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HPNP2023

"Higgs as a Probe of New Physics 2023"

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Constructing all the matrix elements of covariant tensor currents of massless particles in the covariant formulation

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Jaehoon Jeong

jeong229@kias.re.kr
jaehoonjeong229@gmail.com

KIAS
KOREA INSTITUTE FOR ADVANCED STUDY

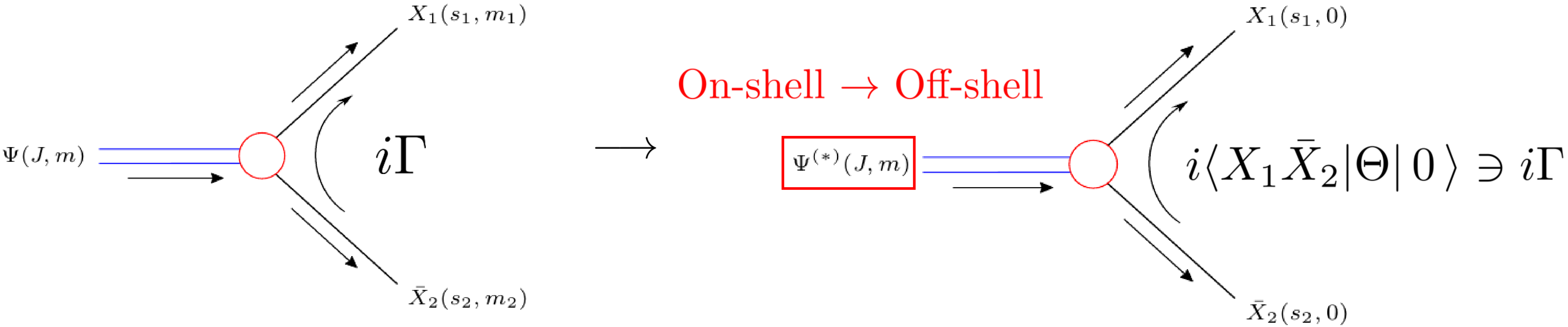
Motivation

Unsolved problems in Standard Model: Dark matter, Neutrino oscillation, Matter antimatter asymmetry, $\cdots$

One powerful strategy for new physics search
in particle physics

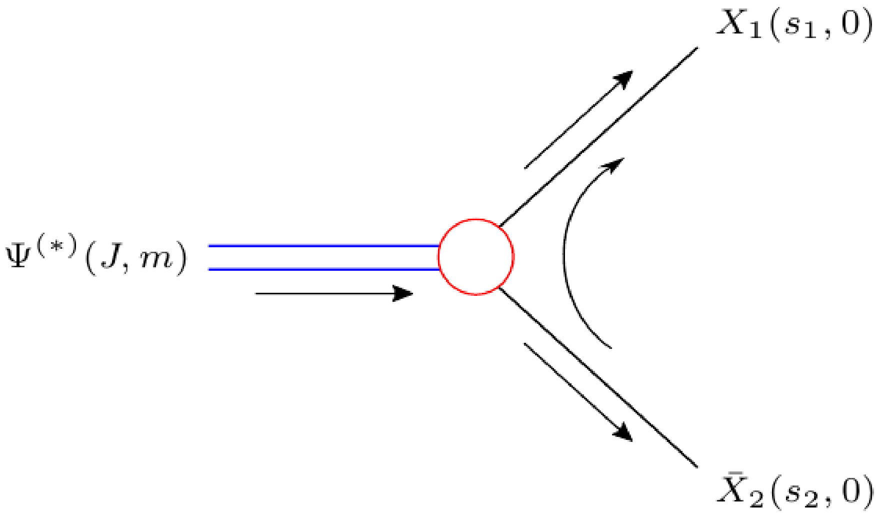
Comprehensive stuties for all the allowed effective interactions
of particles including high-spins in a model-independent way

The first extention of [S. Y. Choi and J. H. Jeong, Phys. Rev. D (2022)]



Outline

1. Decay matrix elements $\langle X_1 \bar{X}_2 | \Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J} | 0 \rangle$



$\longrightarrow$ Algorithm

Integer $s_{1,2}$

2. Summary

Decay matrix elements

Symmetric propagator of Ψ

$$\Pi^{\mu_1\cdots\mu_J\nu_1\cdots\nu_J}(p)$$
$$= \int d^4x\, e^{ip\cdot x}\,\langle 0|T\{\Psi^{\mu_1\cdots\mu_J}(x)\Psi^{\dagger\nu_1\cdots\nu_J}(0)\}|0\rangle$$

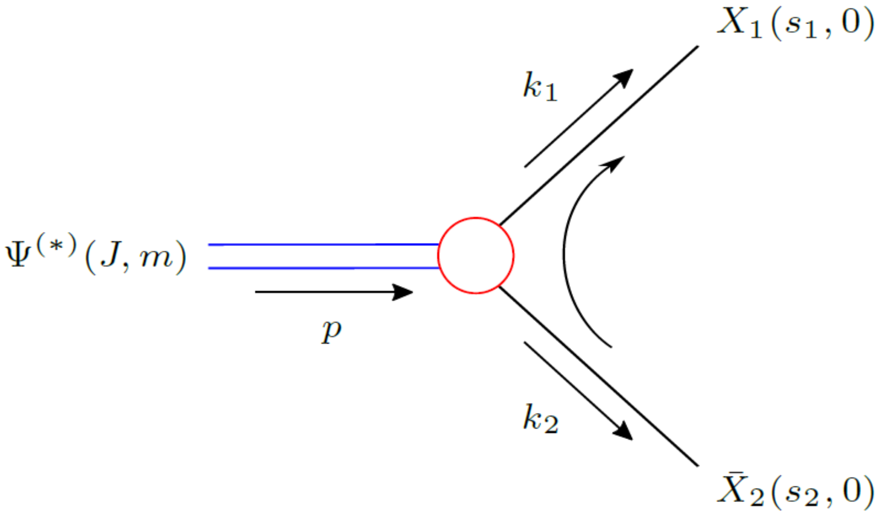
$\rightarrow$ Symmetric tensor current for μ indices

<Matrix elements>

$$\langle k_1,\lambda_1,k_2,\lambda_2|\Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J}|\,0\,\rangle = \Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu_1\cdots\mu_J}(k_1,k_2)$$
$$= \varepsilon^{*\alpha_1\cdots\alpha_{s_1}}(k_1,\lambda_1)\varepsilon^{*\beta_1\cdots\beta_{s_2}}(k_2,\lambda_2)\,\Gamma^{[J;s_1,s_2]}_{\alpha_1\cdots\alpha_{s_1},\beta_1\cdots\beta_{s_2};\mu_1\cdots\mu_J}(k_1,k_2)$$

Integer $s_{1,2}$

$$\varepsilon^{*\alpha_1\cdots\alpha_{s_1}}(k_1,\pm s_1) = \varepsilon^{*\alpha_1}(k_1,\pm 1)\cdots\varepsilon^{*\alpha_{s_1}}(k_1,\pm 1)$$
$$\varepsilon^{*\beta_1\cdots\beta_{s_2}}(k_2,\pm s_2) = \varepsilon^{*\beta_1}(k_2,\pm 1)\cdots\varepsilon^{*\beta_{s_2}}(k_2,\pm 1)$$



Divergence-free	$p_{\mu_i}\,\epsilon^{\mu_1\cdots\mu_i\cdots\mu_J}(p,\sigma)$	$=$	0
Traceless	$g_{\mu_i\mu_j}\,\epsilon^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_J}(p,\sigma)$	$=$	0
Symmetric	$\varepsilon_{\alpha\beta\mu_i\mu_j}\,\epsilon^{\mu_1\cdots\mu_i\cdots\mu_j\cdots\mu_J}(p,\sigma)$	$=$	0

R. E. Behrends and C. Fronsdal, Phys. Rev. (1957)

Decay matrix elements

<Matrix elements>

$$\langle k_1, \lambda_1, k_2, \lambda_2 | \Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_J} | 0 \rangle = \Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu_1\cdots\mu_J}(k_1,k_2)$$

$$= \varepsilon^{*\alpha_1\cdots\alpha_{s_1}}(k_1,\lambda_1)\varepsilon^{*\beta_1\cdots\beta_{s_2}}(k_2,\lambda_2)\,\Gamma^{[J;s_1,s_2]}_{\alpha_1\cdots\alpha_{s_1},\beta_1\cdots\beta_{s_2};\mu_1\cdots\mu_J}(k_1,k_2)$$

Symmetric propagator of Ψ

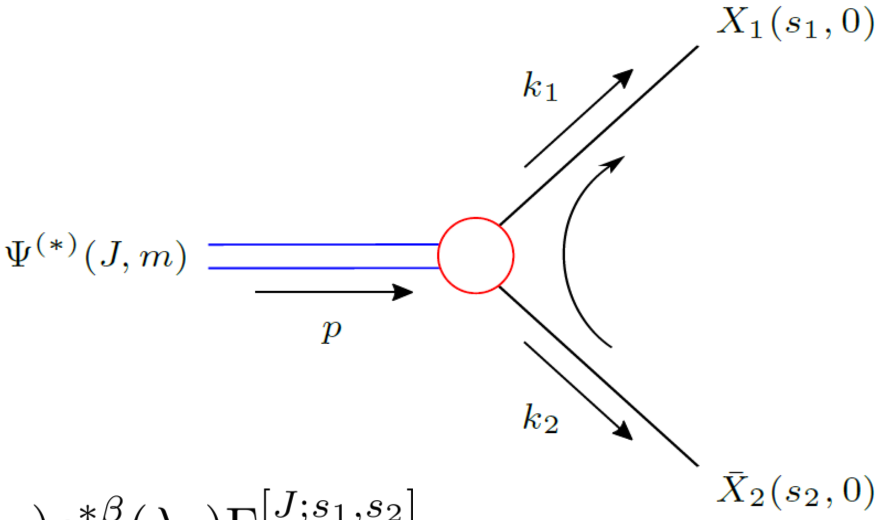
$$\Pi^{\mu_1\cdots\mu_J\nu_1\cdots\nu_J}(p) = \int d^4x\, e^{ip\cdot x}\,\langle 0|T\{\Psi^{\mu_1\cdots\mu_J}(x)\Psi^{\dagger\nu_1\cdots\nu_J}(0)\}|0\rangle$$

