On the Weyl gravity extension of Higgs inflation

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Higgs as a Probe of New Physics (HPNP) 2023

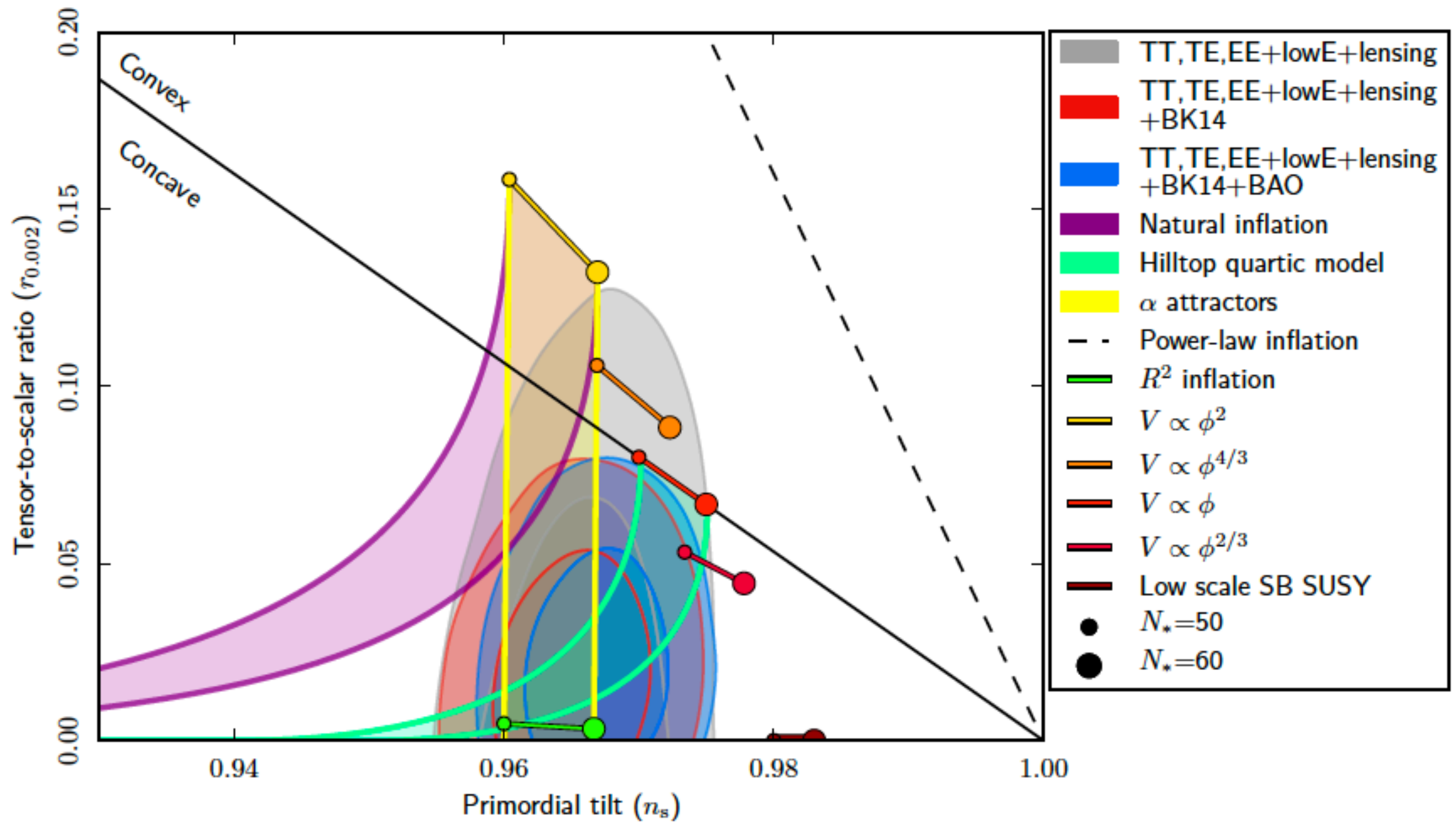
Osaka University, June 9, 2023

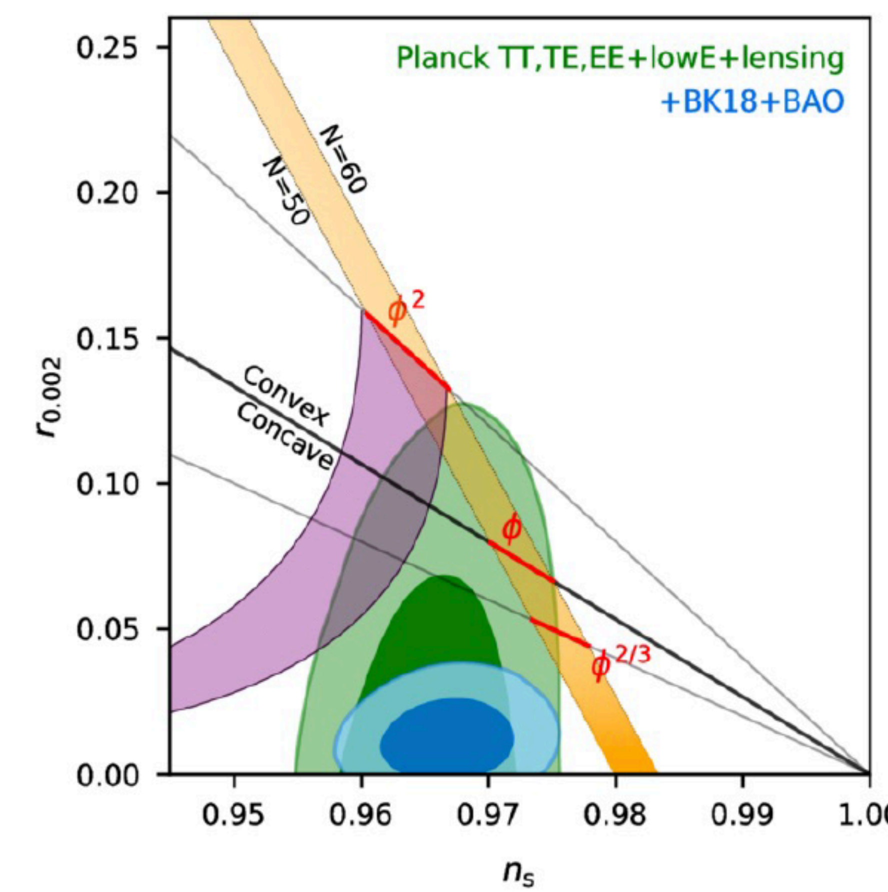
Outline

- Introduction
- Scalar unitarization of Higgs inflation
- Vector unitarization of Higgs inflation
- Conclusions

Introduction

Inflation @ Planck/Bicep





Inflation explains horizon, homogeneity, isotropy, flatness, etc.

Many inflation models ruled out by tensor-to-scalar ratio r:
r<0.035 [Planck18+BK18+BAO]

Forecast for errors $\sigma(r) = 0.009 \Rightarrow 0.003$ Bicep array (2020-)
$\sigma(n_s) = 0.004 \Rightarrow 0.002$ Simon Observatory (2023-)

Higgs/R^2 inflation and reheating dynamics testable soon!

