

A smoking gun signature of 3HDM

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work in progress
and arXiv: 2306.XXXX

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The Standard Model and its shortcomings

- A Higgs boson discovered
- No significant deviation from the SM
- No signs of new physics

But no explanation for

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- Extra sources of CPV
- Fermion mass hierarchy
- Vacuum stability
- Dark Matter & ...

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The recently discovered 125-GeV scalar can be a portal to the dark sector.

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The recently discovered 125-GeV scalar can be a portal to the dark sector.

problem: Current direct and indirect detection as well as relic density bound strongly constrain the simplistic possibilities.

BSMs to the rescue

Scalar extensions with a Z_2 symmetry: 3HDM: SM + 2 scalar doublets

CP-conserving I(2+1)HDM

$$\phi_1, \phi_2, \phi_3$$

$$g_{Z_2} = \text{diag}(-1, -1, +1)$$

$$\text{VEV} = (0, 0, v)$$

[JHEP 1401 (2014) 052], [Phys. Rev. D 90, 075015 (2014)], [arXiv: 1907.12522]

The scalar potential with explicit CPC

$$V_{3HDM} = V_0 + V_{Z_2}$$

$$V_0 = \sum_i^3 \left[-\mu_i^2 (\phi_i^\dagger \phi_i) + \lambda_{ii} (\phi_i^\dagger \phi_i)^2 \right] \\ + \sum_{i,j}^3 \left[\lambda_{ij} (\phi_i^\dagger \phi_i) (\phi_j^\dagger \phi_j) + \lambda'_{ij} (\phi_i^\dagger \phi_j) (\phi_j^\dagger \phi_i) \right]$$

$$V_{Z_2} = -\mu_{12}^2 (\phi_1^\dagger \phi_2) + \lambda_1 (\phi_1^\dagger \phi_2)^2 + \lambda_2 (\phi_2^\dagger \phi_3)^2 + \lambda_3 (\phi_3^\dagger \phi_1)^2 + h.c.$$

The Z_2 symmetry

$$\phi_1 \rightarrow -\phi_1, \quad \phi_2 \rightarrow -\phi_2, \quad \phi_3 \rightarrow \phi_3, \quad \text{SM fields} \rightarrow \text{SM fields}$$

[Phys.Lett.B 695 (2011) 459-462]

Parameters of the model

- All parameters of the potential to be real
- “dark” parameters $\lambda_1, \lambda_{11}, \lambda_{22}, \lambda_{12}, \lambda'_{12}$ (values have been fixed in agreement with the theoretical constraints.)
- $\mu_1^2 = n\mu_2^2, \quad \lambda_3 = n\lambda_2, \quad \lambda_{31} = n\lambda_{23}, \quad \lambda'_{31} = n\lambda'_{23}$
- fixed by the Higgs mass $\mu_3^2 = v^2\lambda_{33} = m_h^2/2$

6 important parameters

- Mass splittings μ_{12}^2, λ_2
- Higgs-DM coupling $\lambda_2, \lambda_{23}, \lambda'_{23}$
- Mass scale of inert particles μ_2^2

[*Eur. Phys. J. C*80(2020)2, 135]

The mass eigenstates

The doublet compositions

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{H_1^0 + iA_1^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{H_2^0 + iA_2^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} G^+ \\ \frac{v + h + iG^0}{\sqrt{2}} \end{pmatrix}$$

The mass eigenstates

$$\begin{aligned} H_1 &= \cos \theta_h H_1^0 + \sin \theta_h H_2^0, & A_1 &= \cos \theta_a A_1^0 + \sin \theta_a A_2^0 \\ H_2 &= \cos \theta_h H_2^0 - \sin \theta_h H_1^0, & A_2 &= \cos \theta_a A_2^0 - \sin \theta_a A_1^0 \\ H_1^\pm &= \cos \theta_c \phi_1^\pm + \sin \theta_c \phi_2^\pm, & H_2^\pm &= \cos \theta_c \phi_2^\pm - \sin \theta_c \phi_1^\pm \end{aligned}$$

H_1 is assumed to be the DM candidate

- Input parameters:**

DM mass m_{H_1} , Mass of second CP-even scalar m_{H_2} ,

Higgs-DM coupling $g_{H_1 H_1 h}$, angles θ_c , θ_a and n .

