couplings.

Motivation

- Motivation
  - If a second Higgs boson (assuming with heavy mass) exists, we would like to verify its existence $\rightarrow$ direct detection or indirect detection.
  - a Two Higgs doublet model (THDM) [1] can explain the small mass of (Dirac) neutrinos without assuming a small Yukawa coupling.

    [1] S.M. Davidson and H.E. Logan, Phys. Rev. D80 (2009).
- Purpose
  - To study the THDM with features like [1] with the viewpoint of the effective theory.
- Method
  - We construct an effective field theory of THDM in the low-energy region by integrating the heavy particles which are not observed on the low energy.

Contents

Effective Field Theory

- Effective field theory (EFT) is one of the useful tools in the research of physical systems.
- In EFT, the theory on the energy scale μ is derived by integrating out particles with a mass higher than the energy scale μ.
- Example of EFT: β decay: $d \to u + e^- + \bar{\nu}_e$

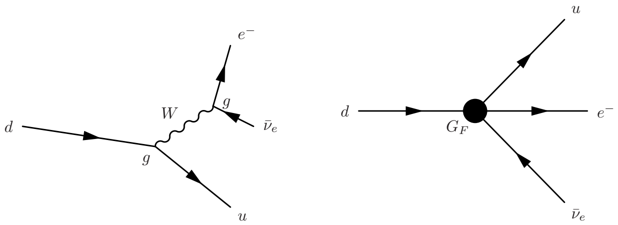


Figure: Feynman diagram for beta decay at quark level is shown (left) and the Feynman diagram in the weak EFT in which the W boson is integrated (right).

The Model

- We consider a simple toy model that can explain the small mass of Dirac neutrinos by the small vacuum expectation value of the second scalar.
- Action (in terms of bare (neutral) fields, bare mass, bare coupling and neutrino)

$$S = \int d^d x \left(-\frac{1}{2} \sum_{i=1}^{2} \rho_{0i} \left(\Box + m_{0i}^2 + \frac{\lambda_{0i}}{2} \rho_{0i}^2 \right) \rho_{0i} - \frac{\lambda_{03}}{4} \rho_{01}^2 \rho_{02}^2 \right.$$
$$\left. -(y_0 \bar{n}_0 n_0 + m_{012}^2 \rho_{01}) \rho_{02} + \sum_{i=1}^{2} h_{0i} m_{0i}^4 + h_{012} m_{012}^4 + 2h_{03} m_{01}^2 m_{02}^2 \right)$$

with m_{012}^2 is the bare mixing mass and y_0 is the Yukawa coupling of the neutrino and the second scalar.

- We assume the following hierarchy for the mass parameters:

$$m_2^2 \gg -m_1^2 \simeq \epsilon \, m_{12}^2 > 0, \quad \epsilon := \frac{m_{12}^2}{m_2^2} \ll 1$$

Symmetry of Model

- We impose the two symmetries (Z_2 and Z_2' symmetries) on the model.
- In order to forbid Majorana mass terms such as $\overline{(n_R)^c} n_R$ and $\overline{(n_L)^c} n_L$, we impose the symmetry under the transformation, $(n_L, n_R) \to e^{i\frac{\pi}{2}}(n_L, n_R)$.

Symmetry	ρ_1	ρ_2	n_L	n_R
Z_2	−	−	$+$	−
Z_2'	−	$+$	$+$	$+$

Table: The charge assignment under Z_2 and Z_2' symmetries.

