Relation between the Parameters of a Gravitational Lens and the Frequencies of Black-hole Quasi-normal Modes

Ivan Zh. Stefanov

Technical university - Sofia

Stoytcho S. Yazadjiev, Galin G. Gyulchev

St.Kliment Ohridski University of Sofia

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Gravitational waves and quasinormal modes

- Gravitational waves could serve as a rich source of information for the universe, supplementary to standard observations in the electromagnetic sector.

- Atoms – distinguished by their electromagnetic spectra. Different sources of gravitational wave (such as stars, black holes, binaries etc.) – distinguished by their gravitational wave spectra.

- Quasinormal modes (QNM) – the resonances. Characteristic frequencies of oscillation – complex, damped oscillations $\omega = \omega_R + i\omega_I$

- QNM – the "fingerprints" of the sources of gravitational waves; information about the physical characteristics of the sources – mass, charges, angular momenta etc.; the type of the object – a black hole, a neutron star, a boson star etc.; information which could help us sift out the viable physical theories – general relativity, $f(R)$ gravity, scalar-tensor theories, string theory and so on.

Connection between QNM and the trajectories of massless particles propagating in a black-hole space-time

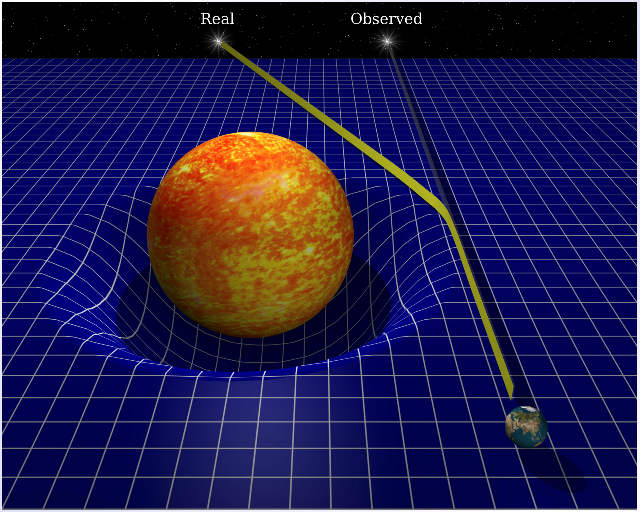
- In the eikonal approximation the frequencies of the QNM of black holes can be parameterized by two parameters.

$$\omega_{\mathrm{QNM}} = \Omega_m \, l - i(n + 1/2) \, |\lambda|$$

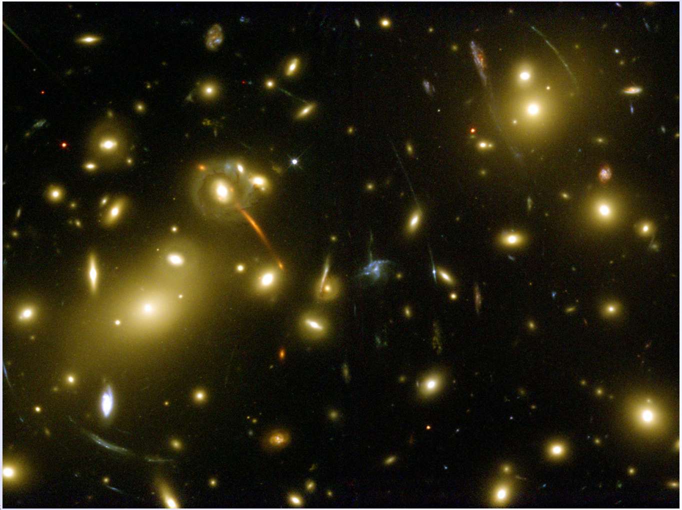
Cardoso et al (2009) where

- $l = 0, 1, 2...$ angular momentum of the mode
- $n = 0, 1, 2...$ the overtone
- Gravitational waves as massless particles propagating along the last, unstable, circular, null orbit and slowly leaking to infinity.
- λ Lyapunov exponent – determines the instability timescale of the last, unstable, circular, null orbit.
- Ω_m angular velocity of a particle propagating along the last, unstable, circular, null orbit.

