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STUDY

Supernova Axion Emissivity with $\Delta(1232)$ Resonance in Heavy Baryon Chiral Perturbation Theory

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The QCD axion

💥A dynamic solution for the strong CP problem in QCD.
(Peccei–Quinn mechanism)

💥A possible candidate of the cold dark matter.
(Coherently oscillating scalar field)

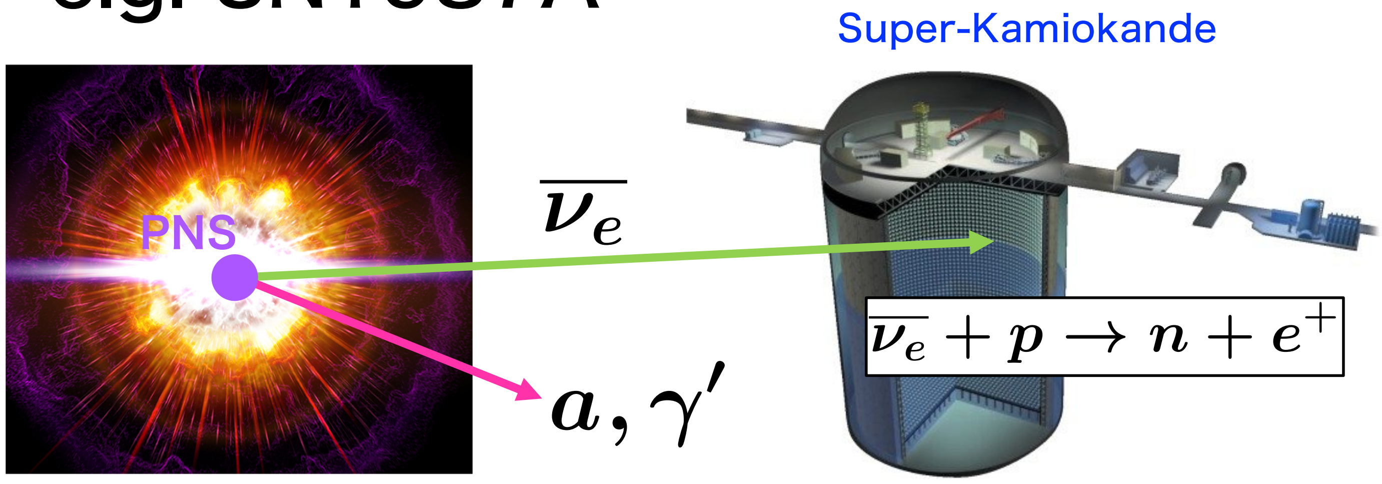
💥It can couple to the SM particles with strength inversely proportional to the decay constant f_a.

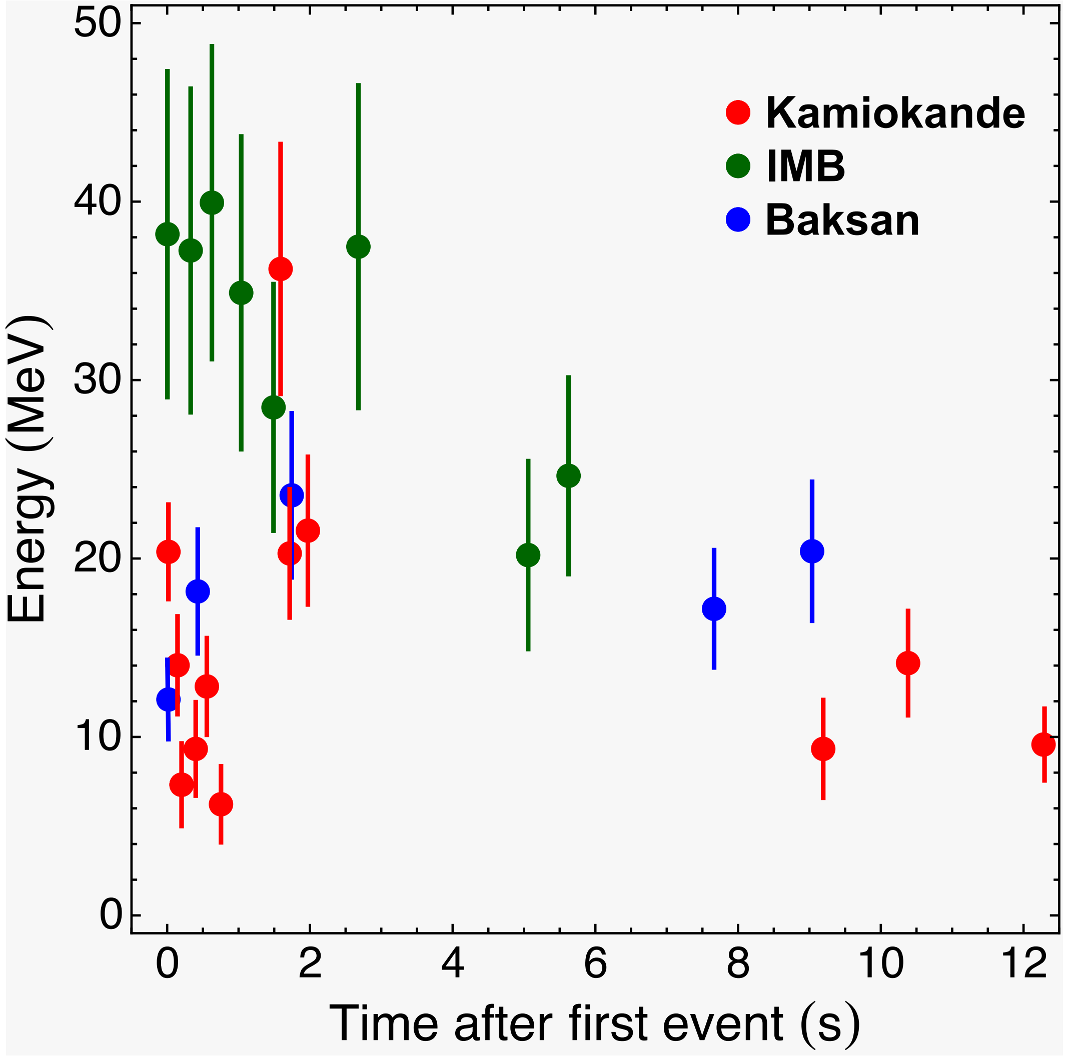
💥It may form a BEC due to its self-gravitational interaction.

Axion emission from celestial bodies

💥The axions can be produced copiously from some and hot dense celestial objects such as supernovae (SNe), neutron stars, and white dwarfs.

