IJS

Univerza *v Ljubljani*
Fakulteta za *matematiko in fiziko*

Heavy neutral leptons
~ theory ~

Miha Nemevšek

with many - Senjanović, Bajc, Nesti, Vissani, Maiezza, …

WG1-SRCH topical meeting - Heavy neutral leptons
February 17th 2023

Mass origin

SM Higgs

Fermion masses in the SM $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$

Electron and other masses are allowed by QED $\boxed{\bar{e}_L e_R}$ 👍

Weak interactions prevent such terms $\boxed{\cancel{\bar{e}_L e_R}}$ ⛔

$$L_L = (2,-1)_F = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L \qquad \ell_R = (1,-2)_F$$

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$$L_L = (2,-1)_F = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L \qquad \ell_R = (1,-2)_F$$

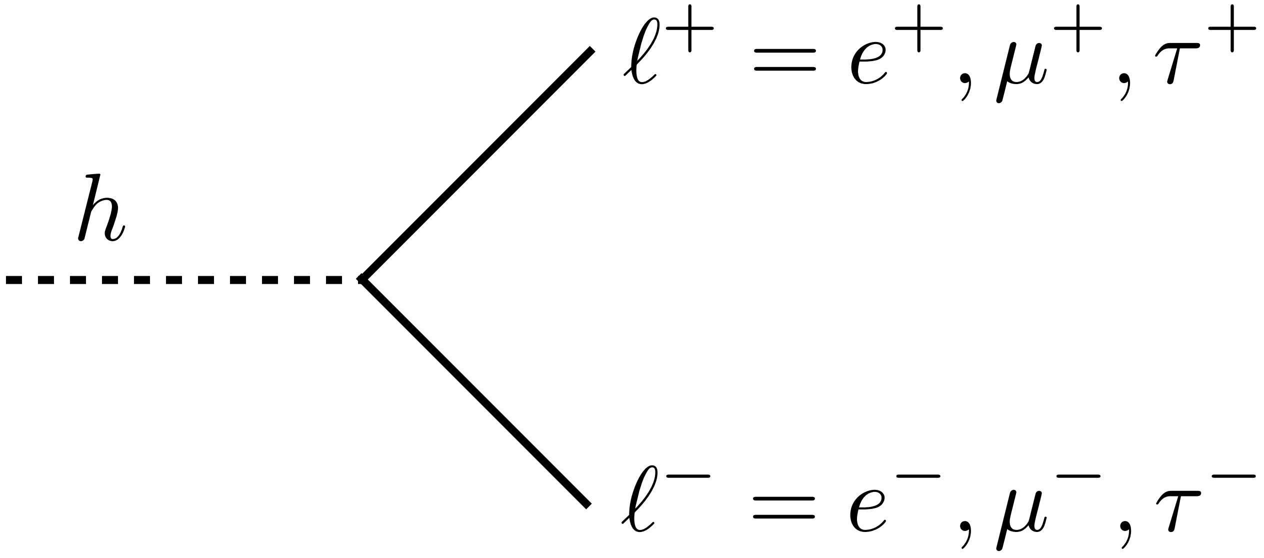
Solution: couple to Higgs $\varphi = (2,1)_B = \begin{pmatrix} 0 \\ \frac{h+v}{\sqrt{2}} \end{pmatrix}$ Weinberg '67

$$\mathcal{L}_D = y_\ell \boxed{\overline{L}_L \varphi \ell_R} + \text{ h.c.}$$ 👍

$$= \frac{y_\ell}{\sqrt{2}}(v+h)\,\overline{e}_L e_R + \text{ h.c.}$$

Mass Higgs coupling

Testing the origin of **Dirac** masses in the SM

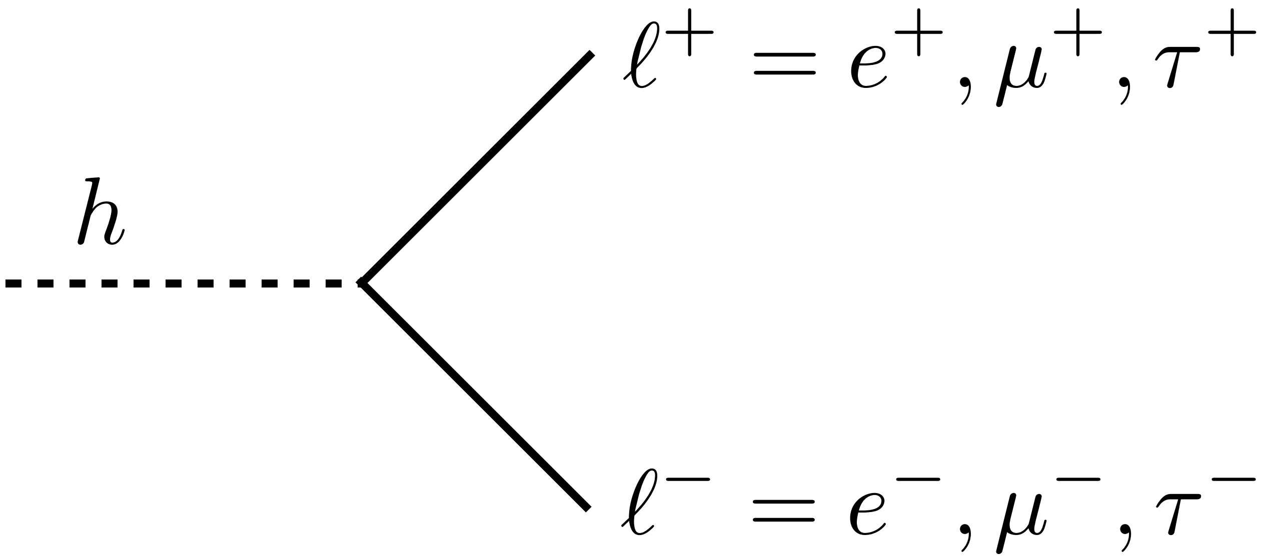


Prediction

$$\boxed{\Gamma_{h \to \ell^+ \ell^-} = \frac{m_h g^2}{32\pi} \left(\frac{m_\ell}{M_W} \right)^2}$$

Testing the origin of **Dirac** masses in the SM

particle != anti-particle

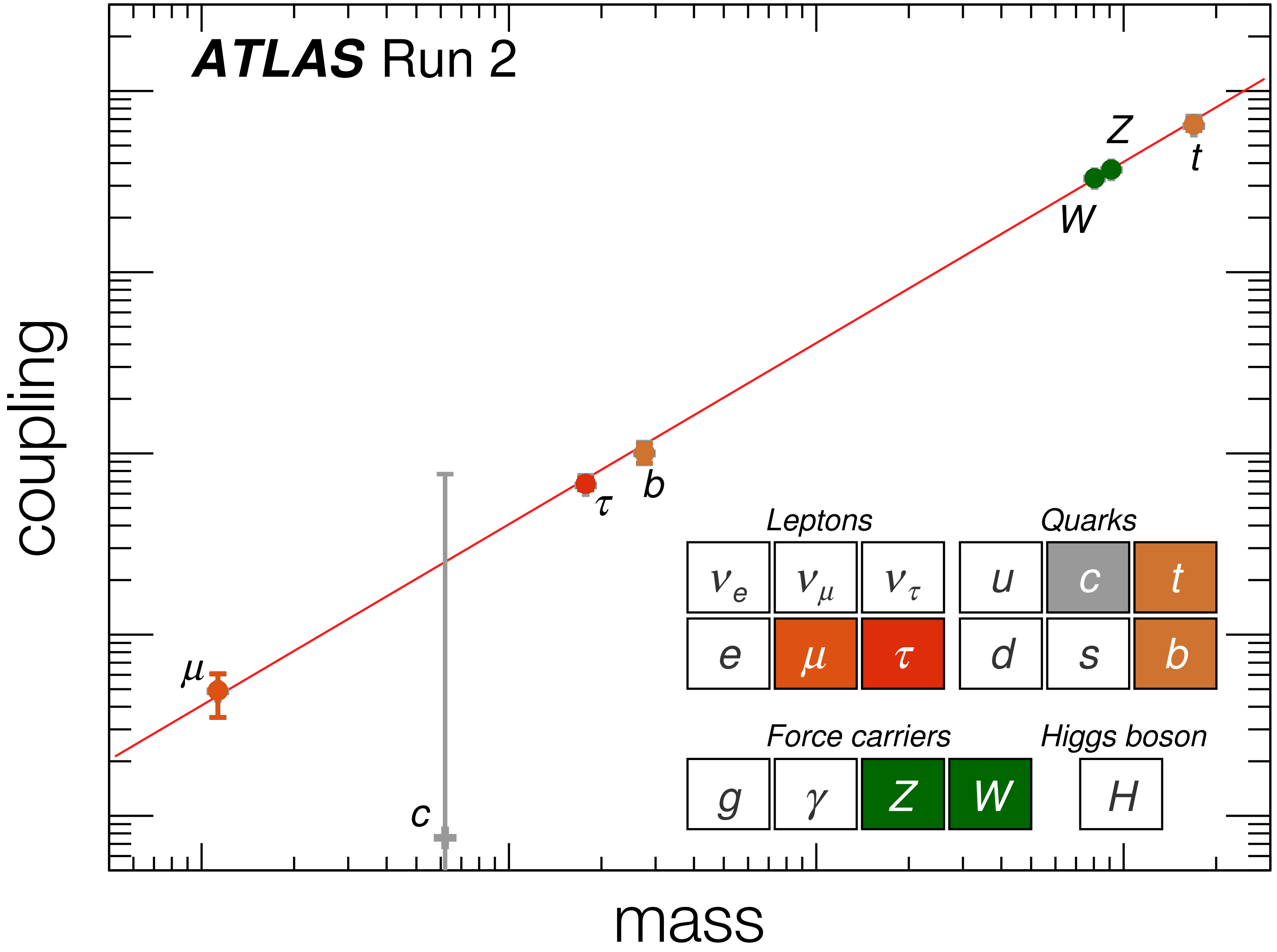


Prediction

$$\Gamma_{h\to\ell^+\ell^-} = \frac{m_h g^2}{32\pi}\left(\frac{m_\ell}{M_W}\right)^2$$

Testing

ATLAS ’22



Neutrino mass in the SM

Weinberg '67

$$L_L = (2,-1)_F = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L \qquad \ell_R = (1,-2)_F \qquad \cancel{\nu_R = (1,0)_F}$$

prediction #1 $\boxed{m_\nu = 0}$

prediction #2 $\boxed{\Delta B = \Delta L = 0}$

Neutrino mass in the SM

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$$L_L = (2,-1)_F = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L \qquad \ell_R = (1,-2)_F \qquad \boxed{\cancel{\nu_R = (1,0)_F}}$$

prediction #1 $\boxed{m_\nu = 0}$ prediction #2 $\boxed{\Delta B = \Delta L = 0}$

Effective theory $$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{n=1}^{\infty} \sum_i \frac{c_i^{(n+4)}}{\Lambda^n} \mathcal{O}_i^{(n+4)}$$ Weinberg ’79
Wilczek ’79

Leading operator $$\frac{c^{(5)}}{\Lambda}\mathcal{O}^{(5)} = \frac{\tilde{y}}{\Lambda_\nu} (L\varphi)(L\varphi)$$

Gives neutrinos **Majorana** mass and breaks lepton number.

particle = anti-particle

Neutrino mass origin

An EFT picture

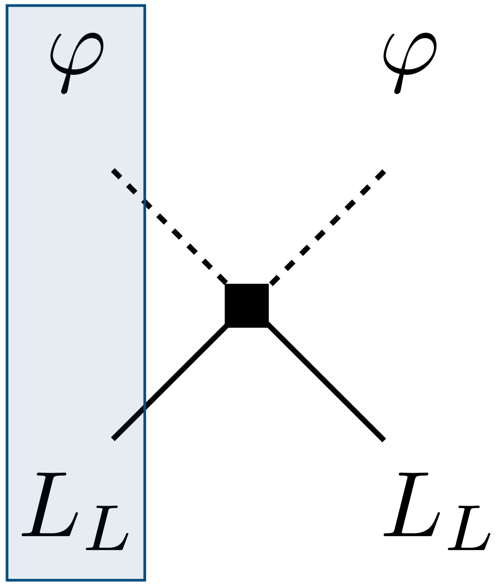
Weinberg operator

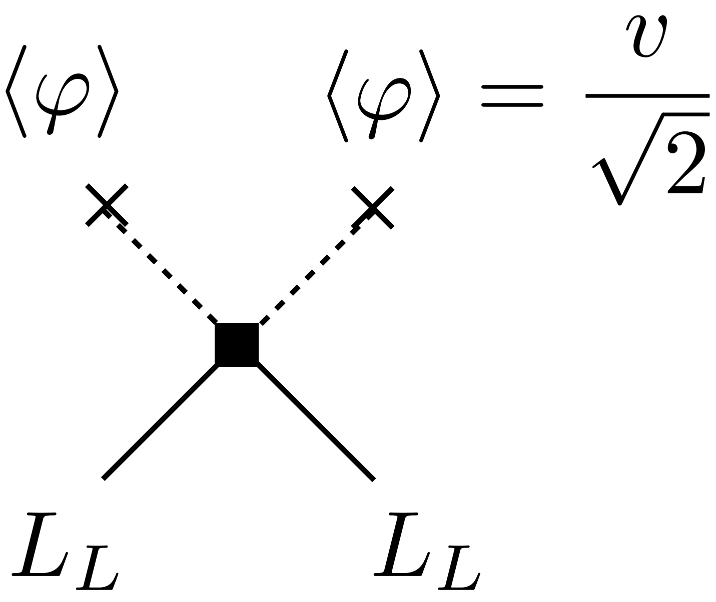
Weinberg ’79

realization(s) of $\boxed{2 \times 2 = 3 + 1}$ ~QM spins

i) Type I seesaw

$$\mathcal{O}^{(5)} = (L\varphi)(L\varphi) = (L^T i\sigma_2\varphi)\mathcal{C}(\varphi i\sigma_2 L) \qquad \mathcal{C} = i\gamma_2\gamma_0$$





Minkowski ’77, Mohapatra, Senjanović ’79, Gell-Mann et al ’79, Yanagida ’79

Weinberg operator

Weinberg ’79

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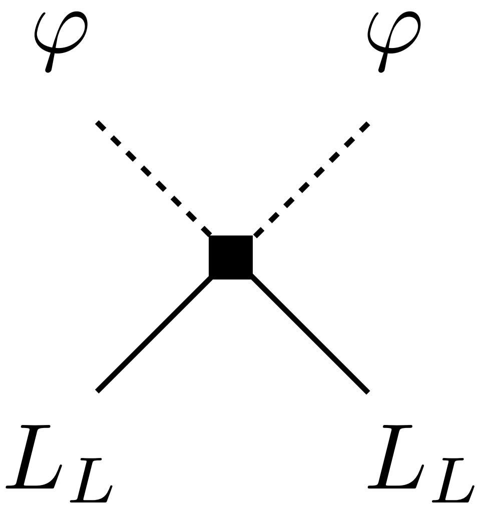
Let’s get neutrino masses and break lepton number

$$L_L^T \mathcal{C} L_L \;\Rightarrow\; \nu_L^T \mathcal{C} \nu_L$$

$$L(L) = 1 = L(\ell) = L(\nu)$$



$\nu_{L,R} \to e^{i\alpha L}\nu_{L,R}$	$\overline{\nu}_L \nu_R$	Dirac	$\Delta L = 0$
$\overline{\nu}_{L,R} \to e^{-i\alpha L}\overline{\nu}_{L,R}$	$\nu_L^T \mathcal{C} \nu_L$	Majorana	$\boxed{\Delta L = 2}$

Weinberg operator

Weinberg '79

realization(s) of $\boxed{2 \times 2 = 3 + 1}$ ~QM spins

i) Type I seesaw

$$\mathcal{O}^{(5)} = (L\varphi)(L\varphi) = (L^T i\sigma_2 \varphi)\mathcal{C}(\varphi i\sigma_2 L)$$

Let's get neutrino masses and break lepton number

$$\begin{aligned}
\frac{\tilde{y}}{\Lambda_\nu}\mathcal{O}^{(5)} &= \frac{\tilde{y}}{\Lambda_\nu}(L\varphi)(L\varphi) \\
&= \frac{\tilde{y}}{\Lambda_\nu}\begin{pmatrix}\nu_L^T & \ell_L^T\end{pmatrix}\begin{pmatrix}0 & 1 \\ -1 & 0\end{pmatrix}\begin{pmatrix}0 \\ \frac{v}{\sqrt{2}}\end{pmatrix}\mathcal{C}\begin{pmatrix}0 & \frac{v}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}0 & 1 \\ -1 & 0\end{pmatrix}\begin{pmatrix}\nu_L^T \\ \ell_L^T\end{pmatrix} \\
&= \frac{\tilde{y}}{\Lambda_\nu}\left(\nu_L^T \frac{v}{\sqrt{2}}\right)\mathcal{C}\left(-\frac{v}{\sqrt{2}}\nu_L\right) = -\frac{\tilde{y}v^2}{2\Lambda_\nu}\nu_L^T \mathcal{C}\nu_L
\end{aligned}$$

$$\boxed{M_\nu = -\frac{\tilde{y}v^2}{\Lambda_\nu}}$$

i) Type I seesaw

$$\mathcal{O}^{(5)} = (L\varphi)(L\varphi) = (L^T i\sigma_2 \varphi)\mathcal{C}\boxed{(\varphi^T i\sigma_2 L)}$$

quantum numbers of a fermionic singlet $(1,0)_F$

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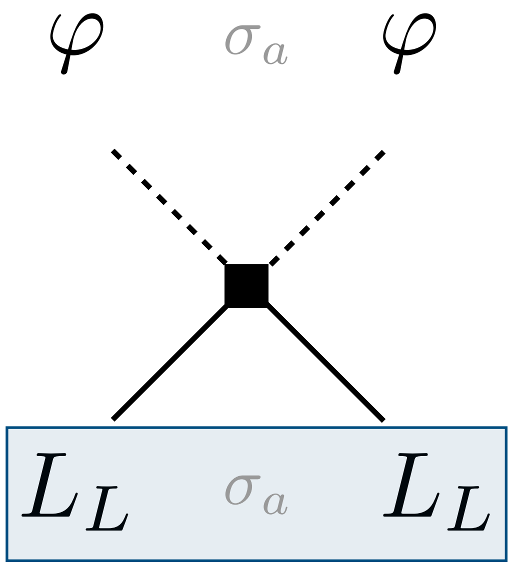
quantum numbers of a fermionic singlet $(1,0)_F$

ii) Type II seesaw

$$\mathcal{O}^{(5)} = (L\varphi)(L\varphi) = (L^T\mathcal{C} i\sigma_2\sigma_a L)\boxed{(\varphi^T i\sigma_2\sigma_a\varphi)}$$



bosonic triplet $(3,2)_B$

σ_3 gives zero

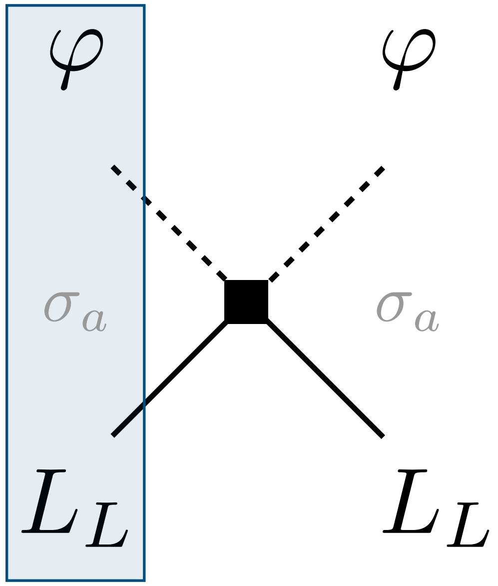
$\sigma_{1,2}$ charged leptons cancel, neutrinos sum up

$\boxed{\cancel{\ell_L^T\mathcal{C}\ell_L}}$ ⛔ $\boxed{\nu_L^T\mathcal{C}\nu_L}$ 👍

$$\boxed{M_\nu = -\frac{2\tilde{y}v^2}{\Lambda_\nu}}$$

$$\boxed{\Delta L = 2}$$

iii) Type III seesaw

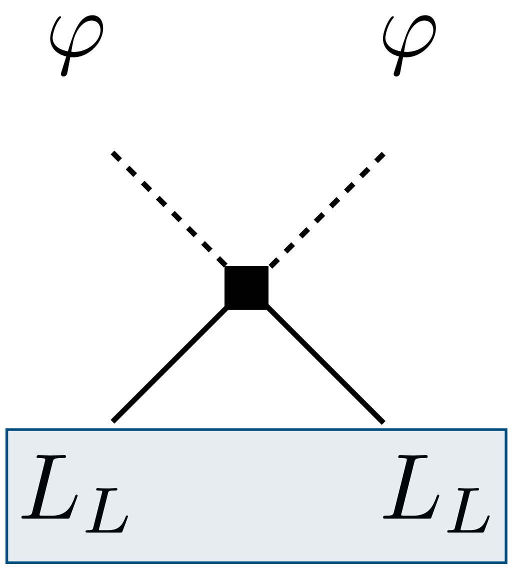


$$\mathcal{O}^{(5)} = (L\varphi)(L\varphi) = (L^T i\sigma_2\sigma_a\varphi)\mathcal{C}\boxed{(\varphi^T i\sigma_2\sigma_a L)}$$

quantum numbers of a fermionic triplet $(3,0)_F$

$$\boxed{M_\nu = -\frac{\tilde{y}v^2}{\Lambda_\nu}}$$

iv) No seesaw?



$$\mathcal{O}^{(5)} = (L\varphi)(L\varphi) = (L^T \mathcal{C} i\sigma_2 L)\boxed{(\varphi^T i\sigma_2\varphi)}$$ ⛔

quantum numbers of a bosonic $(1,2)_B$

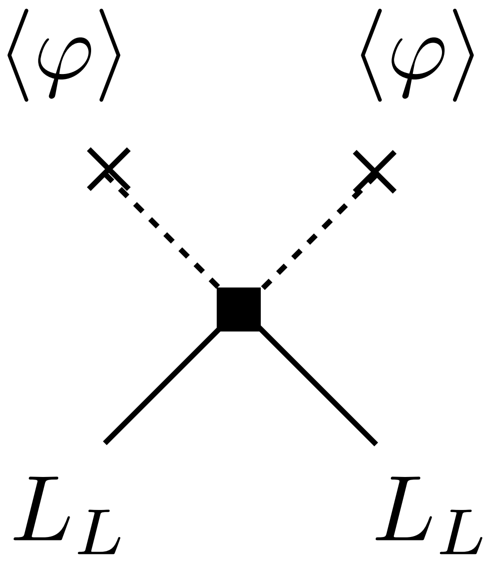
$$\boxed{M_\nu = 0}$$

at tree level (loops nonzero)

Summary

Viewing the BSM as an effective theory we have one operator that…

$\boxed{m_\nu \neq 0}$



$\boxed{\Delta L = 2}$

… gives neutrino mass and breaks lepton number in three ways at tree level.

What if the scale Λ_ν is not too high and we produce the seesaw particles?

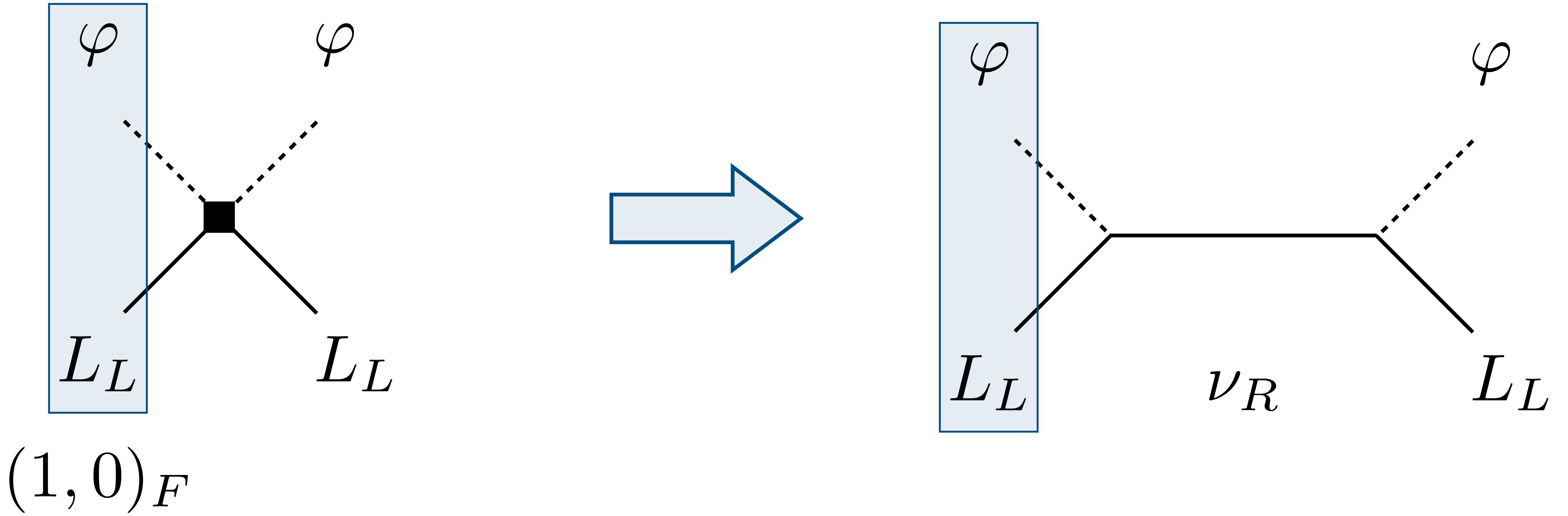
Neutrino mass origin

UV complete models

Type I seesaw bottom up approach, simplicity?

$$\mathcal{O}^{(5)} = (L\varphi)(L\varphi) = (L^T i\sigma_2\varphi)\mathcal{C}(\varphi i\sigma_2 L)$$

Open up the effective operator and write down a UV complete theory



A fermion with no SM quantum numbers = sterile neutrino $\nu_R = (1,0)_F$

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\nu_R}$$

New states, new interactions. How to relate to neutrino mass and exp.?

$$\mathcal{L}_{\nu_R} = \frac{i}{2}\overline{\nu}_R \not{\partial} \nu_R - y_D \overline{\nu}_R \varphi i \sigma_2 L - \frac{1}{2} m_{\nu_R} \overline{\nu}_R \nu_R$$

Kinetic Dirac Majorana

$$\mathcal{L}_{\text{mass}} = \boxed{m_D}\,\overline{\nu}_R \nu_L + \frac{1}{2}\boxed{m_{\nu_R}}\,\overline{\nu}_R \nu_R + \text{ h.c.}$$

Seesaw

$$M_\nu = \begin{pmatrix} 0 & \boxed{m_D} \\ m_D & \boxed{m_{\nu_R}} \end{pmatrix} \qquad m_D = y_D \frac{v}{\sqrt{2}}$$

$$\text{tr} M_\nu = m_\nu + m_N \simeq m_{\nu_R} \qquad \det M_\nu = m_\nu m_{\nu_R}$$

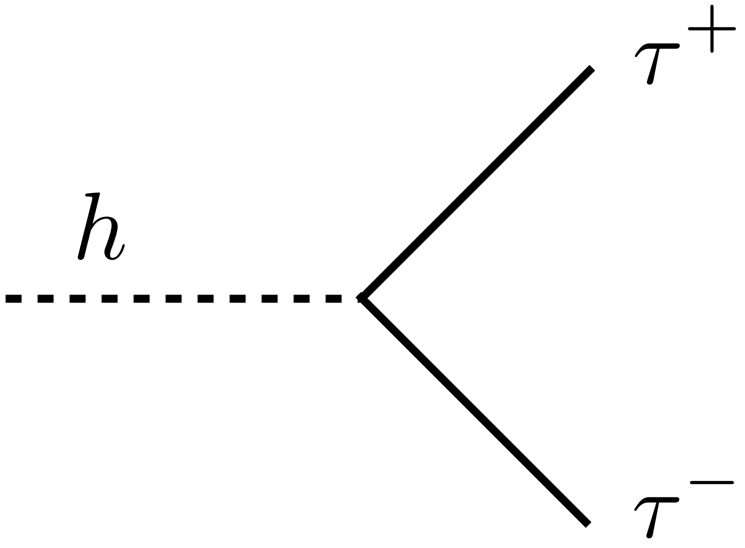
Masses

$$\boxed{m_\nu = -\frac{m_D^2}{m_{\nu_R}}}$$

Mixing

$$\boxed{\theta_{\nu\nu_R} \simeq \frac{m_D}{m_{\nu_R}} \sim \sqrt{\frac{m_\nu}{m_{\nu_R}}}}$$

Testing seesaw or neutrino mass origin



$$\boxed{\Gamma_{h\to\ell^+\ell^-} = \frac{m_h g^2}{32\pi}\left(\frac{m_\tau}{M_W}\right)^2}$$

Dirac fermions only have one source of mass $m_\tau = y_\tau \frac{v}{\sqrt{2}}$

Seesaw $\boxed{M_\nu = -M_D^T M_{\nu_R} M_D}$

$$= OS,\ S^T = S,\ O^T O = 1$$

$$\begin{aligned} M_\nu &\simeq -M_D^T M_{\nu_R} M_D \\ &= -\left(m_s^{-1/2} M_D\right)^T \left(m_s^{-1/2} M_D\right) \\ &= -SO^T OS = -S^2 \end{aligned}$$

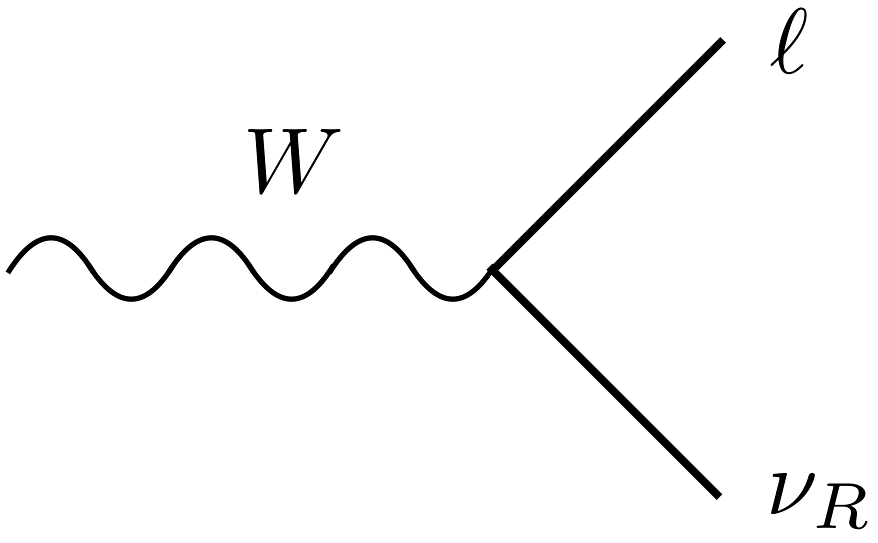
$$\boxed{M_D = i\sqrt{m_S} O \sqrt{M_\nu}}$$

$$S = i\sqrt{M_\nu}$$

Seesaw ambiguity

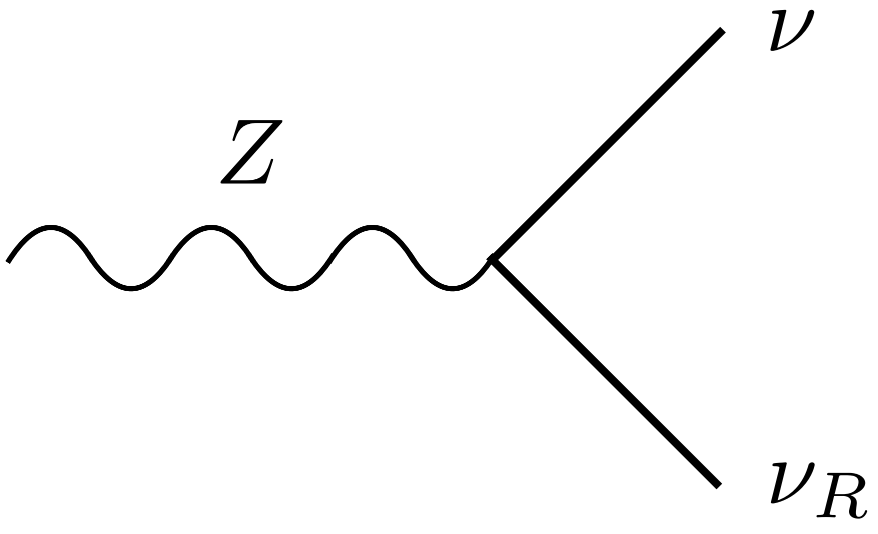
Testing type I seesaw

We can produce heavy sterile neutrinos in

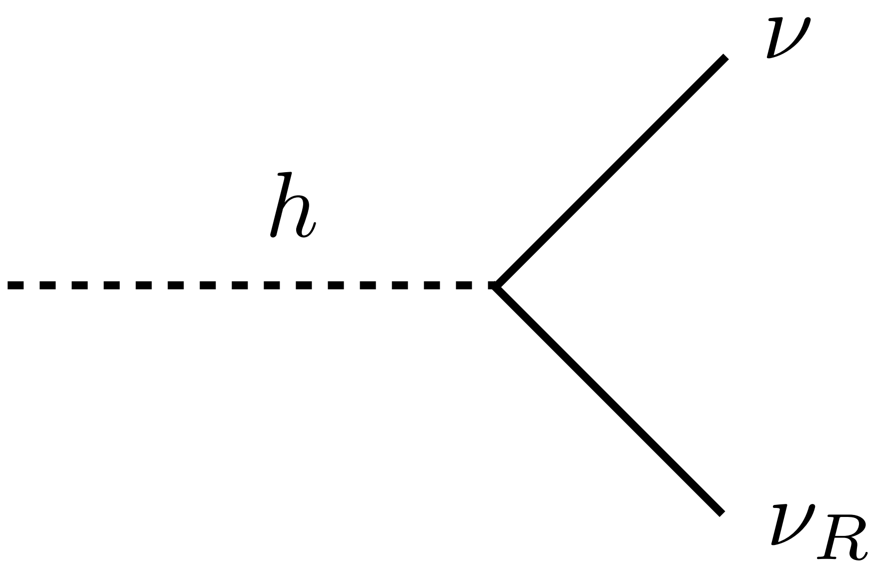


W decays

$$\propto U_{\nu_R \ell} \simeq \sqrt{\frac{m_\nu}{m_{\nu_R}}}$$



Z decays



Higgs decays

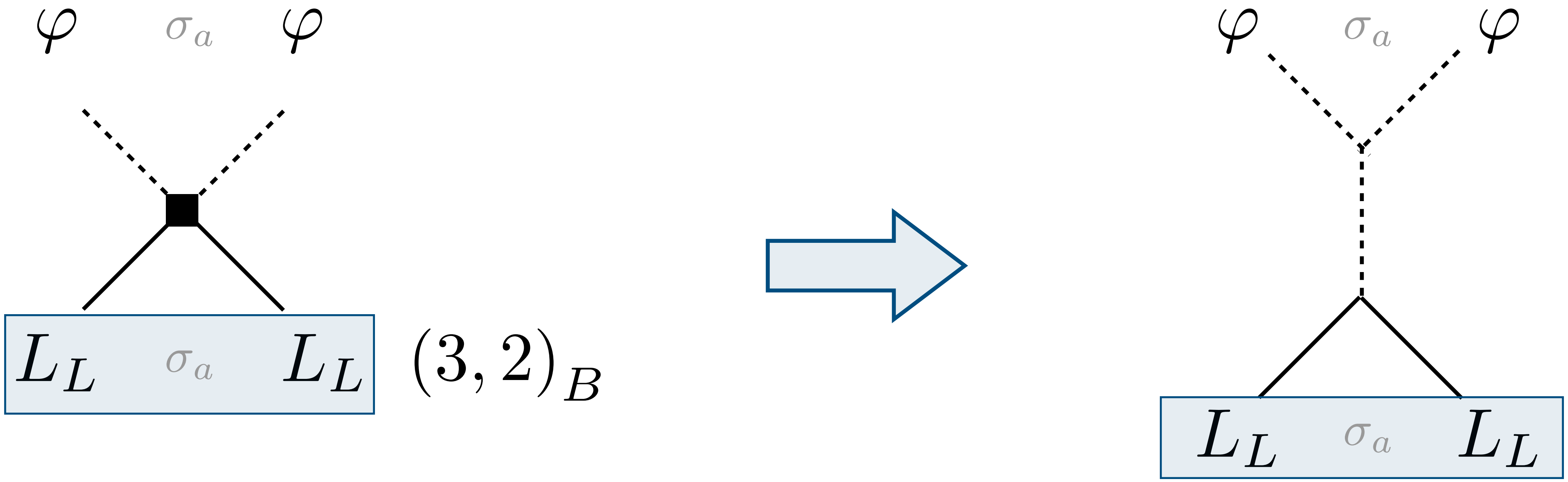
Nuclear processes

Meson decays

LFV, EWPT

Type II seesaw

$$\mathcal{O}^{(5)} = (L\varphi)(L\varphi) = (L^T \mathcal{C} i\sigma_2\sigma_a L)(\varphi^T i\sigma_2\sigma_a\varphi)$$



A bosonic triplet $\Delta_L = (3,2)_B$

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\Delta_L}$$

New states, new interactions. How to relate to neutrino mass and exp.?