Action in terms of Renormalized Quantities

- The relations between bare quantities and renormalized ones are:

$$
\begin{aligned}
\rho_{0i} &= \sqrt{Z_i}\rho_i, \\
n_0 &= \sqrt{Z_n}n, \\
m_{0i}^2 Z_i &= \sum_{j=1}^{2} Z_{mij} m_j^2, \\
m_{012}^2\sqrt{Z_1 Z_2} &= m_{12}^2 Z_{12},
\end{aligned}
$$

$$
\begin{aligned}
\lambda_{0i} Z_i^2 &= \sum_{I=1}^{3} Z_{\lambda_{iI}} \lambda_I \mu^{2\eta}, \\
\lambda_{03} Z_1 Z_2 &= \sum_{I=1}^{3} Z_{\lambda_{3I}} \lambda_I \mu^{2\eta}, \\
y_0 Z_n \sqrt{Z_2} &= Z_y y \mu^{\eta},
\end{aligned}
$$

μ: renormalization scale and $\eta = 2 - \frac{d}{2}$.

Action in terms of Renormalized Quantities

$$
\begin{aligned}
&S[\rho_1, \rho_2, n] \\
=& -\frac{1}{2}\int d^d x \sum_{i=1}^{2}\left(Z_i \rho_i \Box \rho_i + \rho_i^2 Z_{mij} m_j^2 + \frac{\mu^{2\eta}}{2}\sum_{I=1}^{3}\left(\rho_i^4 Z_{\lambda_{iI}} \lambda_I + \rho_1^2 \rho_2^2 Z_{\lambda_3 I} \lambda_I\right)\right) \\
& -\int d^d x \left(Z_y y \mu^\eta \bar{n} n + Z_{12} m_{12}^2 \rho_1 \right) \rho_2 \\
& +\int d^d x \mu^{-2\eta} \left(\sum_{i=1}^{2} Z_{hi} h_i {m_i}^4 + 2 Z_{h3} h_3 m_1^2 m_2^2 + Z_{h12} h_{12} m_{12}^4 \right).
\end{aligned}
$$

Action in terms of Renormalized Quantities

$$
\begin{aligned}
& S[\rho_1, \rho_2, n] \\
= & -\frac{1}{2}\int d^dx \sum_{i=1}^{2}\left(Z_i \rho_i \Box \rho_i + \rho_i^2 Z_{mij} m_j^2 + \frac{\mu^{2\eta}}{2}\sum_{I=1}^{3}\left(\rho_i^4 Z_{\lambda_{iI}}\lambda_I + \rho_1^2\rho_2^2 Z_{\lambda_3 I}\lambda_I\right)\right) \\
& -\int d^dx \left(Z_y y \mu^\eta \bar{n} n + Z_{12} m_{12}^2 \rho_1\right)\rho_2 \\
& +\int d^dx \mu^{-2\eta}\left(\sum_{i=1}^{2} Z_{hi} h_i {m_i}^4 + 2 Z_{h3} h_3 m_1^2 m_2^2 + Z_{h12} h_{12} m_{12}^4\right).
\end{aligned}
$$

We do not include the kinetic term for neutrino and its quantum correction.

Contents

Low-Energy Effective Action

- Introducing the following generating functional $W[J_1, n]$,

$$e^{iW[J_1,n]} = \int d\rho_1 \int d\Delta_2 e^{iS[\rho_1,\Delta_2,n]+i\int \rho_1 J_1 d^4x}$$

- We can define the effective action Γ_{eff}, a functional of $\bar{\rho}_1$ by Legendre transformation of $W[J_1, n]$ as,

$$\begin{aligned}\Gamma_{\text{eff}}[\bar{\rho}_1, n] &= W[J_1, n] - \int J_1 \bar{\rho}_1 d^4x \\ &= -i \log \int d\Delta_1 \int d\Delta_2 e^{iS[\bar{\rho}_1+\Delta_1,\Delta_2,n]-i\int \Delta_1 \frac{\delta\Gamma_{\text{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)} d^4x}\end{aligned}$$

$$\Delta_1 \equiv \rho_1 - \bar{\rho}_1, \quad \Delta_2 = \rho_2, \quad J_1(x) = -\frac{\delta\Gamma_{\text{eff}}[\bar{\rho}_1, n]}{\delta\bar{\rho}_1(x)}$$

- Δ_1 is the quantum fluctuation from the expectation value $\bar{\rho}_1$.

