

# Parallel beam-based alignment methods for storage rings and transport lines

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FCC Optics Tuning and Correction Workshop, CERN

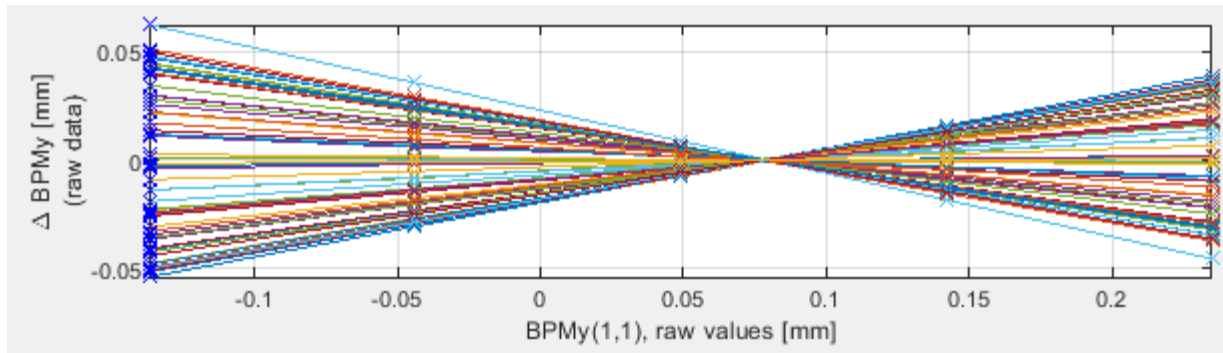
Xiaobiao Huang

SLAC National Accelerator Laboratory

- The need for parallel BBA
- Method 1: correction of induced orbit shifts
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  - Experimental test on SPEAR3
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- Method 2: deduce offsets w/ model and steering
  - Method description
  - Experimental test on SPEAR3
  - Experimental test on LCLS-II
- Summary

# The need for parallel BBA (PBBA)

- In the usual BBA, we target one quadrupole at a time



- Parallel BBA: to determine the centers of multiple quadrupole magnets at the same time
- Scenarios where PBBA is needed or desired
  - Multiple magnets share a common power supply – a common scenario
  - Fast BBA measurements

Currently BBA measurement for SPEAR3 takes ~3 hrs  
APS-U BBA measurement is estimated to take ~50 hrs

# Earlier BBA work for multiple quadrupoles

- Dispersion-free trajectory correction

- Aims at correcting the trajectory and trajectory difference for different energies

$$\sum_{j=1}^{N_q} \frac{(m_j + X_j)^2}{\sigma_{\text{prec}}^2 + \sigma_{\text{BPM}}^2} + \frac{(\Delta m_j + \Delta X_j)^2}{2\sigma_{\text{prec}}^2}$$

T.O. Raubenheimer and R.D. Ruth,  
NIMA 302 (1991) 191-208

- Target all BPMs in a region; does not find quadrupole centers

- Correction of trajectory after quadrupole modulation with correctors at or near quadrupoles

I. Pinayev, NIMA 570 (2007) 351–356

- Deduce quadrupole centers from corrector strengths (kick)

Orbit offset  $\Delta = \frac{\pm \alpha_t \cos(\phi_Q - \phi_t)}{L_Q \delta K_1} \sqrt{\frac{\beta_t}{\beta_Q}}$   $\alpha_t$  kick strength

- Need to have correctors at or near each quadrupole.

# Method 1: PBBA by correcting the induced trajectory/orbit drift

- The induced orbit shift (IOS): orbit changes when the strengths of the group of targeted quadrupoles are modulated
- We can correct the orbit for it to go through the quadrupole centers such that the IOS is zero (or minimized)
  - Correction goal: set IOS to zero
    - Need not to know the orbit at the quadrupoles for correction
  - Actuators: corrector magnets
  - Correction method: the corrector-to-IOS response matrix

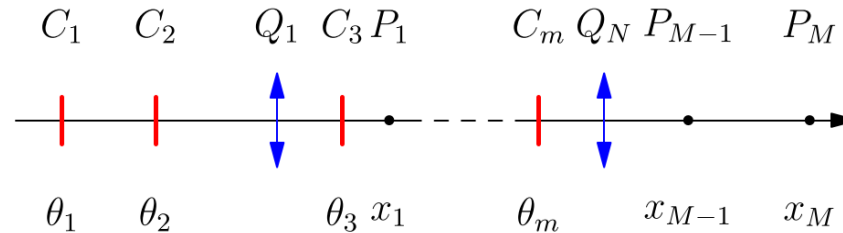
After the IOS correction, the orbit is at the quadrupole centers. Record the orbit reading with nearby BPMs.

