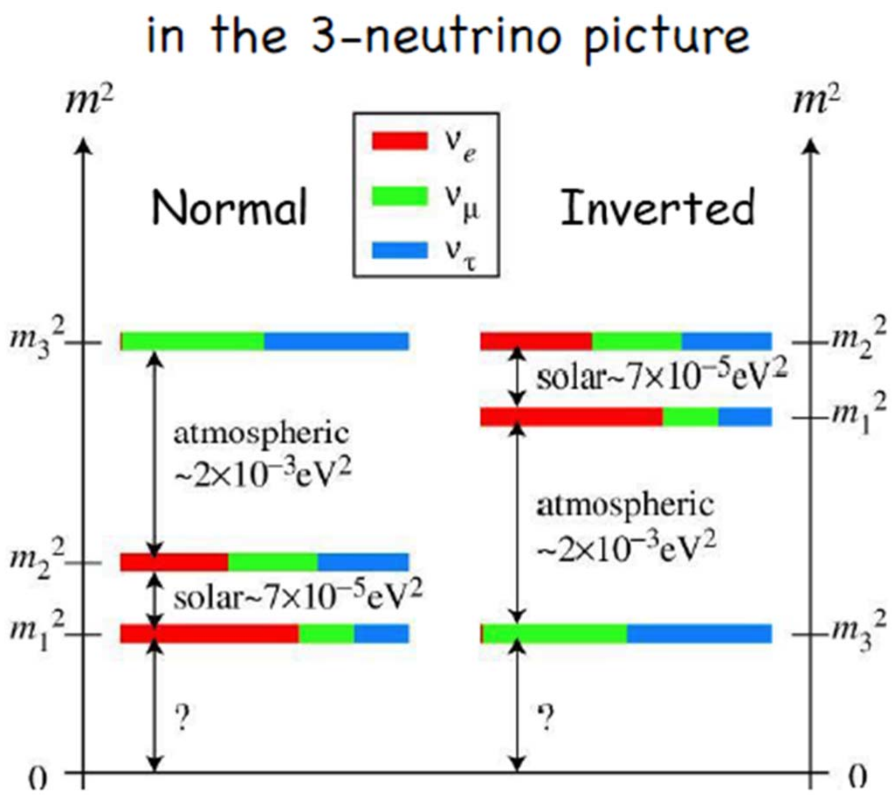
In Honour of Prof. Matey Mateev

Why Neutrinos are different …

- Very low mass
- Large leptonic mixing
- Leptonic number conserved or not ?
  With link to matter antimatter asymetry

J-M. Frère,Dec. 1st, 2010 IIHE, Brussels

What we now from oscillations



Generic prediction of this model:

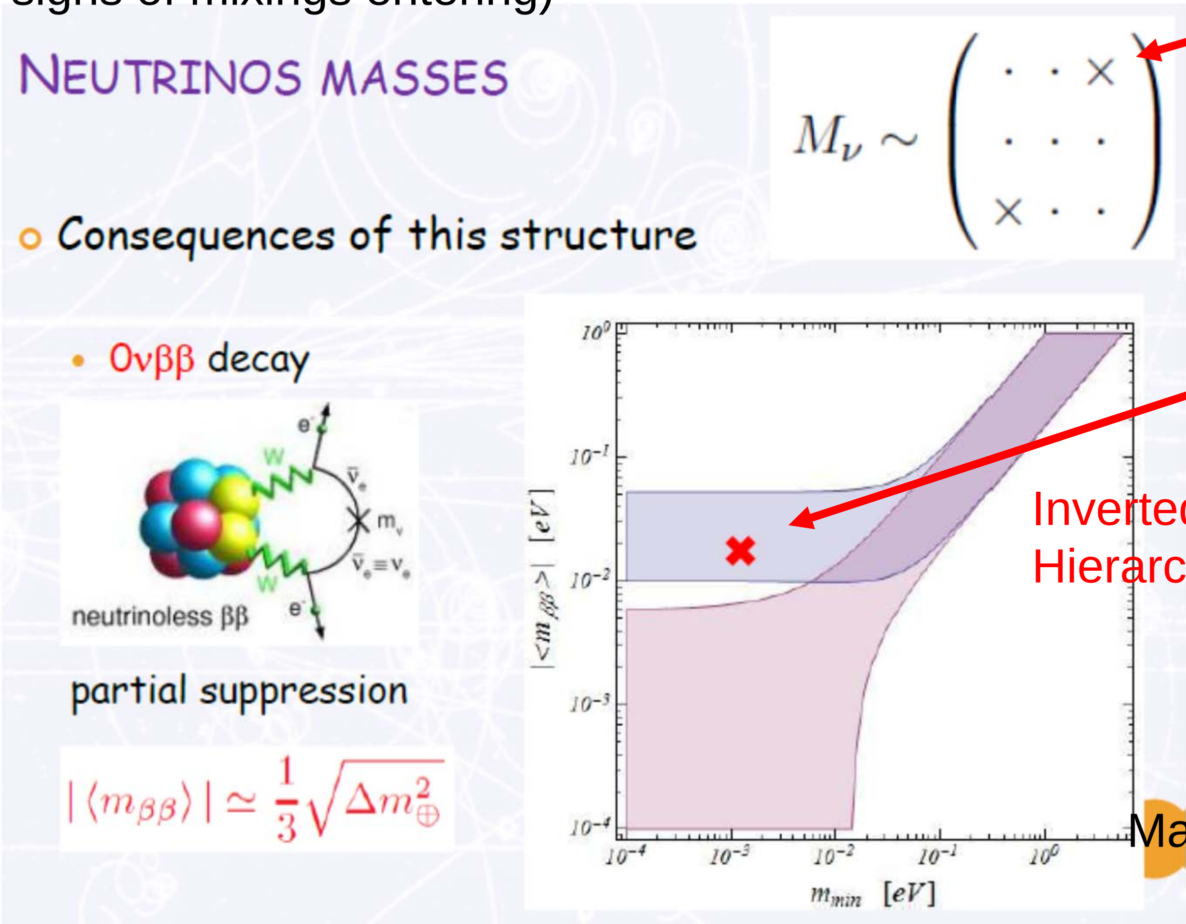
large mixings,

inverted hierarchy

suppressed neutrinoless double beta decay

Generic prediction : **large mixings,**
inverted hierarchy
suppressed neutrinoless double beta decay