Low-Energy Effective Action...

- $\bar{\rho}_1$ is the expectation value of ρ_1 defined by,

$$\bar{\rho}_1\big|_{J_1} = \frac{\delta W[J_1, n]}{\delta J_1} = \frac{\int d\rho_1 \int d\Delta_2 \rho_1 e^{iS[\rho_1,\Delta_2,n]+i\int \rho_1 J_1 d^4x}}{\int d\rho_1 \int d\Delta_2 e^{iS[\rho_1,\Delta_2]+i\int \rho_1 J_1 d^4x}}$$

- Next, we define the following quantity by subtracting the classical action from the effective action,

$$\tilde{\Gamma}_{\text{eff}}[\bar{\rho}_1, n] = \Gamma_{\text{eff}}[\bar{\rho}_1, n] - S[\bar{\rho}_1, 0, 0],$$

$$e^{i\tilde{\Gamma}_{\text{eff}}[\bar{\rho}_1,n]} = \int d\Delta_1 e^{i\left\{\frac{1}{2}\int d^dx \int d^dy \Delta_1(x)\frac{\delta^2 S[\bar{\rho}_1,0,n]}{\delta\bar{\rho}_1(x)\delta\bar{\rho}_1(y)}\Delta_1(y)+S_{1\text{int}}(\Delta_1,\bar{\rho}_1)-\int d^dx\Delta_1(x)\left(\frac{\delta\tilde{\Gamma}_{\text{eff}}[\bar{\rho}_1,n]}{\delta\bar{\rho}_1(x)}\right)\right\}} \times e^{iW_2[J_2,\Delta_1,n]}.$$

- The last factor summarizes the contribution from the heavy field to the effective action.

- Next, the contribution from the heavy field to EA, $e^{iW_2[J_2,\bar{\rho}_1,\Delta_1]}$, is

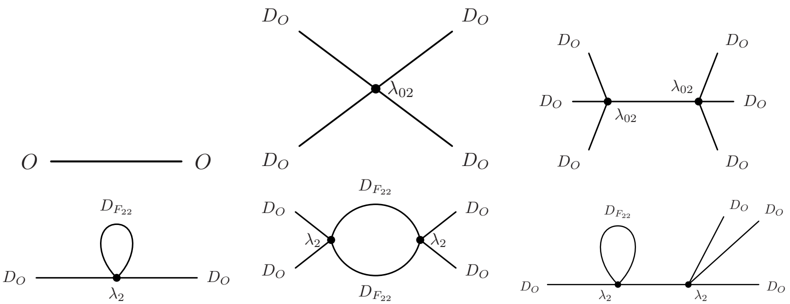
$$e^{iW_2[J_2,\bar{\rho}_1,\Delta_1]} = \left(\det \frac{\delta^2 S[\rho_1,\rho_2,n]}{\delta\rho_2(x)\delta\rho_2(y)}\Big|_{\rho_1=\bar{\rho}_1,\rho_2=0}\right)^{-\frac{1}{2}} \times \langle e^{iS_{2\,\mathrm{int}}(\Delta_2)+iS_{12\,\mathrm{int}}(\Delta_1,\Delta_2,\bar{\rho}_1)}\rangle_0 \; e^{iW_2^c[J_2,\bar{\rho}_1,\Delta_1]},$$

$$e^{iW_2^c[J_2,\bar{\rho}_1,\Delta_1]} = \frac{\left\langle e^{iS_{2\,\mathrm{int}}(\Delta_2)+iS_{12\,\mathrm{int}}(\Delta_1,\Delta_2,\bar{\rho}_1)+i\int d^dx\Delta_2(x)J_2(x)}\right\rangle_0}{\langle e^{iS_{2\,\mathrm{int}}(\Delta_2)+iS_{12\,\mathrm{int}}(\Delta_1,\Delta_2,\bar{\rho}_1)}\rangle_0}.$$

- The first factor is the vacuum graph contribution from the quadratic part of Δ_2 and the second part corresponds to that from the interaction.
- The third factor is the connected Green function contribution of Δ_2 with the source term J_2:

$$J_2(x) = -Z_{12}m_{12}^2(\Delta_1+\bar{\rho}_1) - \mu^\eta y Z_y \bar{n}n.$$

that consists of the fields linearly coupled to the heavy scalar Δ_2.

