Introduction

Quartic Gradient Flow

Numerical results

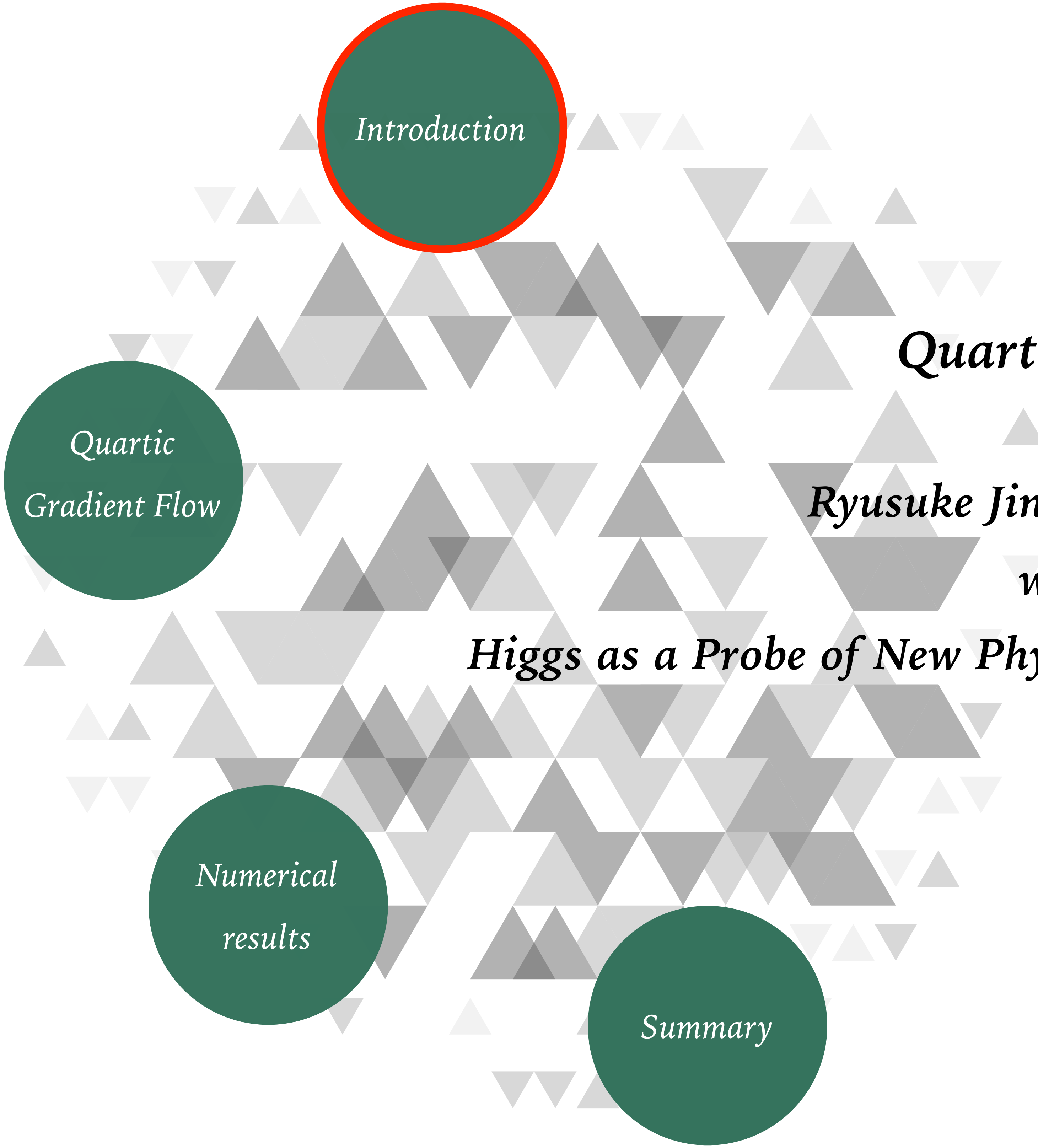
Summary

Quartic Gradient Flow

Ryusuke Jinno (RESCEU, UTokyo)

w/ Muzi Hong

Higgs as a Probe of New Physics @Osaka Univ., June 5-9, 2023



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SADDLE POINTS IN QFT

- Saddle points sometimes appear in QFT

<u>Euclidean bounce</u>

- Saddle point of the Euclidean action
- Determines the tunneling rate of a metastable vacuum

<u>Sphaleron</u>

- Saddle point of the energy in the electroweak theory
- Determines the rate of B+L violating interactions

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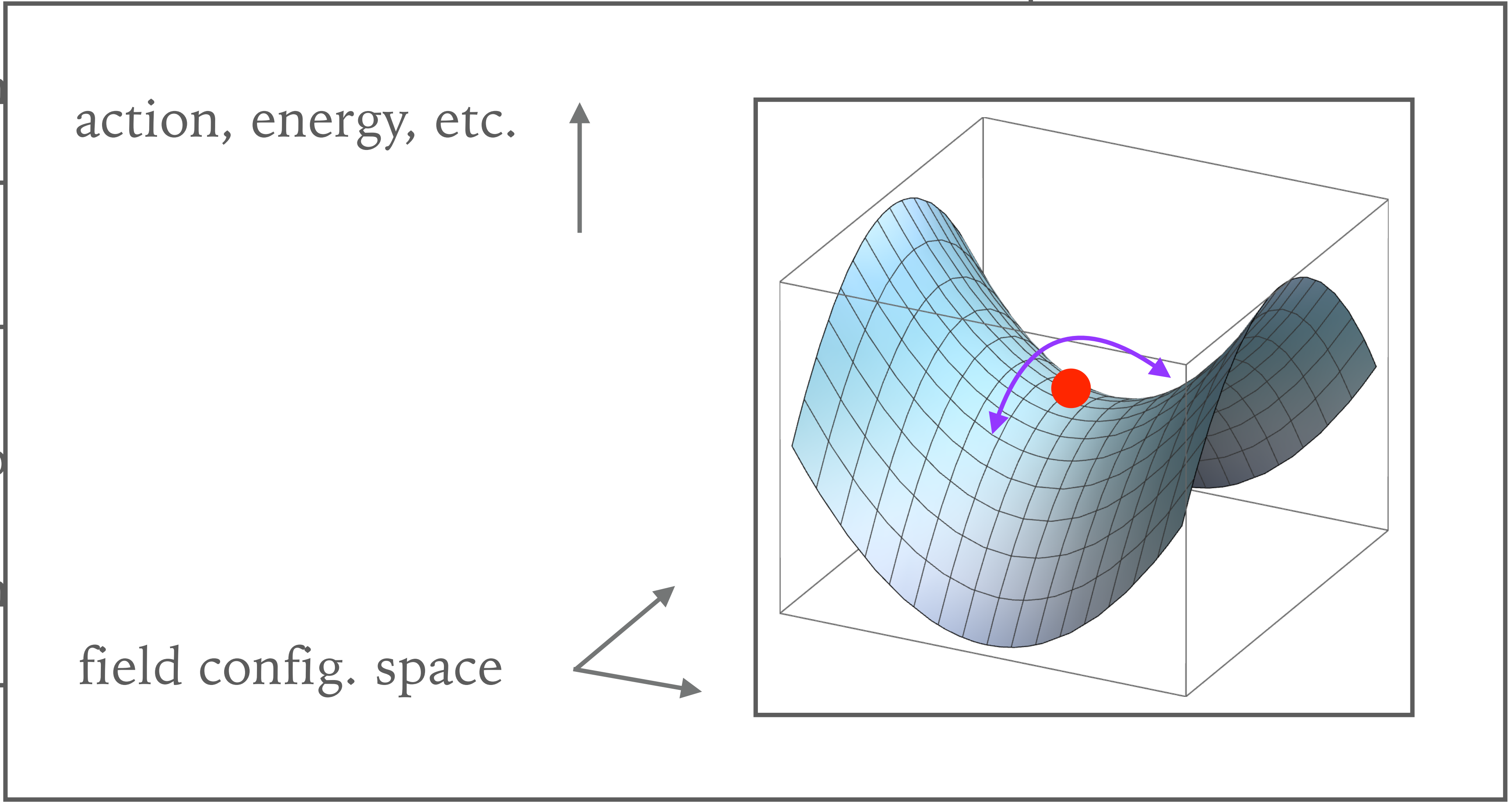