$\rightarrow$ Symmetric tensor current for μ indices

Integer $s_{1,2}$

$$\varepsilon^{*\alpha_1\cdots\alpha_{s_1}}(k_1,\pm s_1) = \varepsilon^{*\alpha_1}(k_1,\pm 1)\cdots\varepsilon^{*\alpha_{s_1}}(k_1,\pm 1)$$

$$\varepsilon^{*\beta_1\cdots\beta_{s_2}}(k_2,\pm s_2) = \varepsilon^{*\beta_1}(k_2,\pm 1)\cdots\varepsilon^{*\beta_{s_2}}(k_2,\pm 1)$$



$$\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu_1\cdots\mu_J}(k_1,k_2) \rightarrow \Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu}$$

$$\Gamma^{[J;s_1,s_2]}_{\alpha_1\cdots\alpha_{s_1},\beta_1\cdots\beta_{s_2};\mu_1\cdots\mu_J}(k_1,k_2) \rightarrow \Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu}$$

$$\varepsilon^{*\alpha_1\cdots\alpha_{s_1}}(k_1,\lambda_1) \rightarrow \varepsilon^{*\alpha}(\lambda_1)$$

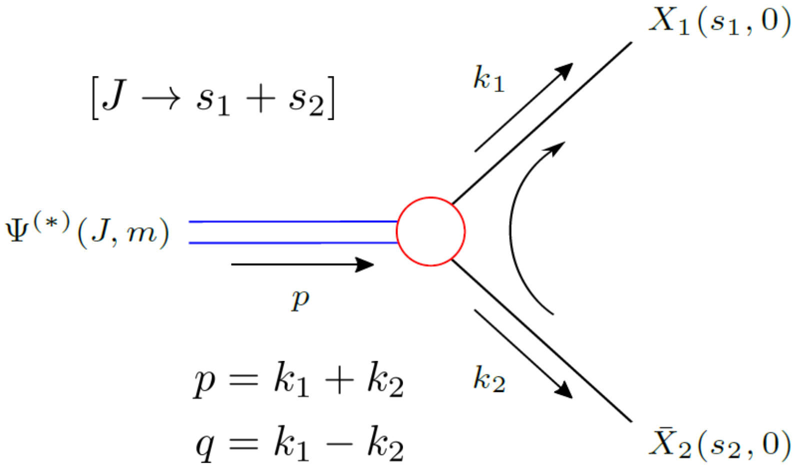
$$\varepsilon^{*\beta_1\cdots\beta_{s_1}}(k_2,\lambda_2) \rightarrow \varepsilon^{*\beta}(\lambda_2)$$

$$\Rightarrow \quad \Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu} = \varepsilon^{*\alpha}(\lambda_1)\varepsilon^{*\beta}(\lambda_2)\Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu}$$

$$\alpha \equiv \alpha_1\cdots\alpha_{s_1}, \quad \beta \equiv \beta_1\cdots\beta_{s_2}, \quad \mu \equiv \mu\cdots\mu_J$$

R. E. Behrends and C. Fronsdal, Phys. Rev. (1957)

Decay matrix elements



Outline of the algorithm

$$\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu} = \varepsilon^{*\alpha}(\lambda_1)\varepsilon^{*\beta}(\lambda_2)\Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu}$$

Building blocks

k_1, k_2 or p, q, $g_{\mu\nu}$, $\varepsilon_{\mu\nu\rho\sigma}$ $\in \Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu}$

$$\Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu} = \sum_n \Big\{ \big[F^{[J;s_1,s_2]}_{(+s_1,+s_2)}\big]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(+s_1,+s_2)\alpha,\beta;\mu} + \big[F^{[J;s_1,s_2]}_{(-s_1,-s_2)}\big]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(-s_1,-s_2)\alpha,\beta;\mu} + \big[F^{[J;s_1,s_2]}_{(+s_1,-s_2)}\big]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(+s_1,-s_2)\alpha,\beta;\mu} + \big[F^{[J;s_1,s_2]}_{(-s_1,+s_2)}\big]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(-s_1,+s_2)\alpha,\beta;\mu} \Big\}$$

Only four helicity configuration! (λ_1, λ_2)

Form factor operators : $F_{(\lambda_1,\lambda_2)\mu}$

$$\big[F^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}\big]^n_\mu$$

Helicity specific operators : $\mathcal{H}_{(\lambda_1,\lambda_2)\alpha,\beta;\mu}$ [Combination of basic operators]

$$\varepsilon^{*\alpha}(\lambda_1)\varepsilon^{*\beta}(\lambda_2)\mathcal{H}^{[J;s_1,s_2]}_{(\lambda'_1,\lambda'_2)\alpha,\beta;\mu} \propto \delta_{\lambda_1,\lambda'_1}\delta_{\lambda_2,\lambda'_2}$$

$\Rightarrow$ Form-factor operators & Basic operators

Decay matrix elements

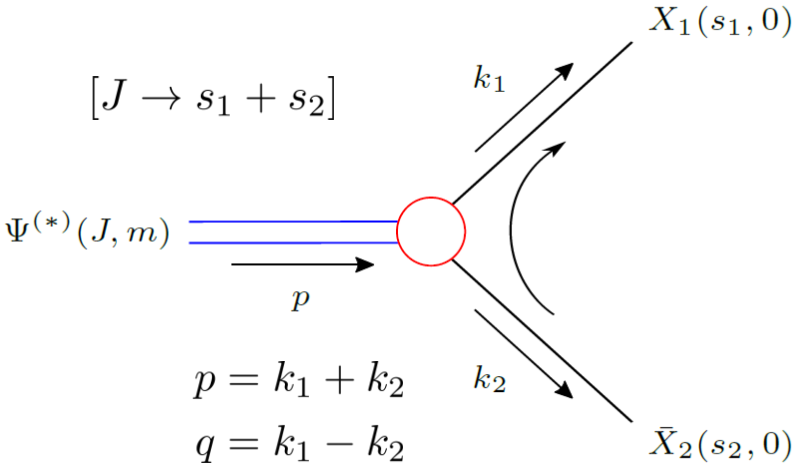
<Form-factor and basic operators>

$[J \to 0 + 0]$

$$\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu} = \varepsilon^{*\alpha}(\lambda_1)\varepsilon^{*\beta}(\lambda_2)\Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu} \quad \to \quad \Theta^{[J;0,0]}_{(0,0)\mu} = \Gamma^{[J;0,0]}_{\mu}$$

Symmetric and covariant tensor currents

$\to \quad q_\mu,\ p_\mu,$ and $g_{\mu_1\mu_2}$



Compact square bracket operator form

$$p_{\mu_1}\cdots p_{\mu_n} \to \boldsymbol{p}^n \qquad \Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu} \to \boldsymbol{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}$$

$$q_{\mu_1}\cdots q_{\mu_n} \to \boldsymbol{q}^n \qquad \left[F^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}\right]^n_\mu \to \left[\boldsymbol{F}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}\right]^n$$

$$g_{\mu_1\mu_2}\cdots g_{\mu_{2n-1}\mu_{2n}} \to \boldsymbol{g}^n$$

$$\boldsymbol{\Theta}^{[J;0,0]}_{(0,0)}(k_1,k_2) = \sum_{n=0}^{J}\left[\boldsymbol{F}^{[J;0,0]}_{(0,0)}(k_1,k_2)\right]^n$$

$$= \sum_{n,m,l} A^{[J;0,0]}_{(0,0)n,m,l}\,\boldsymbol{q}^n\boldsymbol{p}^m\boldsymbol{g}^l \quad \text{with} \quad J = n + m + 2l$$

Form factors

Symmetric propagator of Ψ

$$\Pi^{\mu_1\cdots\mu_J\nu_1\cdots\nu_J}(p) = \int d^4x\, e^{ip\cdot x}\,\langle 0|T\{\Psi^{\mu_1\cdots\mu_J}(x)\Psi^{\dagger\nu_1\cdots\nu_J}(0)\}|0\rangle$$

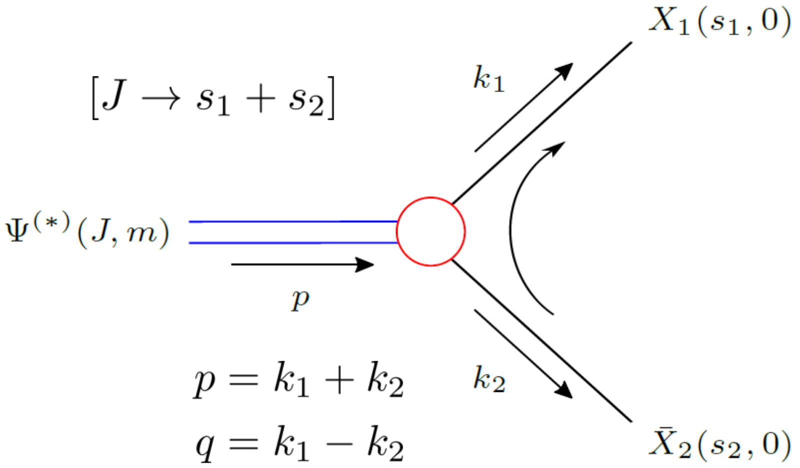
$\to$ Symmetric tensor current for μ indices