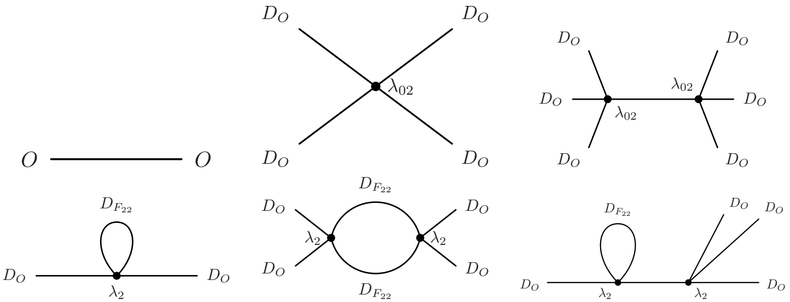


→ All the propagators are heavy scalars and the external source terms denoted by O correspond to the background field $\bar{\rho}_1$ and the bilinear neutrino $\overline{n}n$.

$$iW_2^c[J_2, \bar{\rho}_1, \Delta_1] = iW_2^{c(\text{tree})}[J_2, \bar{\rho}_1, \Delta_1] + i\bar{W}_2^{\,c(1\text{ loop})}[\bar{\rho}_1, n].$$

$$D_O(x) = \int d^d x_i D_{F22}(x, x_i) iO(x_i),$$

$$-J_2(\Delta_1 = 0) = O(x) = Z_{12} m_{12}^2 \bar{\rho}_1 + \mu^\eta y Z_y \bar{n} n$$



→ All the propagators are heavy scalars and the external source terms denoted by O correspond to the background field $\bar{\rho}_1$ and the bilinear neutrino $\overline{n}n$.

$$iW_2^c[J_2, \bar{\rho}_1, \Delta_1] = iW_2^{c(\text{tree})}[J_2, \bar{\rho}_1, \Delta_1] + i\bar{W_2}^{c(1\,\text{loop})}[\bar{\rho}_1, n].$$

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$$-J_2(\Delta_1 = 0) = O(x) = Z_{12} m_{12}^2 \bar{\rho}_1 + \mu^\eta y Z_y \bar{n} n$$

I do not show how we integrate the lighter scalar field in this presentation.

