"The new geometric formulation of quantum field theory according to Matey Mateev`s scenario"

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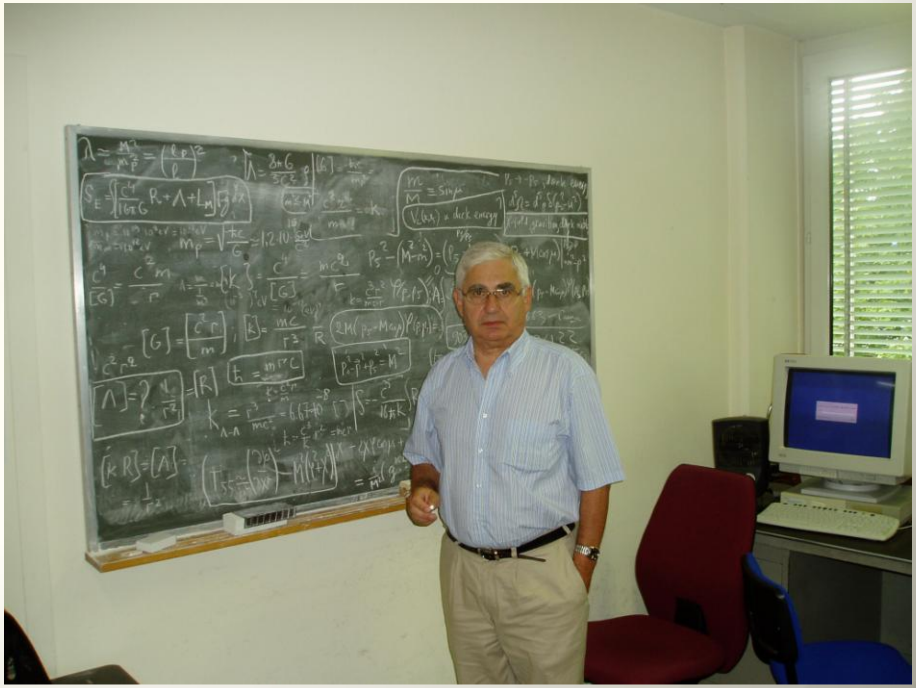

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A.Einstein:

"EXPERIMENT =

GEOMETRY + PHYSICS"

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For decades we have witnessed the impressive success of the Standard Model (SM) in explaining properties and regularities observed in experiments with elementary particles. The mathematical basis of the SM is the local Lagrangian QFT. The very concept of an elementary particle assumes that it does not have a composite structure. In agreement with the contemporary experimental data any fundamental particle of the SM does not manifest a structure like this, up to distances of the order of $10^{-16} - 10^{-17}$ cm. The adequate mathematical images of point like particles are the local quantized fields - boson and fermion. Particles are the quanta of the corresponding fields. In the framework of the SM these are leptons, quarks, vector bosons and the Higgs scalar, all characterized by certain values of mass, spin, electric charge, colour, isotopic spin, hypercharge, etc.

Intuitively it is clear that an elementary particle should carry small enough portions of different "charges" and "spins". In the theory this is guaranteed by assigning the local fields to the lowest representations of the corresponding compact groups.

As for the mass of the particle m, this quantity is the Casimir operator of the ***noncompact*** Poincaré group, and in the unitary representations of this group used in QFT they may have arbitrary values in the interval $0 \leq m < \infty$. In the SM one observes a great variety in mass values. For example, the t-quark is more than 300000 times heavier than the electron. In this situation the question naturally arises: **up to what values of mass one may apply the concept of a local quantum field?**

The free Klein-Gordon equation for the one component real scalar field $\varphi(x)$ has always the form

$$(\Box + m^2)\varphi(x) = 0. \tag{1}$$

Hence, after standard Fourier transform

$$\varphi(x) = \frac{1}{(2\pi)^{3/2}} \int e^{-ip_\mu x^\mu}\, \varphi(p)\, d^4p \qquad (p_\mu x^\mu = p^0 x^0 - \boldsymbol{p}.\boldsymbol{x}) \tag{2}$$

we find the equation of motion in the Minkowski momentum 4-space:

$$(m^2 - p^2)\varphi(p) = 0, \qquad p^2 = p_0^2 - \boldsymbol{p}^2. \tag{3}$$

From a geometric point of view m is the radius of the "mass shell" hyperboloid

$$m^2 = p_0^2 - \boldsymbol{p}^2, \tag{4}$$

where the field $\varphi(p)$ is defined and in the Minkowski momentum space one may embed hyperboloids of type (4) of an arbitrary radius.

