Kramers Diffusive Mechanism of Alpha Decay, Proton/Cluster Radioactivity and Spontaneous Fission, Induced by Vacuum Zero-point Radiation

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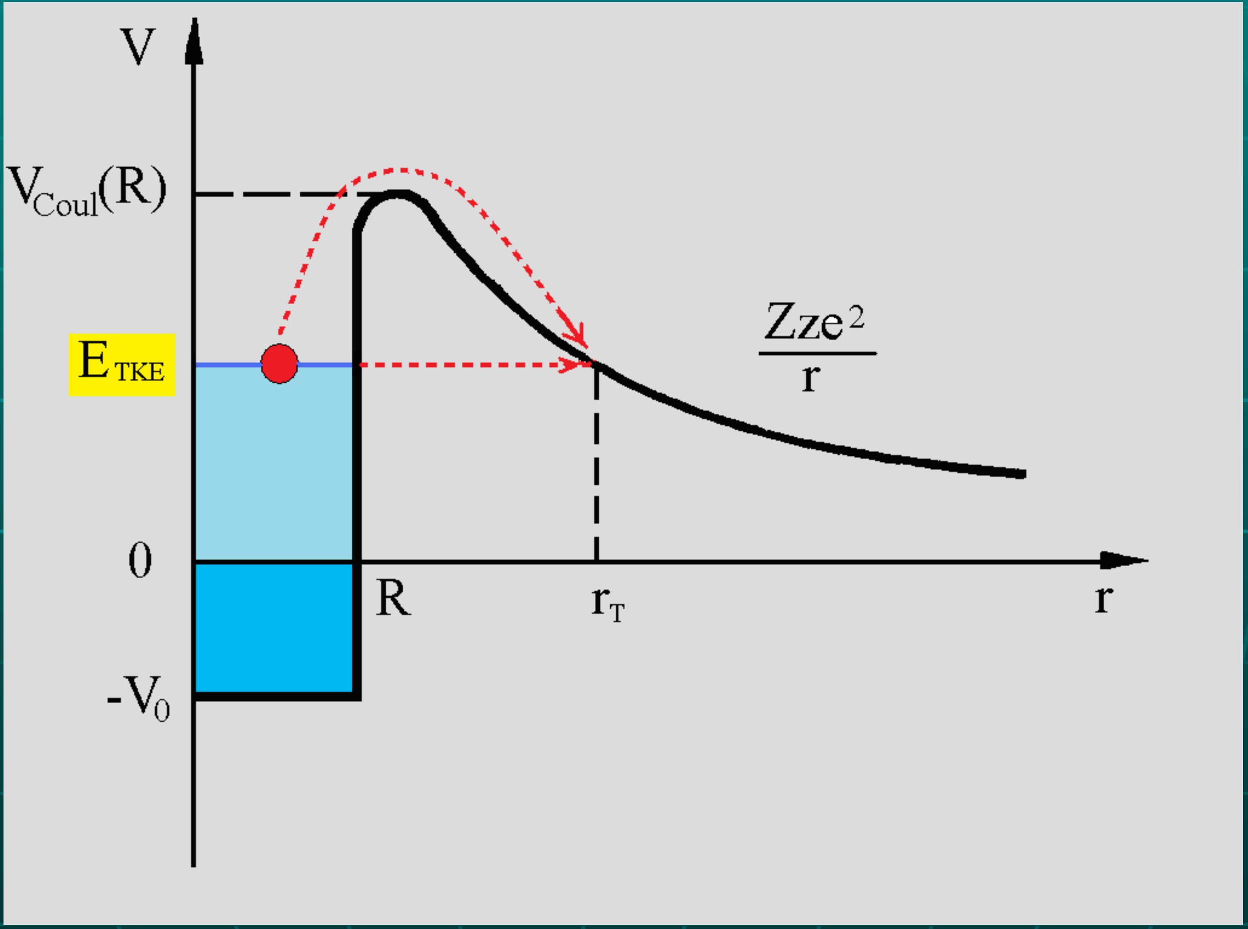
Nucleus nonlinear dynamics

- tunneling
- superfluidity and superconductivity
- Josephson nuclear effect,
- π - condensate
- dynamical supersymmetry and nuclear quantum phase transition
- quantum, dynamical and constructive chaos
- nuclear stochastic resonance

Tunneling or jumping over ?