Magg, Wetterich ’80, Schechter, Valle ’80, Cheng, Li ’80, Mohapatra, Senjanović ’81

$$\mathcal{L}_{\Delta_L} = |D_\mu \Delta_L|^2 - Y_\Delta \boxed{\left(L^T i\sigma_2\sigma_a \mathcal{C} L\right)} \Delta_L^a - V(\Delta,\varphi)$$

Kinetic Majorana Potential

$$\Delta_L = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix} = \sigma_a \Delta_L^a$$

New states = new seesaw particles

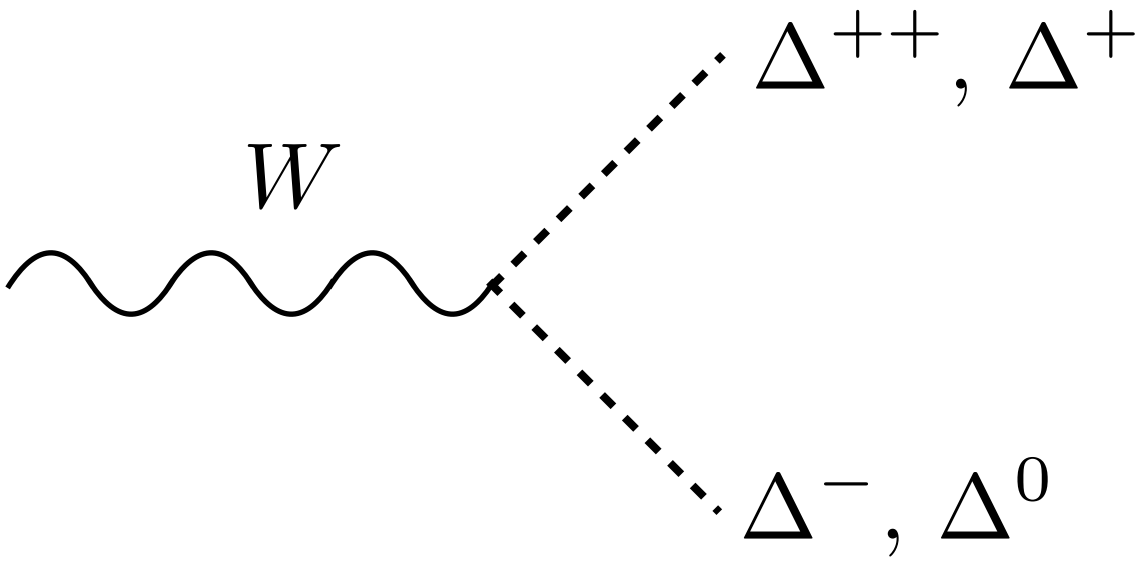
$$Y_\Delta \left(\nu_L^T \quad \ell_L^T\right) \mathcal{C} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ \frac{v_\Delta}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{Y_\Delta v_\Delta}{\sqrt{2}} \nu_L^T \mathcal{C} \nu_L$$

$$\boxed{M_\nu = \sqrt{2}\, Y_\Delta v_\Delta = V_L^* m_\nu V_L^\dagger}$$

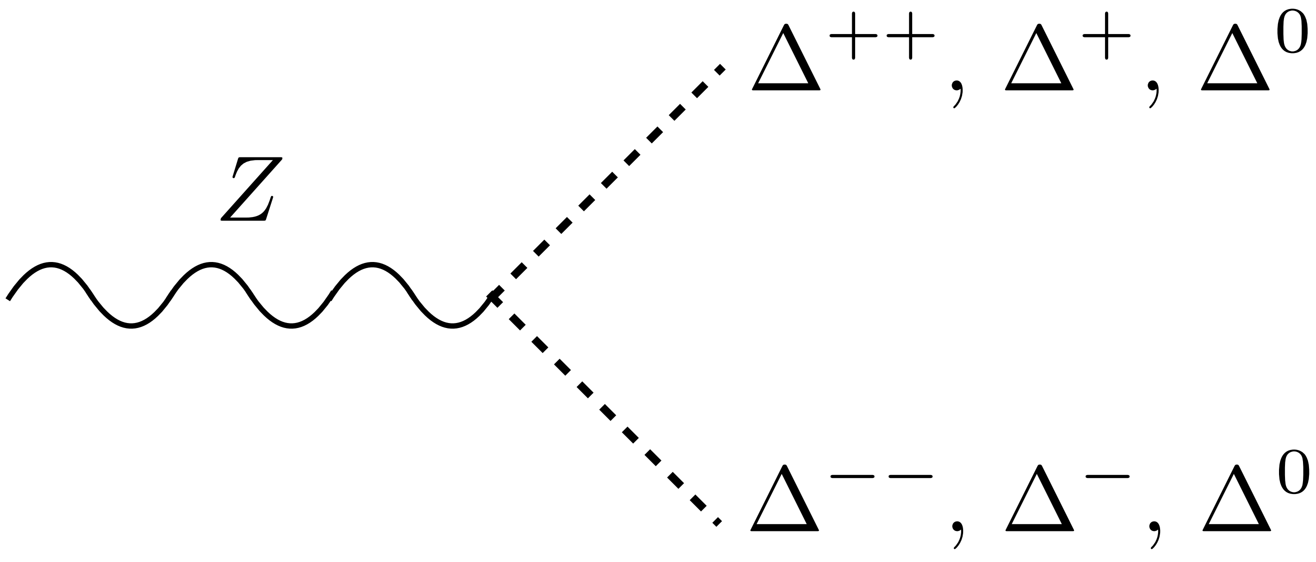
Unambiguous relation to neutrino oscillations

Testing type II seesaw

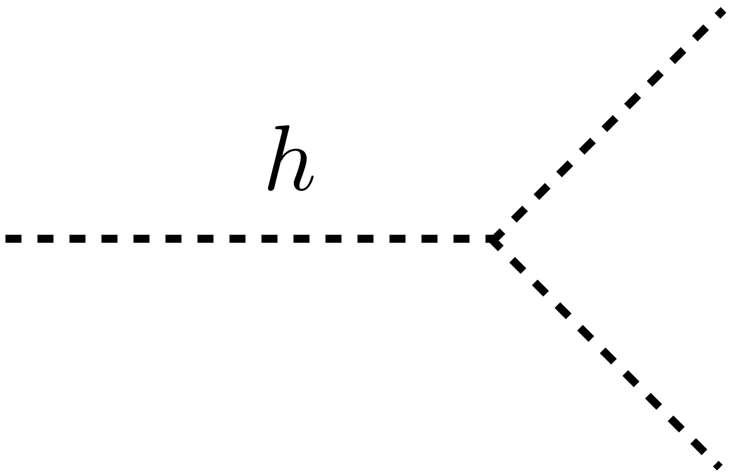
We can produce triplet components via gauge and Higgs



W associated production



Z pair-production



Higgs pair-production

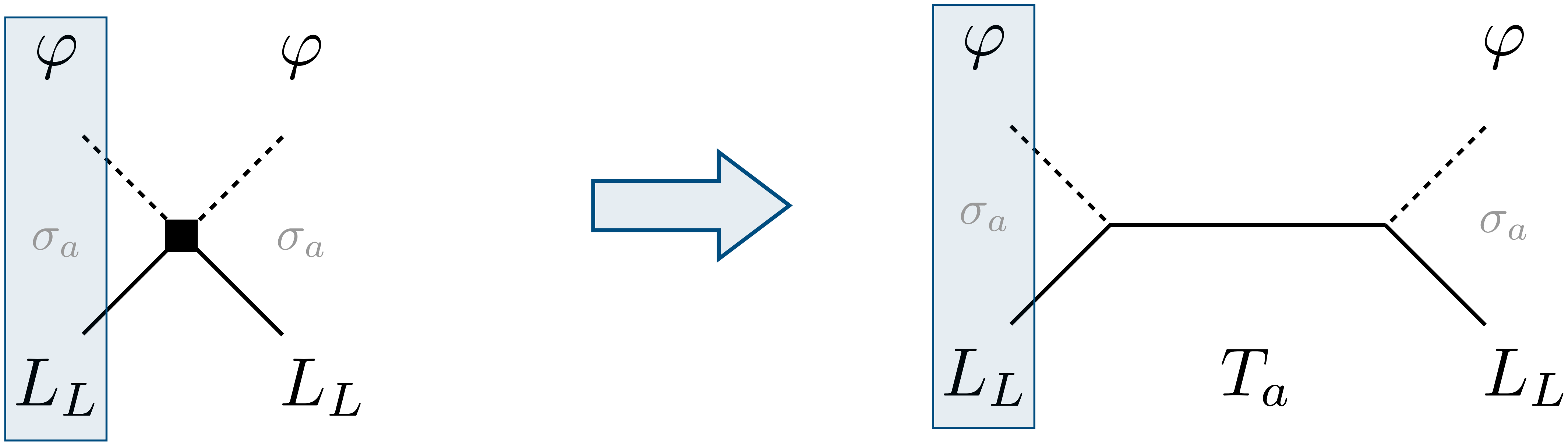
Meson decays

LFV, rare muon decays

EWPT, $h \to \gamma\gamma, Z\gamma$

Type III seesaw

$$\mathcal{O}^{(5)} = (L\varphi)(L\varphi) = (L^T i\sigma_2\sigma_a\varphi)\mathcal{C}\boxed{(\varphi^T i\sigma_2\sigma_a L)}$$



A fermionic triplet $T = (3,0)_F$

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_T$$

Yukawa sector similar to type I; more states and gauge interactions.

$$\mathcal{L}_T = \overline{T} \not{D} T - y_T \left(L^T i \sigma_2 \sigma_a \varphi \right) T_a - \boxed{\frac{m_T}{2} T_a^T \mathcal{C} T_a}$$

Kinetic Dirac Majorana

$T \in \left(T^{\pm}, T^0 \right)$ New states = new seesaw particles

Connection to neutrino masses

$$M_\nu = \begin{pmatrix} 0 & \boxed{m_D} \\ m_D & \boxed{m_T} \end{pmatrix} \qquad \boxed{m_D = y_T \frac{v}{\sqrt{2}}}$$

$$\boxed{M_D = i \sqrt{m_T} O \sqrt{M_\nu}}$$

Seesaw ambiguity

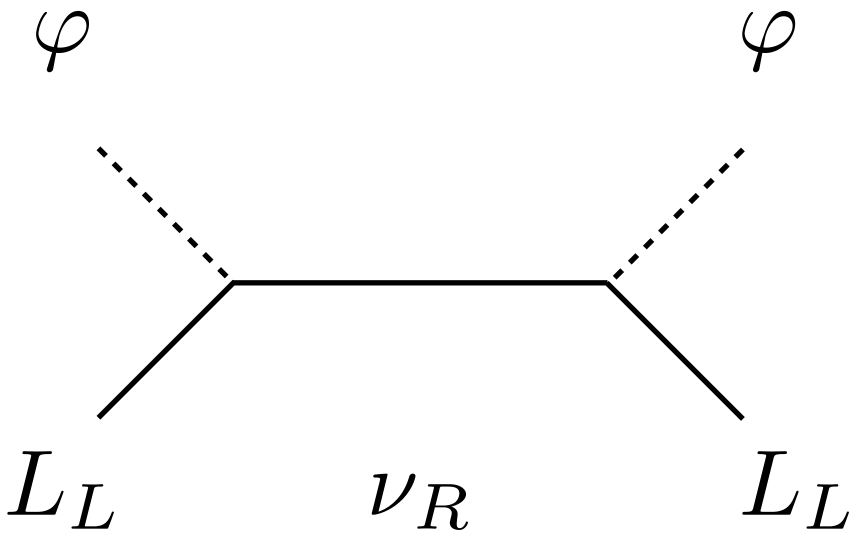
This is the same as for type I seesaw, what is new are gauge interactions.

Phenomenology

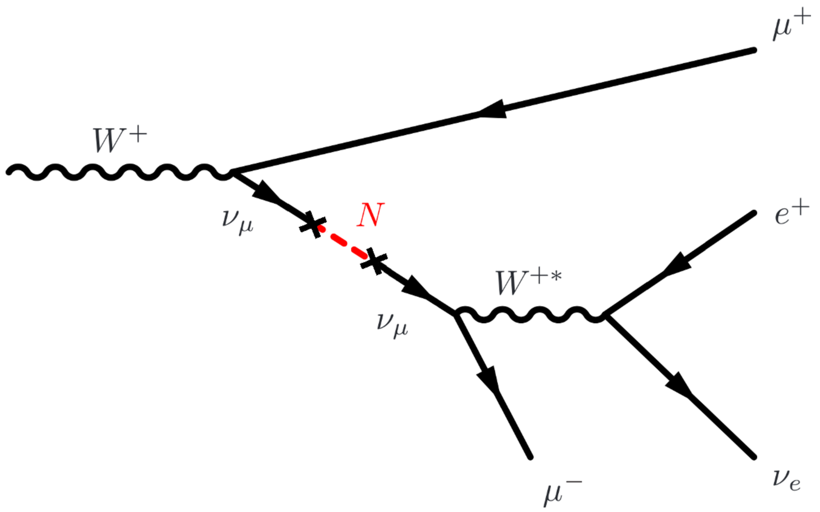
type I seesaw

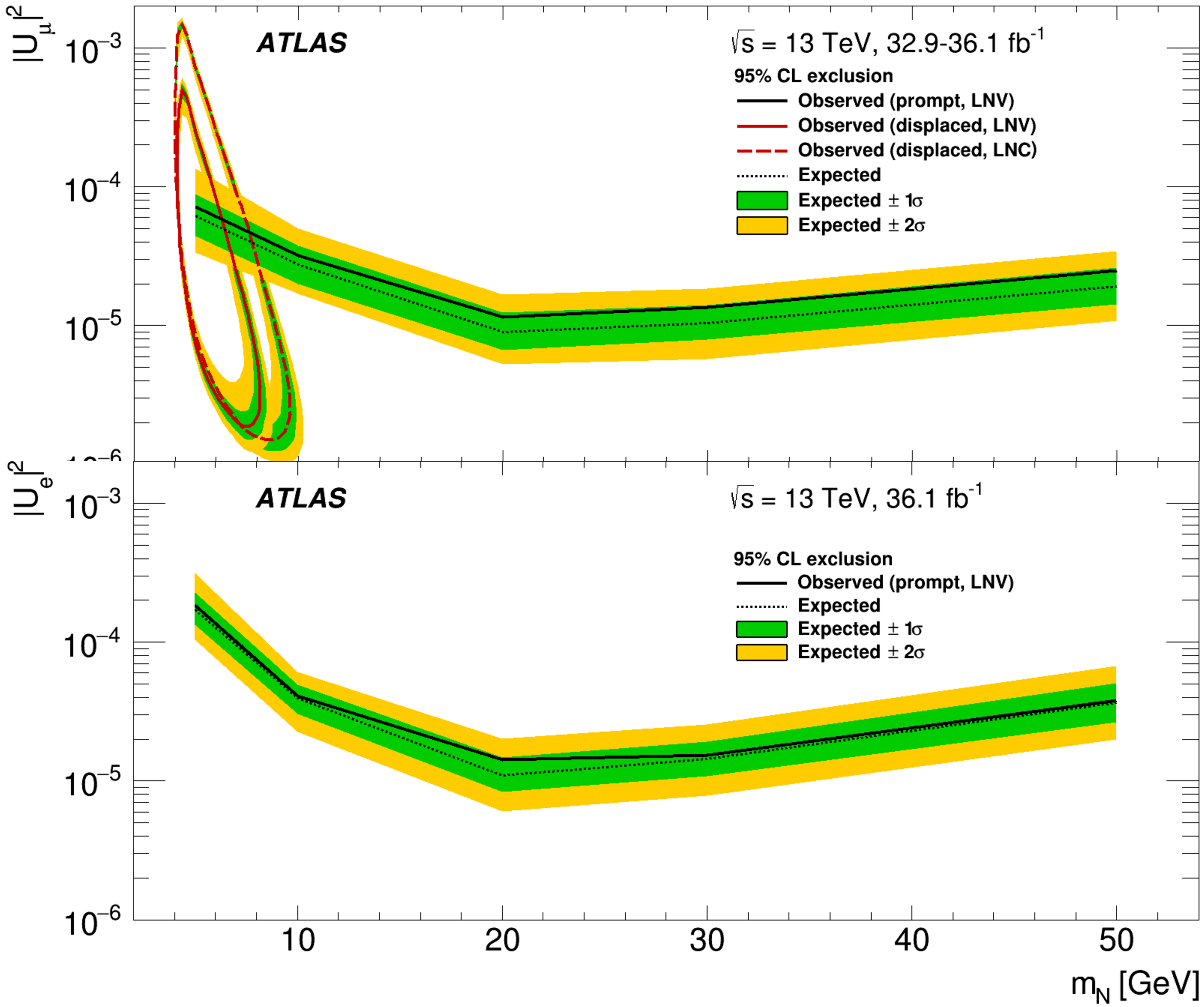
Type I seesaw

ATLAS 1905.09787



Leptonic final states





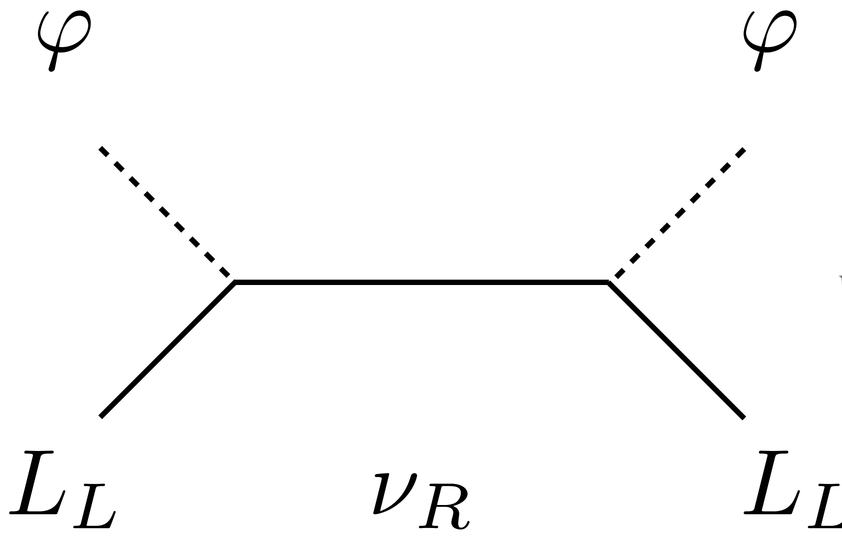
Missing energy

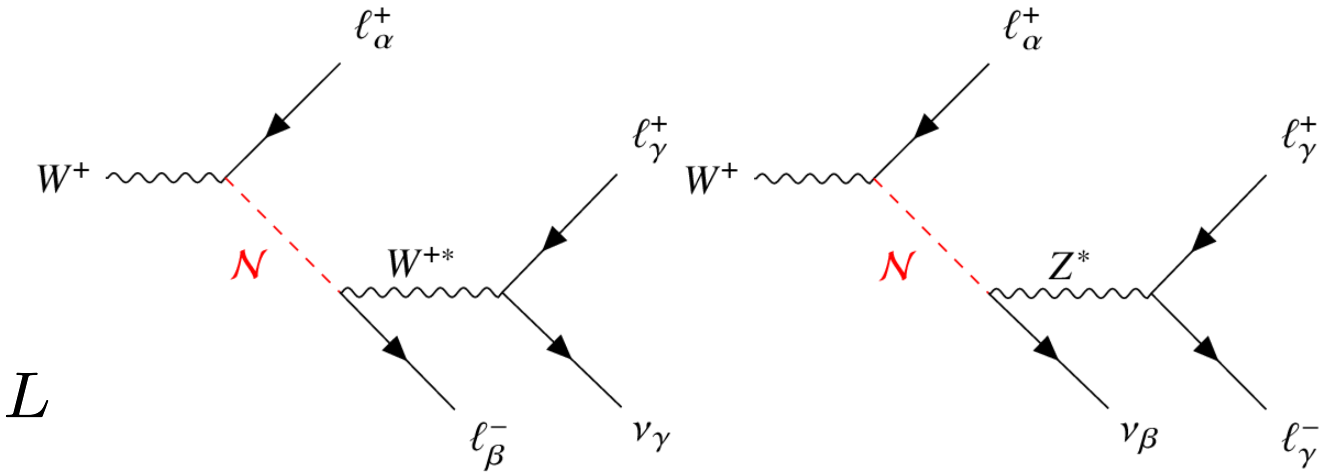
Approx. Mass reconstruction

Flavor of leptons gives info on Dirac couplings

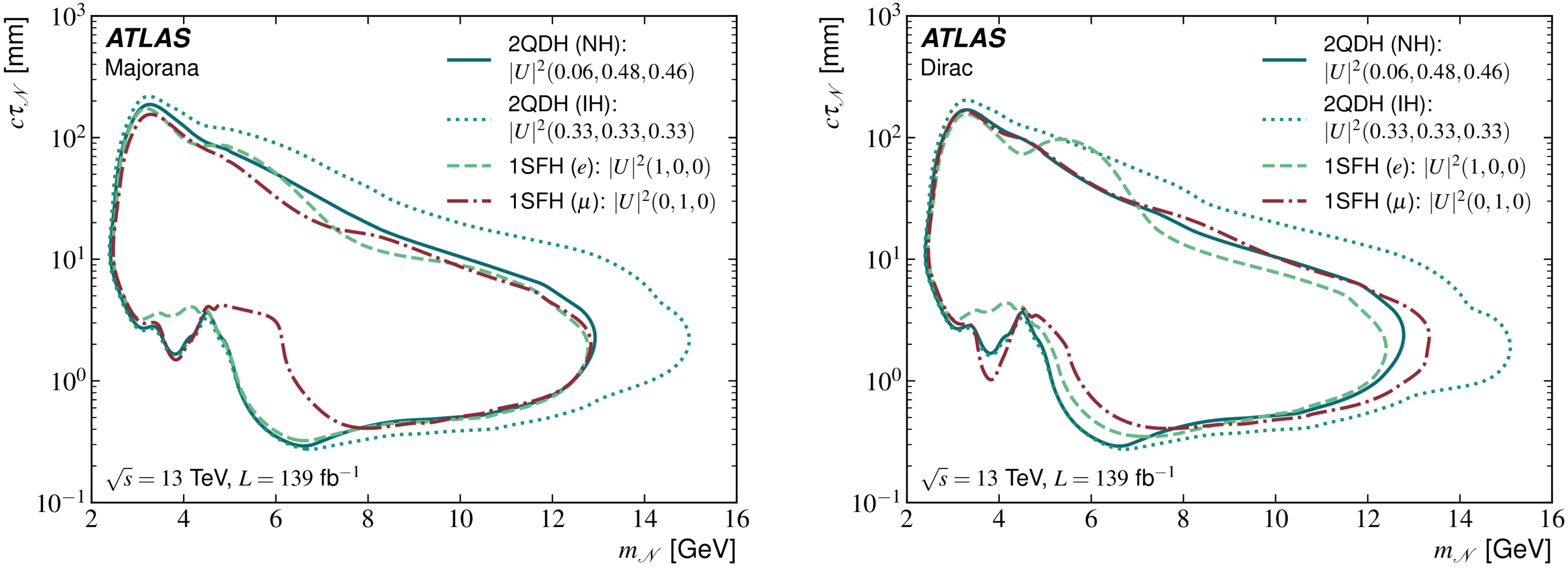
Type I seesaw

ATLAS 2204.11988



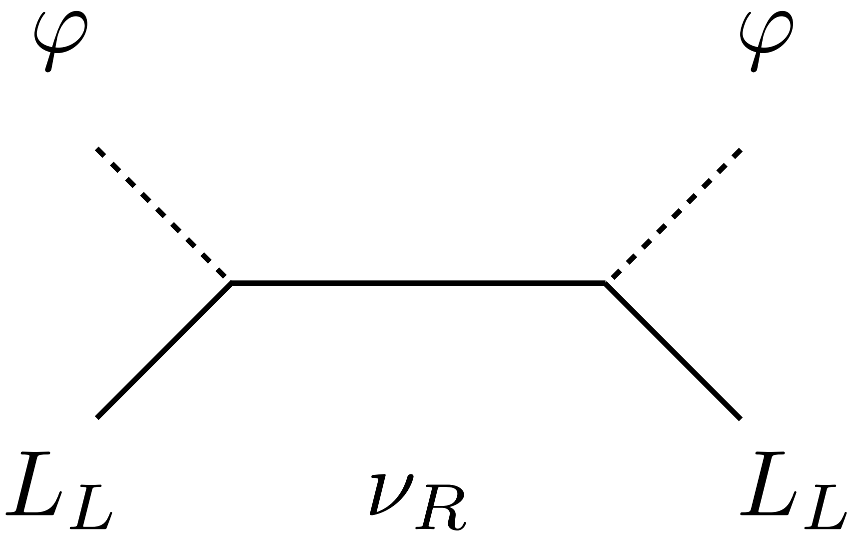


Weak mixing also leads to **displaced** signatures

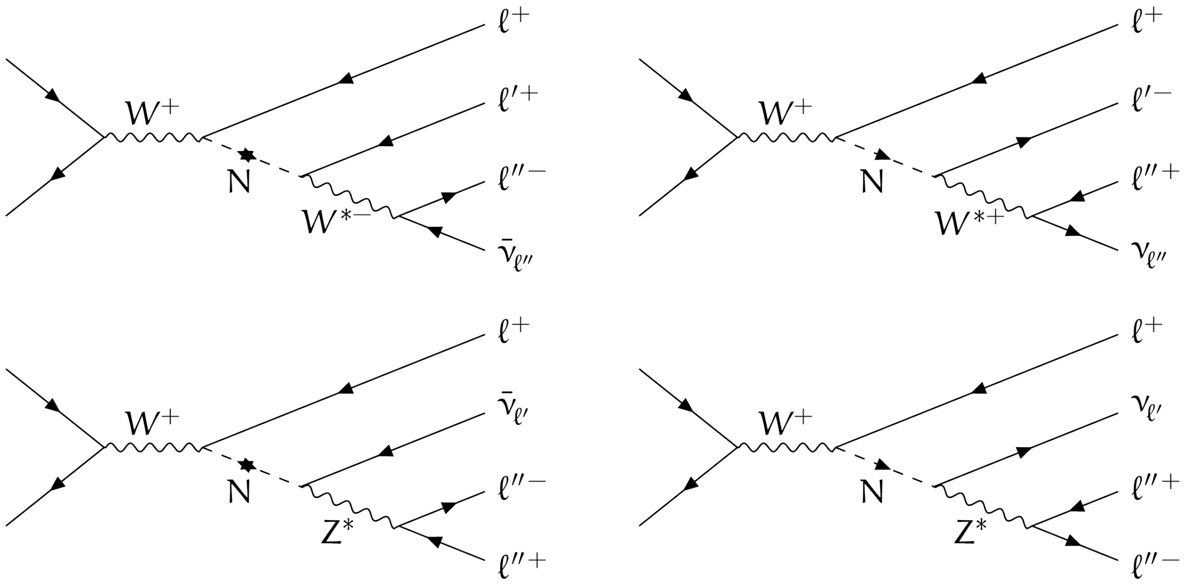


Backgrounds dibosons, jet fakes, secondary photoproduction

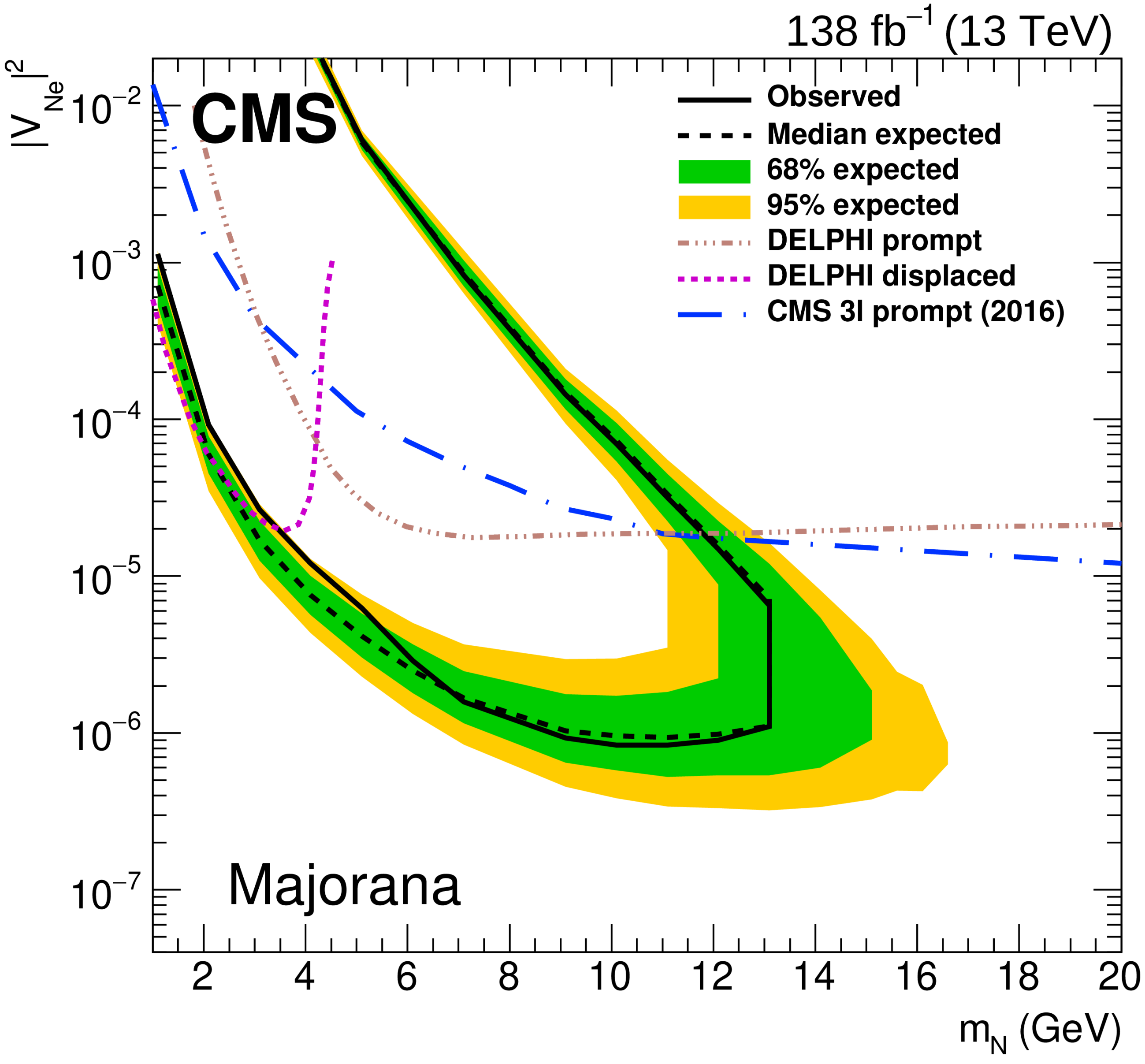
Type I seesaw

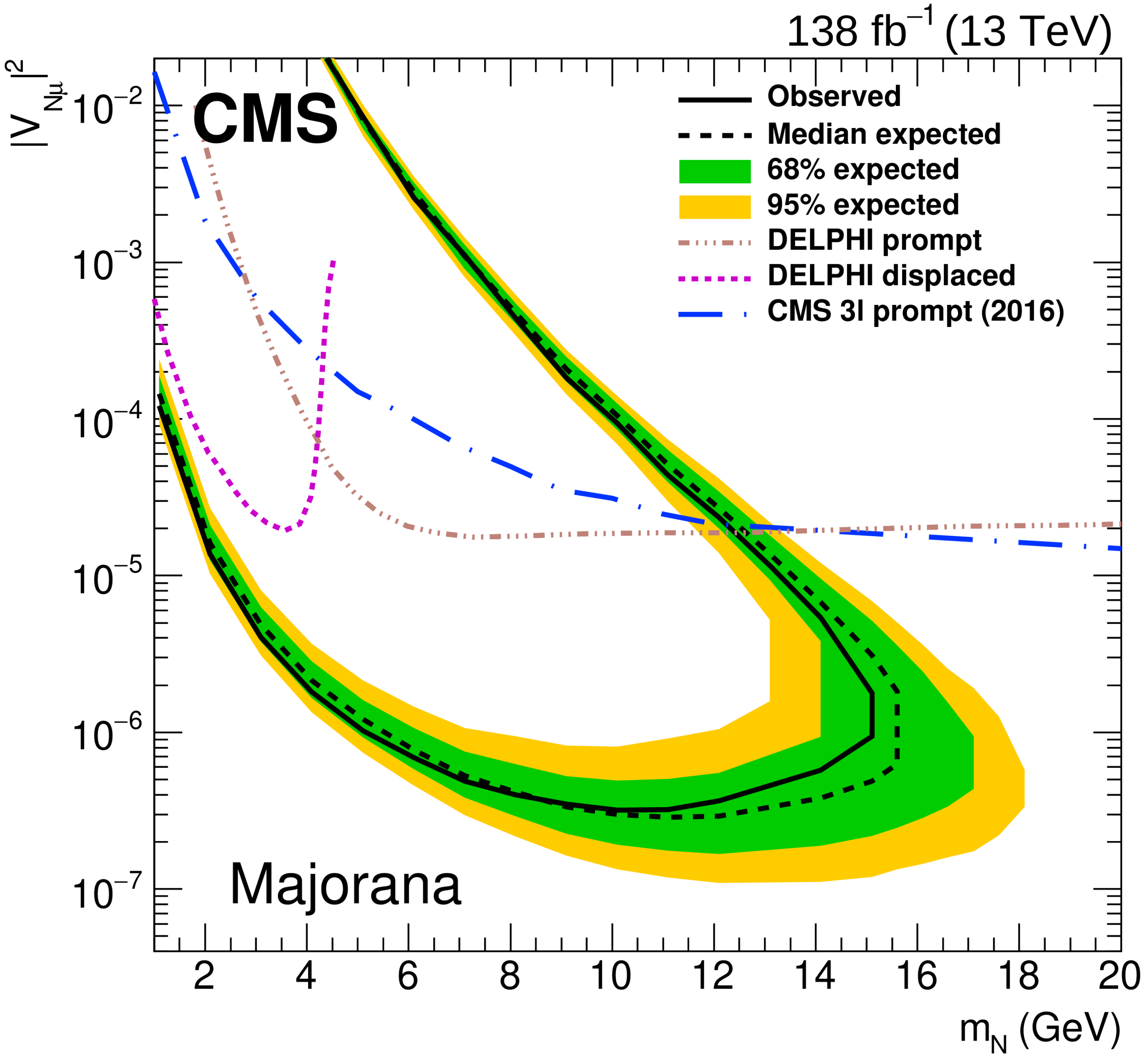


CMS 2201.05578

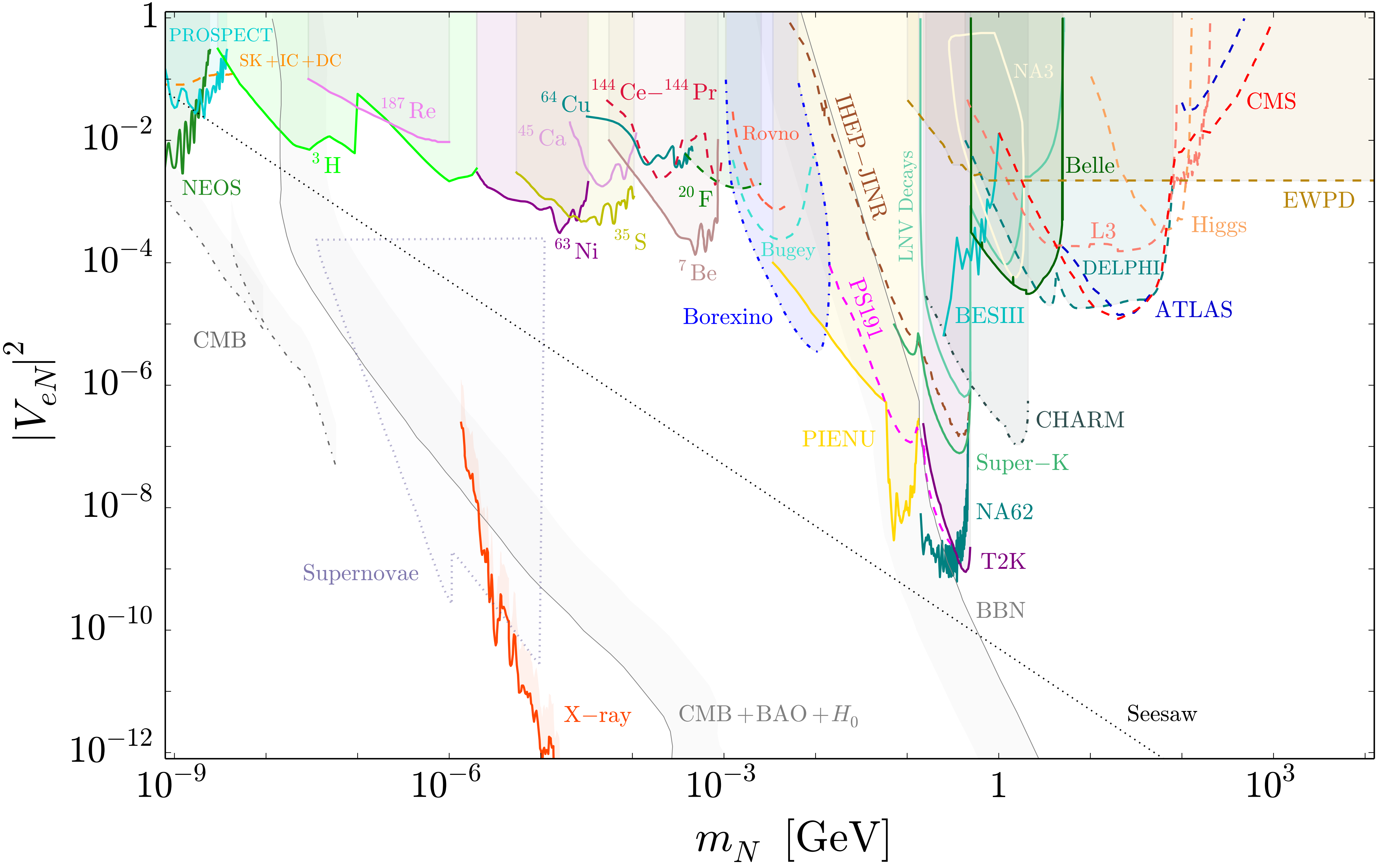


Leptonic final states



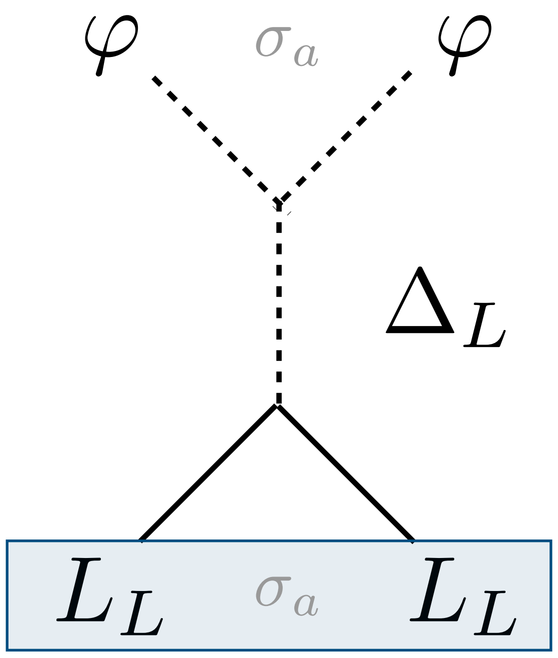


Type I seesaw A host of other constraints Bolton, Deppisch, Dev 2206.01140



type II
seesaw

Type II seesaw



simple(st), one representation only

Cheng, Li '80
Magg, Wetterich '80

possible remnant of

SO(10)	126_H	Lazarides, Shafi, Wetterich '81
SU(5)	15_H	Glashow '79, ...
Left-Right	$\Delta_L(3,1,2)$	Mohapatra, Senjanović '81

tests at the LHC

LHC searches still incomplete

open issues?