Formally, the contemporary QFT remains a logically perfect scheme and its mathematical structure does not change at all up to arbitrarily large values of masses of quanta. **Maybe this pathological property is the Achilles heel of this theory?!** The key idea of the approach developed in [1-10] is the following radical hypothesis: **the mass spectrum of elementary particles, i.e., the objects described by local quantum fields, has to be cut off at a certain value M:**

$$m \leq M, \tag{5}$$

This statement has to be accepted as a new fundamental principle of Nature which similarly to the relativistic and quantum postulates should underlie QFT. The new universal physical constant M is not only the maximal value of particle mass but also plays the role of a new high-energy scale. We shall call this parameter the **fundamental mass**. It is worth emphasizing that here, due to (5), the Compton wave length of a particle $\lambda_C = \hbar/mc$ cannot be smaller than the "fundamental length" $l = \hbar/Mc$. According to [11], the parameter λ_C characterizes the dimensions of the region of space in which a relativistic particle of mass m can be localized. Therefore, the fundamental length l introduces into the theory a universal bound on the accuracy of the localization in space of elementary particles.

Let us go back to the free one-component real scalar field we considered above (see(1)-(4)). We suppose that its mass m satisfies the condition (5). How should one modify the equations of motion in order that the existence of the bound (5) should become as evident as it is the limitation $v \le c$ in the special theory of relativity? In the latter case, everything is explained in a simple way: the relativization of the 3-dimensional velocity space is equivalent to a transition in this space from Euclidean to Lobachevsky geometry realized on the upper sheet of the 4-dimensional hyperboloid (4). Let us act in a similar way and substitute the 4-dimensional Minkowski momentum space, which is used in the standard QFT, to (anti)de Sitter momentum space realized on the 5-hyperboloid:

$$p_0^2 - \boldsymbol{p}^2 + p_5^2 = M^2. \tag{6}$$

We suppose that in the p-representation our scalar field is defined just on the surface (6), i.e., it is a function of five variables $(p_0, \boldsymbol{p}, p_5)$, which are connected by relation (6):

$$\delta(p_0^2 - \boldsymbol{p}^2 + p_5^2 - M^2)\varphi(p_0, \boldsymbol{p}, p_5). \tag{7}$$

Here the energy p_0 and the 3-momentum $\boldsymbol{p}$ preserve their usual meaning and the mass shell relation (4) is satisfied as well. Therefore, for the field considered $\varphi(p_0, \boldsymbol{p}, p_5)$ the condition (5) is always fulfilled.

Clearly, in eq. (7) the specification of a single function $\varphi(p_0, \boldsymbol{p}, p_5)$ of five variables (p_μ, p_5) is equivalent to the definition of two independent functions $\varphi_1(p)$ and $\varphi_2(p)$ of the 4-momentum p_μ:

$$\varphi(p_0, \boldsymbol{p}, p_5) \equiv \varphi(p, p_5) = \begin{pmatrix} \varphi(p, p_5) \\ \varphi(p, -p_5) \end{pmatrix} = \begin{pmatrix} \varphi_1(p) \\ \varphi_2(p) \end{pmatrix}, |p_5| = \sqrt{M^2 - p^2}. \tag{8}$$

The appearance of the new discrete degree of freedom $p_5/|p_5|$ and the associated doubling of the number of field variables is an important feature of the new approach. It must be taken into account in the search for the equation of motion for the free field in the de Sitter momentum space. Due to the mass shell relation (4), the Klein - Gordon equation (3) should also be satisfied by the field $\varphi(p_0, \boldsymbol{p}, p_5)$:

$$(m^2 - p_0^2 + \boldsymbol{p}^2)\varphi(p_0, \boldsymbol{p}, p_5) = 0. \tag{9}$$

From our point of view this relation is unsatisfactory for two reasons:

1. It does not reflect the bounded mass condition (5).

2. It cannot be used to determine the dependence of the field on the new quantum number $p_5/|p_5|$ in order to distinguish between the components $\varphi_1(p)$ and $\varphi_2(p)$.