Higgs inflation

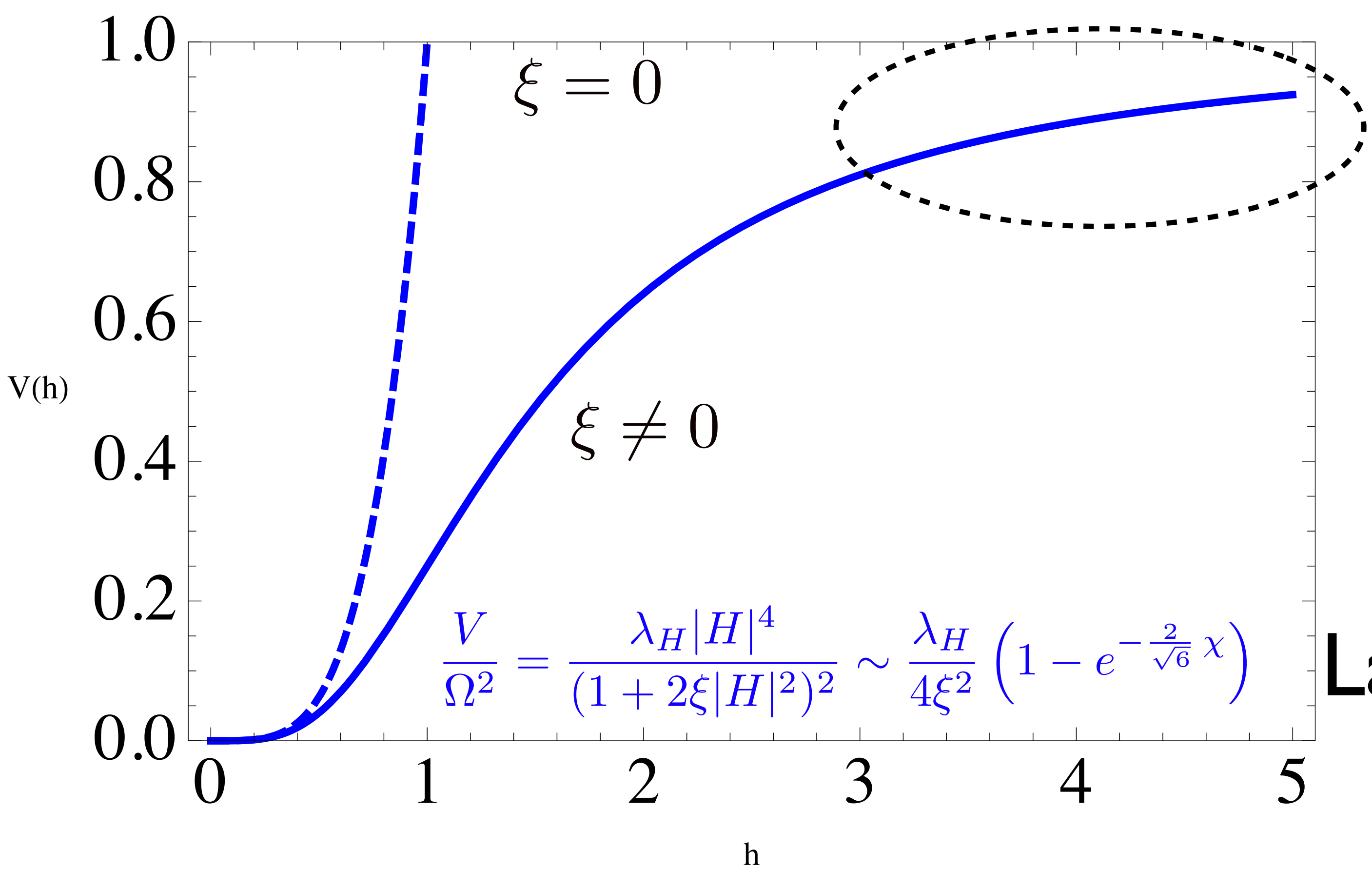
- Einstein gravity + SM Higgs + non-minimal gravity coupling

[Bezrukov, Shaposhnikov (2007)]

$$\mathcal{L} = \sqrt{-g}\Big(\frac{1}{2}\mathcal{R} + \xi|H|^2\mathcal{R} - |D_\mu H|^2 - \lambda_H(|H|^2 - v^2/2)^2\Big)$$

$$\Rightarrow \quad \mathcal{L}_E = \sqrt{-g_E}\Big(\frac{1}{2}\mathcal{R}(g_E) - \frac{3\xi^2}{\Omega^2}(\partial_\mu|H|^2)^2 - \frac{1}{\Omega}|D_\mu H|^2 - \frac{V}{\Omega^2}\Big)$$

$$g_{\mu\nu} = \Omega^{-1} g_{E,\mu\nu}, \quad \Omega = 1 + 2\xi|H|^2$$



Well consistent with CMB data due to flat Higgs potential

$$n_s = 0.966, \quad r = 0.0033$$

But, height of CMB anisotropies

$$\frac{\xi}{\sqrt{\lambda_H}} = 5 \times 10^4$$

Large non-minimal coupling lowers cutoff scale.

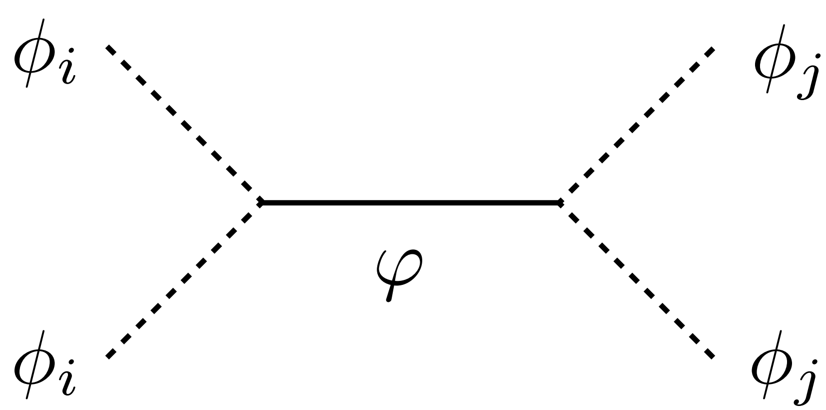
$$E \lesssim \frac{M_P}{\xi}$$

Troubles with large coupling

- Large non-minimal coupling violates perturbative unitarity.

[Burgess, HML, Trott (2009,2010); Barbon, Espinosa (2009); Hertzberg (2010)]

CMB anisotropies: $\frac{\Delta T}{T} \sim 10^{-4}$ $\Rightarrow$ $\frac{\xi}{\sqrt{\lambda_H}} = 5 \times 10^4$



$\mathcal{L}_{\rm int} = 3\xi \phi_i^2 \Box \varphi$ "off-shell" graviton

$\mathcal{M}_{\phi_i\phi_i\to\phi_j\phi_j} \sim \frac{\xi^2 E^2}{M_P^2}$ $\longrightarrow$ $E \lesssim \frac{M_P}{\xi}$

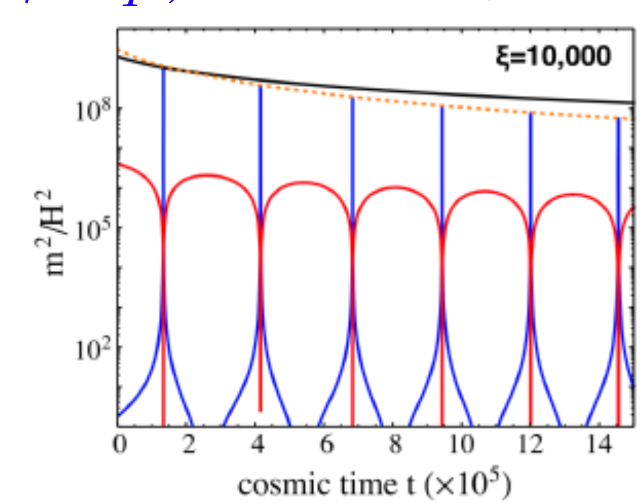
- Unitarity scale is larger during inflation. [F. Bezrukov et al, 2010]

W, Z masses are decoupled; $m_W^2 = \frac{g^2 h^2}{4(1+\xi h^2/M_P^2)} \simeq \frac{1}{4} g^2 \frac{M_P^2}{\xi}.$

- But, preheating violates unitarity.

Particle momenta beyond cutoff scale:

$k_{\rm max} \sim \xi H_{\rm end} \sim \sqrt{\lambda} M_{\rm Pl}$ [e.g. E. Sfakianakis, 2019]



Formulations of Higgs inflation

- Metric formulation: Christoffel symbol solved in terms of Jordan-frame metric, $\Gamma = \Gamma(g_J)$.

[Bezrukov, Shaposhnikov (2007)]

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = \frac{1}{2}(1+\xi_H|H|^2)\mathcal{R}(\Gamma(g_J)) - |D_\mu H|^2 - V(H)$$

$$\rightarrow \frac{\mathcal{L}_E}{\sqrt{-g_E}} = \frac{1}{2}\mathcal{R}(g_E) - \frac{1}{2(1+\xi_H\vec{\phi}^2/M_P^2)}\left(\delta_{ij} + \frac{6\xi_H^2\phi_i\phi_j/M_P^2}{1+\xi_H\vec{\phi}^2/M_P^2}\right)\partial_\mu\phi_i\partial^\mu\phi_j - \frac{V}{(1+\xi_H\vec{\phi}^2/M_P^2)^2}$$

- Palatini formulation: Christoffel symbol solved in terms of Einstein-frame metric, $\Gamma = \Gamma(g_E)$.