X. Huang, PRAB 25, 052802 (2022)

Note mixed use of trajectory and orbit in the following

# The PBBA method explained – the IOS

For a transport line



C corrector,  
Q quadrupole,  
P BPM

Quadrupole modulation:

$$K_i = K_{i0} + k_i$$

Integrated strength  
Original value  $K_{i0}$   
Modulation step  $k_i$

At the quadrupole, beam position reads  $\bar{x}_j$ , magnetic center at  $\Delta_j$ , then beam receives a kick due to modulation

$$\Delta\phi_i = k_i[\Delta_i - \bar{x}_i - \Delta\bar{x}_i(\Delta\phi_1, \Delta\phi_2, \dots, \Delta\phi_{i-1})]$$

Orbit shift due to modulation of upstream quads (optics error)  
Drop this nonlinear term

The IOS at BPM  $i$  :

$$\xi_i = \sum_{j=1}^{Q < P_i} A_{12}^{(ij)} k_j (\Delta_j - \bar{x}_j)$$

$A_{12}^{(ij)}$  The (1,2) element of transfer matrix  $\mathbf{M}(P_i|Q_j)$

In matrix form

$$\boldsymbol{\xi} = \mathbf{A}\mathbf{k}(\boldsymbol{\Delta} - \bar{\mathbf{x}})$$

where  $\mathbf{k}$  is a diagonal matrix whose  $(j, j)$  element is  $k_j$ ,  $\mathbf{A}$  is a matrix of dimension  $M \times N$  with its  $(i, j)$  element being  $A_{12}^{(ij)}$  and zero if quadrupole  $Q_j$  is downstream of BPM  $P_i$ , and  $\boldsymbol{\Delta}$  and  $\bar{\mathbf{x}}$  are vectors formed with  $\Delta_j$  and  $\bar{x}_j$ ,

- Orbit at quadrupole locations

$$\bar{\mathbf{x}}^{(j)} = \bar{\mathbf{x}}_0^{(j)} + \sum_{l=1}^{C < Q_j} \mathbf{M}(Q_j | C_l) \begin{pmatrix} 0 \\ \theta_l \end{pmatrix}$$

In matrix form

$$\bar{\mathbf{x}}(\boldsymbol{\theta}) = \bar{\mathbf{x}}_0 + \mathbf{C}\boldsymbol{\theta}$$

where  $\bar{\mathbf{x}}$  is a  $N$ -dimensional vector with its component being the position coordinates at the quadrupoles,  $\bar{\mathbf{x}}_0 = \bar{\mathbf{x}}(\mathbf{0})$ ,  $\mathbf{C}$  a  $N \times m$  matrix whose  $(j, l)$  element is the  $(1, 2)$  element of  $\mathbf{M}(Q_j | C_l)$  if  $C_l$  is upstream of  $Q_j$  or zero otherwise

The orbit change translates to a change of the IOS, which becomes

$$\boldsymbol{\xi} = \mathbf{A}\mathbf{k}(\boldsymbol{\Delta} - \bar{\mathbf{x}}_0 - \mathbf{C}\boldsymbol{\theta}) = \boldsymbol{\xi}_0 + \mathbf{R}\boldsymbol{\theta}$$

With initial IOS

$$\boldsymbol{\xi}_0 = \mathbf{A}\mathbf{k}(\boldsymbol{\Delta} - \bar{\mathbf{x}}_0)$$

And IOS response matrix

$$\mathbf{R} \equiv \frac{\partial \boldsymbol{\xi}}{\partial \boldsymbol{\theta}} = -\mathbf{A}\mathbf{k}\mathbf{C}$$

We can now correct IOS with

$$\boldsymbol{\theta} = -(\mathbf{R}^T \mathbf{R})^{-1} \mathbf{R}^T \boldsymbol{\xi}_0$$

# Error analysis

- Assume we can choose corrector-quadrupole combinations to avoid degeneracy, and thus correct the IOS to BPM noise level, the precision of quadrupole centers are related to BPM noise via

Covariance matrix of the measured offsets

$$\Sigma_{\Delta\Delta} = \sigma_{\text{BPM}}^2 (\mathbf{R}_Q^T \mathbf{R}_Q)^{-1} \quad \text{for uniform noise sigma across all BPMs}$$

With quadrupole center offset ( $\Delta$ ) to IOS response matrix

$$\mathbf{R}_Q \equiv \frac{\partial \xi}{\partial \Delta} = \mathbf{A} \mathbf{k}$$

Therefore, to reduce error bar

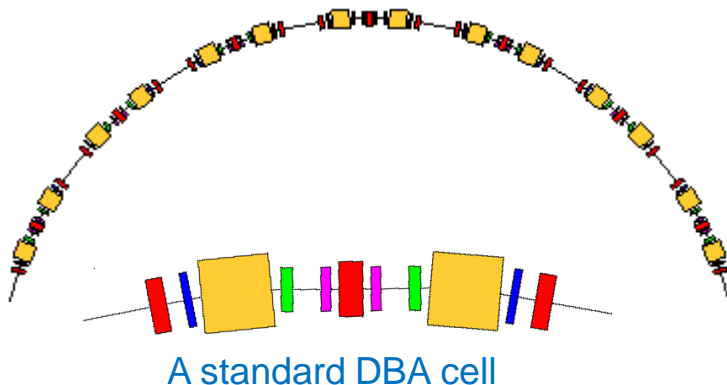
- Use BPMs with large quad-to-BPM response (for IOS target)
- Use more BPMs
- Increase the modulation level