On Chetaev's theorem and its consequences briefly

Chetaev's theorem: Stability condition for Hamiltonian systems in the presence of dissipative forces has the following the form

$$\sum_{ij} \frac{\partial}{\partial q_i}\left(g_{ij} \frac{\partial S}{\partial q_j} \right) = 0 \qquad (1)$$

where S is the action, q is generalized coordinate.

N.G. Chetaev, Scientific proceedings of Kazan Aircraft Institute, № 5, (**1936**) 3;
N.G. Chetaev, Motion stability. Resear. on the analyt. mechanics, Nauka, Moscow **1962**.

The Schrödinger equation as the stability condition of trajectories in classical mechanics

$$\psi = A\exp(ikS)$$

$$\frac{1}{2k^2}\sum_{i,j} g_{ij}\left(\frac{1}{\psi}\frac{\partial\psi}{\partial q_i}-\frac{1}{A}\frac{\partial A}{\partial q_i}\right)\left(\frac{1}{\psi}\frac{\partial\psi}{\partial q_j}-\frac{1}{A}\frac{\partial A}{\partial q_j}\right)=\frac{\partial S}{\partial t}+U+Q$$

$$\sum_{ij}\frac{\partial}{\partial q_i}\left(g_{ij}\frac{\partial S}{\partial q_j}\right)=0$$

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\Delta\psi+U\psi$$

$$Q=-\frac{\hbar^2}{2m}\frac{\Delta A}{A}$$

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The Bohm-Madelung system of equations

$$\frac{\partial A}{\partial t} = -\frac{1}{2m}\left[A\Delta S + 2\nabla A \cdot \nabla S\right] = -\nabla A \cdot \frac{\nabla S}{m}$$

$$\frac{\partial S}{\partial t} = -\left[\frac{(\nabla S)^2}{2m} + U - \frac{\hbar^2}{2m}\frac{\Delta A}{A}\right]$$

Hence it follows that the Bohm-Madelung quantum potential is equivalent to Chetaev's dissipation energy *Q*

$$Q = -\frac{\hbar^2}{2m}\frac{\Delta A}{A}$$

where S is the action; h = $2\pi\hbar$ is Plank constant; A is amplitude, which in the general case is the real function of the coordinates q_i and time t.

Diffusion mechanism of alpha decay, cluster radioactivity and spontaneous fission

$$\frac{\partial Q}{\partial x} = F_{frict}(x,\dot{x}) + F_L(x,t), \quad where \quad \langle F_L \rangle = 0$$

$$\frac{d}{dx}\left(\frac{\partial S}{\partial t} = -\left[\frac{(\nabla S)^2}{2m} + U - \frac{\hbar^2}{2m}\frac{\Delta A}{A}\right]\right)$$

$$m\ddot{x} = -dU_x - F_{frict}(x,\dot{x}) + F_L(x,t)$$

$$\frac{\partial W}{\partial t} = -\frac{p}{m}\frac{\partial W}{\partial x} + \frac{\partial U}{\partial x}\frac{\partial W}{\partial p} + \gamma\frac{\partial}{\partial p}\left[p + mD(T)\frac{\partial}{\partial p}\right]W$$

where $W=W(x,p,t)$ is the probability density distribution in phase space $\{x,p\}$.

$$\frac{\partial W}{\partial t} = -\frac{p}{m}\frac{\partial W}{\partial x} + \frac{\partial U}{\partial x}\frac{\partial W}{\partial p} + \gamma\frac{\partial}{\partial p}\left[p + mD(T)\frac{\partial}{\partial p}\right]W$$