- Substituting the vacuum expectation value v_1 for $\bar{\rho}_1$, we obtain

$$
\begin{aligned}
V_{\rm eff}(v_1) =& V_{\rm cosmo} + \frac{m^2_{1\rm eff}}{2} v_1^2 - \frac{m^2_{12\rm eff}}{2}\epsilon v_1^2 + \frac{\lambda_{1\rm eff}}{4} v_1^4 + \frac{\lambda_{3\rm eff}}{4}\epsilon^2 v_1^4 - y_{\rm eff}\epsilon \bar{n} n v_1 \\
& - \frac{\lambda_3^2}{8m_2^2}\epsilon^2 v_1^6 + \frac{\lambda_3}{2m_2^2}\epsilon(\bar{n}n) v_1^3 - \frac{y^2}{2m_2^2}(\bar{n}n)^2 .\\
V_{\rm cosmo} =& -h_{1\rm eff} {m_1}^4 - h_{2\rm eff} {m_2}^4 - h_{12\rm eff} {m_{12}}^4 - 2h_{3\rm eff} {m_1}^2 {m_2}^2 ,\\
m^2_{1\rm eff} =& m_1^2\left(1 - \frac{3\lambda_1}{16\pi^2}\left(\frac{3}{2} - \log\frac{m_1^2 + 3\lambda_1 v_1^2}{\mu^2}\right)\right) - \frac{\lambda_3 m_2^2}{32\pi^2}\left(1 - \log\frac{m_2^2}{\mu^2}\right),\\
m^2_{12\rm eff} =& m^2_{12}\left\{1 + \frac{3\lambda_1}{16\pi^2}\log\frac{m_1^2 + 3\lambda_1 v_1^2}{m_2^2} + \frac{3\lambda_2}{16\pi^2}\left(1 - \log\frac{m_2^2}{\mu^2}\right) - \frac{\lambda_3}{8\pi^2}\left(\frac{5}{4} - \log\frac{m_2^2}{\mu^2}\right)\right\},\\
\frac{\lambda_{1\rm eff}}{4} =& \frac{\lambda_1}{4}\left\{1 - \frac{9\lambda_1}{16\pi^2}\left(\frac{3}{2} - \log\frac{m_1^2 + 3\lambda_1 v_1^2}{\mu^2}\right) + \frac{1}{64\pi^2}\frac{\lambda_3^2}{\lambda_1}\log\frac{m_2^2}{\mu^2}\right\}\\
\frac{\lambda_{3\rm eff}}{4} =& \frac{\lambda_3}{4}\left\{1 + \frac{3\lambda_2}{16\pi^2}\left(2 - \log\frac{m_2^2}{\mu^2}\right) - \frac{\lambda_3}{4\pi^2}\left(\frac{25}{16} - \log\frac{m_2^2}{\mu^2}\right)\right.\\
& \left. + \frac{3\lambda_1}{16\pi^2}\left(\log\frac{m_1^2 + 3\lambda_1 v_1^2}{\mu^2} + 5\log\frac{m_1^2 + 3\lambda_1 v_1^2}{m_2^2} - 6\frac{\lambda_1}{\lambda_3}\log\frac{m_1^2 + 3\lambda_1 v_1^2}{m_2^2}\right)\right\}\\
h_{1\rm eff} =& h_1 + \frac{1}{64\pi^2}\left(\frac{3}{2} - \log\frac{m_1^2 + 3\lambda_1 v_1^2}{\mu^2}\right), \quad h_{2\rm eff} = h_2 + \frac{1}{64\pi^2}\left(\frac{3}{2} - \log\frac{m_2^2}{\mu^2}\right),\\
h_{3\rm eff} =& h_3 + \frac{\epsilon^2}{32\pi^2}\log\frac{m_1^2 + 3\lambda_1\bar{\rho}_1^2}{\mu^2}, \quad h_{12\rm eff} = h_{12} + \frac{1}{32\pi^2}\left(1 - \log\frac{m_2^2}{\mu^2}\right),\\
y_{\rm eff} =& y\left(1 + \frac{3\lambda_2 - \lambda_3}{16\pi^2}\left(1 - \log\frac{m_2^2}{\mu^2}\right)\right).
\end{aligned}
$$

Contents

Low energy theory without heavy scalar

- Integrate heavy scalars in the original theory and build a low-energy theory without heavy scalars (including its counter terms)