Here we notice that because of (6) eq.(9) can be written as:

$$(p_5 + M\cos\mu)(p_5 - M\cos\mu)\varphi(p, p_5) = 0, \qquad \cos\mu = \sqrt{1 - \frac{m^2}{M^2}}. \tag{10}$$

Now following the Dirac trick we postulate the equation of motion under question in the form:

$$2M(p_5 - M\cos\mu)\varphi(p, p_5) = 0. \tag{11}$$

Clearly, eq. (11) has none of the enumerated defects of the standard Klein-Gordon equation (9). However, equation (9) is still satisfied by the field $\varphi(p, p_5)$.

From eqs. (11) and (8) it follows that

$$2M(|p_5| - M\cos\mu)\varphi_1(p) = 0,$$
$$2M(|p_5| + M\cos\mu)\varphi_2(p) = 0, \tag{12}$$

and we obtain:

$$\varphi_1(p) = \delta(p^2 - m^2)\widetilde{\varphi}_1(p)$$
$$\varphi_2(p) = 0 \tag{13}$$

Therefore, the free field $\varphi(p, p_5)$ defined in the (anti) de Sitter momentum space (6) describes the same free scalar particles of mass m as the field $\varphi(p)$ in the Minkowski p-space, with the only difference that now we necessarily have $m \leq M$. The two-component structure (8) of the new field does not manifest itself on the mass shell, owing to (13). However, it will play an important role when the fields interact, i.e., off the mass shell.

Now we face the problem of constructing the action corresponding to eq. (11) and transforming it to the configuration representation.

In the following, we shall use the Euclidean formulation of the theory which appears as an analytical continuation to purely imaginary energies:

$$p_0 \to ip_4. \tag{14}$$

In this case, instead of the (anti) de Sitter p-space (6), we shall work with de Sitter p-space

$$-p_n^2 + p_5^2 = M^2, \qquad n = 1, 2, 3, 4. \tag{15}$$

Obviously,

$$p_5 = \pm\sqrt{M^2 + p^2}. \tag{16}$$

If one uses eq. (15), the Euclidean Klein-Gordon operator $m^2 + p^2$ may be written, similarly to (10), in the following factorized form:

$$m^2 + p^2 = (p_5 + M\cos\mu)(p_5 - M\cos\mu). \tag{17}$$

Clearly, the nonnegative functional

$$S_0(M) = \pi M \times$$

$$\int \frac{d^4p}{|p_5|}\left[\varphi_1^+(p) 2M(|p_5| - M\cos\mu)\varphi_1(p) + \varphi_2^+(p) 2M(|p_5| + M\cos\mu)\varphi_2(p)\right], \tag{18}$$

$$\varphi_{1,2}(p) \equiv \varphi(p, \pm|p_5|), \tag{19}$$

plays the role of the action integral for the free field $\varphi(p, p_5)$. The action may be written also as a 5 - integral:

$$S_0(M) = 2\pi M \times$$

$$\int \varepsilon(p_5)\delta(p_L p^L - M^2) d^5p \left[\varphi^+(p, p_5) 2M(p_5 - M\cos\mu)\varphi(p, p_5)\right], \tag{20}$$

$$L = 1, 2, 3, 4, 5,$$

where

$$\varepsilon(p_5) = \frac{p_5}{|p_5|}. \tag{21}$$

The Fourier transform and the configuration representation have a special role in this approach. First, we note that in the basic equation (15), which defines the de Sitter p-space, all the components of the 5-momentum enter on equal footing. Therefore, the expression $\delta(p_L p^L - M^2)\varphi(p, p_5)$, which now replaces (7), may be Fourier transformed in the following way:

$$\frac{2M}{(2\pi)^{3/2}} \int e^{-ip_K x^K} \delta(p_L p^L - M^2)\varphi(p, p_5) d^5\, p = \varphi(x, x_5), \quad K, L = 1, 2, 3, 4, 5. \tag{22}$$