➢e.g. SN1987A





➢Raffelt's criteria

$$L_{\text{new particle}} < L_\nu \sim 3 \times 10^{52} \mathrm{erg/s}$$

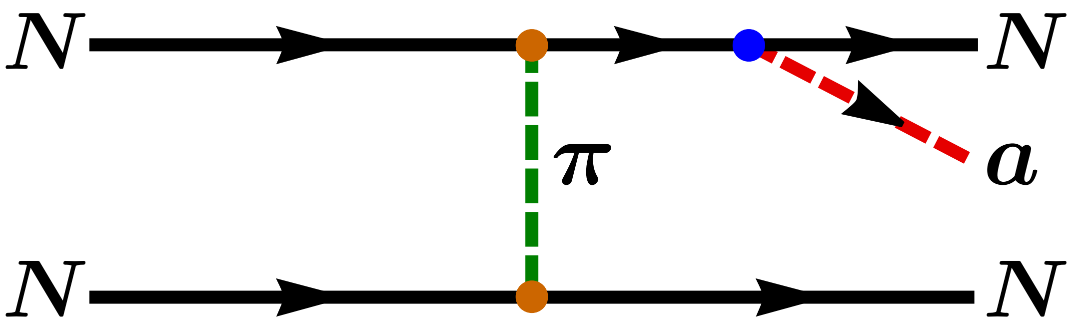
Raffelt `90

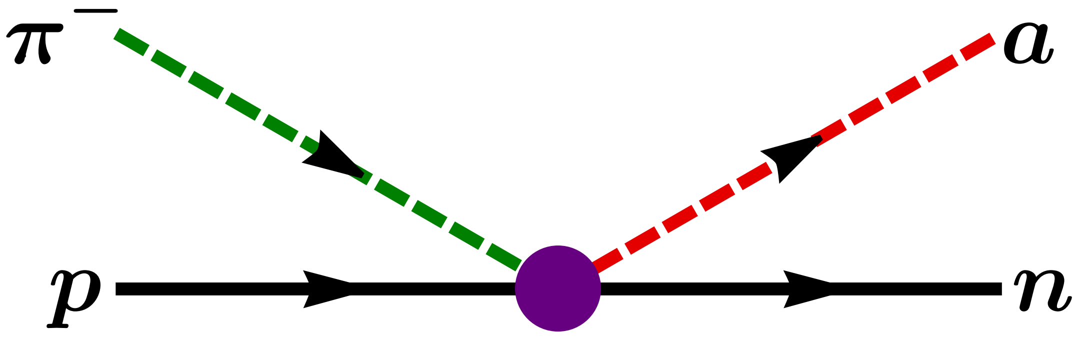
Axion emission process from SNe

💥Two hadronic processes that can create axions inside SNe

➢Nucleon-nucleon bremsstrahlung (NNB) : $NN \to NNa$

➢Pion-induced Compton-like scattering (PCS) : $\pi^- p \to na$





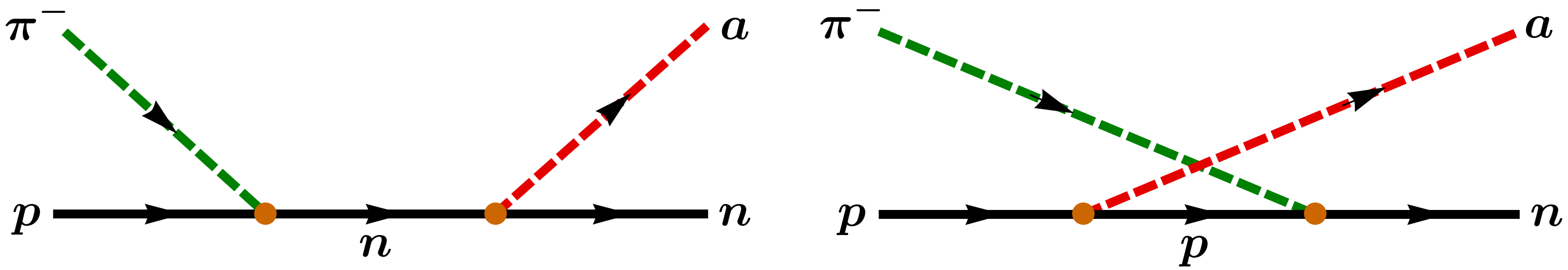
➢It has been thought the NNB as the dominant axion emission for a while due to the underestimate of the n_π inside SNe.

➢Recent studies have shown that the PCS dominates over the NNB to be the main source of the axion emission inside SNe.

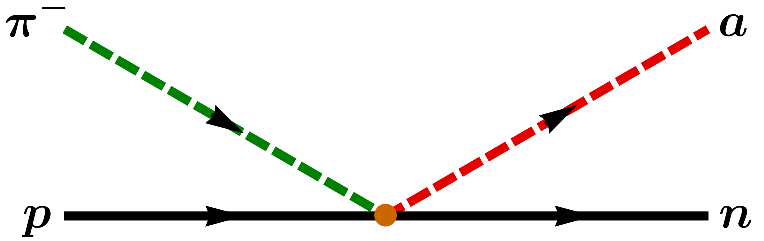
B. Fore and S. Reddy (2020), P. Carenza, et al. (2021), T. Fischer, et al. (2021)

What we did

💥We evaluate the supernova axion emission rate including the Δ resonance in the heavy baryon chiral perturbation theory

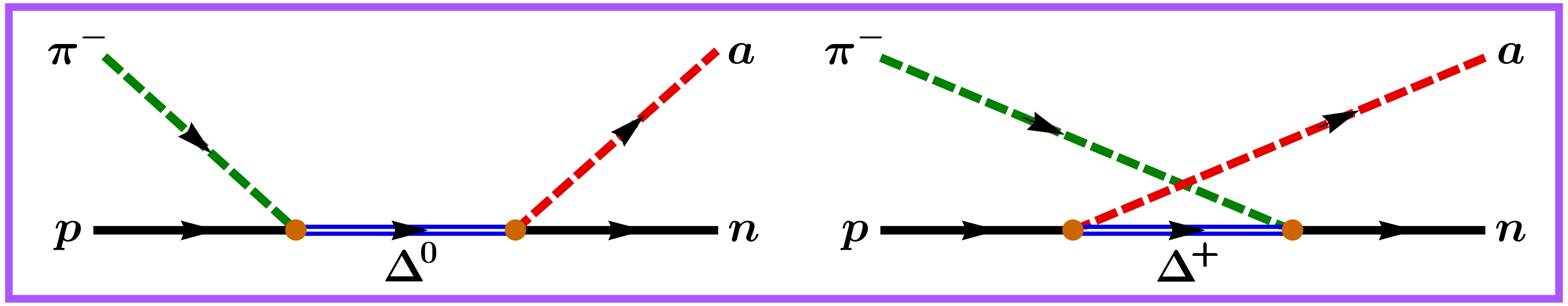


P. Carenza, B. Fore, M. Giannotti, A. Mirizzi and S. Reddy (2021)



K. Choi, H. J. Kim, H. Seong & C. S. Shin (2022)

$|\boldsymbol{k}_\pi| \simeq m_\pi \ll m_p$



In our work

- ➢For $T_{\mathrm{SN}} \sim 30\,\mathrm{MeV}$, $|k_\pi| \simeq \sqrt{3 m_\pi T_{\mathrm{SN}}} \simeq m_\pi, E_\pi \sim 180\,\mathrm{MeV}$
- ➢The $m_{\pi^- p}$ is somewhere in the middle of Δ and N masses.

Heavy Baryon Chiral Perturbation Theory

💥 Interactions of meson octet and baryon octet

Jenkins & Manohar `91

$$\begin{aligned}\mathcal{L}_{\pi B} &= i\left\langle \overline{\mathcal{B}_v} v^\mu \mathcal{D}_\mu \mathcal{B}_v \right\rangle + 2D\left\langle \overline{\mathcal{B}_v} S_v^\mu \{\mathcal{A}_\mu, \mathcal{B}_v\} \right\rangle + 2F\left\langle \overline{\mathcal{B}_v} S_v^\mu [\mathcal{A}_\mu, \mathcal{B}_v] \right\rangle \\ &\quad + \frac{1}{4} f_\pi^2 \left\langle \partial^\mu \mathbf{\Pi} \partial_\mu \mathbf{\Pi}^\dagger \right\rangle + b\left\langle \mathcal{M}_q \left(\mathbf{\Pi} + \mathbf{\Pi}^\dagger\right) \right\rangle + \cdots \ ,\end{aligned}$$

💥 Interactions of meson octet, baryon octet & baryon decuplet

$$\begin{aligned}\mathcal{L}_{\pi BT} &= -i\, \overline{(\mathcal{T}_v^\mu)_{ijk}} v^\rho \mathcal{D}_\rho (\mathcal{T}_{v\mu})_{ijk} + \Delta m_{TB} \overline{(\mathcal{T}_v^\mu)_{ijk}} (\mathcal{T}_{v\mu})_{ijk} \\ &\quad + \mathcal{C}\epsilon_{ijk}\left[\overline{(\mathcal{T}_v^\mu)_{i\ell m}} (\mathcal{A}_\mu)_{\ell j} (\mathcal{B}_v)_{mk} + \overline{(\mathcal{B}_v)_{km}} (\mathcal{A}_\mu)_{j\ell} (\mathcal{T}_{v\mu})_{i\ell m} \right] + \cdots \ ,\end{aligned}$$

→ Conserved axial vector currents $\mathcal{J}_{\pi B}^{A\mu}$ & $\mathcal{J}_{\pi BT}^{A\mu}$

The QCD axion Lagrangian

💥$v_{\text{PQ,EW}} \gg T \gg \Lambda_{\text{QCD}}$ & at leading order in a/f_a

$$\mathcal{L}_{aqg} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g_s^2}{32\pi^2}\frac{a}{f_a} G^c_{\mu\nu}\widetilde{G}^{c\mu\nu} + \overline{q} i\gamma^\mu \partial_\mu q$$

$$- \left(\overline{q_L}\mathcal{M}_q q_R + \text{h.c.}\right) + \frac{\partial_\mu a}{2f_a}\overline{q}\gamma^\mu\gamma^5 \mathcal{X}_q q$$

$\Lambda_{\text{QCD}} \gtrsim T$

💥Chiral transformation : $q \;\rightarrow\; \exp\left(-i\gamma^5 \frac{a}{2f_a}\mathcal{Q}_a\right) q \;,\; \langle \mathcal{Q}_a \rangle \;=\; 1$

$$\mathcal{L}_{aq} = \frac{1}{2}\partial_\mu a \partial^\mu a + \overline{q} i\gamma^\mu \partial_\mu q + \left\langle \mathcal{M}_a q_R \overline{q_L} + \mathcal{M}_a^\dagger q_L \overline{q_R} \right\rangle + \frac{\partial_\mu a}{f_a}\left\langle \left(\mathcal{X}_q + \mathcal{Q}_a\right)\hat{t}^A \right\rangle \mathcal{J}_q^{A\mu}$$

➢Next step is to replace the conserved quark currents with the conserved hadron currents in the HBChPT.