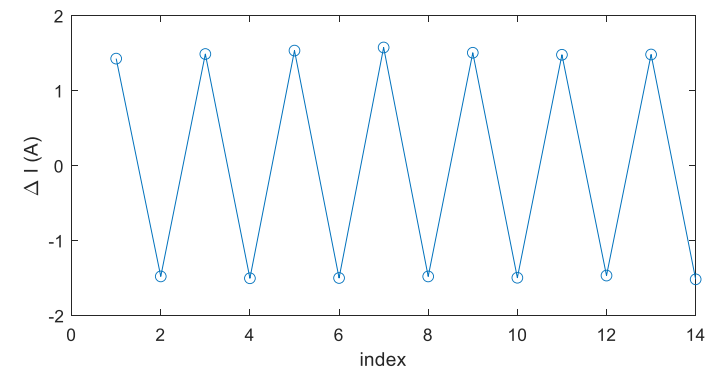
# Application at SPEAR3

- The analysis to a ring is much the same, except now closed orbit is used instead of one-pass trajectory
- A SPEAR3 example: 14 QF magnets
  - The ring has 14 standard DBA cells, each with 2 QF magnets and 2 QD magnets
  - The QF/QDs at the same location in a cell can form a group

Standard DBA cells on half of ring



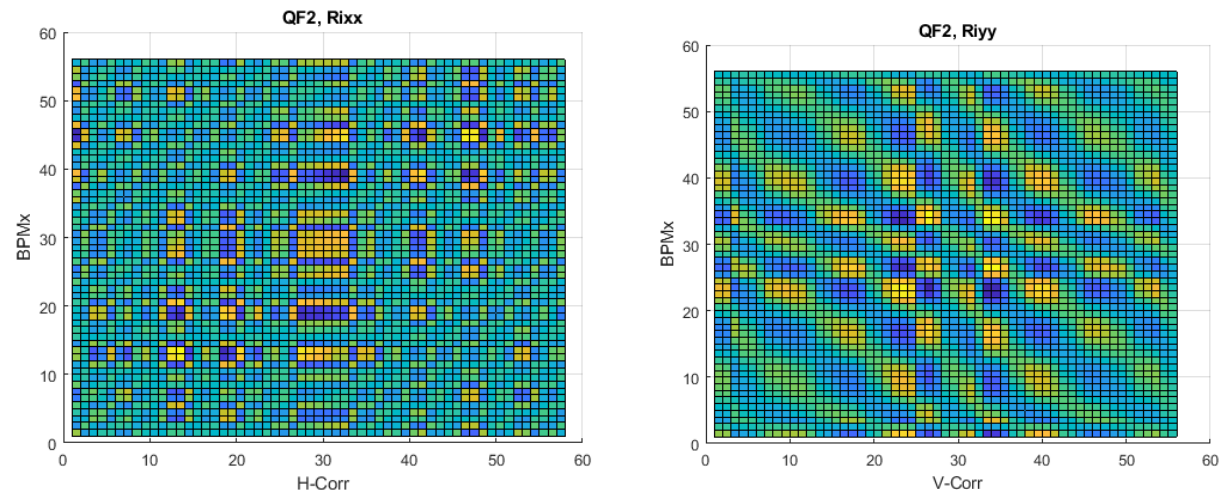
Quadrupole modulation pattern:  
Alternate sign to avoid large shift of tunes



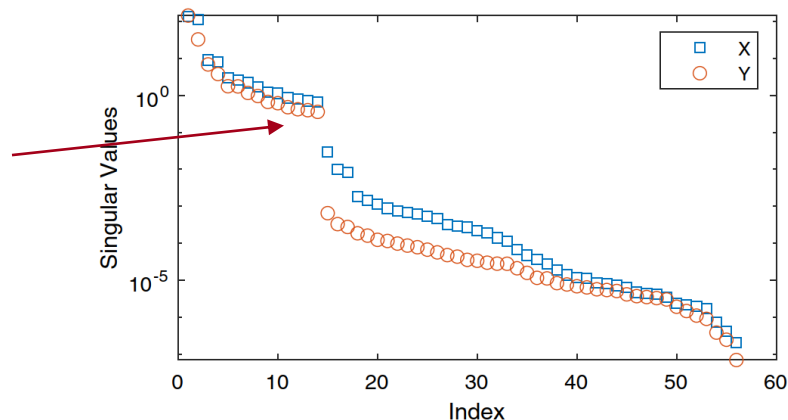
# SPEAR3 example – QF2

- Correct IOS with all correctors: 58 H-Corr and 56 V-Corr
- Use all 56 BPMs to measure IOS