This function obviously satisfies the following differential equation in the ***5-dimensional configuration space***:

$$\left(\frac{\partial^2}{\partial x_5^2} - \Box + M^2\right)\varphi(x, x_5) = 0. \tag{23}$$

Integration over p_5 in (22) gives:

$$\varphi(x,x_5) = \frac{M}{(2\pi)^{3/2}} \int e^{ip_n x^n} \frac{d^4p}{|p_5|} \left[e^{-i|p_5|x^5} \varphi_1(p) + e^{i|p_5|x^5} \varphi_2(p) \right], \qquad (24)$$

$$\varphi^+(x,x_5) = \varphi(x,-x_5),$$

from which we get:

$$\frac{i}{M} \frac{\partial \varphi(x,x_5)}{\partial x_5} = \frac{1}{(2\pi)^{3/2}} \int e^{ip_n x^n} d^4p \left[e^{-i|p_5|x^5} \varphi_1(p) - e^{i|p_5|x^5} \varphi_2(p) \right], \qquad (25)$$

The four dimensional integrals (24) and (25) transform the fields $\varphi_1(p)$ and $\varphi_2(p)$ to the configuration representation. The inverse transforms have the form:

$$\varphi_1(p) = \frac{-i}{2M(2\pi)^{5/2}} \int e^{-ip_n x^n} d^4x \left[\varphi(x,x_5) \frac{\partial e^{i|p_5|x^5}}{\partial x_5} - e^{i|p_5|x^5} \frac{\partial \varphi(x,x_5)}{\partial x_5} \right], \qquad (26)$$

$$\varphi_2(p) = \frac{i}{2M(2\pi)^{5/2}} \int e^{-ip_n x^n} d^4x \left[\varphi(x,x_5) \frac{\partial e^{-i|p_5|x^5}}{\partial x_5} - e^{-i|p_5|x^5} \frac{\partial \varphi(x,x_5)}{\partial x_5} \right].$$

We note that the independent field variables

$$\varphi(x,0) \equiv \varphi(x) = \frac{M}{(2\pi)^{3/2}} \int e^{ip_n x^n} d^4 p \frac{\varphi_1(p) + \varphi_2(p)}{|p_5|} \tag{27}$$

and

$$\frac{i}{M}\frac{\partial \varphi(x,0)}{\partial x_5} \equiv \chi(x) = \frac{1}{(2\pi)^{3/2}} \int e^{ip_n x^n} d^4 p \, [\varphi_1(p) - \varphi_2(p)] \tag{28}$$

can be treated as initial Cauchy data on the surface $x_5 = 0$ for the hyperbolic-type equation (23).

Now substituting eq.(26) into the action (18) we obtain

$$S_0(M) = \frac{1}{2} \int d^4 x \left[\left| \frac{\partial \varphi(x,x_5)}{\partial x_n} \right|^2 + m^2 |\varphi(x,x_5)|^2 + \left| i \frac{\partial \varphi(x,x_5)}{\partial x_5} - M \cos \mu \varphi(x,x_5) \right|^2 \right] \equiv$$

$$\equiv \int L_0(x,x_5) d^4 x. \tag{29}$$

It is easily verified that due to eq. (23) the action (29) is independent of x_5:

$$\frac{\partial S_0(M)}{\partial x_5} = 0. \tag{30}$$

Therefore, the variable x_5 can be arbitrarily fixed and $S_0(M)$ can be viewed as a functional of the corresponding initial Cauchy data for the equation (23). For example, for $x_5 = 0$ we have:

$$S_0(M) = \frac{1}{2}\int d^4 x\left[\left(\frac{\partial\varphi(x)}{\partial x_n}\right)^2 + m^2(\varphi(x))^2 + M^2\left(\chi(x) - \cos\mu\varphi(x)\right)^2\right] \equiv$$

$$\equiv \int L_0(x, M) d^4 x. \tag{31}$$

Thus, we have shown that in the developed approach the property of locality of the theory does not disappear, moreover it becomes even deeper, as it is extended to dependence on the extra fifth dimension x_5.