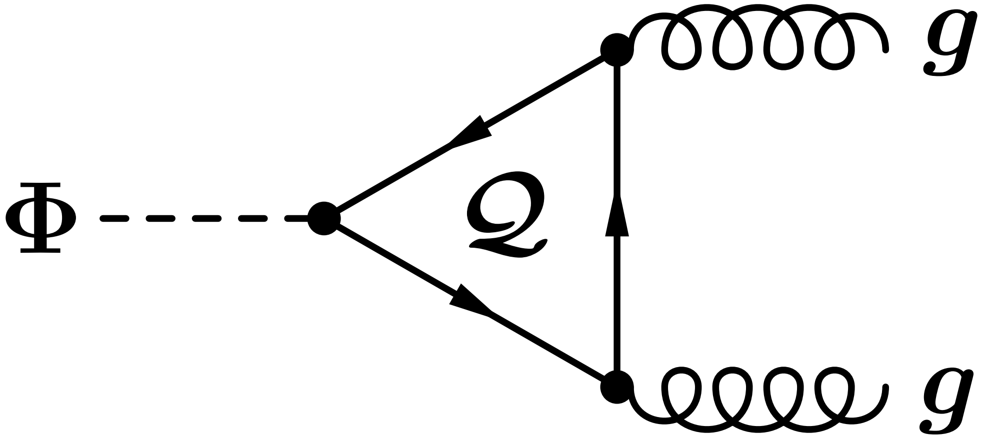
Axion models

💥KSVZ model

Kim `79,
Shifman, Vainshtein, Zakharov `80

- The QCD anomaly is realized by introducing a heavy vector-like fermion.

$$\mathcal{Q} = \mathcal{Q}_L + \mathcal{Q}_R \sim (\mathbf{3},\mathbf{1})_0$$



- Interactions : $y_Q \Phi \overline{\mathcal{Q}_L} \mathcal{Q}_R + \mathrm{h.c.}$
- Under PQ symmetry

$$\Phi \to e^{iq_{\mathsf{PQ}}} \Phi \qquad \mathcal{Q}_L \to e^{iq_{\mathsf{PQ}}/2} \mathcal{Q}_L \qquad \mathcal{Q}_R \to e^{-iq_{\mathsf{PQ}}/2} \mathcal{Q}_R$$

- Only $\mathbf{\Phi}$ and $\boldsymbol{\mathcal{Q}}$ have PQ charges : $X_u = X_d = X_s = 0$

(at tree level)

Axion models

💥DFSZ model

Dine, Fischler, Srednicki `81
Zhitnitsky `80

- ➢The QCD anomaly is induced by assuming 2HDM $\boldsymbol{H_u}$ & $\boldsymbol{H_d}$ couples to the SM quark fields.
- ➢Interactions : $H_u^\dagger H_d (\Phi^*)^2$ $\overline{Q_L}(\mathcal{Y}_u \widetilde{H}_u U_R + \mathcal{Y}_d H_d D_R) + \mathrm{h.c.}$
- ➢Under PQ symmetry

$$\Phi \to e^{iq_{\mathsf{PQ}}}\Phi \quad H_u \to e^{-iq_{\mathsf{PQ}}} H_u \quad H_d \to e^{iq_{\mathsf{PQ}}} H_d$$

$$Q_L \to Q_L \quad U_R \to e^{-iq_{\mathsf{PQ}}} U_R \quad D_R \to e^{-iq_{\mathsf{PQ}}} D_R$$

- ➢The axion as a linear combination of the CP-odd scalars can couple to the SM quarks : $X_u = \dfrac{\cos^2\beta}{3} \; , \; X_{d,s} = \dfrac{\sin^2\beta}{3}$

$\tan\beta = \dfrac{\langle \boldsymbol{H_u} \rangle}{\langle \boldsymbol{H_d} \rangle}$

Axion couplings to hadrons

💥 Axion couplings to pions and baryons : $\mathcal{J}_q^{A\mu} \sim \mathcal{J}_{\text{hadron}}^{A\mu}$

$$\frac{\partial_\mu a}{f_a}\left\langle\left(\mathcal{X}_q+\mathcal{Q}_a\right)\hat{t}^A\right\rangle\mathcal{J}_q^{A\mu} \longrightarrow \mathcal{L}_{a\pi B} = \frac{\partial_\mu a}{f_a}\left[\left\langle\left(\mathcal{X}_q+\mathcal{Q}_a\right)t^A\right\rangle\mathcal{J}_{\pi B}^{A\mu}+\frac{1}{3}S\left\langle\mathcal{X}_q+\mathcal{Q}_a\right\rangle\mathcal{J}_{\pi B}^{0\mu}\right]$$

➢ Axion couplings to pions and nucleons :

$$\mathcal{L}_{a\pi N} = \frac{\partial_\mu a}{f_a}\left[C_{ap}\,\overline{p_v}S_v^\mu p_v + C_{an}\,\overline{n_v}S_v^\mu n_v + \frac{i}{2f_\pi}C_{a\pi N}\left(\pi^+\overline{p_v}v^\mu n_v - \pi^-\overline{n_v}v^\mu p_v\right)\right]$$

$$C_{ap} = X_u\Delta u + X_d\Delta d + X_s\Delta s + \frac{\Delta u + z\Delta d + w\Delta s}{1+z+w}$$

$$C_{an} = X_d\Delta u + X_u\Delta d + X_s\Delta s + \frac{z\Delta u + \Delta d + w\Delta s}{1+z+w}$$

$$C_{a\pi N} = \frac{1}{\sqrt{2}}\left(X_u - X_d + \frac{1-z}{1+z+w}\right) = \frac{C_{ap}-C_{an}}{\sqrt{2}g_A}$$

$z \equiv m_u/m_d \simeq 0.485$

$w \equiv m_u/m_s \simeq 0.025$

$\Delta u = 0.847$

$\Delta d = -0.407$

$\Delta s = -0.035$

Axion couplings to hadrons

💥Axion couplings to pions and baryons : $\mathcal{J}_q^{A\mu} \sim \mathcal{J}_{\text{hadron}}^{A\mu}$

$$\frac{\partial_\mu a}{f_a}\left\langle(\mathcal{X}_q+\mathcal{Q}_a)\hat{t}^A\right\rangle \mathcal{J}_q^{A\mu} \longrightarrow \mathcal{L}_{a\pi BT} = \frac{\partial_\mu a}{f_a}\left\langle(\mathcal{X}_q+\mathcal{Q}_a)t^A\right\rangle \mathcal{J}_{\pi BT}^{A\mu}$$

➢Axion couplings to pions, nucleons and Delta baryons :

$$\mathcal{L}_{aN\Delta} = \frac{\partial_\mu a}{2f_a}\left[C_{ap\Delta}\left(\overline{p_v}\Delta_\mu^+ + \overline{\Delta_\mu^+}p_v\right) + C_{an\Delta}\left(\overline{n_v}\Delta_\mu^0 + \overline{\Delta_\mu^0}n_v\right)\right]$$

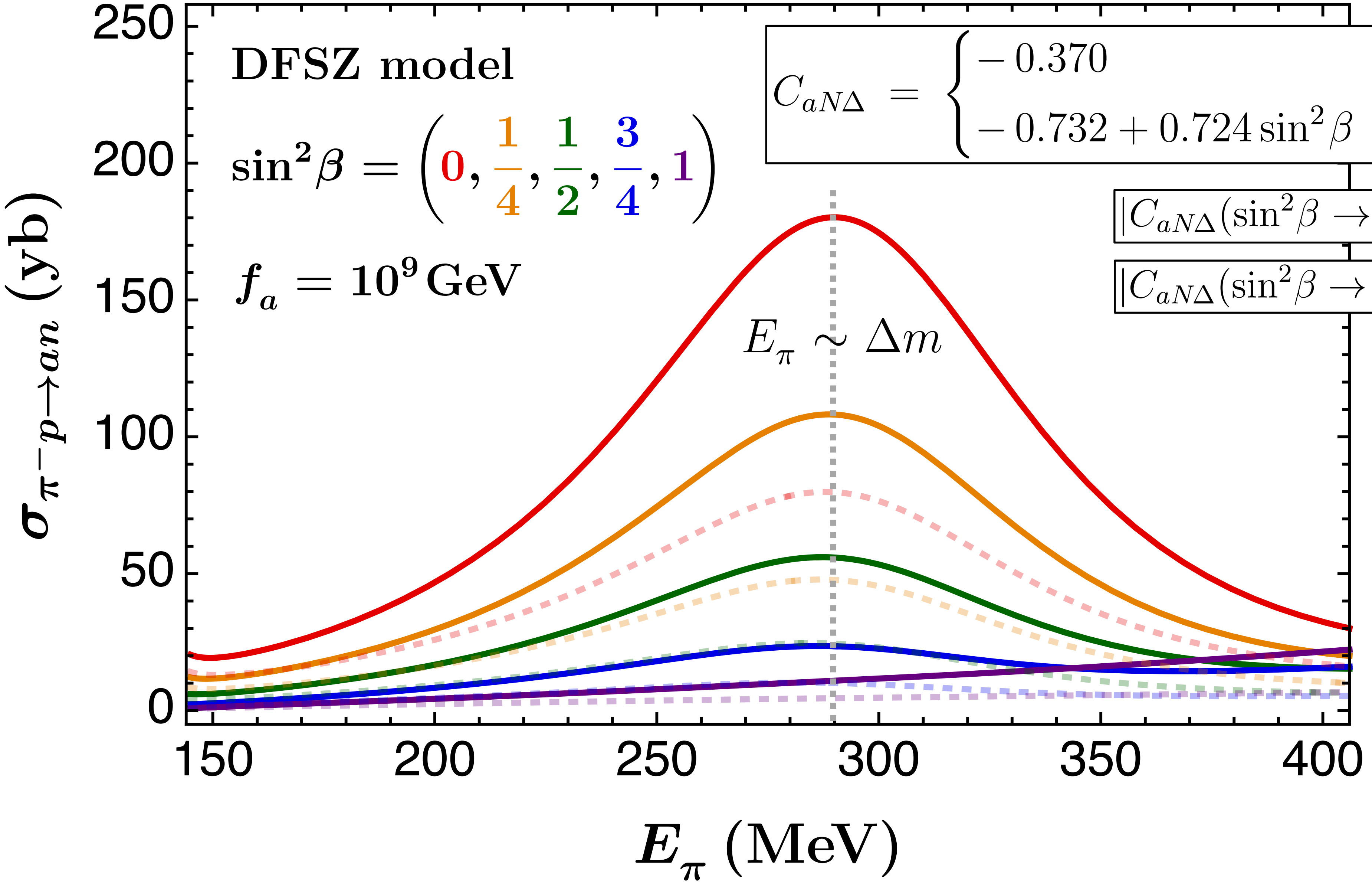
$$C_{ap\Delta} = C_{an\Delta} \equiv C_{aN\Delta} = -\frac{\mathcal{C}}{\sqrt{3}}\left(X_u - X_d + \frac{1-z}{1+z+w}\right) = -\frac{\sqrt{3}}{2}\left(C_{ap} - C_{an}\right)$$

Derive for the first time

➢Note that $C_{a\pi N}$ and $C_{aN\Delta}$ are not independent parameters since they can be expressed in terms of $C_{ap} - C_{an}$.