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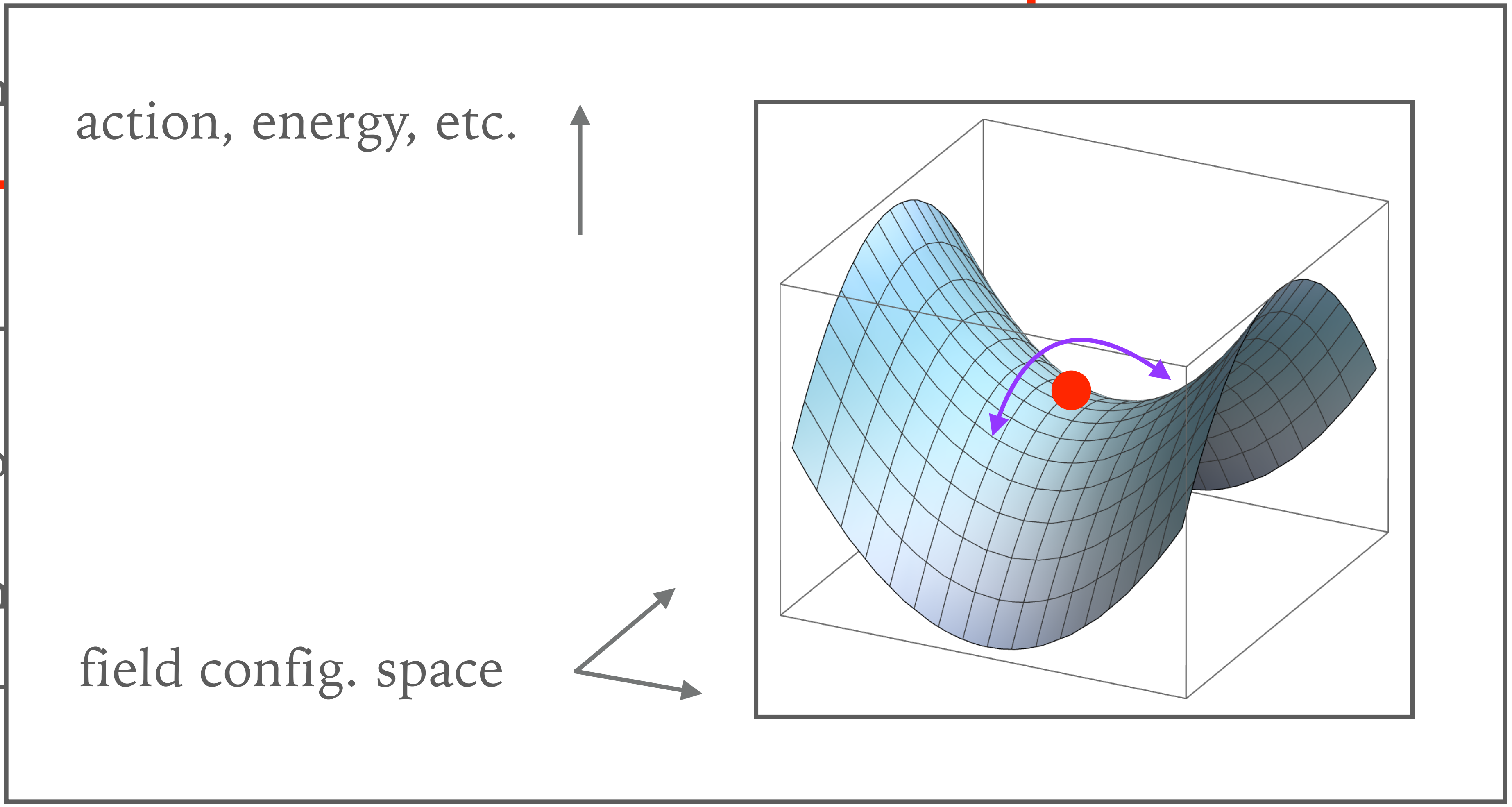
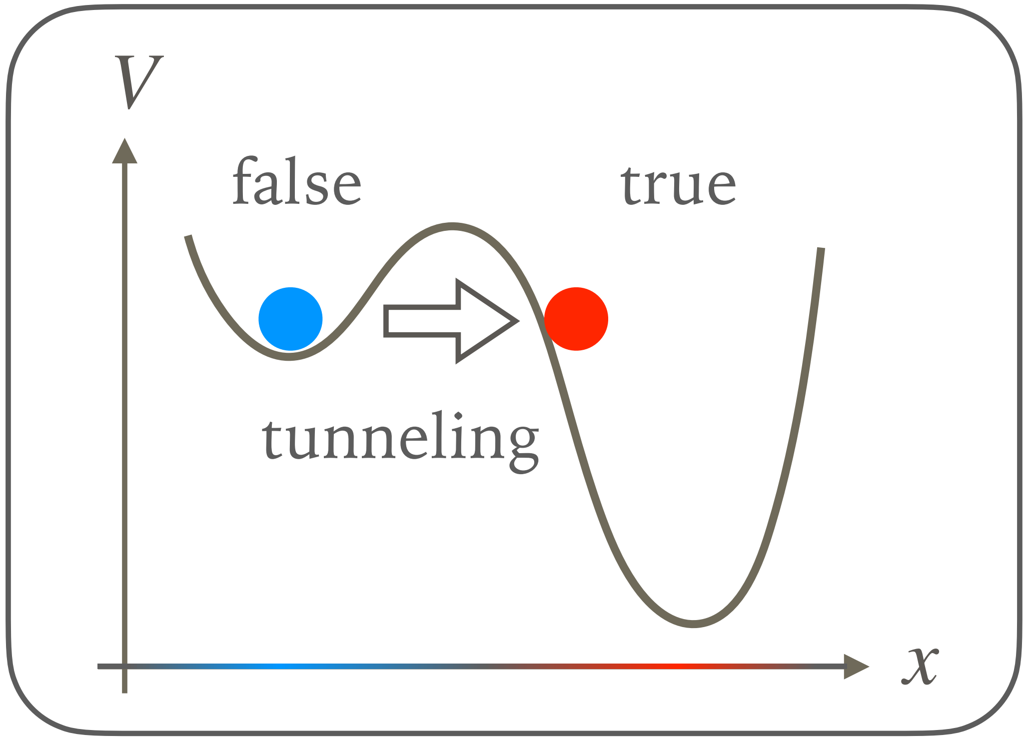