[Bauer, Dmir (2008)]

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = \frac{1}{2}(1+\xi_H|H|^2)\mathcal{R}(\Gamma) - |D_\mu H|^2 - V(H)$$

$$\rightarrow \frac{\mathcal{L}_E}{\sqrt{-g_E}} = \frac{1}{2}\mathcal{R}(g_E) - \frac{1}{2(1+\xi_H\vec{\phi}^2/M_P^2)^2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{V}{(1+\xi_H\vec{\phi}^2/M_P^2)^2}$$

Palatini ≠ Metric : discrete choices of constraints in EFT.

Towards UV complete models

- Linearize the non-linear Higgs kinetic term.

$$\mathcal{L}_{\rm kin} = \frac{1}{2(1+\xi_H \vec{\phi}^2/M_P^2)}\left(\delta_{ij} + \frac{6\xi_H^2 \phi_i\phi_j/M_P^2}{1+\xi_H\vec{\phi}^2/M_P^2}\right)\partial_\mu\phi_i\partial^\mu\phi_j$$

$$\longrightarrow \mathcal{L}_{\rm linear} = \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\phi_i)^2$$

"Linear sigma models"
New scalar?

- Cancel the non-linear Higgs kinetic term.

$$\mathcal{L}_{\rm non-linear} = \frac{3\xi_H^2}{M_P^2}\frac{(\partial_\mu|H|^2)^2}{(1+2\xi_H|H|^2/M_P^2)^2}$$

$$\longrightarrow \mathcal{L}_{\rm eff} = \frac{3\xi_H^2}{M_P^2}\frac{(\partial_\mu|H|^2)^2}{(1+2\xi_H|H|^2/M_P^2)^2} - \frac{K_\mu K^\mu}{48M_P^2} = 0$$

"Weyl symmetry" New gauge boson?

Scalar unitarization of Higgs inflation

Higgs inflation in conformal frame

- Original Higgs inflation with non-minimal coupling.

$$\mathcal{L} = \sqrt{-\hat{g}}\Big[-\frac{1}{2}(1+\xi\hat{\phi}_i^2)\hat{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i - \frac{\lambda}{4}(\hat{\phi}_i^2)^2\Big]$$

Conformal mode: $\hat{g}_{\mu\nu} = e^{2\varphi}g_{\mu\nu}$

$\phi_i = e^{\varphi}\hat{\phi}_i,\ \sqrt{6}e^{\varphi} = \phi + \sigma$ [Y. Ema et al (2020)]

- Higgs inflation = non-linear sigma model in disguise:

$$\Rightarrow \mathcal{L} = \sqrt{-g}\Big[-\frac{1}{2}\Big(1-\frac{1}{6}\phi_i^2-\frac{1}{6}\sigma^2\Big)R + \frac{1}{2}(\partial_\mu\phi_i)^2 + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{\lambda}{4}(\phi_i^2)^2\Big]$$

$\phi = \sqrt{6}$

[Giudice, HML (2010)]; Y. Ema et al (2020)]

$$\Big(\sigma+\frac{\sqrt{6}}{2}\Big)^2 + 3\Big(\xi+\frac{1}{6}\Big)\phi_i^2 - \frac{3}{2} = 0$$

Introduce a dynamical field exciting the vacuum.

⇒ Frame-independent UV completion for Higgs inflation.

Higgs-Starobinsky model

- Lagrangian for Starobinsky model + Higgs inflation:

$$\mathcal{L}_{R2} = \sqrt{-\hat{g}}\Big[-\frac{1}{2}(1+\xi\hat{\phi}_i^2)\hat{R} + \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i - \frac{\lambda}{4}(\hat{\phi}_i^2)^2 + \underline{\alpha\hat{R}^2}\Big]$$

Scalar-dual: $\alpha\hat{R}^2 \Longleftrightarrow -2\alpha\hat{\chi}\hat{R} - \alpha\hat{\chi}^2$

$$\frac{\delta S}{\delta\hat{\chi}} = 0: \quad \hat{\chi} = -\hat{R}$$

Scalar-dual Lagrangian:

$$\mathcal{L}_{\hat{\chi}} = \sqrt{-\hat{g}}\Big[-\frac{1}{2}(1+\underline{4\alpha\hat{\chi}}+\xi\hat{\phi}_i^2)\hat{R} + \frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i \underline{- \frac{\lambda}{4}(\hat{\phi}_i^2)^2 - \alpha\hat{\chi}^2}\Big]$$

Extra non-minimal coupling

Extended scalar potential

Unitarity in Jordan frame

- R^2 term and perturbative unitarity in Jordan frame.

Conformal mode in flat metric: $\delta g_{\mu\nu} = 2\varphi\eta_{\mu\nu}$

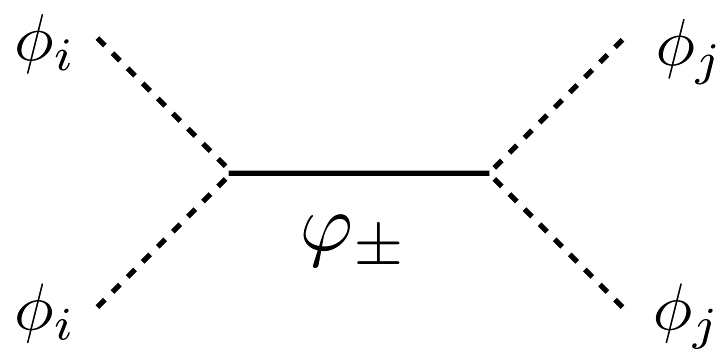
Kinetic mixing between conformal mode & scalaron:

$$\mathcal{L} = 3(\varphi\Box\varphi + \underline{4\alpha\hat{\chi}\Box\varphi}) - \alpha\hat{\chi}^2 + 3\xi\hat{\phi}_i^2\Box\varphi$$

$$\alpha \gtrsim 1: \quad \varphi_\pm \simeq \frac{1}{\sqrt{2}}(\varphi \mp \hat{\chi}), \quad \varphi_\pm \to \frac{1}{\sqrt{12\alpha}}\varphi_\pm \text{ (rescaling)}$$

$$\Rightarrow \quad \mathcal{L} \simeq \frac{1}{2}(\partial_\mu\varphi_+)^2 - \frac{1}{2}(\partial_\mu\varphi_-)^2 + \underline{\frac{3\xi}{2\sqrt{6\alpha}}\hat{\phi}_i^2\Box(\varphi_+ + \varphi_-)} - \frac{1}{24}(\varphi_+ - \varphi_-)^2$$

Higgs-scalar interactions



$$\Rightarrow \quad \frac{\xi}{\sqrt{\alpha}} \lesssim 1: \text{ Perturbative unitarity!}$$

Individually large $\xi,\ \alpha$

Unitarity in conformal frame

- Perturbative unitarity in conformal frame.