- mass splittings
- gauge boson final states

Type II seesaw

$$\Delta_L = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & -\frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{L}_{\Delta_L} = \left|D_\mu \Delta_L\right|^2 - Y_\Delta \left(L^T i\sigma_2 \sigma_a \mathcal{C} L\right) \Delta_L^a - V(\Delta, \varphi)$$

gauge production

decays to g.b.s

neutrino mass

decays to leptons

mass spectrum

cascade decays

$W, Z \to \Delta\Delta$

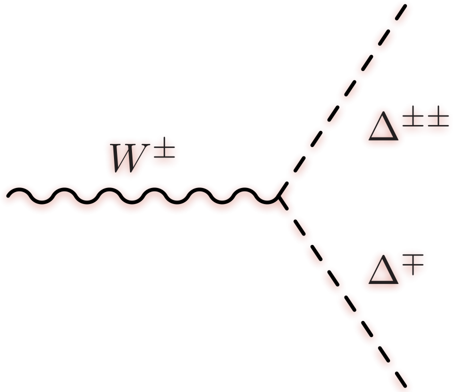
$$\boxed{M_\nu = \sqrt{2}\, Y_\Delta v_\Delta = V_L^* m_\nu V_L^\dagger}$$

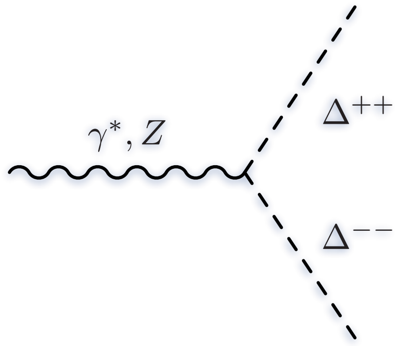
$$m_+^2 - m_{++}^2 \simeq$$
$$m_0^2 - m_+^2 \simeq \beta v^2/4$$

$v_\Delta < \mathcal{O}(\mathrm{GeV})$

Need Yukawa and v_Δ to break LNV

Type II seesaw production

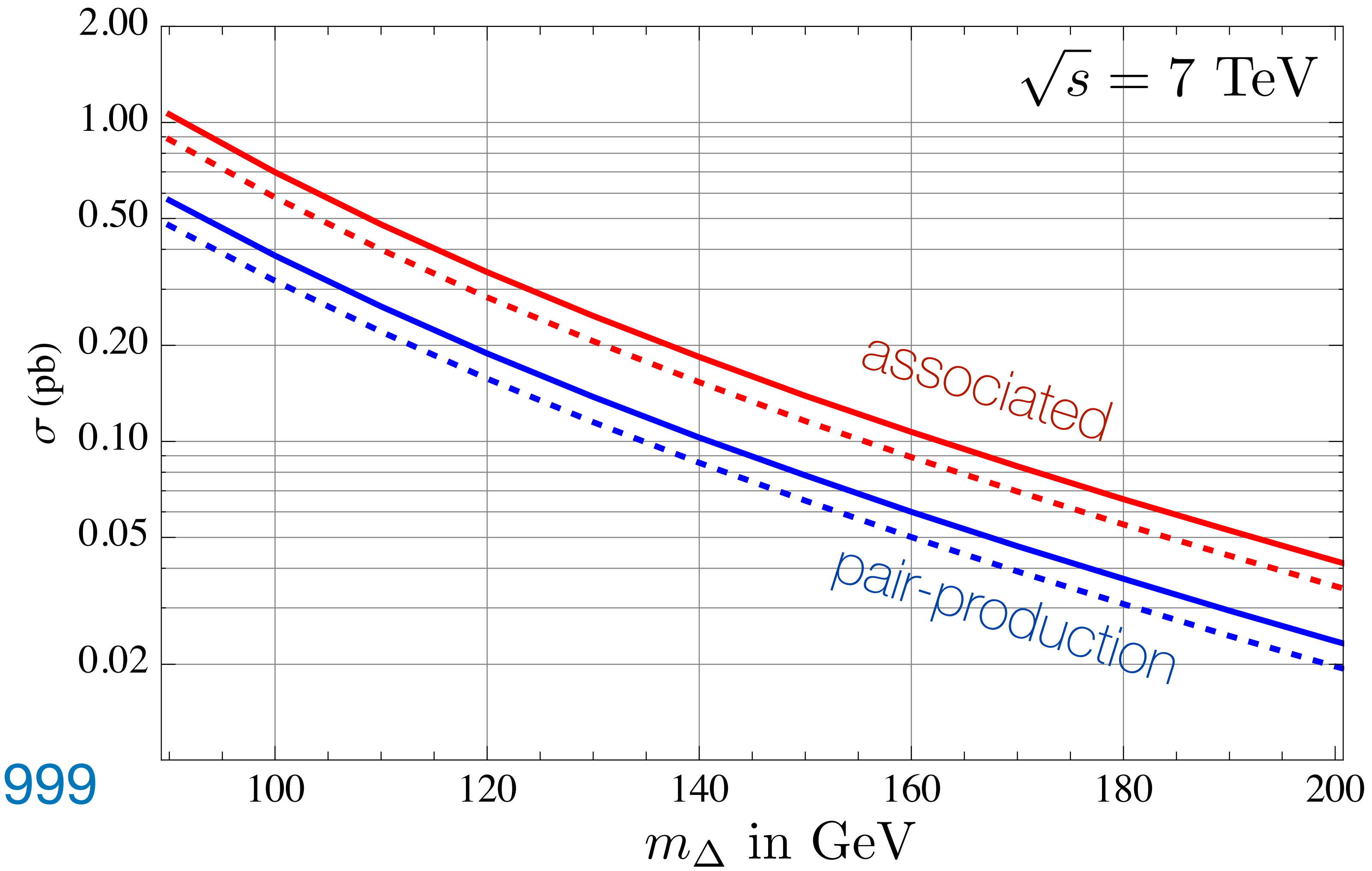




Azuelos et al. ’05
Akeroyd, Aoki ’05

Single production small

Godfrey, Moats ’10
Bhambaniya et al. 1504.03999



Production studies

del Aguila, Aguilar-Saavedra, Pittau ’07, ’08
Han, Fileviez-Perez, Huang, Li, Wang ’08

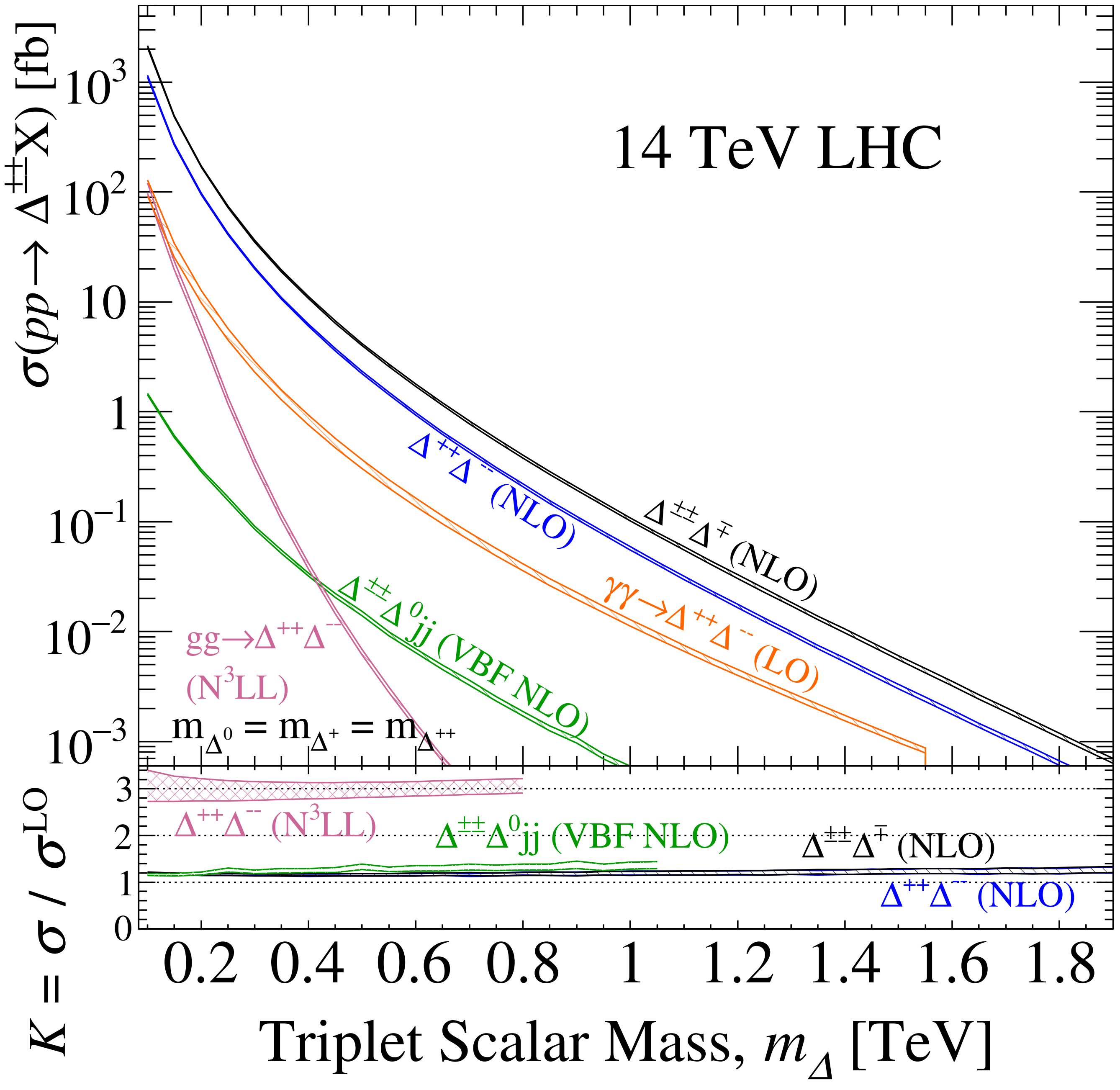
LHC reach @ 14 TeV

$$m_\Delta \lesssim 700 \text{ GeV} - 1 \text{ TeV}$$

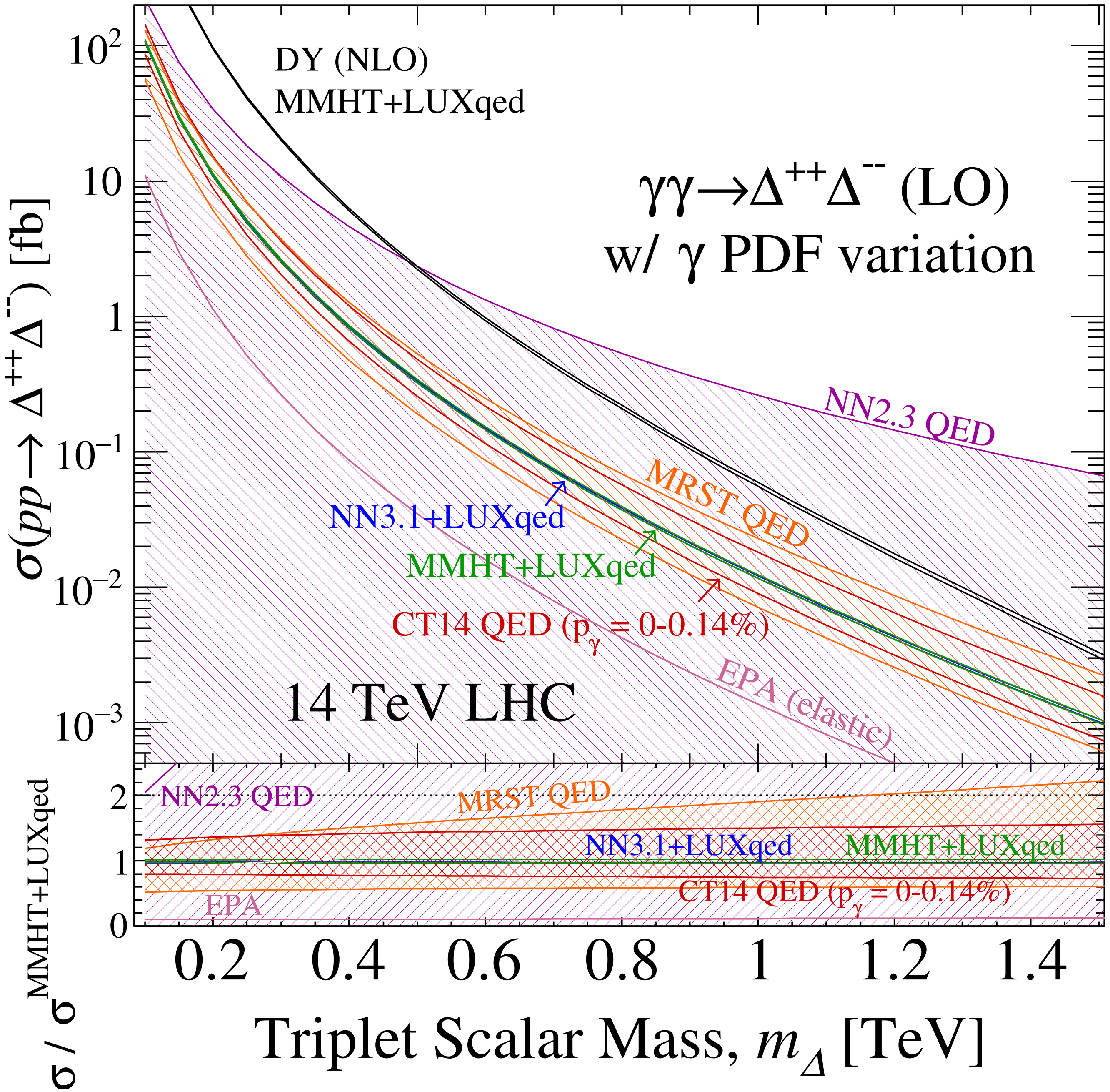
Type II seesaw production

precise production studies

Fuks, MN, Ruiz '19



One loop QCD corrections

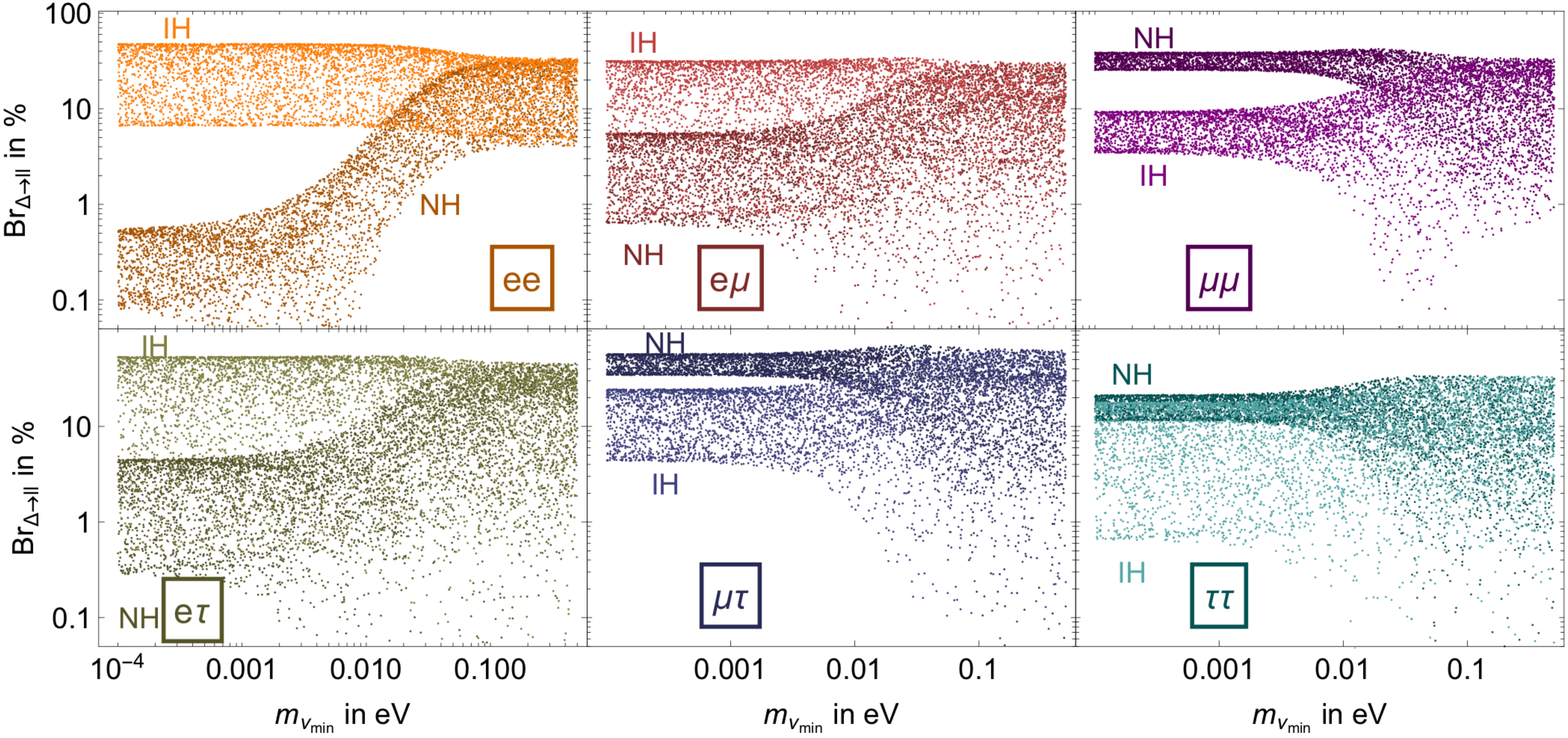


Photon induced production

Type II seesaw: decays

Connection to flavor

Fuks, MN, Ruiz '19



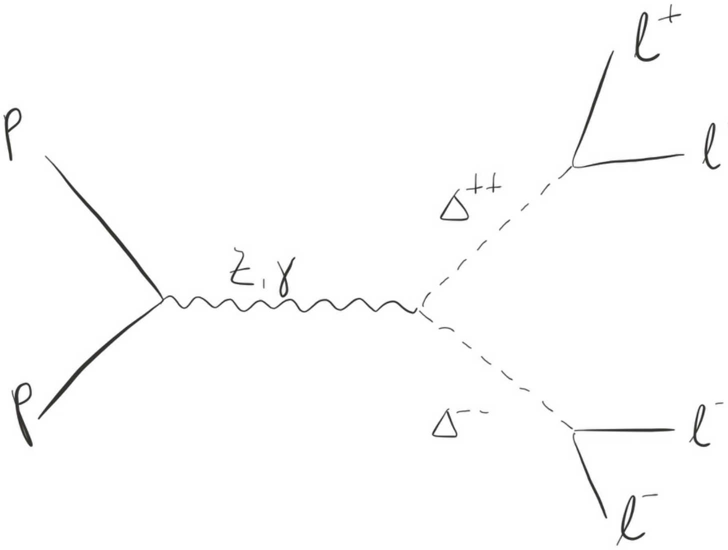
Direct probe of neutrino mass hierarchy

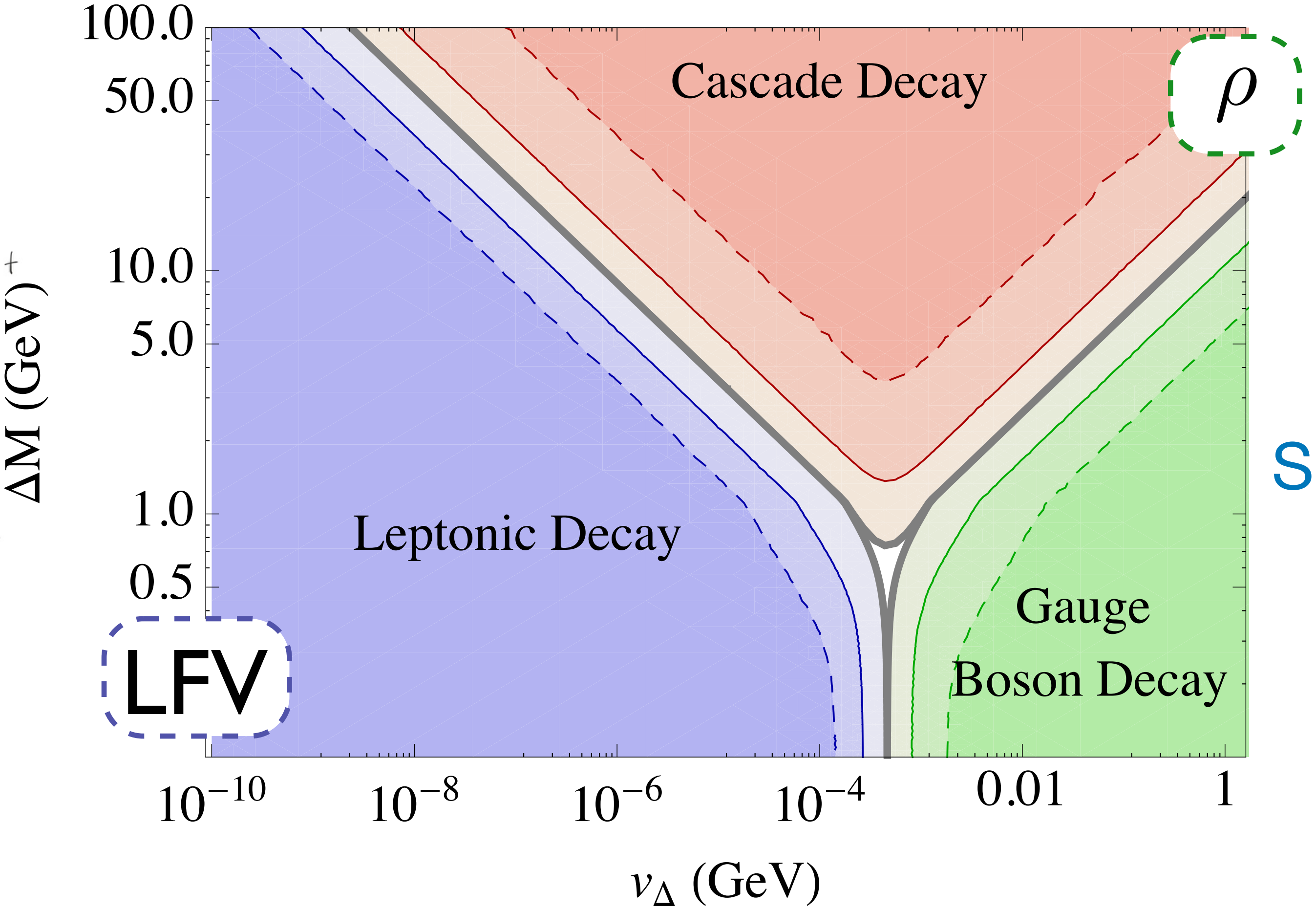
Sensitive to mixing angles and CP (Majorana) phases

Type II seesaw: decays

$$\Delta m^2 = m_+^2 - m_{++}^2$$
$$\simeq m_0^2 - m_+^2 \simeq \beta v^2/4$$

spectacular two same-sign di-leptons





Phase space

Melfo, MN, Nesti, Senjanović, Zhang '11

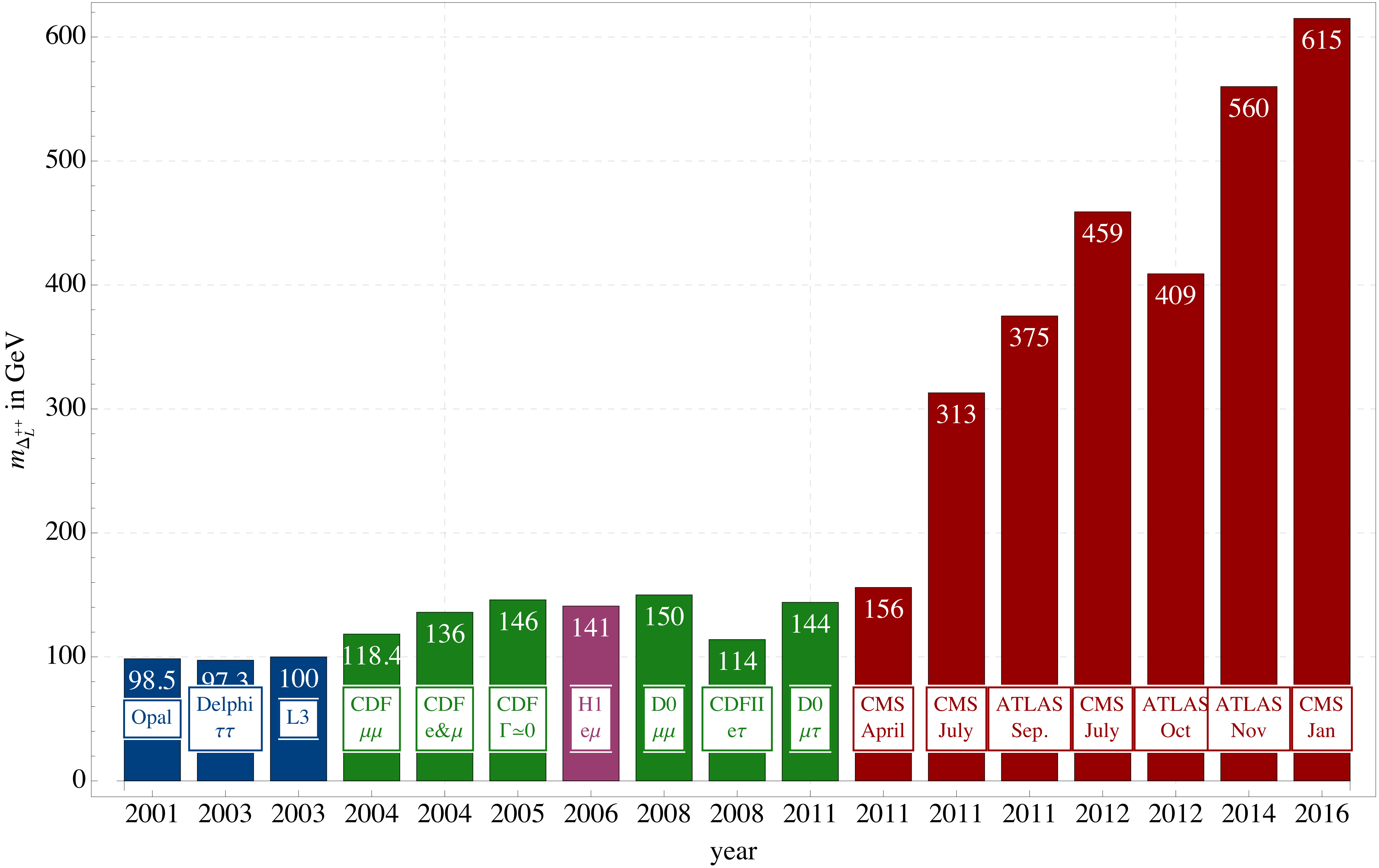
all flavors - LFV

no $\not{E}$ and no LNV

probes Majorana phases & hierarchy $\Gamma \propto |M_\nu|^2$

Chun, Lee, Park '03
Garayoa, Schwetz '07
Kadastik, Raidal, Rebane '07

Type II seesaw: *history* of searches - assumes degeneracy, small v_Δ



1207.2666 CMS (7 TeV / 4.9)
1201.1091 ATLAS (7 TeV / 1.6)

1412.0237 ATLAS (8 TeV / 20.3)
1210.5070 ATLAS (7 TeV / 4.8)

CMS-PAS-HIG-14-039 (8 TeV/19.7)

Type II seesaw

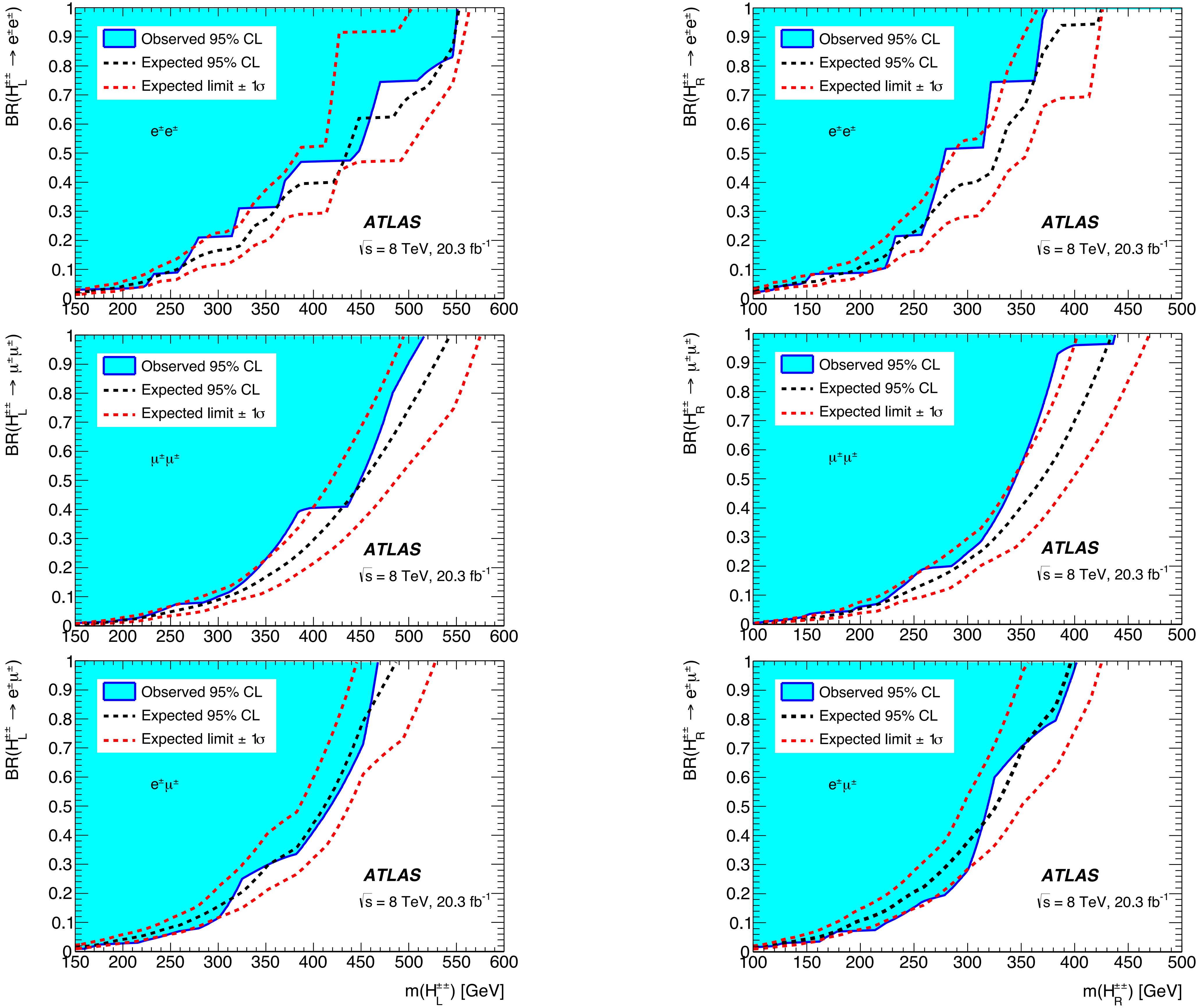
Δ_L^{++} Δ_R^{++}

1412.0237 ATLAS
(8 TeV / 20.3)

ee

$\mu\mu$

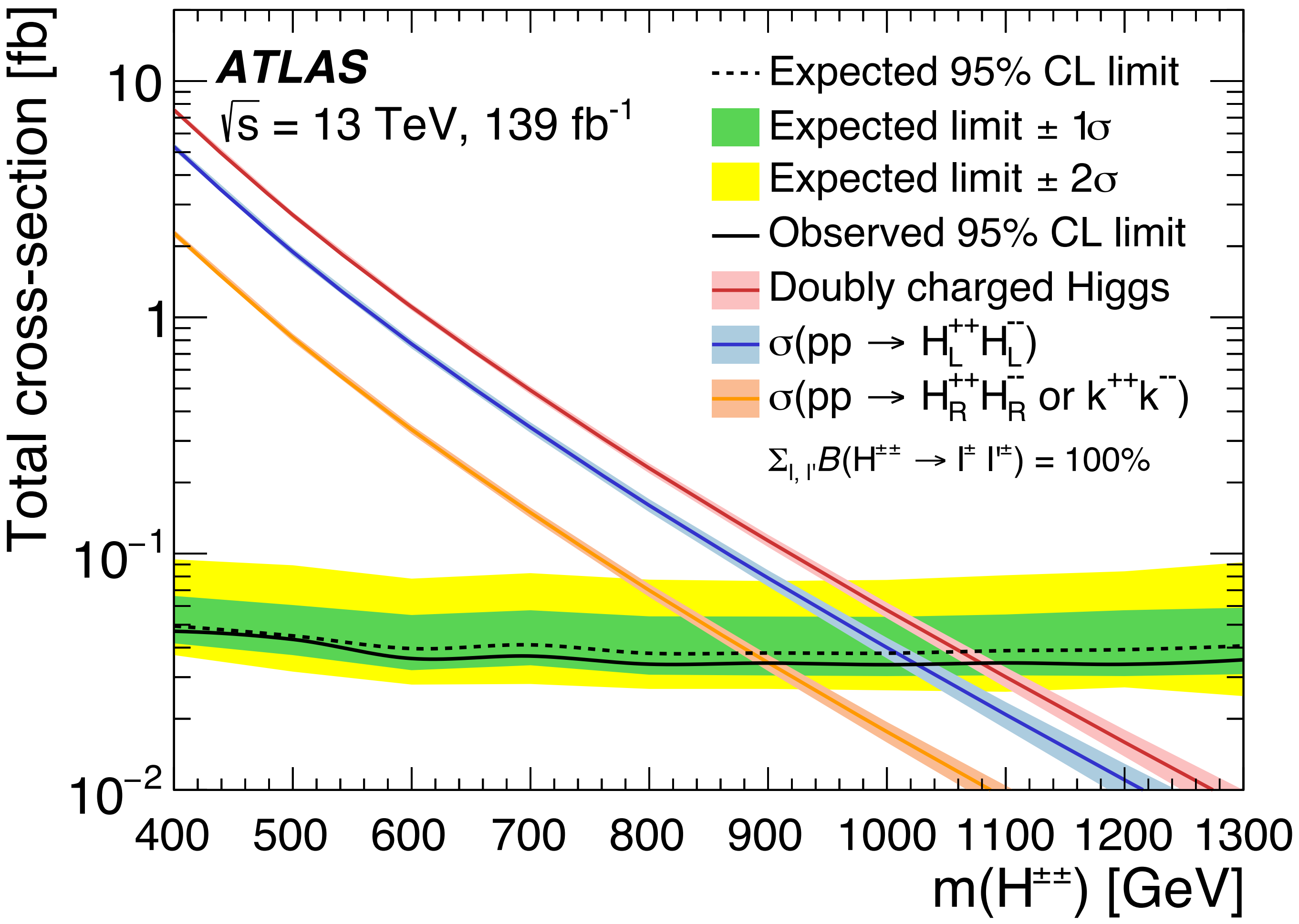
$e\mu$

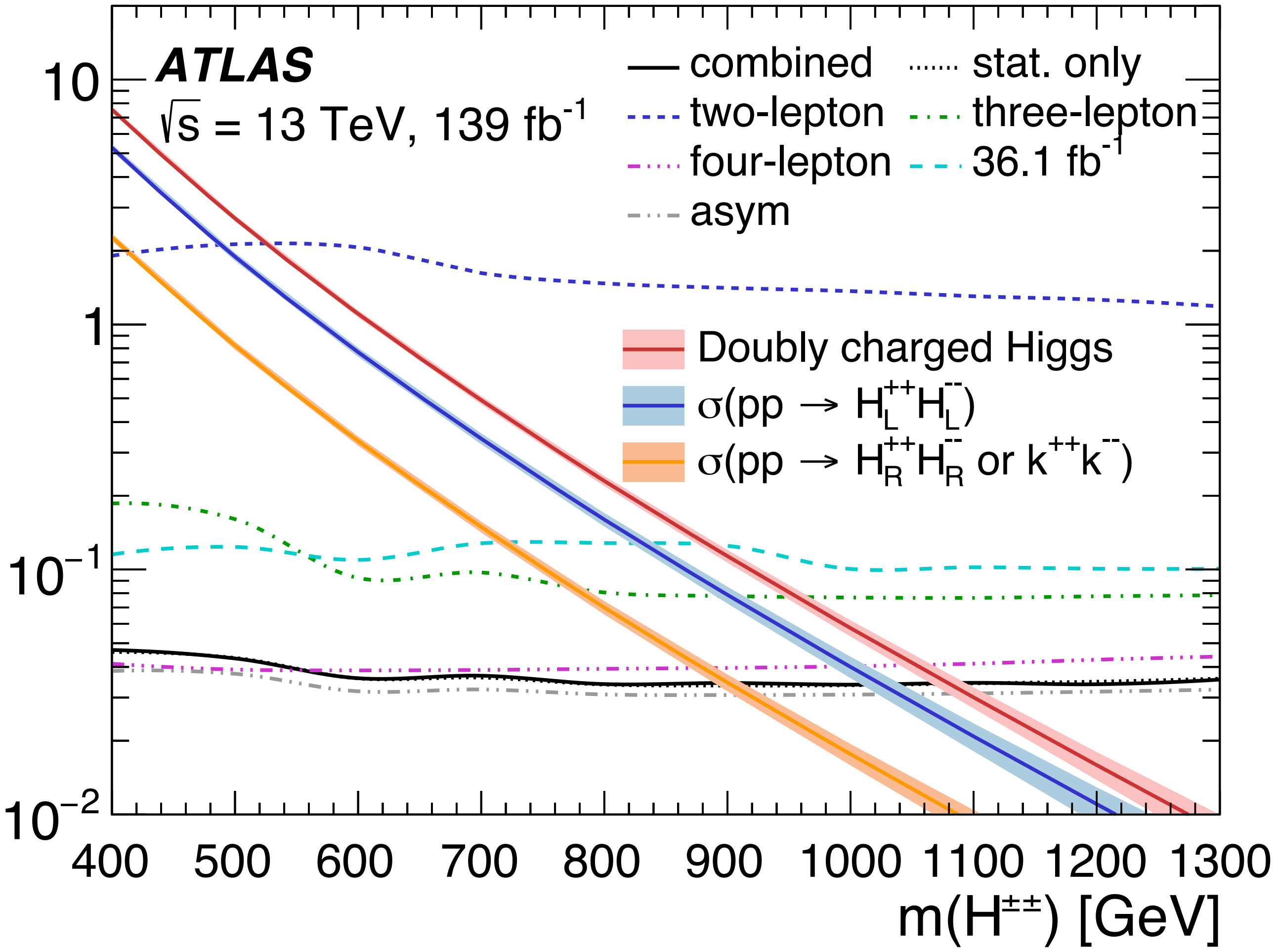


CMS also has τ s CMS 1207.2666

Type II seesaw Δ_L^{++}

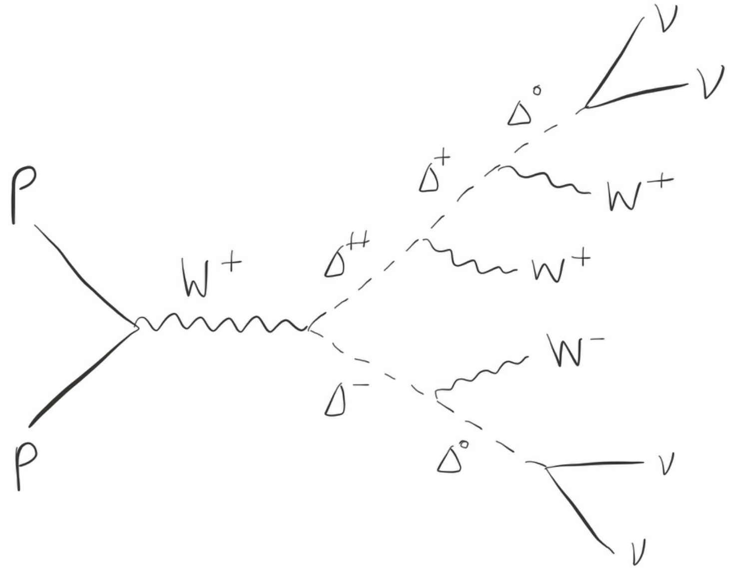
ATLAS 2211.07505





Leptonic channel breaks the TeV barrier!