Neutrino-less double beta decay controlled by weighted sum of masses (with signs of mixings entering)



based on **arXiv:1006.5196** , to appear in JHEP
And work with M Libanov, S. Troitsly, E Nugaev, FS Ling

Why Neutrinos are Different

How LHC can confirm the model

How LHC can compete with
fixed target Lepton Flavour Violation expts

From now on …

An « ordinary » Z’
(with same
couplings as Z)

e+ e- pairs

for later …

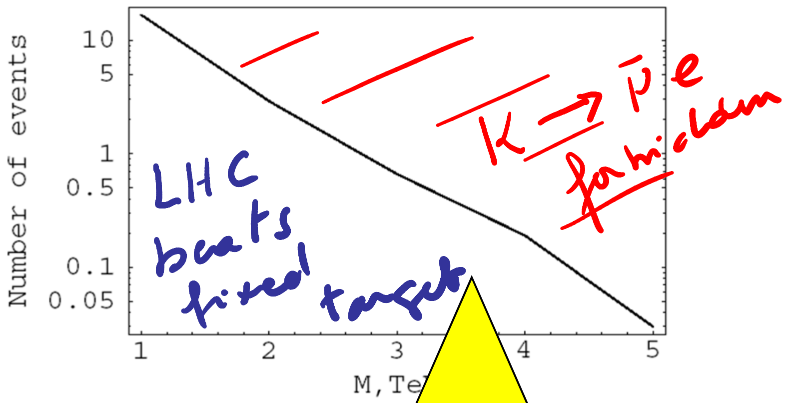


Fig. 1. Num… …uc-tion as a function … M, with $\kappa = M/(100\text{TeV})$.

μ+ e- pairs

In a nutshell:

- One family in 6D and proper boundary conditions → 3 families in 6D
- At lowest order in Cabibbo mixing , Charged fermion masses are
  - diagonal
  - strongly hierarchical

At LHC, this can result in rare exotic signals ($Z' \rightarrow \mu^+ e^- >> \mu^- e^+$)
but also a « standard » Z' , more readily observed

- At same order, we get 4D Majorana neutrinos with
  - Antidiagonal mass matrix
- This yields, in a generic way:
  - Large mixings in the neutrino sector
  - Inverted Hierarchy
  - Pseudo- Dirac structure (further suppression of neutrinoless double beta decay)
- Not as automatic, but typical : measurable Θ_{13}

A very few words about extra dimensions … start with ONE extra spatial dim.

What are Zero Modes ?

$$A = 0,1,2,3,4,5$$
$$\mu,\nu = 0,1,2,3$$

Dirac equation in N+1 dimensions,
For a fermion interacting with a field Φ:

$$i\partial_A \gamma^A \Psi = \Phi \Psi$$

(or $m \Psi$)

For ONE compact extra dim

$0 \leq y \leq 2\pi R$

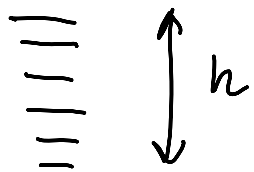
$$\Psi(x^\mu, y) = \sum \Psi_n(x^\mu) e^{i\frac{ny}{2\pi R}}$$

$$i\partial_\nu \gamma^\nu \Psi_n(x^\mu) e^{i\frac{ny}{2\pi R}} = \left(\frac{n}{2\pi R} i\gamma_5 + \Phi\right) \Psi_n(x^\mu) e^{i\frac{ny}{2\pi R}}$$

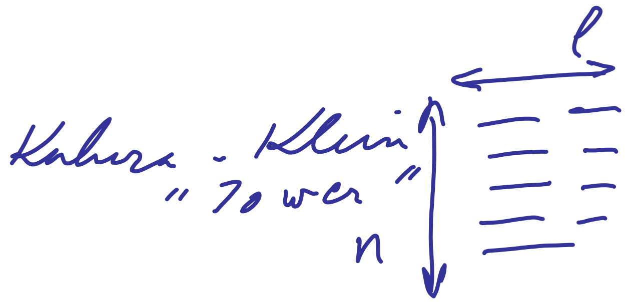
→ Kaluza-Klein tower

effective mass ($\gg$ 1 TeV)

= 0 ⇒ zero modes.



For 2 compact extra dim



$$\Psi(x^\mu, x^4, x^5) = \sum_{n,l} \psi_{n,l}(x^\mu)\ f_{n,l}(x^4, x^5)$$

$$i\partial_\nu\ \gamma^\nu \psi_{n,l}(x^\mu)\ f_{n,l}(x^4, x^5) = \psi_{n,l}(x^\mu)(\Phi - i\partial_4\ \gamma^4\ - i\partial_5\ \gamma^5)\ f_{n,l}(x^4, x^5)$$

4-d Dirac eq.