Conformal transform: $\hat{g}_{\mu\nu} = \Omega^{-2} g_{\mu\nu}\ , \quad \hat{\phi}_i = \Omega\phi_i$ and $\hat{\chi} = \Omega^2\chi$

[Y. Ema et al (2020)] $\Omega^{-2} = \left(1 + \frac{\sigma}{\sqrt{6}}\right)^2$

Linear sigma model with scalar σ:

$$\frac{\mathcal{L}_{R2}}{\sqrt{-g}} = -\frac{1}{2}R\left(1 - \frac{1}{6}\phi_i^2 - \frac{1}{6}\sigma^2\right) + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\phi_i)^2 - \alpha\chi^2 - \frac{\lambda}{4}\phi_i^4$$

Redefined scalaron: $4\alpha\chi = \frac{1}{2} - \frac{1}{3}\left(\sigma + \frac{\sqrt{6}}{2}\right)^2 - \left(\xi + \frac{1}{6}\right)\phi_i^2$

→ Extra scalar potential: $\alpha\chi^2 = \frac{1}{4}\kappa\left(\sigma(\sigma + \sqrt{6}) + 3\left(\xi + \frac{1}{6}\right)\phi_i^2\right)^2, \quad \kappa = \frac{1}{36\alpha}$

$\kappa = \frac{1}{36\alpha} \lesssim \mathcal{O}(1), \ \ \lambda_{\rm eff} = \lambda + 9\kappa\left(\xi + \frac{1}{6}\right)^2 \lesssim \mathcal{O}(1)$ **Perturbativity!**

Higher curvature terms

- General higher curvature terms + Higgs inflation

$$\sum_k \frac{2(-1)^{k+1}\alpha_k}{k+1}\hat{R}^{k+1} \Longleftrightarrow -2\sum_k \alpha_k \hat{\chi}_k \hat{R} \; -\sum_k 2\left(\frac{k}{k+1}\right)\alpha_k \hat{\chi}_k^{\frac{k+1}{k}}$$

One Lagrange multiplier per each curvature term

Conformal transform: $\hat{g}_{\mu\nu} = \Omega^{-2} g_{\mu\nu}$, $\hat{\phi}_i = \Omega\phi_i$ and $\hat{\chi}_k = \Omega^2 \chi_k$

General sigma-model Lagrangian: [HML, Menkara (2021)]

$$\frac{\mathcal{L}_{\rm gen}}{\sqrt{-g}} = -\frac{1}{2}R\left(1-\frac{1}{6}\phi_i^2-\frac{1}{6}\sigma^2\right)+\frac{1}{2}(\partial_\mu\sigma)^2+\frac{1}{2}(\partial_\mu\phi_i)^2-\frac{\lambda}{4}\phi_i^4 - \sum_k \Omega^{-2+\frac{2}{k}}\left(\frac{2k}{k+1}\right)\alpha_k \chi_k^{1+\frac{1}{k}}$$

Linearized kinetic terms — dual-scalar potential

$$+\, y(x) \cdot \left[\sum_k 4\alpha_k\chi_k - \frac{1}{2}+\frac{1}{3}\left(\sigma+\frac{\sqrt{6}}{2}\right)^2+\left(\xi+\frac{1}{6}\right)\phi_i^2\right]$$

Lagrange multipler — Constraint on vacuum manifold

F(R) on vacuum manifold

- Solutions for dual scalars and Lagrange multipliers:

$$\frac{\delta \mathcal{L}_{\rm gen}}{\delta \chi_k} = 0 \quad \Rightarrow \quad \chi_k = 2^k \Omega^{2k-2}\, y^k\,, \quad k = 1, 2, \cdots, N$$

$$\frac{\delta \mathcal{L}_{\rm gen}}{\delta y} = 0 \quad \Rightarrow \quad \sum_k 4\alpha_k \chi_k = \frac{1}{2} - \frac{1}{3}\left(\sigma + \frac{\sqrt{6}}{2}\right)^2 - \left(\xi + \frac{1}{6}\right)\phi_i^2$$

$$= \sum_k 4\alpha_k\, 2^k \Omega^{2k-2}\, y^k$$

[HML, Menkara (2021)]

y : the solution to the N-th order algebraic equation.

- Effective sigma-model potential with Higgs couplings:

$$U(\sigma, \phi_i) = \sum_k \Omega^{-2+\frac{2}{k}} \left(\frac{2k}{k+1}\right) \alpha_k\, \chi_k^{1+\frac{1}{k}} = \sum_k \left(\frac{2^{k+2} k}{k+1}\right) \alpha_k (\Omega(\sigma))^{2k-2} (y(\sigma, \phi_i))^{k+1}$$

$$\frac{\partial U}{\partial \sigma} = 0 \quad \Rightarrow \quad y = 0 : \quad \left(\sigma + \frac{\sqrt{6}}{2}\right)^2 + 3\left(\xi + \frac{1}{6}\right)\phi_i^2 - \frac{3}{2} = 0$$

UV complete perturbations of Starobinsky model.

Higgs-Starobinsky inflation

- Einstein frame Lagrangian for Higgs-Starobinsky:

$$\mathcal{L}_E = \sqrt{-g_E}\Big\{ -\frac{1}{2}R(g_E) + \frac{1}{2\Omega'^4}\Big[\Big(1-\frac{1}{6}\sigma^2\Big)(\partial_\mu h)^2 + \Big(1-\frac{1}{6}h^2\Big)(\partial_\mu\sigma)^2 + \frac{1}{3}h\,\sigma\,\partial_\mu h\partial^\mu\sigma\Big] - V(\sigma,h)\Big\}$$

$$V(\sigma,h) = \frac{1}{(1-\frac{1}{6}h^2-\frac{1}{6}\sigma^2)^2}\Big[\frac{1}{4}\kappa_1\Big(\sigma(\sigma+\sqrt{6})+3\Big(\xi+\frac{1}{6}\Big)h^2\Big)^2 + \frac{1}{4}\lambda h^4\Big]$$

- Effective theory for inflation [HML, A. Menkara (2021)]

Decoupled Higgs direction:

$$\frac{\partial V}{\partial h} = 0 \qquad \text{i.e.} \qquad h^2 = \frac{\kappa_1\sigma(\sigma+\sqrt{6})(\sigma-3(\xi+\frac{1}{6})(\sigma-\sqrt{6}))}{\lambda(\sigma-\sqrt{6})-3\kappa_1(\xi+\frac{1}{6})(\sigma-3(\xi+\frac{1}{6})(\sigma-\sqrt{6}))}$$

$$\Rightarrow \quad \frac{\mathcal{L}_{\rm eff}}{\sqrt{-g_E}} = -\frac{1}{2}R(g_E) + \underline{\frac{(\partial_\mu\sigma)^2}{2(1-\sigma^2/6)^2} - V_{\rm eff}(\sigma)},$$

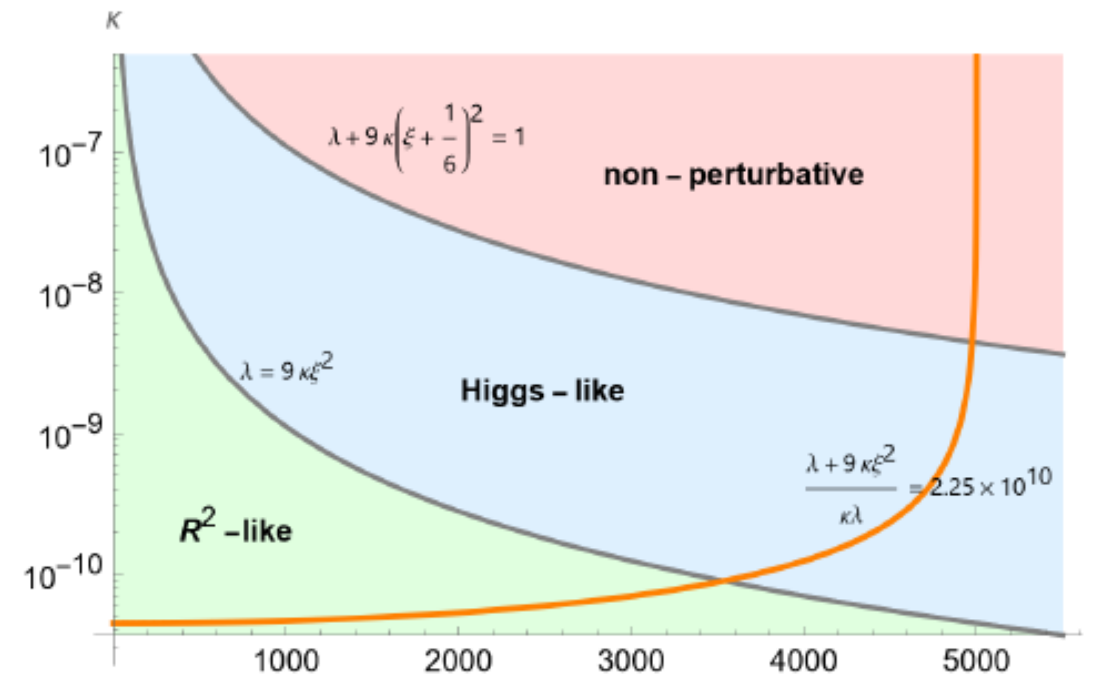
$$V_{\rm eff}(\sigma) = 9\lambda\,\kappa_1\sigma^2\Big[\lambda(\sigma-\sqrt{6})^2 + \kappa_1\Big(\sigma-3\Big(\xi+\frac{1}{6}\Big)(\sigma-\sqrt{6})\Big)^2\Big]^{-1}$$

Limits of UV Higgs inflation

- Perturbative Higgs inflation

$$\sigma = -\sqrt{6}\tanh\left(\frac{\chi}{\sqrt{6}}\right)$$

$$V \approx \begin{cases} \frac{9\kappa_1}{4}\left(1-e^{-2\chi/\sqrt{6}}\right)^2, & 9\kappa_1\xi^2 \ll \lambda, \\ \frac{\lambda}{4\xi^2}\left(1-e^{-2\chi/\sqrt{6}}\right)^2, & 9\kappa_1\xi^2 \gg \lambda. \end{cases}$$



CMB normalization: $\frac{\sqrt{\lambda + 9\kappa_1\xi^2}}{\sqrt{\kappa_1 \lambda}} = 1.5\times 10^5$

- Similar predictions in Higgs or Starobinsky models.