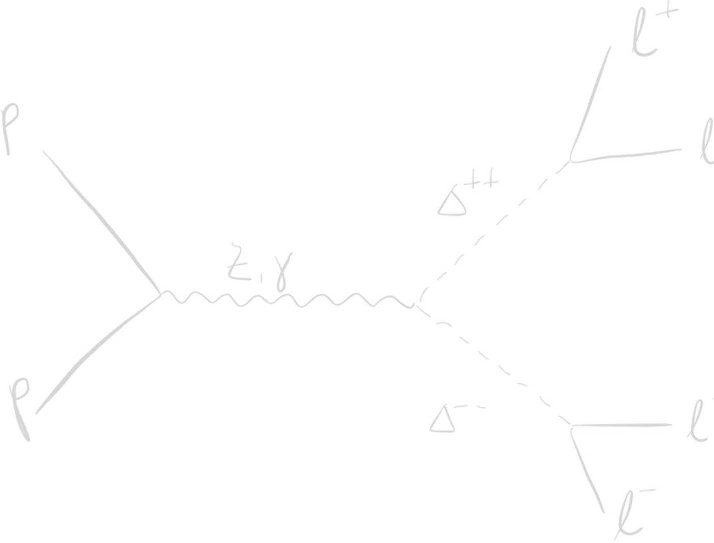
Type II seesaw: decays

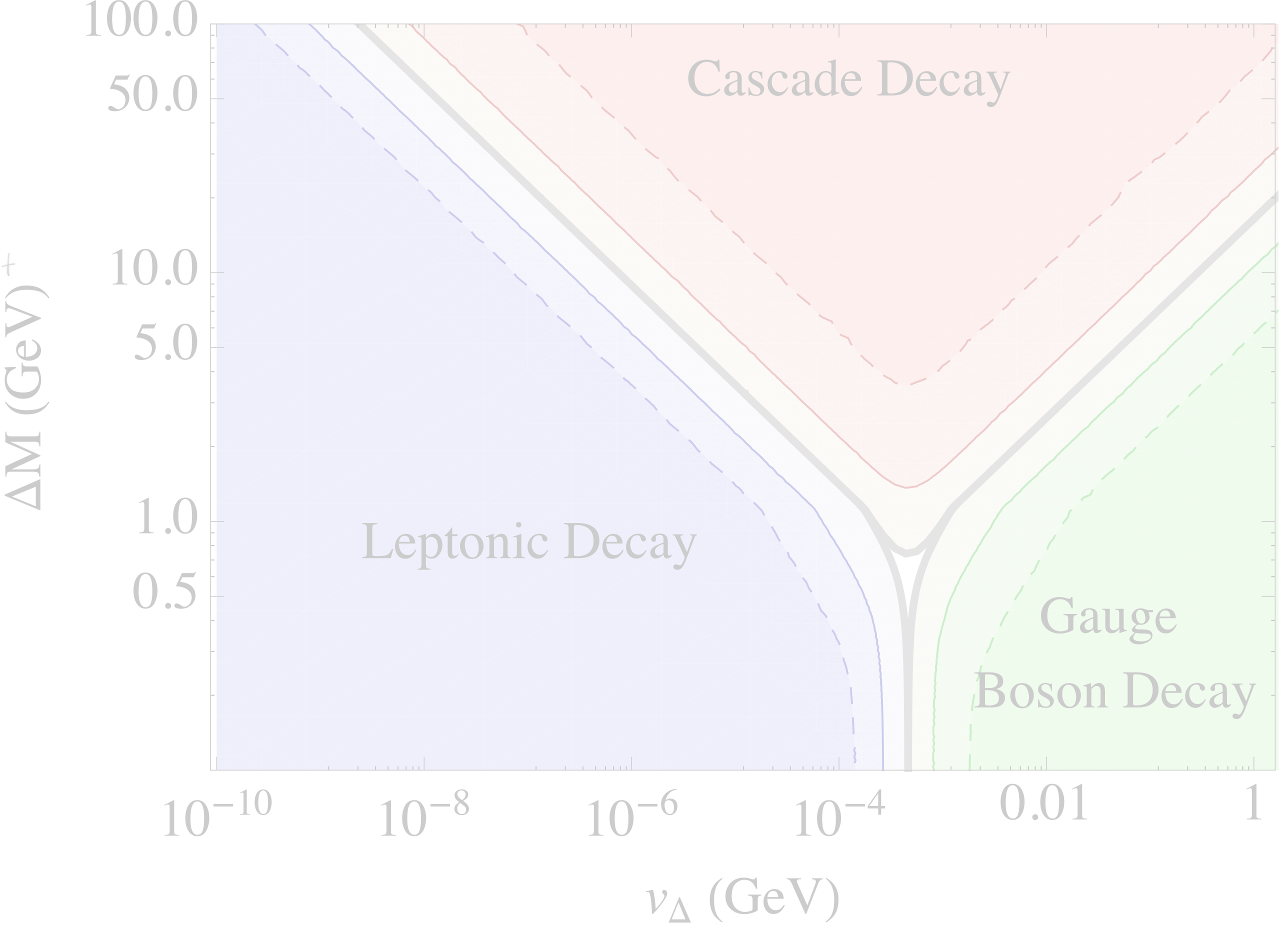


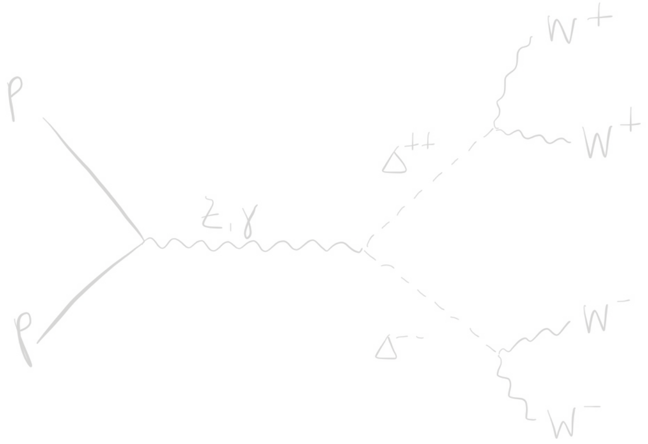
~soft jets/leptons

missing energy - large bckg

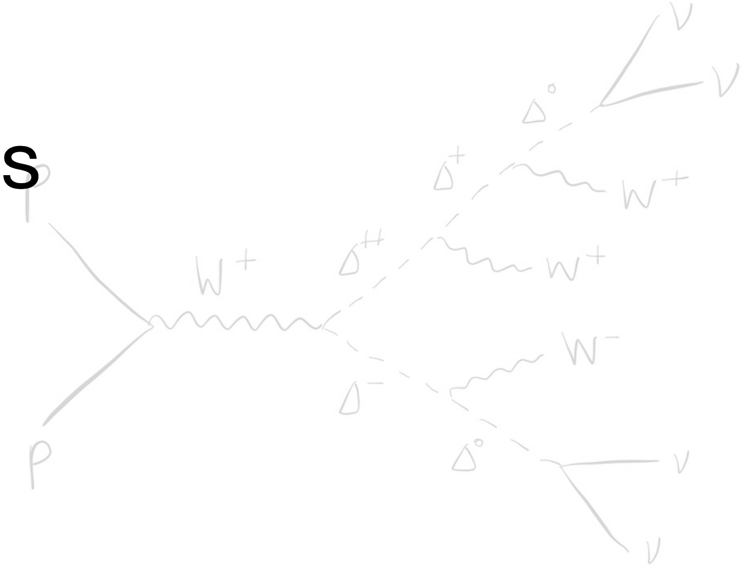
bounds strengthen/weaken

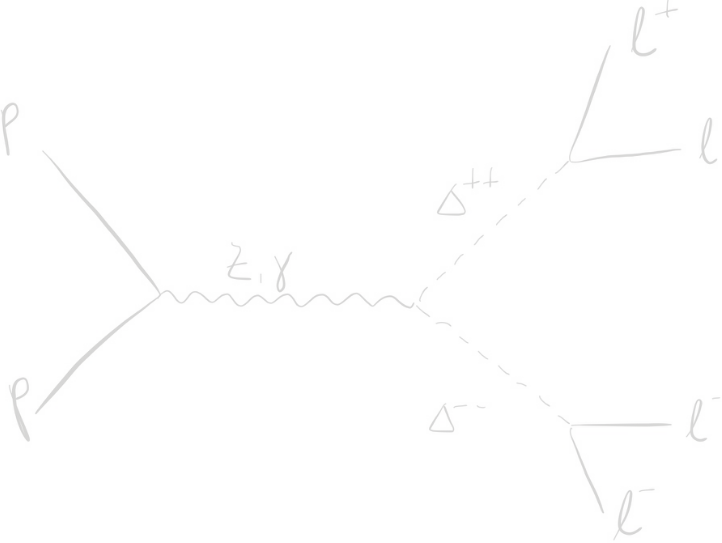


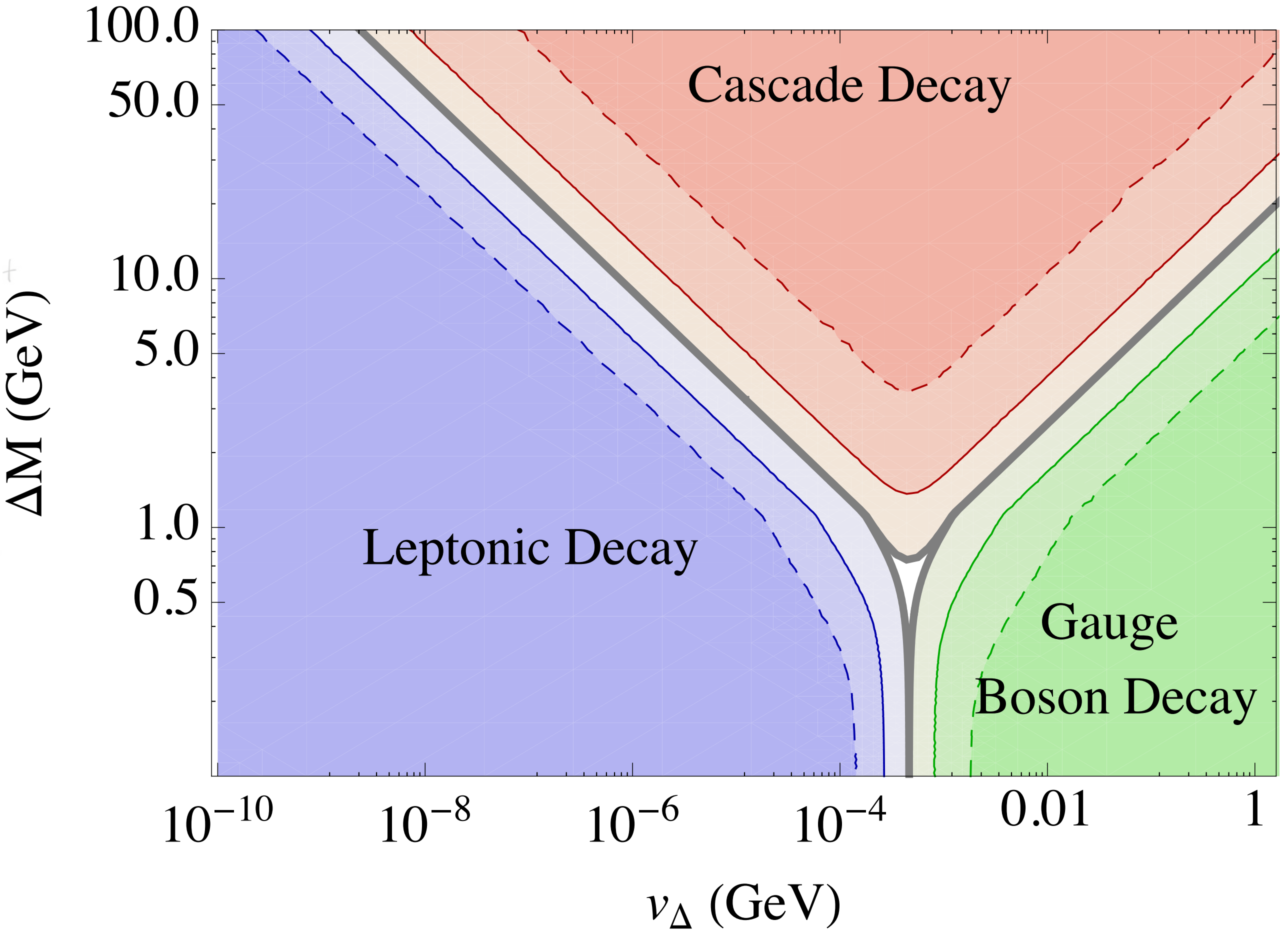


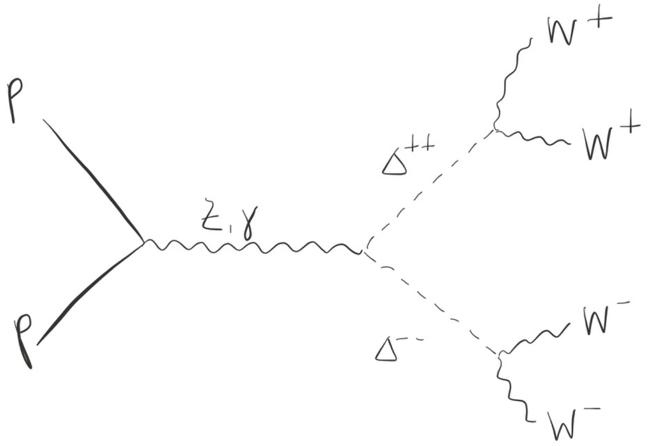


Type II seesaw: decays









soft Ws, bcgk

some sensitivity

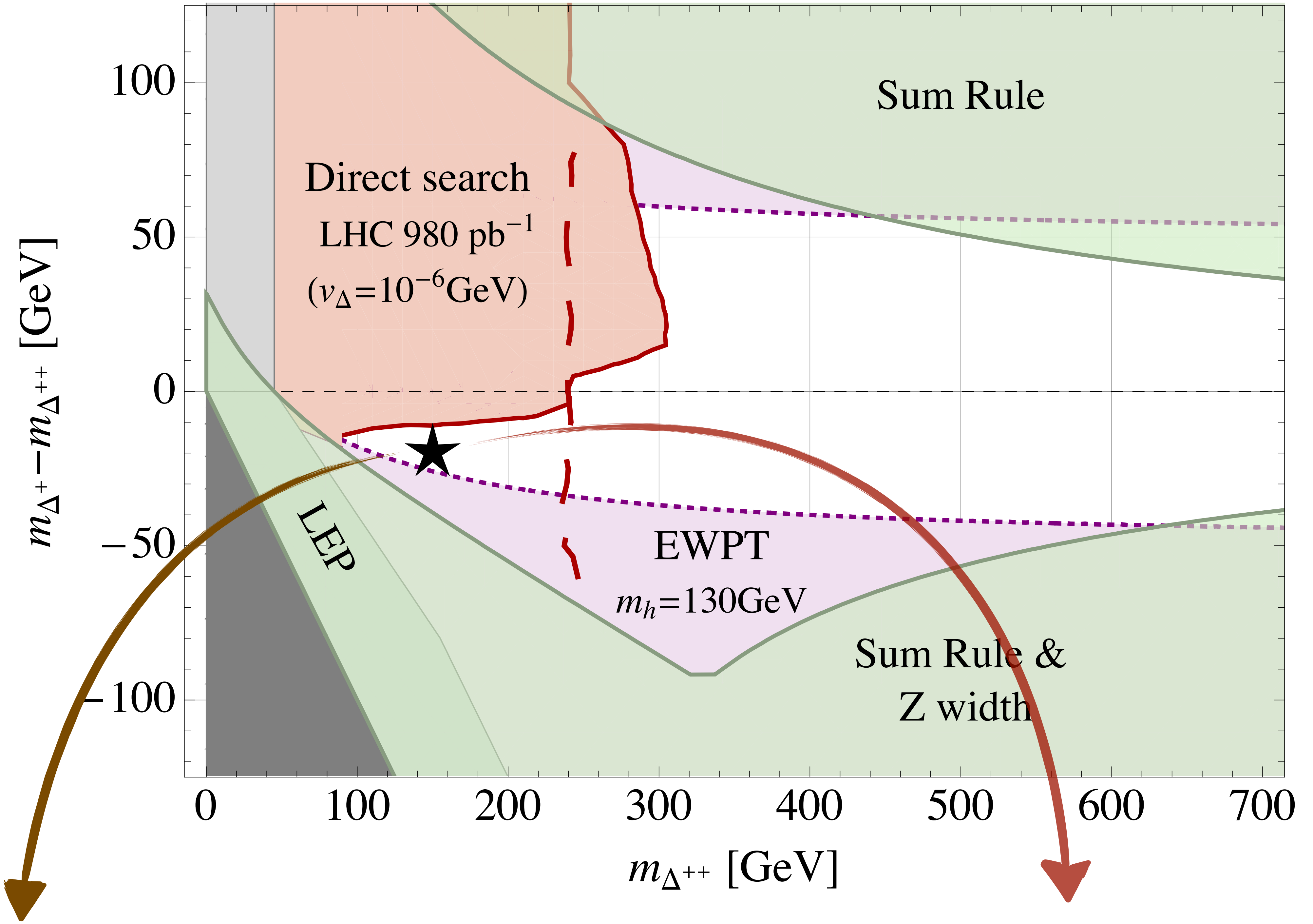
$m_\Delta \lesssim 90$ GeV

Kanemura et al. 1407.6547

ATLAS 2101.11961 $m_\Delta \gtrsim 350(230)$ GeV

Type II seesaw: LHC roadmap

Melfo, MN, Nesti, Senjanović, Zhang '11



limited sensitivity

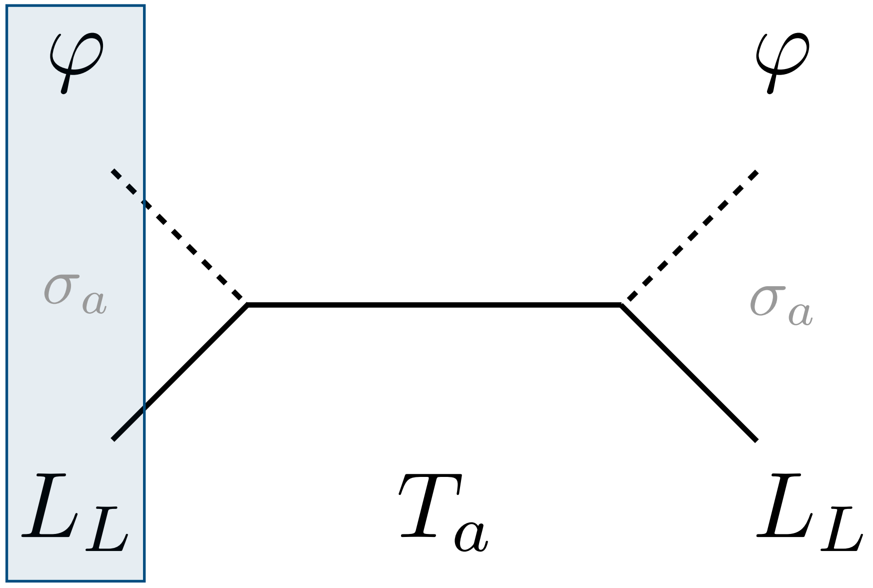
Han, Ding, Liao 1502.05242, 1506.08996

enhanced $h \to \gamma\gamma$

Melfo, MN, Nesti, Senjanović, Zhang '11
..., Akeroyd, Moretti 1206.0535

type III
seesaw

Type III seesaw



same logic as type I, need two reps Foot, Lew, Joshi ’89

predicted light remnant of

SU(5) 24_F Bajc, Senjanović ’06
Bajc, MN, Senjanović ’07

$$\mathcal{L}_T = i\overline{T}\not{D}T - y_T\left(L^T i\sigma_2\sigma_a\varphi\right)T_a - \frac{m_T}{2}T_a^T C T_a$$

gauge production

mass splitting & cascades

neutrino mass

need m_T and y_T for LNV

$$M_\nu = -v^2\, y_T^T\, m_T^{-1}\, y_T$$

Tests at the LHC

LNV @ LHC ~automatic

Type III seesaw: production

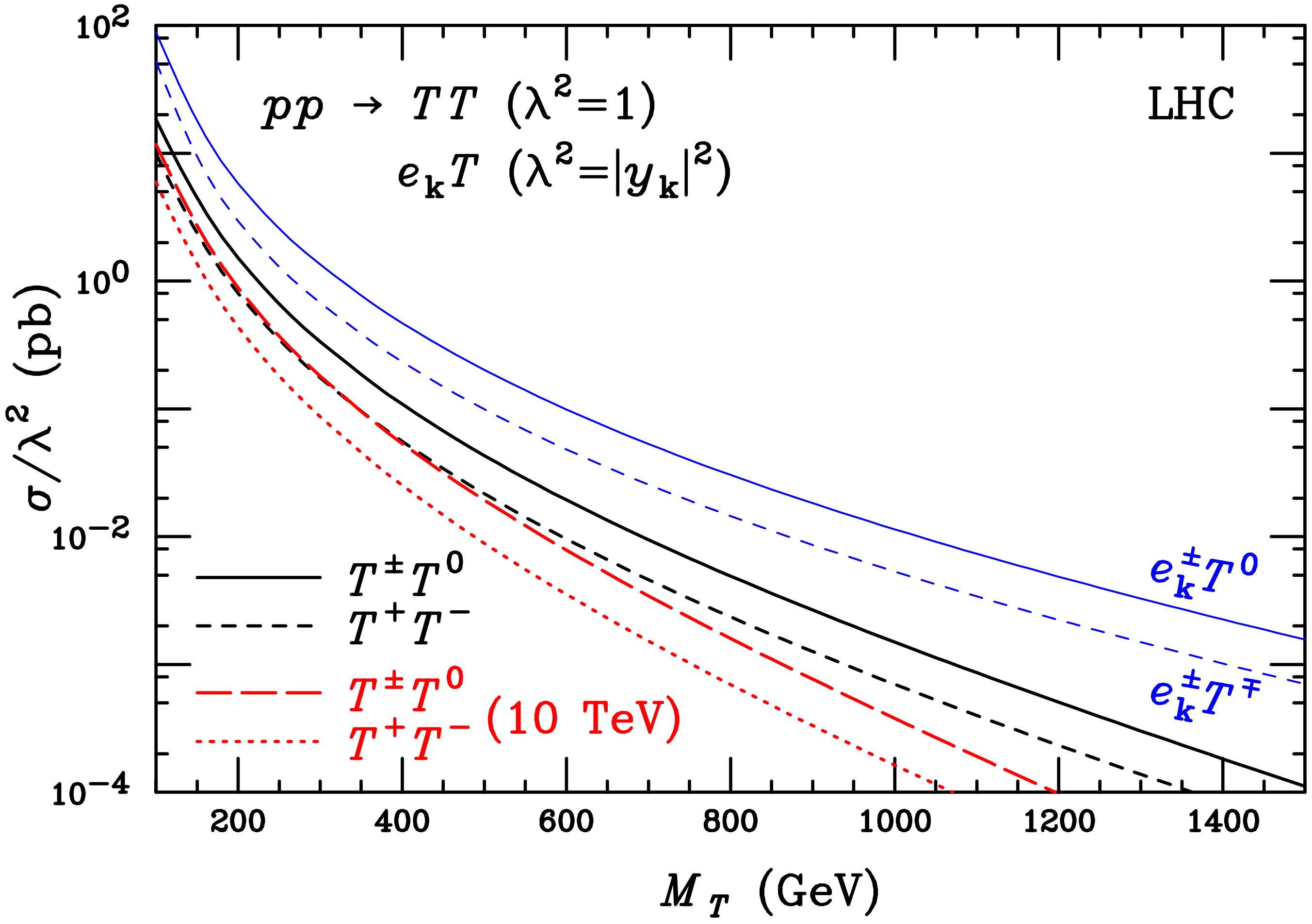
Franceschini, Hambye, Strumia ’08
Arhrib, Bajc, Ghosh, Han, Huang, Puljak, Senjanović ’09

gauge pairs

$$W, Z \to TT$$

single production suppressed

$$W, Z \to \ell T \propto y^2$$



Type III seesaw: mass spectrum

m_T arbitrary, from minimal SU(5) $\lesssim 10^{3.5}$ GeV

$T^\pm$ and T^0 ~degenerate $\Delta m_T \sim 160$ MeV

Type III seesaw: decays

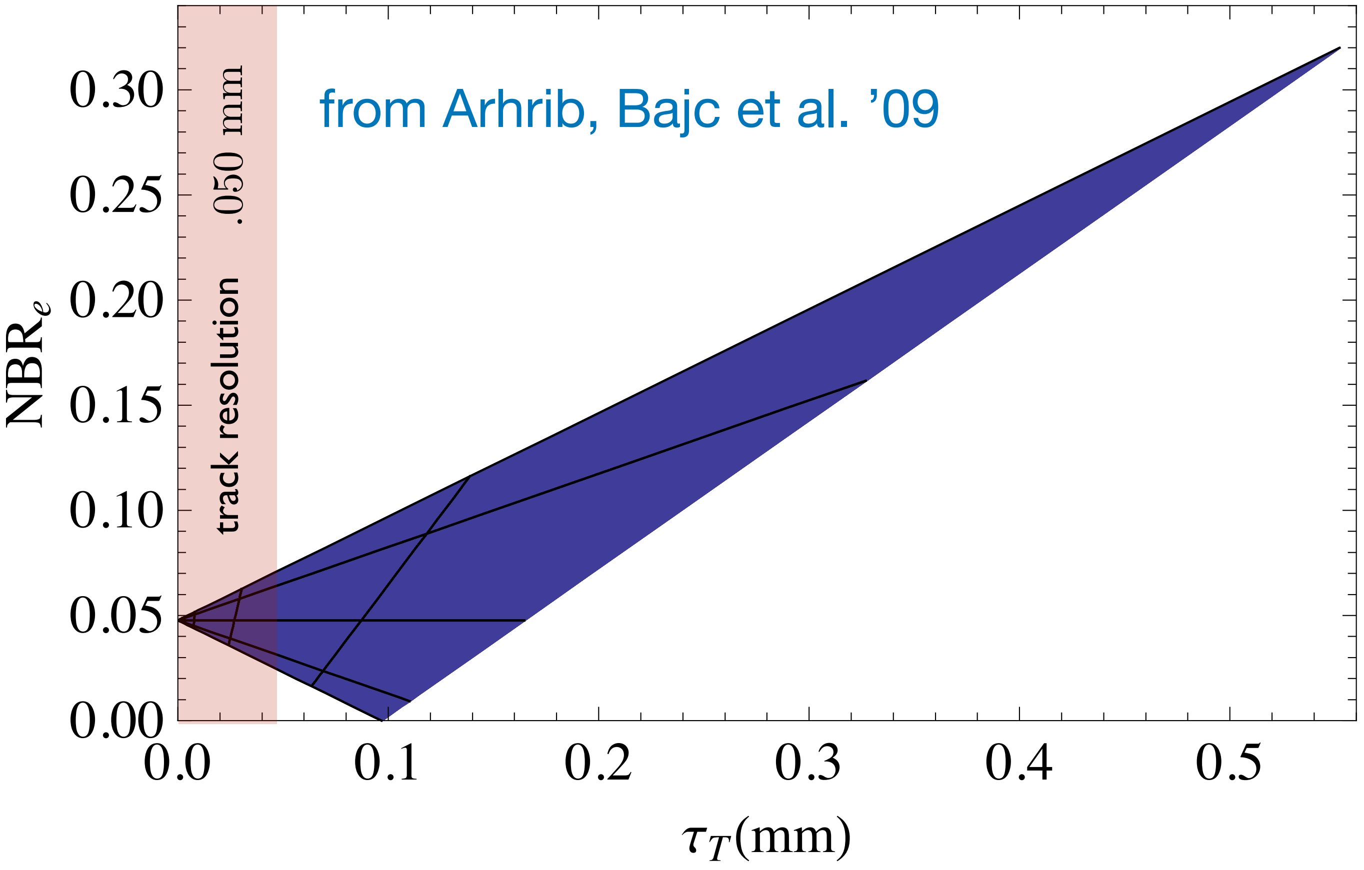
Bajc, MN, Senjanović '07

$$\left.\boxed{\begin{array}{c}\Gamma(T^- \to Z\ell^-)\\ \wr \\ \Gamma(T^0 \to W\ell)\end{array}}\begin{array}{l}\sim \frac{1}{2}\Gamma(T^- \to W^-\nu)\\ \\ \sim 2\Gamma(T^0 \to Z\nu)\end{array}\right\} \propto |y_T|^2$$

ambiguous, sensitive to Majorana phases and the hierarchy

$V \to jj$ explicit LNV

$$y_T^i = \frac{\sqrt{m_T}}{v}\begin{cases}U_{i2}\sqrt{m_2^\nu}\,c \pm U_{i3}\sqrt{m_3^\nu}\,s\\ U_{i1}\sqrt{m_1^\nu}\,c \pm U_{i2}\sqrt{m_2^\nu}\,s\end{cases}$$



from Arhrib, Bajc et al. '09

possibly displaced even in minimal I+III

cascades less important

$$\Gamma(T^\pm \to T^0\pi^\pm) < \Gamma(T \to V\ell)$$

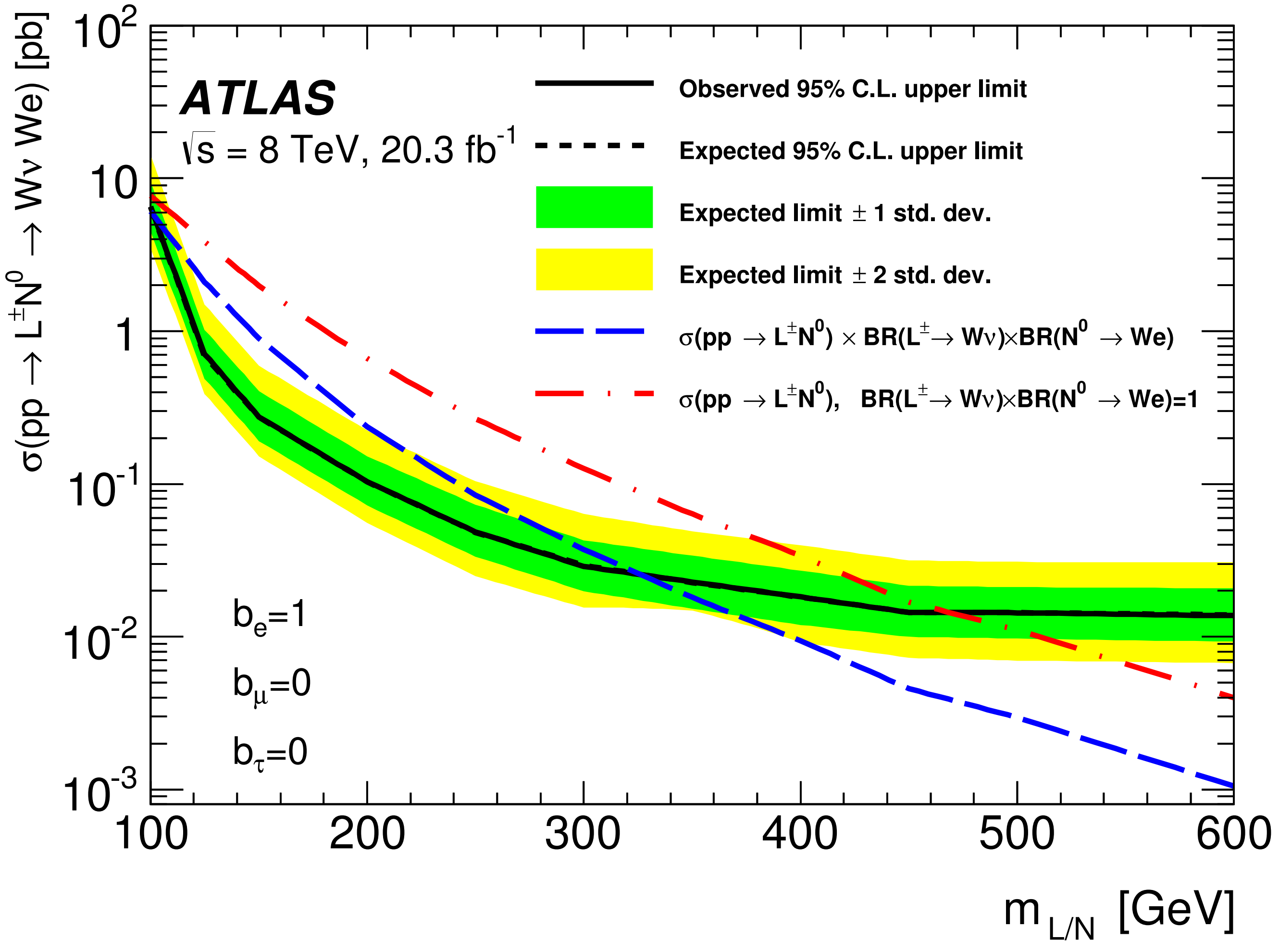
tri-leptons & seesaw comparisons

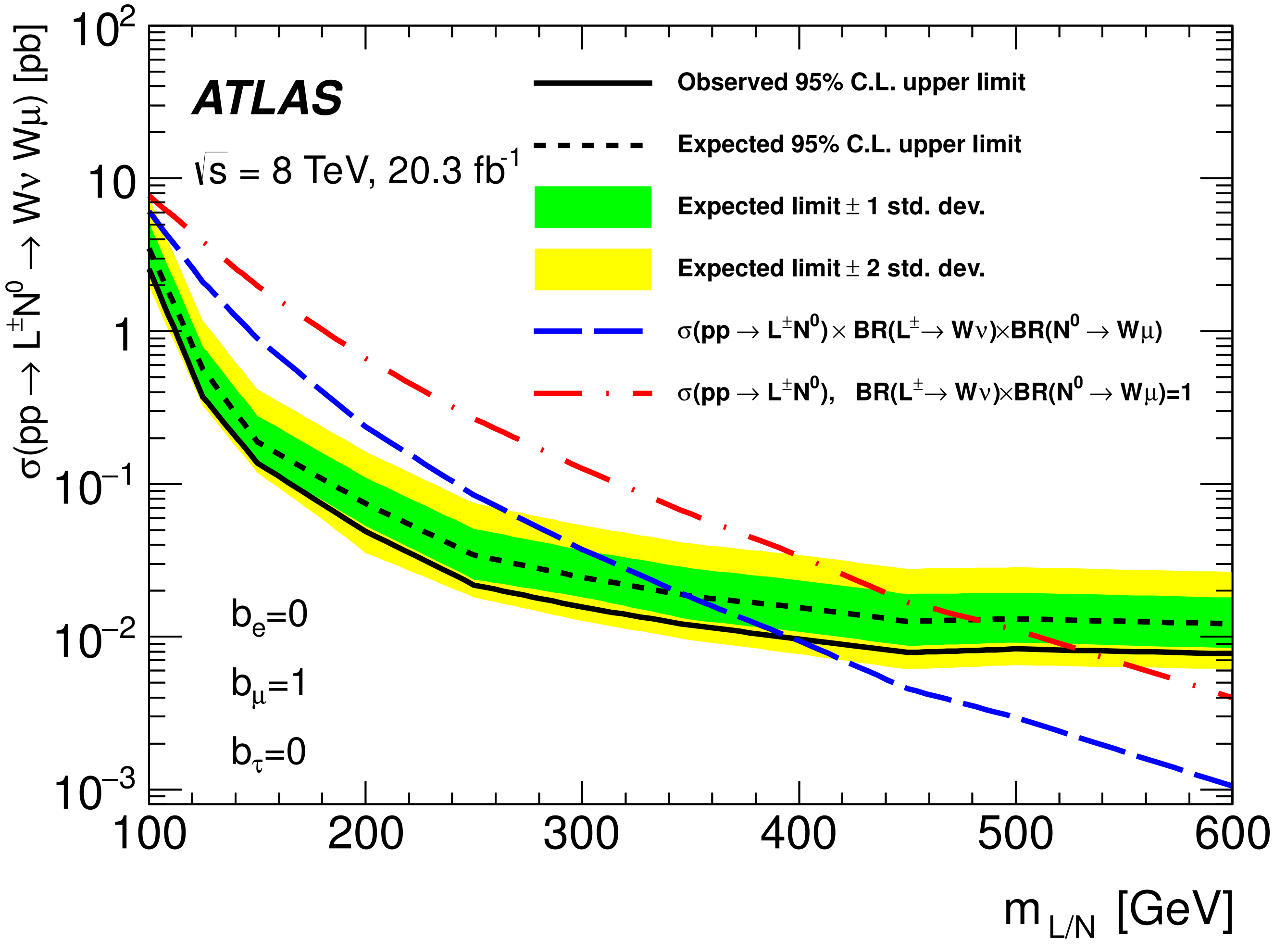
Del Aguila, Aguilar-Saavedra 0808.2468

Type III seesaw: decays

$$\left.\begin{array}{c}\Gamma(T^-\to Z\ell^-)\sim\frac{1}{2}\underline{\Gamma(T^-\to W^-\nu)}\\ \wr \\ \underline{\Gamma(T^0\to W\ell)}\sim 2\Gamma(T^0\to Z\nu)\end{array}\right\}\propto\left|y_T\right|^2$$

ATLAS 1506.01839

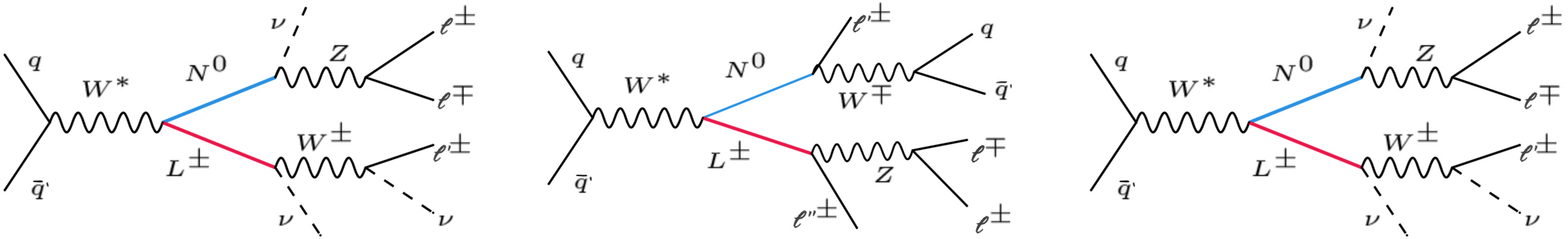


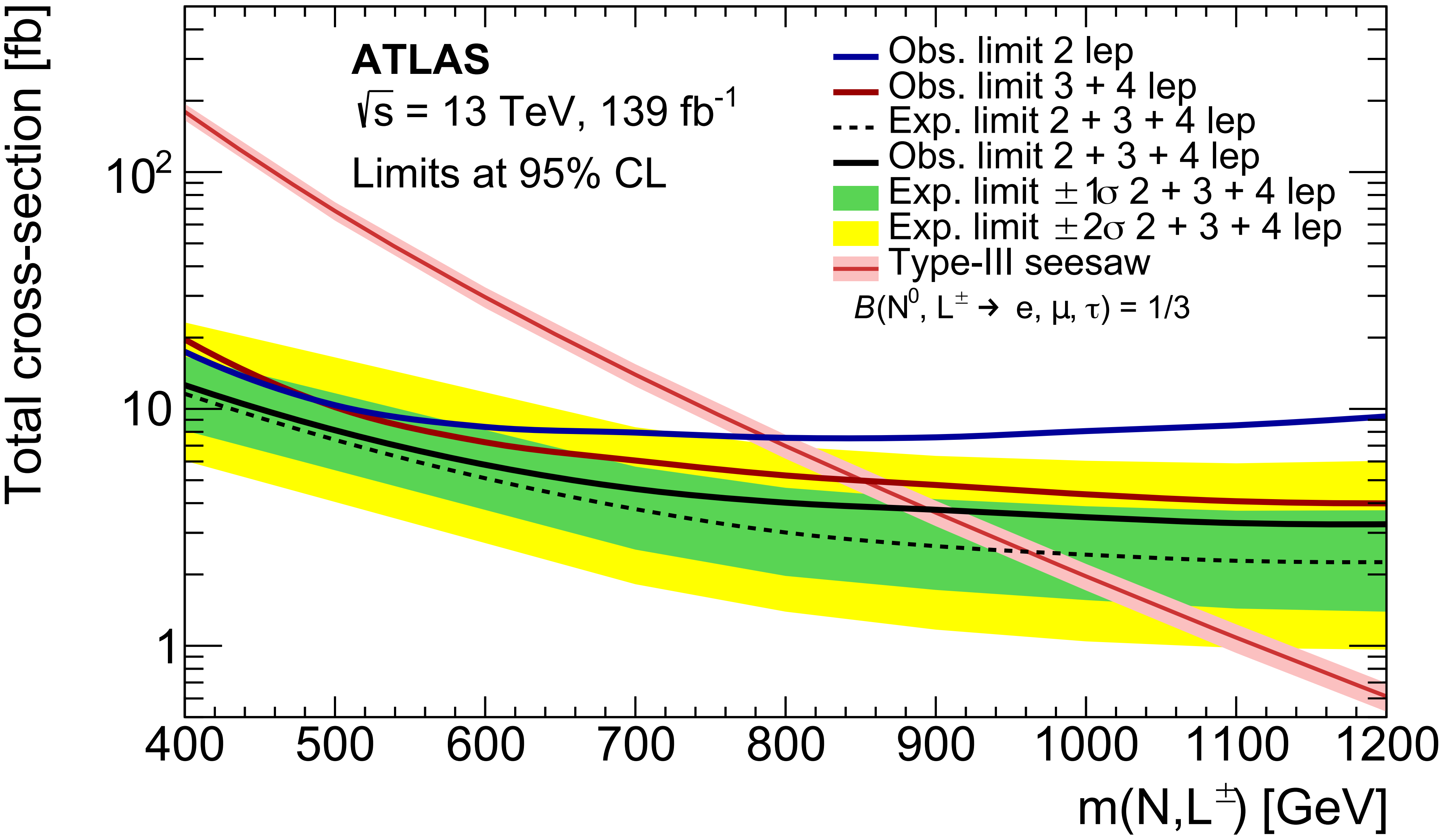


large Br, but $\not{E} \neq 0$ and LNV not obvious

Type III seesaw: trileptons and dileptons+jets

ATLAS 2202.02039





Type II seesaw

direct flavor relation with neutrino mass

involved phase space, limited experimental searches

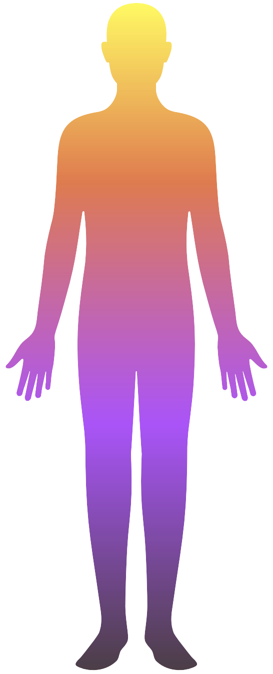
Type III seesaw

ambiguous relation to neutrino mass

simpler searches, bounds more robust

LNV ubiquitous, possibly displacement

Left-Right



weak force (a)symmetry

K-decay $K_L \to e\mu$

$$SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$$

$$M_{PS} \gtrsim 10^8 \text{ GeV}$$

$$L_R = \begin{pmatrix} N \\ \ell_R \end{pmatrix}$$

$$m_N \sim v_R$$

K & B oscillations

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

$$M_{W_R} \gtrsim 3-4 \text{ TeV}$$

The minimal Left-Right symmetric model

Minkowski ’77
Mohapatra, Senjanović ’80

$$\Delta_L(3,1,2),\ \Phi(2,2,0),\ \Delta_R(1,3,2) \qquad \langle\Phi\rangle = \begin{pmatrix} v_1 & 0 \\ 0 & e^{i\alpha} v_2 \end{pmatrix} \qquad \tan\beta = v_2/v_1$$

Parity / Charge

$$\mathcal{P}: \begin{cases} \psi_L \to \psi_R \\ \Phi \to \Phi^\dagger \end{cases} \qquad \mathcal{C}: \begin{cases} \psi_L \to \psi_R^c \\ \Phi \to \Phi^T \end{cases}$$