Gravitational lenses



Gravitational lenses



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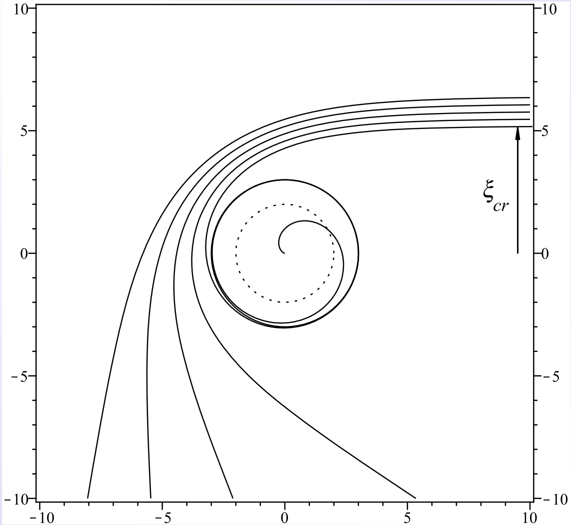
Gravitational lenses

- Objects acting as gravitational lenses : galaxies, galaxy clusters, compact objects (micro-lenses)
- Information about the physical characteristics of the lens: mass, mass distribution, angular momentum, charge, type of object, type of theory
- Photon sphere – the last, unstable, circular, null orbit.
- Relativistic images
- Properties of the images: number, distribution, deformation, magnification, time delay

Gravitational lenses



Strong deflection limit

$$\alpha(\theta) = -\overline{a} \ln \left(\frac{\theta D_{OL}}{u_m} - 1 \right) + \overline{b},$$

V. Bozza (2002)

- α deflection angle (becomes divergent with the approach of the light source to the optical axes)
- θ angular position of the light source
- D_{OL} distance observer-lens
- u_m impact parameter
- $\overline{a}$, $\overline{b}$ parameters characterizing the lens

Connection between the parameters of the last unstable, circular orbit and the lens parameters

$$\lambda = \frac{c}{u_m \overline{a}}.$$

$$\Omega_m = \frac{c}{u_m}.$$

$$\omega_{\rm QNM} = \Omega_m \, l - i(n + 1/2) \, |\lambda|$$

I. Stefanov, S. Yazadjiev, G. Gyulchev (2010)

Observables

$$\bar{a} = \frac{2\pi}{\ln \tilde{r}},$$

where

$$\tilde{r} = \frac{\mu_1}{\sum_{n=2}^{\infty} \mu_n}.$$

$r_m = 2.5\,\mathrm{Log}\ \tilde{r}.$

$$u_m = D_{OL}\theta_\infty,$$

where θ_∞ is the angular position that is closes to the black hole . Actually, only the first image can be observed separately and the rest of the images are packed togather.

The QNM frequencies in terms of the observables

$$\lambda = \frac{c \ln \tilde{r}}{2\pi D_{OL}\theta_\infty}.$$

$$\Omega_m = \frac{c}{D_{OL}\theta_\infty}.$$

or

$$\lambda = \frac{\ln \tilde{r}}{\triangle T_{2,1}}$$

$$\Omega_m = \frac{2\pi}{\triangle T_{2,1}}.$$

where $\triangle T_{2,1}$ is the time-delay between the first and the second relativistic image (of the order of days).

Possible applications

- Physical interpretation of the lens parameters
- Method for the measuring of the QNM frequencies of black holes through observation in the electromagnetic sector
- What frequencies to expect from a black hole acting as a gravitational lens, tune detectors
- Method for the localization of the source of gravitational waves (thousands of galaxy clusters in the error box of LISA).

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THANK YOU FOR YOUR ATTENTION!