Spectral index: $n_s = 1 - \frac{2}{N} - \frac{9}{2N^2} + \frac{3\kappa_1}{N^2}\frac{(-\lambda + 12\lambda\xi + 18\kappa_1\xi^2(1+6\xi))}{(2\lambda + 3\kappa_1\xi(1+6\xi))^2},$

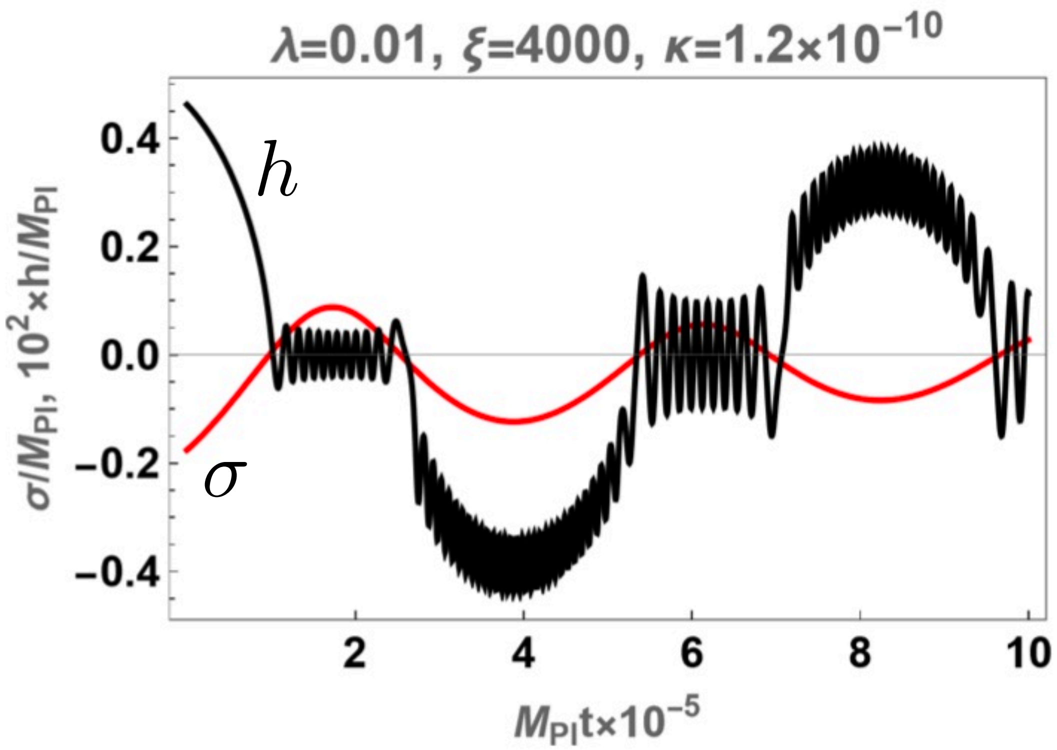
Tensor-to-scalar: $r = \frac{12}{N^2}$

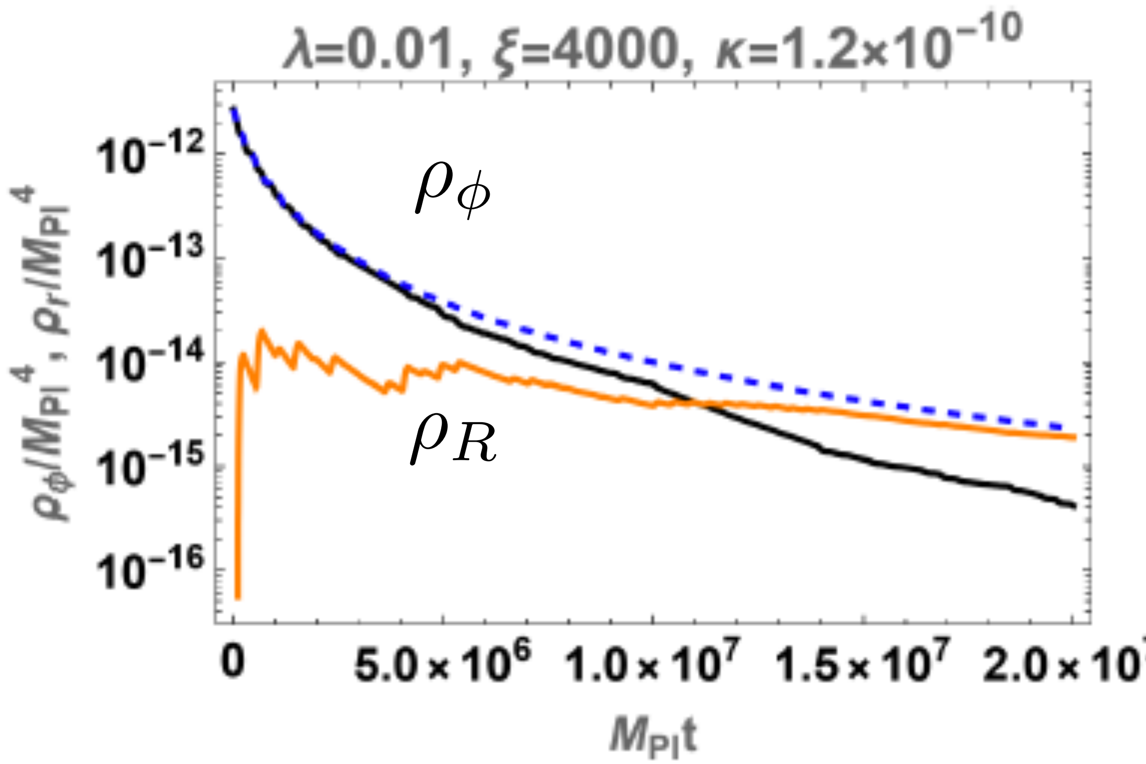
small corrections to Higgs inflation $(\kappa_1\xi^2 \lesssim 1)$

Reheating: if Higgs-like, $T_{\rm RH} \sim 10^{14}\,{\rm GeV}$ [S.Aoki, HML et al (2022)]