Decay matrix elements

<Form-factor and basic operators>

$[J \to s_1 + s_2]$

$$\Theta^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\mu} = \boxed{\varepsilon^{*\alpha}(\lambda_1)\varepsilon^{*\beta}(\lambda_2)}\Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu}$$



Boosts of spin-1 polarization vectors of massless particles

$$\varepsilon_{\alpha,\beta}(k_{1,2}) \to \varepsilon_{\alpha,\beta}(k'_{1,2}) + \eta\, k'_{1\alpha,2\beta}$$

Polarization-covariant operators

$$[k_1\rho\alpha\mu]_+ = i(k_{1\rho}g_{\alpha\mu} - k_{1\alpha}g_{\rho\mu} - k_{1\mu}g_{\rho\alpha}) \quad \to \quad F^\dagger_{1\rho\mu} = \partial_\rho X^\dagger_{1\mu} - \partial_\mu X^\dagger_{1\rho}$$

$$[k_2\sigma\beta\nu]_+ = i(k_{2\sigma}g_{\beta\nu} - k_{2\beta}g_{\sigma\nu} - k_{2\nu}g_{\sigma\beta}) \quad \to \quad F_{2\sigma\nu} = \partial_\sigma X_{2\nu} - \partial_\nu X_{2\sigma}$$

$$[k_1\rho\alpha\mu]_- = \varepsilon_{\gamma\rho\alpha\mu}\, k_1^\gamma \quad \to \quad -i\tilde{F}^\dagger_{1\rho\mu} = -i\varepsilon_{\rho\mu\gamma\alpha}F^{\dagger\gamma\alpha}/2$$

$$[k_2\sigma\beta\nu]_- = \varepsilon_{\delta\sigma\beta\nu}\, k_2^\delta \quad \to \quad -i\tilde{F}_{2\sigma\nu} = -i\varepsilon_{\rho\mu\delta\beta}F^{\delta\beta}/2$$

Fundamental operators

$$[abcd]_+ = i(g_{ab}g_{cd} - g_{ac}g_{bd} - g_{ad}g_{bc})$$

$$[abcd]_- = \varepsilon_{abcd}$$

Contraction symbols

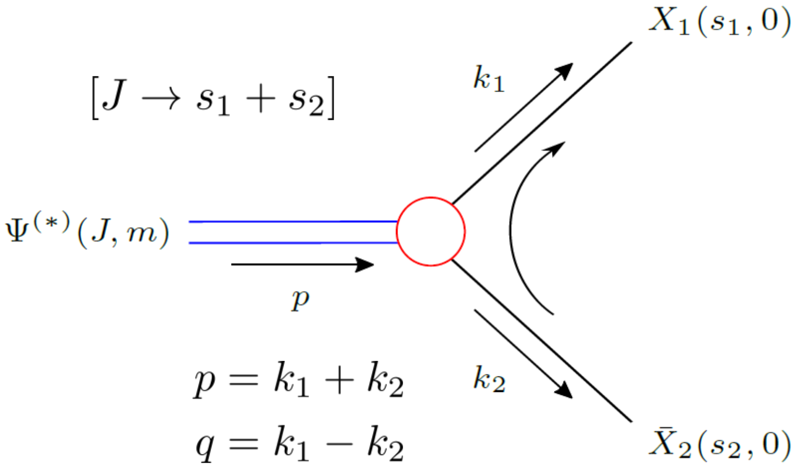
$$[a\overbrace{bcd]_i[ef}gh]_j = [abcd]_i[efgh]_j\, g^{bf}$$

Decay matrix elements

<Form-factor and basic operators>

$[0 \to 1 + 1]$

$$\Theta^{[0;1,1]}_{(\sigma_1,\sigma_2)} = \varepsilon^{*\alpha}(\sigma_1)\varepsilon^{*\beta}(\sigma_2)\Gamma^{[0;1,1]}_{\alpha,\beta} \qquad (\sigma_{1,2} = \pm 1)$$



$p = k_1 + k_2$

$q = k_1 - k_2$

Even-parity scalar operator

$$\frac{1}{2}[k_1\rho\alpha\mu]_\pm[k_2\sigma\beta\nu]_\pm \overset{\text{eff}}{=} i[k_1k_2\alpha\beta]_+ \quad \to \quad \Theta^{[0;1,1]}_{(\sigma_1,\sigma_2)} = \delta_{\sigma_1,\sigma_2}(k_1 \cdot k_2)$$

Odd-parity scalar operator

$$\frac{1}{2}[k_1\rho\alpha\mu]_\pm[k_2\sigma\beta\nu]_\mp \overset{\text{eff}}{=} i[k_1k_2\alpha\beta]_- \quad \to \quad \Theta^{[0;1,1]}_{(\sigma_1,\sigma_2)} = \sigma_1\delta_{\sigma_1,\sigma_2}(k_1 \cdot k_2)$$

Polarization-covariant operators

$$[k_1\rho\alpha\mu]_+ = i(k_{1\rho}g_{\alpha\mu} - k_{1\alpha}g_{\rho\mu} - k_{1\mu}g_{\rho\alpha})$$
$$[k_2\sigma\beta\nu]_+ = i(k_{2\sigma}g_{\beta\nu} - k_{2\beta}g_{\sigma\nu} - k_{2\nu}g_{\sigma\beta})$$
$$[k_1\rho\alpha\mu]_- = \varepsilon_{\gamma\rho\alpha\mu}\, k_1^\gamma$$
$$[k_2\sigma\beta\nu]_- = \varepsilon_{\delta\sigma\beta\nu}\, k_2^\delta$$

Basic scalar operators

$$S^\pm_{\alpha,\beta} = \frac{i}{2}\left([k_1k_2\alpha\beta]_+ \pm [k_1k_2\alpha\beta]_-\right) \quad \to \quad \Theta^{[0;1,1]}_{(\sigma_1,\sigma_2)} = \delta_{\sigma_1,\pm}\delta_{\sigma_1,\sigma_2}(k_1 \cdot k_2)$$

Decay matrix elements

<Form-factor and basic operators>

$$[1 \to 1+0] \oplus [1 \to 0+1]$$

$$\Theta^{[1;1,0]}_{(\sigma_1,0)\mu} = \varepsilon^{*\alpha}(\sigma_1)\Gamma^{[1;1,0]}_{\alpha;\mu} \quad \oplus \quad \Theta^{[1;0,1]}_{(0,\sigma_2)\mu} = \varepsilon^{*\beta}(\sigma_2)\Gamma^{[1;0,1]}_{\beta;\mu} \qquad (\sigma_{1,2} = \pm 1)$$

Even-parity vector operator

$$i[k_1 k_2 \alpha\mu]_+ \quad \to \quad \Theta^{[1;1,0]}_{(\sigma_1,0)\mu} = -(k_1 \cdot k_2)\varepsilon^*_{1\perp\mu}(\sigma_1)$$

$$i[k_2 k_1 \beta\nu]_+ \quad \to \quad \Theta^{[1;0,1]}_{(0,\sigma_2)\mu} = -(k_1 \cdot k_2)\varepsilon^*_{2\perp\mu}(\sigma_2)$$

Odd-parity vector operator

$$i[k_1 k_2 \alpha\mu]_- \quad \to \quad \Theta^{[1;1,0]}_{(\sigma_1,0)\mu} = -\sigma_1(k_1 \cdot k_2)\varepsilon^*_{1\perp\mu}(\sigma_1)$$

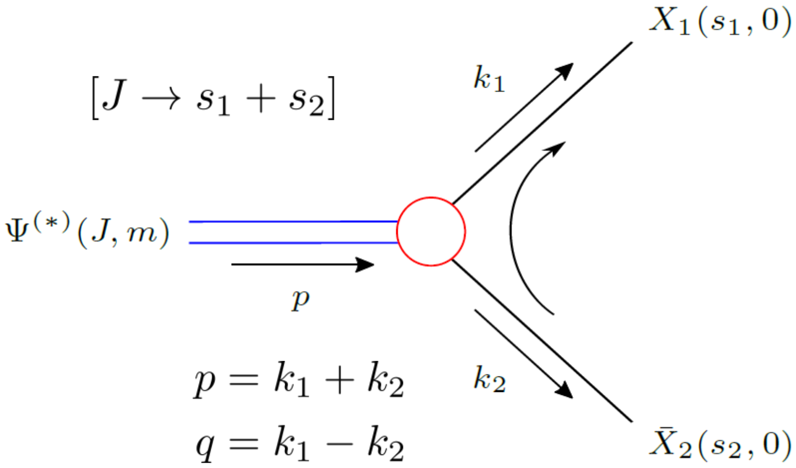
$$i[k_2 k_1 \beta\nu]_- \quad \to \quad \Theta^{[1;0,1]}_{(0,\sigma_2)\mu} = -\sigma_2(k_1 \cdot k_2)\varepsilon^*_{2\perp\mu}(\sigma_2)$$