large effective mass $\gg 1$ TeV

if = 0 → zero mode

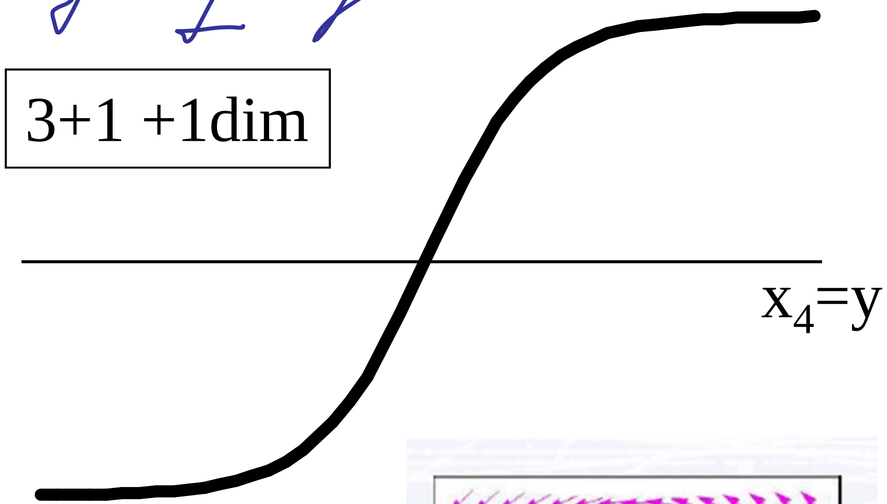
→ Look for zero modes …

Use of dimensional reduction obtain 3+1-dim chiral spinors : use of topological singularities in the extra dimensions to get zero modes,

Shape of Φ field :

3+1 +1dim

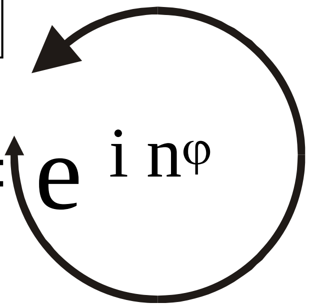
e.g

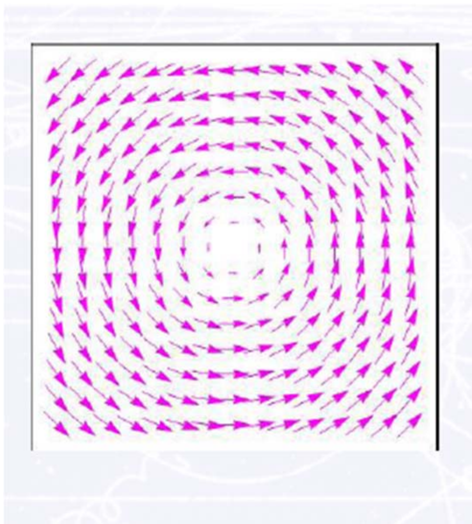


Solitonic background: index theorem localizes one chiral Fermion ; Alternatively, orbifold

3+1 +2 dim

$$\Phi = e^{\, i\, n\varphi}$$

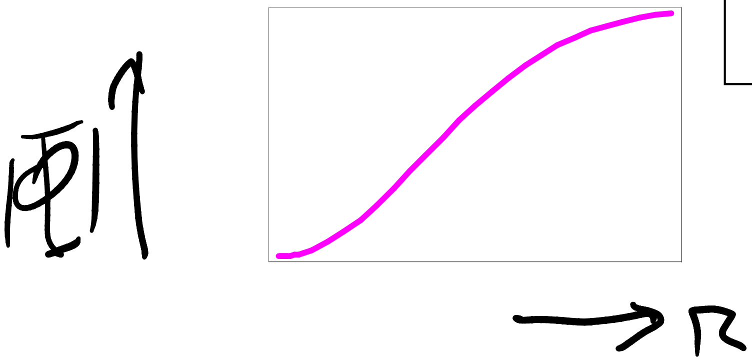




Vortex with winding number n localizes n chiral massless fermion modes in 3+1

3 families from one in 5+1 dim

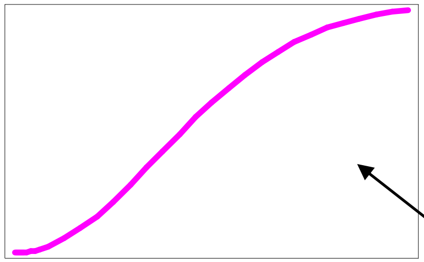
$$\Phi = e^{i n \varphi}$$

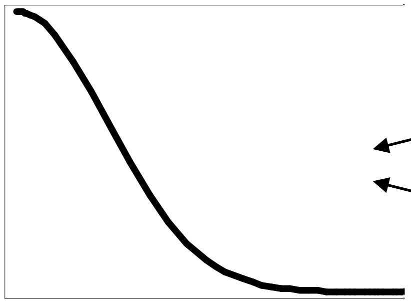


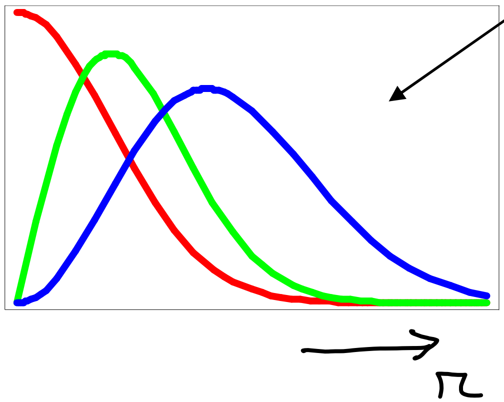
we assume a background scalar field Φ providing a vortex in the 2 extra dimensions;
It vanishes at the origin– where we live!

The 3 fermion modes have different shapes, and different winding properties in the extra dimension variable ϕ

For some reason, n=3 !!!