Constraints

- **Vacuum stability:** scalar potential V bounded from below
- **Perturbative unitarity:** eigenvalues Λ_i of the high-energy scattering matrix fulfill the condition $|\Lambda_i| < 8\pi$
- **Collider:** bounds on masses of the scalars
 - Limits from gauge bosons width:
$$m_{H_i} + m_{H_j^\pm} \geq m_W, \quad m_{H_i} + m_{H_j} \geq m_Z, \quad 2 m_{H_{1,2}^\pm} \geq m_Z$$
 - Limits on charged scalar mass and lifetime:
$$m_{H_i^\pm} \geq 70 \text{ GeV}, \quad \tau \leq 10^{-7} \text{ s} \rightarrow \Gamma_{\text{tot}} \geq 10^{-18} \text{ GeV}$$
 - Allowed by Higgs invisible branching ratio, $Br(h \rightarrow \text{inv.}) < 19\%$
 - Allowed by Higgs total decay width, $\mu^{\text{tot}}(h)$ as well as Higgs signal strength data.
- **DM constraints:** Relic density, Direct and indirect detection bounds.

Relevant DM scenario

In the low mass region ($m_{H_1} < m_Z$)

We can have multiple scenarios but we are interested in

- **Scenario I: coannihilation** with $H_2, A_{1,2}$:

$$M_{H_1} \approx M_{A_1} \approx M_{H_2} \approx M_{A_2} < M_{H_1^\pm, H_2^\pm}$$

[JHEP 09 (2018) 059]

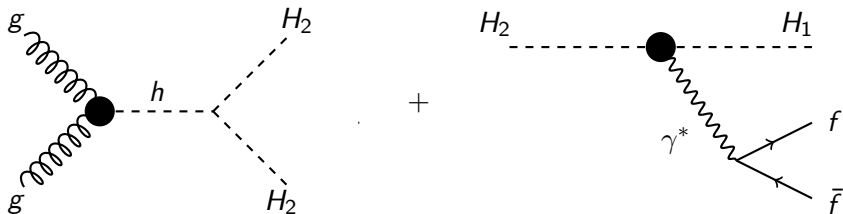
CPC DM at the LHC

Looking for a **smoking-gun** signal of the 3HDM which is not allowed in the 2HDM with one inert doublet.

The $\cancel{E}_T + 4l$ signature at the LHC

Smoking gun Signal

- We focused on,



In the CPC I(2+1)HDM, a process contributing to the $\cancel{E}_T l^+ l^- l^+ l^-$ signature is

$$gg \rightarrow h \rightarrow H_2 H_2 \rightarrow H_1 H_1 \gamma^* \rightarrow H_1 H_1 l^+ l^- l^+ l^-,$$

where the off-shell γ^* splits into $l^+ l^-$ and the H_1 states escape detection and will give \cancel{E}_T .

Smoking gun Signal

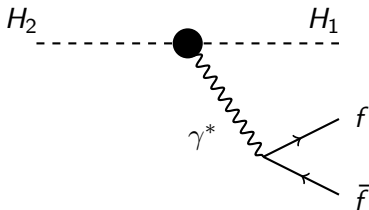


Figure: Radiative decay of the heavy neutral particle $H_2 \rightarrow H_1 \gamma^* \rightarrow H_1 l^+ l^-$.

The $\cancel{E}_T + 4l$ signature at the LHC

Smoking gun Signal

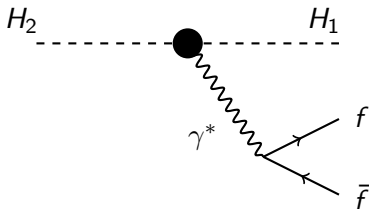


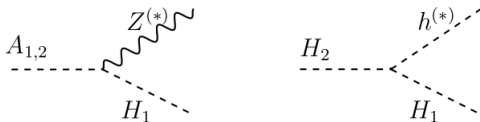
Figure: Radiative decay of the heavy neutral particle $H_2 \rightarrow H_1 \gamma^* \rightarrow H_1 l^+ l^-$.