Maiezza, MN, Nesti,
Senjanović ’10

Yukawa sector

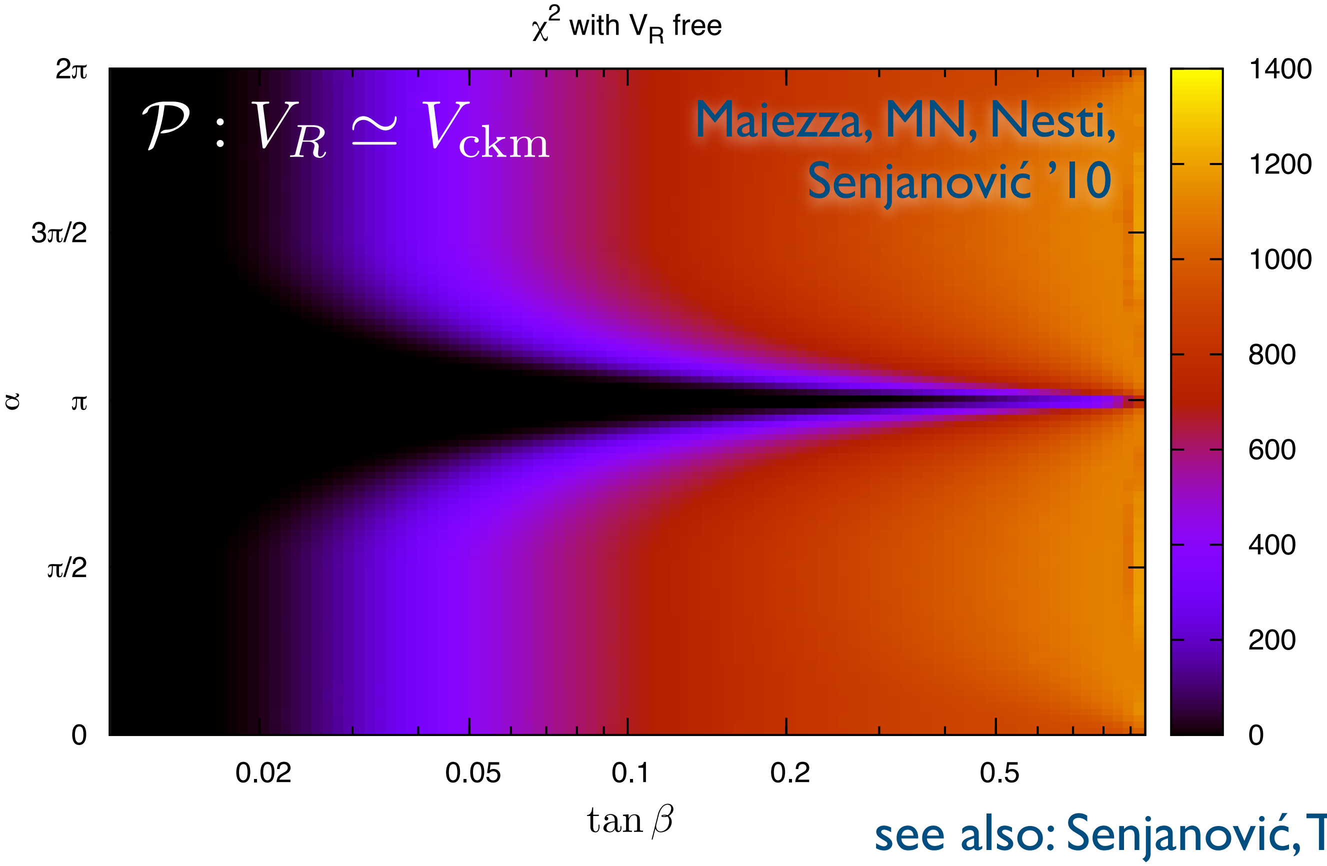
$$\mathcal{L}_Y \ni \overline{\psi}_L \left(Y\,\Phi + \tilde{Y}\,\tilde{\Phi} \right) \psi_R + \text{h.c.} \qquad \mathcal{P}: Y = Y^\dagger\,, \quad \mathcal{C}: Y = Y^T$$

quarks

$$M_u = v\left(Y c + \tilde{Y} s e^{-i\alpha}\right)$$

$$M_d = v\left(Y s e^{i\alpha} + \tilde{Y} c\right)$$

$$\mathcal{C}: V_R = K_u V_{\text{ckm}}^* K_d$$



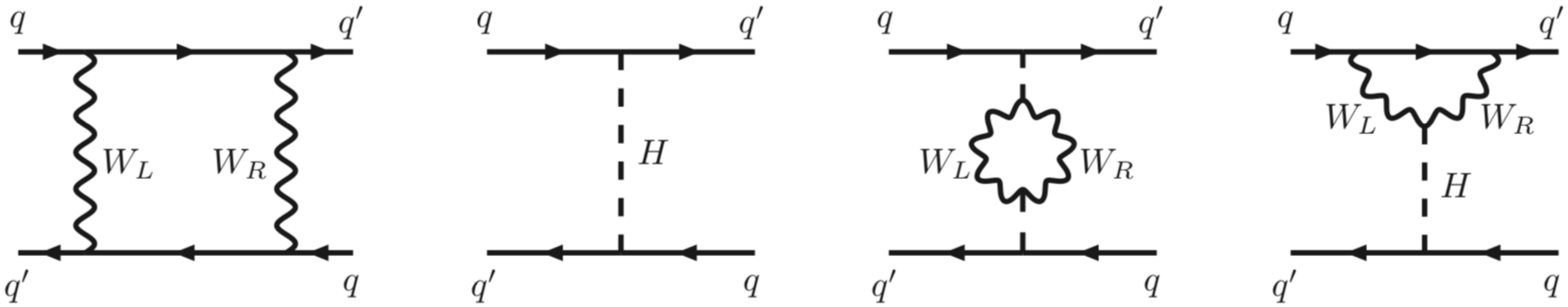
for $\mathcal{P}$

see also: Senjanović, Tello ’15, ’16

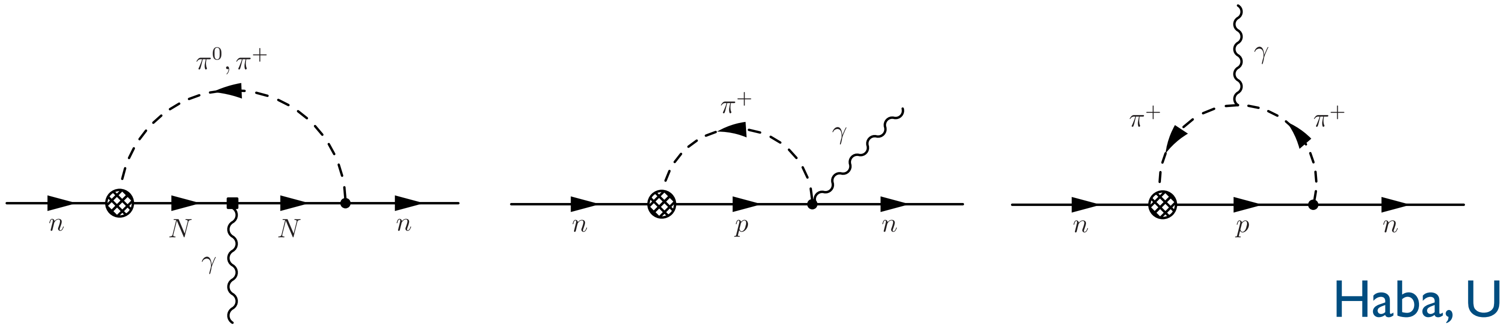
Flavor limits

Maiezza, Nesti, Bertolini ’14
Cirigliano, Dekens, de Vries, Mereghetti ’16

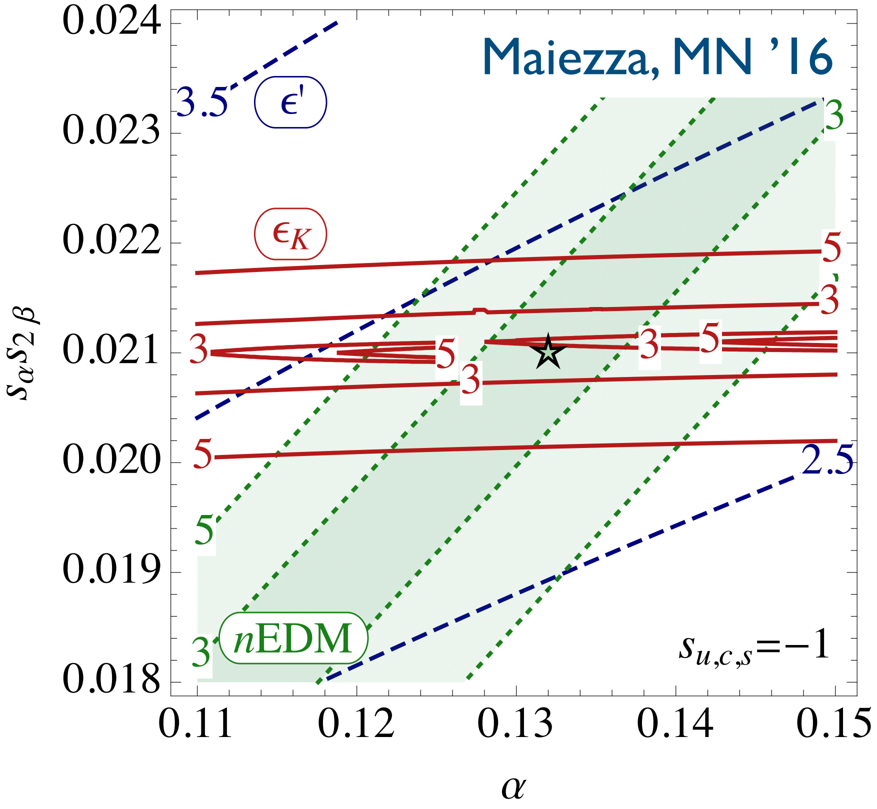
CP-even *K* and *B* meson mixing

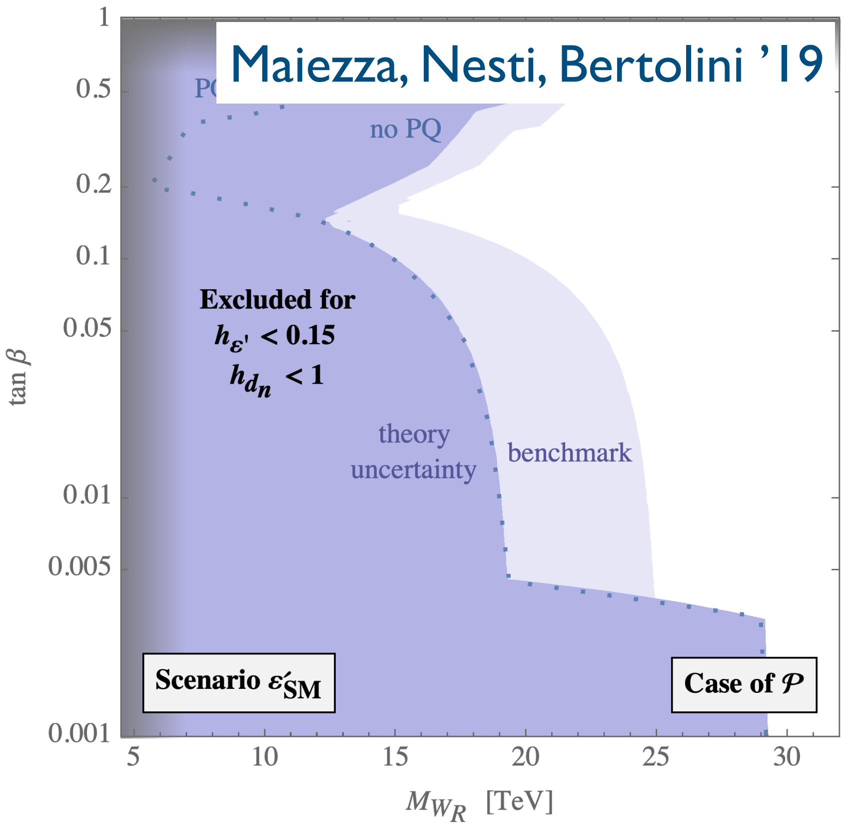


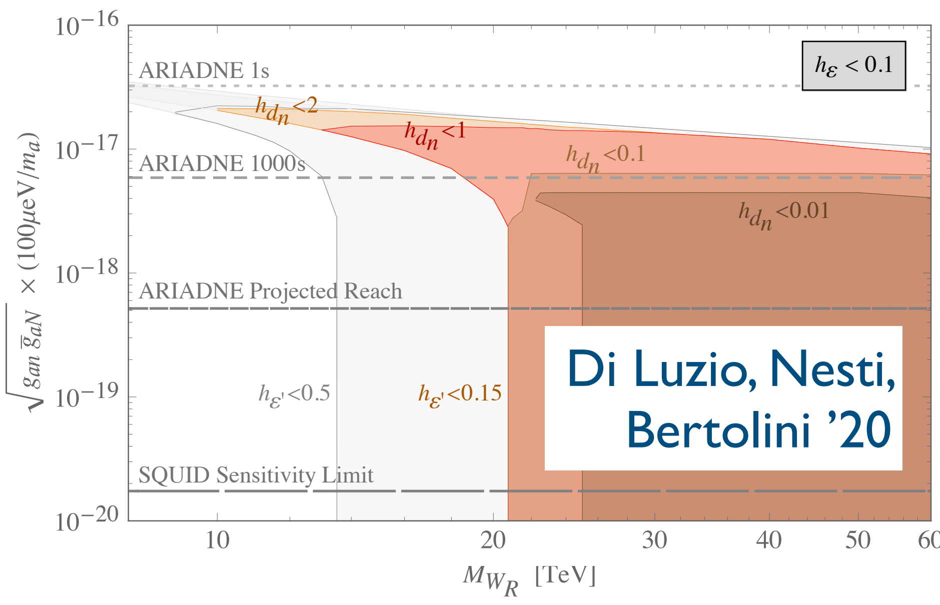
CP-odd $\varepsilon, \varepsilon'$ and nEDM



An, Ji, Xu ’09,
Maiezza, MN ’16,
Haba, Umeeda, Yamada ’17, ’18, ’18


Maiezza, MN ’16

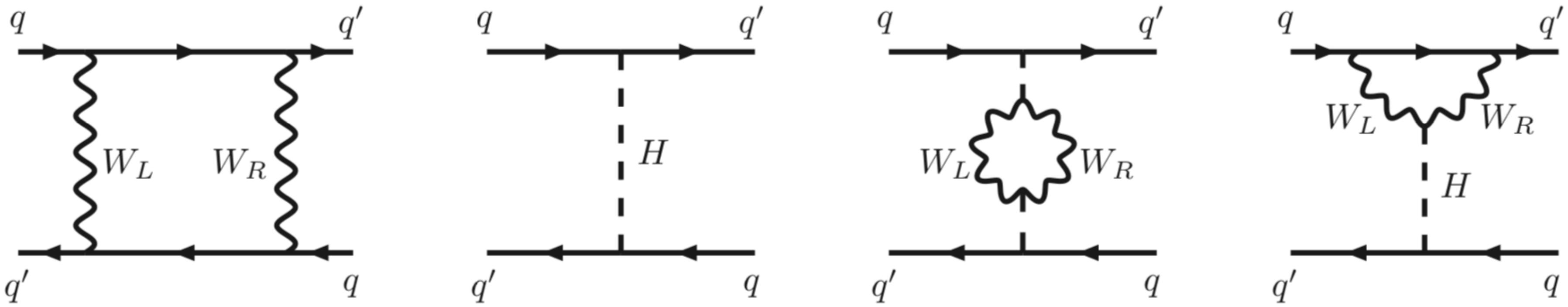

Maiezza, Nesti, Bertolini ’19


Di Luzio, Nesti, Bertolini ’20

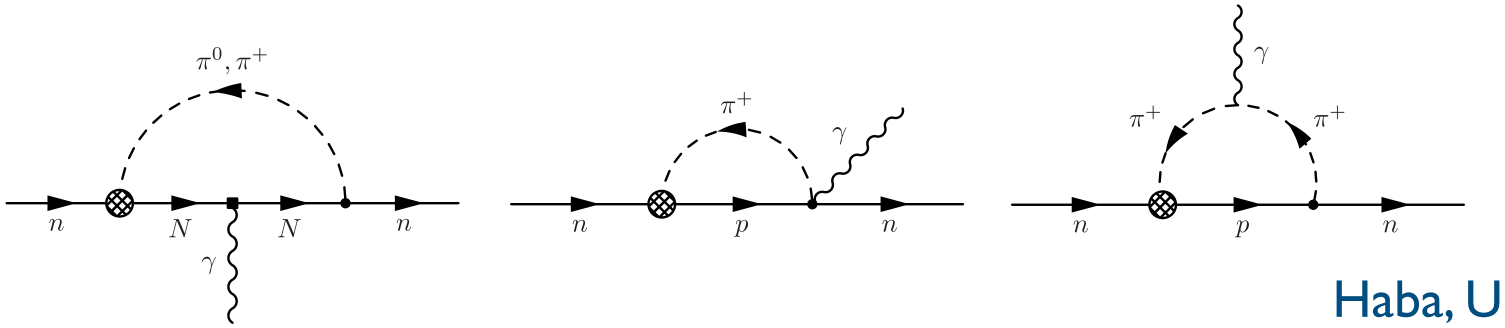
Flavor limits

Maiezza, Nesti, Bertolini ’14
Cirigliano, Dekens, de Vries, Mereghetti ’16

CP-even *K* and *B* meson mixing



CP-odd $\varepsilon, \varepsilon'$ and nEDM



An, Ji, Xu ’09,
Maiezza, MN ’16,
Haba, Umeeda, Yamada ’17, ’18, ’18

M_{W_R} limit in TeV

*Di Luzio, Nesti, Bertolini ’20

	Present	Future
C	3	8
P	12 (*16)	> 20
P and PQ	6 (*14)	> 20

Majorana meets Dirac

$$\langle \Delta_{L,R} \rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix} \qquad v_L \ll v_R$$

$$\mathcal{L}_Y \ni \overline{L}_L \left(Y \, \Phi + \tilde{Y} \, \tilde{\Phi} \right) L_R + Y_L L_L^T C \Delta_L L_L + Y_R L_R^T C \Delta_R L_R + \text{h.c.}$$

Seesaw $M_\nu = -M_D^T M_N^{-1} M_D + M_L$

$$\mathcal{C} : M_D^T = M_D , \quad M_L = \frac{v_L}{v_R} M_N$$

MN, Tello, Senjanović ’12

$\mathcal{P}$: Tello, Senjanović ’16, ’18

$$M_\nu = -M_D M_N^{-1} M_D + \frac{v_L}{v_R} M_N$$

Colliders, wDM, 0nu2b

No ambiguity

$$M_D = M_N \sqrt{\frac{v_L}{v_R} - M_N^{-1} M_\nu}$$

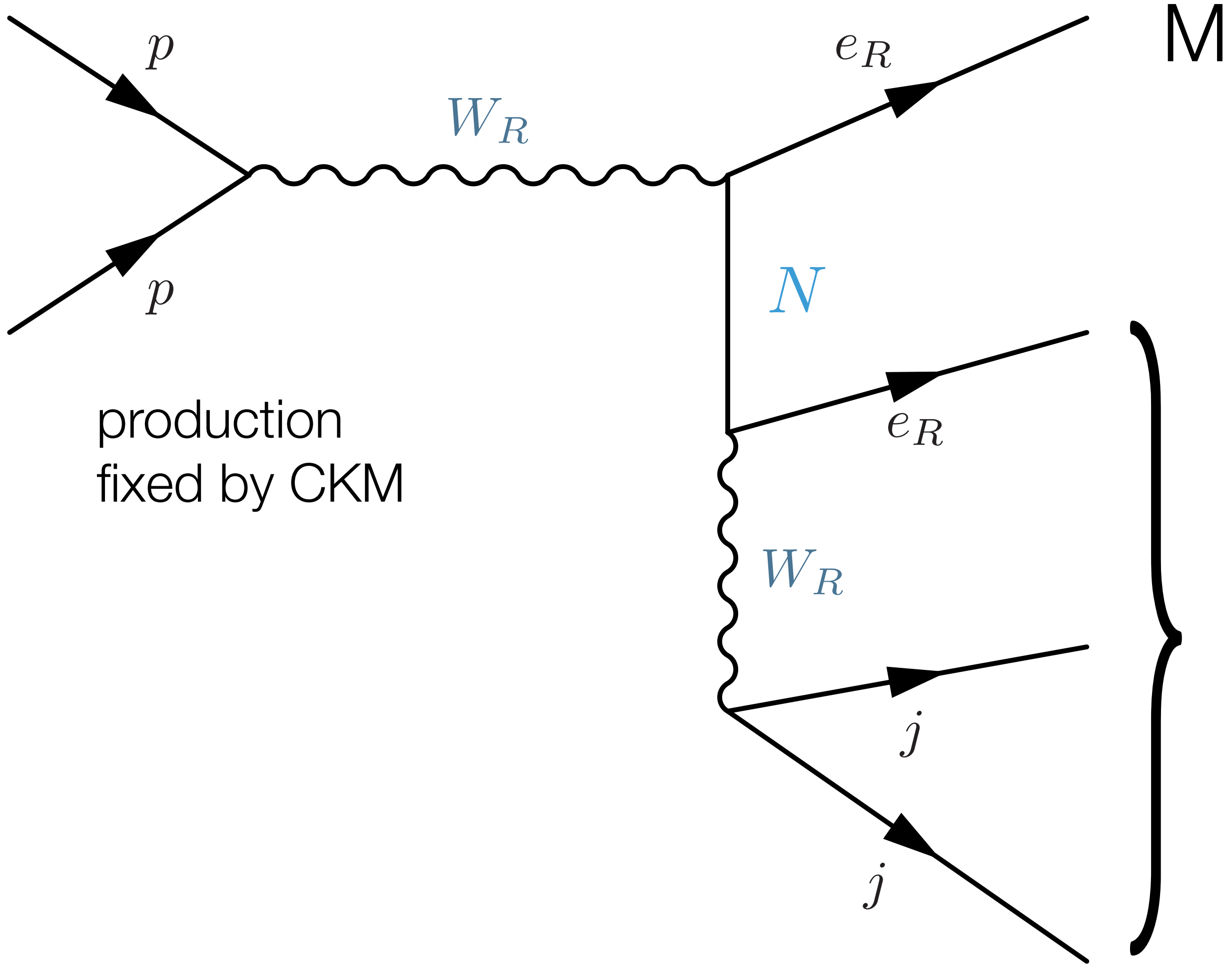
Dirac mass large with extreme fine-tuning

Colliders, eEDM, 0nu2b, X-rays,…

Oscillations, cosmology, KATRIN, CnuB, …

Collider probes

Keung, Senjanović ’83



production
fixed by CKM

Main feature: **L**epton **N**umber **V**iolation

On-shell Majorana fermion

$N \to \ell^{\pm} jj$ 50-50% same-opposite sign

$m_{\ell jj} = m_N$

narrow mass peaks for $m_N < M_{W_R}$

flavor states measure V_R (free) $M_N = V_R^* m_N V_R^\dagger$

~no missing energy

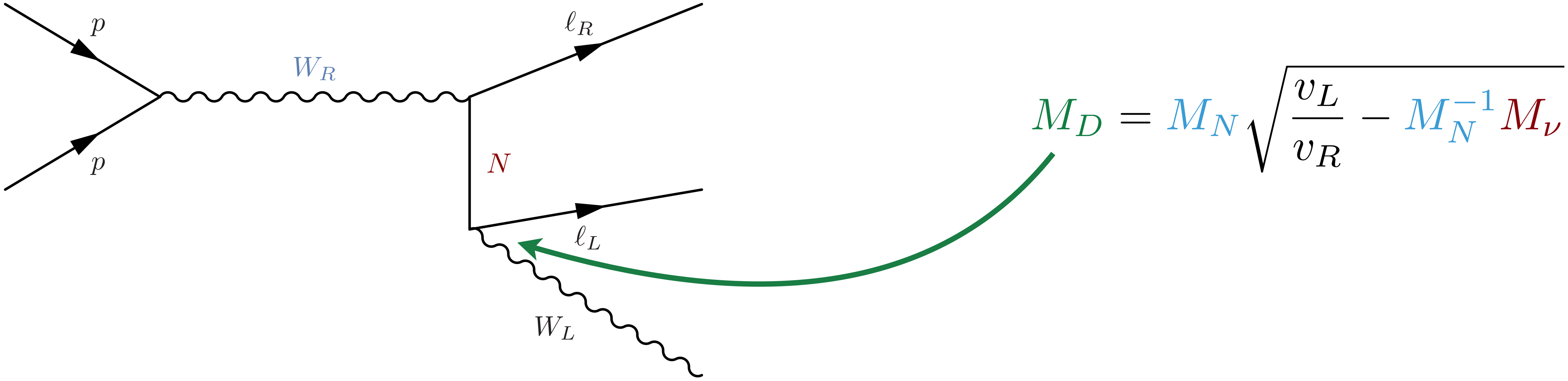
more on the Majorana nature

LNV vs. LNC states

Gluza, Jelinski ’15 ’16, Das, Dev, Mohapatra ’17, Godbole et al. ’20

Collider signatures of Dirac masses

MN, Senjanović, Tello, '12



$$M_D = M_N \sqrt{\frac{v_L}{v_R} - M_N^{-1} M_\nu}$$

Sub-dominant decays, needs high luminosity

Helo, Li, Neill, Ramsey-Musolf, Vasquez '18

Six flavor channels for M_D

Polarimetry Han, Luiz, Ruiz, Si '12

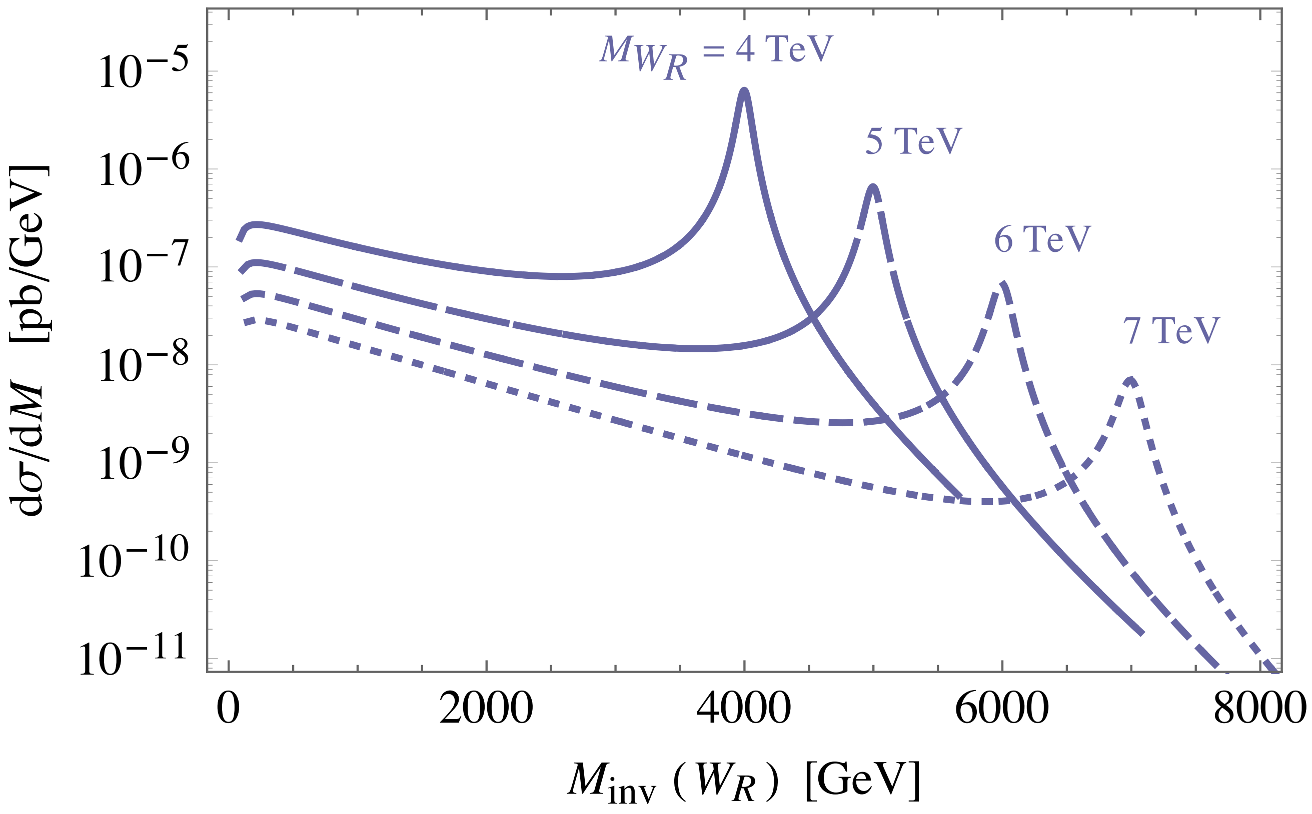
Golden channel: $pp \to W_R \to \ell_R N$

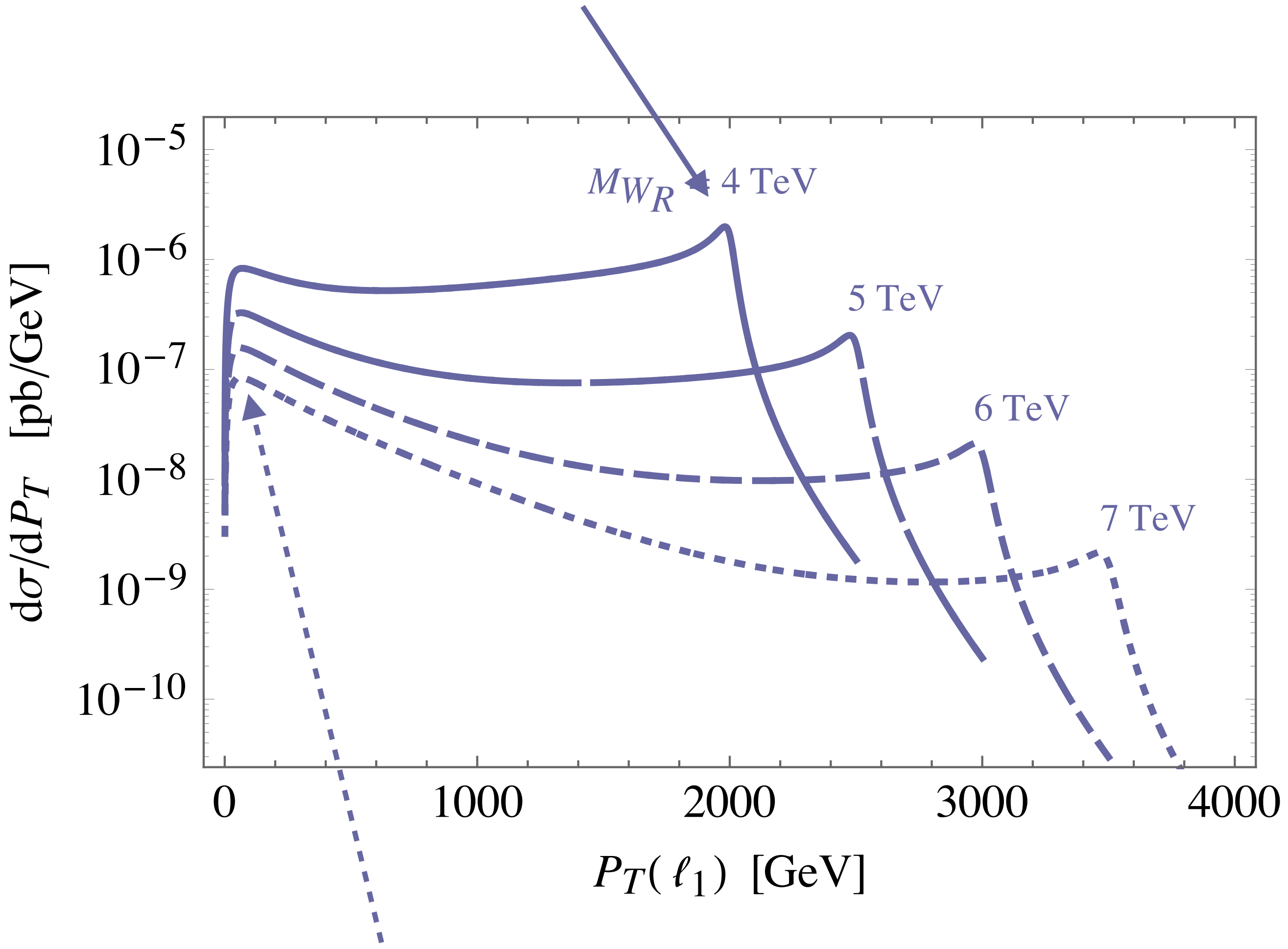
Keung, Senjanović '83

$$\hat{\sigma}_{ij}^{\ell N}(\hat{s}) = \frac{\alpha_2^2 \pi}{72 \hat{s}^2} \left| V_{ij}^{\mathrm{CKM}} \right|^2 \frac{\left(\hat{s} - m_N^2\right)^2 \left(2\hat{s} + m_N^2\right)}{\left(\hat{s} - M_{W_R}^2\right)^2 + M_{W_R}^2 \Gamma_{W_R}^2}$$

clear peaks

mostly on-shell, N boosted





$\mathrm{m}_{\mathrm{inv}}$ disappears

off-shell = soft lepton and N

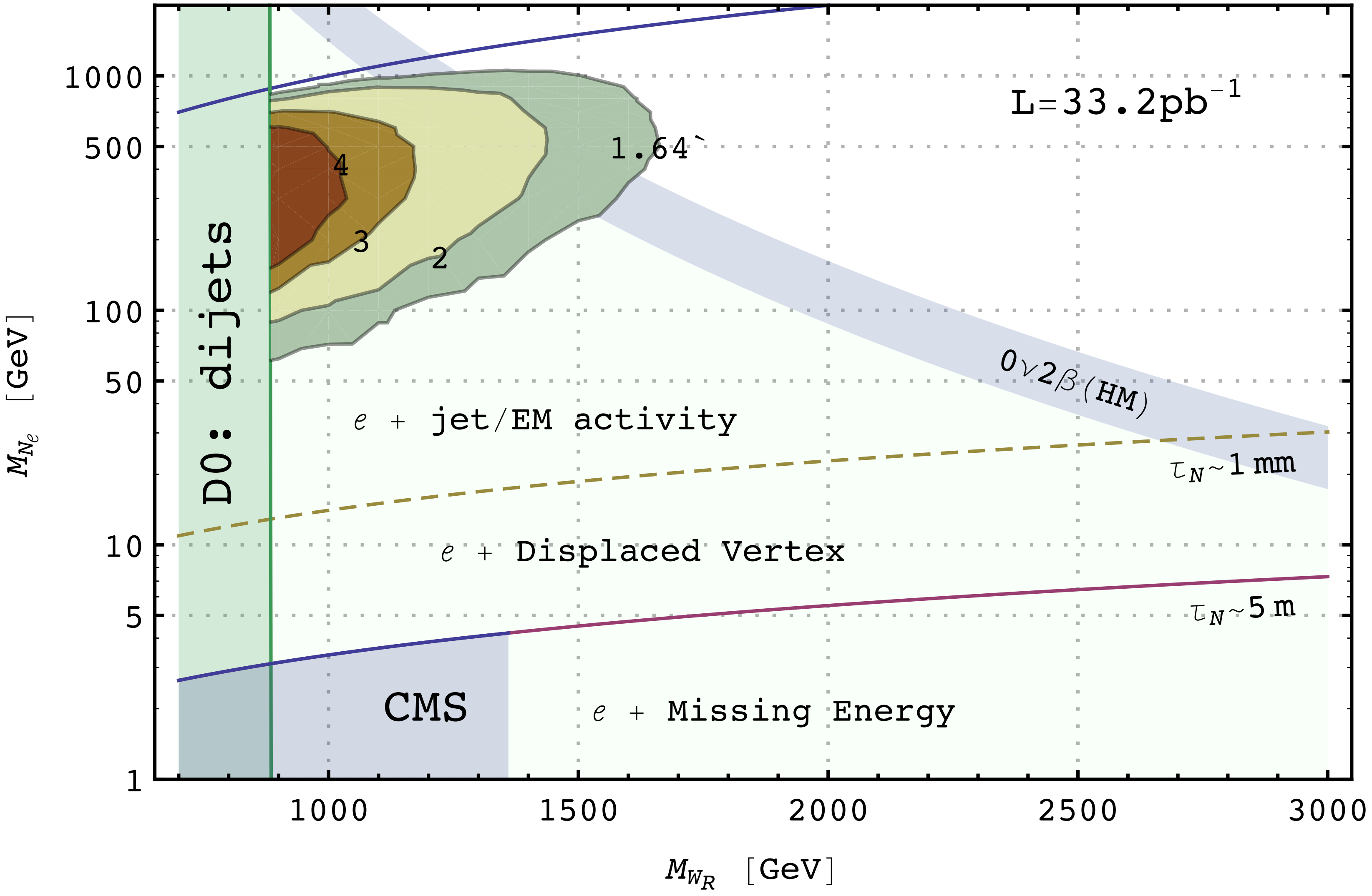
Ruiz '17

Sketch of a search : $pp \to W_R \to \ell_R N$

MN, Nesti, Senjanović, Zhang '11

separated
eejj

first LHC data,
low bound



merged
neutrino jet

Mattelaer, Mitra,
Ruiz '16

LNV relation to
0ν2β

displaced
jet

missing
energy

Reach of 5-6 TeV at 14 TeV

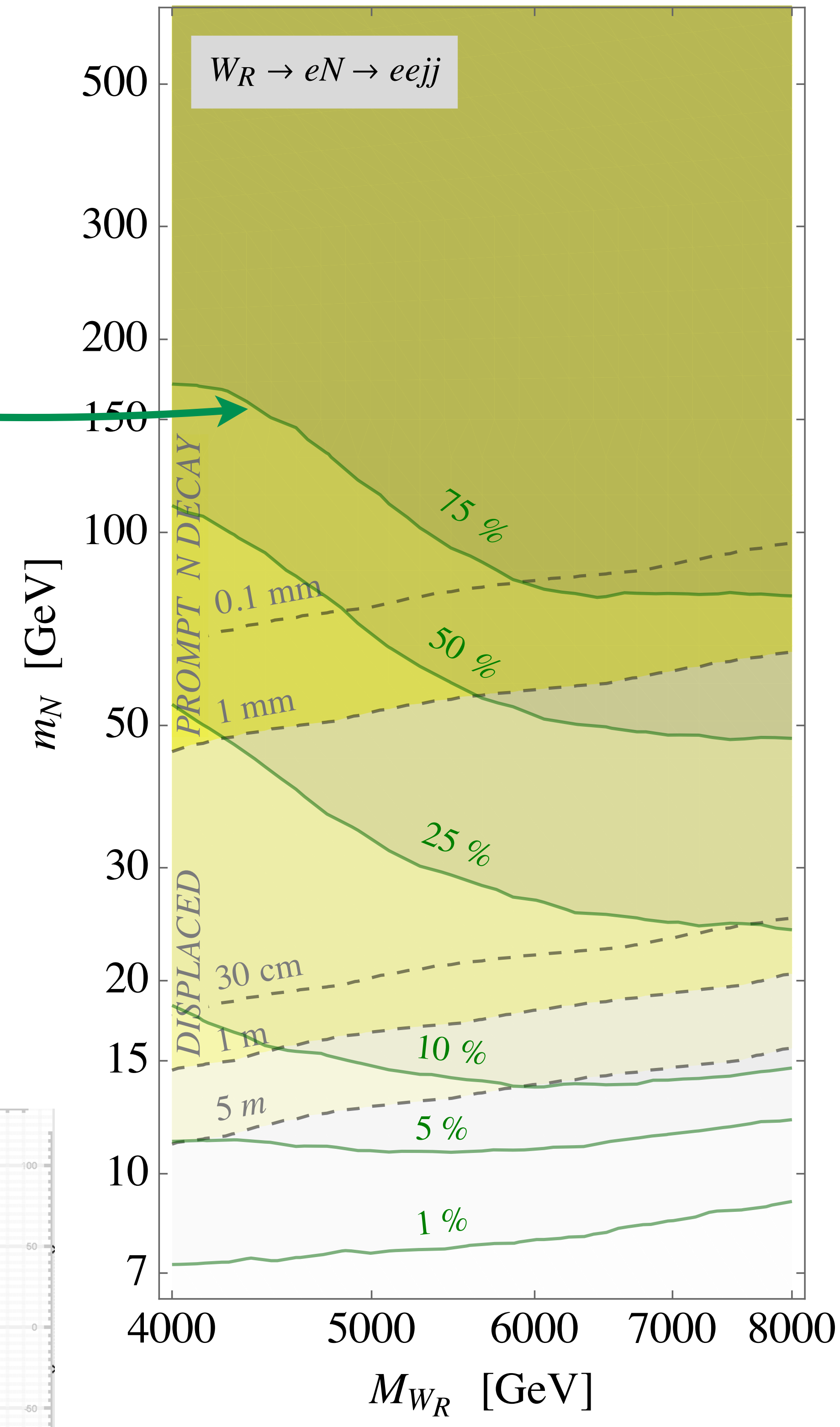
ATLAS: Ferrari et al. '00
CMS: Gninenko et al. '07