The transition rate over the potential barrier looks like

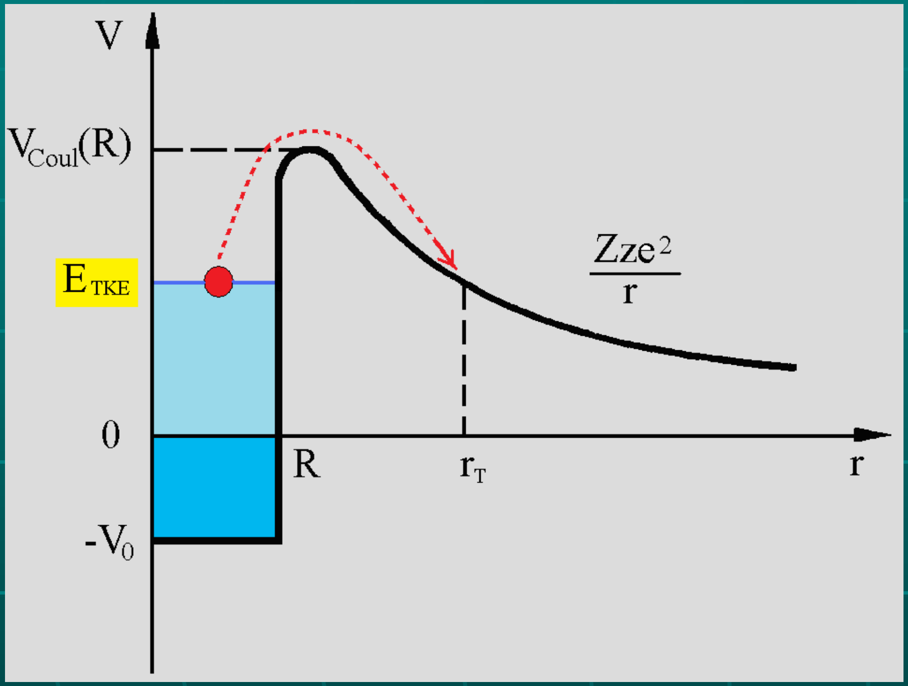
$$w_K = \frac{\omega_{\min}}{2\pi}\left\{\left[1+\left(\frac{\beta}{2\omega_{\max}}\right)^2\right]^{1/2} - \frac{\beta}{2\omega_{\max}}\right\}\exp\left(-\frac{\Delta U}{D(T)}\right), \quad \beta = \frac{\gamma}{m} \geq \frac{\omega_{\max}}{10}$$

$$D(T) = \frac{\hbar\omega_{\min}}{2}\coth\left(\frac{\hbar\omega_{\min}}{2k_B T}\right) = \begin{cases} k_B T \quad for \quad high \quad T, \\ \hbar\omega_{\min}/2, \quad when \quad T \to 0 \end{cases}$$

$$D(T) = (E^*/a)^{1/2}$$

where E^* is the heat excitation energy; $a = A/(8 \pm 1)$ MeV^{-1} is the parameter of the density of one-particle levels.

The Kramers's channel of α-decay, cluster radioactivity and spontaneous fission



The dependence of nuclear particle potential energy on distance to nuclear center

Kramers transition rate →

$$w_K = \frac{\langle \omega \rangle_{Kramers}}{2\pi} \exp\left(- \frac{V_{Coul} - E_{TKE}}{D(T)} \right)$$

Kramers's transition time

$$T_{1/2} = (w_k)^{-1} \rightarrow \lg T_{1/2} = -\lg \frac{\langle\omega\rangle_{Kramers}}{2\pi} + \lg e \frac{V_{Coul} - E_{TKE}}{D(T)}$$

where

$$V_{Coul} = \frac{(Z - Z_{cl}) z_{cl}}{R_{Coul}} = \frac{(Z - Z_{cl}) z_{cl}}{R_{A-A_{cl}, Z-Z_{cl}} + R_{cl} + R_{rf}}, \quad [MeV]$$

$T_{1/2}$ is half-life; $\langle\omega\rangle_{Kramers}$ is the effective frequency of daughter particle appearance on the nuclear surface of radius *R; A and Z* are mass number and the charge of parent nucleus; Z_{cl} is the charge of outgoing particle; $(Z-Z_{cl})$ is the charge of the daughter nucleus; R_{Coul} is minimal Coulomb radius, *Fm.*

Comparing theory with experiment

It is necessary to solve the inverse nonlinear problem, which represents the system of nonlinear equations of following type:

$$\lg T_{1/2}^{exp} = \lg T_{1/2}^{Kramers}\left(E_{cl}, A, Z, Z_{cl}, R_{Kramers}(A, Z, A_{cl}, Z_{cl}), \omega_{Kramers}(R_{Kramers}), \mu(Z, A_{cl}, Z_{cl})\right)$$

for which we have applied parameterization of functions $R_{Kramers}$, $\omega_{Kramers}$, μ with respect to quantum numbers A, Z, A_{cl}, Z_{cl}, which determine the mass numbers and the charges of parent nucleus and cluster, and energies E_{TKE}, Q_{cl}, which determine the kinetic and total energy of decay.