- $m_{H_2} - m_{H_1}$ is very small
- H_2 , into the lightest inert state, H_1 , and a virtual photon, which then would split into a light $l\bar{l}$ pair.

The $\cancel{E}_T + 4l$ signature at the LHC

Inert cascade decays at the LHC

When there is a **large mass splitting** between DM and other inert particles:



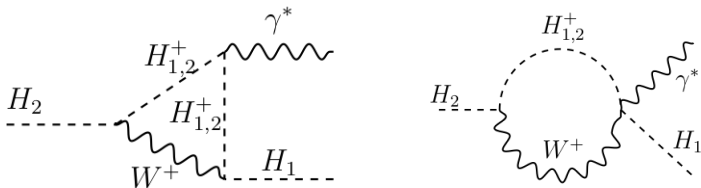
It can give the **tree level** process $E_{miss}^T + l^+l^-l^+l^-$:

$$pp \rightarrow H_2H_2/A_{1,2}A_{1,2} \rightarrow H_1H_1Z^*Z^* \rightarrow H_1H_1l^+l^-l^+l^-$$

The $\cancel{E}_T + 4l$ signature at the LHC

Inert cascade decays at the LHC

When there is a **small mass splitting** between DM and other inert particles (winning scenarios):

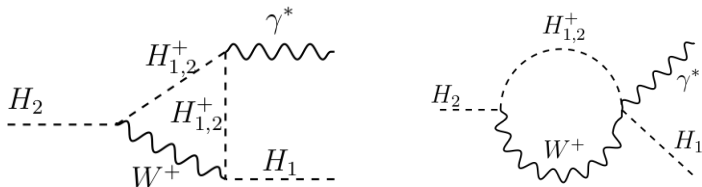


It can give the **loop level** process $E_{miss}^T + l^+l^-l^+l^-$:

$$pp \rightarrow H_2 H_2 / A_{1,2} A_{1,2} \rightarrow H_1 H_1 \gamma^* \gamma^* \rightarrow H_1 H_1 l^+ l^- l^+ l^-$$

Inert cascade decays at the LHC

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The **smoking gun** channel

- We are looking for Benchmarks with small mass gap (Δm) between H_2 and H_1

BPs	m_{H_1}	m_{H_2}	Δm	n	$g_{H_1 H_1 h}$	θ_h	$\sigma(pp \rightarrow H_1 H_1 2\mu^+ 2\mu^-)$
I_5^{50}	50	55	5	0.83	0.01	0.105	6.923 fb
I_{10}^{50}	50	60	10	0.70	0.01	0.103	4.0 fb

Table: Parameter choices of our Benchmark points (BPs)

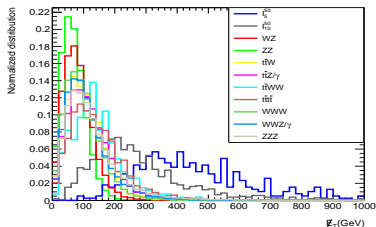
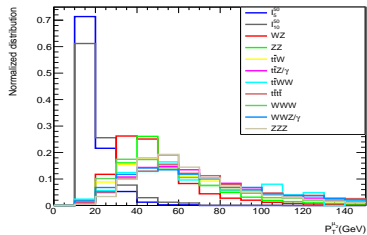
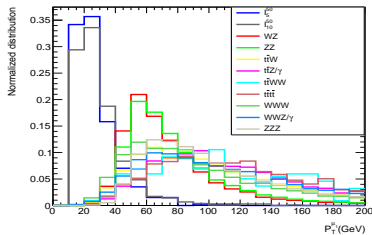
Signal and backgrounds

- **Signal:** At least 3-lepton with at least one pair of Opposite sign $\mu + \bar{E}_T$.