IOS response matrix  
Modulation +/-2%



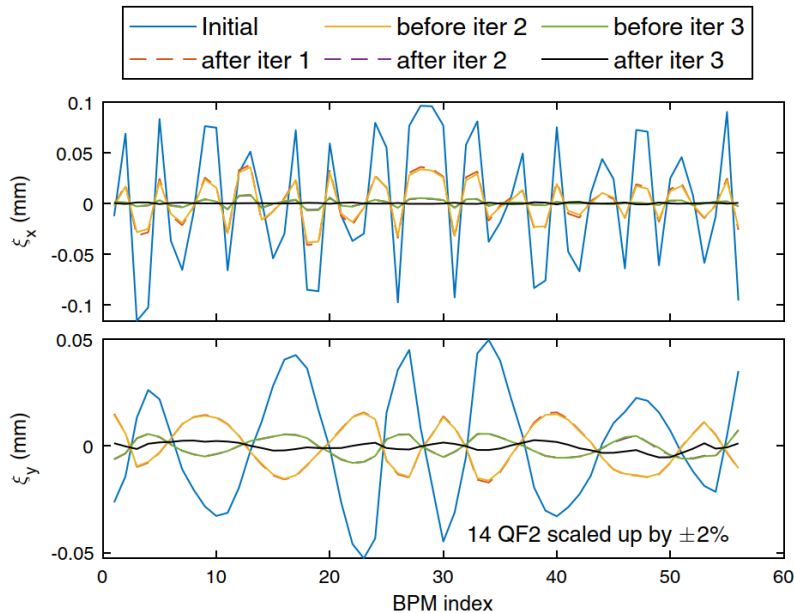
There are 14 non-trivial singular values of the IOS response matrices



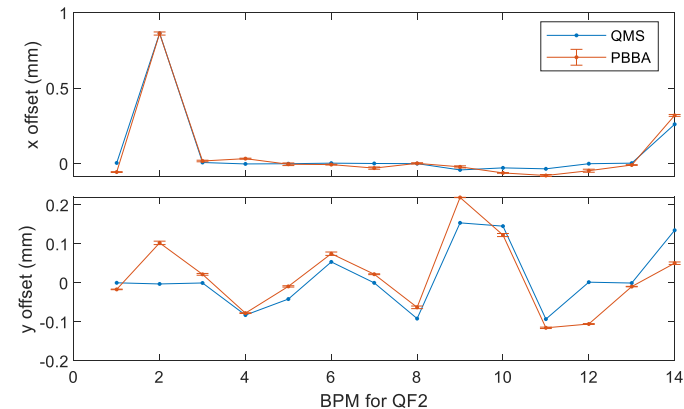
# SPEAR example –correction result

The orbit is steered toward the quadrupole centers (at the 14 QF quads)

The IOS before and after correction

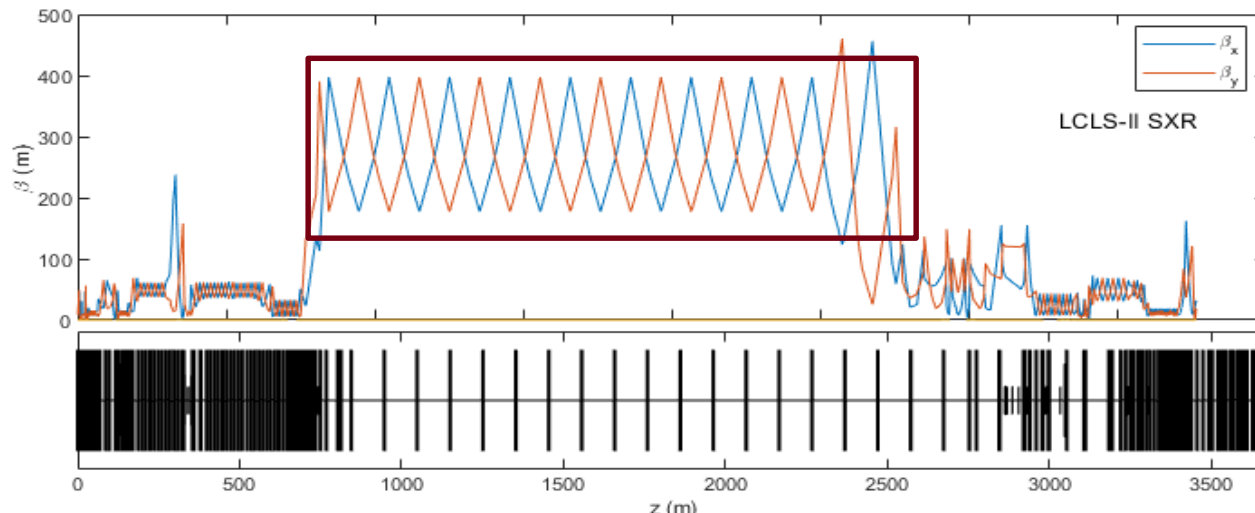


The quadrupole centers agree with the usual BBA method (QMS)