It is clear that the dependence of the action (31) on the two functional arguments $\varphi(x)$ and $\chi(x)$ is a direct consequence of the fact that in momentum space the field has a doublet structure $\begin{pmatrix} \varphi_1(p) \\ \varphi_2(p) \end{pmatrix}$ due to two possible signs of p_5. However, the Lagrangian $L_0(x, M)$ does not contain a kinetic term corresponding to the field $\chi(x)$. Therefore, this variable is just **auxiliary**. In advance let us point out that the special role of the 5-dimensional configuration space in the new formalism is determined by the fact that the gauge symmetry transformations are now localized in it.

Let us discuss the question about the conditions for the transition of the new scheme into the standard Euclidean QFT (the so called "correspondence principle"). The Euclidean momentum 4-space is the "flat limit" of the de Sitter p-space (15) and may be associated with the approximation

$$\begin{gathered} |p_n| \ll M \\ p_5 \simeq M \\ . \end{gathered} \tag{32}$$

In configuration space we have, respectively,

$$\begin{gathered} \varphi(x, x_5) \simeq e^{-iMx_5}\varphi(x) \\ \chi(x) \simeq \varphi(x) \\ . \end{gathered} \tag{33}$$

In the next approximation

$$\varphi(x) - \chi(x) \simeq \frac{\Box\varphi(x)}{2M^2}. \tag{34}$$

Taking into account (10), (31) and (34) one may conclude that in the "flat limit" (formally when $M \to \infty$) the Lagrangian $L_0(x, M)$ from (31) coincides with its Euclidean counterpart.

Let us briefly consider the new version of the free Maxwell field theory based on the de Sitter p-space (15). The electromagnetic potential, similarly to the 5-momentum, now becomes a 5-vector

$$A_L(p,p_5) = \{A_l(p,p_5), A_5(p,p_5)\} = \{A_l^\dagger(-p,p_5), -A_5^\dagger(-p,p_5)\}$$
$$l = 1,2,3,4. \tag{35}$$

Its 5-dimensional Fourier transform looks like

$$A_L(x,x_5) = \tfrac{2M}{(2\pi)^{3/2}} \int e^{-ip_N x^N} \delta(p_K p^K - M^2) A_L(p,p_5) d^5 p,$$
$$K, L, N = 1,2,3,4,5. \tag{36}$$

It is evident that (36) satisfies equation (23):

$$\left(\frac{\partial^2}{\partial x_5^2} - \Box + M^2\right) A_L(x,x_5) = 0. \tag{37}$$

The action is given by the integrals (compare with (20) and (29))

$$S_0(M) = 2\pi M \times$$

$$\times \int \varepsilon(p_5)\delta(p_L p^L - M^2) d^5 p\, 2M(p_5 - M) \left| A_n(p,p_5) - \frac{p_n A_5(p,p_5)}{p_5 - M} \right|^2 =$$

$$= \int d^4 x L_0(x,x_5) = \frac{1}{4}\int d^4 x F^*_{KL}(x,x_5) F^{KL}(x,x_5) + \tag{38}$$

$$+ \frac{1}{2}\int d^4 x \left| \frac{\partial(e^{iMx_5} A_L(x,x_5))}{\partial x_L} - 2iMe^{iMx_5} A_5(x,x_5) \right|^2 ,$$

$$n = 1,2,3,4; \qquad K,L = 1,2,3,4,5,$$

where the "field strength 5-tensor":

$$F^{KL}(x,x_5) = \frac{\partial(e^{iMx_5} A_K(x,x_5))}{\partial x_L} - \frac{\partial(e^{iMx_5} A_L(x,x_5))}{\partial x_K}. \tag{39}$$

is introduced. This quantity is obviously expressed in terms of the commutator of the 5-dimensional covariant derivatives

$$D_L = \frac{\partial}{\partial x^L} - iqe^{iMx_5} A_L(x, x_5), \tag{40}$$

where q is the electric charge. It is easy to verify that the integral (38) is invariant under gauge transformations of the 5-potential $A_L(x, x_5)$:

$$e^{iMx_5} A_L(x, x_5) \to e^{iMx_5} A_L(x, x_5) - \frac{\partial(e^{iMx_5}\lambda(x, x_5))}{\partial x^L} \tag{41}$$

with the condition

$$\left(\frac{\partial^2}{\partial x_5^2} - \Box + M^2\right)\lambda(x, x_5) = 0, \quad \lambda^\dagger(x, x_5) = \lambda(x, -x_5). \tag{42}$$