Signal and backgrounds

- **Signal:** At least 3-lepton with at least one pair of Opposite sign $\mu + \bar{\mu}$.
- **Backgrounds:**
 - 1) **Di-boson**, $VV(V : W, Z, \gamma)$: Mainly WZ/γ and ZZ have large contribution where both V can decay leptonically.
 - 2) **Tri-boson**, $VVV(V : W, Z, \gamma)$: Mainly consider WWZ/γ , WWW and ZZZ . All vector bosons are supposed to decay leptonically.
 - 3) $t\bar{t}X$, ($X: W, Z, \gamma, WW, t\bar{t}$): The fully leptonic decay mode of $t\bar{t}X$ can give us atleast three lepton with at least one pair of μ with opposite charge.

Distributions



Distributions

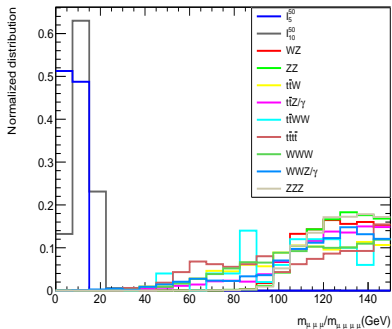
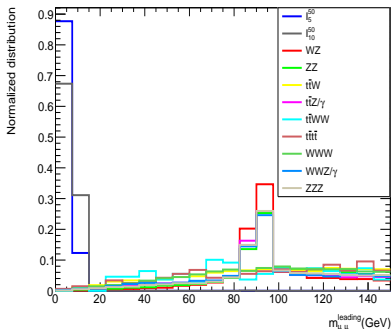


Figure: Normalized distribution of invariant mass of two leading muons and invariant mass of all muons for signal BPs and backgrounds.

Distributions

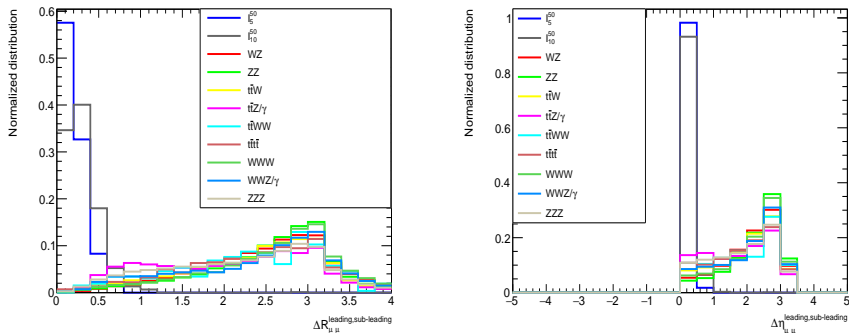


Figure: Normalized Distribution of ΔR and $\Delta\eta$ of leading and sub-leading muon for signal BPs and backgrounds.

Results

Cuts:

1) **Pre-selection Cut:** We are looking for events where we can have at least three or four muons in final state with no $b - jet$.

2) **Cut-A:**

- $m_{\mu\mu}^{leading}$ and $m_{\mu\mu}^{\Delta R_{min}}$ has to be less than 50 GeV.
- $m_{\mu\mu\mu}/m_{\mu\mu\mu\mu}$ has to be less than 70 GeV.

3) **Cut-B:**

- $m_{\mu\mu}^{leading}$ and $m_{\mu\mu}^{\Delta R_{min}}$ has to be less than 20 GeV.
- $m_{\mu\mu\mu}/m_{\mu\mu\mu\mu}$ has to be less than 30 GeV.
- $\Delta R_{\mu\mu}^{leading,sub-leading} < 1.0$ and $\Delta R_{\mu\mu}^{leading,sub-sub-leading} < 1.2$.
- $\Delta\eta_{\mu\mu}^{leading,sub-leading} < 1.0$ and $\Delta\eta_{\mu\mu}^{leading,sub-sub-leading} < 1.0$.