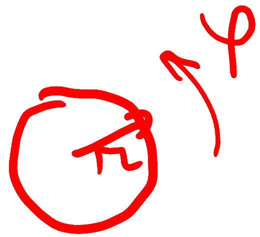
Field Content

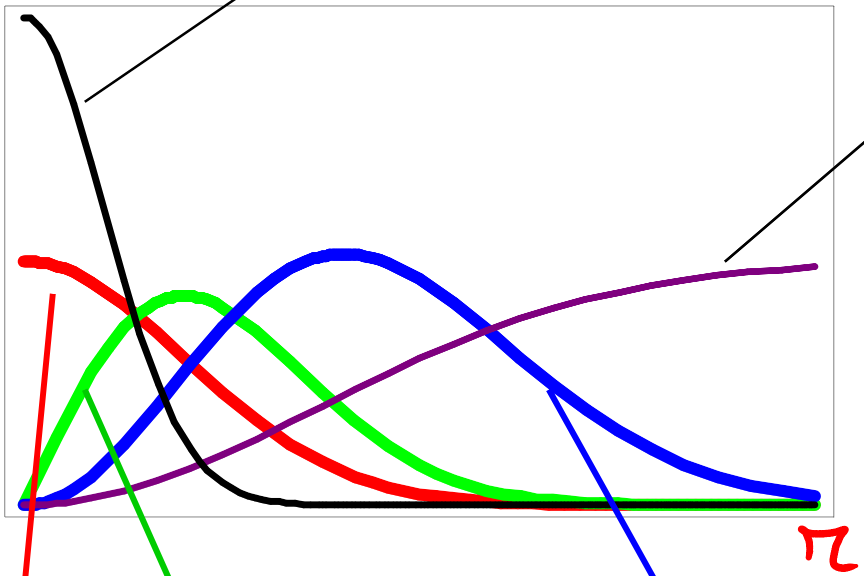
fields		profiles	charges		representations	
			$U(1)_g$	$U(1)_Y$	$SU(2)_W$	$SU(3)_C$
scalar	Φ	$F(r)e^{i\theta}$ $F(0)=0,\ F(\infty)=v_\Phi$	$+1$	0	**1**	**1**
scalar	X	$X(r)$ $X(0)=v_X,\ X(\infty)=0$	$+1$	0	**1**	**1**
scalar	H	$H(r)$ $H(0)=v_H,\ H(\infty)=0$	-1	$+1/2$	**2**	**1**
fermion	Q	3 L zero modes	axial $+3/2$	$+1/6$	**2**	**3**
fermion	U	3 R zero modes	axial $-3/2$	$+2/3$	**1**	**3**
fermion	D	3 R zero modes	axial $-3/2$	$-1/3$	**1**	**3**
fermion	L	3 L zero modes	axial $+3/2$	$-1/2$	**2**	**1**
fermion	E	3 R zero modes	axial $-3/2$	-1	**1**	**1**

Table 1: Scalars and fermions with their gauge quantum numbers. For convenience, we describe here also the profiles of the classical scalar fields and fermionic wave functions in extra dimensions.

Brout-Englert-Higgs field H

Vortex Profile $e^{i\,3\,\phi}$





$e^{i\,0\,\phi}$ $e^{i\,1\,\phi}$ $e^{i\,2\phi}$

The 3 fermion modes have different shapes, and different winding properties in the extra dimension variable ϕ

$$\int_0^{2\pi}\int_0^{R} \bar{\Psi}_n \Psi_n \cdot H \, dr \, d\varphi$$

The 4D mass matrices are obtained by integrating r and ϕ, and are the convolution of these curves

We get a mass matrix like :

$$\begin{pmatrix} small & \varepsilon & \\ & medium & \varepsilon \\ & & large \end{pmatrix}$$

Generation number n =	Winding
1	2
2	1
3	0

An auxiliary scalar X , with winding $e^{i\phi}$ can give the small Cabibbo mixings ε

The scheme is very constrained, as the profiles are dictated by the equations, instead of being imposed by hand, like in multilocalisation; Yet, several schemes possible …

Neutrinos ARE different

In the same context (0th order in Cabibbo mixing), we get indeed:

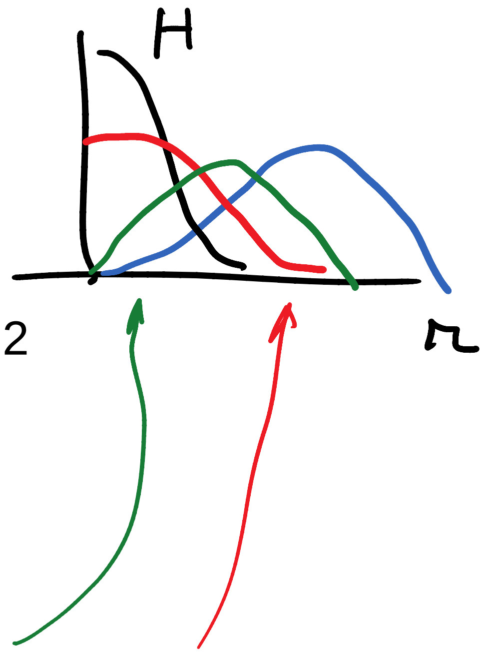
$$M_\nu \sim \begin{pmatrix} \cdot & \cdot & m \\ \cdot & \mu & \cdot \\ m & \cdot & \cdot \end{pmatrix}$$

Where m >> μ
After 45° 1-3 rotation and 23 permutation, this leads to
an inverted hierarchy, (solar mass difference between the heavier

$$M_\nu \sim \begin{pmatrix} m & \cdot & \cdot \\ \cdot & -m & \cdot \\ \cdot & \cdot & \mu \end{pmatrix}$$

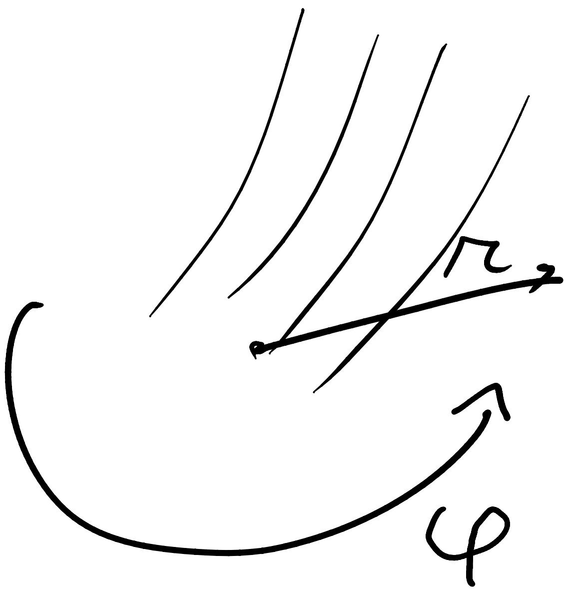
The – sign may be absorbed in the mixing matrix, but contributes destructively to the effective mass for neutrinoless double beta decay (Pseudo-Dirac structure when full Cabibbo-like mixing is introduced)

WHY the difference?