$$
\begin{aligned}
V_{\rm eff}^{\rm Low} =& \frac{m_1^2}{2}\left\{1-\frac{3\lambda_1}{16\pi^2}\left(\frac{3}{2}-\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right)\right\}v_1^2 \\
&-\frac{m_{12}^2}{2}\left\{1-\frac{3\lambda_1}{16\pi^2}\left(1-\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right)\right\}\epsilon v_1^2-\epsilon y\bar{n}n v_1-\frac{1}{2m_2^2}\left(\frac{\lambda_3\epsilon}{2}v_1^3-y\bar{n}n\right)^2 \\
&+\frac{\lambda_1}{4}\left\{1-\frac{9\lambda_1}{16\pi^2}\left(\frac{3}{2}-\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right)\right\}v_1^4 \\
&+\frac{\lambda_3}{4}\left\{1+\frac{9}{8\pi^2}\frac{\lambda_1(\lambda_1-\lambda_3)}{\lambda_3}+\frac{9}{8\pi^2}\frac{\lambda_1(\lambda_3-\lambda_1)}{\lambda_3}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right\}\epsilon^2 v_1^4 \\
&-m_1^4\left\{h_1+\frac{1}{64\pi^2}\left(\frac{3}{2}-\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right)\right\} \\
&+2\epsilon^2 m_1^2 m_2^2\left\{-\frac{h_3}{\epsilon^2}+\frac{1}{64\pi^2}\left(1-\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right)\right\} \\
&-\epsilon^2 m_{12}^4\left\{\frac{h_{12}}{\epsilon^2}-\frac{1}{64\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{\mu^2}\right)\right\}-h_2 {m_2}^4 .
\end{aligned}
$$

RG Improved Effective Potential

- Since we can improve the low energy effective potential by RG, we are able to obtain the RG improved effective potential defined by,

$$V_{\text{eff}}^{\text{Improved}} = V_{\text{eff}}(\mu = m_2) - V_{\text{eff}}^{\text{Low}}(\mu = m_2) + V_{\text{eff}}^{\text{Low RGimproved}},$$

- $V_{\text{eff}}^{\text{Low RGimproved}}$ is obtained by using the RG improved coupling and masses (after solving the RG equations)

$$\lambda_1'(\mu_0) = \frac{\lambda_1'(\mu)}{1 + \frac{9\lambda_1'(\mu)}{16\pi^2}\log\frac{\mu^2}{\mu_0^2}},$$

$$m_1'^2(\mu_0) = \frac{m_1'^2(\mu)}{\left(1 + \frac{9\lambda_1'(\mu)}{16\pi^2}\log\frac{\mu^2}{\mu_0^2}\right)^{\frac{1}{3}}},$$

$$\bar{h}(\mu_0) = \bar{h}(\mu) - \frac{1}{64\pi^2}\log\frac{\mu_0^2}{\mu^2}.$$

$$V_{\rm eff}^{\rm Improved} = V_{\textsf{cosmo}}^{\textsf{(Imp.)}} + \frac{m_{\textsf{1eff}}^{2,\textsf{(Imp.)}}}{2} v_1^2 - \frac{m_{\textsf{12eff}}^{2,\textsf{(Imp.)}}}{2} \epsilon v_1^2 + \frac{\lambda_{\textsf{1eff}}^{\textsf{(Imp.)}}}{4} v_1^4 + \frac{\lambda_{\textsf{3eff}}^{\textsf{(Imp.)}}}{4} \epsilon^2 v_1^4 - \left(1 + \frac{3\lambda_2 - \lambda_3}{16\pi^2}\right) y\epsilon\bar{n}n v_1 - \frac{1}{2m_2^2}\left(\frac{\lambda_3}{2}\epsilon v_1^3 - y\bar{n}n\right)^2$$

- Cosmological term of RG improved effective potential

$$V_{\textsf{cosmo}}^{\textsf{(Imp.)}} = -m_1^4\left\{h_1(m_2) + \frac{1}{64\pi^2}\left(\frac{3}{2} - \log\frac{m_1^2 + 3\lambda_1 v_1^2}{m_2^2}\right)\right\} - m_2^4\left(h_2(m_2) + \frac{3}{128\pi^2}\right) - m_{12}^4\left(h_{12}(m_2) + \frac{1}{32\pi^2}\right) - 2m_1^2 m_2^2\left\{h_3(m_2) + \frac{\epsilon^2}{64\pi^2}\log\left(\frac{m_1^2 + 3\lambda_1 v_1^2}{m_2^2}\right)\right\}$$