Isolation and displacement $pp \to W_R \to \ell_R N$

MN, Nesti, Popara '18

2nd lepton isolation depends on the boost of *N*

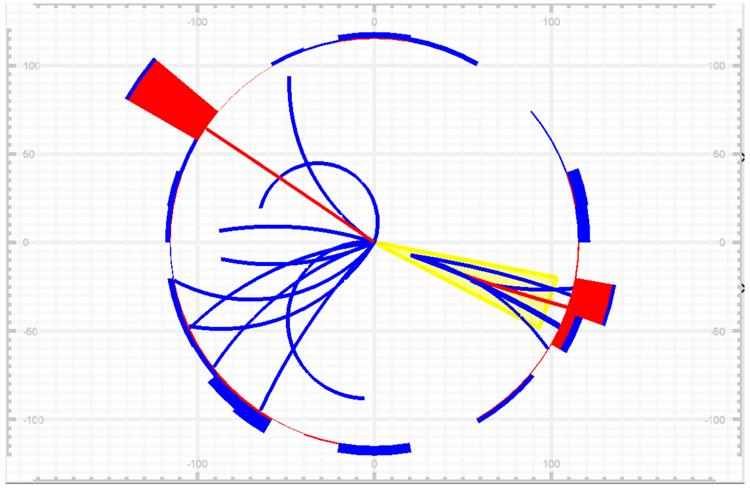
$$\gamma_N \simeq \begin{cases} \frac{M_{W_R}}{2m_N}, & W_R \to \text{on-shell}, \\ \frac{1\ \text{TeV}}{m_N}, & W_R \to \text{off-shell} \end{cases}$$



Lab decay length very sensitive to m_N

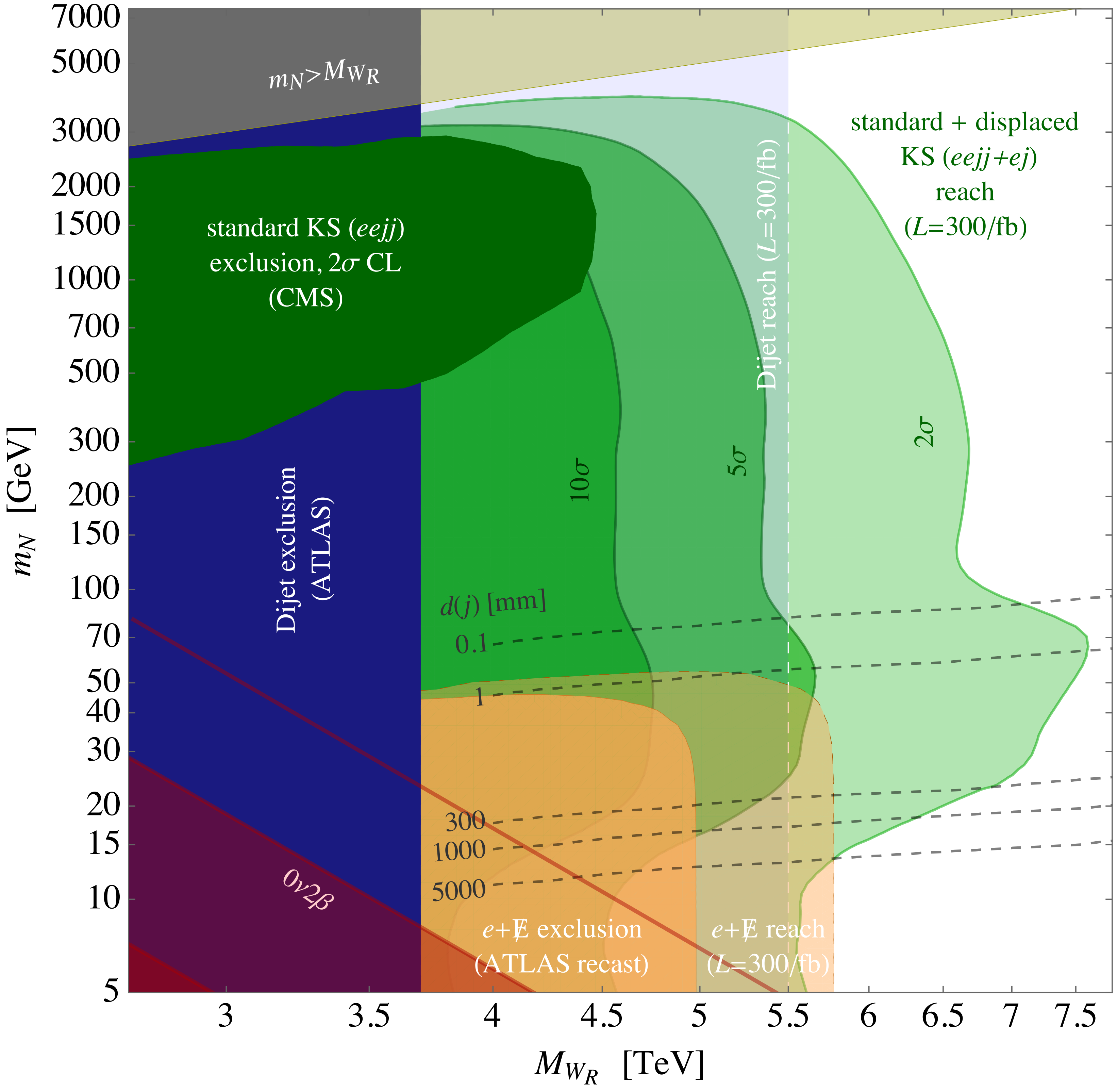
$$\Gamma_N^0 \sim \frac{\alpha_2^2 m_N^5}{64\pi M_{W_R}^4} \simeq \frac{1}{2.5\,\text{mm}} \frac{(m_N/10\ \ \text{GeV})^5}{(M_{W_R}/3\ \ \text{TeV})^4}$$

Simultaneous transition from prompt isolated to displaced merged - look for displaced merged jets (tracks)



Search overview $pp \to W_R \to \ell_R N$

MN, Nesti, Popara '18



standard prompt isolated mode

Ng et al.'15, Ruiz '17

merged neutrino jet $\ell\, j_N$

Mitra, Ruiz, Spannowsky '16

displaced jet $\ell\, j_N^d$

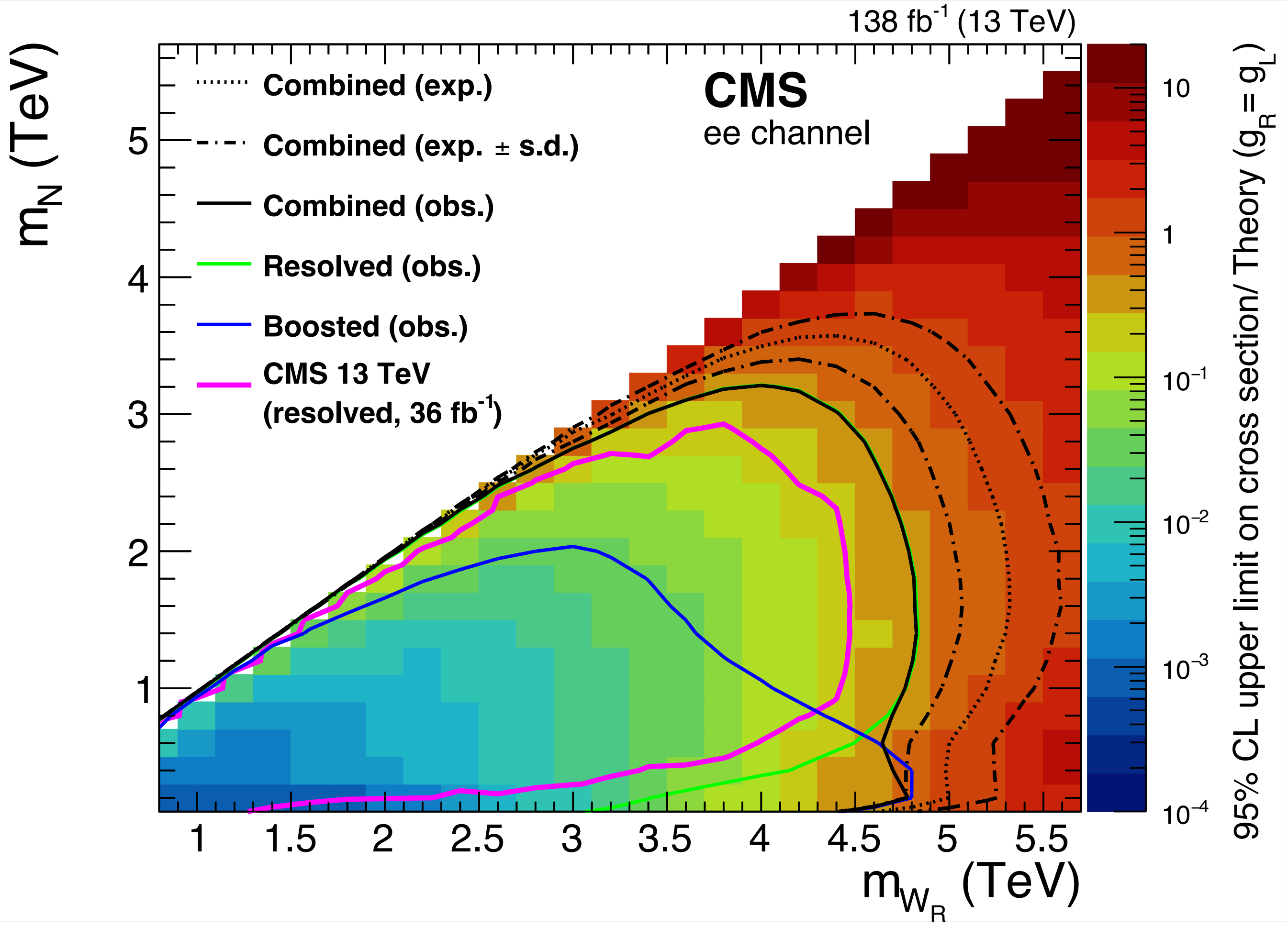
Cottin, Helo, Hirsch '18

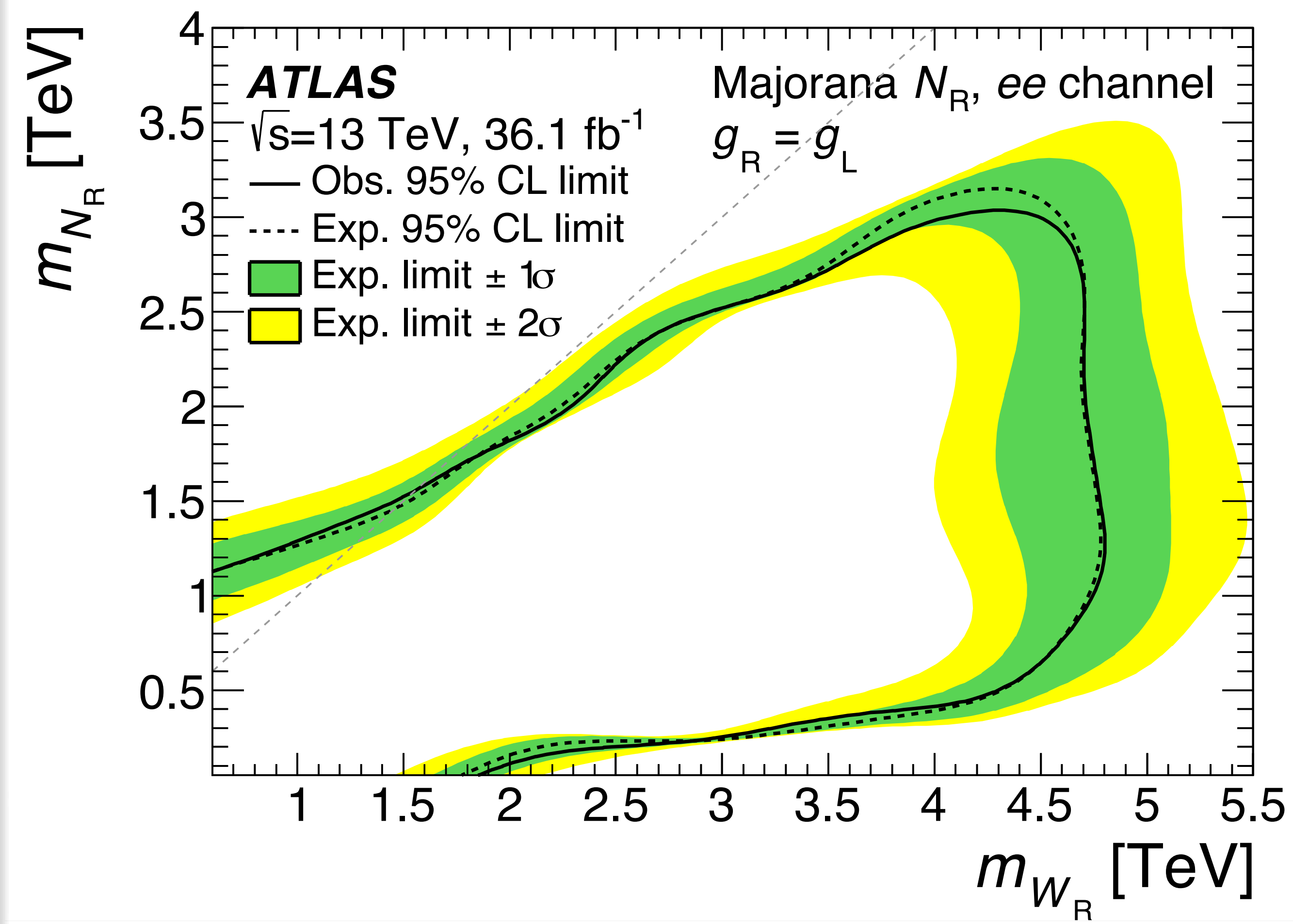
Cottin, Helo, Hirsch, Silva '19

invisible: prompt $\ell + E_{miss}$

relevant for any light N search (SHIP, FASER, MATHUSLA, etc.)

Resolved leptons and jets: search status in 2023



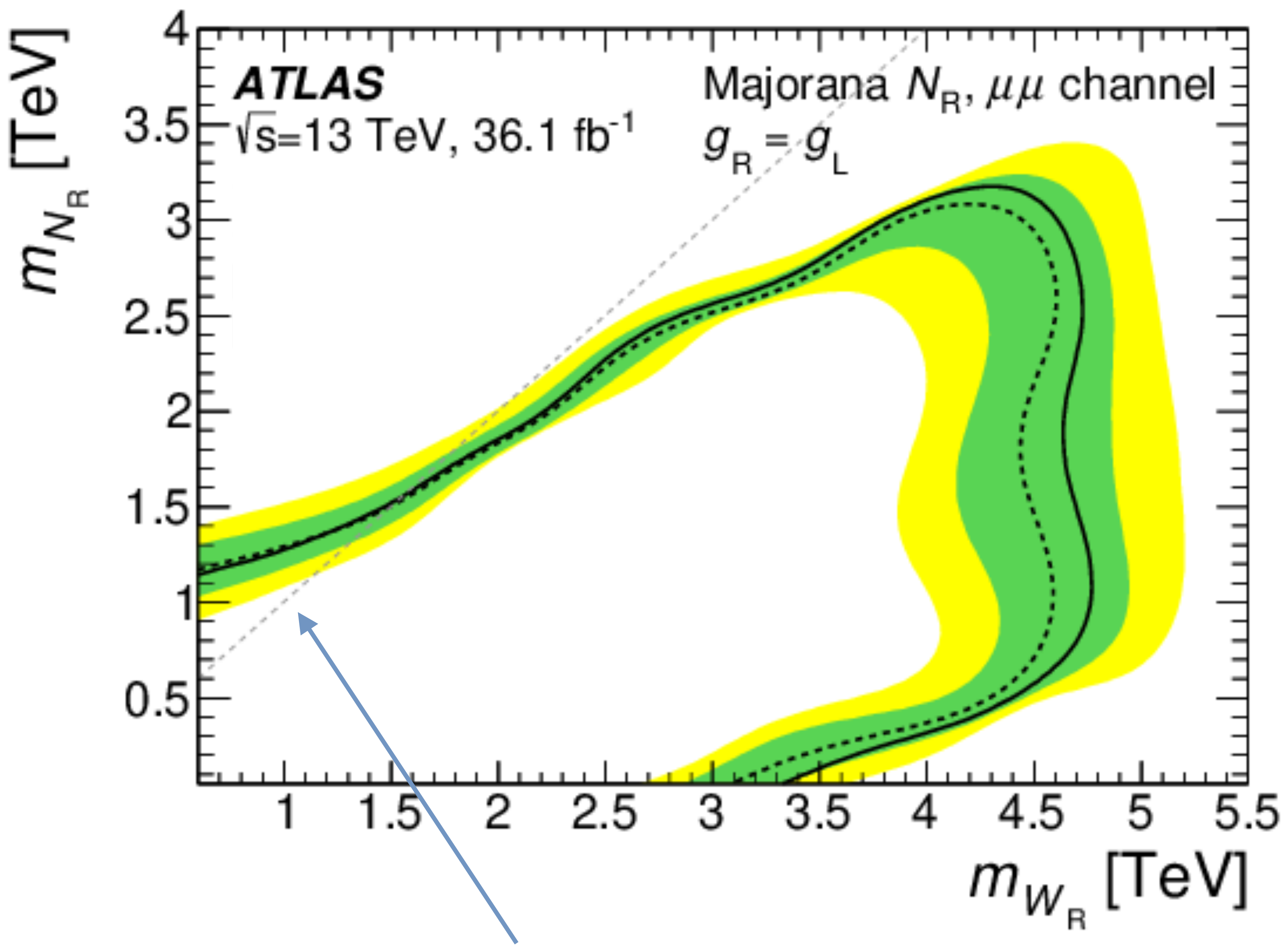


similar for muons

CMS 2112.03949

ATLAS 1809.11105

Experimental limits review in 2023



probes the off-shell N, too

standard prompt isolated mode

*e, mu	4.8 TeV	ATLAS 1809.11105
tau	3.5 TeV	CMS 2112.03949

lepton + missing energy

*e, mu	$M_{W_R} \gtrsim 5.1(6)$ TeV	ATLAS 1906.05609
tau$_h$	$M_{W_R} > 3.7$ TeV	ATLAS 1801.06992

*interplay with $0\nu2\beta$

Mohapatra Senjanović ’79
Tello, MN, Nesti, Senjanović, Vissani ‘10

dijets

$M_{W_R} > 4$ TeV	ATLAS 1910.08447

tb

$M_{W_R} > 3$ TeV	ATLAS 1801.07893

di-boson WZ mode $\propto \xi_{LR}$

~ 4.8 TeV	ATLAS 2007.05293 CMS PAS B2G-20-009

LNV Higgses

$$\Delta_L(3,1,2),\ \Phi(2,2,0),\ \Delta_R(1,3,2)$$

Minkowski ’77
Mohapatra, Senjanović ’79

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \qquad \langle\Phi\rangle = \begin{pmatrix} v & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Delta_R = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}_R \quad \langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

SSB of parity

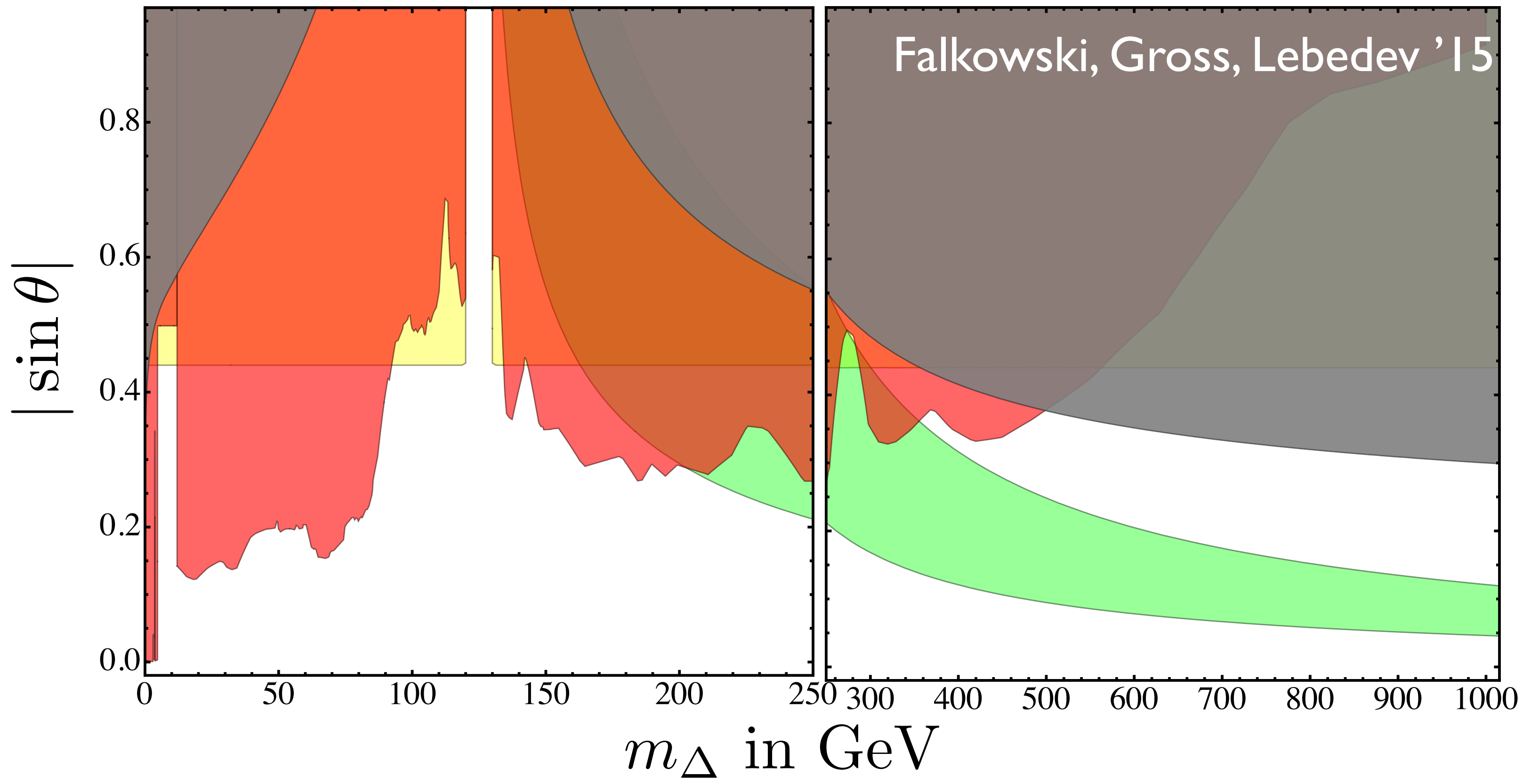
$$\mathcal{P}: \begin{cases} \Delta_L \leftrightarrow \Delta_R,\ \Phi \to \Phi^\dagger \\ Q_L \leftrightarrow Q_R,\ L_L \leftrightarrow L_R \end{cases}$$

Senjanović,
Mohapatra ’75

$$V \in \lambda\,(\Phi^\dagger\Phi)^2 + \alpha(\Phi^\dagger\Phi)(\Delta_R^\dagger\Delta_R) + \rho\,(\Delta_R^\dagger\Delta_R)^2$$

same for $\mathcal{C}$-symmetry

$h - \Delta$ mixing: $\theta \simeq \left(\frac{\alpha}{2\,\rho}\right)\left(\frac{v}{v_R}\right) \lesssim .44$



Future collider outlook

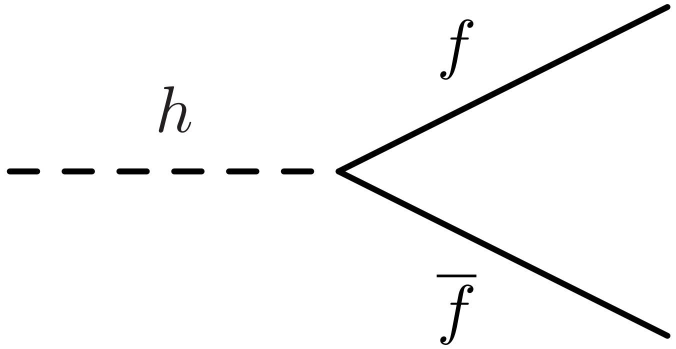
$$|\sin\theta| < .34$$

Buttazzo, Sala, Tesi ’15

SM Higgs and mass origin

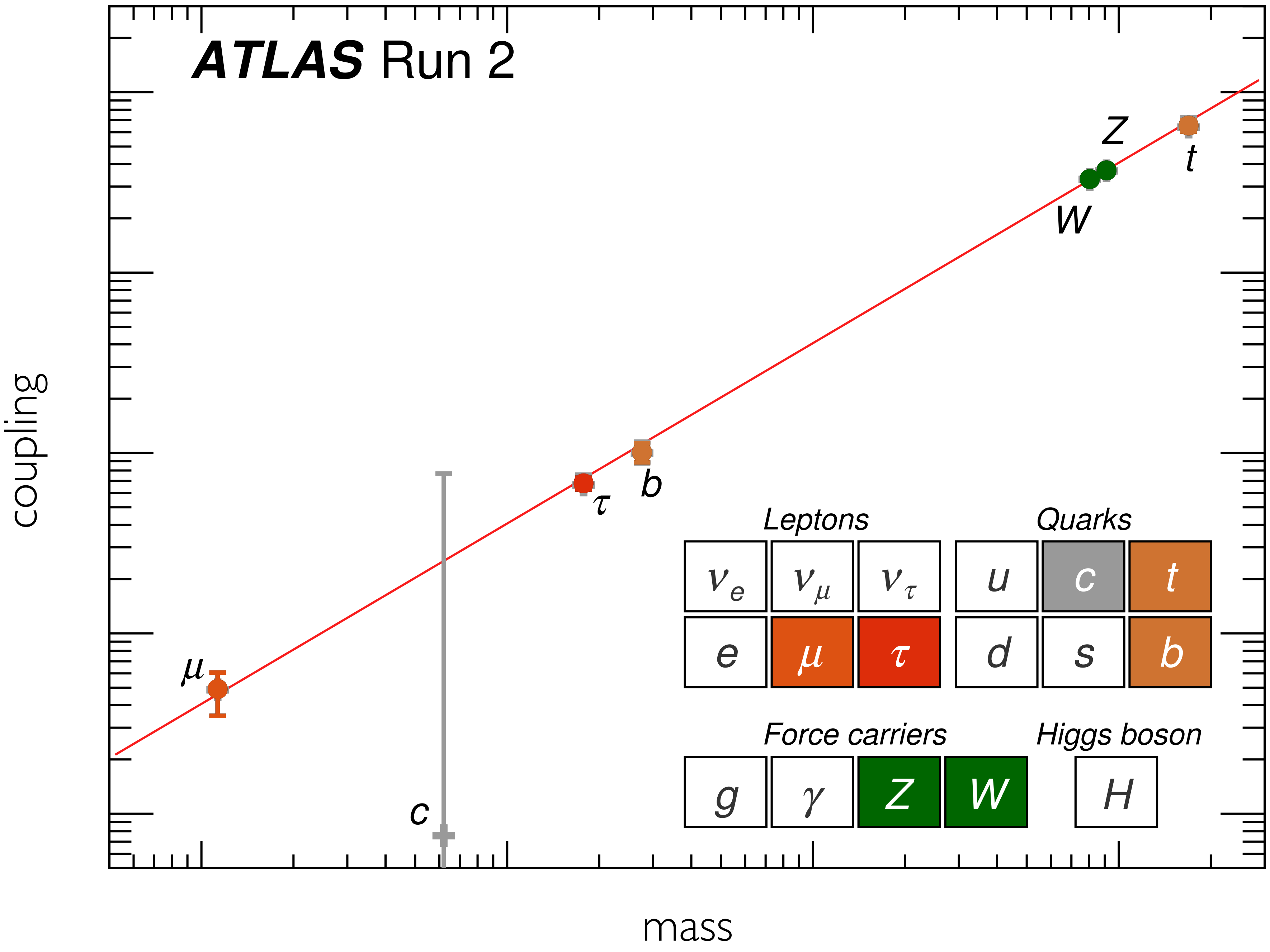
h decays

$$\Gamma_{h\to f\overline{f}} \propto c_\theta^2 m_f^2$$

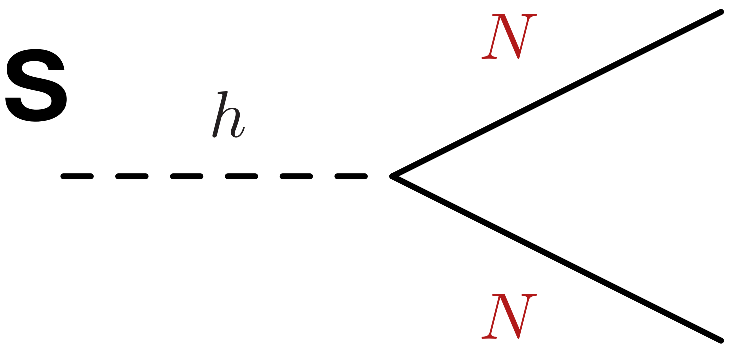


Weinberg ‘67

Dirac only



‘Majorana’ SM Higgs



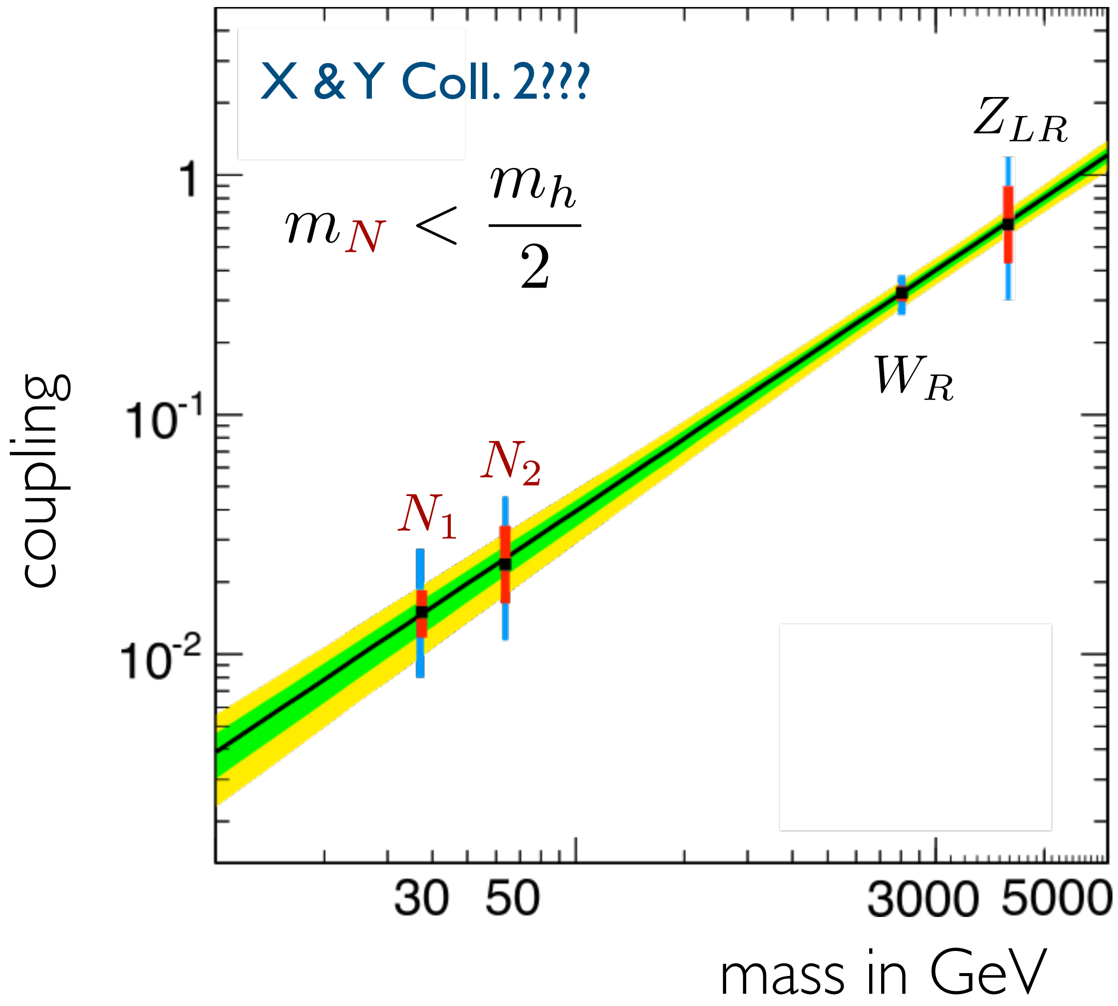
h decays

$$\Gamma_{h\to NN} \propto s_\theta^2\, m_N{}^2$$

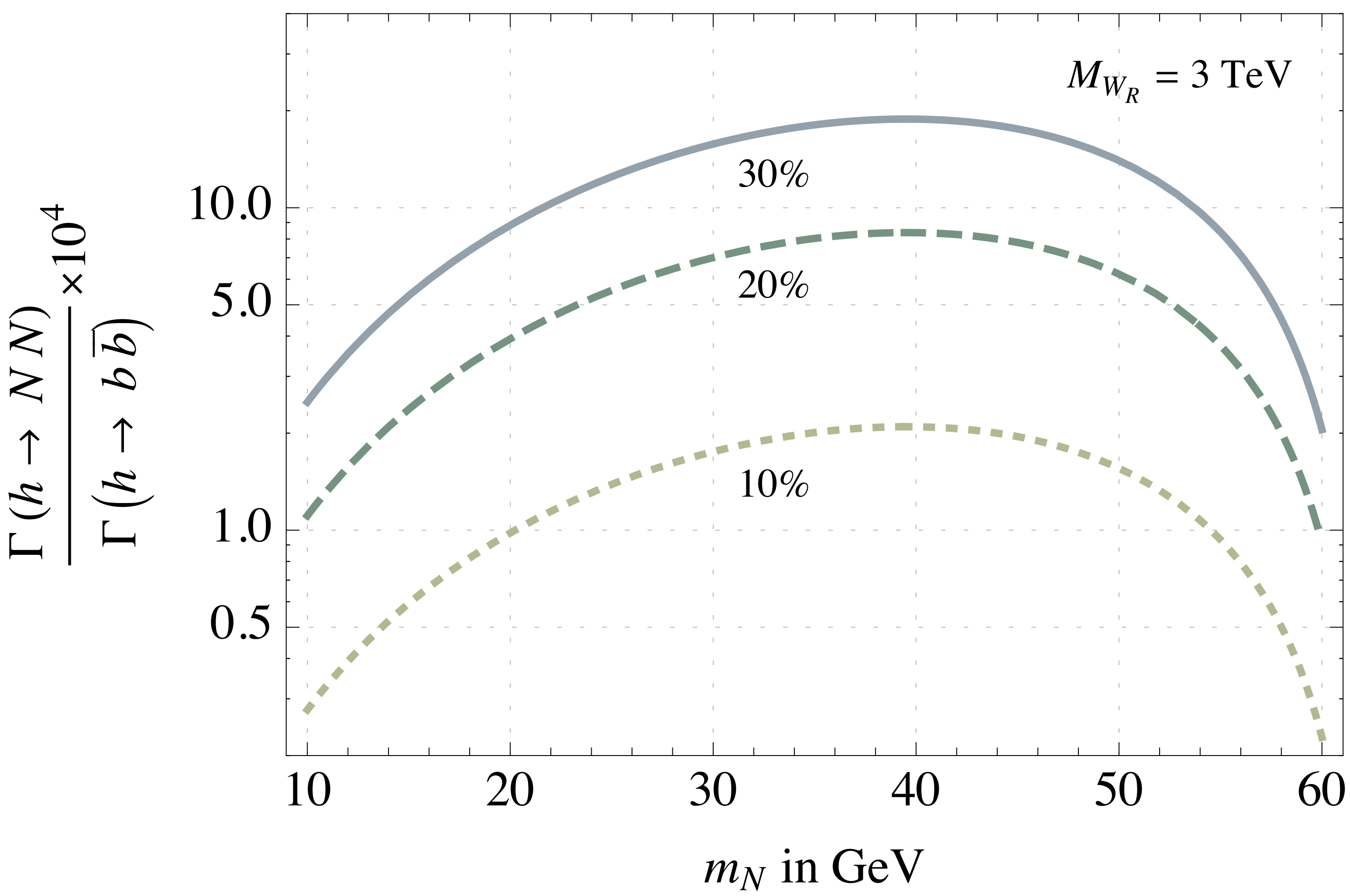
$$\frac{\Gamma_{h\to NN}}{\Gamma_{h\to b\bar{b}}} \simeq \frac{\theta^2}{3}\left(\frac{m_N}{m_b}\right)^2\left(\frac{M_W}{M_{W_R}}\right)^2$$

Gunion et al. Snowmass ’86

EFT SM+h+N Graesser ’07



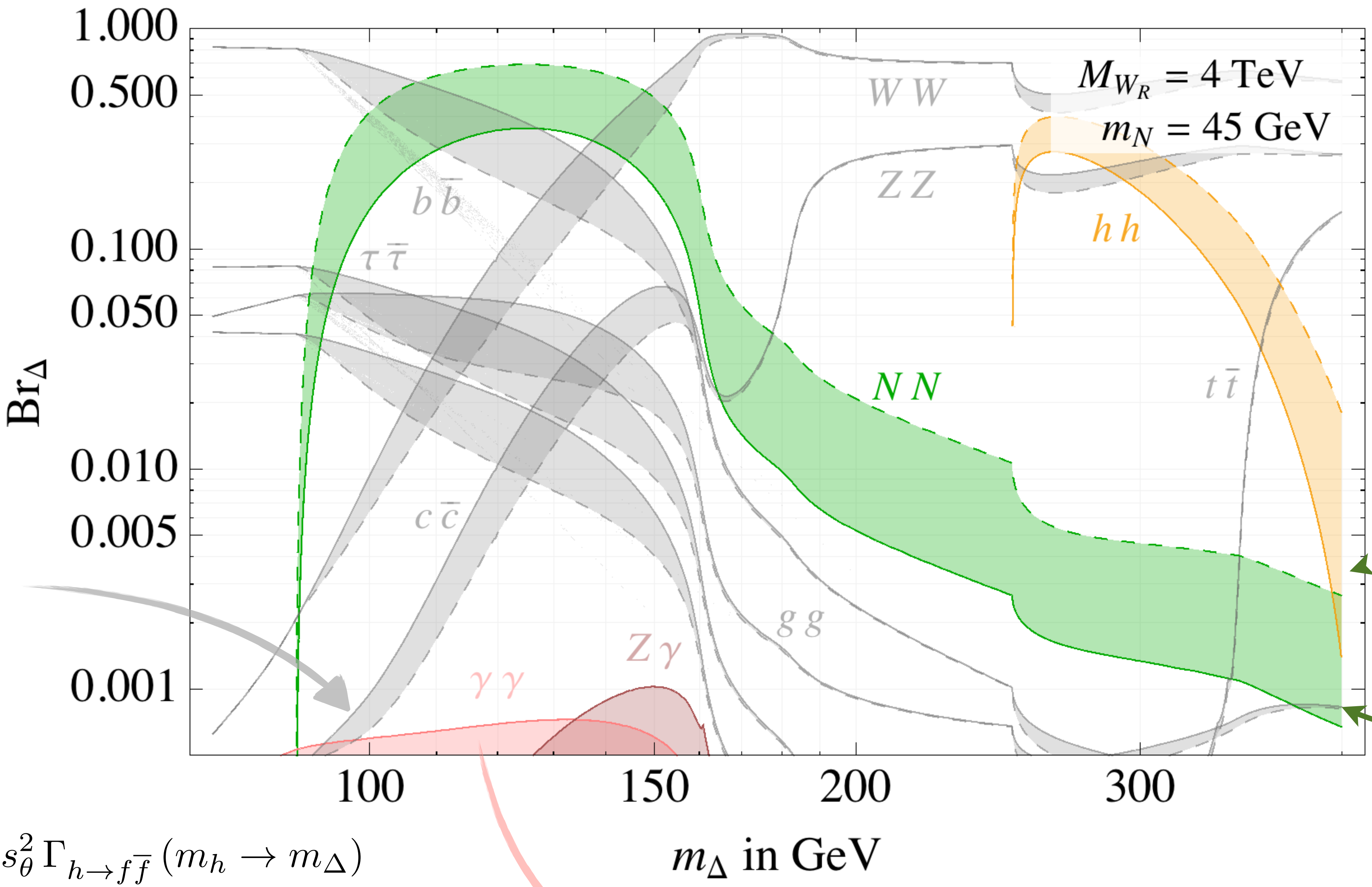
Maiezza, MN, Nesti ’15



‘Right-handed’ Higgs

Δ_R^0 decays

MN, Nesti, Vasquez ’16



to SM via mixing

$s_\theta = 5\%$

10%

$$\Gamma_{\Delta\to f\bar{f}} = s_\theta^2\,\Gamma_{h\to f\bar{f}}\,(m_h \to m_\Delta)$$

radiative loops

$(\mathrm{SM}, W_R, \Delta_{L,R}^{++})$

Displaced photons

Dev, Mohapatra, Zhang ’16

$$\Gamma_{\Delta\to\gamma\gamma} = \frac{m_\Delta^3}{64\pi}\left(\frac{\alpha}{4\pi}\right)^2 |F_\Delta|^2$$