Results

Datasets	Cross-section (fb)	Pre-selection Cut	Cut – A	Cut – B
I_5^{50}	6.961	17	16	16
I_{10}^{50}	3.733	59	58	58
WZ	163.4068	97691	9	0
ZZ	16.554	22614	2	0
WWW	0.248862	185	3	0
WWZ/γ	0.04978	96	1	0
ZZZ	9.3516×10^{-3}	16	0	0
$t\bar{t}W$	0.606	114	2	0
$t\bar{t}Z/\gamma$	0.3045	136	1	0
$t\bar{t}WW$	1.279×10^{-3}	0	0	0
$t\bar{t}t\bar{t}$	1.51359×10^{-3}	0	0	0

Table: Signal and background events cross-section at and Number of Events after cuts at $\sqrt{s} = 14$ TeV and $\mathcal{L} = 3000fb^{-1}$ for $\geq 3\text{-}\mu + E_T$ final state.

Significance

- we calculated the projected significance (\mathcal{S}) in the $3\mu + \cancel{E}_T$ channel for each benchmark point, for **14 TeV LHC** with **3000 fb⁻¹**. The significance \mathcal{S} is defined as follows:

$$\mathcal{S} = \sqrt{2[(S + B)\text{Log}(1 + \frac{S}{B}) - S]} \quad (1)$$

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$$\mathcal{S} = \sqrt{2[(S + B)\text{Log}(1 + \frac{S}{B}) - S]} \quad (1)$$

BP	$\mathcal{S}(\text{Pre} - \text{selection})$	$\mathcal{S}(\text{Cut} - \text{A})$
I_5^{50}	0.05 σ	3.77 σ
I_{10}^{50}	0.17 σ	13.67 σ

- with **Cut – B** we will end up with only signal events
- $L = 300 \text{ fb}^{-1}$ also can give us fully background eliminated signals after **Cut – A** itself.

Summary

- Inert Doublet Model
 - a good DM model with rich phenomenology, however, **very constrained**.
- CP-Conserving in **I(2+1)HDM**
 - SM-like active sector: $H_3 \equiv h^{SM}$
 - The inert sector: $H_{1,2}, A_{1,2}, H_{1,2}^\pm$, $H_1 \rightarrow \text{DM}$
 - less constrained DM sector with low mass DM particle
 - New **Smoking-gun** signature at the LHC: m_{H_2} and m_{H_1} are close
 - Good signal significance in $3l + \cancel{E}_T$ channel over backgrounds at HL-LHC.

Back-up slides

BSMs to the rescue

Solution: Scalar extensions with a Z_2 symmetry:

- 2HDM: SM + scalar doublet
 - Type-I, Type-II, ...: $\phi_1, \phi_2 \Rightarrow$ CPV, DM
 - IDM - I(1+1)HDM: $\phi_1, \phi_2 \Rightarrow$ DM, CPV
- 3HDM: SM + 2 scalar doublets
 - Weinberg model: $\phi_1, \phi_2, \phi_3 \Rightarrow$ CPV, DM
 - I(1+2)HDM: $\phi_1, \phi_2, \phi_3 \Rightarrow$ DM, CPV
 - I(2+1)HDM: $\phi_1, \phi_2, \phi_3 \Rightarrow$ CPV, DM

Dark Matter (DM)

around 25 % of the Universe is:

- cold
- non-baryonic
- neutral
- very weakly interacting
 - ⇒ **Weakly Interacting Massive Particle**
- stable due to the discrete symmetry

$$\underbrace{\text{DM DM} \rightarrow \text{SM SM}}_{\text{pair annihilation}}, \quad \underbrace{\text{DM} \not\rightarrow \text{SM}, \dots}_{\text{stable}}$$

Higgs-portal DM

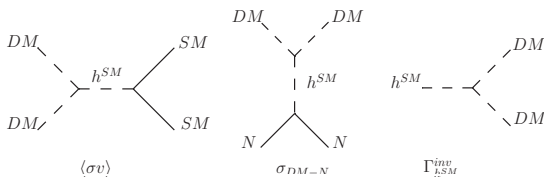
Simplest realisation: the SM with $\Phi_{SM} + Z_2$ -odd scalar S :

$$S \rightarrow -S, \quad \text{SM fields} \rightarrow \text{SM fields}$$

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}(\partial S)^2 - \frac{1}{2}m_{DM}^2 S^2 - \lambda_{DM} S^4 - \lambda_{hDM} \Phi_{SM}^2 S^2$$