Basic vector operators

$$V^{\pm}_{1\alpha;\mu} = \frac{i}{2}\left([k_1 k_2 \alpha\mu]_+ \pm [k_1 k_2 \alpha\mu]_-\right) \quad \to \quad \Theta^{[1;1,0]}_{(\sigma_1,0)\mu} = -\delta_{\sigma_1,\pm}(k_1 \cdot k_2)\varepsilon^*_{1\perp\mu}(\sigma_1)$$

$$V^{\pm}_{2\beta;\nu} = \frac{i}{2}\left([k_2 k_1 \beta\nu]_+ \pm [k_2 k_1 \beta\nu]_-\right) \quad \to \quad \Theta^{[1;0,1]}_{(0,\sigma_2)\mu} = -\delta_{\sigma_2,\pm}(k_1 \cdot k_2)\varepsilon^*_{2\perp\mu}(\sigma_2)$$

$$[J \to s_1 + s_2]$$



$$p = k_1 + k_2$$
$$q = k_1 - k_2$$

Polarization-covariant operators

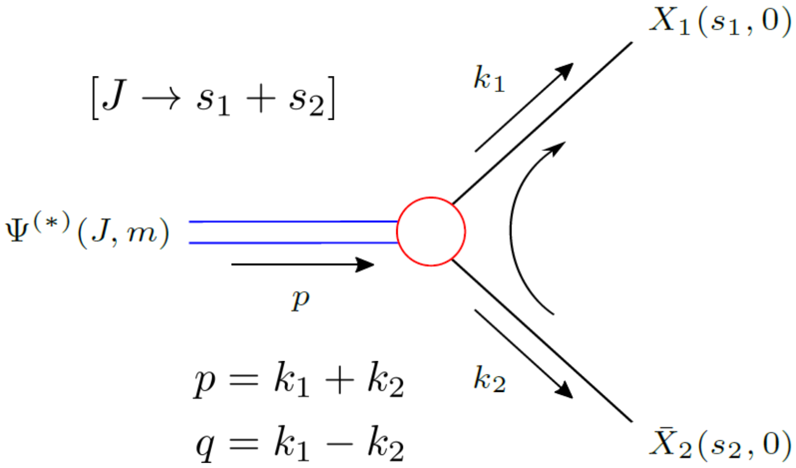
$$[k_1 \rho\alpha\mu]_+ = i(k_{1\rho} g_{\alpha\mu} - k_{1\alpha} g_{\rho\mu} - k_{1\mu} g_{\rho\alpha})$$
$$[k_2 \sigma\beta\nu]_+ = i(k_{2\sigma} g_{\beta\nu} - k_{2\beta} g_{\sigma\nu} - k_{2\nu} g_{\sigma\beta})$$
$$[k_1 \rho\alpha\mu]_- = \varepsilon_{\gamma\rho\alpha\mu}\, k_1^\gamma$$
$$[k_2 \sigma\beta\nu]_- = \varepsilon_{\delta\sigma\beta\nu}\, k_2^\delta$$

Covariant polarization vectors

$$\varepsilon_{1,2\perp\mu}(\sigma_{1,2}) = \varepsilon_{1,2}(\sigma_{1,2}) - \left(\frac{k_{2,1} \cdot \varepsilon(\sigma_{1,2})}{(k_1 \cdot k_2)}\right)$$

Decay matrix elements

<Form-factor and basic operators>



$[2 \to 1 + 1]$

$$\Theta^{[2;1,1]}_{(\sigma_1,\sigma_2)\mu_1\mu_2} = \varepsilon^{*\alpha}(\sigma_1)\varepsilon^{*\beta}(\sigma_2)\,\Gamma^{[2;1,1]}_{\alpha,\beta;\mu_1\mu_2} \qquad (\sigma_{1,2} = \pm 1)$$

Even-parity tensor operator

$$[k_1 \overline{\rho\alpha\mu]_+[k_2\sigma}\beta\nu]_+ - [k_1 \overline{\rho\alpha\mu]_-[k_2\sigma}\beta\nu]_-$$

$$\to \quad \Theta^{[2;1,1]}_{(\sigma_1,\sigma_2)\mu_1\mu_2} = -\delta_{\sigma_1,-\sigma_2}(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\sigma_1)\varepsilon^*_{2\perp\mu_2}(\sigma_2) + \mu_1 \leftrightarrow \mu_2\right]$$

Odd-parity tensor operator

$$[k_1 \overline{\rho\alpha\mu]_+[k_2\sigma}\beta\nu]_- - [k_1 \overline{\rho\alpha\mu]_-[k_2\sigma}\beta\nu]_+$$

$$\to \quad \Theta^{[2;1,1]}_{(\sigma_1,\sigma_2)\mu_1\mu_2} = -\lambda_1\delta_{\sigma_1,-\sigma_2}(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\sigma_1)\varepsilon^*_{2\perp\mu_2}(\sigma_2) + \mu_1 \leftrightarrow \mu_2\right]$$

Basic tensor operators

$$T^{\pm}_{\alpha,\beta;\mu\nu} = \frac{1}{2}\sum_{\tau=\pm}\tau\Big([k_1 \overline{\rho\alpha\mu]_\tau[k_2\sigma}\beta\nu]_\tau \pm [k_1 \overline{\rho\alpha\mu]_\tau[k_2\sigma}\beta\nu]_{-\tau}\Big)$$

$$\to \quad \Theta^{[2;1,1]}_{(\sigma_1,\sigma_2)\mu_1\mu_2} = -\delta_{\sigma_1,\pm}\delta_{\sigma_1,-\sigma_2}(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\sigma_1)\varepsilon^*_{2\perp\mu_2}(\sigma_2) + \mu_1 \leftrightarrow \mu_2\right]$$

Polarization-covariant operators

$$[k_1\rho\alpha\mu]_+ = i(k_{1\rho}g_{\alpha\mu} - k_{1\alpha}g_{\rho\mu} - k_{1\mu}g_{\rho\alpha})$$
$$[k_2\sigma\beta\nu]_+ = i(k_{2\sigma}g_{\beta\nu} - k_{2\beta}g_{\sigma\nu} - k_{2\nu}g_{\sigma\beta})$$
$$[k_1\rho\alpha\mu]_- = \varepsilon_{\gamma\rho\alpha\mu}\,k_1^\gamma$$
$$[k_2\sigma\beta\nu]_- = \varepsilon_{\delta\sigma\beta\nu}\,k_2^\delta$$

Covariant polarization vectors

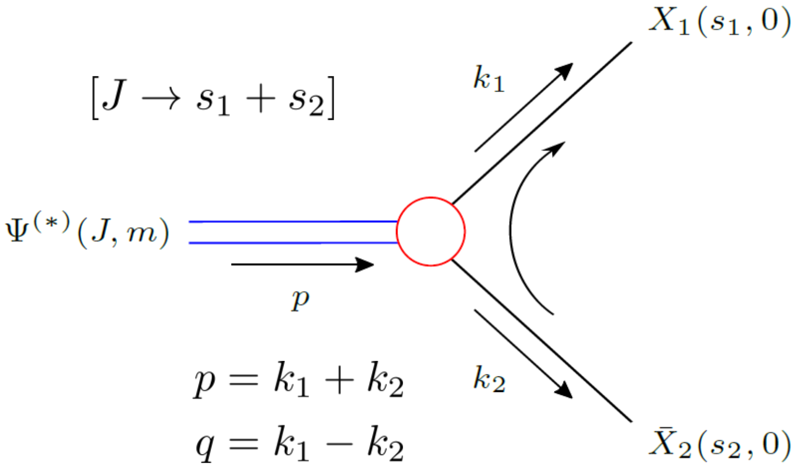
$$\varepsilon_{1,2\perp\mu}(\sigma_{1,2}) = \varepsilon_{1,2}(\sigma_{1,2}) - \left(\frac{k_{2,1}\cdot\varepsilon(\sigma_{1,2})}{(k_1\cdot k_2)}\right)$$

Decay matrix elements

How to construct the 3p vertices

$$\Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu} = \sum_n \Big\{ \boxed{\left[F^{[J;s_1,s_2]}_{(+s_1,+s_2)}\right]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(+s_1,+s_2)\alpha,\beta;\mu}} + \left[F^{[J;s_1,s_2]}_{(-s_1,-s_2)}\right]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(-s_1,-s_2)\alpha,\beta;\mu}$$

$$+ \left[F^{[J;s_1,s_2]}_{(+s_1,-s_2)}\right]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(+s_1,-s_2)\alpha,\beta;\mu} + \left[F^{[J;s_1,s_2]}_{(-s_1,+s_2)}\right]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(-s_1,+s_2)\alpha,\beta;\mu} \Big\}$$