$$V_{\text{eff}}^{\text{Improved}} = V_{\text{cosmo}}^{(\text{Imp.})} + \frac{m_{1\text{eff}}^{2,(\text{Imp.})}}{2} v_1^2 - \frac{m_{12\text{eff}}^{2,(\text{Imp.})}}{2} \epsilon v_1^2 + \frac{\lambda_{1\text{eff}}^{(\text{Imp.})}}{4} v_1^4 + \frac{\lambda_{3\text{eff}}^{(\text{Imp.})}}{4} \epsilon^2 v_1^4$$
$$- \left(1 + \frac{3\lambda_2 - \lambda_3}{16\pi^2}\right) y\epsilon\bar{n}nv_1 - \frac{1}{2m_2^2}\left(\frac{\lambda_3}{2}\epsilon v_1^3 - y\bar{n}n\right)^2$$

- Mass squared term of RG improved effective potential

$$m_{1\text{eff}}^{2,(\text{Imp.})} = m_1^2(m_2)\left[-\frac{\lambda_3}{32\pi^2}\frac{m_2^2}{m_1^2} + \frac{1 - \frac{9\lambda_1(m_2)}{32\pi^2}}{\left(1 + \frac{9\lambda_1(m_2)}{16\pi^2}\log\left(\frac{m_2^2}{m_1^2 + 3\lambda_1 v_1^2}\right)\right)^{\frac{1}{3}}}\right]$$

$$m_{12\text{eff}}^{2,(\text{Imp.})} = m_{12}^2(m_2)\left[-\frac{5\lambda_3}{32\pi^2} + \frac{3\lambda_2}{16\pi^2} + \frac{3\lambda_1}{16\pi^2} + \frac{1 - \frac{3\lambda_1(m_2)}{16\pi^2}}{\left(1 + \frac{9\lambda_1(m_2)}{16\pi^2}\log\left(\frac{m_2^2}{m_1^2 + 3\lambda_1 v_1^2}\right)\right)^{\frac{1}{3}}}\right]$$

$$
\begin{aligned}
V_{\text{eff}}^{\text{Improved}} =& V_{\text{cosmo}}^{(\text{Imp.})} + \frac{m_{1\text{eff}}^{2,(\text{Imp.})}}{2} v_1^2 - \frac{m_{12\text{eff}}^{2,(\text{Imp.})}}{2} \epsilon v_1^2 + \frac{\lambda_{1\text{eff}}^{(\text{Imp.})}}{4} v_1^4 + \frac{\lambda_{3\text{eff}}^{(\text{Imp.})}}{4} \epsilon^2 v_1^4 \\
& - \left(1 + \frac{3\lambda_2 - \lambda_3}{16\pi^2}\right) y\epsilon \bar{n} n v_1 - \frac{1}{2m_2^2} \left(\frac{\lambda_3}{2} \epsilon v_1^3 - y\bar{n}n\right)^2
\end{aligned}
$$

- Quartic term of RG improved effective potential

$$
\begin{aligned}
\lambda_{1\text{eff}}^{(\text{Imp.})} =& \lambda_1(m_2) \left[\frac{1 - \frac{27\lambda_1(m_2)}{32\pi^2}}{1 + \frac{9\lambda_1(m_2)}{16\pi^2} \log\left(\frac{m_2^2}{m_1^2 + 3\lambda_1 v_1^2}\right)} \right] \\
\lambda_{3\text{eff}}^{(\text{Imp.})} =& \lambda_3(m_2) \left[\frac{6\lambda_2}{16\pi^2} - \frac{25\lambda_3}{64\pi^2} + \frac{9\lambda_1}{8\pi^2}\left(1 - \frac{\lambda_1}{\lambda_3}\right) \right. \\
& \left. + \frac{1 - \frac{9\lambda_1(m_2)}{8\pi^2}\left(1 - \frac{\lambda_1(m_2)}{\lambda_3(m_2)}\right)}{1 + \frac{9\lambda_1(m_2)}{8\pi^2}\left(1 - \frac{\lambda_1(m_2)}{\lambda_3(m_2)}\right) \log\left(\frac{m_2^2}{m_1^2 + 3\lambda_1 v_1^2}\right)} \right]
\end{aligned}
$$