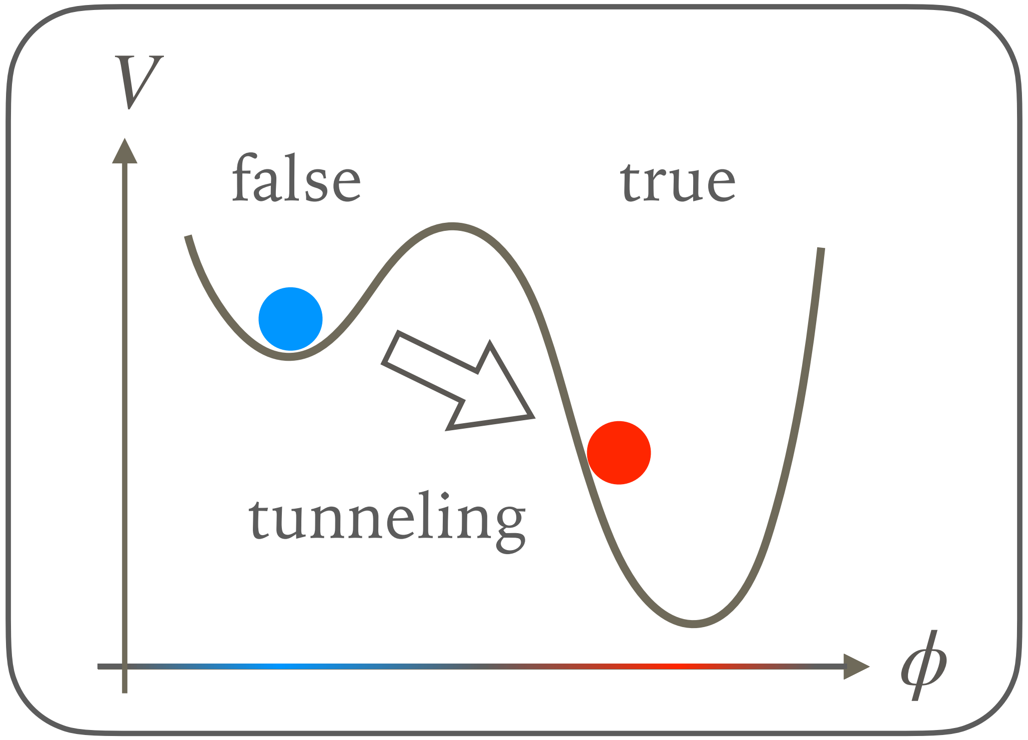
TUNNELING IN QUANTUM MECHANICS AND QFT

- Tunneling in quantum mechanics & quantum field theory

Quantum mechanics



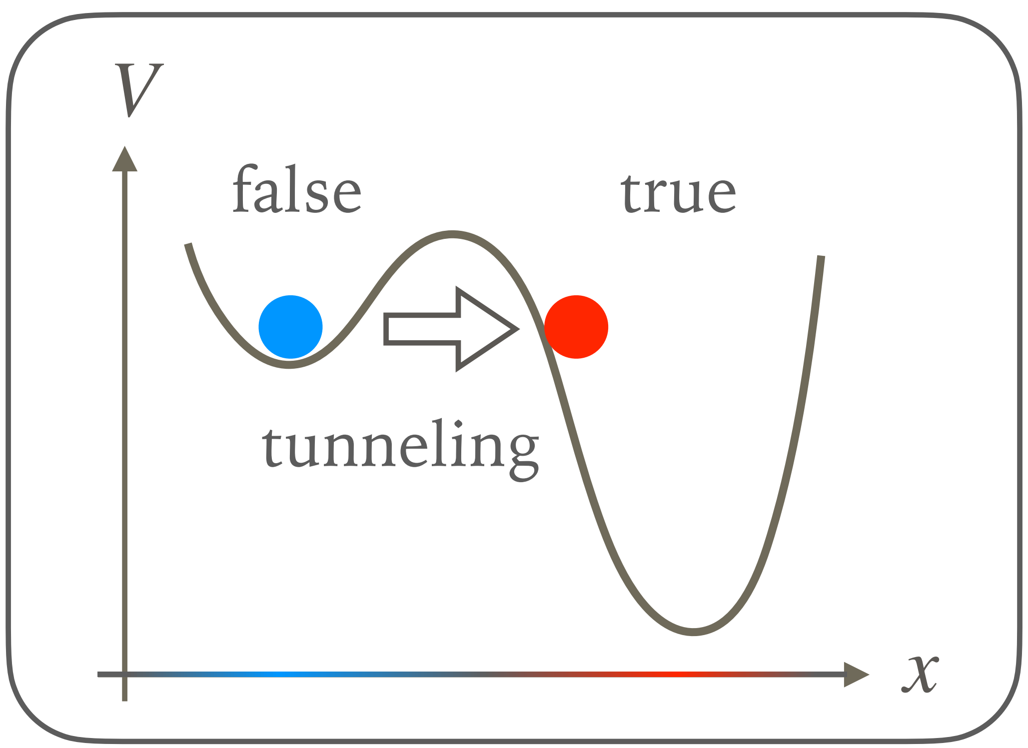
QFT



TUNNELING IN QUANTUM MECHANICS AND QFT

- Tunneling in quantum mechanics & quantum field theory

<u>Quantum mechanics</u>



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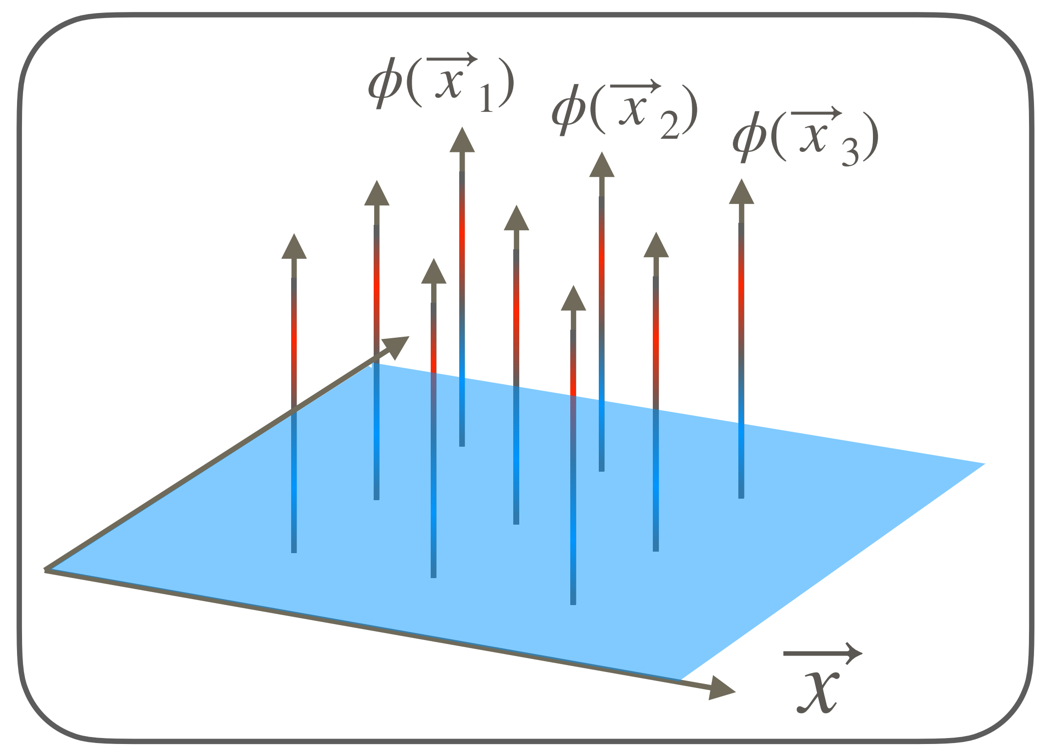

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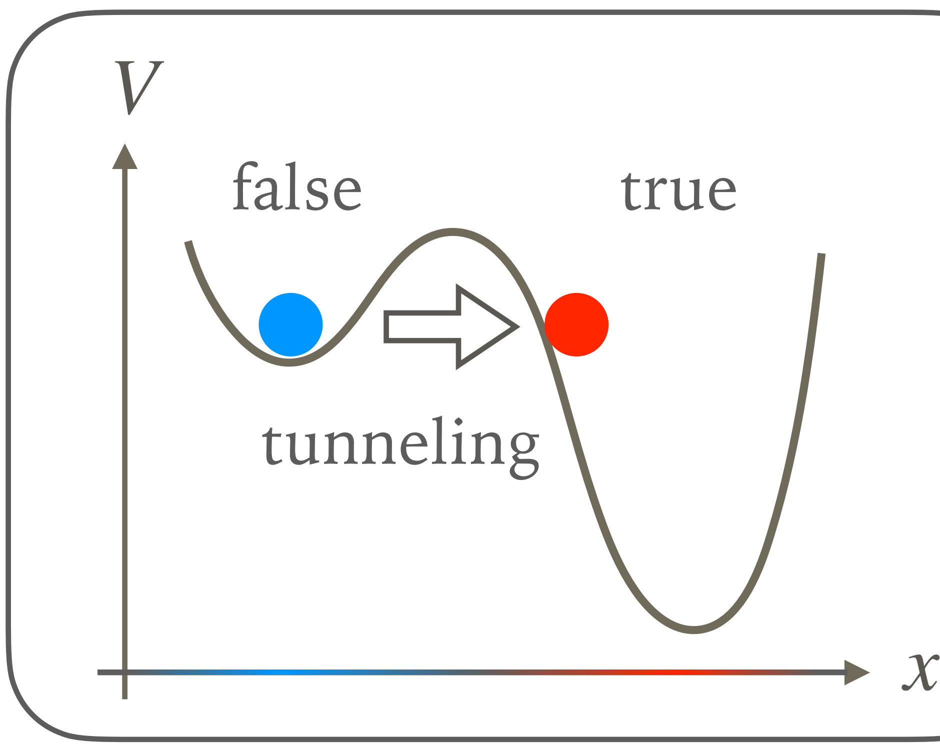
<u>Quantum mechanics</u>



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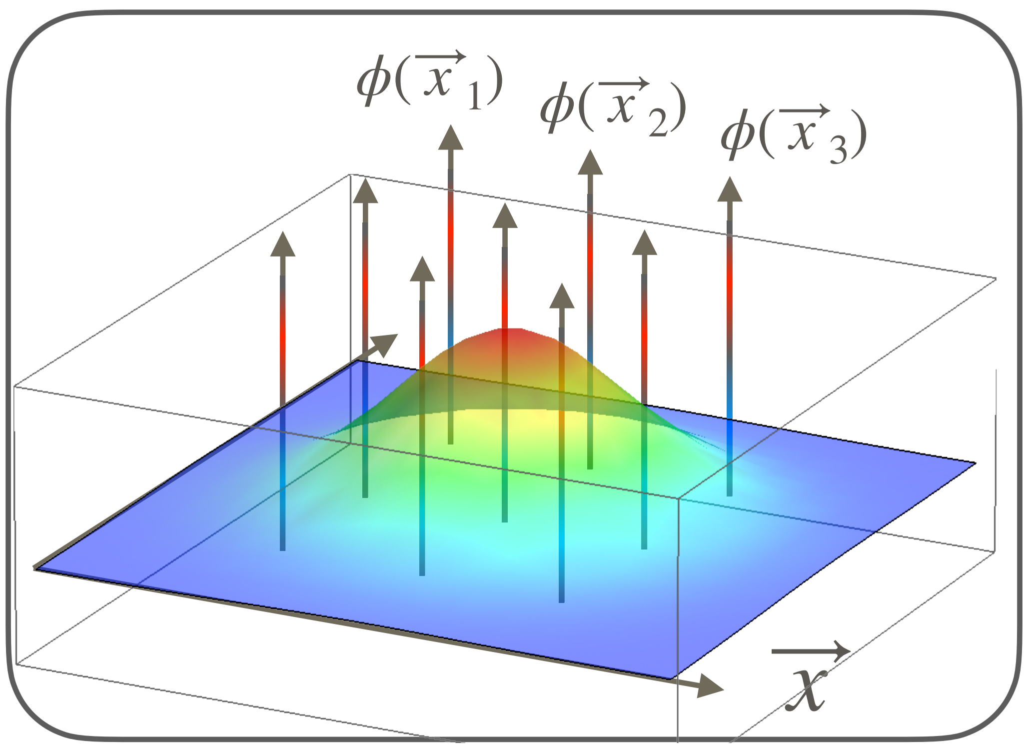
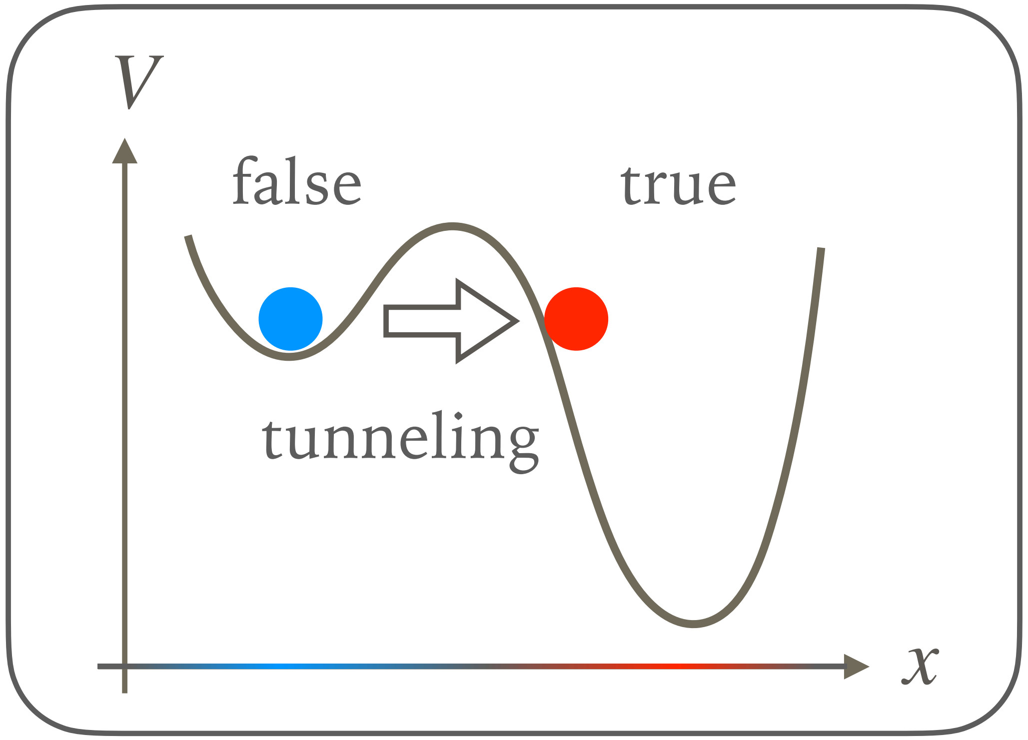