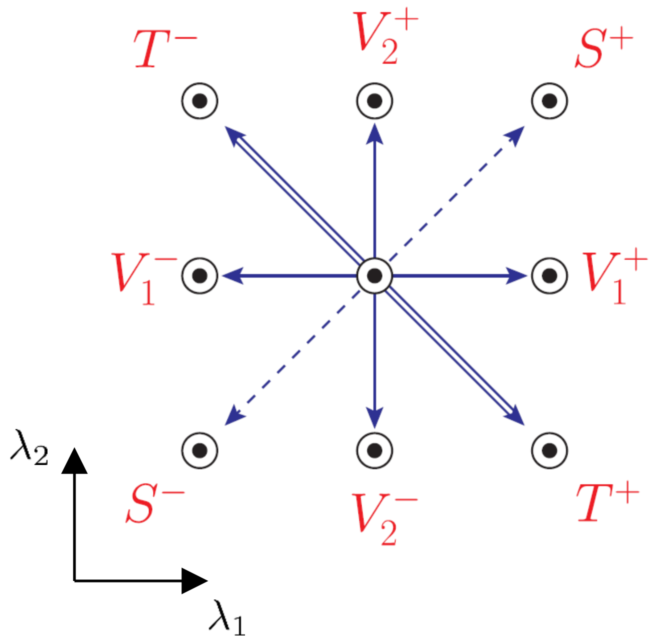


For $s_1 > s_2$

$$\mathcal{H}_{(+s_1,+s_2)\alpha,\beta;\mu} = \underbrace{(S^+ \cdots S^+)}_{s_2} \times \underbrace{(V_1^+ \cdots V_1^+)}_{s_1 - s_2}$$

(s_2, s_2) $\quad$ $(s_1 - s_2 + s_2, s_2)$

Roles of basic operators



Compact square bracket operator form

$$S^\pm_{\alpha_1,\beta_1} \cdots S^\pm_{\alpha_n\,\beta_n} \to (\boldsymbol{S}^\pm)^n$$

$$V^\pm_{1\alpha_1;\mu_1} \cdots V^\pm_{1\alpha_n;\mu_n} \to (\boldsymbol{V}_1^\pm)^n$$

$$V^\pm_{2\beta_1;\mu_1} \cdots V^\pm_{2\beta_n;\mu_n} \to (\boldsymbol{V}_2^\pm)^n$$

$$T^\pm_{\alpha_1,\beta_1;\mu_1\mu_2} \cdots T^\pm_{\alpha_n,\beta_n;\mu_{2n-1}\mu_{2n}} \to (\boldsymbol{T}^\pm)^n$$

$$\mathcal{H}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\alpha,\beta;\mu} \to \boldsymbol{\mathcal{H}}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}$$

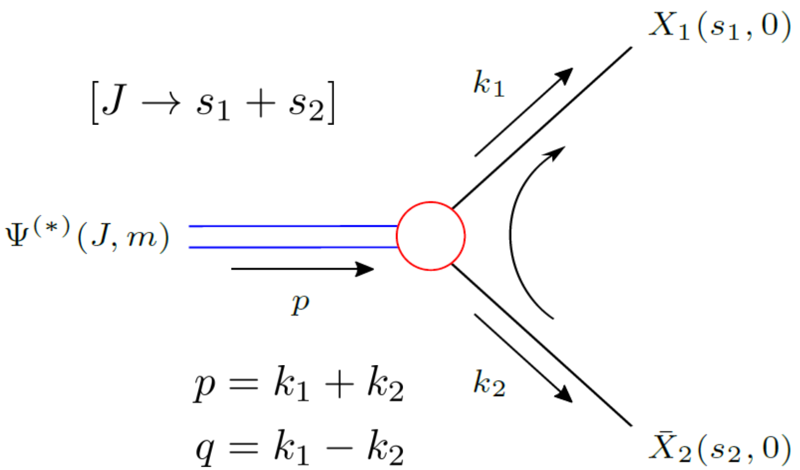
$$\left[F^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}\right]^n_\mu \to \left[\boldsymbol{F}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}\right]^n$$

$$\Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu} \to \boldsymbol{\Gamma}^{[J;s_1,s_2]}$$

Decay matrix elements

How to construct the 3p vertices

$$\Gamma^{[J;s_1,s_2]}_{\alpha,\beta;\mu} = \sum_n \Big\{ \big[F^{[J;s_1,s_2]}_{(+s_1,+s_2)}\big]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(+s_1,+s_2)\alpha,\beta;\mu} + \big[F^{[J;s_1,s_2]}_{(-s_1,-s_2)}\big]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(-s_1,-s_2)\alpha,\beta;\mu} + \big[F^{[J;s_1,s_2]}_{(+s_1,-s_2)}\big]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(+s_1,-s_2)\alpha,\beta;\mu} + \big[F^{[J;s_1,s_2]}_{(-s_1,+s_2)}\big]^n_\mu \mathcal{H}^{[J;s_1,s_2]}_{(-s_1,+s_2)\alpha,\beta;\mu} \Big\}$$

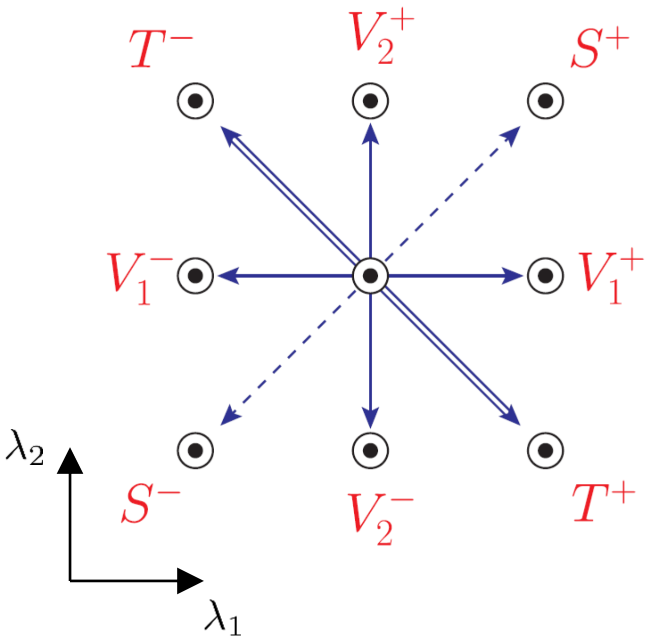


For $s_1 > s_2$

$$\mathcal{H}_{(+s_1,+s_2)\alpha,\beta;\mu} = \underbrace{(S^+ \cdots S^+)}_{s_2} \times \underbrace{(V_1^+ \cdots V_1^+)}_{s_1 - s_2}$$

(s_2, s_2) $\quad$ $(s_1 - s_2 + s_2, s_2)$

Roles of basic operators



<General expression of the covariant 3p vertices>

$$\boldsymbol{\Gamma}^{[J;s_1,s_2]} = \sum_{\lambda=\pm} \Big\{ \theta(J_-) \sum_{n=0}^{J_-} \big[\boldsymbol{F}^{[J;s_1,s_2]}_{(\lambda s_1,\lambda s_2)}\big]^n (\boldsymbol{S}^\lambda)^{s_{\min}} \Big[(\boldsymbol{V}_1^\lambda)^{s_1 - s_{\min}} + (\boldsymbol{V}_2^\lambda)^{s_2 - s_{\min}}\Big] + \gamma_{s_{\min}} \theta(J_+) \sum_{n=0}^{J_+} \big[\boldsymbol{F}^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)}\big]^n (\boldsymbol{T}^\lambda)^{s_{\min}} \Big[(\boldsymbol{V}_1^\lambda)^{s_1 - s_{\min}} + (\boldsymbol{V}_2^{-\lambda})^{s_2 - s_{\min}}\Big] \Big\}$$

$(J_\mp = J - |s_1 \mp s_2|)$, $(\gamma_{s_{\min}} = 1 - \delta_{s_{\min},0}$ with $s_{\min} = \min[s_1, s_2])$

Summary

- ✓ An efficient algorithm in the covariant formulation. $\langle X_1 \bar{X}_2 | \Theta^{[J;s_1,s_2]}_{\mu_1\cdots\mu_n} | 0 \rangle$
- ✓ Covariance conditions on form factors. $A^{[J;0,0]}_{(0,0)n,m,l}$
- ✓ Identical particles, Gauge invariance, Discrete symmetries, $\cdots$
- ✓ Validity of the Landau-Yang and Weinberg-Witten theorems.
- ✓ Covariant three-point vertices for all the off-shell particles case.
- ✓ Covariant four-point vertices.
- ✓ Program for generating covariant vertices and Lagrangian operators.
- ✓ Algorithm $\leftrightarrow$ Spinor helicity formalism.

Thank you

Backup

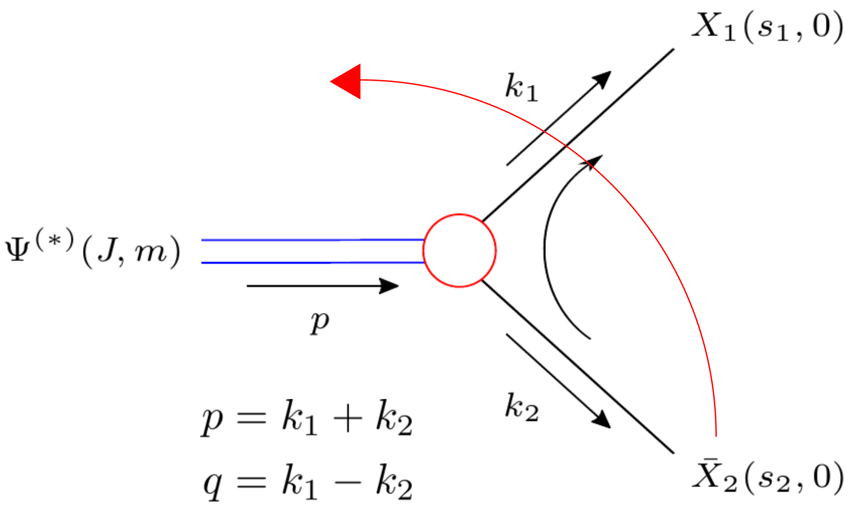
Spin-1 polarization vectors in the $\Psi^{(*)}$ rest frame

$$k_1 = \frac{\sqrt{p^2}}{2}(1, \hat{n}) \quad \text{and} \quad k_2 = \frac{\sqrt{p^2}}{2}(1, -\hat{n})$$