Let us emphasize that the solution of equation (42) is defined by the initial data

$$\begin{aligned} \lambda(x, 0) &\equiv \lambda(x) = \lambda^\dagger(x) \\ \tfrac{i}{M}\tfrac{\partial\lambda(x,0)}{\partial x_5} &\equiv \mu(x) = \mu^\dagger(x) \end{aligned} \tag{43}$$

The action (38), due to (37), similarly to its scalar analogue (29), does not depend on the coordinate x_5. For that reason it may be considered as a functional of the Cauchy data for equation (37):

$$A_L(x, 0) = A_L(x), \qquad \frac{i}{M}\frac{\partial A_L(x, 0)}{\partial x_5} \equiv X_L(x). \tag{44}$$

According to (41),(43) and (44) the gauge transformations of these functions are the following:

$$\begin{gathered} A_l(x) \rightarrow A_l(x) - \frac{\partial\lambda(x)}{\partial x^l} \\ A_5(x) \rightarrow A_5(x) - iM(\lambda(x) - \mu(x)) \\ X_l(x) \rightarrow X_l(x) - \frac{\partial\mu(x)}{\partial x^l} \\ X_5(x) \rightarrow X_5(x) + iM(\lambda(x) - \mu(x)) - \frac{i}{M}\Box\lambda(x) \\ l = 1, 2, 3, 4. \end{gathered} \tag{45}$$

Let us emphasize that in the gauge

$$A_5(x) = 0 \tag{46}$$

the transformations (45) shrink up to a standard gauge group parameterized by the function $\lambda(x)$.

If one considers the charged scalar particles in our formalism, the corresponding action integral takes the form(cf(3.1))

$$S_0(M) = \int d^4x \left[\left| \frac{\partial\varphi(x)}{\partial x_n} \right|^2 + m^2 \left|\varphi(x)\right|^2 + M^2 \left|\chi(x) - \cos\mu\varphi(x)\right|^2 \right], \tag{47}$$

where $\varphi(x)$ and $\chi(x)$ are complex functions.

It is easy to realize that the Abelian gauge group (45) has the following representation in the charged scalar field basis:

$$\begin{aligned} &\varphi \to e^{iq\lambda(x)}\varphi(x) \\ &\chi(x) \to e^{iq\lambda(x)}\left[i(\mu(x)-\lambda(x))\varphi(x)+\chi(x)\right] \end{aligned} \tag{48}$$

(q is the electric charge). The technique developed allows one to formulate in our terms a unique prescription for construction of the action integral for the Euclidean scalar electrodynamics consistent with the requirements of locality, gauge invariance, and the de Sitter structure of the momentum space:

1. In the action integral (47) for the complex scalar field it is necessary to replace the simple derivatives (including $\frac{\partial}{\partial x_5}$ in $\chi(x) = \frac{i}{M}\frac{\partial}{\partial x_5}\varphi(x,0)$) by the covariant ones (see (40)).

2. Add to the obtained expression the action integral of the electromagnetic field (38) putting $x_5 = 0$ in it.

   The total action integral remains invariant under simultaneous transformations (45) and (48).

As far as the new QFT is elaborated on the basis of the de Sitter momentum space (15), it is natural to suppose that in the developed approach the fermion fields $\psi_\alpha(p, p_5)$ have to be de Sitter spinors, i.e., to transform under the four-dimensional representation of the group SO(4, 1). Further on, we shall use the following γ - matrix basis ($\gamma^4 = i\gamma^0$):

$$\gamma^L = (\gamma^1, \gamma^2, \gamma^3, \gamma^4, \gamma^5)$$

$$\{\gamma^L, \gamma^M\} = 2g^{LM},$$

$$g^{LM} = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{49}$$

In the ordinary formalism the free Euclidean Dirac operator

$$D(p) = m + p_n\gamma^n; \quad n = 1, 2, 3, 4 \tag{50}$$

appears as a result of factorization of the Euclidean K.-G.wave operator:

$$p_n^2 + m^2 = (m + p_n\gamma^n)(m - p_n\gamma^n). \tag{51}$$

Now instead of (51) we obtain the following factorization formula:

$$2M(p_5 - M\cos\mu) = \tag{52}$$
$$\left[2M\sin\frac{\mu}{2} + p_n\gamma^n + (p_5 - M)\gamma^5\right]\left[2M\sin\frac{\mu}{2} - p_n\gamma^n - (p_5 - M)\gamma^5\right]$$

and, correspondingly, instead of (50) the new expression for the Dirac operator

$$D(p, M) = p_n\gamma^n + (p_5 - M)\gamma^5 + 2M\sin\frac{\mu}{2}. \tag{53}$$

It is easy to check that in the "flat approximation"

$$|p_n| \ll M,\ \ m \ll M,\ \ p_5 \simeq M$$

both expressions (53) and (50) coincide. The operator (53) allows us to develop the local spinor field formalism in configuration space that can be considered as a generalization of the Euclidean Dirac theory along our lines. But the amusing point is that the new KG-operator $2M(p_5 - M\cos\mu)$ has one more decomposition into matrix factors:

$$2M(p_5 - M\cos\mu) = \tag{54}$$
$$\left[p_n\gamma^n + \gamma^5(p_5 + M) + 2M\cos\frac{\mu}{2}\right]\left[p_n\gamma^n + \gamma^5(p_5 + M) - 2M\cos\frac{\mu}{2}\right].$$

Therefore, if our approach is considered to be realistic, it may be assumed that in Nature there exists some **exotic fermion field** associated with the wave operator

$$D_{exotic}(p, M) = p_n\gamma^n + \gamma^5(p_5 + M) + 2M\cos\frac{\mu}{2} \tag{55}$$

In contrast to $D(p, M) = p_n\gamma^n + (p_5 - M)\gamma^5 + 2M\sin\frac{\mu}{2}$ the operator $D_{exotic}(p, M)$ does not have a limit when $M \to \infty$, that justifies the name chosen for the field considered. The polarization properties of the exotic fermion field differ sharply from the standard ones. **It is tempting to think that the quanta of the exotic fermion field have a relation to the structure of the "dark matter".**

Using the matrix basis $(\gamma^1, \gamma^2, \gamma^3, \gamma^4, \gamma^5)$ one may represent (15) as

$$(M + p_L\gamma^L)(M - p_K\gamma^K) = M^2 - p_Kp^K = 0, \quad K, L = 1, 2, 3, 4, 5.$$

For spinor field $\psi(p, p_5)$, which is defined on the de Sitter surface (15), the matrix operators

$$\begin{array}{c} \frac{1}{2M}(M + p_K\gamma^K) \equiv \Pi_R(p, p_5) \\ \frac{1}{2M}(M - p_K\gamma^K) \equiv \Pi_L(p, p_5) \end{array} \tag{56}$$

are projection operators. In other words,

$$\begin{array}{c} \Pi_R^2 = \Pi_R, \quad \Pi_L^2 = \Pi_L \\ \Pi_R\Pi_L = \Pi_L\Pi_R = 0 \\ \Pi_R + \Pi_L = 1. \end{array} \tag{57}$$

So in the de Sitter momentum space the fermion field $\psi(p, p_5)$ can be represented as a sum of two fields

$$\begin{array}{c} \psi(p, p_5) = \psi_R(p, p_5) + \psi_L(p, p_5) \\ \psi_R(p, p_5) = \Pi_R\psi(p, p_5) \\ \psi_L(p, p_5) = \Pi_L\psi(p, p_5) \end{array} \tag{58}$$

which obey the following 5-dimensional Dirac equations:

$$\begin{array}{c} (M - p_K\gamma^K)\psi_R(p, p_5) = 0 \\ (M + p_K\gamma^K)\psi_L(p, p_5) = 0. \end{array} \tag{59}$$