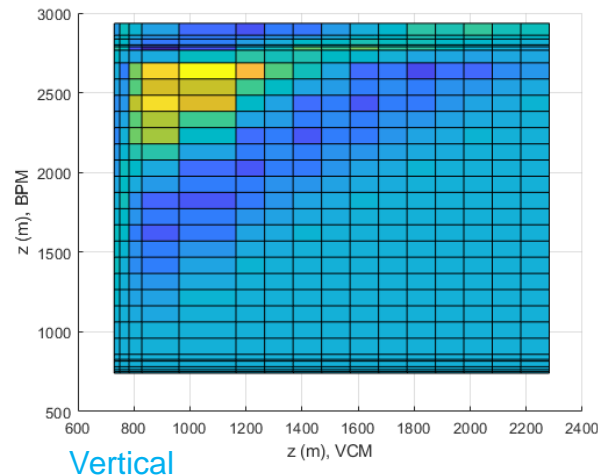
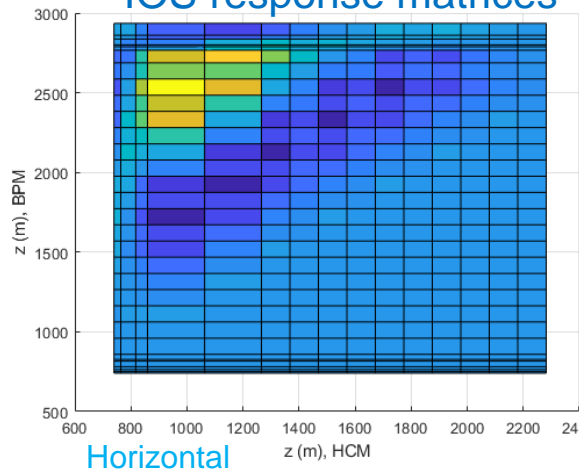
# Application at LCLS-II

- The method has been applied to LCLS-II commissioning  
The bypass line has 15 quadrupoles powered by one serial PS.

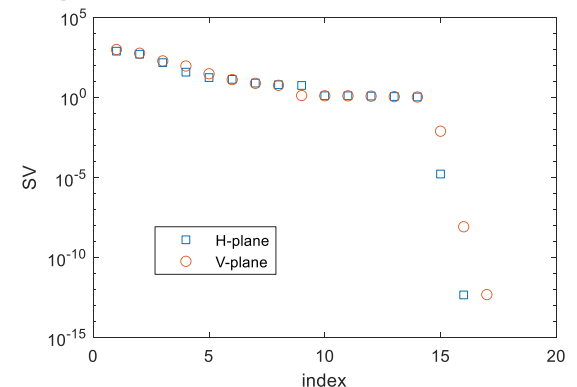


31 BPMs  
16 H-Corr  
17 V-Corr  
15 quads

IOS response matrices



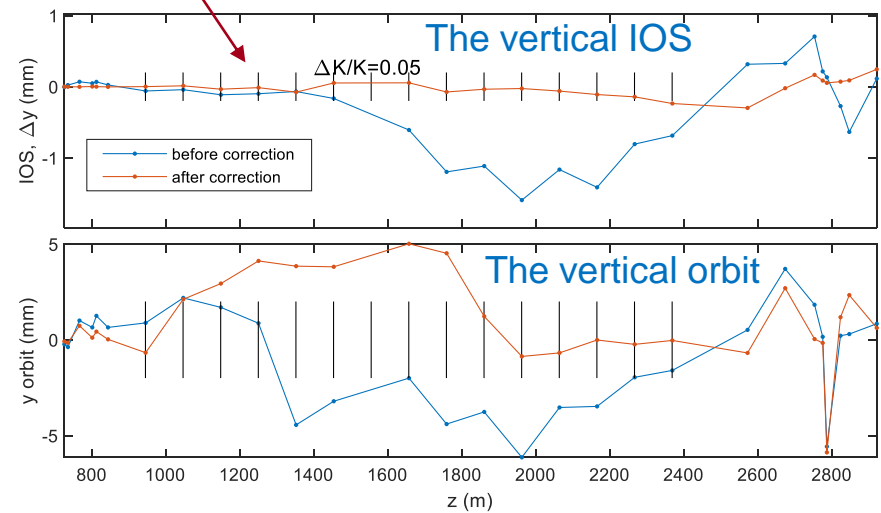
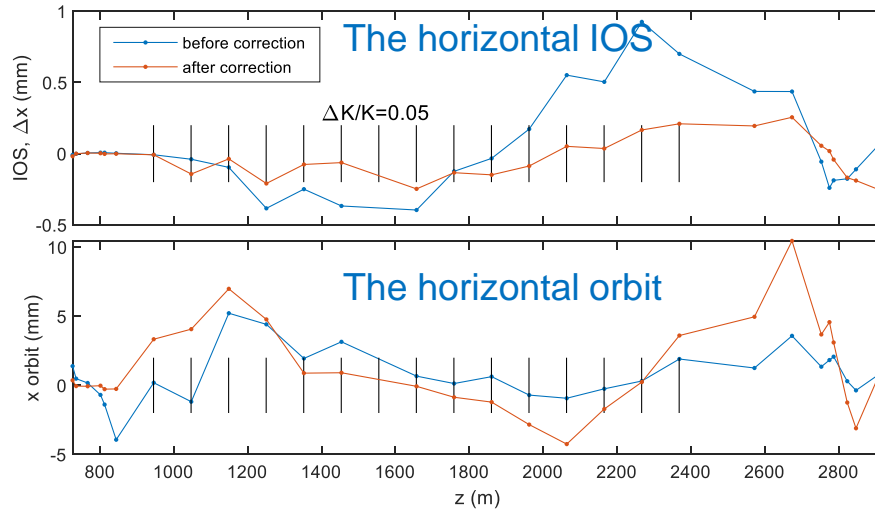
Singular values of IOS response matrices