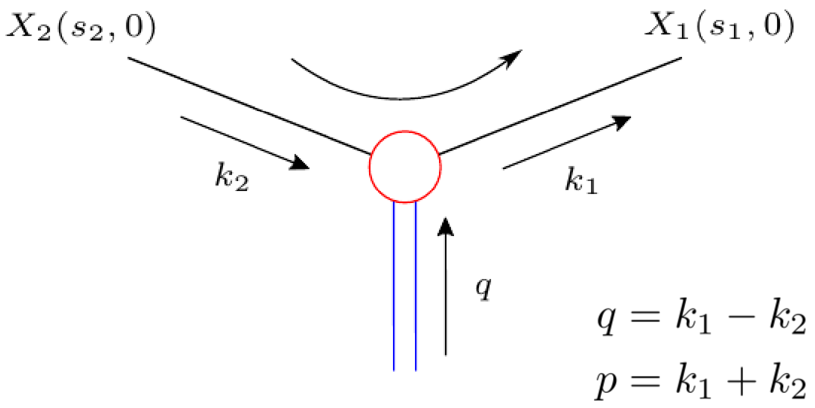
$$\varepsilon(k_1, \pm) = \frac{1}{\sqrt{2}}(0, \mp\hat{\theta} - i\hat{\phi}) \quad \text{and} \quad \varepsilon(k_2, \pm) = \frac{1}{\sqrt{2}}(0, \mp\hat{\theta} + i\hat{\phi})$$

$$\begin{aligned}
\hat{n} &= (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \\
\hat{\theta} &= (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta) \\
\hat{\phi} &= (\ -\sin\phi\ ,\ \cos\phi\ ,\ 0\ )
\end{aligned}$$

Scattering matrix elements



$\longrightarrow$



Decay matrix elements $\longrightarrow$ Scattering matrix elements

$$\langle k_1,\lambda_1;k_2,\lambda_2|\mathbf{\Theta}^{[J;s_1,s_2]}|\,0\,\rangle=\mathbf{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)} \longrightarrow \langle k_1,\lambda_1|\mathbf{\Theta}^{[J;s_1,s_2]}|k_2,\lambda_2\rangle=\bar{\mathbf{\Theta}}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)}$$

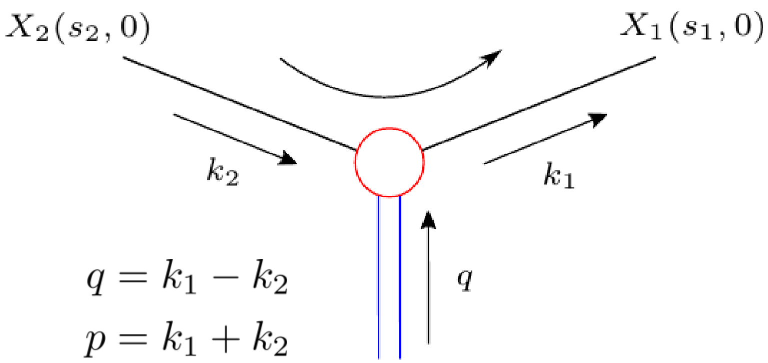
In the three-point vertices	$+k_2\to-k_2$
In the X_2 wave tensors	$\varepsilon^*(k_2,\pm\lambda_2)\to-\varepsilon(k_2,\mp\lambda_2)$

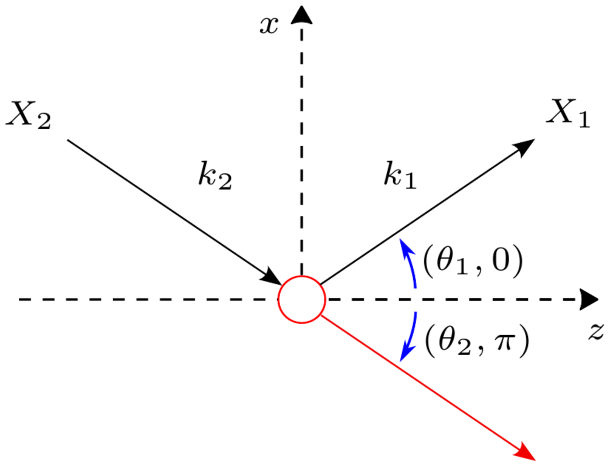
$$\mathbf{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,-\lambda_2)}\longleftrightarrow\bar{\mathbf{\Theta}}^{[J;s_1,s_2]}_{(\lambda_1,+\lambda_2)}$$

Scattering matrix elements

$$\xrightarrow{\theta_{1,2}\to 0} \quad k_{1,2} = k = k^0(1,0,0,1) \quad \longrightarrow$$

$$\lim_{\theta_{1,2}\to 0} \sqrt{k_1\cdot k_2}\;\varepsilon^*_{1\perp}(\pm) = \pm k$$
$$\lim_{\theta_{1,2}\to 0} \sqrt{k_1\cdot k_2}\;\varepsilon_{2\perp}(\pm) = \pm k$$





$$\bar S^{\pm}_{\alpha,\beta} = -S^{\pm}_{\alpha,\beta} \quad\rightarrow\quad \bar\Theta^{[0;1,1]}_{(\sigma_1,\sigma_2)}(k_1,k_2) = +\delta_{\sigma_1,\pm}\delta_{\sigma_1,-\sigma_2}\,(k_1\cdot k_2)$$

$$\bar V^{\pm}_{1\alpha;\mu} = -V^{\pm}_{1\alpha;\mu} \quad\rightarrow\quad \bar\Theta^{[1;1,0]}_{(\sigma_1,0)\mu}(k_1,k_2) = +\delta_{\sigma_1,\pm}(k_1\cdot k_2)\,\varepsilon^*_{1\perp\mu}(\sigma_1) \qquad (\sigma_{1,2}=\pm 1)$$

$$\bar V^{\mp}_{2\beta;\nu} = -V^{\pm}_{2\beta;\nu} \quad\rightarrow\quad \bar\Theta^{[1;0,1]}_{(0,\sigma_2)\nu}(k_1,k_2) = -\delta_{\sigma_2,\mp}(k_1\cdot k_2)\,\varepsilon_{2\perp\nu}(\sigma_2)$$

$$\bar T^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad\rightarrow\quad \bar\Theta^{[2;1,1]}_{(\sigma_1,\sigma_2)\mu\nu}(k_1,k_2) = -\delta_{\sigma_1,\pm}\delta_{\sigma_1,\sigma_2}(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu\leftrightarrow\nu\right]$$

$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu),(p_\mu),(g_{\mu\nu}) \quad\rightarrow\quad \bar F_{(\lambda_1,-\lambda_2)} \sim (p_\mu),(q_\mu),(g_{\mu\nu})$$

Covariant polarization vectors

$$\varepsilon_{1,2\perp\mu}(\sigma_{1,2}) = \varepsilon_{1,2}(\sigma_{1,2}) - \left(\frac{k_{2,1}\cdot\varepsilon(\sigma_{1,2})}{(k_1\cdot k_2)}\right)$$

Scattering matrix elements

$$\xrightarrow{\theta_{1,2}\to 0} \quad k_{1,2} = k = k^0(1,0,0,1) \quad \longrightarrow$$

$$\lim_{\theta_{1,2}\to 0} \sqrt{k_1\cdot k_2}\;\varepsilon^*_{1\perp}(\pm) = \pm k$$
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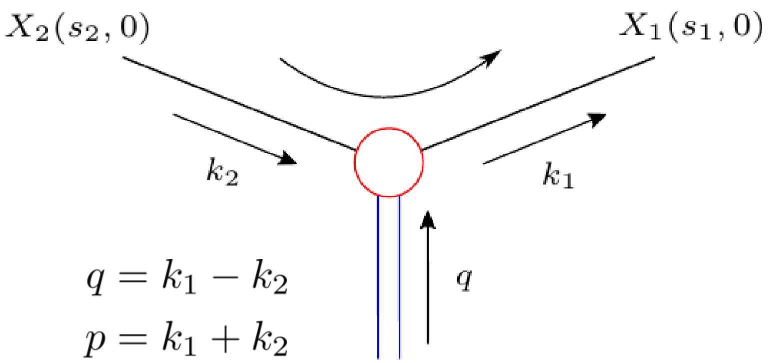
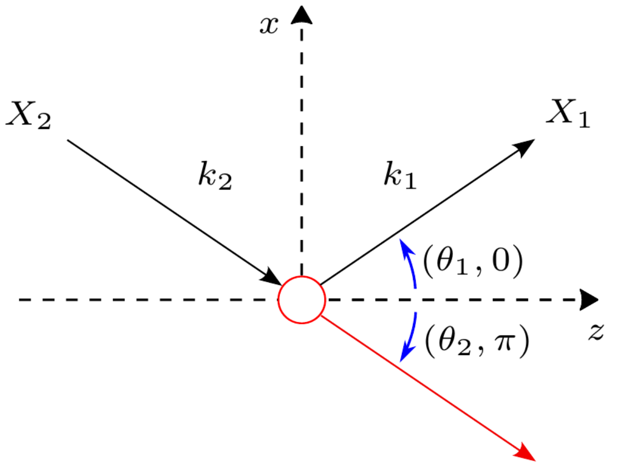