tunneling (nucleation)

TUNNELING IN QUANTUM MECHANICS AND QFT

➤ Tunneling in quantum mechanics & quantum field theory

Quantum mechanics



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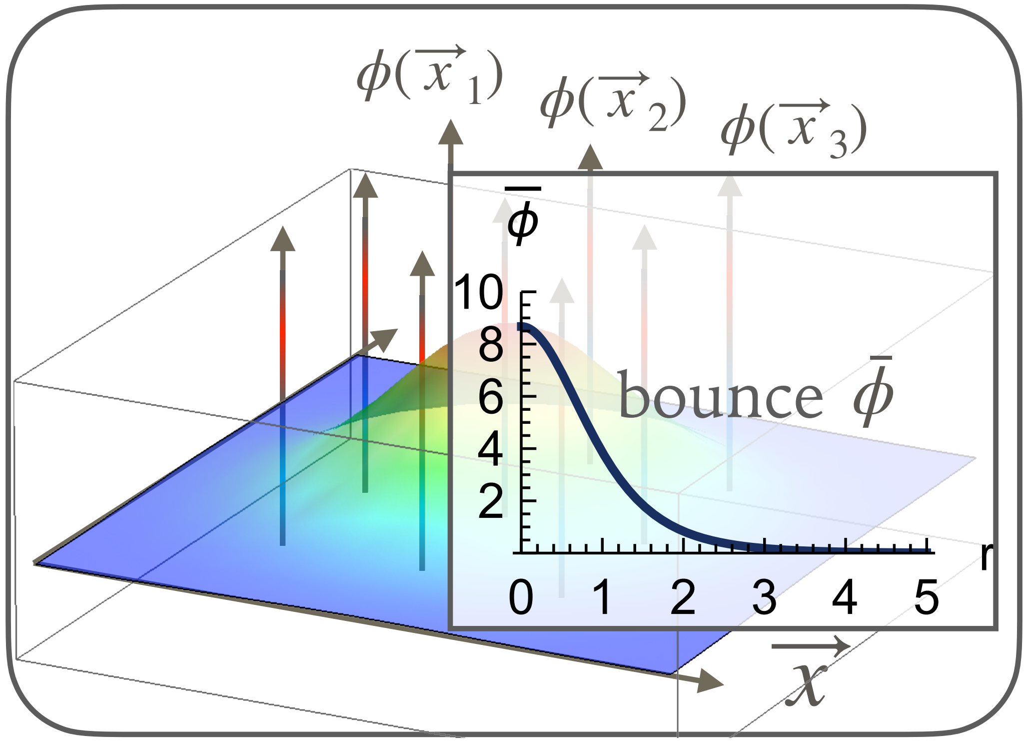


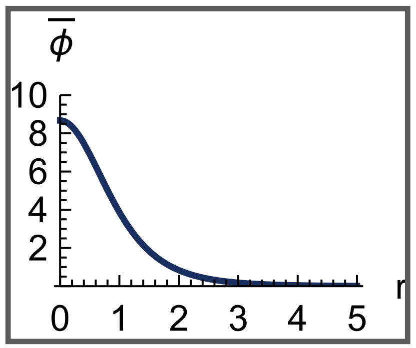
tunneling (nucleation)

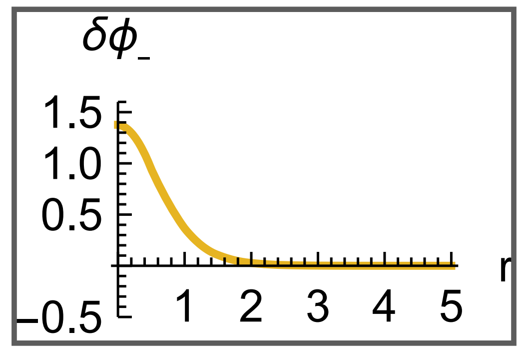
BOUNCE IS A SADDLE POINT OF THE EUCLIDEAN ACTION

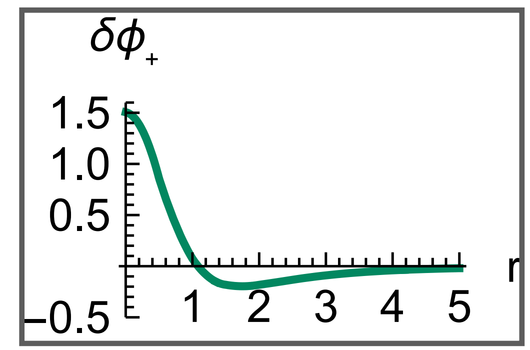
[Coleman '77]
[Callan, Coleman '77]

➤ Saddle point – what does it mean?

- When ϕ is perturbed around $\bar{\phi}$ as

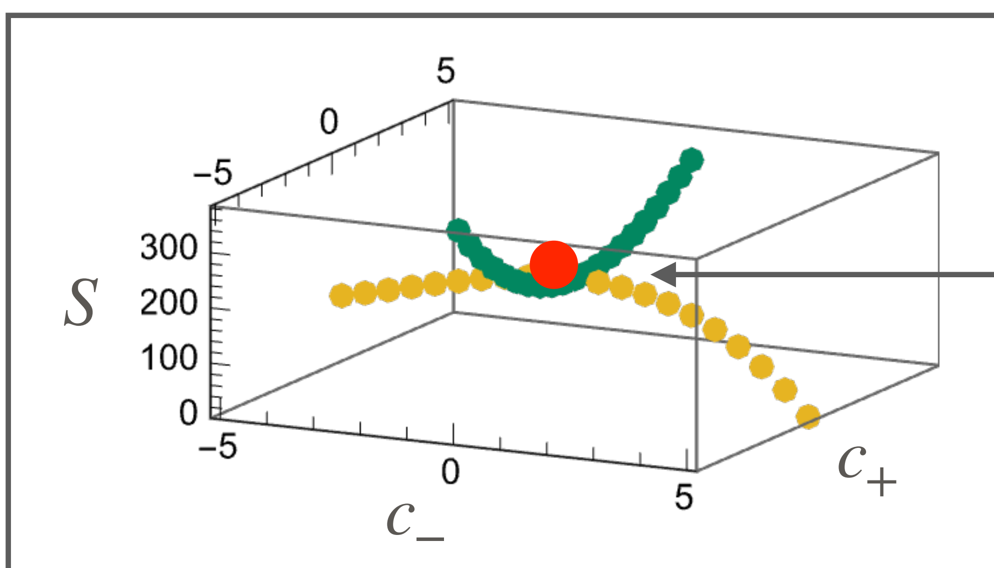
$\phi =$ 

$+ c_- \times$ 

$+ c_+ \times$ 

$+ \cdots$

Euclidean action $S = \int d^4x \left[\frac{1}{2}(\partial_E \phi)^2 + V(\phi)\right] = 2\pi^2 \int dr\ r^3 \left[\frac{1}{2}(\partial_r \phi)^2 + V(\phi)\right]$ behaves as