# LCLS-II example: IOS correction for bypass quads

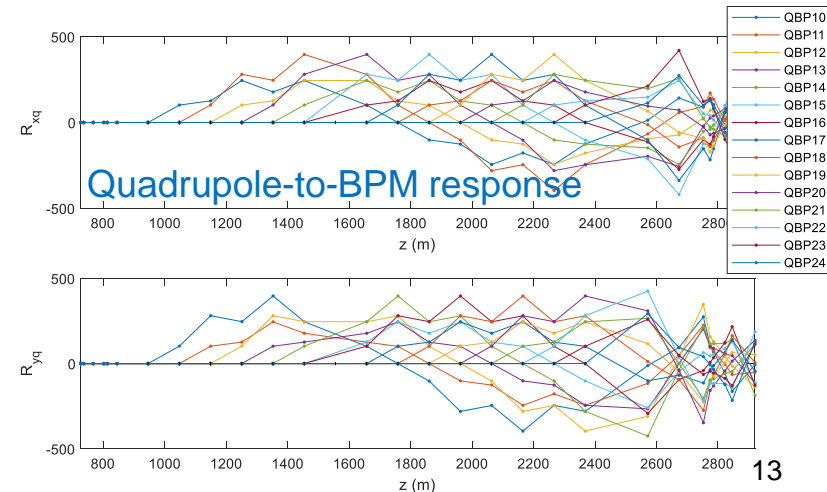
- Correction of IOS steered the beam toward quadrupoles centers

Vertical bars mark quadrupole locations



- Correction utilizes 7 SVs for both planes
  - Excluding some SVs to avoid stressing the correctors

BPM readings near the bypass line quadrupoles for the new orbit can be used as steering target

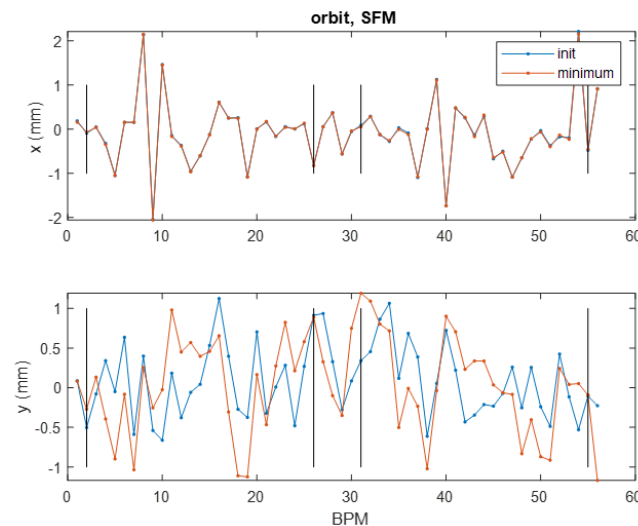


# Minimizing IOS for sextupoles

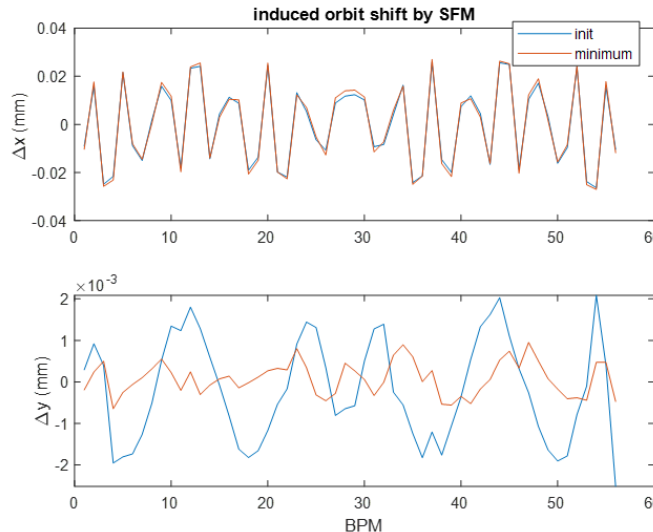
- The approach of steering orbit to minimize IOS can be applied to sextupoles
  - An optimization algorithm can be used, instead of using response matrix

SPEAR3 example: SFM family (w/ 4 pairs of magnets in 4 matching cells)