Using the Alexandrov dynamic regularization method we have obtained the parameterization of functions $R_{Kramers}$, $\omega_{Kramers}$ *and* μ :

$$\lg \frac{\langle \omega \rangle_{Kramers}}{2\pi} = a_{20} + \frac{1}{R_{Kramers}}$$

$$\mu = \exp\left[a_1 + a_2 \frac{(A-2Z)^2}{A^2} + a_3 \frac{A-A_{cl}}{A}\left(1-\frac{E_{TKE}}{Q_{cl}}\right) + \left(a_4 \frac{A-A_{cl}}{A} + a_5 \frac{1}{Z_{cl}}\right)\left(1-\frac{1}{Z_{cl}}\right)\right]$$

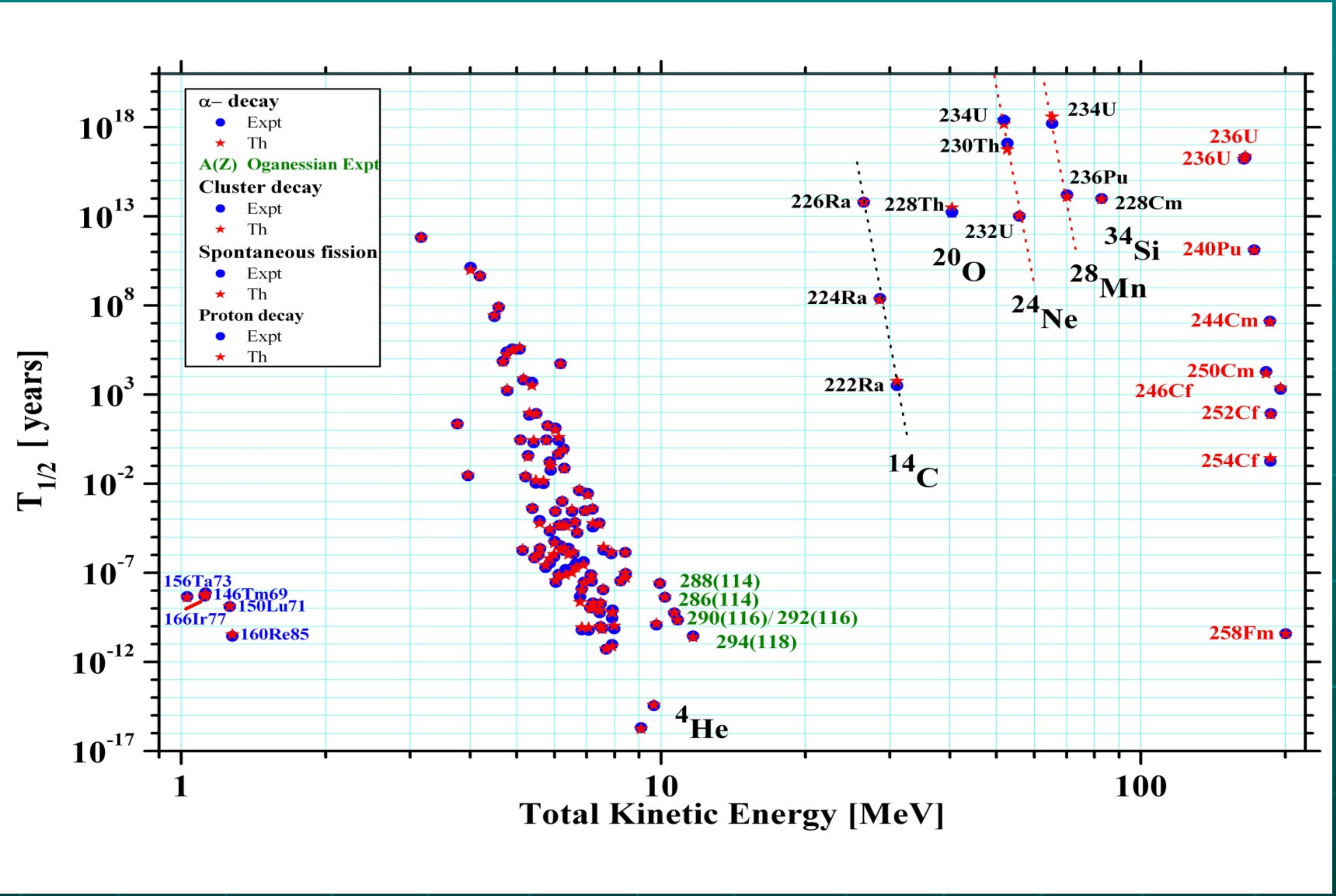
$$R_{Kramers} = \left[B_1 (A-Z_{cl})^{1/3} + B_1 A_{cl}^{1/3} - 1\right] B_2 , \quad [fm]$$