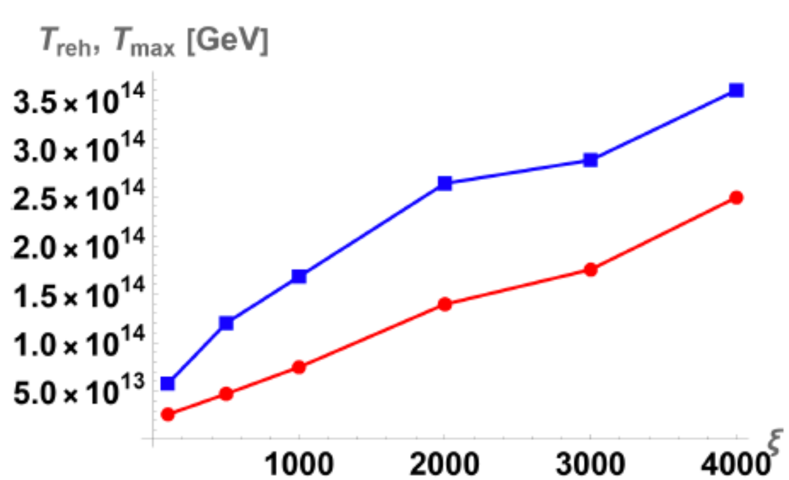
Reheating and dark matter

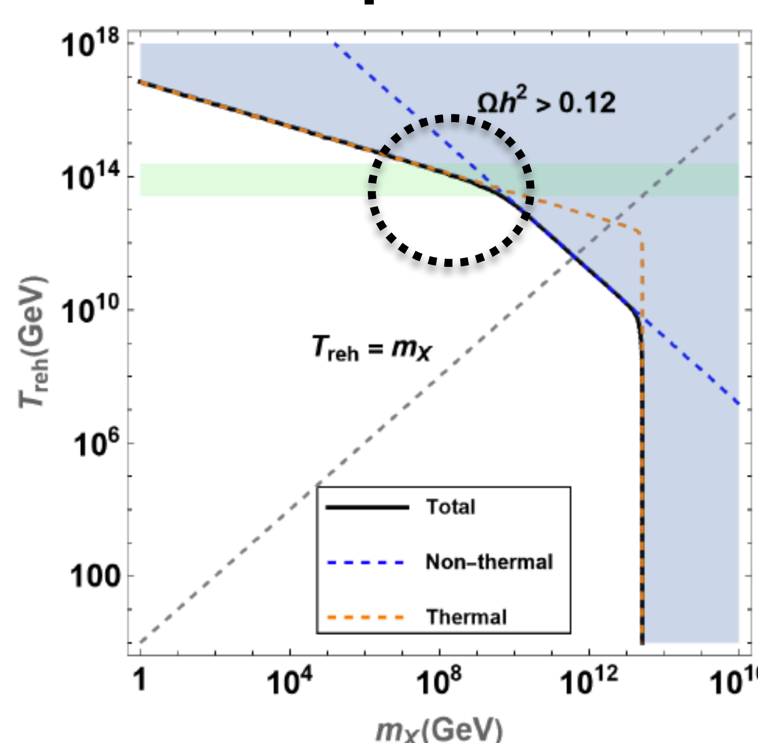
- Two inflaton oscillations quasi-decoupled for large ξ





- Reheating & dark matter production by sigma/Higgs fields





Higgs condensate decay:

$T_{\rm RH} \sim 10^{14}\,{\rm GeV}$

Inflaton/thermal scattering:

Production of conformally coupled heavy scalar DM $\sim 10^8$ GeV

Vector unitarization of Higgs inflation

Unitarizing with gauge field

- Non-canonical Higgs kinetic terms in Higgs inflation are of Weyl current-current type.

$$\frac{\mathcal{L}_{H,\rm eff}}{\sqrt{-g_E}} = \frac{3\xi_H^2}{M_P^2}\frac{(\partial_\mu |H|^2)^2}{\Omega^2} = \frac{1}{48M_P^2}\frac{K_\mu K^\mu}{\Omega^2}, \qquad K_\mu = \partial_\mu K_H \text{ with } K_H = 12\xi_H |H|^2.$$

- Introduce a Weyl gauge field in Jordan frame.

$$\frac{\mathcal{L}_J}{\sqrt{-g}} = -\frac{1}{4}w_{\mu\nu}w^{\mu\nu} + \frac{1}{2}m_w^2 w_\mu w^\mu - \frac{1}{2}g_w w_\mu K^\mu + \frac{1}{2}g_w^2 w_\mu w^\mu K_H, \qquad m_w^2 = 6g_w^2 M_P^2$$

Integrate out the Weyl gauge field by $w_\mu = \frac{g_w}{2}\frac{K_\mu}{m_w^2 + g_w^2 K_H}$

$$\Rightarrow \frac{\mathcal{L}_{J,\rm eff}}{\sqrt{-g}} = -\frac{1}{48M_P^2}\frac{K_\mu K^\mu}{\Omega} \Rightarrow \boxed{\frac{\mathcal{L}_{E,\rm eff}}{\sqrt{-g_E}} = -\frac{1}{48M_P^2}\frac{K_\mu K^\mu}{\Omega^2}}$$

Effective interaction in Einstein frame cancels non-canonical Higgs kinetic terms!

Higgs inflation + Weyl photon

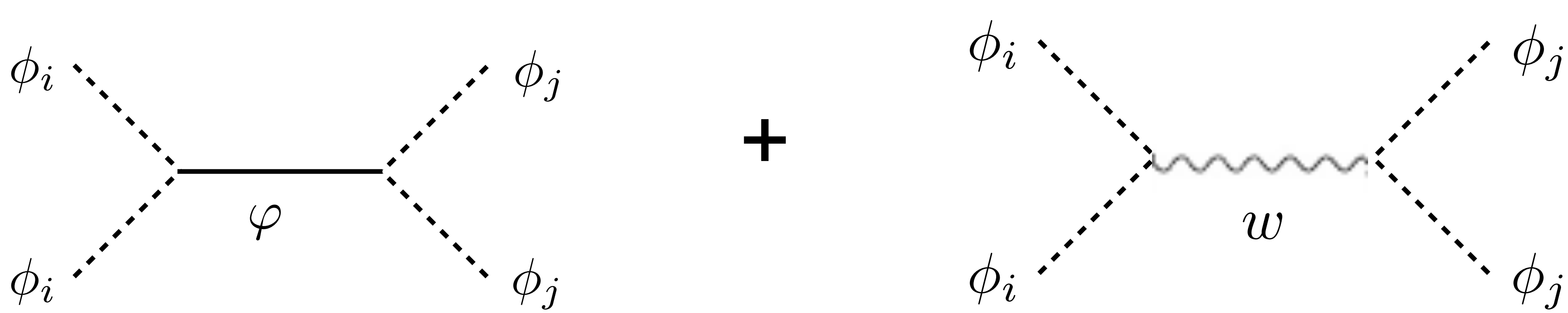
- Minimal extension of Higgs inflation in Weyl gravity:

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}(M_P^2 + 2\xi_H|H|^2)R + |D_\mu H|^2 - V(H)$$
$$-\frac{1}{4}w_{\mu\nu}w^{\mu\nu} + \frac{1}{2}m_w^2 w_\mu w^\mu - \frac{1}{2}g_w w_\mu K^\mu + \frac{1}{2}g_w^2 w_\mu w^\mu K_H$$

Local Weyl symmetry

$$\frac{\mathcal{L}_{\rm min}}{\sqrt{-g}} = -\frac{1}{2}(\underline{\xi_\phi\phi^2} + 2\xi_H|H|^2)\underline{\tilde{R}(\tilde{\Gamma})} - \frac{1}{4}w_{\mu\nu}w^{\mu\nu} - V(H,\phi)$$
$$\underline{+|D_\mu H|^2}$$

Weyl photon in Weyl gravity restores unitarity.



More on Weyl gravity

- Weyl geometry

$\tilde{\nabla}_\rho g_{\mu\nu} = 0$, in Weyl gravity, $\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + g_w\Big(\delta^\rho_\mu w_\nu + \delta^\rho_\nu w_\mu - g_{\mu\nu} w^\rho\Big)$

$\longrightarrow$ $\nabla_\rho g_{\mu\nu} = 2 g_w \omega_\rho g_{\mu\nu}$ in Einstein gravity.