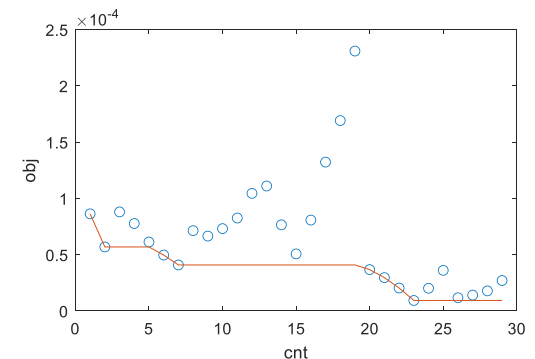
For the vertical offsets



The beam orbit before and after optimization



The measured IOS before and after optimization



Minimize IOS (y-plane by SFM) w/ RCDS

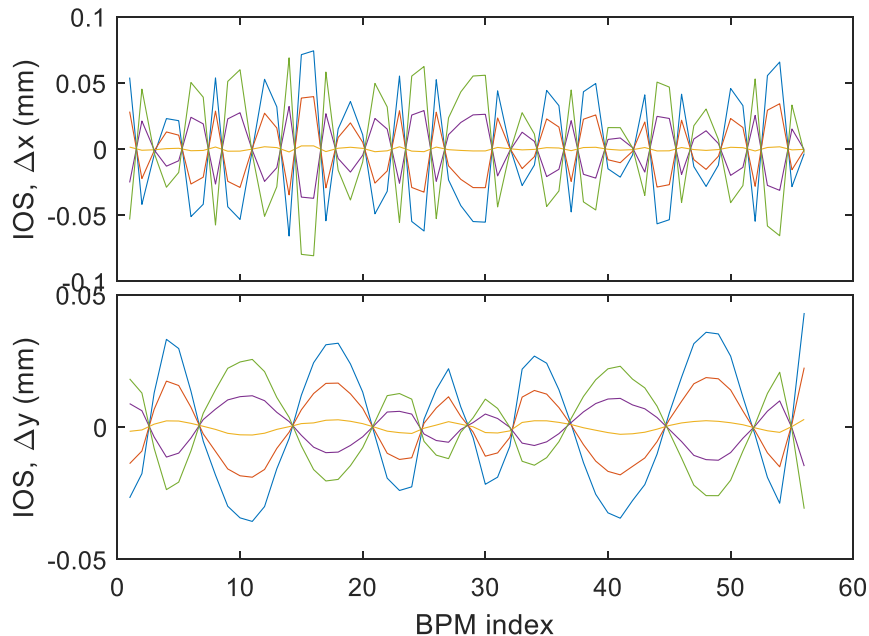
## Method 2: deduce quadrupole kicks from IOS w/ model, use steering to find quadrupole centers

- Assuming the machine lattice is close to the model, we can calculate the kicks by the quads from IOS measurement
  - By inverting the quadrupole-to-BPM response matrix
- By steering the beam orbit and repeating the measurements, we can determine the quadrupole centers
  - In the same fashion as the usual 'bowtie' method
  - A kick-vs-orbit plot for each quadrupole is obtained. Quadrupole center is the zero-crossing of IOS.
  - Two correctors (w/  $\sim 90^\circ$  phase advance) are used to steer the beam (instead of one in the usual method), so that orbit is shifted at all quadrupoles

This method may be called 'parallel QMS' since the usual bowtie method is called QMS.

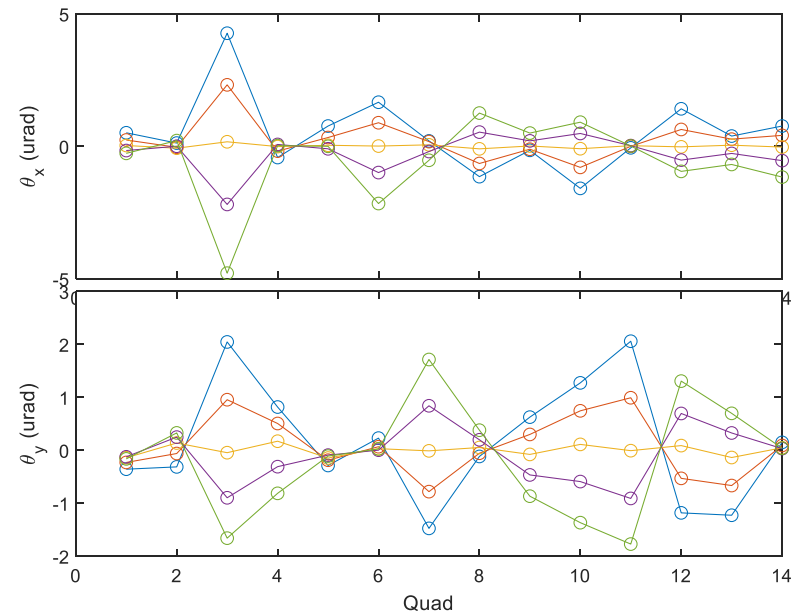
# Application to SPEAR3: from IOS to quadrupole kicks