It comes from rotation invariance in the 2 extra dimensions…

Vortex



$$\int_0^{2\pi} d\varphi \, \bar{\Psi}_i(\varphi)\, \Psi_j(\varphi)\, H$$

WHY the difference? --- return in more detail to the 6D spinors,

For the charged spinors, we have both L and R spinors bound to the vortex.

$$\Psi = \begin{pmatrix} \psi_{+R} \\ \psi_{+L} \\ \psi_{-L} \\ \psi_{-R} \end{pmatrix}$$

L → $\bar{R}$

$$L \sim \sum_n \begin{pmatrix} 0 \\ f_{3-n}(r)\ e^{i(3-n)\phi}\psi_{Ln}(x^\mu) \\ f_{n-1}[r)\ e^{i(1-n)\phi}\psi_{Ln}(x^\mu) \\ 0 \end{pmatrix} \qquad R \sim \sum_n \begin{pmatrix} f_{n-1}[r)\ e^{i(1-n)\phi}\chi_{Rn}(x^\mu) \\ 0 \\ 0 \\ f_{3-n}(r)\ e^{i(3-n)\phi}\chi_{Rn}(x^\mu) \end{pmatrix}$$

Dirac mass

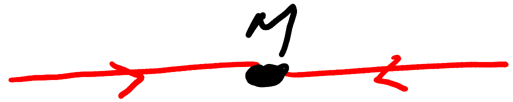
$$\bar{R} L = \sum_{n,n'} \bar{R}_{n'} L_n$$

$$\Rightarrow \int dr \int_0^{2\pi} d\varphi\, f_{3-n}\, f^{*}_{3-n'}\, e^{i(n-n')\varphi} \rightarrow \delta(n-n')$$

diagonal

Effective Lagrangian : integrate over r and φ, For neutrinos (using only Majorana-type 4D mass term) we get

$$\bar{L}^c L \Rightarrow \sum_{n n'} \overline{(L_{n'})^c} L_n$$



$$L \sim \sum_n \begin{pmatrix} 0 \\ f_{3-n}(r)\ e^{i(3-n)\phi}\psi_{Ln}(x^\mu) \\ f_{n-1}[r)\ e^{i(1-n)\phi}\psi_{Ln}(x^\mu) \\ 0 \end{pmatrix}$$

$$\Rightarrow \int_0^{2\pi} d\varphi \ e^{i(4-n-n')\varphi}$$

$$\Rightarrow \delta(n+n'-4)$$

$$\Rightarrow \begin{pmatrix} & \mu^m \\ m & \end{pmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

NUMERICAL EXAMPLE

- With a good selection of Yukawa operators, we can get

$$M_\nu \sim \begin{pmatrix} \cdot & \times & \times \\ \times & \cdot & \cdot \\ \times & \cdot & \cdot \end{pmatrix}$$

→ *Possibility to have a bimaximal mixing*

$$S_+ = \boxed{\Phi^*,\ X^*,} X^{*2}\Phi,\ \ldots$$
$$S_- = X^2, \boxed{X\Phi,} \Phi^2,\ \ldots$$

$$\tilde{Y}_\nu^+ = y_\nu \{1, 1.7\} \qquad y_\nu = 2.8 \cdot 10^{-2}$$
$$\tilde{Y}_\nu^- = y_\nu \qquad M = 1/R = 70 \text{ TeV}$$

Numerical example

$$M_\nu = \begin{pmatrix} 0 & 3.62\cdot 10^{-2} & 3.50\cdot 10^{-2} \\ 3.62\cdot 10^{-2} & 1.46\cdot 10^{-3} & 0 \\ 3.50\cdot 10^{-2} & 0 & 0 \end{pmatrix} \quad [\mathrm{eV}]$$

$$\Delta m^2_{21} = 7.63\times 10^{-5}\ \mathrm{eV}^2$$
$$\Delta m^2_{13} = 2.50\times 10^{-3}\ \mathrm{eV}^2$$

$\rightarrow$ $\Delta m^2_{21}/\Delta m^2_{13} = 3.05\%$

$$M_l = \begin{pmatrix} 4.21\cdot 10^{-4} & 1.08\cdot 10^{-3} & 0 \\ 0 & 4.19\cdot 10^{-3} & 5.98\cdot 10^{-2} \\ 0 & 0 & 1.71 \end{pmatrix} \quad [\mathrm{GeV}]$$

$$U_l^\dagger M_l V_l = D_l = \mathrm{diag}\{4.07\cdot 10^{-4}, 4.33\cdot 10^{-3}, 1.71\} \quad [\mathrm{GeV}]$$

Semi-realistic example
(including extra winding introduced by scalar field combination,
like for the charged fermions):

Neutrino masses are:
(INVERTED HIERARCHY)

-50.03 meV
50.79 meV
0.7089 meV

$$|\langle m_{\beta\beta}\rangle| = 17.0 \quad \text{meV}$$

(pseudo-Dirac suppression
Approx 1/3)

$$U_{MNS} = \begin{pmatrix} 0.808 & 0.555 & 0.196 \\ -0.286 & 0.662 & -0.693 \\ -0.514 & 0.504 & 0.694 \end{pmatrix}$$

$$\tan^2\theta_{12} = 0.471,\ \tan^2\theta_{23} = 0.997,\ \text{and}\ \sin^2\theta_{13} = 3.85\cdot 10^{-2}.$$