Higgs-portal interaction:

SM sector $\overset{\text{Higgs}}{\longleftrightarrow}$ DM sector



given by the same coupling

2HDM with CP-violation (DM)

The general scalar potential

$$\begin{aligned}
 V = & \mu_1^2(\phi_1^\dagger\phi_1) + \mu_2^2(\phi_2^\dagger\phi_2) - \left[\mu_3^2(\phi_1^\dagger\phi_2) + h.c. \right] \\
 & + \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
 & + \left[\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + h.c. \right].
 \end{aligned}$$

$$Z_2 \text{ symmetry} \Rightarrow \lambda_6 = \lambda_7 = 0$$

The doublets composition with $\tan\beta = v_2/v_1$

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + h_1^0 + ia_1^0}{\sqrt{2}} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 + h_2^0 + ia_2^0}{\sqrt{2}} \end{pmatrix}$$

CP-mixed mass eigenstates

- 2×2 charged mass-squared matrix

$$\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} \Rightarrow \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

- 4×4 neutral mass-squared matrix

$$\begin{pmatrix} a_1^0 \\ h_1^0 \\ a_2^0 \\ h_2^0 \end{pmatrix} \Rightarrow \begin{pmatrix} G^0 \\ H_1 \\ H_2 \\ H_3 \end{pmatrix}$$

CPV severely constrained from SM data

The Inert Doublet Model (CPV)

Scalar potential V invariant under a Z_2 -transformation:

$$Z_2 : \quad \phi_1 \rightarrow \phi_1, \quad \phi_2 \rightarrow -\phi_2, \quad \text{SM fields} \rightarrow \text{SM fields}$$

$$\begin{aligned} V = & -\frac{1}{2} \left[m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 \right] + \frac{1}{2} \left[\lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 \right] \\ & + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \frac{1}{2} \lambda_5 \left[(\phi_1^\dagger \phi_2)^2 + (\phi_2^\dagger \phi_1)^2 \right] \end{aligned}$$

- All parameters are real \rightarrow no CP violation
- Only ϕ_1 couples to fermions
- The whole Lagrangian is explicitly Z_2 -symmetric

DM in the IDM

The Inert minimum

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Z_2 -symmetry survives the EWSB

$$g_{Z_2} = \text{diag}(+1, -1)$$

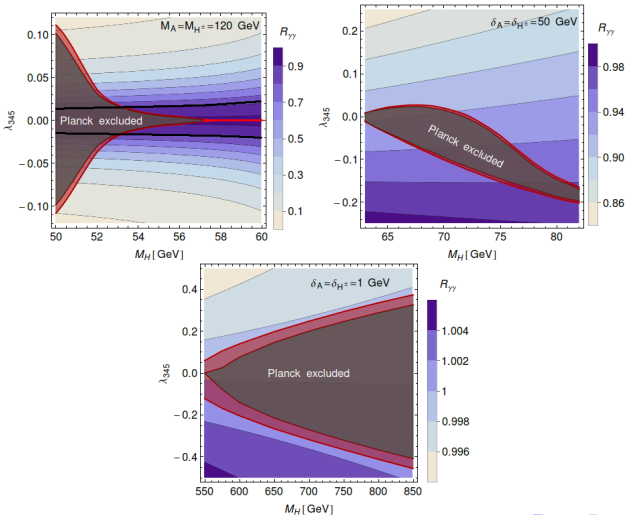
$$\text{VEV} = (v, 0)$$

- ϕ_1 is active (plays the role of the SM-Higgs)
- ϕ_2 is “dark” or inert (with 4 dark scalars H, A, H^\pm)

→ the lightest scalar is a candidate for the DM

$h \rightarrow \gamma\gamma$ signal strength

(JHEP 09 (2013) 055)