Obviously, decomposition (58) is de Sitter invariant. It is easy to verify that in the "flat approximation" $|p_n| \ll M$, $p_5 \simeq M$ one has

$$\Pi_{R,L} = \frac{1 \pm \gamma^5}{2}. \tag{60}$$

This is the reason that we can consider the fields $\psi_R(p, p_5)$ and $\psi_L(p, p_5)$ as "chiral" components in our approach. The new chirality operator $\frac{p_L \gamma^L}{M}$, similarly to its "flat counterpart", has eigenvalues equal to ± 1 but depends on the energy and momentum. It is well known that the chiral fermions are the basic spinor field variables in the SM. The new geometrical nature of these quantities has to manifest itself at high energies $E \geq M$. In configuration space the 5-dimensional Dirac equations (59) take the form

$$\begin{gathered} \left[M - i\frac{\partial}{\partial x^K}\gamma^K\right] \psi_R(x, x_5) = 0 \\ \left[M + i\frac{\partial}{\partial x^K}\gamma^K\right] \psi_L(x, x_5) = 0 \\ K = 1, 2, 3, 4, 5. \end{gathered} \tag{61}$$

Introducing the corresponding initial conditions at $x_5 = 0$

$$\psi_R(x,0) \equiv \psi_{(R)}(x)$$
$$\psi_L(x,0) \equiv \psi_{(L)}(x)$$

one obtains the local fields which can undergo chiral gauge transformations. The new geometric concept of chirality allows us to think that the parity violation in weak interactions discovered more than 50 years ago was a manifestation of the de Sitter nature of the momentum 4-space.

It was demonstrated that there exists a local field formalism respecting the gauge invariance principle and being consistent with our main hypothesis $m \leq M$.

A nontrivial generalization of the Standard Model based on our geometric approach, in particular, on a new concept of chirality, is now being worked out. Unfortunately, already without Mag ...

In conclusion, I will demonstrate just one fragment of the new SM version concerning the Higgs boson. It is clear that for description of this particle in our framework, one needs two $\mathrm{SU_L}(2)$ -doublets of complex scalar fields $\varphi(x)$ and $\chi(x)$:

$$\varphi(x) = \begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \end{pmatrix}, \quad \chi(x) = \begin{pmatrix} \chi_1(x) \\ \chi_2(x) \end{pmatrix} \tag{62}$$

which are defined in the Euclidean 4-dimensional x-space.

Let us consider in this space the following Lagrangian:

$$L_{\mathrm{HIGGS}}(\varphi,\chi) = \tag{63}$$
$$\left(\frac{\partial\varphi(x)}{\partial x^n}\right)^\dagger\left(\frac{\partial\varphi(x)}{\partial x^n}\right) + M^2(\varphi(x)-\chi(x))^\dagger(\varphi(x)-\chi(x))$$
$$+\frac{\lambda^2}{2}\left\{\left[\frac{\varphi^\dagger(x)\varphi(x)+\chi^\dagger(x)\chi(x)}{2}-v^2\right]-v^2(\varphi(x)-\chi(x))^\dagger(\varphi(x)-\chi(x))\right\}$$
$$\equiv\left(\frac{\partial\varphi(x)}{\partial x^n}\right)^\dagger\left(\frac{\partial\varphi(x)}{\partial x^n}\right)+U(\varphi(x),\chi(x)),$$

where

$$M^2 > \frac{\lambda^2 v^2}{2}. \tag{64}$$

The potential $U(\varphi(x),\chi(x))$ admits an infinite set of degenerate ground states with minimum energy satisfying the following condition (cf. [12], p. 16):

$$|\varphi_2(x)| = |\chi_2(x)| = \tfrac{v}{2}.$$

The standard procedure based on the fixation of the ground state leads to the spontaneous breakdown of the $SU(2)_L \bigotimes U(1)_Y$ - symmetry. As a result, we obtain the following expression for the Higgs boson mass:

$$m_{\rm H} = \sqrt{2}\lambda v \sqrt{1 - \frac{\lambda^2 v^2}{2M^2}} \tag{65}$$

(see (64)). From (65) one easily finds

$$1 - \frac{m_{\rm H}^2}{M^2} = \left(1 - \frac{\lambda^2 v^2}{M^2}\right)^2 \geq 0. \tag{66}$$

So our main principle (5) is not violated. In the "flat limit" $M \to \infty$ relation (65) gives us the familiar formula:

$$m_{\rm H} = \sqrt{2}\lambda v. \tag{67}$$

Instead of epilogue

Eternal memory to Rumi and Mag…