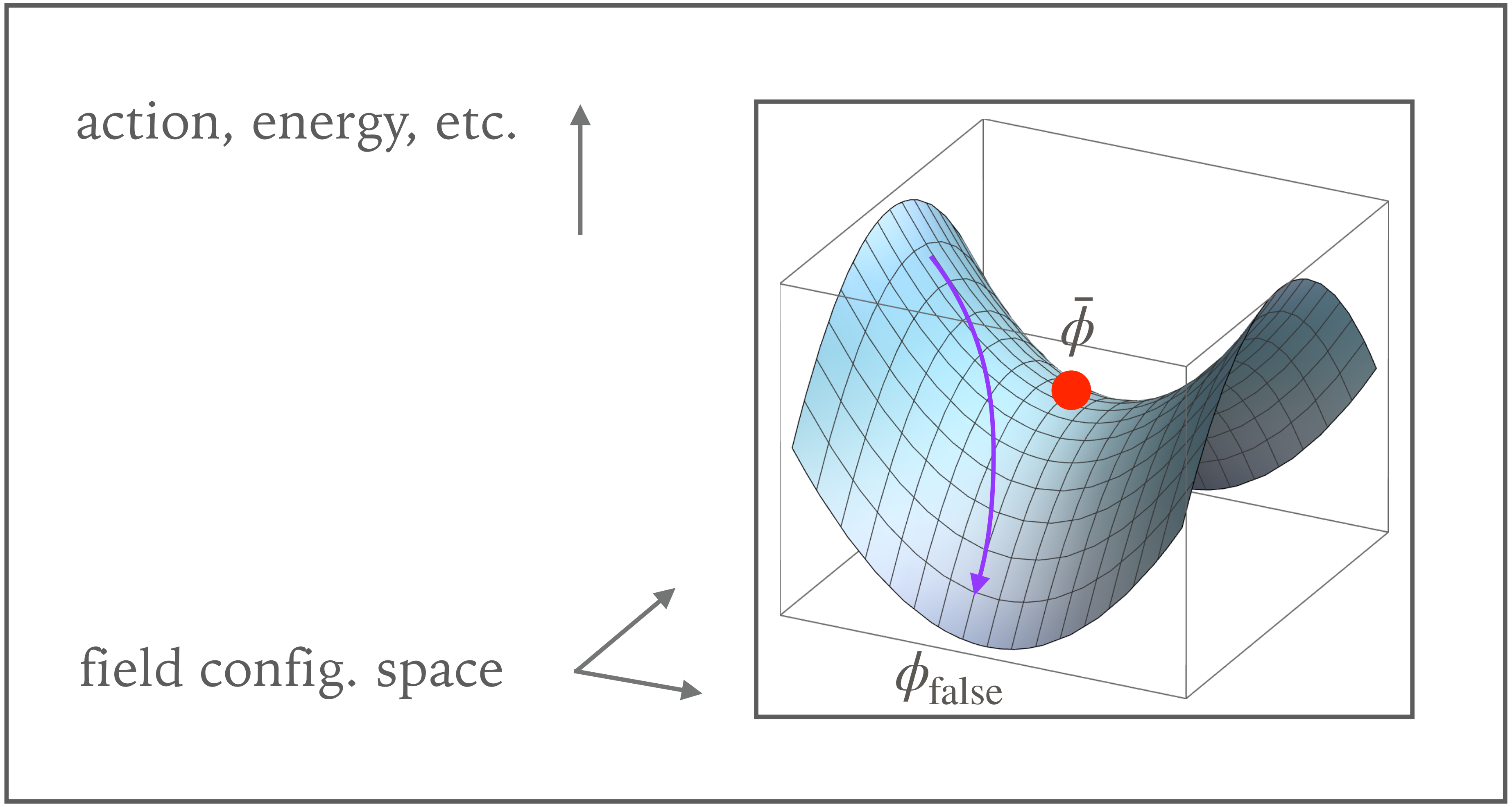


bounce $\partial_r^2 \phi + \frac{3}{r}\partial_r \phi - V'(\phi) = 0$

DIFFICULTY ABOUT THE BOUNCE

- Difficulty in calculating the bounce and other saddle points in general

  - If you naively minimize the Euclidean action, you'll get $\phi = \phi_{\rm false}$ (trivial sol.)



DIFFICULTY ABOUT THE BOUNCE

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- A more mathematical description of the difficulty

  - Suppose the configuration is close to the bounce $\phi = \bar{\phi} + \delta\phi$

  - Expand the action around the bounce: $S[\phi] \simeq S[\bar{\phi}] + \frac{1}{2}\int d^4x\ \delta\phi\, \mathscr{M}[\bar{\phi}]\, \delta\phi$

    w/ the operator $\mathscr{M}[\bar{\phi}] = \left.\frac{\delta^2 S}{\delta\phi\delta\phi}\right|_{\phi=\bar{\phi}} = -\partial_r^2 - \frac{3}{r}\partial_r + V''(\bar{\phi})$

  - The operator $\mathscr{M}[\bar{\phi}]$ has one negative eigenvalue [Callan & Coleman '77]

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GRADIENT FLOW

➤ What's gradient flow?

- Euler-Lagrange eq. is transformed into a diffusive form with fictitious time t

$$\partial_r^2 \phi + \cdots = \not{0}\ \partial_t \phi$$

- If $\partial_t \phi \to 0$ for $t \to \infty$, the resulting configuration satisfies the original equation

GRADIENT FLOW

- However, for saddle-points, naive gradient flow does not work

  - To understand this, decompose fluctuations around the bounce

$$\phi(t,r) = \bar{\phi}(r) + \delta\phi(t,r) = \bar{\phi}(r) + \sum_n c_n(t)\,\delta\phi_n(r)$$

$\delta\phi_n$: n-th eigenfunction of $\mathscr{M}[\bar{\phi}]$ with eigenvalue λ_n

  - Naive gradient flow $\partial_t\phi = \partial_r^2\phi + \frac{3}{r}\partial_r\phi - V'(\phi)$ behaves as

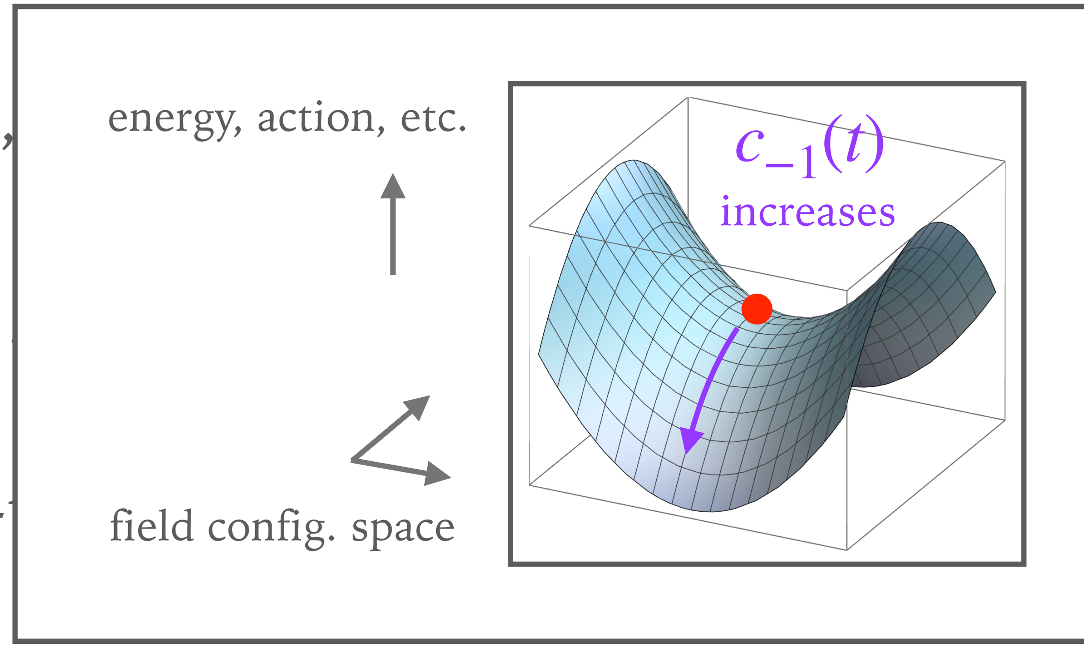
$$\sum_n \partial_t c_n(t)\,\delta\phi_n(r) \quad = \quad -\sum_n \lambda_n c_n(t)\,\delta\phi_n(r) \quad \rightarrow \quad \boxed{\partial_t c_n(t) = -\lambda_n c_n(t)}$$

  - Due to the negative eigenvalue (call it $\lambda_{-1} < 0$), $c_{-1}(t)$ does ***not*** converge to zero

GRADIENT FLOW

- However, for saddle-points, naive gradient flow does not work

  - To understand this, … bounce



$\phi(t,$ … $\phi_n(r)$

$\delta\phi_n$: n- … value λ_n

  - Naive gradient flow $\partial_t \phi = \partial_r^2 \phi + \frac{3}{r}\partial_r \phi - V'(\phi)$ behaves as

$$\sum_n \partial_t c_n(t)\, \delta\phi_n(r) \;=\; -\sum_n \lambda_n c_n(t)\, \delta\phi_n(r) \quad \to \quad \boxed{\partial_t c_n(t) = -\lambda_n c_n(t)}$$

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QUARTIC GRADIENT FLOW

- We consider a variant of gradient flow

$$\boxed{\partial_t \phi + \mathscr{M}\left[\frac{\delta S}{\delta \phi}\right] = 0} \quad \text{with} \quad \mathscr{M} = \frac{\delta^2 S}{\delta\phi\delta\phi}$$

meaning $$\partial_t \phi + \left(-\partial_r^2 - \frac{3}{r}\partial_r + V''(\phi)\right)\left(-\partial_r^2\phi - \frac{3}{r}\partial_r\phi + V'(\phi)\right) = 0$$