Weyl transfs: $g_{\mu\nu} \to e^{2\alpha} g_{\mu\nu}, \quad \phi \to e^{-\alpha}\phi, \quad H \to e^{-\alpha} H, \quad w_\mu \to w_\mu - \frac{1}{g_w}\partial_\mu \alpha$

- General Weyl invariant Lagrangian

$$\frac{\mathcal{L}_{\rm gen}}{\sqrt{-g}} = -\frac{1}{2}(\xi_\phi \phi^2 + 2\xi_H |H|^2)\tilde{R}(\tilde{\Gamma}) + 3\xi_\phi (r_\phi - 1)(D_\mu \phi)^2 + 6\xi_H (r_H - 1)|D_\mu H|^2 - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} - V(H,\phi)$$

[D. Ghilencea, H. M. Lee (2018)]

Covariant derivatives: $D_\mu H = (\partial_\mu - g_w w_\mu) H$, $D_\mu \phi = (\partial_\mu - g_w w_\mu)\phi$

Weyl invariant potential: $V(H,\phi) = \frac{1}{2}\lambda_{\phi H}\phi^2 |H|^2 + \frac{1}{4}\lambda_\phi \phi^4 + \lambda_H |H|^4$

Canonical Higgs kinetic term: $r_H = 1 + \frac{1}{6\xi_H}$

Spontaneous scale generation

- Spontaneous breaking of scale symmetry: $\langle\phi^2\rangle = M_P^2/\xi_\phi$

$$\frac{\mathcal{L}_{\rm gen,\,fix}}{\sqrt{-g}} = -\frac{1}{2}(M_P^2+2\xi_H|H|^2)R+|\partial_\mu H|^2-V(H)$$

[S. Aoki, H. M. Lee (2022)]

$$-\frac{1}{4}w_{\mu\nu}w^{\mu\nu}+\frac{1}{2}m_w^2 w_\mu w^\mu-\frac{1}{2}g_w w_\mu K^\mu+\frac{1}{2}g_w^2 w_\mu w^\mu K_H$$

"wanted terms kept for unitarity"

$m_w^2 = 6r_\phi g_w^2 M_P^2$, $\quad K_H = 12 r_H \xi_H |H|^2$: general Weyl couplings

- Dimensionless couplings of hierarchy:

$$V(H) = \frac{\lambda_\phi M_P^4}{4\xi_\phi^2}+\frac{\lambda_{\phi H}M_P^2}{2\xi_\phi}|H|^2+\lambda_H|H|^2$$

$$\begin{cases} \dfrac{|m_H^2|}{M_P^2} = \dfrac{|\lambda_{\phi H}|}{2\xi_\phi} \ll 1 & \longrightarrow \text{Small Higgs mass} \\ \dfrac{\Lambda}{M_P^4} = \dfrac{\lambda_\phi}{4\xi_\phi^2} \ll 1 & \longrightarrow \text{Small cosmological constant} \end{cases}$$

Higgs inflation in Weyl gravity

- Higgs kinetic terms in Einstein frame

$$\frac{\mathcal{L}_{\rm kin}}{\sqrt{-g_E}} = \underbrace{6\xi_H(r_H-1)}_{=1}\frac{|\partial_\mu H|^2}{\Omega} + \underline{\frac{3\xi_H^2}{M_P^2}\frac{(\partial_\mu|H|^2)^2}{\Omega^2}} - \underline{\frac{g_w^2 r_H^2}{8\Omega}\frac{K_\mu K^\mu}{m_w^2+g_w^2 r_H K_H}}$$

Non-minimal coupling — Weyl currents

$$\frac{\mathcal{L}_{\rm kin}}{\sqrt{-g_E}} = \frac{1}{\Omega}|\partial_\mu H|^2 + \frac{1}{M_P^2\Omega^2}\frac{3r_\phi\xi_H^2 - 3(\xi_H+\frac{1}{6})^2 - \xi_H(\xi_H+\frac{1}{6})|H|^2/M_P^2}{r_\phi + 2(\xi_H+\frac{1}{6})|H|^2/M_P^2}(\partial_\mu|H|^2)^2$$

$$= |\partial_\mu H|^2 + \frac{1}{\Lambda^2}(\partial_\mu|H|^2)^2 + \cdots$$

[S. Aoki, H. M. Lee (2022)]

Cutoff-scale: $\Lambda = \frac{M_P}{\left|\xi_H(3\xi_H+1)(1-\frac{1}{r_\phi}) - \frac{1}{12r_\phi}\right|^{1/2}}$ $\xrightarrow{r_\phi=1}$ $\Lambda \sim M_P$ insensitive to ξ_H

But, "Hilltop-like" inflation is possible only for very small ξ_H, Λ_H.

$$V_E = \frac{\lambda_H M_P^4}{4\xi_H^2}\Big(1+\frac{M_P^2}{\xi_H h^2}\Big)^{-2} \simeq \frac{\lambda_H M_P^4}{4\xi_H^2}\Big(1+\frac{\xi_H\chi^2}{M_P^2}\Big)^{-2}$$

χ : canonical inflaton

[D. Ghilencea, H. M. Lee (2018)]

Double Weyl gravity

- Doubled diffeomorphism and two Weyl gauge fields:

$$\mathcal{L} = \sum_{i=1,2} \sqrt{-g_i}\Big[-\frac{1}{2}\xi_i\phi_i^2 \tilde{R}(\tilde{\Gamma}_i) - \frac{1}{4} w_{i,\mu\nu} w_i^{\mu\nu}\Big] + \Delta\mathcal{L}\,,$$

Weyl symmetry: $g_{i,\mu\nu} \to e^{2\alpha_i} g_{i,\mu\nu}$, $\phi_i \to e^{-\alpha_i}\phi_i$, $w_{i,\mu} \to w_{i,\mu} - \frac{1}{g_{w_i}}\partial_\mu \alpha_i$

Broken diffeom: $\Delta\mathcal{L} = \sum_{i=1,2} \sqrt{-g_i}\,(-3\xi_i a_i \phi_i^2)\Big(g_{w_1} w_{1,\mu} + \kappa_i g_{w_2} w_{2,\mu}\Big)^2$

- Effective Weyl gravity: $g_{1,\mu\nu} = g_{2,\mu\nu} \equiv g_{\mu\nu}\,,\quad \phi_1 = \phi_2 = \phi$

$$\mathcal{L}_{\rm eff} = \sqrt{-g}\Big[-\frac{1}{2}\xi_\phi \phi^2 \tilde{R}(\tilde{\Gamma}) - \frac{1}{4} w_{\mu\nu} w^{\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu}\Big]$$

[S. Aoki, H. M. Lee(2022)]

$$\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + g_w\Big(\delta^\rho_\mu w_\nu + \delta^\rho_\nu w_\mu - g_{\mu\nu} w^\rho\Big), \quad g_w = \tfrac{1}{2}\sqrt{g_{w_1}^2 + g_{w_2}^2},$$

$$w_\mu = (g_{w_1} w_{1,\mu} + g_{w_2} w_{2,\mu})/\sqrt{g_{w_1}^2 + g_{w_2}^2}\,,$$ X_μ : orthogonal combination.

⇒ Choose extra Weyl photon X, decoupled from dilaton.

Palatini limit of Higgs inflation

- Effects of the light Weyl photon X: $(r_\phi = r_H = 1)$

$$\frac{\mathcal{L}_2}{\sqrt{-g}} = -\frac{1}{2}(\xi_\phi \phi^2 + 2\xi_H |H|^2)\tilde{R}(\tilde{\Gamma}) - \frac{1}{4}w_{\mu\nu}w^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \underline{|D'_\mu H|^2} - V(H,\phi)$$

Effective Weyl symmetry: $g_{\mu\nu} \to e^{2\alpha} g_{\mu\nu}, \quad \phi \to e^{-\alpha}\phi, \quad H \to e^{-\alpha} H,$

$$D'_\mu H = (\partial_\mu - g_X X_\mu) H \qquad w_\mu \to w_\mu - \frac{1}{g_w}\partial_\mu \alpha, \quad X_\mu \to X_\mu - \frac{1}{g_X}\partial_\mu \alpha$$

Add the X-covariant derivative term in Higgs-Weyl gravity:

$$\frac{\mathcal{L}_2}{\sqrt{-g}} = -\frac{1}{2}(M_P^2 + 2\xi_H |H|^2)R + \underline{|D'_\mu H|^2} - V(H) - \frac{1}{4}X_{\mu\nu}X^{\mu\nu}$$
$$-\frac{1}{4}w_{\mu\nu}w^{\mu\nu} + \frac{1}{2}m_w^2 w_\mu w^\mu - \frac{1}{2}g_w w_\mu K^\mu + \frac{1}{2}g_w^2 w_\mu w^\mu K_H$$

➡ Effective Lagrangian: $\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{M_P^2}{2}R + \underline{\frac{|D'_\mu H|^2}{\Omega} - \frac{V(H)}{\Omega^2}} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu}$

~ Palatini formulation, but differ by light Weyl photon.