Scattering cross section v.s. E_π

💥DFSZ model

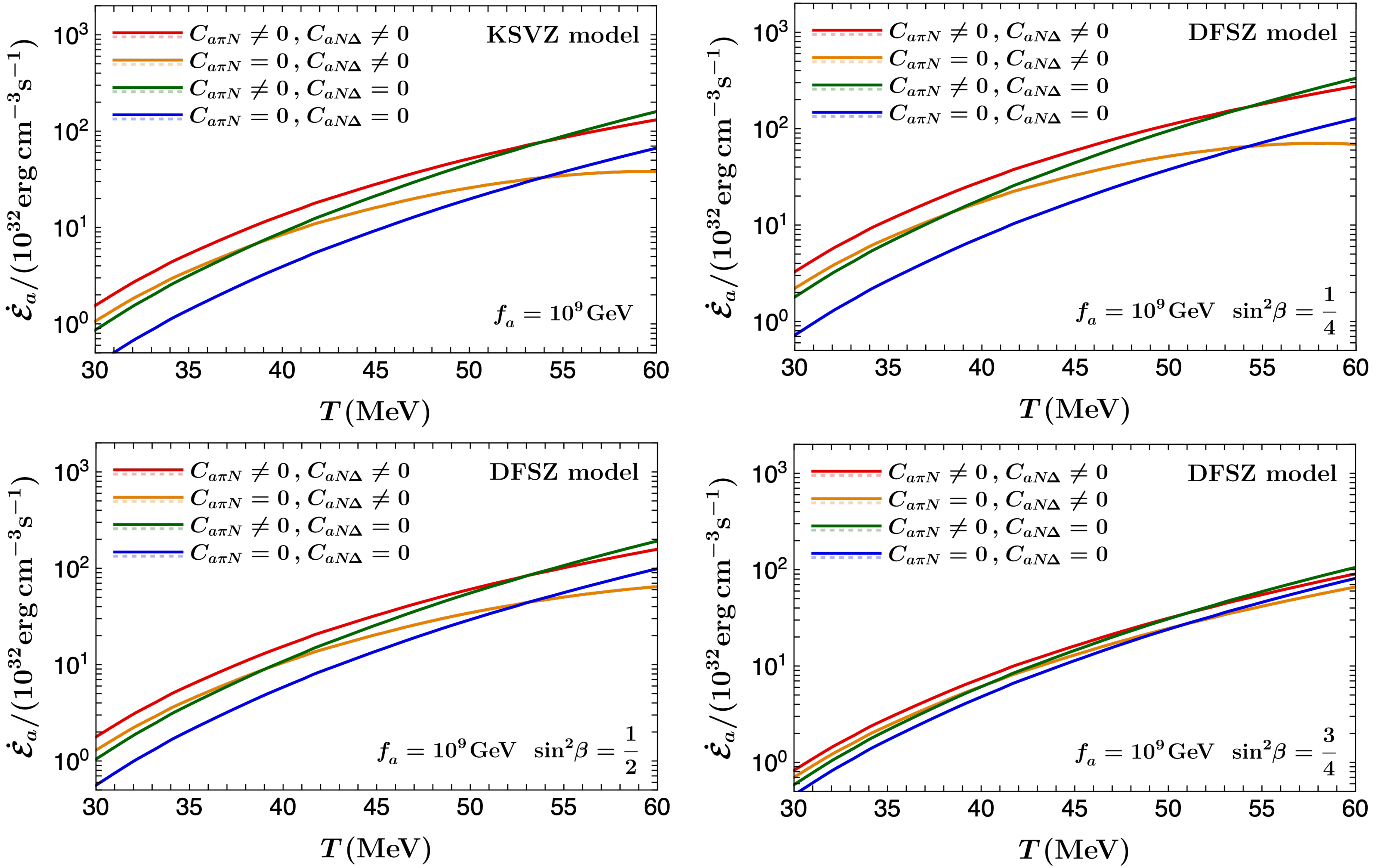


$$C_{aN\Delta} = \begin{cases} -0.370 & \text{KSVZ model} \\ -0.732 + 0.724\sin^2\beta & \text{DFSZ model} \end{cases}$$

$|C_{aN\Delta}(\sin^2\beta \to 0)| \sim 1$

$|C_{aN\Delta}(\sin^2\beta \to 1)| \sim 0.01$

Supernova Axion Emissivity v.s. T



Summary

💥We have estimated the supernova axion emissivity with the Δ(1232) resonance in the HBChPT.

💥We have shown that the supernova axion emissivity can be enhanced by a factor of ∼4 in the KSVZ model and up to a factor of ∼5 in the DFSZ model with $\tan\beta \to 0$ compared to the case without the $C_{a\pi N}$ and $C_{aN\Delta}$.

💥We have found that the Δ resonance can give a destructive contribution to the supernova axion emissivity at high $T_{\rm SN}$.

Thank you for your listening

Back up

The Strong CP Problem in QCD

- The CP-violating term in QCD

$$\mathcal{L}_\theta = \underbrace{\theta}_{\text{strong CP phase}} \frac{g_s^2}{32\pi} G^{c\mu\nu} \widetilde{G}^c_{\mu\nu}$$

- Experimental bound from neutron EDM : $|\theta| < 10^{-10}$
- Theoretically, this problem even more puzzling

$$\theta = \underbrace{\theta_0}_{\text{theta vacuum}} + \underbrace{\arg\det(M_u M_d)}_{\text{chiral transformation}}$$

Why θ is so small is the strong CP problem.

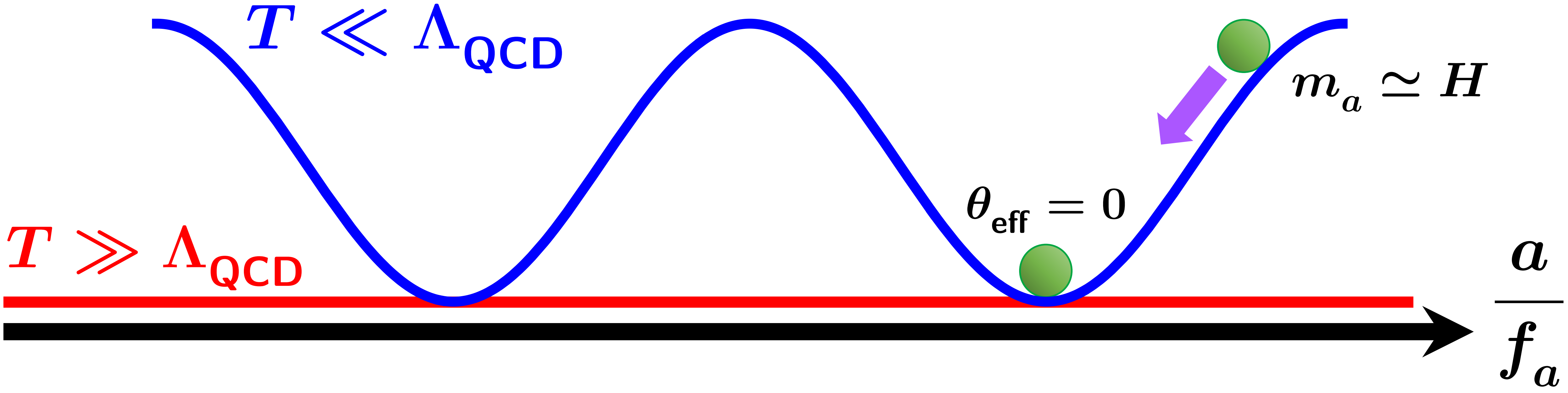
The QCD axion

💥Peccei-Quinn (PQ) mechanism : Strong CP phase is promoted to a dynamical variable :

Peccei, Quinn `77, Weinberg `78, Wilczek `78

$$\mathcal{L}_\theta = \underbrace{\left[\theta + \frac{a(x)}{f_a}\right]}_{\theta_{\rm eff}(x)} \frac{g_s^2}{32\pi} G^{c\mu\nu}\widetilde{G}^c_{\mu\nu}$$

f_a : decay constant



Axion Interactions with matter

💥Axion-electron interaction

$$\mathcal{L}_{aee} = -iC_{ae}\frac{m_e}{f_a}a\overline{\psi}_e\gamma^5\psi_e \overset{\text{E.O.M. \& I.P.}}{=} C_{ae}\frac{\partial_\mu a}{2f_a}\overline{\psi}_e\gamma^\mu\gamma^5\psi_e$$

C_{ae} : model-dependent coefficient

💥Axion-nucleons interaction

$$\mathcal{L}_{aNN} = \sum_{N=p,n} C_{aN}\frac{\partial_\mu a}{2f_a}\overline{\psi}_N\gamma^\mu\gamma^5\psi_N$$

(related to our work)

💥The axion couples to the SM particles with strength inversely proportional to the decay constant. Hence, the axion feebly couples to the SM particles due to the large decay constant.