'Right-handed' Higgs

Δ decays

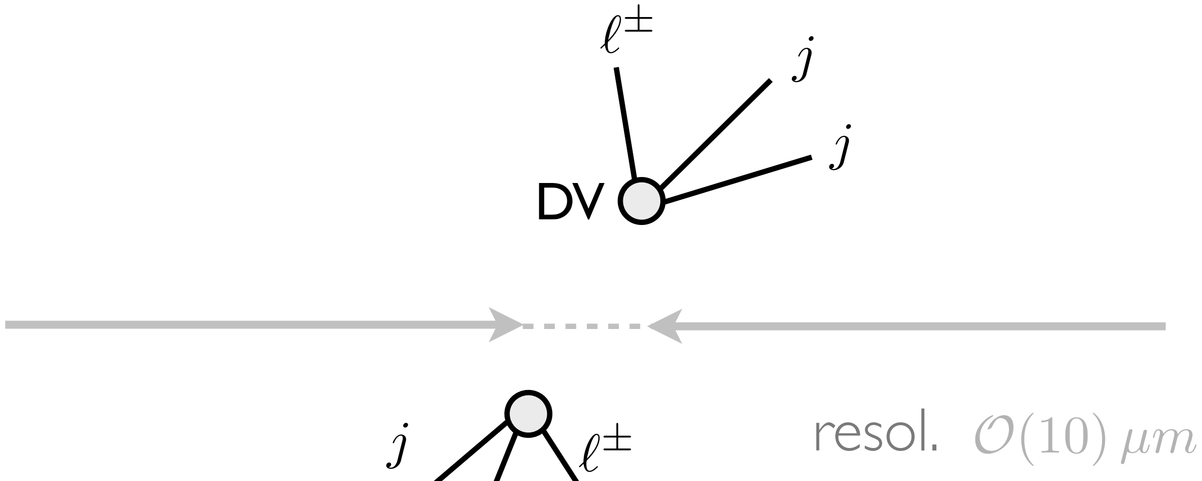
Region of interest for $\Delta \to NN$

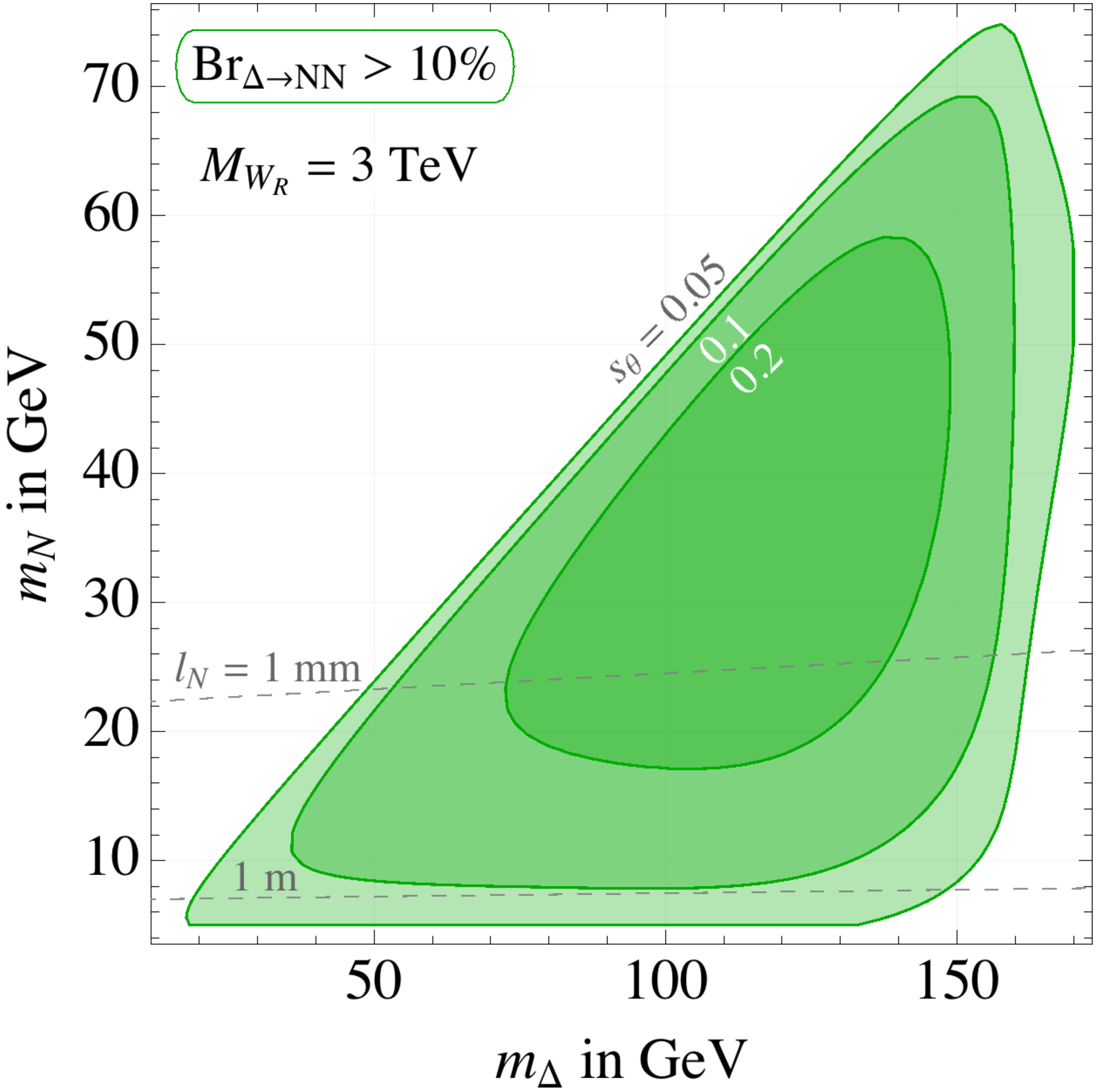
$$20\ \text{GeV} \lesssim m_\Delta \lesssim 170\ \text{GeV}$$

Decay length

$$c\tau_N^0 \simeq 0.1\ \text{mm}\left(\frac{40\ \text{GeV}}{m_N}\right)^5\left(\frac{M_{W_R}}{5\ \text{TeV}}\right)^4$$

Leads to two DV with LNV





‘Majorana’ Higgses at LHC

ggF production $\sigma_{gg \to h} \simeq 45 \text{ pb}$ N^{3}LO

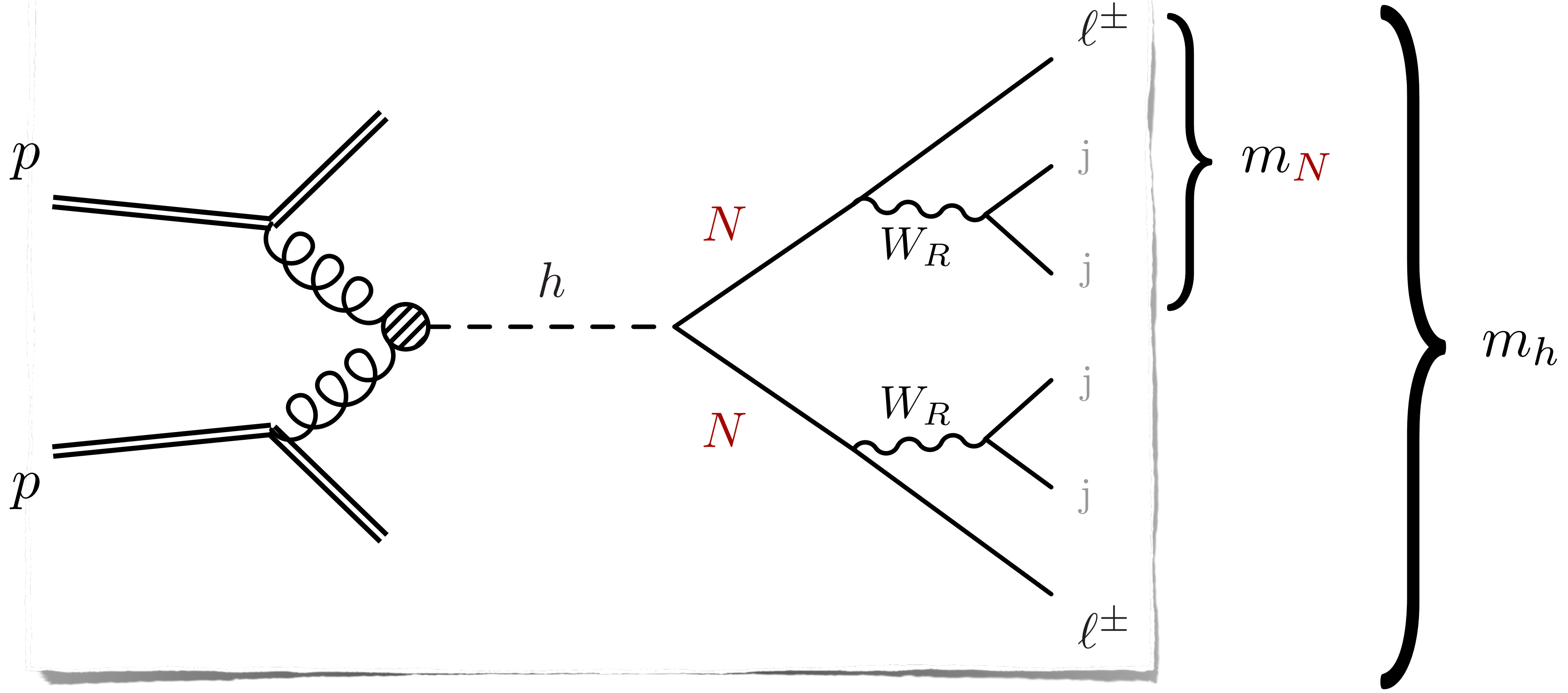
Anastasiou et al. ’14

MN, Nesti, Vasquez ’16

$\Delta L = 0, \boxed{2}$

$\Gamma_{h \to NN} \propto s_\theta^2 \, m_N{}^2$

$\text{Br}_{h \to NN} \simeq 10^{-3}$



small couplings, no tuning

no missing energy

light jets only $V_L^q = V_R^q$

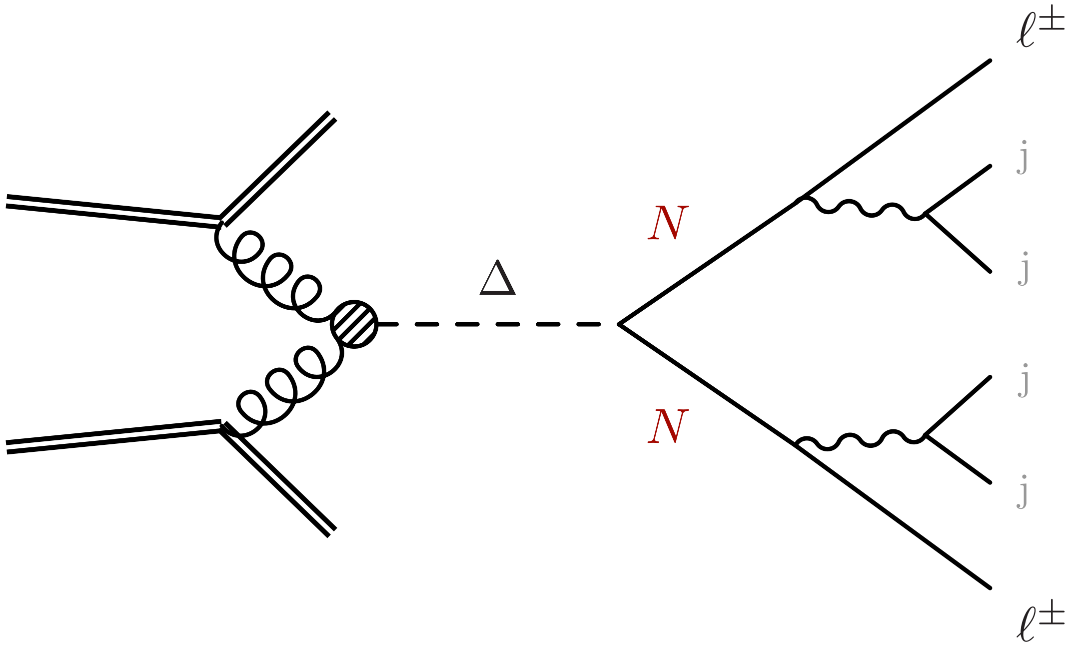
soft products $p_T \simeq m_h/6 \sim 20 \text{ GeV}$

Kiers et al. ’02, Zhang et al. ’07
Maiezza et al. ’10, Senjanović, Tello ’14

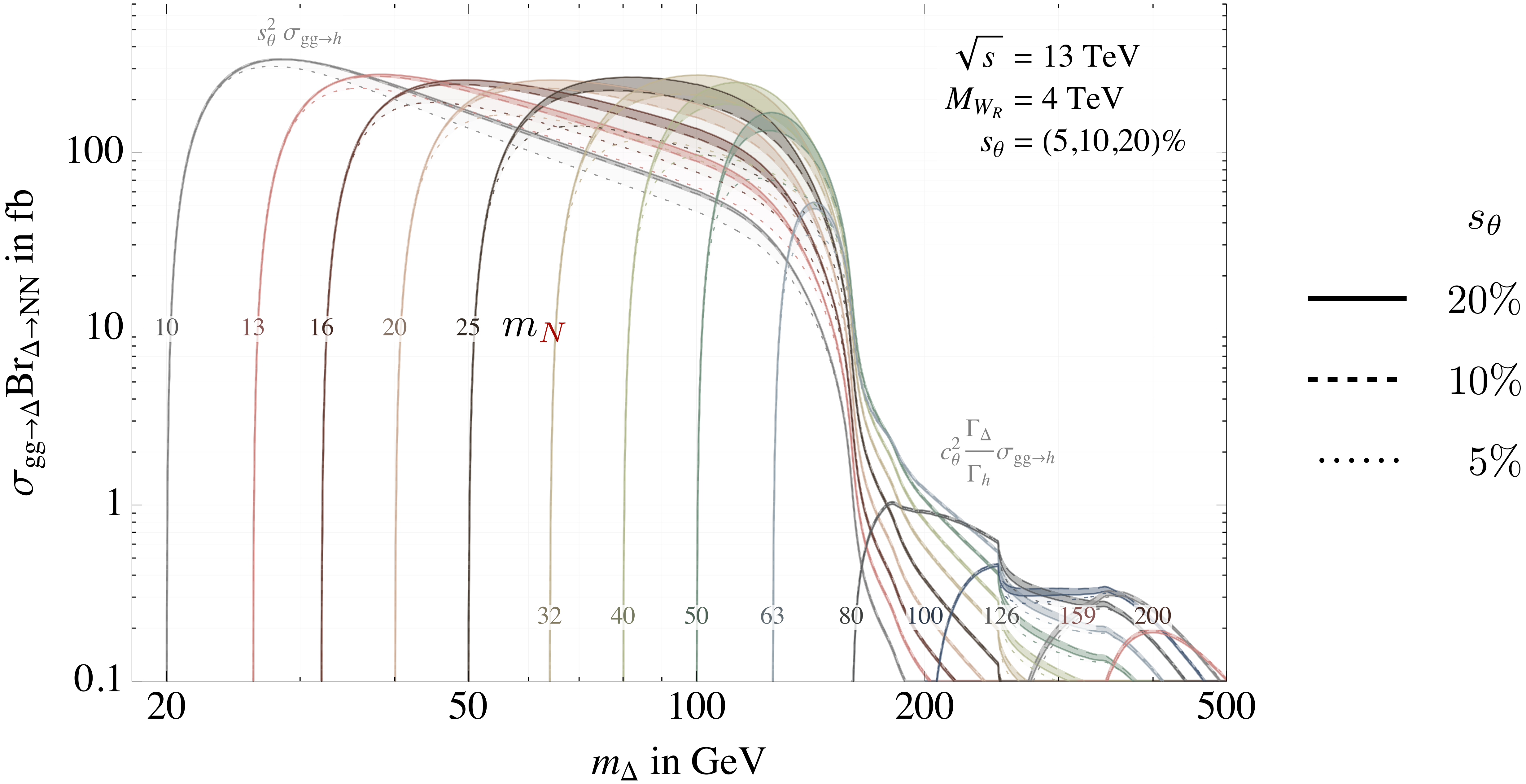
low background (LNV)

Δ signals

single



$\Delta L = 0, 2$



Backgrounds

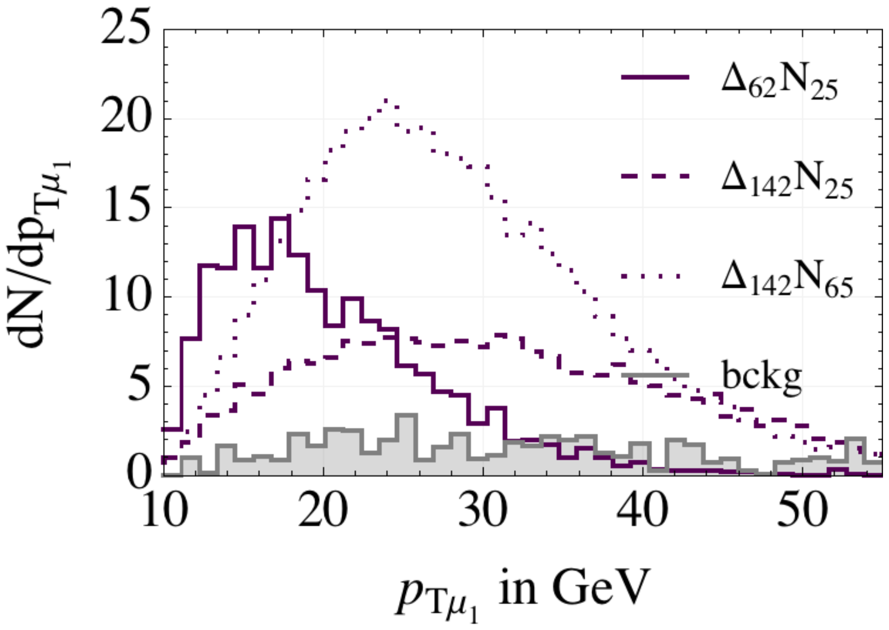
$\ell^{\pm}\ell^{\pm} + n_j$

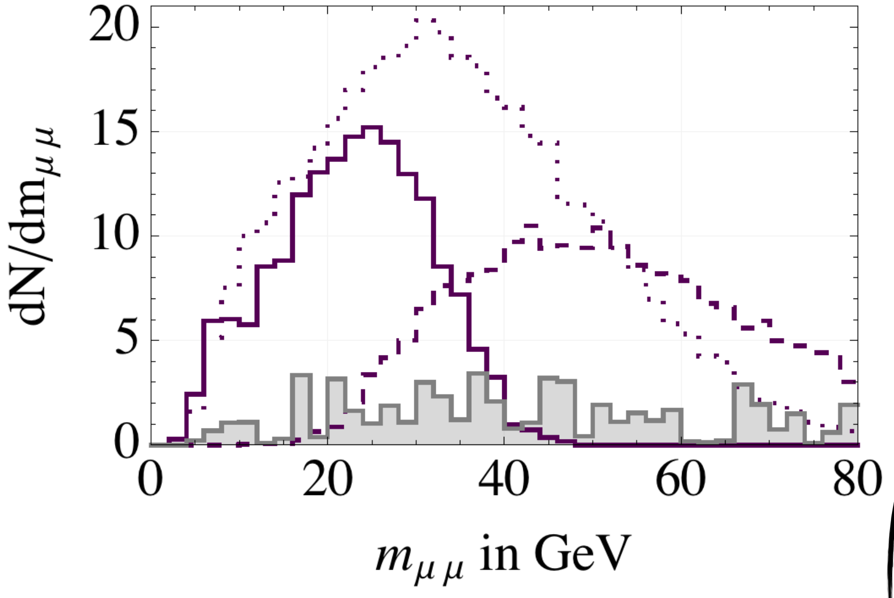
	$t\bar{t}$	$t\bar{t}h$	$t\bar{t}Z$	$t\bar{t}W$	WZ	Wh	ZZ	Zh	$WWjj$	fakes
select	806	4	5	26	1241	87	147	16	1.5	2651
$\not{E}_T$	313	0.5	0.7	3	400	21	129	7	0.2	782
p_T	112	0.2	0.1	0.7	174	8.4	63	4	0.05	284
m_T	60	0.1	0.04	0.3	80	4	56	2	0.03	106
m^{inv}	35	0.03	0.03	0.2	25	2	36	2	0	80
l_{Te}	0	0	0	0	0.7	0.1	0.9	0.05	0.001	2
	$t\bar{t}$	$t\bar{t}h$	$t\bar{t}Z$	$t\bar{t}W$	WZ	Wh	ZZ	Zh	$WWjj$	fakes
select	670	4	6	32	750	133	68	16	2	1676
$\not{E}_T$	130	0.5	0.9	3.5	200	32	33	6	0.3	391
p_T	57	0.2	0.2	1	95	17	16	3	0.1	152
m_T	32	0.1	0.1	0.5	51	9	12	2	0.05	49
m^{inv}	17	0.04	0.04	0.2	23	5	8	1	0.01	40
$l_{T\mu}$	0	0	0	0	1.4	0.4	1	0.15	0.005	3

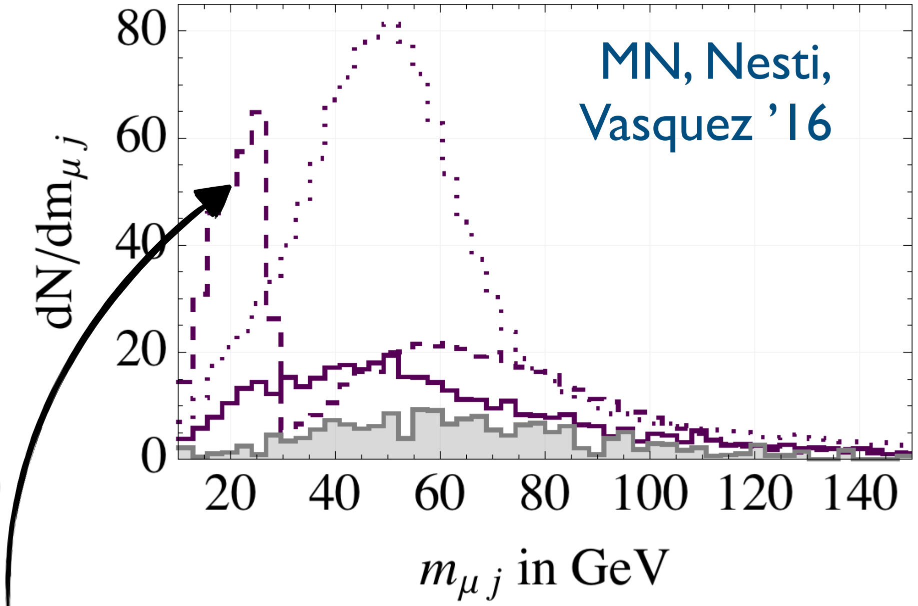
all contain missing energy

one prompt, one displaced lepton

Kinematics



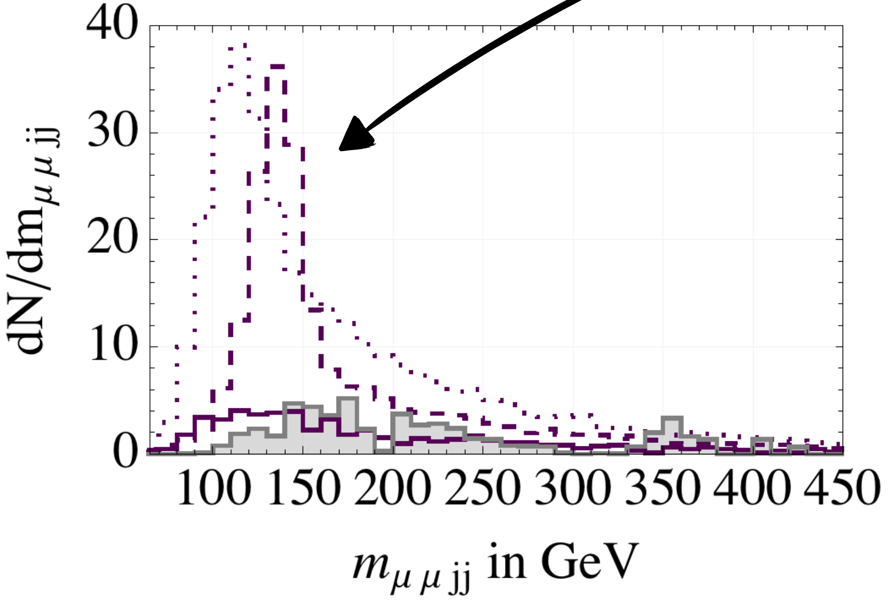


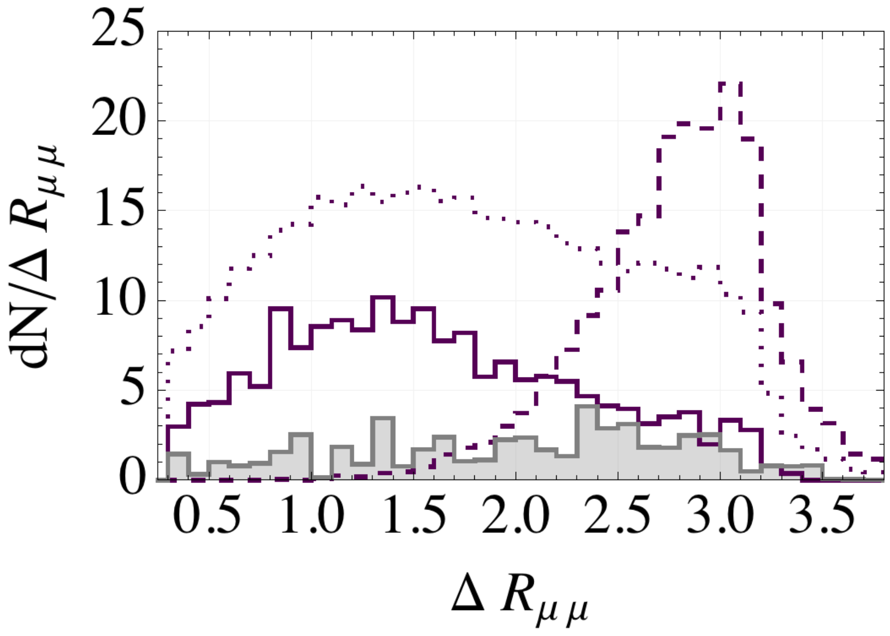


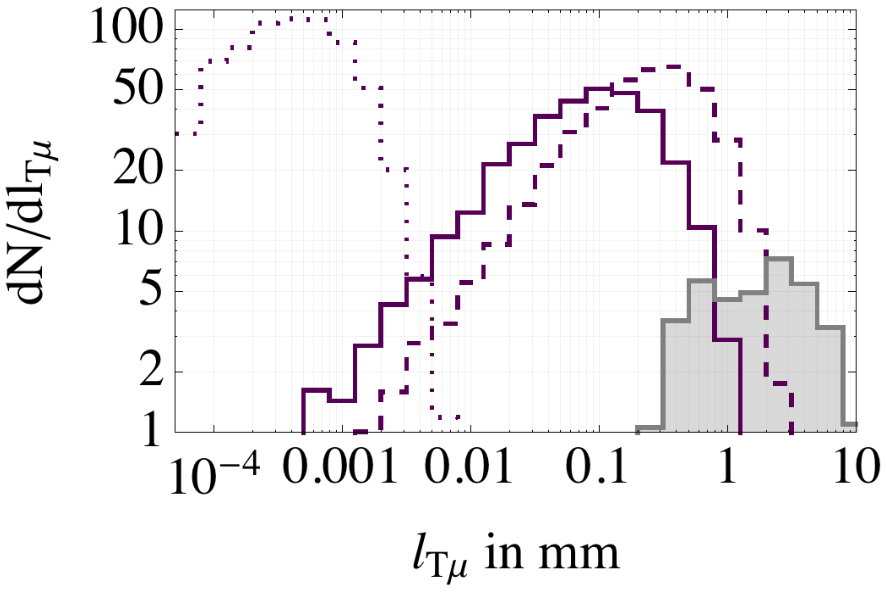
MN, Nesti, Vasquez '16

~soft leptons

characteristic mass peaks







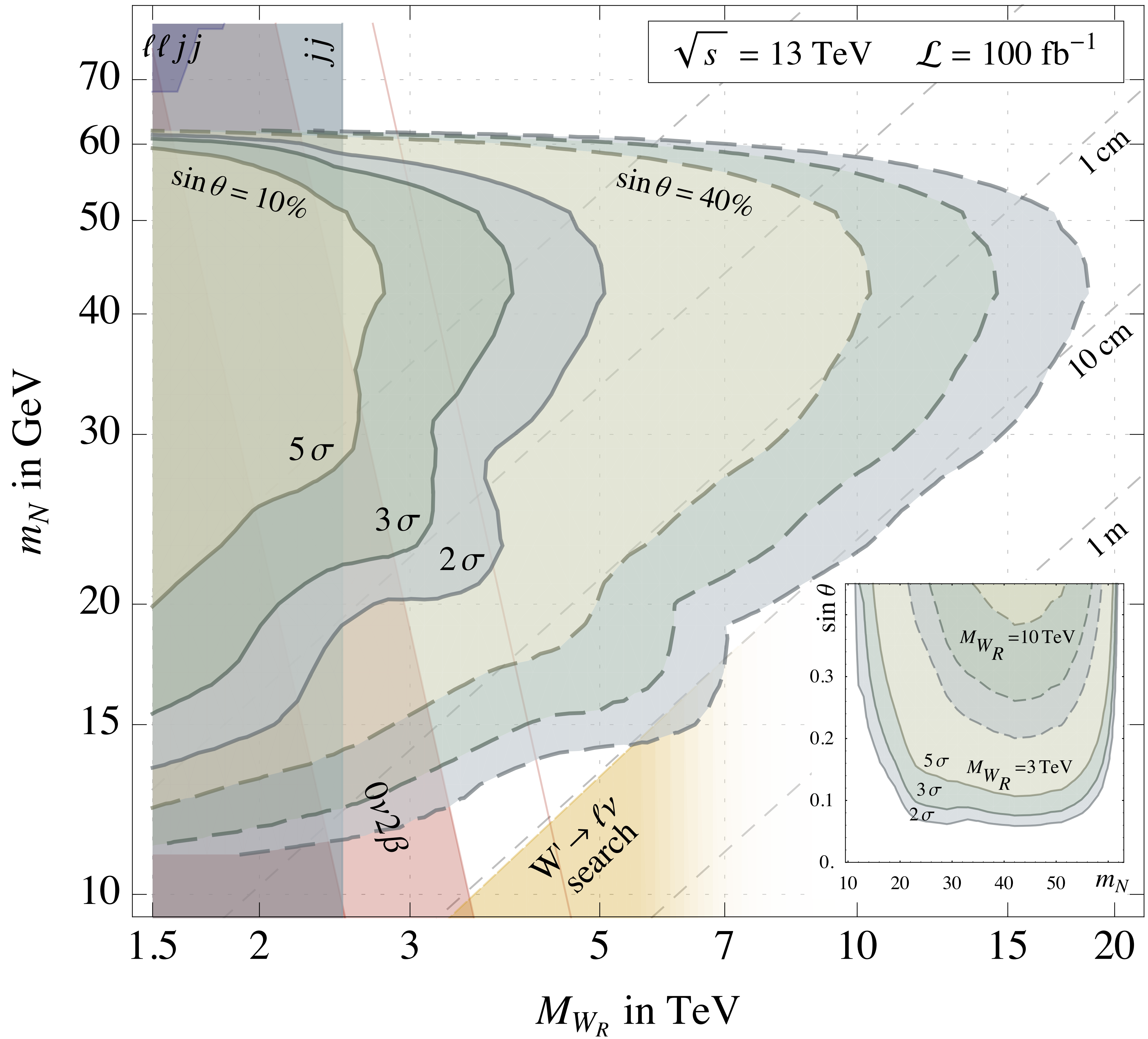
angular separation

displacement

Sensitivity

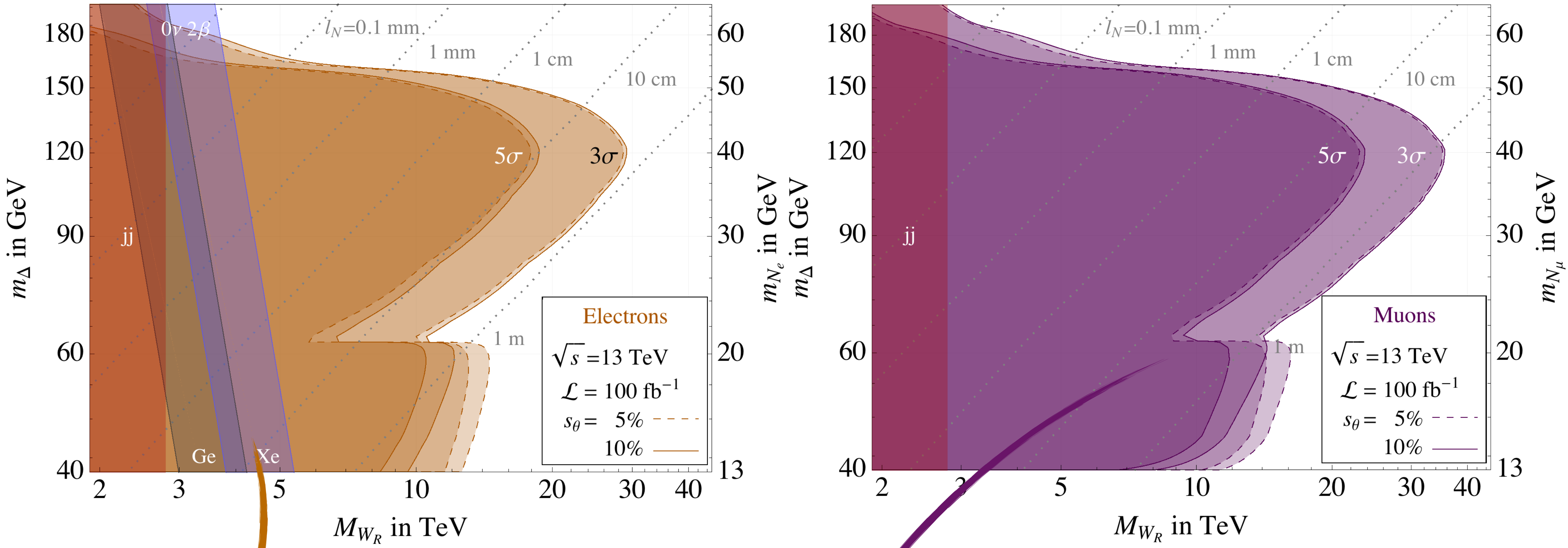
$h \to NN$

Maiezza, MN, Nesti '14



Sensitivity

Combined $h \to NN \quad \Delta \to NN \quad \Delta\Delta \to NNNN$



connection to $0\nu 2\beta$

GERDA, Neutrino '16

KamLAND-Zen '16

displaced 0.01 mm - >1m

$h \to \Delta\Delta \to NNNN$

discovery reach beyond direct searches

Summary

Minimal LR model

viable framework for understanding parity restoration and neutrino mass origin

strongly constrained by flavor, CP-even and CP-odd observables

complete theory of neutrino mass, Dirac predicted from Majorana

Majorana signatures

breaking of lepton number in nuclear processes and high energy colliders

Higgs sector under-explored, opportunity for displaced searches and future colliders

Backup

Displaced jet discrimination

MN, Nesti, Popara '18

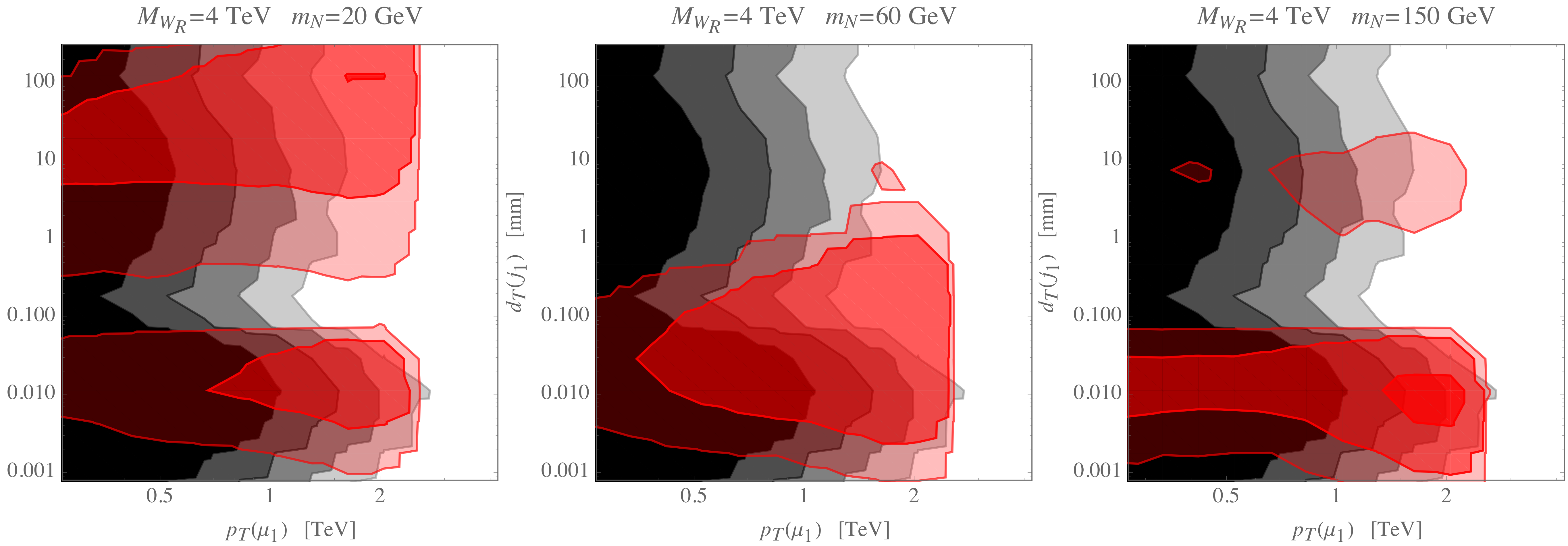
Event generation: custom generator KSEG, small width issues with MG5

also Feynrules model file and Delphes,
MadAnalysis displacement hack

sites.google.com/site/leftrighthep

Main bckgs

background	# generator	weight	# detector
$V+012j$	22.46 M	0.021	9.93M
$VV+012j$	10.55 M	0.0028	4.61M
$t\bar{t}+012j$	10.47 M	0.024	4.38M