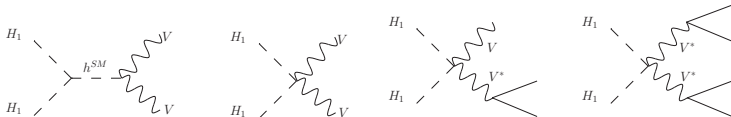
CP-conserving $I(2+1)$ HDM

Dark Matter Annihilation

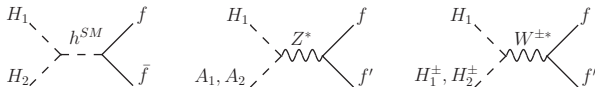
- annihilation through Higgs into fermions; dominant channel for $M_{DM} < M_h/2$



- annihilation to gauge bosons; crucial for heavy masses



- coannihilation; when particles have similar masses



DM Annihilation Scenarios

(A) **no coannihilation effects:**

$$M_{H_1} < M_{H_2, A_1, A_2, H_1^\pm, H_2^\pm}$$

(I) **coannihilation** with $H_2, A_{1,2}$:

$$M_{H_1} \approx M_{A_1} \approx M_{H_2} \approx M_{A_2} < M_{H_1^\pm, H_2^\pm}$$

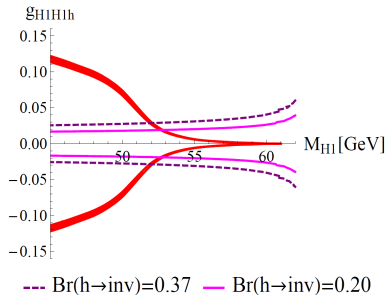
(G) **coannihilation** with $H_2, A_{1,2}, H_{1,2}^\pm$:

$$M_{H_1} \approx M_{A_1} \approx M_{H_2} \approx M_{A_2} \approx M_{H_1^\pm, H_2^\pm}$$

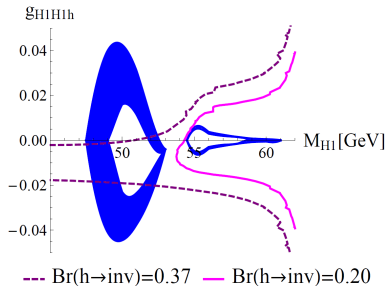
(H) **coannihilation** with A_1, H_1^\pm :

$$M_{H_1} \approx M_{A_1} \approx, H_1^\pm < M_{H_2, A_2, H_2^\pm}$$

LHC vs Planck $M_{DM} < M_h/2$



case A



case I

• $Br(h \rightarrow inv) < 37\%$ & $\Omega_{DM} h^2 \Rightarrow$

- Case A: $M_{DM} \gtrsim 53$ GeV
- Case I: most masses are OK

Masses and mixing angles

- **The CP-even neutral inert fields**

The pair of inert neutral scalar gauge eigenstates, H_1^0, H_2^0 , are rotated by

$$R_{\theta_h} = \begin{pmatrix} \cos \theta_h & \sin \theta_h \\ -\sin \theta_h & \cos \theta_h \end{pmatrix}, \text{ with } \tan 2\theta_h = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda_{\phi_1} - \mu_2^2 + \Lambda_{\phi_2}}$$

into the mass eigenstates, H_1, H_2 , with squared masses

$$m_{H_1}^2 = (-\mu_1^2 + \Lambda_{\phi_1}) \cos^2 \theta_h + (-\mu_2^2 + \Lambda_{\phi_2}) \sin^2 \theta_h - 2\mu_{12}^2 \sin \theta_h \cos \theta_h,$$

$$m_{H_2}^2 = (-\mu_1^2 + \Lambda_{\phi_1}) \sin^2 \theta_h + (-\mu_2^2 + \Lambda_{\phi_2}) \cos^2 \theta_h + 2\mu_{12}^2 \sin \theta_h \cos \theta_h,$$

$$\text{where } \Lambda_{\phi_1} = \frac{1}{2}(\lambda_{31} + \lambda'_{31} + 2\lambda_3)v^2, \quad \Lambda_{\phi_2} = \frac{1}{2}(\lambda_{23} + \lambda'_{23} + 2\lambda_2)v^2.$$