Heavy Baryon Formalism

Jenkins & Manohar `91

💥 In this formalism, the nucleon is almost on-shell with a nearly unchanged velocity when it exchanges some tiny momentum with the pion

$$m_N/\Lambda_\chi \sim 1$$

$$p_N^\mu = m_N v^\mu + \delta k_\pi^\mu \qquad v^2 = 1$$

$$\delta k_\pi^\mu/\Lambda_\chi \ll 1$$

- ➢ Velocity-dependence baryon field

$$\Lambda_\chi \sim 1\,\mathrm{GeV}$$

$$\mathcal{B}_v(x) = e^{im_B v\cdot x}\mathcal{B}(x) \longrightarrow \overline{\mathcal{B}}(i\not{\partial} - m_B)\mathcal{B} \to \overline{\mathcal{B}_v}\, i\not{\partial}\,\mathcal{B}_v$$

- ➢ The power counting expansion of the effective field theory for pions and baryons can be systematic and well-behaved.
- ➢ The algebra of the spin operator formalism can be much simpler than that of the gamma matrix formalism.

Effective Chiral Lagrangian

Jenkins & Manohar `91

💥Interaction between meson octet and baryon octet

$$\mathcal{L}_{\pi B} = i\left\langle \overline{\mathcal{B}_v} v^\mu \mathcal{D}_\mu \mathcal{B}_v \right\rangle + 2D\left\langle \overline{\mathcal{B}_v} S_v^\mu \left\{ \mathcal{A}_\mu , \mathcal{B}_v \right\} \right\rangle + 2F\left\langle \overline{\mathcal{B}_v} S_v^\mu \left[\mathcal{A}_\mu , \mathcal{B}_v \right] \right\rangle$$

$$+ \frac{1}{4} f_\pi^2 \left\langle \partial^\mu \boldsymbol{\Pi} \partial_\mu \boldsymbol{\Pi}^\dagger \right\rangle + b\left\langle \mathcal{M}_q \left(\boldsymbol{\Pi} + \boldsymbol{\Pi}^\dagger \right) \right\rangle + \cdots ,$$

$$\langle \cdots \rangle = \mathrm{tr}(\cdots)$$

$$f_\pi \simeq 92.4\,\mathrm{MeV}$$

➤ $$\mathcal{B}_v = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma_v^0 + \frac{1}{\sqrt{6}}\Lambda_v & \Sigma_v^+ & \enclose{circle}{p_v} \\ \Sigma_v^- & -\frac{1}{\sqrt{2}}\Sigma_v^0 + \frac{1}{\sqrt{6}}\Lambda_v & \enclose{circle}{n_v} \\ \Xi_v^- & \Xi_v^0 & -\frac{2}{\sqrt{6}}\Lambda_v \end{pmatrix} , \quad \mathcal{D}_\mu \mathcal{B}_v = \partial_\mu \mathcal{B}_v + \left[\mathcal{V}_\mu , \mathcal{B}_v \right]$$

baryon
octet

$$\mathcal{V}_\mu = \frac{1}{2}\left(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right) , \quad \mathcal{A}_\mu = \frac{i}{2}\left(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right) ,$$

$$\xi = \exp\left(\frac{i\boldsymbol{\pi}}{f_\pi} \right) , \quad \boldsymbol{\Pi} = \xi^2 , \quad \boldsymbol{\pi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \enclose{circle}{\pi^+} & K^+ \\ \enclose{circle}{\pi^-} & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}_0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

meson
octet

Effective Chiral Lagrangian

Jenkins & Manohar `91

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$$\mathcal{L}_{\pi B} = i\left\langle \overline{\mathcal{B}_v} v^\mu \mathcal{D}_\mu \mathcal{B}_v \right\rangle + 2D\left\langle \overline{\mathcal{B}_v} S_v^\mu \left\{ \mathcal{A}_\mu , \mathcal{B}_v \right\} \right\rangle + 2F\left\langle \overline{\mathcal{B}_v} S_v^\mu \left[\mathcal{A}_\mu , \mathcal{B}_v \right] \right\rangle$$
$$+ \frac{1}{4} f_\pi^2 \left\langle \partial^\mu \mathbf{\Pi} \partial_\mu \mathbf{\Pi}^\dagger \right\rangle + b\left\langle \mathcal{M}_q \left(\mathbf{\Pi} + \mathbf{\Pi}^\dagger \right) \right\rangle + \cdots \ ,$$

➢Spin operator : $S_v^\mu = \gamma^5 \left[\not{v} , \gamma^\mu \right] / 4$ $\quad v \cdot S_v = 0$

➢Quark mass matrix : $\mathcal{M}_q = \mathrm{diag}\left(m_u , m_d , m_s \right)$ $\quad \mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \rightarrow \mathrm{SU}(3)_V$

➢Under the $\mathbf{SU(3)}_{\boldsymbol{L}} \otimes \mathbf{SU(3)}_{\boldsymbol{R}}$ symmetry

$$\mathcal{B}_v \to \mathcal{U}_H \mathcal{B}_v \mathcal{U}_H^\dagger \ , \quad \mathcal{D}_\mu \mathcal{B}_v \to \mathcal{U}_H \left(\mathcal{D}_\mu \mathcal{B}_v \right) \mathcal{U}_H^\dagger \ , \quad \mathbf{\Pi} \to \mathcal{U}_L \mathbf{\Pi} \, \mathcal{U}_R^\dagger \ ,$$

$$\xi \to \mathcal{U}_L \xi \mathcal{U}_H^\dagger = \mathcal{U}_H \xi \mathcal{U}_R^\dagger \ , \quad \mathcal{V}_\mu \to \mathcal{U}_H \mathcal{V}_\mu \mathcal{U}_H^\dagger + \mathcal{U}_H \partial_\mu \mathcal{U}_H^\dagger \ , \quad \mathcal{A}_\mu \to \mathcal{U}_H \mathcal{A}_\mu \mathcal{U}_H^\dagger$$

$$\mathcal{U}_{L,R} \in \mathrm{SU}(3)_{L,R} \qquad \mathcal{U}_H = \mathcal{U}_H(x) \in \mathrm{SU}(3)_H$$ (local)

Effective Chiral Lagrangian

Jenkins & Manohar `91

💥 Interaction between meson octet and baryon octet

$$\begin{aligned}\mathcal{L}_{\pi B} &= i\left\langle \overline{\mathcal{B}_v} v^\mu \mathcal{D}_\mu \mathcal{B}_v \right\rangle + 2D\left\langle \overline{\mathcal{B}_v} S_v^\mu \left\{\mathcal{A}_\mu, \mathcal{B}_v\right\}\right\rangle + 2F\left\langle \overline{\mathcal{B}_v} S_v^\mu \left[\mathcal{A}_\mu, \mathcal{B}_v\right]\right\rangle \\ &\quad + \frac{1}{4} f_\pi^2 \left\langle \partial^\mu \mathbf{\Pi} \partial_\mu \mathbf{\Pi}^\dagger \right\rangle + b\left\langle \mathcal{M}_q \left(\mathbf{\Pi} + \mathbf{\Pi}^\dagger\right)\right\rangle + \cdots ,\end{aligned}$$

➢ To the first order in $\boldsymbol{\pi}/f_\pi$

$$\xi = \mathbb{I}_{3\times 3} + i\boldsymbol{\pi}/f_\pi \qquad \mathcal{A}_\mu = \partial_\mu \boldsymbol{\pi}/f_\pi \qquad \mathcal{V}_\mu = 0$$

$$\longrightarrow \mathcal{L}_{\pi B} \supset \underline{\frac{2(D+F)}{f_\pi}\left\langle \overline{\mathcal{B}_v} S_v^\mu \left(\partial_\mu \boldsymbol{\pi}\right) \mathcal{B}_v \right\rangle} + \frac{2(D-F)}{f_\pi}\left\langle \overline{\mathcal{B}_v} S_v^\mu \mathcal{B}_v \left(\partial_\mu \boldsymbol{\pi}\right)\right\rangle$$

$$\downarrow$$

$$g_A = D + F \simeq 1.254$$

$$\mathcal{L}_{\pi N} = \frac{\sqrt{2} g_A}{f_\pi}\left(\overline{p_v} S_v^\mu n_v \partial^\mu \pi^+ + \overline{n_v} S_v^\mu p_v \partial^\mu \pi^-\right)$$

pion-nucleon interaction

Effective Chiral Lagrangian

Jenkins & Manohar `91

💥Interactions of meson octet, baryon octet & baryon decuplet

$$\begin{aligned}\mathcal{L}_{\pi BT} &= -i\,\overline{(\mathcal{T}_v^\mu)}_{ijk}\, v^\rho \mathcal{D}_\rho (\mathcal{T}_{v\mu})_{ijk} + \Delta m_{TB}\,\overline{(\mathcal{T}_v^\mu)}_{ijk} (\mathcal{T}_{v\mu})_{ijk} \\ &\quad + \mathcal{C}\epsilon_{ijk}\left[\overline{(\mathcal{T}_v^\mu)}_{i\ell m}(\mathcal{A}_\mu)_{\ell j}(\mathcal{B}_v)_{mk} + \overline{(\mathcal{B}_v)}_{km}(\mathcal{A}_\mu)_{j\ell}(\mathcal{T}_{v\mu})_{i\ell m}\right] + \cdots \ ,\end{aligned}$$