$$B_1 = \exp\left[a_6 \left(\frac{A-2Z}{A}\right)^2 + a_7 \frac{Z}{A} + \left(a_8 + a_9 \frac{A-A_{cl}}{A} + a_{10} \frac{1}{A_{cl}}\right)\left(1-\frac{E_{TKE}}{Q_{cl}}\right) + \left(a_{11} + a_{12} \frac{A-A_{cl}}{A} + a_{13} \frac{1}{Z_{cl}}\right)\left(1-\frac{1}{Z_{cl}}\right)\right]$$

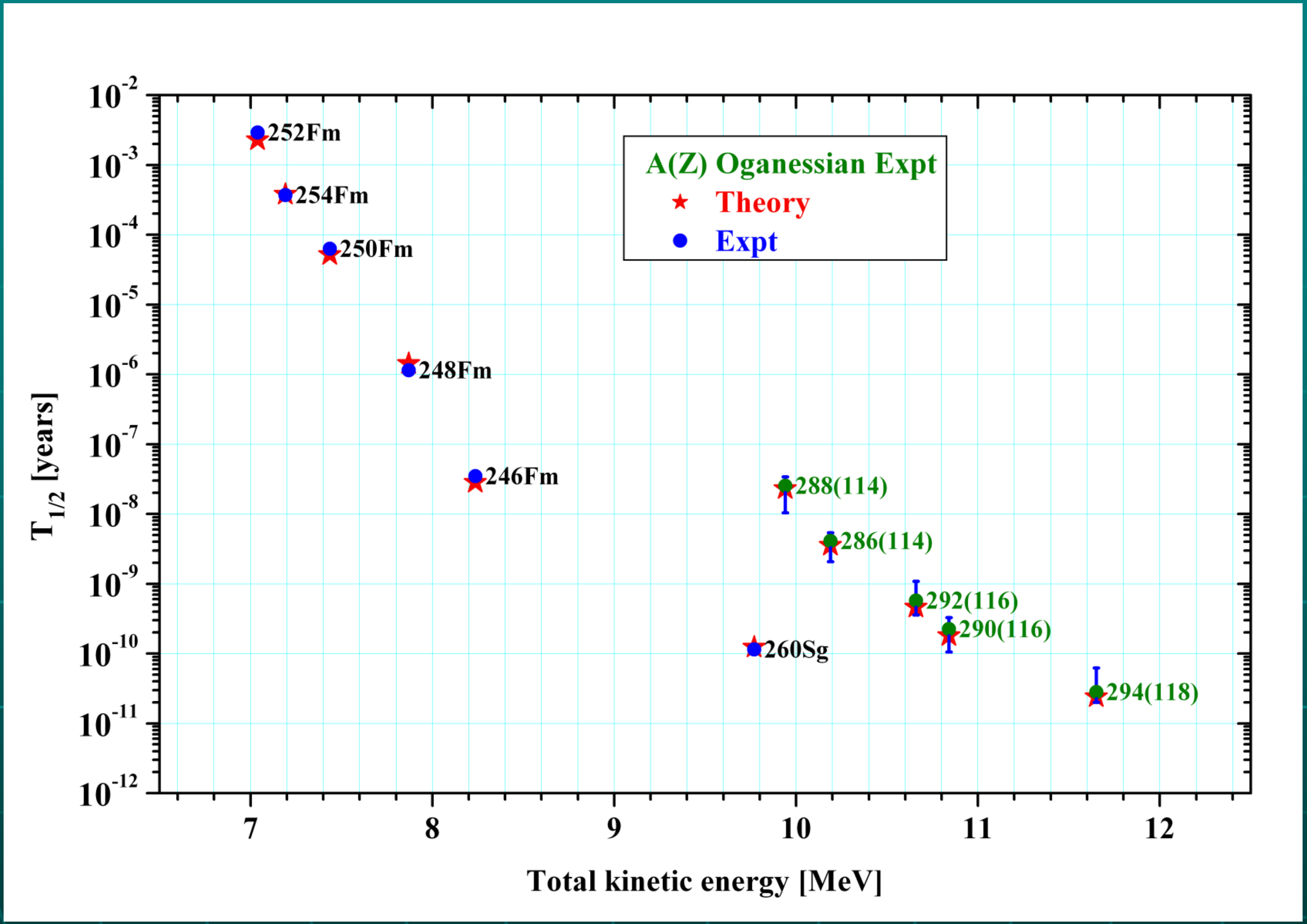
$$B_2 = \exp\left[a_{14} \frac{1}{Z} + a_{15} \left(\frac{A-2Z}{A}\right)^2 + a_{16} \frac{Z}{A} + a_{17} \frac{A-A_{cl}}{A}\left(1-\frac{E_{TKE}}{Q_{cl}}\right) + \left(a_{18} + a_{19} \frac{Z-Z_{cl}}{Z}\right)\left(1-\frac{1}{Z_{cl}}\right)\right]$$