QUARTIC GRADIENT FLOW

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$$\boxed{\partial_t \phi + \mathscr{M}\left[\frac{\delta S}{\delta \phi}\right] = 0} \qquad \text{with} \qquad \mathscr{M} = \frac{\delta^2 S}{\delta\phi\delta\phi}$$

- Intuitive explanation on how it works

  - Again, decompose the fluctuation around the bounce $\phi(t,r) = \bar{\phi}(r) + \sum_n c_n(t)\,\delta\phi_n(r)$

  - Previously, the naive gradient flow failed because $\frac{\delta S}{\delta \phi} \sim \sum_n \lambda_n c_n(t)\,\delta\phi_n(r)$

    but now the corresponding term behaves $\mathscr{M}\left[\frac{\delta S}{\delta \phi}\right] \sim \sum_n \lambda_n^2 c_n(t)\,\delta\phi_n(r)$

  - So, the coefficients behave $\boxed{\partial_t c_n(t) = -\lambda_n^2 c_n(t)}$ $\quad c_n(t)$ decreases for all n

- *Introduction*
- *Quartic Gradient Flow*
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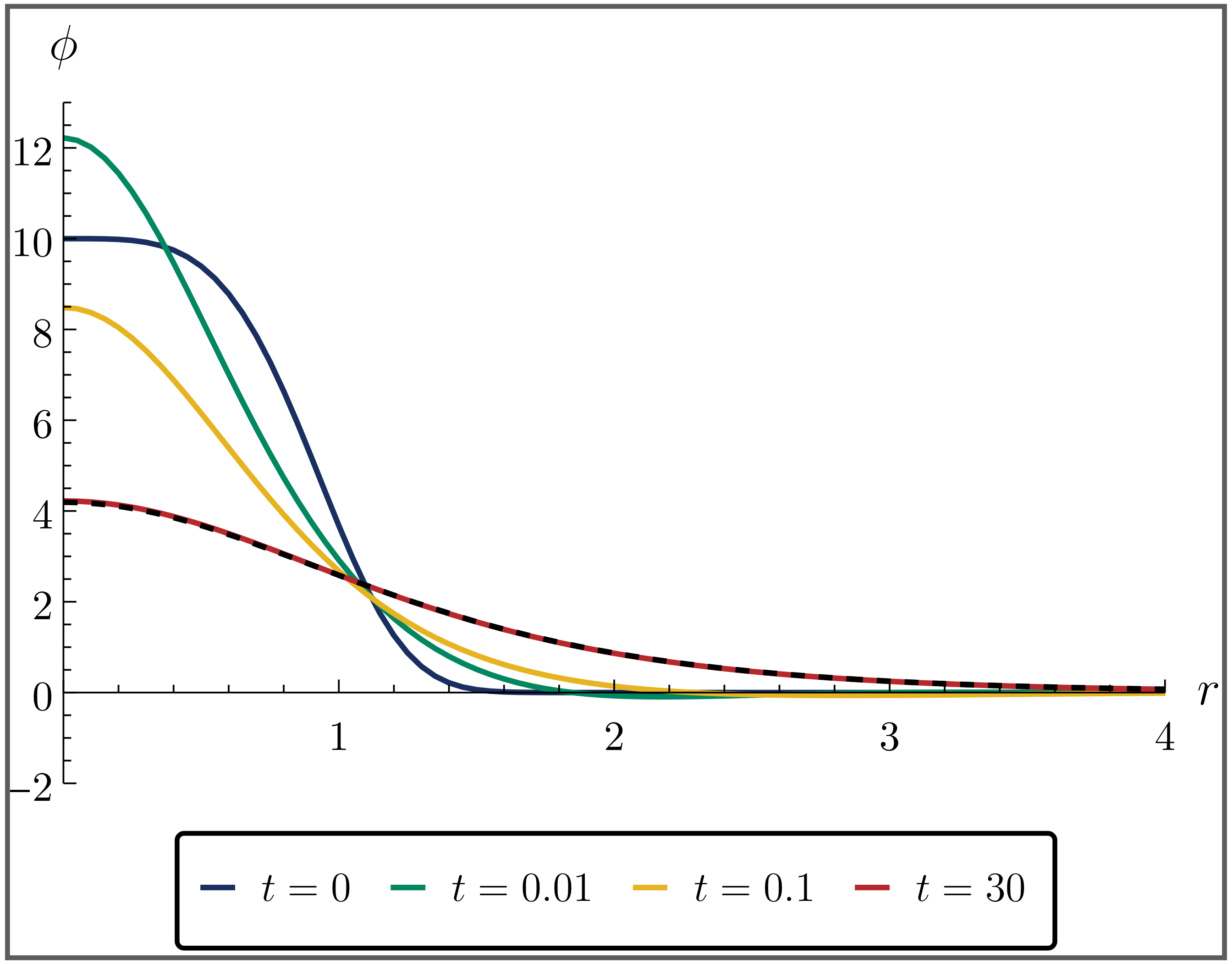
- *Summary*

NUMERICAL RESULTS

➤ Example 1: $V(\phi) = \dfrac{\phi^2}{2} - \dfrac{\phi^3}{3}$

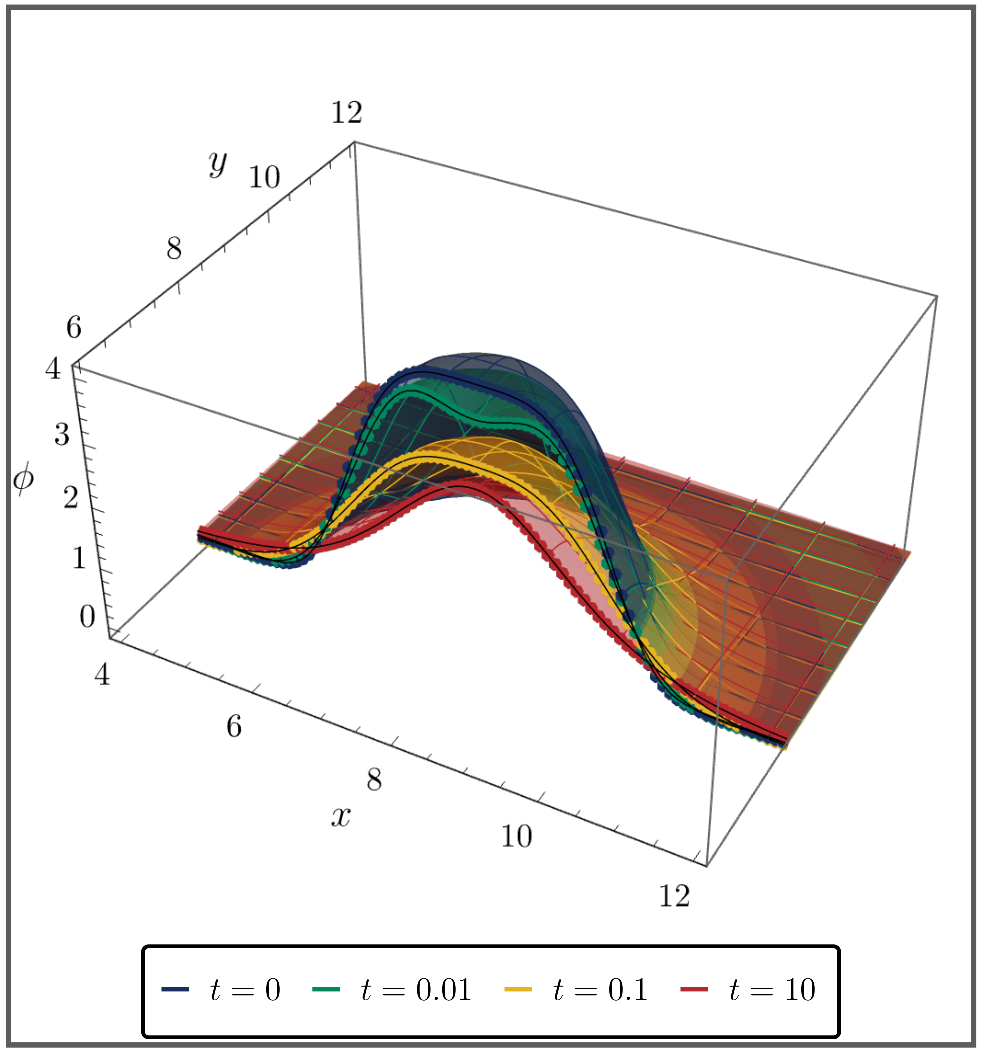
- We first assume spherical symmetry, $S = \int 2\pi^2 r^3\, dr \left[\dfrac{1}{2}(\partial_r \phi)^2 + V(\phi)\right]$