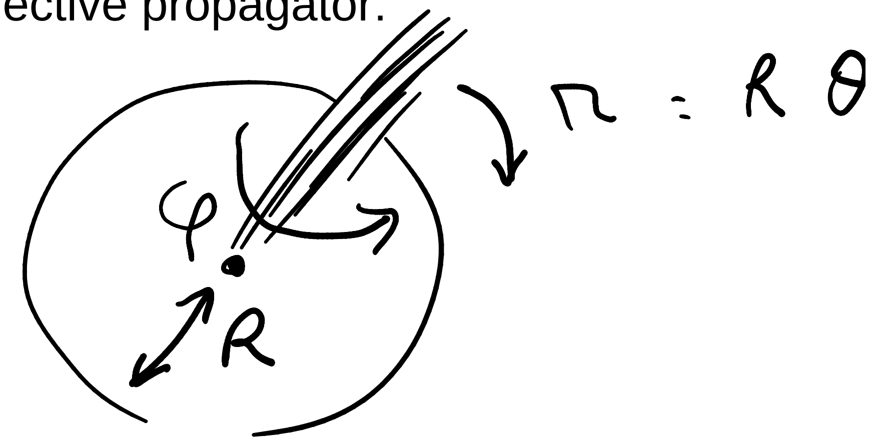
Some other developments :

- compactification of the 2 extra dim on a sphere instead of a plane (avoid localisation of gauge bosons) – spinors modified, but conclusions kept (already mentioned) with extra scale 1/R

- phenomenological implications of the excited modes..

- constraint on B-E-H boson (Libanov and Nugaev: LIGHT)

The Majorana term can be traced to the « Majorana mass term » in the Lagrangian (not to be confused with a non-existant 6D Majorana spinor
It leads to a contribution proportional to the effective propagator:



$$M \, \overline{N^c} N \;\underset{4D}{\Longrightarrow}\; m \sim \frac{M}{\not p - \left(\frac{2\pi}{R}\right)^2 - M^2}$$

Small m → $M \gg 1/R$
OR
→ $M \ll 1/R$ (GeV ok)

IMPORTANT : « family number » (n) is approximatively conserved ! - $e^{in\phi}$ plays somewhat like a U(1) horizontal symmetry

2 extra dim : → ll gauge bosons, possess 2 types of Kaluza- Klein excitations in particular, Z and Gluons

- radial Z'_0 (approx. flavour conserving) → From now on …

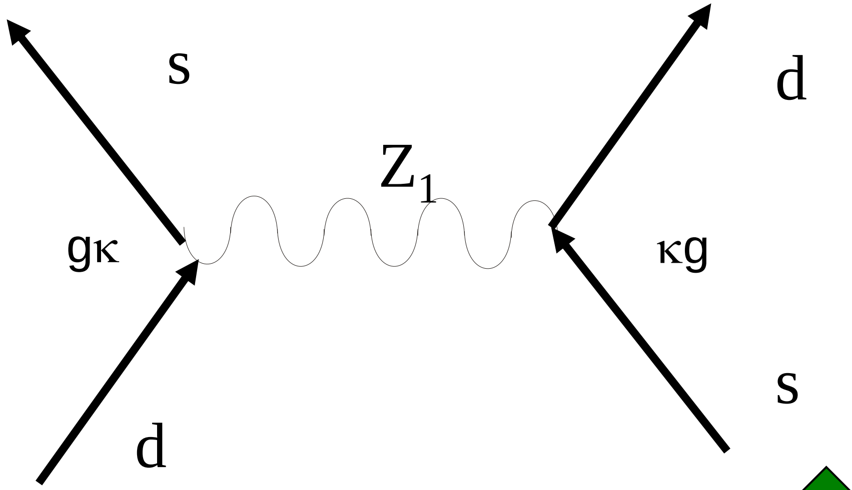
Almost the same couplings as Z

- angular : $Z'_{\S 1}$ behaves like $e^{i\phi}$ → for later …

Flavour violating

$Z_{\S 1}$ thus carries « family number »

« family number » (n) is approximatively conserved ! - somewhat like U(1) horizontal symmetry $e^{i\phi}$



u e

c μ

$Z_{\S\ 1}$ thus carries « family number »

Flavour conserving

Flavour violating,

LIMITS

Family number conserving

→ LIMITS

Typical limit

$K_L \rightarrow \mu^- e^+$ or $\mu^+ e^-$ B.R. $< 10^{-12}$

Expect thus typical mass scale $M_{Z1} / \kappa > (10^{12})^{1/4} M_Z = \kappa\, 100$ TeV