Masses and mixing angles

- The charged inert fields**

The pair of inert charged gauge eigenstates, ϕ_1^\pm, ϕ_2^\pm , are rotated by

$$R_{\theta_c} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}, \text{ with } \tan 2\theta_c = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda'_{\phi_1} - \mu_2^2 + \Lambda'_{\phi_2}}$$

into the mass eigenstates, H_1^\pm, H_2^\pm , with squared masses

$$m_{H_1^\pm}^2 = (-\mu_1^2 + \Lambda'_{\phi_1}) \cos^2 \theta_c + (-\mu_2^2 + \Lambda'_{\phi_2}) \sin^2 \theta_c - 2\mu_{12}^2 \sin \theta_c \cos \theta_c,$$

$$m_{H_2^\pm}^2 = (-\mu_1^2 + \Lambda'_{\phi_1}) \sin^2 \theta_c + (-\mu_2^2 + \Lambda'_{\phi_2}) \cos^2 \theta_c + 2\mu_{12}^2 \sin \theta_c \cos \theta_c,$$

where $\Lambda'_{\phi_1} = \frac{1}{2}(\lambda_{31})v^2, \quad \Lambda'_{\phi_2} = \frac{1}{2}(\lambda_{23})v^2.$

Masses and mixing angles

- **The CP-odd neutral inert fields**

The pair of inert pseudo-scalar gauge eigenstates, A_1^0, A_2^0 , are rotated by

$$R_{\theta_a} = \begin{pmatrix} \cos \theta_a & \sin \theta_a \\ -\sin \theta_a & \cos \theta_a \end{pmatrix}, \text{ with } \tan 2\theta_a = \frac{2\mu_{12}^2}{\mu_1^2 - \Lambda''_{\phi_1} - \mu_2^2 + \Lambda''_{\phi_2}},$$

into the mass eigenstates, A_1, A_2 , with squared masses

$$m_{A_1}^2 = (-\mu_1^2 + \Lambda''_{\phi_1}) \cos^2 \theta_a + (-\mu_2^2 + \Lambda''_{\phi_2}) \sin^2 \theta_a - 2\mu_{12}^2 \sin \theta_a \cos \theta_a,$$

$$m_{A_2}^2 = (-\mu_1^2 + \Lambda''_{\phi_1}) \sin^2 \theta_a + (-\mu_2^2 + \Lambda''_{\phi_2}) \cos^2 \theta_a + 2\mu_{12}^2 \sin \theta_a \cos \theta_a,$$

where $\Lambda''_{\phi_1} = \frac{1}{2}(\lambda_{31} + \lambda'_{31} - 2\lambda_3)v^2$, $\Lambda''_{\phi_2} = \frac{1}{2}(\lambda_{23} + \lambda'_{23} - 2\lambda_2)v^2$.

Dependent parameters in terms of input parameters

$$\Lambda_{\phi_2} = \frac{v^2 g_{H_1 H_1 h}}{4(\sin^2 \theta_h + n \cos^2 \theta_h)},$$

$$\Lambda'_{\phi_2} = \frac{2\mu_{12}^2}{(1-n)\tan 2\theta_c} + \mu_2^2,$$

$$\Lambda''_{\phi_2} = \frac{2\mu_{12}^2}{(1-n)\tan 2\theta_a} + \mu_2^2,$$

$$\mu_2^2 = \Lambda_{\phi_2} - \frac{m_{H_1}^2 + m_{H_2}^2}{1+n},$$

$$\mu_{12}^2 = \frac{1}{2} \sqrt{(m_{H_1}^2 - m_{H_2}^2)^2 - (-1+n)^2 (\Lambda_{\phi_2} - \mu_2^2)^2},$$

$$\lambda_2 = \frac{1}{2v^2} (\Lambda_{\phi_2} - \Lambda''_{\phi_2}),$$

$$\lambda_{23} = \frac{2}{v^2} \Lambda'_{\phi_2},$$

$$\lambda'_{23} = \frac{1}{v^2} (\Lambda_{\phi_2} + \Lambda''_{\phi_2} - 2\Lambda'_{\phi_2})$$