NUMERICAL RESULTS

- Example 1: $V(\phi) = \dfrac{\phi^2}{2} - \dfrac{\phi^3}{3}$

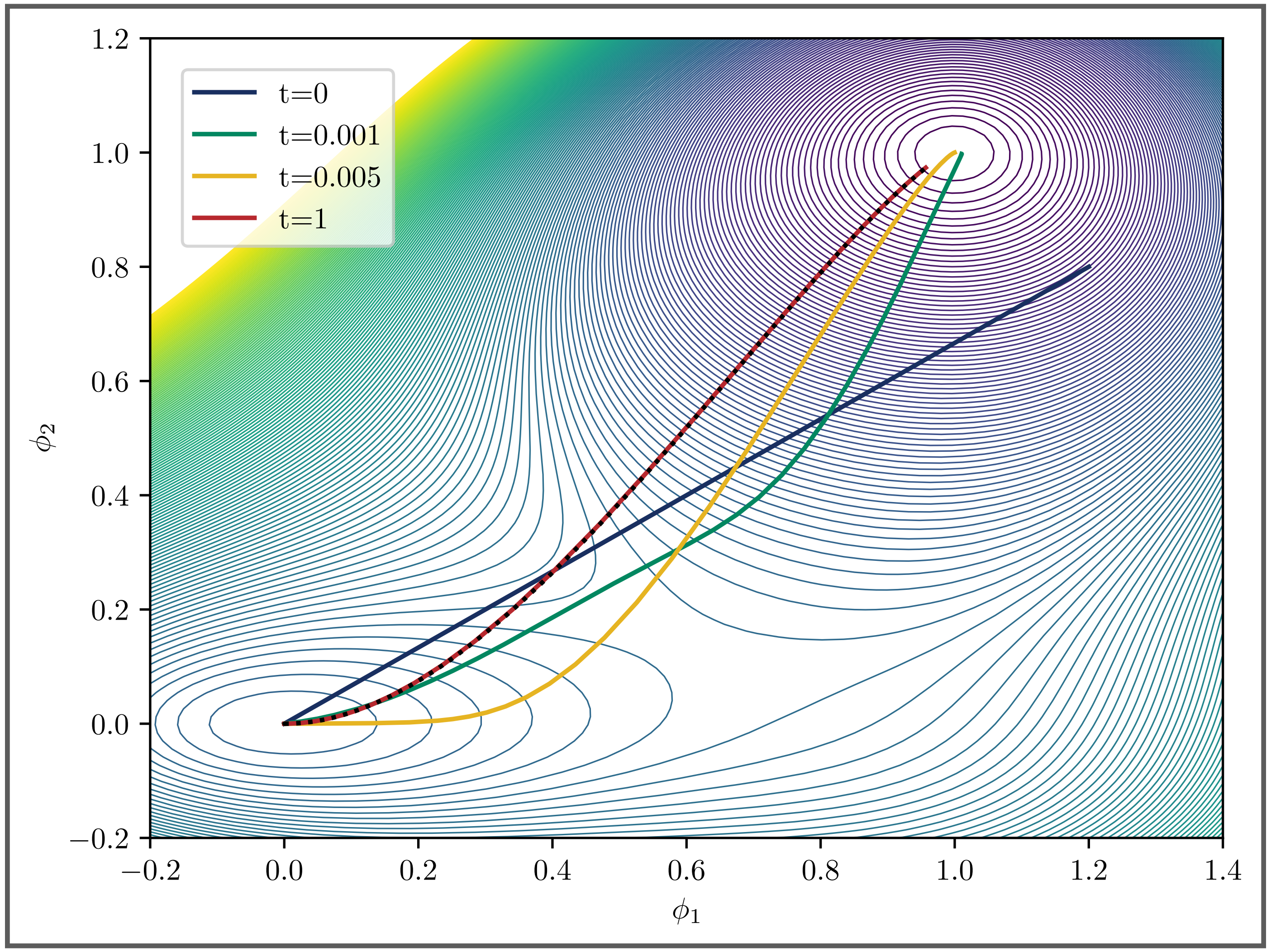
  - We can also play without spherical symmetry, $S = \int dxdy \left[\frac{1}{2}(\partial\phi)^2 + V(\phi)\right]$



NUMERICAL RESULTS

➤ Example 2: $V(\phi_1, \phi_2) = (\phi_1^2 + 5\phi_2^2)[5(\phi_1 - 1)^2 + (\phi_2 - 1)^2] + 80\left(\frac{\phi_2^4}{4} - \frac{\phi_2^3}{3}\right)$

- With spherical symmetry $S = \int 2\pi^2 r^3\, dr \left[\frac{1}{2}(\partial\phi_1)^2 + \frac{1}{2}(\partial\phi_2)^2 + V(\phi)\right]$



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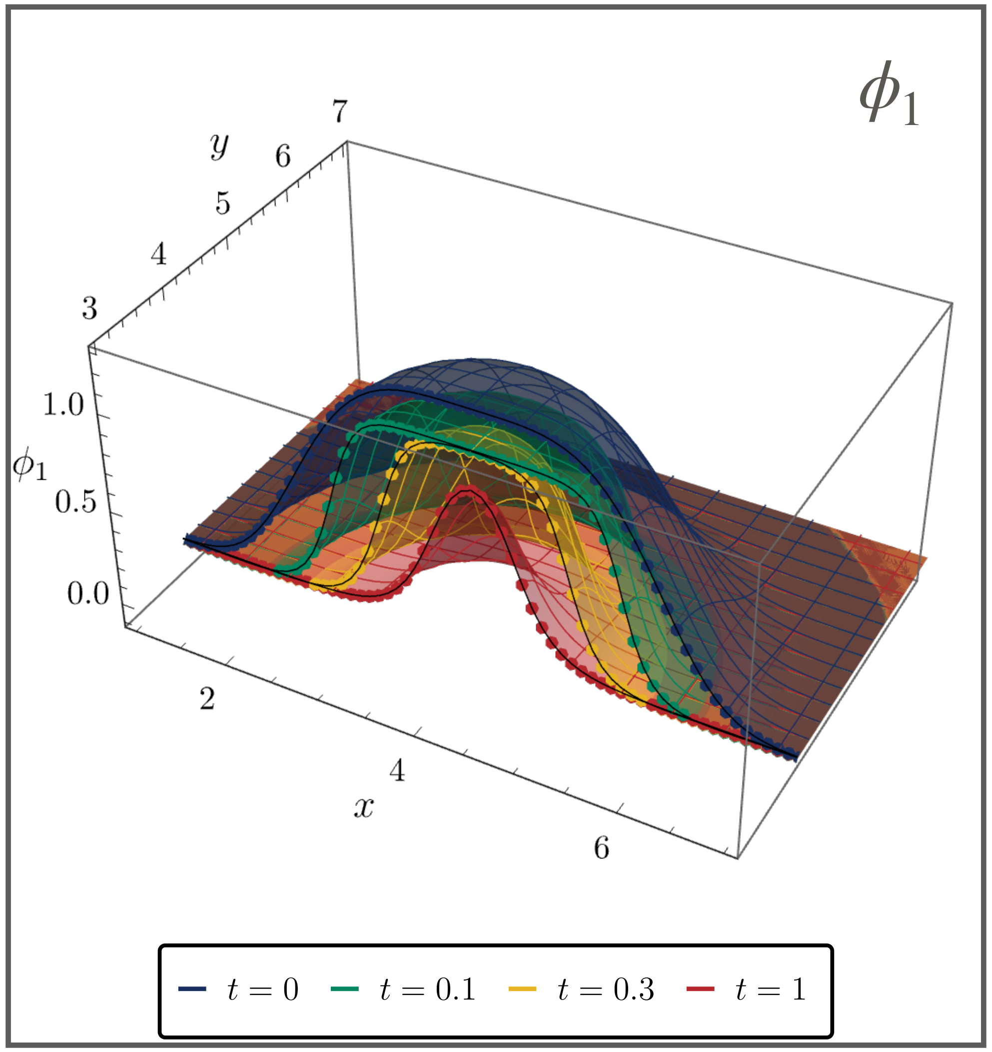
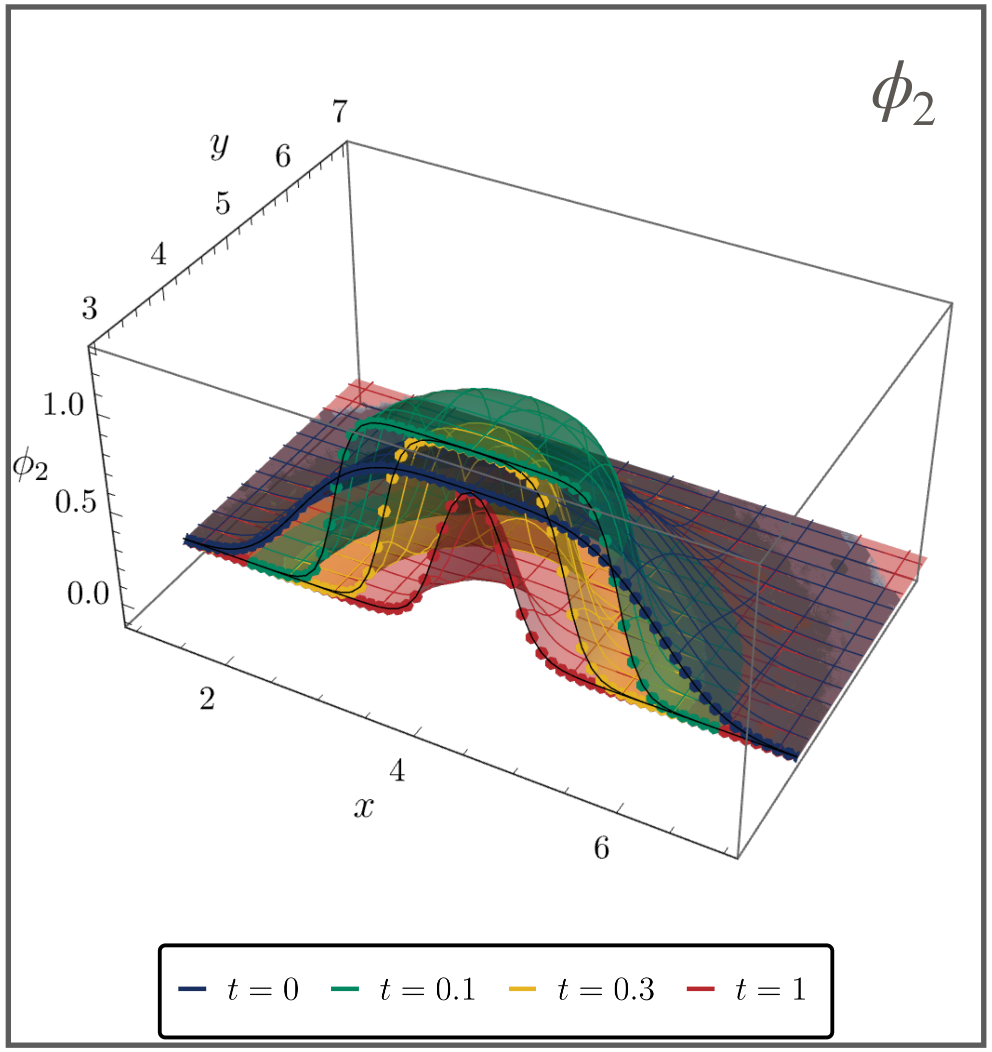
- Without spherical symmetry $S = \int dxdy \left[\frac{1}{2}(\partial\phi_1)^2 + \frac{1}{2}(\partial\phi_2)^2 + V(\phi)\right]$