In fact, the small overlap of wave functions implies

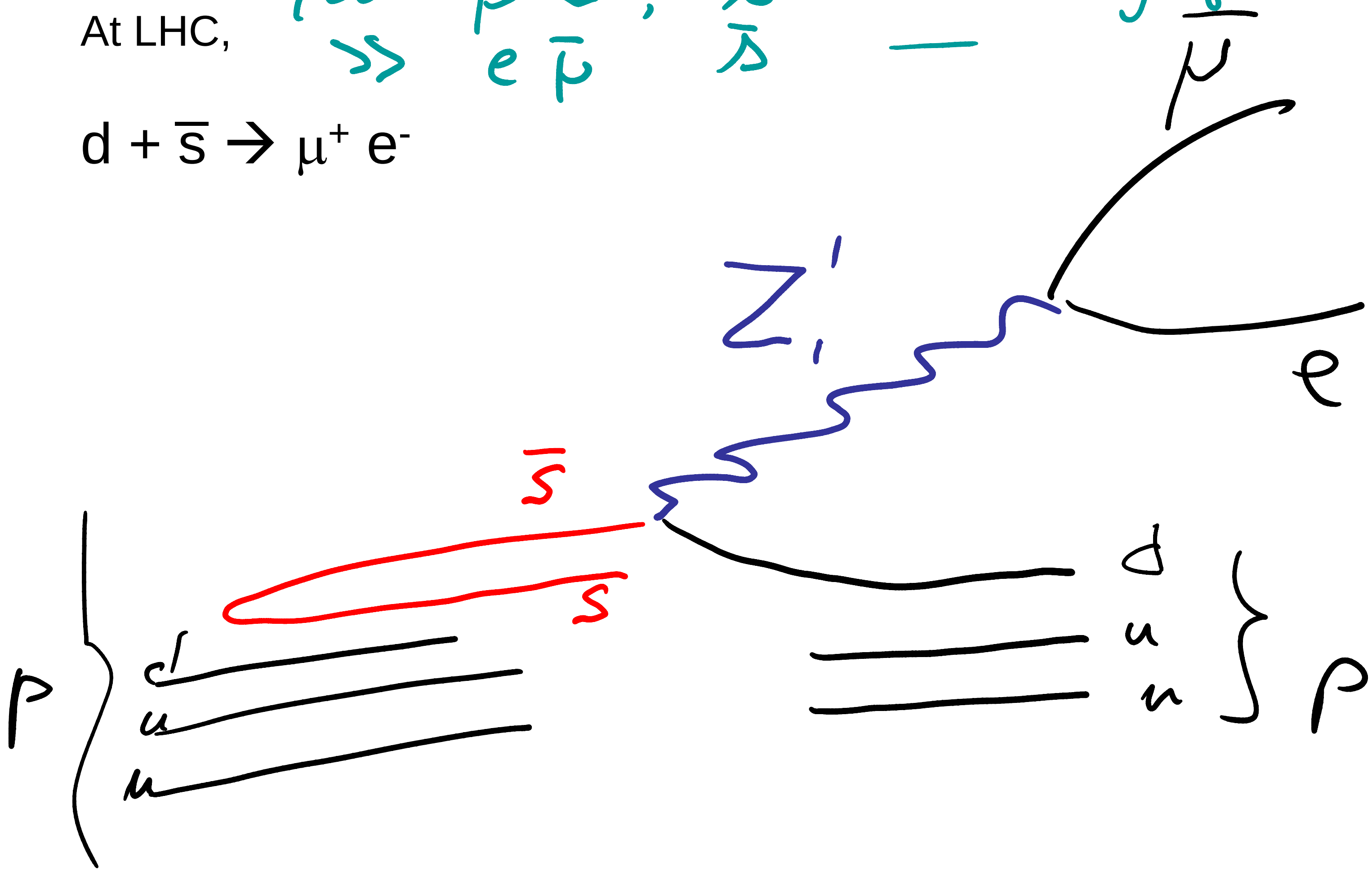
some suppression of the coupling; $\kappa \ll 1$

→ bound becomes $M(Z_1) > \kappa$ 100 TeV

→ Take κ from .01 to 0.5 → Plot for $M(Z_1)$>1TeV--

At LHC, nes $\bar{\mu}$ e , s in underlying event
>> e $\bar{\mu}$ $\bar{s}$ —

$d + \bar{s} \rightarrow \mu^{+} e^{-}$



$pp \longrightarrow \bar{\mu}\, e + s + X$
↳ Λ, $\bar{K}$, ..
nes.

We saturate the bound on κ

$\kappa = 100\ \mathrm{TeV}/M_{Z1}$

$(100\ \mathrm{fb}^{-1}, 14\mathrm{TeV})$

numbers for $\mu^- e^+$ are ONE ORDER below at LHC,due to quark content of protons

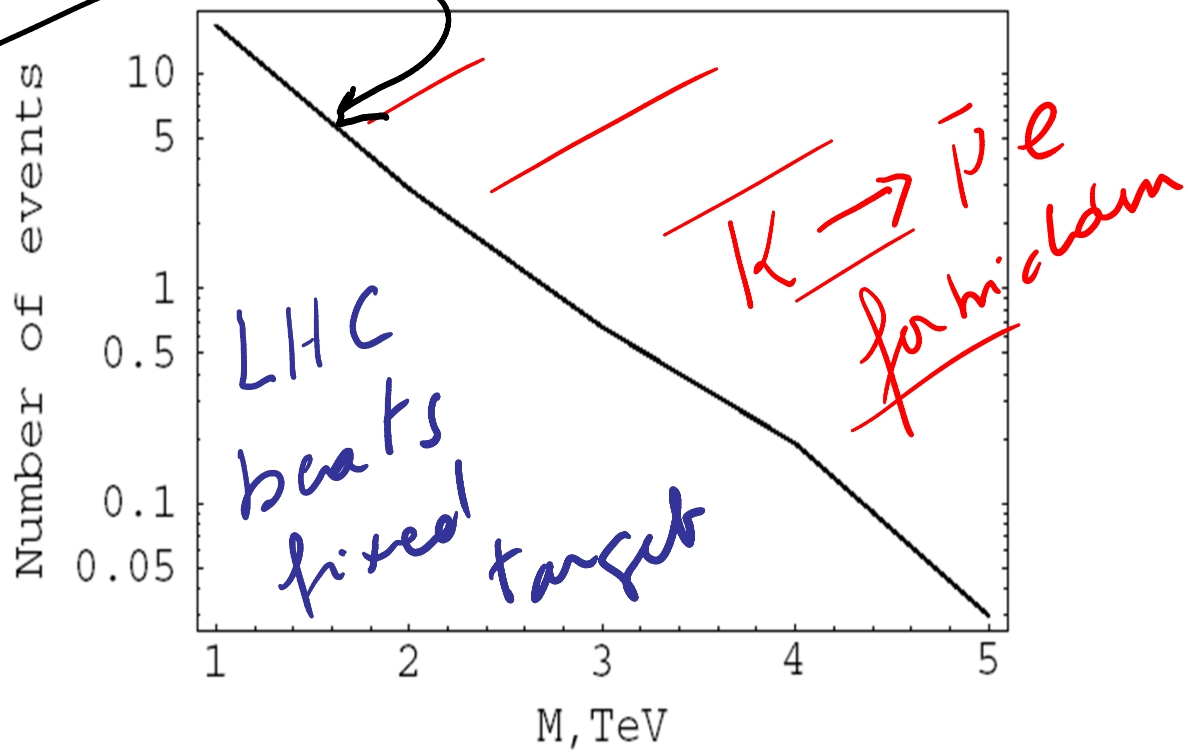


Fig. 1. Number of events for $(\mu^+ e^-)$ pairs production as a function of the vector bosons mass M, with $\kappa = M/(100\mathrm{TeV})$. (also s left in underlying event)

See.**JETP Lett.79:598-601,2004, Pisma Zh.Eksp.Teor.Fiz.79:734-737,2004**.
JMF, M Libanov, S Troitsky, E Nugaev **hep-ph/0404139**

LHC thus has the potential (in a specific model, of course) to beat even the very sensitive fixed-target K → μ e limit!