Heavy mass dependence on VEV

- We study the stationary condition of the improved effective potential,

$$\frac{\partial V_{\text{eff}}^{\text{Improved}}}{\partial v_1^2} = 0.$$

- The solution satisfies,

$$\frac{v_1}{v_{10}} = \sqrt{\left[-\frac{\lambda_3 m_2^2}{32\pi^2 m_1^2} + \frac{1}{\left\{1 - \frac{9\lambda_1}{16\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right\}^{\frac{1}{3}}}\right]\left[1 - \frac{9\lambda_1}{16\pi^2}\log\left(\frac{m_1^2+3\lambda_1 v_1^2}{m_2^2}\right)\right]},$$

$$v_{10} = \sqrt{-\frac{m_1^2}{\lambda_1}}.$$

- We keep the leading logarithmic correction and the correction proportional to the heavy scalar mass squared.

Illustration for heavy mass dependence on VEV

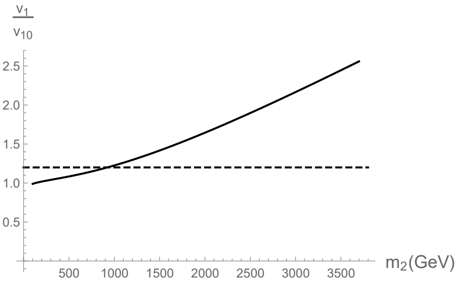

Figure: The vev's ratio as a function of heavy scalar mass m_2. The dashed line corresponds to $\frac{v_1}{v_{10}} = 1.2$. We fix the parameters as $m_1^2 = -(100)^2$(GeV)^2 and $\lambda_1 = \lambda_3 = 1$ which corresponds to $v_{10} = 100$ (GeV).

- The correction to the vev increases as the heavy scalar mass increases.

Illustration for heavy mass dependence on VEV

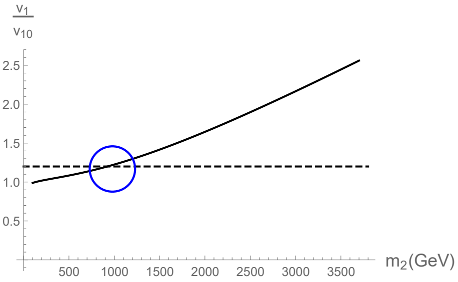

Figure: The vev's ratio as a function of heavy scalar mass m_2. The dashed line corresponds to $\frac{v_1}{v_{10}} = 1.2$. We fix the parameters as $m_1^2 = -(100)^2(\text{GeV})^2$ and $\lambda_1 = \lambda_3 = 1$ which corresponds to $v_{10} = 100$ (GeV).

- The correction to the vev increases as the heavy scalar mass increases.
- If we require the correction to be within 20% compared to the vev without the radiative correction, the upper bound on the heavy scalar mass is about 1000 (GeV).

Contents

Summary and Outlook

- We have constructed a low-energy effective potential by integrating heavy and light scalar particles.
- We found that the effective Yukawa coupling constant for neutrino mass is inversely proportional to the heavy scalar mass squared as $y_{\rm eff} \simeq y\frac{m_{12}^2}{m_2^2}$ and is naturally suppressed.
- The effective coupling constant of the quartic interaction of Higgs boson and the quartic interaction term of neutrino is given up to the dimension six operators.
- As an outlook, we will apply the present analysis in the low-energy region for Davidson and Logan model (Ongoing work). In addition, we will discuss the effects that experiments can verify.

Thank you!