$$\bar S^{\pm}_{\alpha,\beta} = -S^{\pm}_{\alpha,\beta} \quad \to \quad \bar\Theta^{[0;1,1]}_{(\sigma_1,\sigma_2)}(k_1,k_2) = +\delta_{\sigma_1,\pm}\delta_{\sigma_1,-\sigma_2}\cancel{(k_1\cdot k_2)}$$

$$\bar V^{\pm}_{1\alpha;\mu} = -V^{\pm}_{1\alpha;\mu} \quad \to \quad \bar\Theta^{[1;1,0]}_{(\sigma_1,0)\mu}(k_1,k_2) = +\delta_{\sigma_1,\pm}\cancel{(k_1\cdot k_2)}\,\varepsilon^*_{1\perp\mu}(\sigma_1) \qquad (\sigma_{1,2} = \pm 1)$$

$$\bar V^{\mp}_{2\beta;\nu} = -V^{\pm}_{2\beta;\nu} \quad \to \quad \bar\Theta^{[1;0,1]}_{(0,\sigma_2)\nu}(k_1,k_2) = -\delta_{\sigma_2,\mp}\cancel{(k_1\cdot k_2)}\,\varepsilon_{2\perp\nu}(\sigma_2)$$

$$\bar T^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \to \quad \bar\Theta^{[2;1,1]}_{(\sigma_1,\sigma_2)\mu\nu}(k_1,k_2) = -\delta_{\sigma_1,\pm}\delta_{\sigma_1,\sigma_2}(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu\leftrightarrow\nu\right]$$

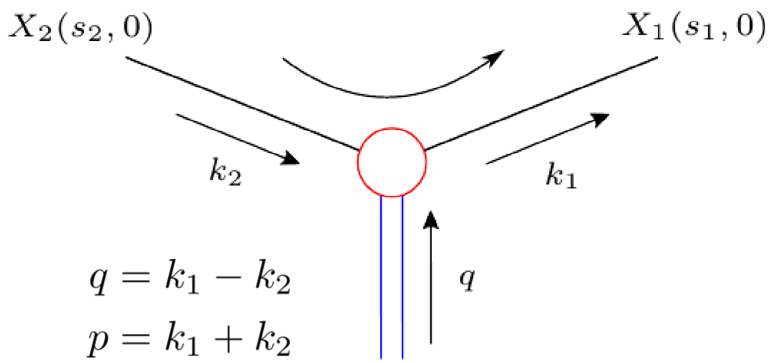
$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu), (p_\mu), (g_{\mu\nu}) \quad \to \quad \bar F_{(\lambda_1,-\lambda_2)} \sim (p_\mu), \cancel{(q_\mu)}, (g_{\mu\nu})$$

Covariant polarization vectors

$$\varepsilon_{1,2\perp\mu}(\sigma_{1,2}) = \varepsilon_{1,2}(\sigma_{1,2}) - \left(\frac{k_{2,1}\cdot\varepsilon(\sigma_{1,2})}{(k_1\cdot k_2)}\right)$$

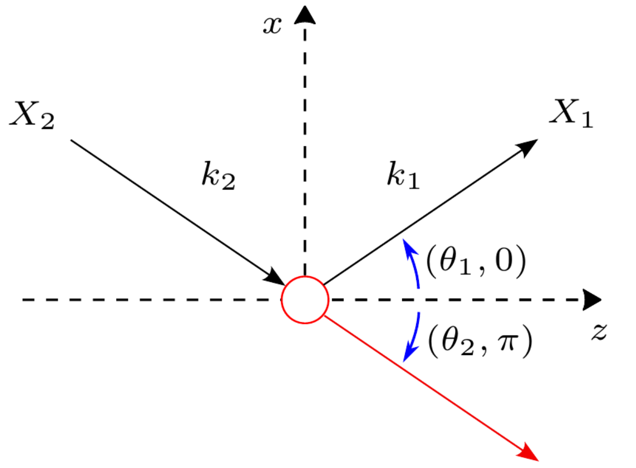
Scattering matrix elements

$$\xrightarrow{\theta_{1,2}\to 0} \quad k_{1,2} = k = k^0(1,0,0,1) \quad \longrightarrow \quad \begin{array}{l} \lim_{\theta_{1,2}\to 0} \sqrt{k_1\cdot k_2}\,\varepsilon^*_{1\perp}(\pm) = \pm k \\ \lim_{\theta_{1,2}\to 0} \sqrt{k_1\cdot k_2}\,\varepsilon_{2\perp}(\pm) = \pm k \end{array}$$



Form factors including $(k_1\cdot k_2)$ in denominators

$$\left.\begin{array}{l} \bar{S}^{\pm}_{\alpha,\beta}/(k_1\cdot k_2) \\ \bar{V}^{\pm}_{1\alpha;\mu}/\sqrt{k_1\cdot k_2} \\ \bar{V}^{\mp}_{2\beta;\nu}/\sqrt{k_1\cdot k_2} \end{array}\right\} \to \boldsymbol{\Theta}^{[J,s_1,s_2]}_{(\lambda_1,\lambda_2)} \neq 0$$



→ *Violate the covariance of tensor currents!*

$$\bar{S}^{\pm}_{\alpha,\beta} = -S^{\pm}_{\alpha,\beta} \quad \to \quad \bar{\Theta}^{[0;1,1]}_{(\sigma_1,\sigma_2)}(k_1,k_2) = +\delta_{\sigma_1,\pm}\delta_{\sigma_1,-\sigma_2}\cancel{(k_1\cdot k_2)}$$

$$\bar{V}^{\pm}_{1\alpha;\mu} = -V^{\pm}_{1\alpha;\mu} \quad \to \quad \bar{\Theta}^{[1;1,0]}_{(\sigma_1,0)\mu}(k_1,k_2) = +\delta_{\sigma_1,\pm}\cancel{(k_1\cdot k_2)}\,\varepsilon^*_{1\perp\mu}(\sigma_1) \qquad (\sigma_{1,2} = \pm 1)$$

$$\bar{V}^{\mp}_{2\beta;\nu} = -V^{\pm}_{2\beta;\nu} \quad \to \quad \bar{\Theta}^{[1;0,1]}_{(0,\sigma_2)\nu}(k_1,k_2) = -\delta_{\sigma_2,\mp}\cancel{(k_1\cdot k_2)}\,\varepsilon_{2\perp\nu}(\sigma_2)$$

$$\bar{T}^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad \to \quad \bar{\Theta}^{[2;1,1]}_{(\sigma_1,\sigma_2)\mu\nu}(k_1,k_2) = -\delta_{\sigma_1,\pm}\delta_{\sigma_1,\sigma_2}(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu\leftrightarrow\nu\right]$$

$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu),(p_\mu),(g_{\mu\nu}) \quad \to \quad \bar{F}_{(\lambda_1,-\lambda_2)} \sim (p_\mu),\cancel{(q_\mu)},(g_{\mu\nu})$$

Covariant polarization vectors

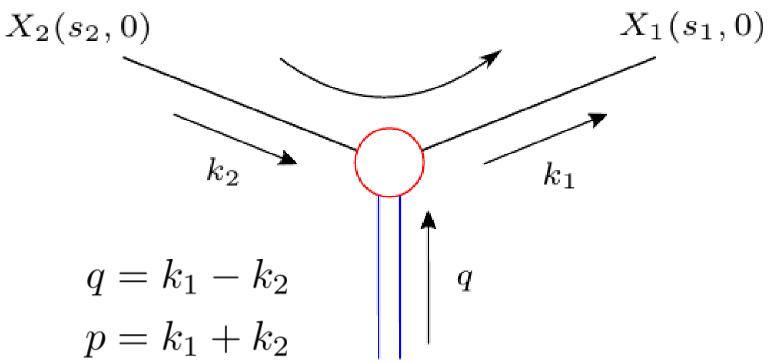
$$\varepsilon_{1,2\perp\mu}(\sigma_{1,2}) = \varepsilon_{1,2}(\sigma_{1,2}) - \left(\frac{k_{2,1}\cdot\varepsilon(\sigma_{1,2})}{(k_1\cdot k_2)}\right)$$

Scattering matrix elements

$$\xrightarrow{\theta_{1,2}\to 0} \quad k_{1,2} = k = k^0(1,0,0,1) \quad \longrightarrow$$

$$\lim_{\theta_{1,2}\to 0} \sqrt{k_1\cdot k_2}\;\varepsilon^*_{1\perp}(\pm) = \pm k$$

$$\lim_{\theta_{1,2}\to 0} \sqrt{k_1\cdot k_2}\;\varepsilon_{2\perp}(\pm) = \pm k$$

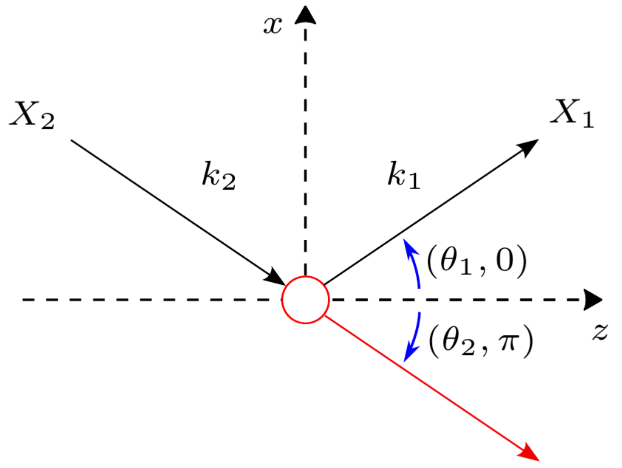


<Covariance conditions on form factors>

$$\lim_{k_{1,2}\to k} \bar{A}^{[J;s_1,s_2]}_{(\lambda s_1,-\lambda s_2)n,m,l} \times (k_1\cdot k_2)^{\frac{s_1+s_2}{2}} = 0$$

$$\lim_{k_{1,2}\to k} \bar{A}^{[J;s_1,s_2]}_{(\lambda s_1,+\lambda s_2)n,m,l} \times (k_1\cdot k_2)^{\frac{|s_1-s_2|}{2}} = 0$$