➢Spin-3/2 Rarita-Schwinger field : $(\mathcal{T}_v^\mu)_{ijk}$

$$\Delta m_{TB} = m_T - m_B$$
$$\mathcal{C} \simeq 3g_A/2$$

➢Under the $\mathbf{SU(3)}_L \otimes \mathbf{SU(3)}_R$ symmetry

$$(\mathcal{T}_v^\mu)_{ijk} \to (\mathcal{U}_H)_{i\ell}(\mathcal{U}_H)_{jm}(\mathcal{U}_H)_{kn}(\mathcal{T}_v^\mu)_{lmn}$$

➢Rep. of the Delta baryon : $(\mathcal{T}_{v\mu})_{112} = \frac{1}{\sqrt{3}}\Delta^+_{v\mu}\ , \quad (\mathcal{T}_{v\mu})_{122} = \frac{1}{\sqrt{3}}\Delta^0_{v\mu}$

$$\Longrightarrow \quad \mathcal{L}_{\pi N\Delta} = \frac{\mathcal{C}}{\sqrt{6}f_\pi}\left(\overline{n_v}\,\Delta^+_{v\mu}\partial^\mu\pi^- + \overline{\Delta^+_{v\mu}}\,n_v\partial^\mu\pi^+ - \overline{p_v}\,\Delta^0_{v\mu}\partial^\mu\pi^+ - \overline{\Delta^0_{v\mu}}\,p_v\partial^\mu\pi^-\right)$$

pion-nucleon-delta interaction

Hadronic Axial Vector Currents

💥The Lagrangian invariant under the local $\mathrm{SU}(3)_H$ symmetry

$$\mathcal{L}_{\pi B} \supset i\left\langle \overline{\mathcal{B}_v} v^\mu \mathcal{D}_\mu \mathcal{B}_v \right\rangle + 2D\left\langle \overline{\mathcal{B}_v} S_v^\mu \{\mathcal{A}_\mu, \mathcal{B}_v\} \right\rangle + 2F\left\langle \overline{\mathcal{B}_v} S_v^\mu [\mathcal{A}_\mu, \mathcal{B}_v] \right\rangle$$

$$\mathcal{L}_{\pi BT} \supset \mathcal{C}\epsilon_{ijk}\left[\overline{(\mathcal{T}_v^\mu)_{i\ell m}} (\mathcal{A}_\mu)_{\ell j} (\mathcal{B}_v)_{mk} + \overline{(\mathcal{B}_v)_{km}} (\mathcal{A}_\mu)_{j\ell} (\mathcal{T}_{v\mu})_{i\ell m} \right]$$

$$\mathcal{B}_v \to \mathcal{U}_H \mathcal{B}_v \mathcal{U}_H^\dagger \ , \quad \mathcal{D}_\mu \mathcal{B}_v \to \mathcal{U}_H (\mathcal{D}_\mu \mathcal{B}_v) \mathcal{U}_H^\dagger \ , \quad \mathcal{A}_\mu \to \mathcal{U}_H \mathcal{A}_\mu \mathcal{U}_H^\dagger$$

$$(\mathcal{T}_v^\mu)_{ijk} \to (\mathcal{U}_H)_{i\ell} (\mathcal{U}_H)_{jm} (\mathcal{U}_H)_{kn} (\mathcal{T}_v^\mu)_{lmn}$$

➤Noether's theorem : $\xi \to \mathcal{U}_H \xi \mathcal{U}_R^\dagger \to (1+i\epsilon^A t^A)\xi$ $\epsilon^A \to 0$

$$\boxed{\mathcal{J}_{\pi B}^{A\mu}} = D\left\langle \overline{\mathcal{B}_v} S_v^\mu \{\xi^\dagger t^A \xi + \xi t^A \xi^\dagger, \mathcal{B}_v\} \right\rangle + F\left\langle \overline{\mathcal{B}_v} S_v^\mu [\xi^\dagger t^A \xi + \xi t^A \xi^\dagger, \mathcal{B}_v] \right\rangle + \frac{1}{2} v^\mu \left\langle \overline{\mathcal{B}_v} [\xi^\dagger t^A \xi - \xi t^A \xi^\dagger, \mathcal{B}_v] \right\rangle \ ,$$

$t^A \ (A = 1, 2, \cdots, 8)$

Gell-Mann matrices

$$\boxed{\mathcal{J}_{\pi BT}^{A\mu}} = \frac{\mathcal{C}}{2}\epsilon_{ijk}\left[\overline{(\mathcal{T}_v^\mu)_{i\ell m}} \left(\xi^\dagger t^A \xi + \xi t^A \xi^\dagger\right)_{\ell j} (\mathcal{B}_v)_{mk} + \overline{(\mathcal{B}_v)_{km}} \left(\xi^\dagger t^A \xi + \xi t^A \xi^\dagger\right)_{\ell j} (\mathcal{T}_{v\mu})_{i\ell m} \right]$$

The QCD axion Lagrangian

💥 $v_{\text{PQ,EW}} \gg T \gg \Lambda_{\text{QCD}}$ & at leading order in a/f_a

$$\mathcal{L}_{aqg} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g_s^2}{32\pi^2}\frac{a}{f_a}G^c_{\mu\nu}\widetilde{G}^{c\mu\nu} + \overline{q} i \gamma^\mu \partial_\mu q$$

$$- \left(\overline{q_L}\mathcal{M}_q q_R + \text{h.c.}\right) + \frac{\partial_\mu a}{2f_a}\overline{q}\gamma^\mu\gamma^5\mathcal{X}_q q$$

axion derivative interactions

- Light quark fields : $q = (u, d, s)^{\mathsf{T}}$
- Axion coupling matrix : $\mathcal{X}_q = \text{diag}(X_u, X_d, X_s)$
- Typically, one introduces an SM-singlet complex scalar field $\mathbf{\Phi} \sim (1, 1)_0$ with a PQ charge in UV models. After the $\cancel{v}_{\text{PQ}}$, the phase of $\mathbf{\Phi} \propto e^{ia/f_a}$ is then identified as the axion.

Axion couplings to hadrons

- 💥 Below the QCD confinement scale, one can remove the axion-gluon coupling by the chiral trans. on the light quark fields

$$q \;\to\; \mathcal{R}_a q \;=\; \exp\left(-i\gamma^5 \frac{a}{2f_a}\mathcal{Q}_a\right) q \;, \qquad \langle \underline{\mathcal{Q}_a} \rangle \;=\; 1$$

act on the quark flavor space

$$\longrightarrow \int \mathcal{D}q\mathcal{D}\bar{q} \to \int \mathcal{D}q\mathcal{D}\bar{q}\, \exp\left[i\int \mathrm{d}^4x \left(-\frac{g_s^2}{32\pi^2}\frac{a}{f_a} G^c_{\mu\nu}\widetilde{G}^{c\mu\nu}\langle \mathcal{Q}_a \rangle\right)\right]$$

$\epsilon^{0123} = +1$

- 💥 To avoid the axion-π^0 mass mixing, the customary choice is

$$\mathcal{Q}_a \;=\; \frac{\mathcal{M}_q^{-1}}{\mathrm{tr}\left(\mathcal{M}_q^{-1}\right)} \;=\; \frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\,\mathrm{diag}\left(\frac{1}{m_u}, \frac{1}{m_d}, \frac{1}{m_s}\right)$$

Axion couplings to hadrons

💥Under the chiral trans., the quark kinetic term is shifted as

$$\bar{q} i \gamma^\mu \partial_\mu q \rightarrow \bar{q} i \gamma^\mu \partial_\mu q + \frac{\partial_\mu a}{2 f_a} \bar{q} \gamma^\mu \gamma^5 \mathcal{Q}_a q + \mathcal{O}\left(\frac{a^2}{f_a^2} \right)$$

💥The light quark mass term becomes

$$\overline{q_L} \mathcal{M}_q q_R \rightarrow \overline{q_L} \mathcal{M}_a q_R \ , \quad \overline{q_R} \mathcal{M}_q q_L \rightarrow \overline{q_R} \mathcal{M}_a^\dagger q_L$$

$$\mathcal{M}_a \equiv \mathcal{R}_a \mathcal{M}_q \mathcal{R}_a$$

Up to the second order in the axion field

$$\mathcal{M}_a = \mathcal{M}_q - i \frac{a}{2 f_a} \{ \mathcal{M}_q, \mathcal{Q}_a \} - \frac{a^2}{8 f_a^2} \{ \{ \mathcal{M}_q, \mathcal{Q}_a \}, \mathcal{Q}_a \} + \mathcal{O}\left(\frac{a^3}{f_a^3} \right)$$

Axion couplings to hadrons

💥The resulting Lagrangian with only the axion and quark fields

$$\mathcal{L}_{aq} = \frac{1}{2}\partial_\mu a \partial^\mu a + \bar{q} i\gamma^\mu \partial_\mu q - \left(\overline{q_L}\mathcal{M}_a q_R + \overline{q_R}\mathcal{M}_a^\dagger q_L\right) + \frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma^5\left(\mathcal{X}_q + \mathcal{Q}_a\right) q$$

➢Use $M_{3\times3} = 2\langle M_{3\times3}\,\hat{t}^A\rangle \hat{t}^A$ for any 3x3 Hermitian matrix $M_{3\times3}$

$$\{\hat{t}^A\} = \{t^A\} \cup \{t^0\} \quad t^0 = \mathbb{I}_{3\times3}/\sqrt{6}$$

$$\mathcal{L}_{aq} = \frac{1}{2}\partial_\mu a \partial^\mu a + \bar{q} i\gamma^\mu \partial_\mu q + \left\langle \mathcal{M}_a q_R \overline{q_L} + \mathcal{M}_a^\dagger q_L \overline{q_R} \right\rangle + \frac{\partial_\mu a}{f_a}\left\langle \left(\mathcal{X}_q + \mathcal{Q}_a\right)\hat{t}^A \right\rangle \mathcal{J}_q^{A\mu}$$

quark axial vector currents $\mathcal{J}_q^{A\mu} = \bar{q}\gamma^\mu\gamma^5\hat{t}^A q$

➢The next step is to replace the light quark fields with the corresponding hadron fields in the HBChPT.