$\bar{t}$ + c or $\bar{b}$ + s are similarly produced by the **gluon excitations**,

Expect a **few 1000's events ---** but must consider background!

From now on …

$M(Z'_0) = M(Z'\pm) > \kappa\ 100$ TeV

$\kappa\ 0.01 \ldots 0.3$

Find the Z'_0, W'_0,

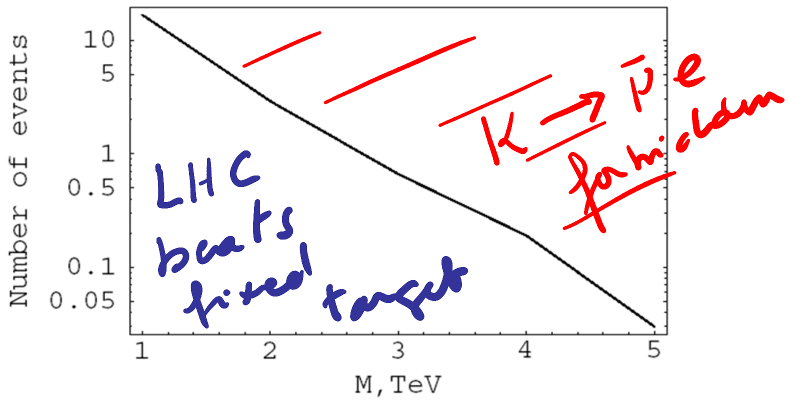
…also expect gluon recurrences

An « ordinary » Z' (with same couplings as Z)

No κ suppression

for later …

Find the $Z'\pm$, κ suppressed



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$(100\ \text{fb}^{-1}, 14\text{TeV})$

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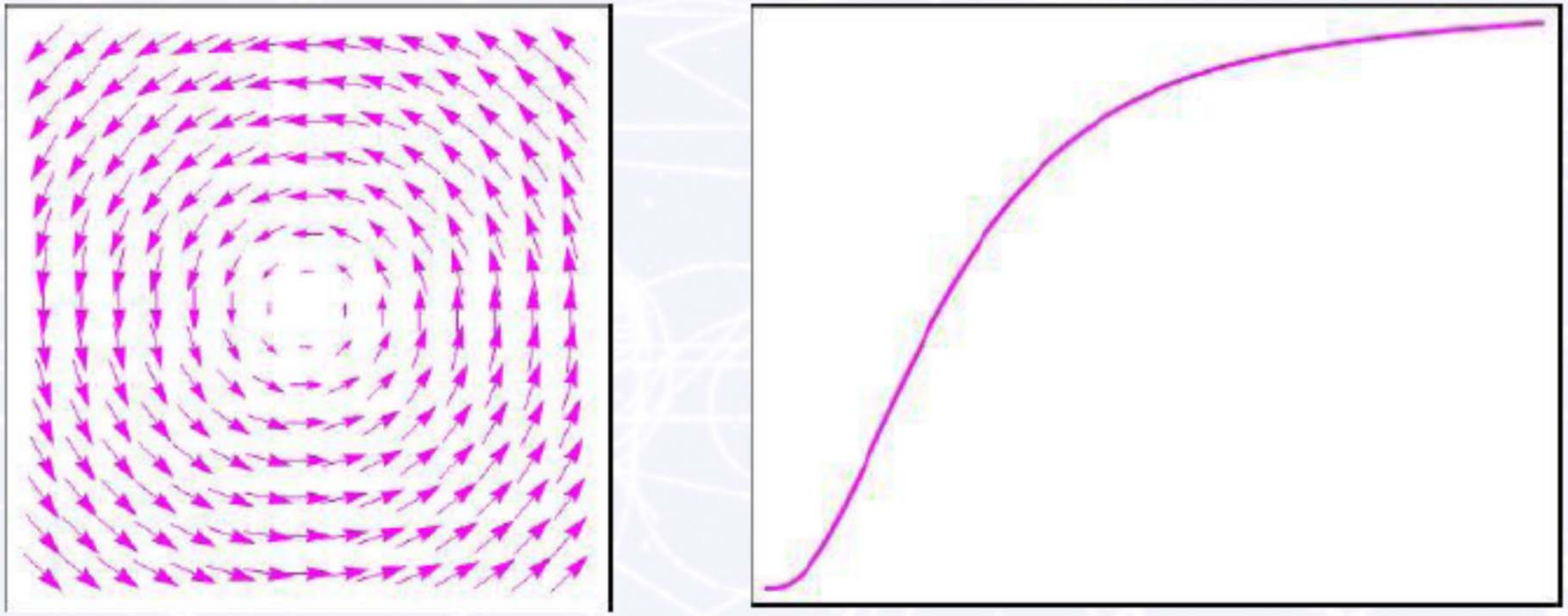
Some kinematical details

3 FAMILIES IN 4D FROM 1 FAMILY IN 6D

- **Vortex in 6D**

  $U(1)_g$ gauge field A + background scalar field Φ



ABIKOSOV-NIELSEN-OLESEN VORTEX

- A vortex on a sphere is in fact like a magnetic monopole configuration in 3D