The values of parameters a_i and their relative errors $\Delta a_i/a_i$

i	a_i	$\Delta a_i/a_i$, %
1	-0.5786501537235E+01	2.90
2	-0.2096263480335E+02	1.90
3	-0.3814591516659E+02	2.70
4	0.6900587198207E+01	2.70
5	0.7660345598675E+01	5.00
6	0.1908313257301E+02	1.40
7	0.1826397833295E+02	1.30
8	0.5919551276390E+01	1.60
9	-0.1028171722816E+02	1.50
10	-0.4411225968202E+02	3.50
11	-0.1131089043128E+02	1.00
12	0.1314247777365E+01	1.20
13	0.2865440882262E+01	2.60
14	0.4240393211738E+01	18.00
15	-0.1229614115313E+02	2.20
16	-0.1772081454140E+02	1.20
17	0.1689691120764E+02	0.75
18	0.1666949024191E+02	0.82
19	-0.8643631861055E+01	0.78
20	0.2749864919484E+02	1.20



The theoretical and experimental values of half-life for even-even nuclei as a function of the total kinetic energy E_{TKE} for α–decay, cluster and proton radioactivity, spontaneous fission.



The theoretical and experimental values of the half-life of even-even nuclei as function of fission total kinetic energy E_{TKE} for α–decay of superheavy nuclei with Z=114, 116, 118.

CONCLUSIONS

- In the framework of Bohmian quantum mechanics supplemented with the Chetaev theorem on stable trajectories in dynamics in the presence of dissipative forces we have shown the possibility of the classical (without tunneling) universal description of radioactive decay of heavy nuclei, in which under certain conditions so called noise-induced transition is generated or, in other words, the stochastic channel of alpha decay, cluster radioactivity and spontaneous fission conditioned by the Kramers diffusion mechanism.

- Based on the ENSDF database we have found the parametrized solutions of the Kramers equation of Langevin type by Alexandrov dynamic auto-regularization method (FORTRAN program REGN-Dubna). These solutions describe with high-accuracy the dependence of the half-life (decay probability) of heavy radioactive nuclei on total kinetic energy of daughter decay products.

- The verification of inverse problem solution in the framework of the universal Kramers description of the alpha decay, cluster radioactivity and spontaneous fission, which was based on the newest experimental data for alpha-decay of even-even super heavy nuclei (**Z=114, 116, 118**) have shown the good coincidence of the experimental and theoretical half-life depend on of alpha-decay energy.

The principle of least action of dissipative forces

The statement that $P(x, y, z, t)$ indeed is the probability density function of particle trajectory number is substantiated as follows. Let us assume that the influence of the perturbation forces generated by the potential Q on the wave packet in an arbitrary point in the phase space is proportional to the density of the particle trajectories ($\psi\psi^*=A^2$) at this point. From where follows that the wave packet is practically not perturbed when the following condition is fulfilled

$$\int Q\psi\psi^* dV \Rightarrow \min, \quad where \quad \int \psi\psi^* dV = 1$$

$$\delta\int Q\psi\psi^* dV = \delta Q = 0$$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + U\psi$$

$$Q = -\frac{\hbar^2}{2m}\frac{\Delta A}{A}$$