Axion couplings to hadrons

💥 Axion couplings to pions : $\mathcal{U}_L(q_L\overline{q_R})\mathcal{U}_R^\dagger \sim \mathcal{U}_L\mathbf{\Pi}\,\mathcal{U}_R^\dagger$

$$\langle \mathcal{M}_a q_R\overline{q_L} + \mathcal{M}_a^\dagger q_L\overline{q_R}\rangle \longrightarrow \mathcal{L}_{a\pi} = \frac{1}{2}f_\pi^2 B_0\langle \mathcal{M}_a\mathbf{\Pi}^\dagger + \mathcal{M}_a^\dagger\mathbf{\Pi}\rangle$$

➢ $$\mathcal{M}_a = \underbrace{\mathcal{M}_q}_{\text{pion mass}} - \underbrace{i\frac{a}{2f_a}\{\mathcal{M}_q,\mathcal{Q}_a\}}_{\text{axion-}\pi^0\text{ mass mixing}} - \underbrace{\frac{a^2}{8f_a^2}\{\{\mathcal{M}_q,\mathcal{Q}_a\},\mathcal{Q}_a\}}_{\text{axion mass}}$$

$$\boxed{\mathcal{Q}_a = \frac{\mathcal{M}_q^{-1}}{\mathrm{tr}\left(\mathcal{M}_q^{-1}\right)}}$$

➢ **Axion mass :** $$m_a = \sqrt{\frac{z}{(1+z)(1+z+w)}}\frac{f_\pi m_\pi}{f_a} \simeq 6\,\mathrm{meV}\left(\frac{10^9\,\mathrm{GeV}}{f_a}\right)$$

$$z \equiv m_u/m_d \simeq 0.485$$

$$w \equiv m_u/m_s \simeq 0.025$$

$$m_\pi = \sqrt{B_0(m_u+m_d)} \simeq 139.57\,\mathrm{MeV}$$

Axion couplings to hadrons

💥Numerically, we obtain

$$C_{ap} = \begin{cases} +0.430 & \text{KSVZ model} \\ +0.712 - 0.430 \sin^2\beta & \text{DFSZ model} \end{cases}$$

$$C_{an} = \begin{cases} +0.002 & \text{KSVZ model} \\ -0.134 + 0.406 \sin^2\beta & \text{DFSZ model} \end{cases}$$

$$C_{a\pi N} = \begin{cases} +0.241 & \text{KSVZ model} \\ +0.477 - 0.471 \sin^2\beta & \text{DFSZ model} \end{cases}$$

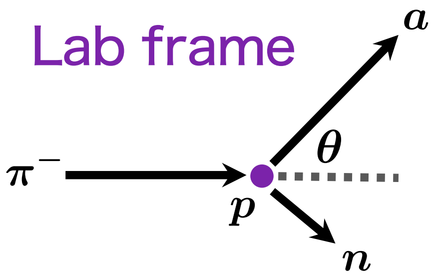
$$C_{aN\Delta} = \begin{cases} -0.370 & \text{KSVZ model} \\ -0.732 + 0.724 \sin^2\beta & \text{DFSZ model} \end{cases}$$

Squared matrix element

💥 $$\overline{\left|\mathcal{M}_{\pi^- p\to na}\right|^2} = \frac{2m_N^2}{f_\pi^2 f_a^2}\left\langle P_+\Omega^\dagger P_+\Omega\right\rangle$$

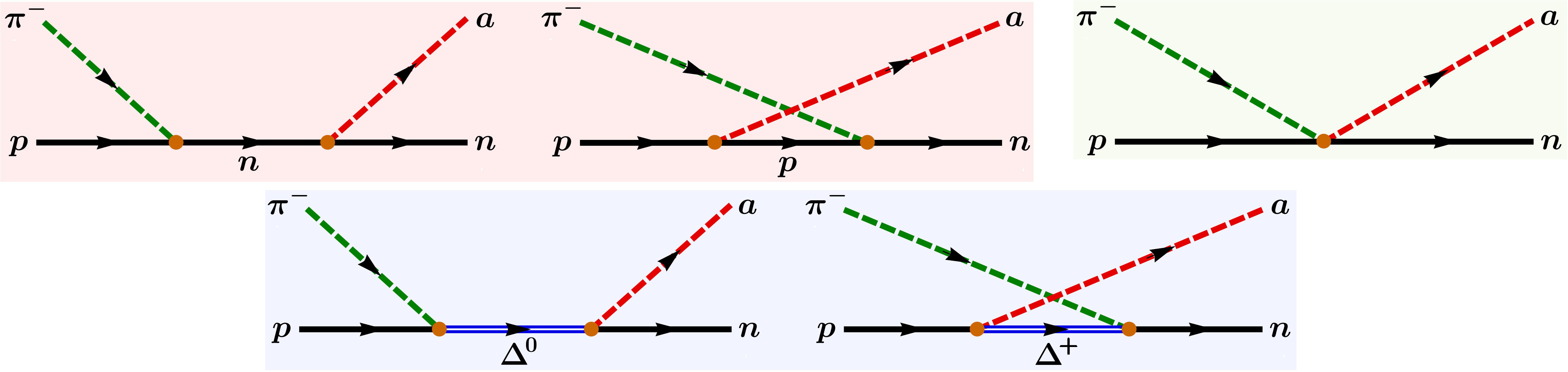
$$P_+ = \mathrm{diag}(1,1,0,0)$$
$$\Theta = \mathrm{diag}\left(e^{+i\theta}, e^{-i\theta}, e^{+i\theta}, e^{-i\theta}\right)$$

Lab frame



$$\begin{aligned}\Omega = {} & \frac{\sqrt{2}g_A|\boldsymbol{k}_\pi||\boldsymbol{k}_a|}{4E_\pi}\left(C_{ap}\Theta - C_{an}\Theta^\dagger\right) + \frac{C_{a\pi N}|\boldsymbol{k}_a|}{2}\mathbb{I}_{4\times4} \\ & + \frac{\mathcal{C}|\boldsymbol{k}_\pi||\boldsymbol{k}_a|}{6\sqrt{6}}\left[\frac{C_{an\Delta}\left(3\cos\theta\,\mathbb{I}_{4\times4} - \Theta^\dagger\right)}{E_\pi - \Delta m + i\Gamma_\Delta/2} + \frac{C_{ap\Delta}\left(3\cos\theta\,\mathbb{I}_{4\times4} - \Theta\right)}{E_\pi + \Delta m - i\Gamma_\Delta/2}\right]\end{aligned}$$

$$\Delta m = m_\Delta - m_N \simeq 293\,\mathrm{MeV}$$
$$\Gamma_\Delta \simeq 117\,\mathrm{MeV}$$



Scattering cross section

💥Cross section formula

$$\sigma_{\pi^- p \to na} = \int \frac{\mathrm{d}^3\boldsymbol{k}_a}{(2\pi)^3 2E_a} \frac{\mathrm{d}^3\boldsymbol{k}_n}{(2\pi)^3 2E_n} (2\pi)^4 \delta^{(4)}\left(k_\pi + k_p - k_a - k_n\right) \frac{\overline{\left|\mathcal{M}_{\pi^- p \to na}\right|^2}}{4\left[(k_\pi \cdot k_p)^2 - (m_\pi m_N)^2\right]^{1/2}}$$