Second key result



$$\bar{S}^{\pm}_{\alpha,\beta} = -S^{\pm}_{\alpha,\beta} \quad\to\quad \bar{\Theta}^{[0;1,1]}_{(\sigma_1,\sigma_2)}(k_1,k_2) = +\delta_{\sigma_1,\pm}\delta_{\sigma_1,-\sigma_2}\cancel{(k_1\cdot k_2)}$$

$$\bar{V}^{\pm}_{1\alpha;\mu} = -V^{\pm}_{1\alpha;\mu} \quad\to\quad \bar{\Theta}^{[1;1,0]}_{(\sigma_1,0)\mu}(k_1,k_2) = +\delta_{\sigma_1,\pm}\cancel{(k_1\cdot k_2)}\,\varepsilon^*_{1\perp\mu}(\sigma_1) \qquad (\sigma_{1,2} = \pm 1)$$

$$\bar{V}^{\mp}_{2\beta;\nu} = -V^{\pm}_{2\beta;\nu} \quad\to\quad \bar{\Theta}^{[1;0,1]}_{(0,\sigma_2)\nu}(k_1,k_2) = -\delta_{\sigma_2,\mp}\cancel{(k_1\cdot k_2)}\,\varepsilon_{2\perp\nu}(\sigma_2)$$

$$\bar{T}^{\pm}_{\alpha,\beta;\mu\nu} = -T^{\pm}_{\alpha,\beta;\mu\nu} \quad\to\quad \bar{\Theta}^{[2;1,1]}_{(\sigma_1,\sigma_2)\mu\nu}(k_1,k_2) = -\delta_{\sigma_1,\pm}\delta_{\sigma_1,\sigma_2}(k_1\cdot k_2)\left[\varepsilon^*_{1\perp\mu}(\pm)\varepsilon_{2\perp\nu}(\pm) + \mu\leftrightarrow\nu\right]$$

$$F_{(\lambda_1,\lambda_2)} \sim (q_\mu),(p_\mu),(g_{\mu\nu}) \quad\to\quad \bar{F}_{(\lambda_1,-\lambda_2)} \sim (p_\mu),\cancel{(q_\mu)},(g_{\mu\nu})$$

Covariant polarization vectors

$$\varepsilon_{1,2\perp\mu}(\sigma_{1,2})$$
$$= \varepsilon_{1,2}(\sigma_{1,2}) - \left(\frac{k_{2,1}\cdot\varepsilon(\sigma_{1,2})}{(k_1\cdot k_2)}\right)$$

Generalized Landau-Yang theorem

For the identical particles, X_1 and $\bar{X}_2$ ($X_1 = \bar{X}_2$)

$$\Big[\langle X_1; k_1, \lambda_1 | \langle \bar{X}_2; k_2, \lambda_2 |\Big] \mathbf{\Theta}^{[J;s,s]} | 0 \rangle = \Big[\langle \bar{X}_2; k_1, \lambda_1 | \langle X_1; k_2, \lambda_2 |\Big] \mathbf{\Theta}^{[J;s,s]} | 0 \rangle$$

$$\downarrow \quad k_1 \leftrightarrow k_2$$

$$\varepsilon^{*\alpha}(k_1,\lambda_1)\varepsilon^{*\beta}(k_2,\lambda_2)\ \Gamma^{[J;s,s]}_{\alpha,\beta;\mu}(k_1,k_2) = \varepsilon^{*\alpha}(k_2,\lambda_2)\varepsilon^{*\beta}(k_1,\lambda_1)\ \Gamma^{[J;s,s]}_{\alpha,\beta;\mu}(k_2,k_1)$$

$$\downarrow \quad \alpha \leftrightarrow \beta$$

Identical particle condition

$$\Gamma^{[J;s,s]}_{\alpha,\beta;\mu}(p,q) = \Gamma^{[J;s,s]}_{\beta,\alpha;\mu}(p,-q)$$

Selction rules

1. $(J = 1, 3, \cdots) \not\to (0+0)$
2. $(1) \to$ ~~$(0+0)$~~, $(1/2+1/2)$ ~~$(1+1)$~~, ~~$(\cdots)$~~

$\vdots$

$$\mathbf{\Gamma}^{[J;s,s]}_{\rm on} = \sum_{\lambda=\pm}\Big\{\theta(J)(1-\eta_J)\big[\boldsymbol{F}^{[J;s,s]}_{(\lambda s,\lambda s)}\big]^J (\boldsymbol{S}^\lambda)^s + \gamma_s\,\theta(J-2s)\big[\boldsymbol{F}^{[J;s,s]}_{(\lambda s,-\lambda s)}\big]^{J-2s}(\boldsymbol{T}^\lambda)^s\Big\}$$

with $\big[\boldsymbol{F}^{[J;s,s]}_{(+s,-s)}\big]^{J-2s} = \pm\big[\boldsymbol{F}^{[J;s,s]}_{(-s,+s)}\big]^{J-2s}$ for even (+) or odd (-) integers $J-2s$

and $\eta_J = [1-(-1)^J]/2$

Weinberg-Witten theorem

Conserved tensor currents

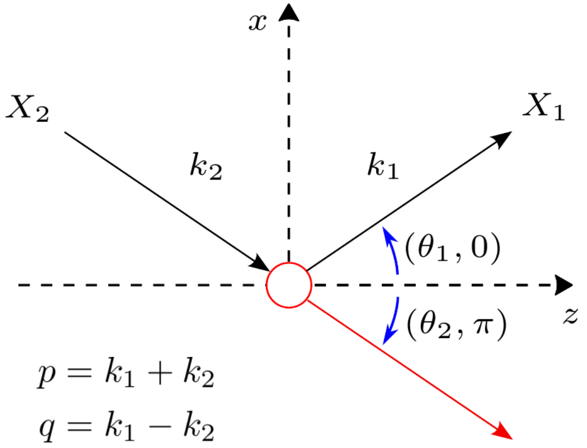
$$q^{\mu_i}\,\bar{\Theta}^{[J;s_1,s_2]}_{(\lambda_1,\lambda_2)\,\mu_1\cdots\mu_i\cdots\mu_J}(k_1,k_2) = 0$$

Identical-momentum limit ($k_1 = k_2$)

$$\boldsymbol{\Theta}^{[J;s,s]}_{\pm s,\pm s} = \Theta(J-2s)\,2^{J-s}\,\bar{A}^{[J;s,s]}_{(\pm s,\pm s)}\,\boldsymbol{k}^J$$

Zeroth components of charged current and energy-momentum tensor

$$\int d^3\vec{x}\,\frac{\langle k,\lambda|\Theta^{[J;s,s]}_{\mu}(x)|k',\lambda'\rangle}{\sqrt{\langle k,\lambda|k,\lambda\rangle}\sqrt{\langle k',\lambda'|k',\lambda'\rangle}} = \frac{\langle k,\lambda|\Theta^{[J;s,s]}_{\mu}(0)|k,\lambda\rangle}{2k^0}\delta_{\lambda,\lambda'},$$



with $\langle k,\lambda|k',\lambda'\rangle = 2k^0(2\pi)^3\delta^3(\vec{k}-\vec{k}\,')\delta_{\lambda,\lambda'}$

$\bar{A}_{(\pm,\pm)} \equiv Q$ or 2^{s-1} for J_μ or $T_{\mu\nu}$

$$\frac{\langle k,\pm|J_0|k,\pm\rangle}{2k^0} = \Theta(1-2s)\,Q \qquad \frac{\langle k,\pm|T_{0\mu}|k,\pm\rangle}{2k^0} = \Theta(2-2s)\,k_\mu$$

$$
\begin{aligned}
B^{\pm}(0,5.2) &\rightarrow K^{*}(1,0.9)^{\pm} + \gamma(1,0) \\
H(0,125) &\rightarrow \gamma(1,0) + \gamma(1,0) \\
H(0,125) &\rightarrow g(1,0) + g(1,0) \\
H(0,125) &\rightarrow Z(1,91) + \gamma(1,0) \\
H(0,125) &\rightarrow Z^{*}(1,\text{virtual}) + Z(1,91) \\
t(1/2,173) &\rightarrow b(1/2,4) + W^{+}(1,80) \\
\tau(1/2,1.7) &\rightarrow \pi(0,0.15) + \nu_{\tau}(1/2,0) \\
\tau(1/2,1.7) &\rightarrow \rho(1,0.77) + \nu_{\tau}(1/2,0) \\
Z(1,91) &\rightarrow \tau(1/2,1.7) + \bar{\tau}(1/2,1.7) \\
V^{*}(1,\text{virtual}) &\rightarrow W^{-}(1,80) + W^{+}(1,80) \\
J/\psi(1,3) &\rightarrow a_{2}(1320)(2,1.3) + \rho(1,0.77) \\
J/\psi(1,3) &\rightarrow f_{4}(2050)(4,2) + \gamma(1,0)
\end{aligned}
$$