Metric to Palatini Higgs inflation

- General kinetic terms with two Weyl photons:

$$\frac{\mathcal{L}_G}{\sqrt{-g}} = -\frac{1}{2}(\xi_\phi \phi^2 + \xi_H |H|^2)\tilde{R} + \underline{3\xi_\phi(r_\phi - 1)(D_\mu \phi)^2 + 6\xi_H(r_H - 1)|D_\mu H|^2} \\ -\frac{1}{4}w_{\mu\nu}w^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \underline{(1 - 6\xi_H(r_H - 1))|D'_\mu H|^2} - V(H,\phi).$$

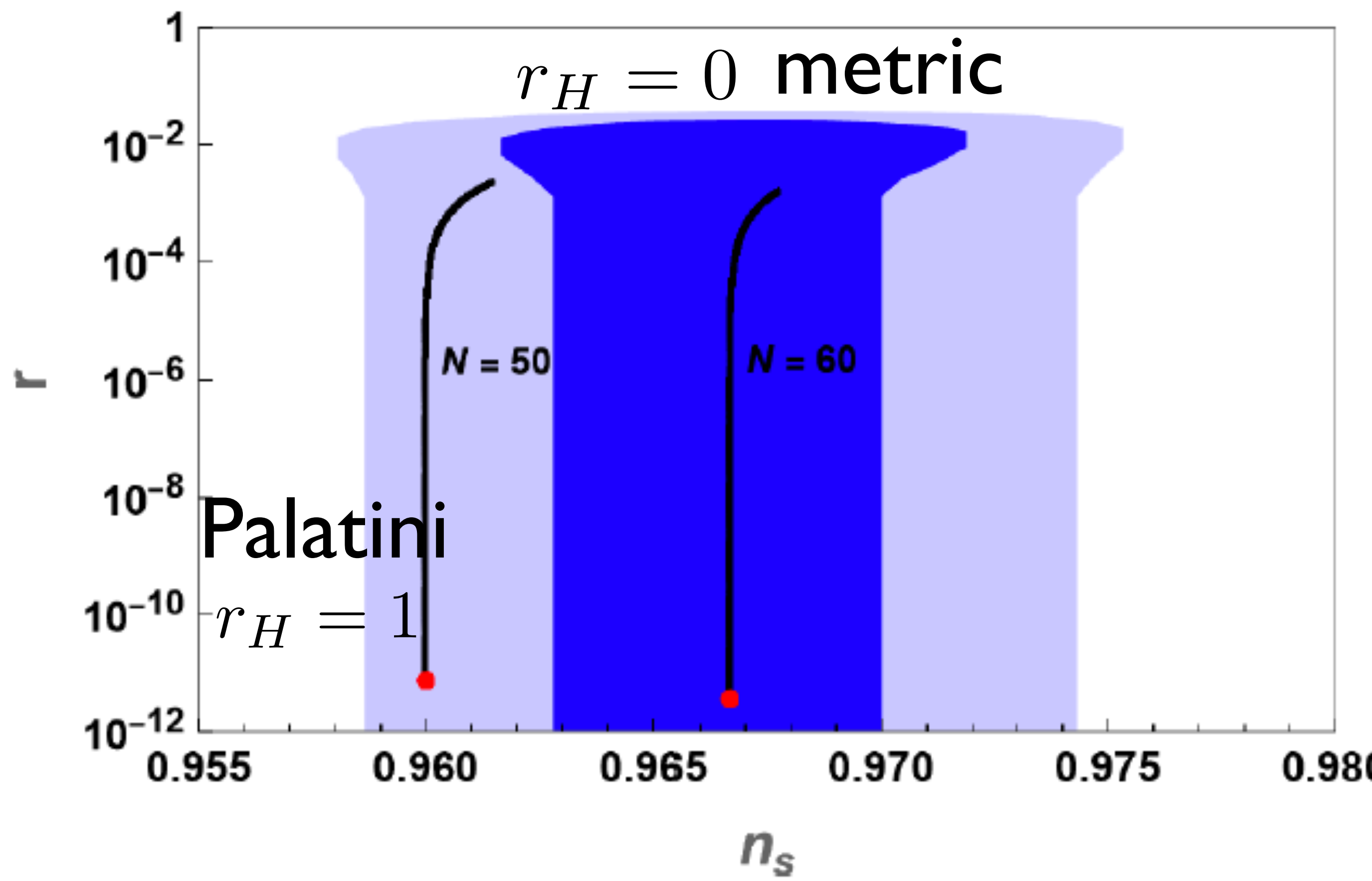
$r_\phi = r_H$

$$\frac{\mathcal{L}_E}{\sqrt{-g_E}} = -\frac{M_P^2}{2}R + \underline{6\xi_H(r_H - 1)\frac{|\partial_\mu H|^2}{\Omega} + (1 - 6\xi_H(r_H - 1))\frac{|D'_\mu H|^2}{\Omega}} \\ + \underline{\frac{3\xi_H^2(1 - r_H)}{\Omega^2 M_P^2}(\partial_\mu |H|^2)^2} - \frac{V(H)}{\Omega^2} \quad -\frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{4}\tilde{w}_{\mu\nu}\tilde{w}^{\mu\nu} + \frac{1}{2}m_w^2 \tilde{w}_\mu \tilde{w}^\mu$$

- Higgs inflation varies between metric and Palatini cases for $0 < r_\phi = r_H < 1$

Cutoff scale varies from M_P/ξ_H to M_P.

$$\Lambda_1 = \frac{M_P}{\left|3\xi_H^2\left(1 - \frac{r_H^2}{r_\phi}\right) + \xi_H\right|^{1/2}}$$



Higgs mechanism of Weyl photon

- "X" Weyl photon is light, if it couples only to SM Higgs and light singlet "s":

$$\frac{\Delta\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}(D'_\mu s)^2 - \lambda_H\big(|H|^2 + \tfrac{1}{2a_H}s^2\big)^2 + \underbrace{(1-6\xi_H(r_H-1))}_{a_H}\frac{|D'_\mu H|^2}{\Omega}$$

Weyl photon interactions:
$$\mathcal{L}_{X,\mathrm{int}} = a_H\Big(-g_X X_\mu \partial^\mu |H|^2 + g_X^2 X_\mu X^\mu |H|^2\Big) - \frac{1}{2}g_X X_\mu \partial^\mu s^2 + \frac{1}{2}g_X^2 X_\mu X^\mu s^2,$$

$$H = (0, v+h)^T/\sqrt{2}, \quad s = v_s + \tilde{s}.$$

→ Weyl photon mass: $m_X^2 = g_X^2(a_H v^2 + v_s^2)$

- Higgs-singlet scalar mixing is necessary.

$$\begin{pmatrix} h \\ \tilde{s} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} h_{\rm SM} \\ G_X \end{pmatrix}$$

$$G_X \sim a_H v h + v_s \tilde{s}$$

Higgs precision at 10(1)%

→ $\sin\theta \simeq (a_H v)/v_s \lesssim 0.3(0.03)$

Interactions of Weyl photon

- Higgs and X-Goldstone interactions:

$$\mathcal{L}_{X,\rm int} = -a_H g_X X_\mu h \partial^\mu h + \frac{1}{2} a_H g_X^2 X_\mu X^\mu (h^2 + 2vh) \\ -g_X X_\mu \tilde{s} \partial^\mu \tilde{s} + \frac{1}{2} g_X^2 X_\mu X^\mu (\tilde{s}^2 + 2v_s \tilde{s})$$

Weyl photons interact with a pair of SM Higgs bosons.

- \+ Gauge kinetic mixing $\mathcal{L}_{\rm gmix} = -\frac{1}{2} \sin\xi \, X_{\mu\nu} B^{\mu\nu}$

$$\longrightarrow \quad \mathcal{L}_{\rm EM/NC} \simeq e\tilde{A}_\mu J^\mu_{\rm EM} + \tilde{Z}_\mu \left[\frac{e}{2 s_W c_W} J^\mu_Z + \underline{\varepsilon g_X t_W J^\mu_X} \right] + \tilde{X}_\mu \left[g_X J^\mu_X - \underline{e\varepsilon J^\mu_{\rm EM}} \right]$$

{ Z-boson couplings to Weyl current $\quad J^\mu_X = -a_H \partial_\mu |H|^2 + \frac{1}{2} \partial_\mu s^2$

Weyl photon couplings to EM current

Conclusions

- Higgs inflation is a non-linear sigma model or described by the effective Weyl current-current interaction in Einstein gravity.

- Non-linear Higgs kinetic terms can be linearized with a massive singlet scalar field in linear sigma models or can be cancelled by a massive Weyl photon.

- We considered a Weyl gravity theory for Higgs inflation with one or two Weyl photons. There are new testable predictions for inflation and/or Higgs physics with light Weyl photon.