➤ $\sigma_{\pi^- p \to na} = \dfrac{E_\pi m_N^2}{16\pi f_\pi^2 f_a^2 |\boldsymbol{k}_\pi|} \mathcal{G}_a(|\boldsymbol{k}_\pi|)$ $\qquad C_\pm \equiv \left(C_{ap} \pm C_{an}\right)/2$ $\qquad \bar{\Gamma}_\Delta = \Gamma_\Delta/2$

$$\mathcal{G}_a(|\boldsymbol{k}_\pi|) = \underset{🔴🔴}{\frac{2g_A^2\left(2C_+^2 + C_-^2\right)}{3}} \left(\frac{|\boldsymbol{k}_\pi|}{m_N}\right)^2 + \underset{🟢🟢}{C_{a\pi N}^2} \left(\frac{E_\pi}{m_N}\right)^2 + \frac{8\sqrt{2} g_A C_{a\pi N} C_-}{3} \left(\frac{|\boldsymbol{k}_\pi|}{m_N}\right)^2 \left(\frac{E_\pi}{m_N}\right) \text{🔴🟢}$$

$$+ \frac{4C_{aN\Delta}^2 \mathcal{C}^2}{81} \frac{E_\pi^2\left(\Delta m^2 + 2E_\pi^2 + \bar{\Gamma}_\Delta^2\right)}{\left[\left(\underset{\text{Delta resonance}}{\Delta m - E_\pi}\right)^2 + \bar{\Gamma}_\Delta^2\right]\left[\left(\Delta m + E_\pi\right)^2 + \bar{\Gamma}_\Delta^2\right]} \left(\frac{|\boldsymbol{k}_\pi|}{m_N}\right)^2 \text{🔵🔵}$$

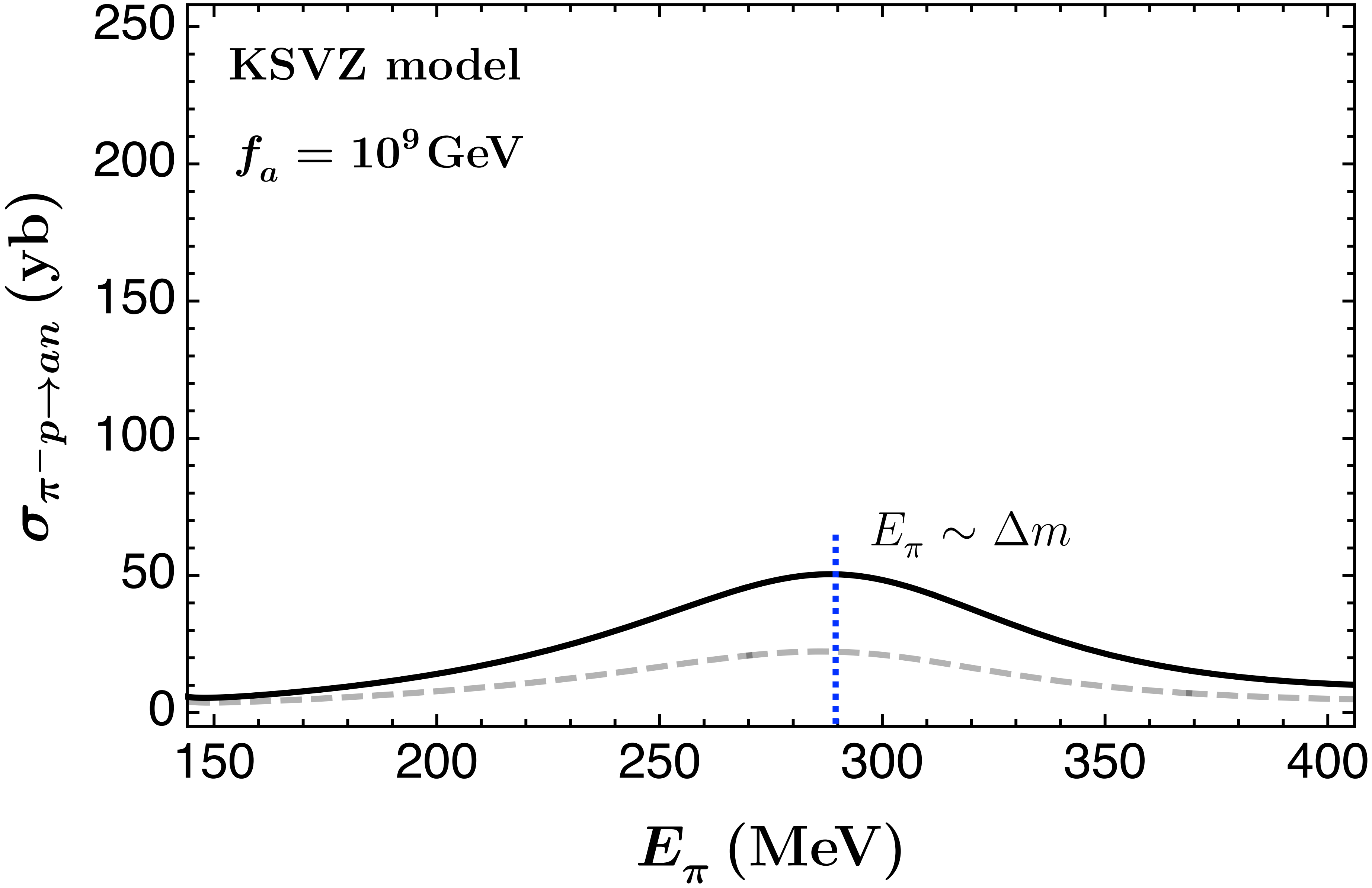
$$- \frac{8\sqrt{3} g_A C_{aN\Delta} \mathcal{C}}{27} \frac{E_\pi\left[\left(\Delta m^2 - E_\pi^2\right)\left(C_+ \Delta m + C_- E_\pi\right) + \bar{\Gamma}_\Delta^2\left(C_+ \Delta m - C_- E_\pi\right)\right]}{\left[\left(\Delta m - E_\pi\right)^2 + \bar{\Gamma}_\Delta^2\right]\left[\left(\Delta m + E_\pi\right)^2 + \bar{\Gamma}_\Delta^2\right] \text{🔴🔵}} \left(\frac{|\boldsymbol{k}_\pi|}{m_N}\right)^2$$

$\boldsymbol{m_N \gg |k_\pi|, E_\pi}$

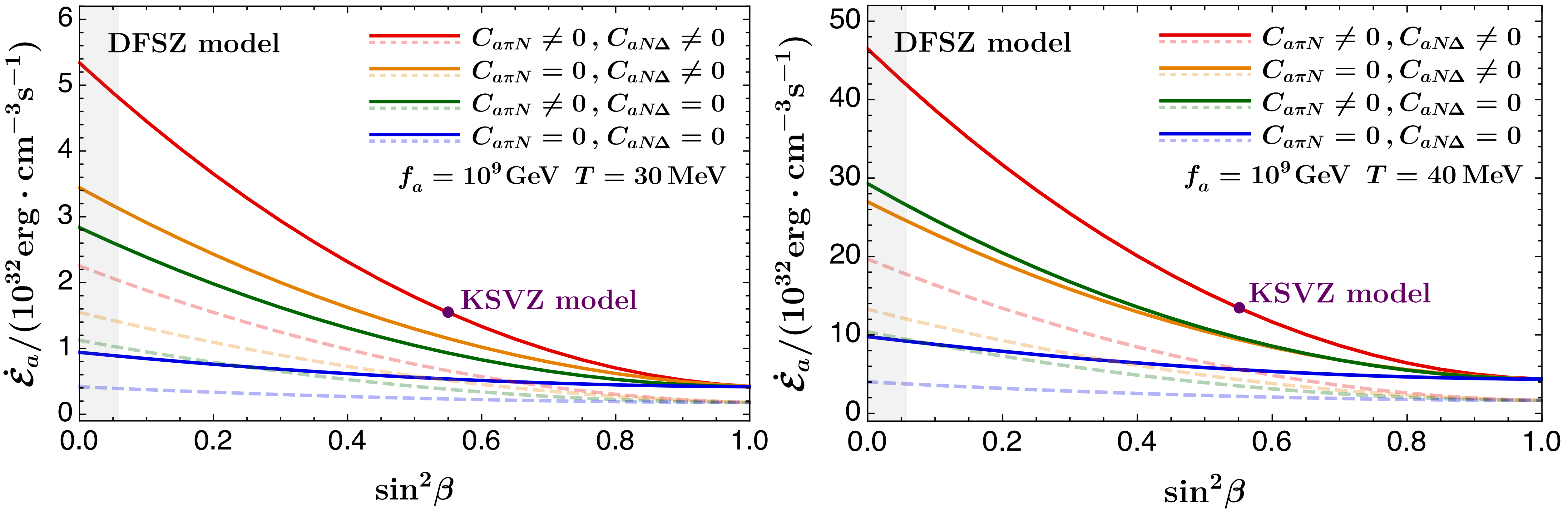
$$- \frac{16\sqrt{6} C_{a\pi N} C_{aN\Delta} \mathcal{C}}{27} \frac{E_\pi^2\left(\Delta m^2 - E_\pi^2 - \bar{\Gamma}_\Delta^2\right)}{\left[\left(\Delta m - E_\pi\right)^2 + \bar{\Gamma}_\Delta^2\right]\left[\left(\Delta m + E_\pi\right)^2 + \bar{\Gamma}_\Delta^2\right]} \left(\frac{|\boldsymbol{k}_\pi|}{m_N}\right)^2 \left(\frac{E_\pi}{m_N}\right) \text{🟢🔵}$$

Scattering cross section v.s. E_π

💥KSVZ model



Supernova Axion Emissivity v.s. $\sin^2\beta$



- 💥The gray band is excluded by tree-level unitarity of fermion scattering : $0.25 \lesssim \tan\beta \lesssim 170$ Luzio, et al. 2021
- 💥Supernova axion emissivity can be enhanced at most by a factor of $\sim$5 for $\beta \to 0$ compared to the earlier studies.

Supernova Axion Emissivity v.s. T