DISCUSSION

- ➤ Various methods have been proposed for the bounce calcualtion

  - Squared EOM [Moreno, Quiros, Seco '98, John '98]

    Saddle points become local minima for the new action $S' = \int d^4x \left(\frac{\delta S}{\delta \phi} \right)^2$

  - Gradient flow [Moroi, Chigusa, Shoji '19] see also [Sato '19, Hamada, Kikuchi '20]

    To aviod flowing to $\phi = 0$, add a new term $\partial_t \phi + \frac{\delta S}{\delta \phi} - \beta \left\langle \frac{\delta S}{\delta \phi} \middle| g \right\rangle g = 0$

  - Many others:

    dilatational maximization [Claudson, Hall, Hinchliffe '83]

    improved action [Kusenko '95, +Langacker, Segre '96, Dasgupta '96]

    backstep [Cline, Espinosa, Moore, Riotto '98, Cline, Moore, Servant '99]

    improved poetntial [Konstandin, Huber '06, Park '10]

    path deformation [Wainwright '11]

    perturbative method [Akula, Balazs, White '16, Athron et al. '19]

    multiple shooting [Masoumi, Olum, Shlaer '16]

    tunneling potential [Espinosa '18]

    polygon approximation [Guada, Maiezza, Nemevsek '18]

    machine learning [Jinno '18, Piscopo, Spanowski, Waite '19]

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- ➤ Quartic Gradient Flow can be understood as

  "gradient flow for the squared-EOM action"

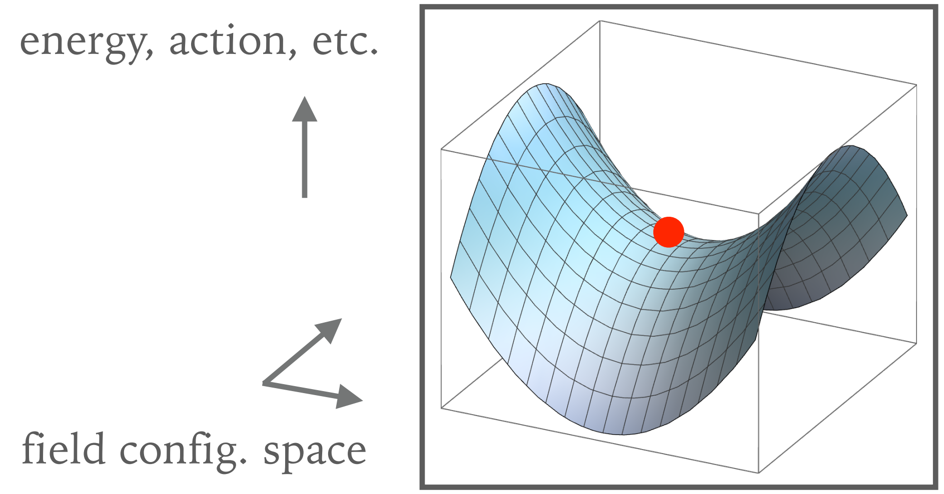
$$\partial_t \phi + \frac{\delta S'}{\delta \phi} = \partial_t \phi + \frac{\delta^2 S}{\delta \phi \delta \phi} \frac{\delta S}{\delta \phi} = 0$$

SUMMARY

➤ Saddle points often appear in QFT

- Euclidean bounce

- Sphalerons



➤ We propose a gradient-flow-like method to calculate such configurations

$$\boxed{\partial_t \phi + \mathscr{M}\left[\frac{\delta S}{\delta \phi}\right] = 0} \qquad \text{with} \quad \mathscr{M} = \frac{\delta^2 S}{\delta\phi\delta\phi}$$

➤ In this method, deviation from the saddle converges to zero

$$\boxed{\partial_t c_n(t) = -\lambda_n^2 c_n(t)} \qquad \text{with} \quad \phi(t,r) = \bar{\phi}(r) + \sum_n c_n(t)\,\delta\phi_n(r)$$

Backup

CALCULATION OF TUNNELING RATE À LA COLEMAN

[Coleman '77]
[Callan, Coleman '77]

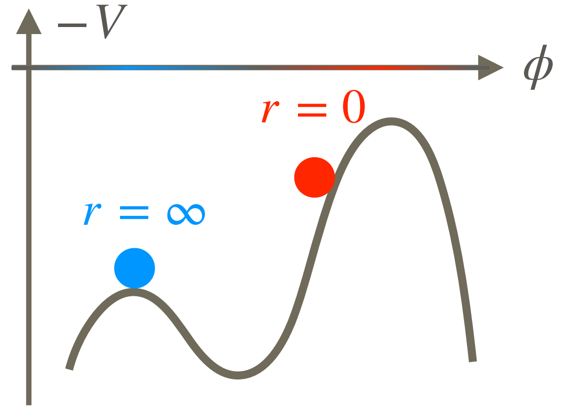
- Tunneling rate is estimated from the Euclidean partition function

$$Z = \langle \phi_{\rm false} | e^{-HT} | \phi_{\rm false} \rangle \simeq \int_{\phi(t_i)=\phi_{\rm false}}^{\phi(t_f)=\phi_{\rm false}} \mathscr{D}\phi \ e^{-S[\phi]}$$

w/ Euclidean action $\quad S = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 + V(\phi) \right]$

- The path integral is evaluated with the saddle point $\bar{\phi}$, called "bounce"

bounce $\bar{\phi}$ =



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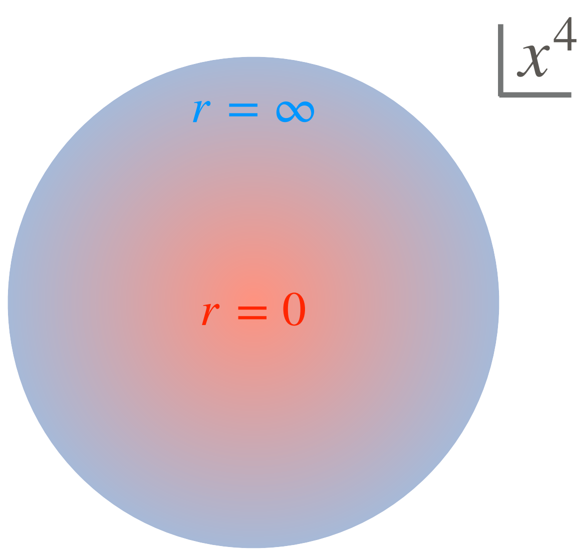

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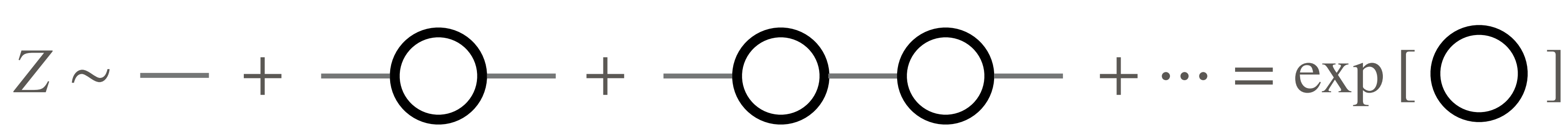

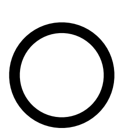 $= K \cdot VT \cdot e^{-S[\bar{\phi}]}$: contribution from one-bounce