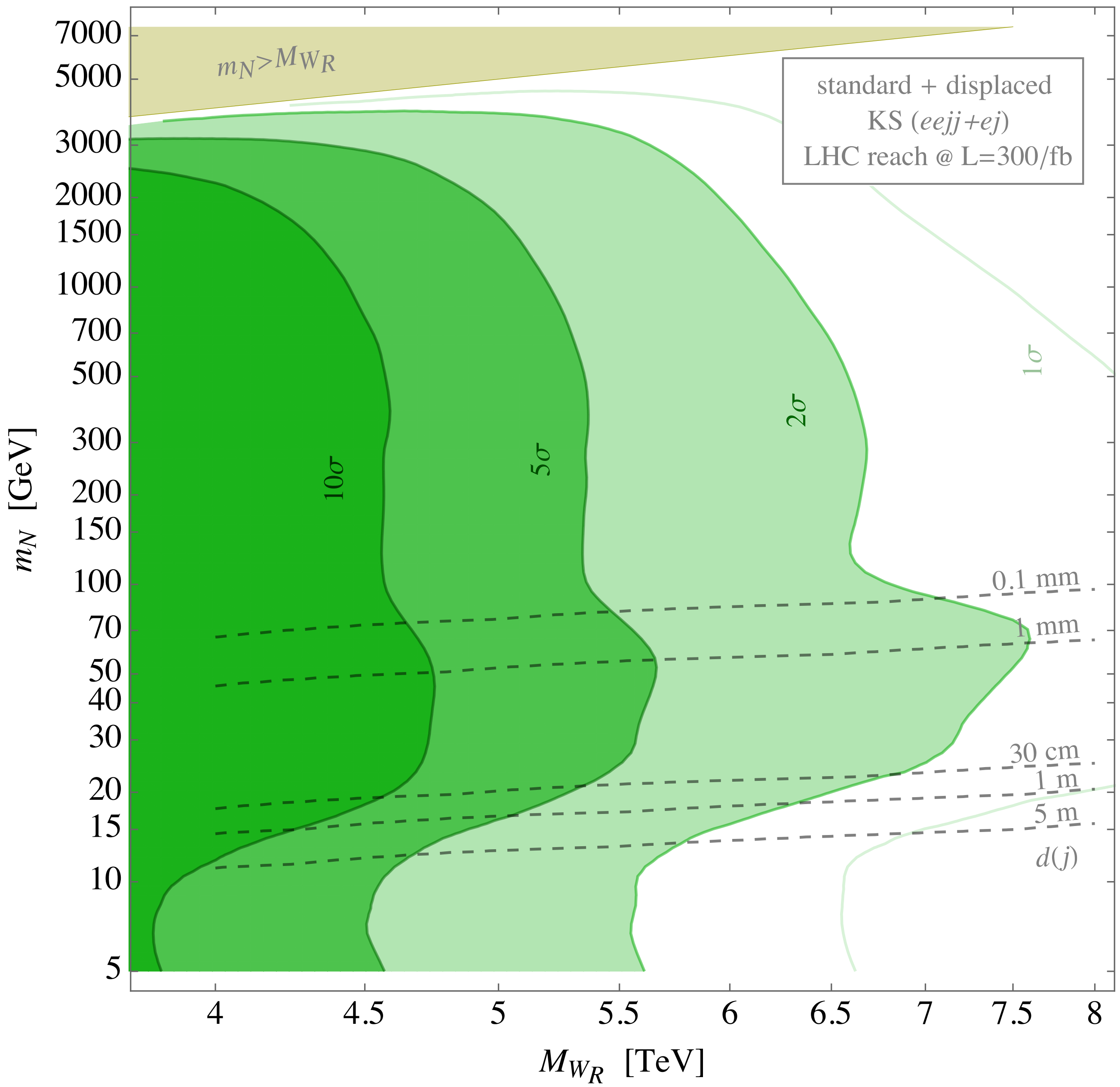
Sensitivity estimate

MN, Nesti, Popara '18

rough pre-selection

bin over 6 variables below

reach to 6-7 TeV

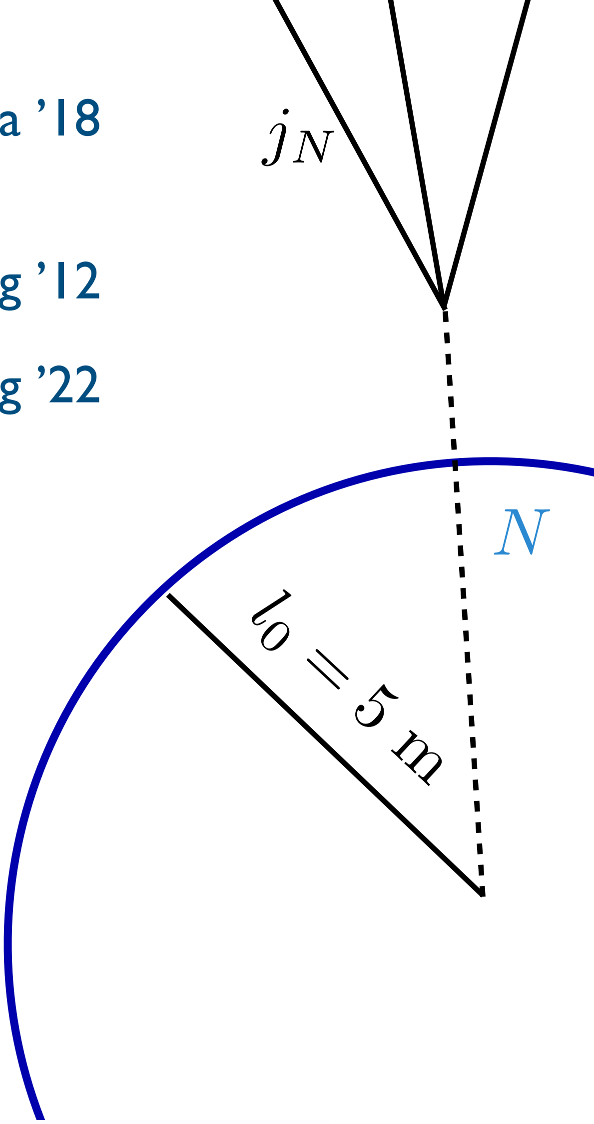


Sensitivities

Electron Channel	$\mathcal{L} = 300\,\mathrm{fb}^{-1}$		M_{W_R}:	4 TeV	4 TeV	4 TeV	6 TeV	6 TeV
variable	range	# bins	m_N:	20 GeV	300 GeV	2 TeV	20 GeV	300 GeV
$p_T(\ell_1)$	$\{150, 4500\}$ GeV	35		14.19	13.82	7.19	1.03	1.77
$d_T(j_1)$	$\{0.001, 300\}$ mm	100		17.57	14.04	7.60	2.02	1.91
#(jets)	1, 2, 3, 4	4		17.88	14.20	7.94	2.24	2.04
#(leptons)	1, 2	2		17.97	14.90	9.08	2.30	2.23
#(same sign)	0, 1	2		18.00	15.71	9.85	2.32	2.61
$m^{\mathrm{inv}}_{\ell_1 j_1}$	$\{200, 8500\}$ GeV	20		18.82	17.24	10.91	2.81	3.03

Recasting the $W' \to \ell\nu$

MN, Nesti, Popara '18



motivated by WDM

MN, Senjanović, Zhang '12

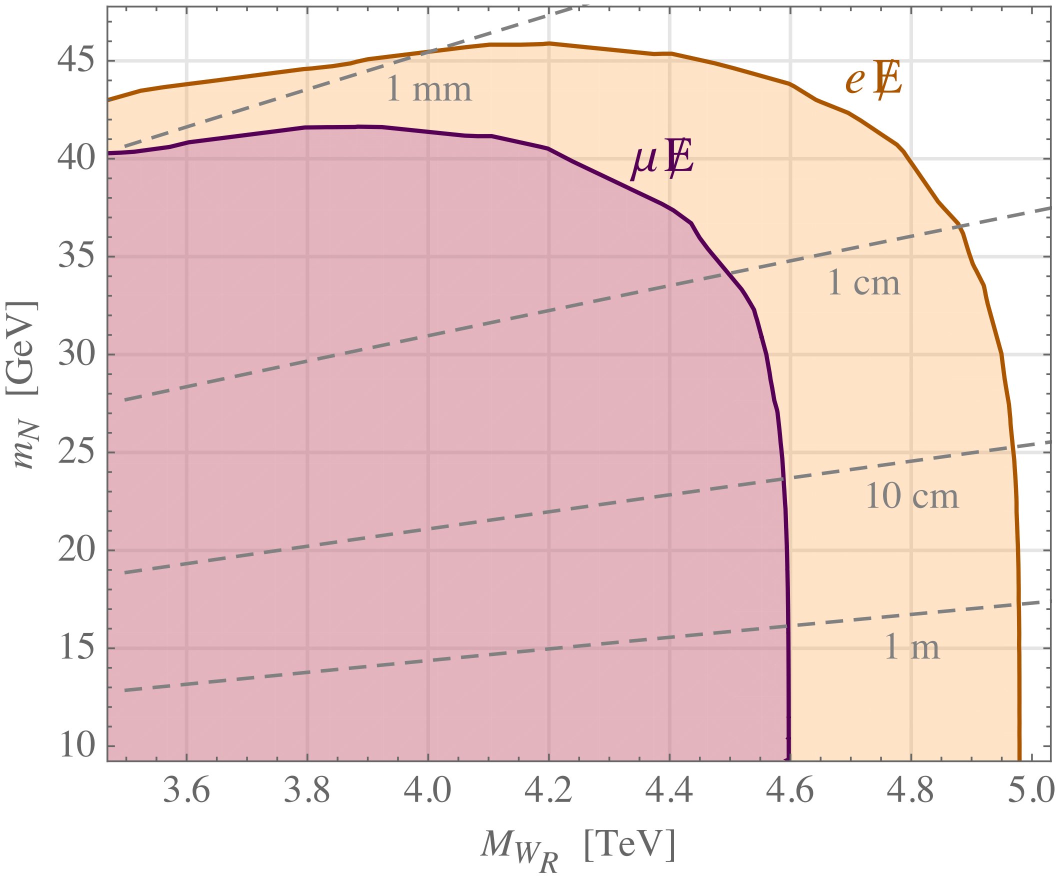
MN, Zhang '22

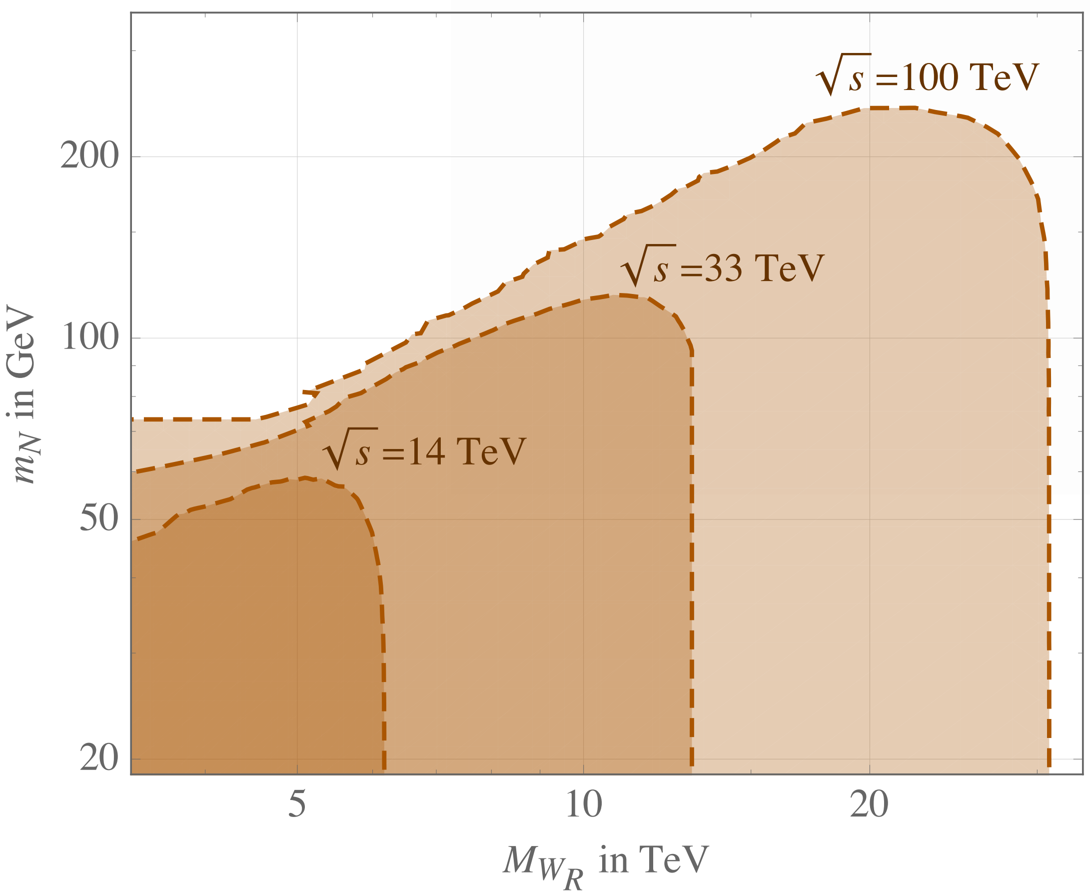
prompt hard leading lepton and significant missing energy

$$\frac{\mathrm{d}\sigma}{\mathrm{d}m_T} = \alpha_2^2 \frac{\pi}{24} p_T \int_{\tau_-}^{1} \int_{\frac{\tau_-}{x_1}}^{1} \mathrm{d}x_{1,2} \frac{\left(\hat{s} - m_N^2 - 2p_T^2\right) \pm 1}{\sqrt{\left(\hat{s} - m_N^2\right)^2 - 4p_T^2 \hat{s}}}$$

$$\frac{\varepsilon_\ell^{\pm}(p_T, \eta_\ell)}{\left(\hat{s} - M^2\right)^2 + (\Gamma M)^2} \left|V_{ud} V_{\ell N}\right|^2 f_u(x_{1,2}) f_{\overline{d}}(x_{2,1}) \boxed{e^{-l_0/L_\pm}}$$

exponential distributions have tails





‘Right-handed’ Higgs

Δ production

single $\sigma(gg \to \Delta) = s_\theta^2\, \sigma(gg \to h)$ N^{3}LO Anastasiou et al. ’16

$$\sigma(pp \to V\Delta) = s_\theta^2\, \sigma(pp \to Vh)$$

pair & associated

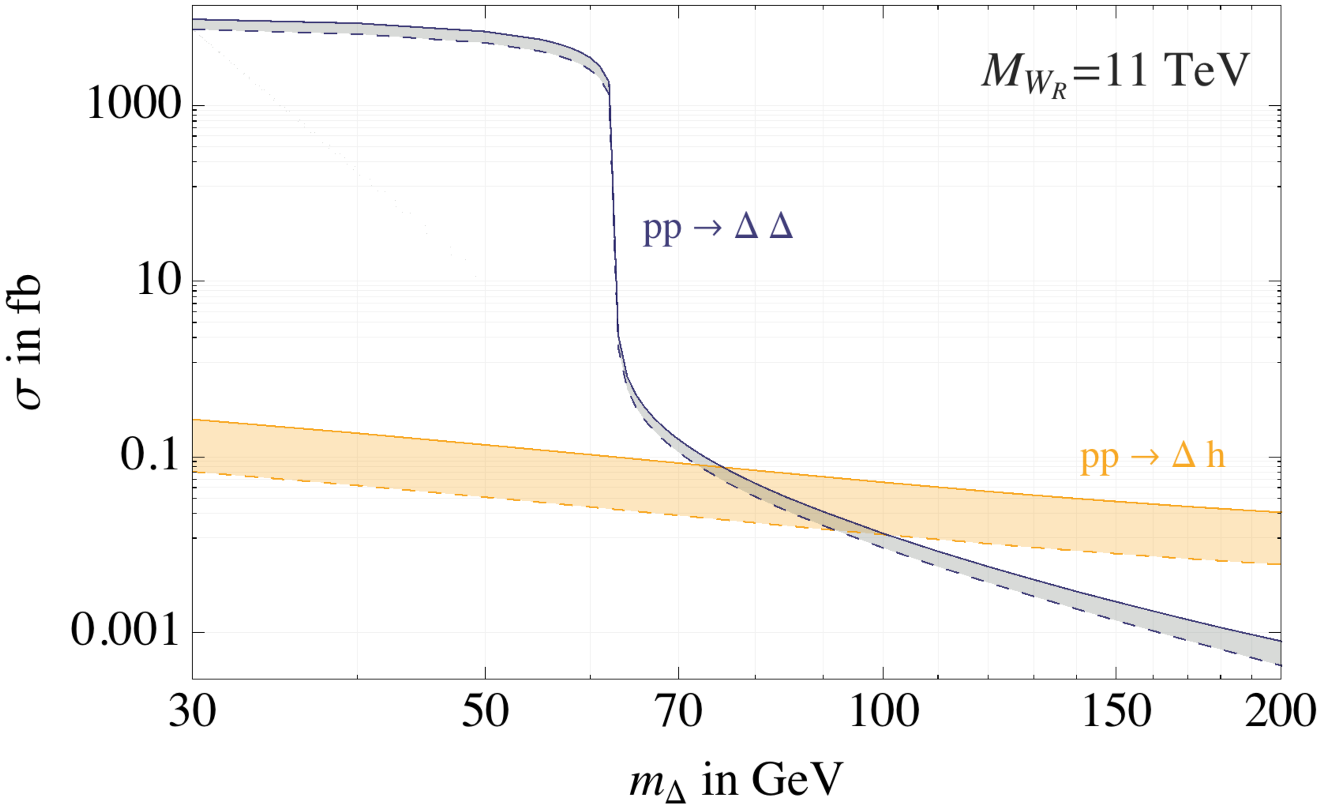
$$\hat{\sigma}_{gg \to \Delta S} \simeq \frac{c_\theta^2}{64\pi(1+\delta_{\Delta S})}\, \hat{s} \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{v_{hS\Delta}^2}{(\hat{s}-m_h^2)^2 + \hat{s}\Gamma_h^2} \left|F_b + F_t\right|^2 \sqrt{\beta_{\hat{s}\Delta S}}$$

large rate for $m_\Delta < m_h/2$

$$\sigma_{gg \to \Delta\Delta} \simeq \sigma_{gg \to h}\, \mathrm{Br}_{h \to \Delta\Delta}$$

not very significant

(accidental cancellation)



Backgrounds

Selection criteria

	$t\bar{t}$	$t\bar{t}h$	$t\bar{t}Z$	$t\bar{t}W$	WZ	Wh	ZZ	Zh	$WWjj$	fakes

Selection	$\ell^{\pm}\ell^{\pm} + n_j$
$\not{E}_T$	$\not{E}_T < 30\,\mathrm{GeV}$
p_T	$p_T(\ell_1) < 55\ \mathrm{GeV}$
m_T	$m^T_{\ell\not{p}_T} < 30\,\mathrm{GeV}$
m_{inv}	$m_{\ell\ell} < 80\,\mathrm{GeV}$
	$m_{\ell\not{p}_T} < 60\,\mathrm{GeV}$
$l_{T\ell}$	$l_{T\ell} > 0.1\ \mathrm{mm}$

all contain missing energy

one prompt, one displaced lepton

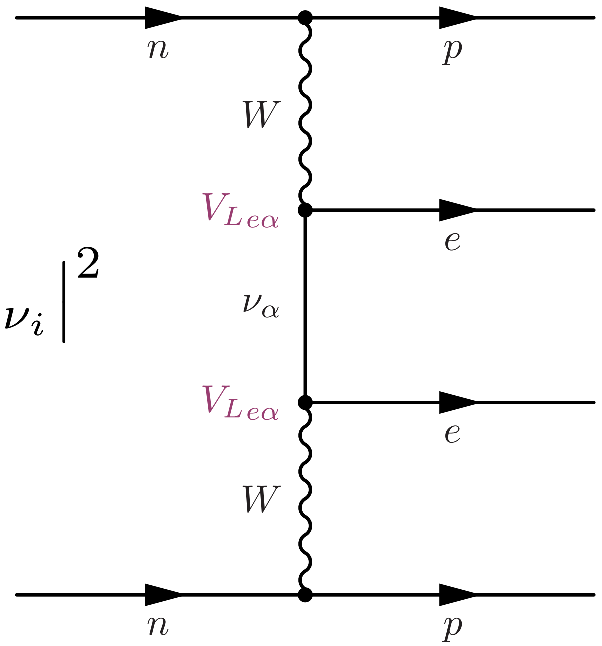
LNV interplay

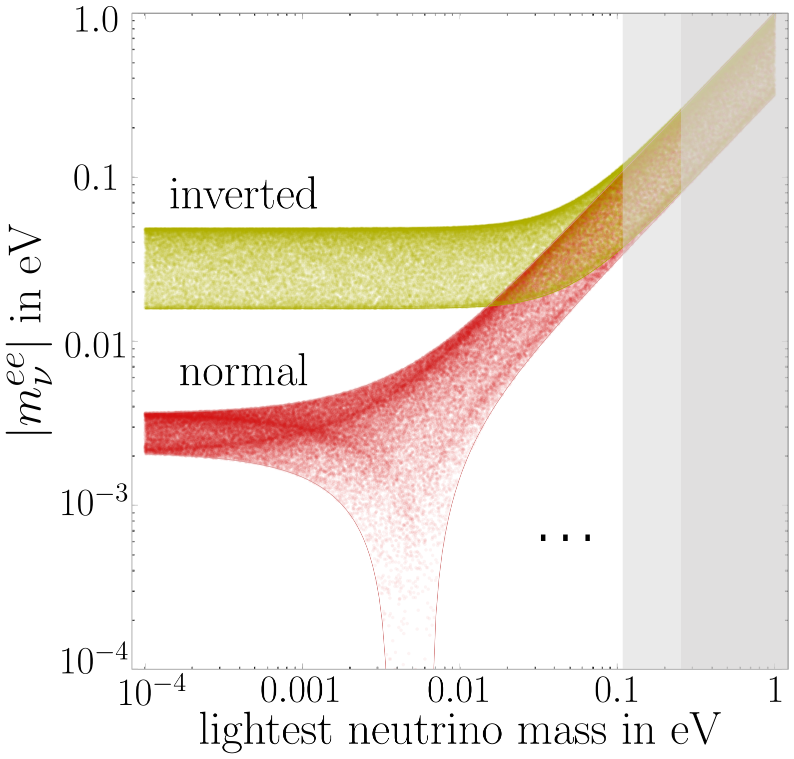
Nuclear vs. collider physics

Immediate proposal after Majorana: $0\nu 2\beta$ via light neutrino exchange

Furry '39

$$\Gamma \propto \left| m_\nu^{ee} = V_{ei}^2 m_{\nu_i} \right|^2$$





Cosmology, KATRIN, …

Possible tension

Vissani '02

LNV interplay

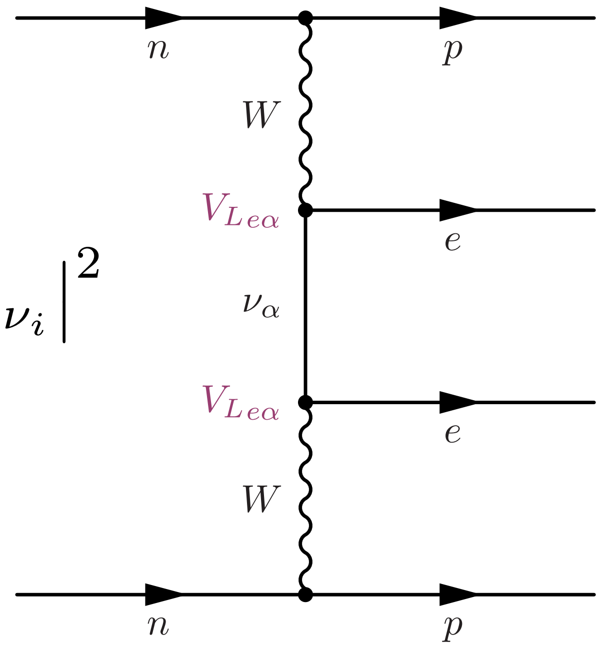
Nuclear vs. collider physics

Immediate proposal after Majorana: $0\nu 2\beta$ via light neutrino exchange

Furry '39

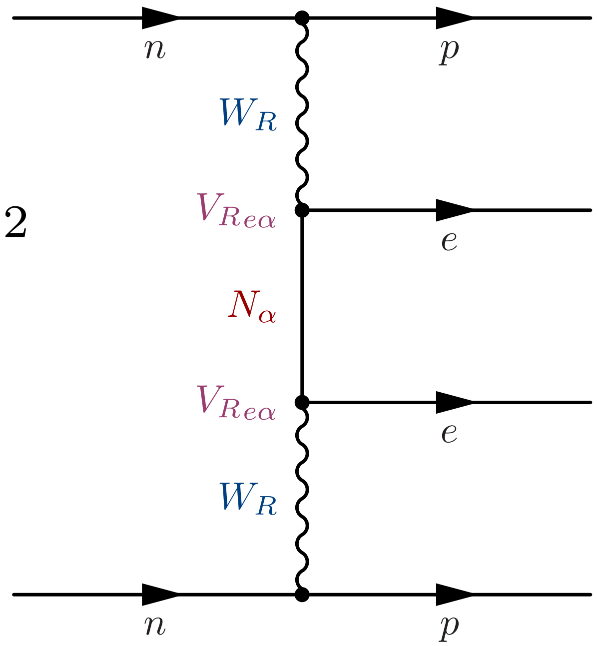
Mohapatra, Senjanović '79

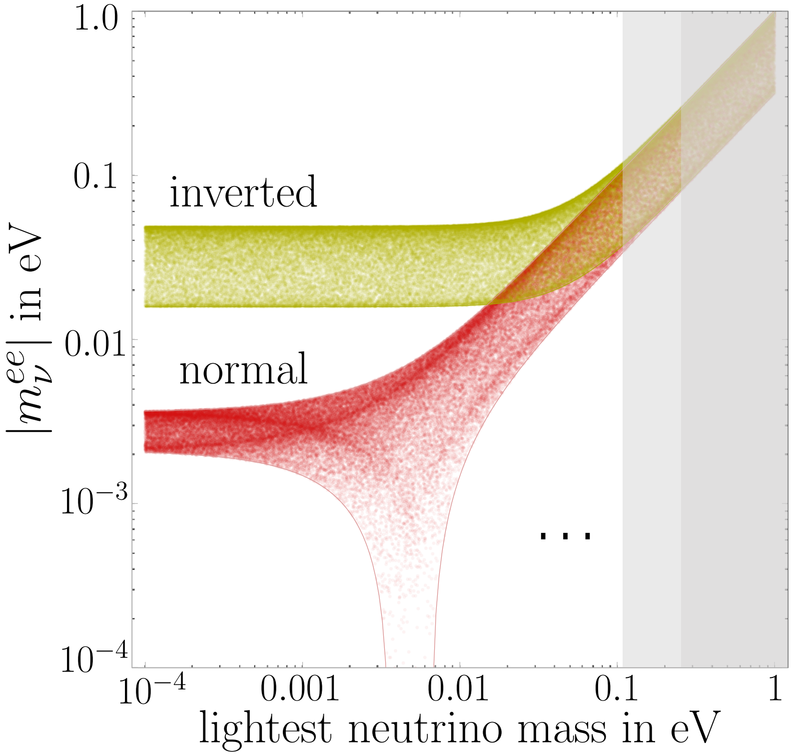
$$\Gamma \propto \left| m_\nu^{ee} = V_{ei}^2 m_{\nu_i} \right|^2$$



$$\Gamma \propto \left| m_N^{ee} = \frac{V_{ei}^{R2}}{m_{N_i}} \right|^2$$

'heavy' neutrinos

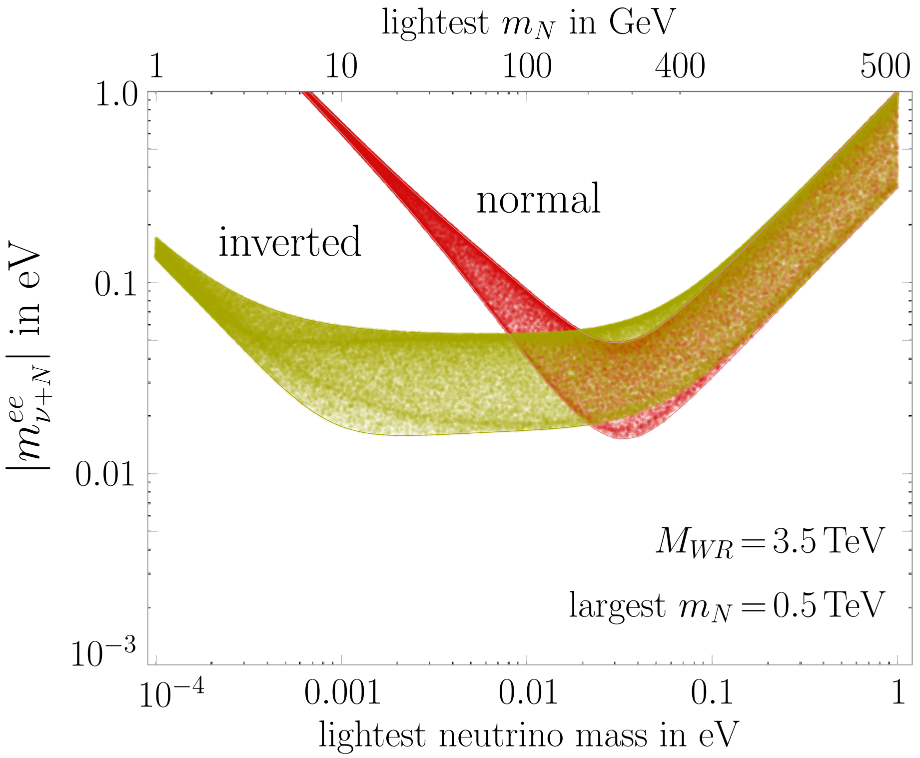




Cosmology, KATRIN, …

Possible tension

Vissani '02



Tello, MN, Nesti, Senjanović, Vissani '11

no tension in LR

$$V_R = V_{PMNS}$$

connection to LFV

Fermions and Dirac

Relativistic equation $\boxed{i\gamma^\mu \partial_\mu \psi - m_D \psi = 0}$ Dirac '28

What is ψ? A representation of the Lorentz group, of course!

$$\psi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}$$

Four component spinor made of two Weyl spinors ψ_L and ψ_R.

$$\boxed{\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}}$$

$$\boxed{\begin{array}{ll} \sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{array}}$$

Chiral representation

$$i\sigma^\mu \partial_\mu \psi_R = m\psi_L$$
$$i\overline{\sigma}^\mu \partial_\mu \psi_L = m\psi_R$$

L and R mix

Weyl spinors ψ_L and ψ_R transform separately, reducibly

$$\boxed{\psi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}} \qquad \boxed{\gamma^5} = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1_{2\times 2} & 0 \\ 0 & 1_{2\times 2} \end{pmatrix}$$

Transformations under the Lorentz group

- three rotations θ_i
- three boosts φ_i

$$\psi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix} \xrightarrow[\text{transform}]{\text{Lorentz}} \psi' = \begin{pmatrix} \chi'_L \\ \chi'_R \end{pmatrix}$$

$$\boxed{P_{L,R} = \frac{1}{2}\left(1 \mp \gamma^5\right)}$$

projectors

$$P_X^2 = 1\,,\, P_L P_R = 0$$

$$P_L\psi = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix} \qquad P_R\psi = \begin{pmatrix} 0 \\ \chi_R \end{pmatrix}$$

Lorentz

transform

$$\chi_L' = \exp\left(i\frac{\sigma^i}{2}\left(\theta_i \boxed{+} i\varphi_i\right)\right)\chi_L$$

$$\chi_R' = \exp\left(i\frac{\sigma^i}{2}\left(\theta_i \boxed{-} i\varphi_i\right)\right)\chi_R$$

Opposite boosts!

The Dirac fermion Lagrangian is Lorentz invariant

$$\boxed{\mathcal{L}_D = i\chi_L^\dagger\overline{\sigma}^\mu\partial_\mu\chi_L + i\chi_R^\dagger\sigma^\mu\partial_\mu\chi_R - m_D\left(\chi_L^\dagger\chi_R + \chi_R^\dagger\chi_L\right)}$$

Dirac mass term $\chi_L^\dagger\chi_R \to \chi_L^\dagger\exp\left(-i\frac{\sigma^i}{2}\left(\theta_i - i\varphi_i\right)\right)\exp\left(i\frac{\sigma^i}{2}\left(\theta_i - i\varphi_i\right)\right)\chi_R = \chi_L^\dagger\chi_R$

Four component $\overline{\psi} = \psi^\dagger\gamma^0 = \begin{pmatrix}\chi_L^\dagger \\ \chi_R^\dagger\end{pmatrix}\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix} = \begin{pmatrix}\chi_R^\dagger \\ \chi_L^\dagger\end{pmatrix}$

$$\boxed{m_D\left(\chi_L^\dagger\chi_R + \chi_R^\dagger\chi_L\right) = m_D\overline{\psi}\psi}$$

LI

Adding interactions - coupling to photons $U(1)$ $\psi \to e^{iQ\alpha(x)}\psi$

$$\boxed{\mathcal{L}_D = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\not{D}\psi - m_D\overline{\psi}\psi}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \not{D} = \gamma^\mu\left(\partial_\mu + ieA_\mu\right)$$

QED current $j^\mu = \overline{\psi}\gamma^\mu\psi$

Four component spinor contains particles and anti-particles Dirac ’31

$$\psi^c = C\overline{\psi}^T = i\gamma^2\psi^* \qquad Q(\psi^c) = -Q(\psi) \qquad m(e^+) = m(e^-)$$

Anti-matter prediction confirmed immediately Anderson ’32

Now confirmed for all charged particles $p^+, \mu^+, \tau^+, q, \ldots$

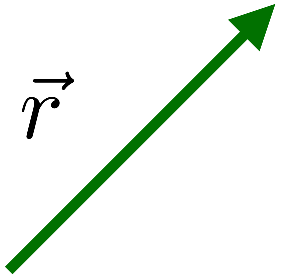
Fermions and Majorana

Q: Have we found all possible reps and invariants?

Indeed no, says Majorana.

Dirac ’37

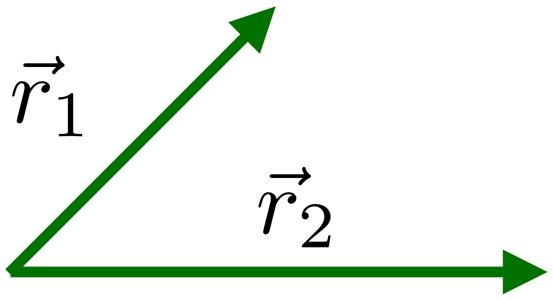
Hint from rotations $SO(2)$



length is invariant

$$\boxed{\vec{r}\cdot\vec{r} \sim r^T r}$$

Scalar product



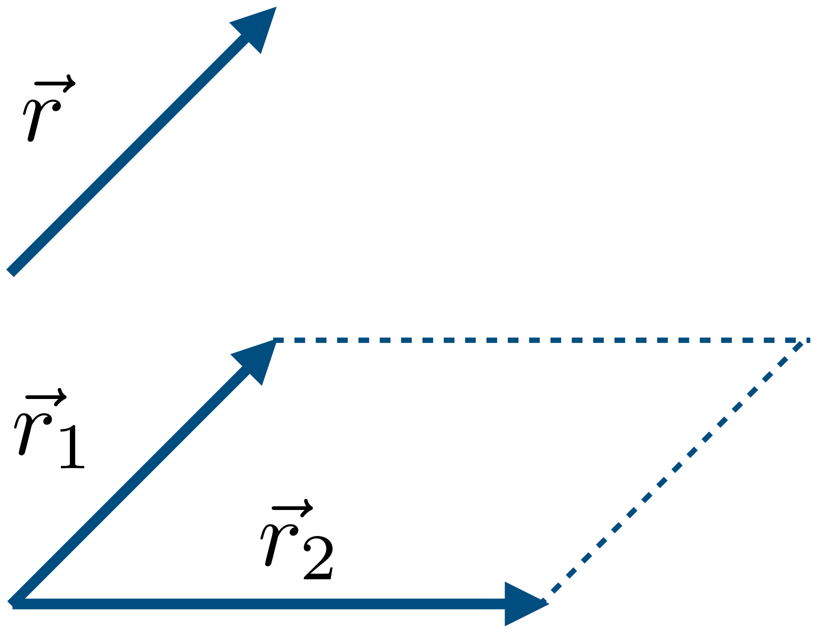
$$\boxed{\vec{r}_1 \cdot \vec{r}_2}$$

The Dirac mass term is analogous

$$\boxed{\overline{\psi}\psi \sim \vec{r}\cdot\vec{r}}$$

Hint from $SO(2)$

Vector product



zero for commuting objects

$$\boxed{\vec{r} \times \vec{r}}$$

area is invariant

$$\boxed{\vec{r}_1 \times \vec{r}_2}$$

Majorana mass term from $\chi'_L = \exp\left(i\frac{\sigma^i}{2}(\theta_i + i\varphi_i)\right)\chi_L$

$$\boxed{\chi_L^T(i\sigma_2)\chi_L} = \chi_L^a \varepsilon^{ab} \chi_L^b$$

antisymmetric, too

Lorentz invariant! A new mass term with one component unlike Dirac.

$$\boxed{\mathcal{L}_M = i\chi_L^\dagger \overline{\sigma}^\mu \partial_\mu \chi_L - \frac{1}{2} m_M \left(\chi_L^T(i\sigma_2)\chi_L + \chi_L^\dagger(i\sigma_2)\chi_L^*\right)}$$

Majorana fermions in four components $\psi_M = \begin{pmatrix} \chi_L \\ -i\sigma_2 \chi_L^* \end{pmatrix}$

$$\boxed{\mathcal{L}_M = \frac{1}{2}\left(i\overline{\psi}_M \partial\!\!\!/ \psi_M - m_M \overline{\psi}_M \psi_M\right)}$$

Easy to derive propagators and Feynman rules

Neutral only!

Looks the same, but no $U(1)$ charge $\boxed{\chi_L \nrightarrow e^{i\alpha}\chi_L}$ b.c. $\boxed{\chi_L^T (i\sigma_2)\chi_L}$

$$\psi^c = i\gamma^2\psi^* = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} \chi_L^* \\ \chi_R^* \end{pmatrix} = \begin{pmatrix} i\sigma_2 \chi_R^* \\ -i\sigma_2 \chi_L^* \end{pmatrix}$$

$$\psi_M = P_L \psi + P_R \psi^c = \begin{pmatrix} \chi_L \\ -i\sigma_2 \chi_L^* \end{pmatrix}$$

Majorana fermions in four components

$$\boxed{\mathcal{L}_M = \frac{1}{2}\left(i\overline{\psi}_M \not{\partial}\psi_M - m_M\overline{\psi}_M\psi_M\right)}$$

Kinetic

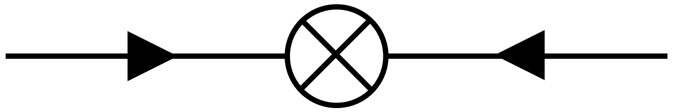
$$\begin{aligned}
\overline{\psi}_M\not{\partial}\psi_M &= \left(\chi_L^\dagger, \quad \chi_L^T(i\sigma_2)\right)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^\mu\\ \overline{\sigma}^\mu & 0\end{pmatrix}\begin{pmatrix}\partial_\mu\chi_L\\ -i\sigma_2\partial_\mu\chi_L^*\end{pmatrix}\\
&= \chi_L^\dagger\overline{\sigma}^\mu\partial_\mu\chi_L + \chi_L^T\sigma_2\sigma^\mu\sigma_2\partial_\mu\chi_L^*\\
&= \chi_L^\dagger\overline{\sigma}^\mu\partial_\mu\chi_L - \left(\partial_\mu\chi_L^T\overline{\sigma}^\mu\chi_L^\dagger\right)^T\\
&= 2\chi_L^\dagger\overline{\sigma}^\mu\partial_\mu\chi_L
\end{aligned}$$

Mass

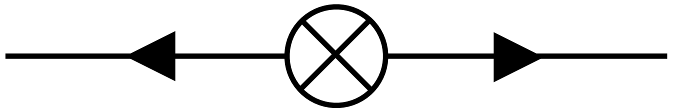
$$\begin{aligned}
\overline{\psi}_M\psi_M &= \left(\chi_L^T i\sigma_2, \quad \chi_L^\dagger\right)\begin{pmatrix}\chi_L\\ -i\sigma_2\chi_L^*\end{pmatrix}\\
&= \chi_L^T i\sigma_2\chi_L - \chi_L^\dagger i\sigma_2\chi_L^* = \chi_L^T i\sigma_2\chi_L + \mathrm{cc}
\end{aligned}$$

$$\overline{\psi}_M\psi_M = \psi_M^T\mathcal{C}\psi_M\,,$$
$$\mathcal{C} = i\gamma_2\gamma_0$$

Mass insertions…



…break fermion number flow



$$\psi_M = \begin{pmatrix} \chi_L \\ -i\sigma_2\chi_L^* \end{pmatrix}$$

Majorana has no anti-particle…

$$\boxed{\psi_M^c} = i\gamma^2\psi_M^* = \begin{pmatrix} 0 & i\sigma_2 \\ -i\sigma_2\chi_L^* & \end{pmatrix} \begin{pmatrix} \chi_L \\ -i\sigma_2\chi_L^* \end{pmatrix} = \boxed{\psi_M}$$

… and *U(1)* currents vanish by construction

$$\boxed{\overline{\psi}_M\gamma^\mu\psi_M} = (\chi_L^T i\sigma_2, \quad \chi_L^\dagger) \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \begin{pmatrix} \chi_L \\ -i\sigma_2\chi_L^* \end{pmatrix} \boxed{= 0}$$

Mass terms necessarily symmetric, unlike Dirac

$$\boxed{M_M = M_M^T}$$

$$\begin{aligned} (M_M)_{ij}\overline{\psi}_{Mi}\psi_{Mj} &= M_{M\,ij}\left(\chi_{Li}^T i\sigma_2\chi_{Lj} + \mathrm{cc}\right) \\ &= M_{M\,ij}\left(-\chi_{Lj}^T i\sigma_2^T\chi_{Li} + \mathrm{cc}\right) \\ &= M_{M\,ij}\left(\chi_{Lj}^T i\sigma_2\chi_{Li} + \mathrm{cc}\right) \\ &= M_{M\,ji}\left(\chi_{Li}^T i\sigma_2\chi_{Lj} + \mathrm{cc}\right) \end{aligned}$$

$$\boxed{M_D \to U_L^\dagger m_D U_R}$$

$$\boxed{M_M \to U_L^T m_M U_L}$$

additional CP phases