- The same QF2 family as an example



Measured IOS at 5 quadrupole modulation levels, up to 4%

For each IOS measurement, we 'solve' for kicks at the quads

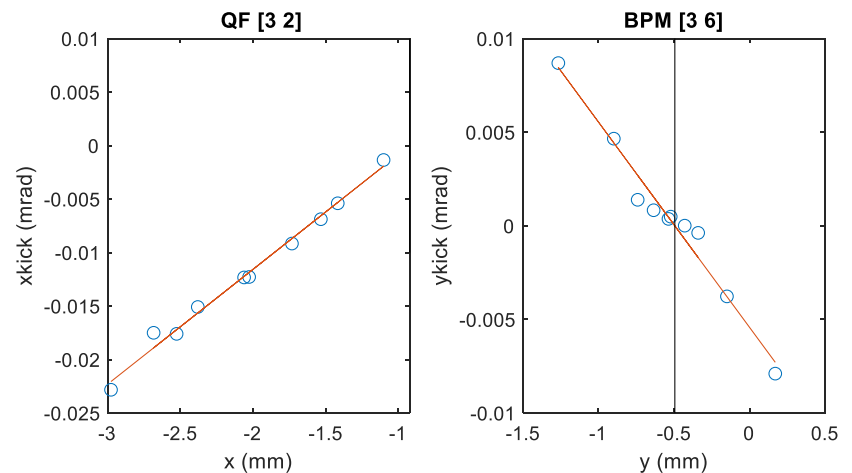
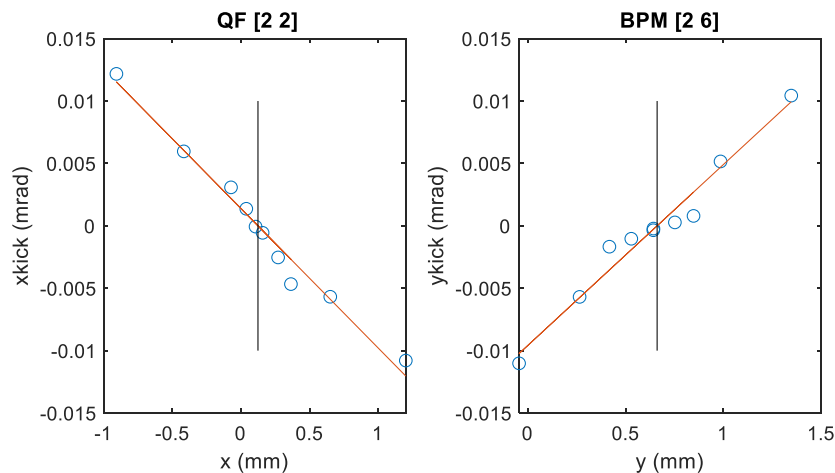


The corresponding quadrupole kicks



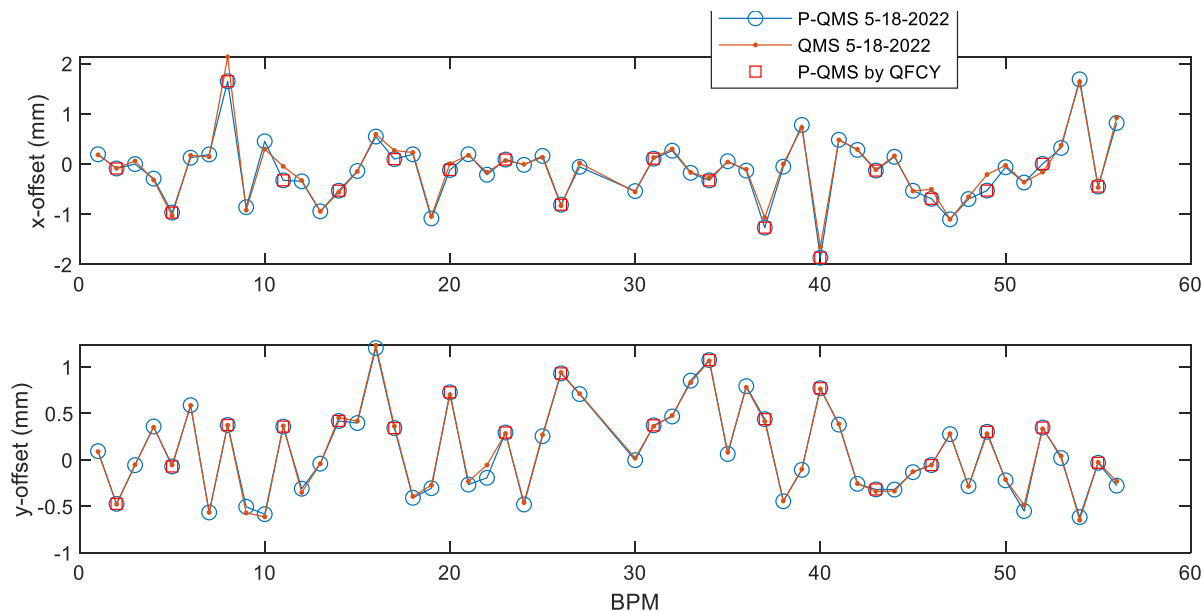
# SPEAR3 example – determination of quads centers

- The kicks vary with orbits at the quadrupole location. Fit for zero crossing
  - The center is as registered by the nearby BPM. It is model independent (same as the usual QMS method).



# SPEAR3 example – compare to QMS

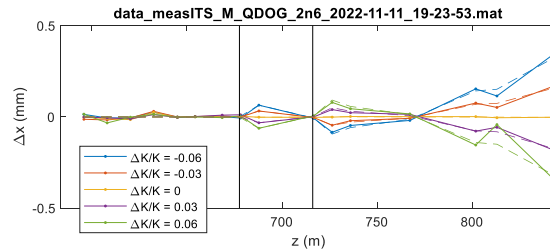
- The P-QMS results are in good agreement with the usual QMS
  - P-QMS is 5 times faster!
  - (But not as fast as P-BBA since it repeats IOS measurement multiple times)



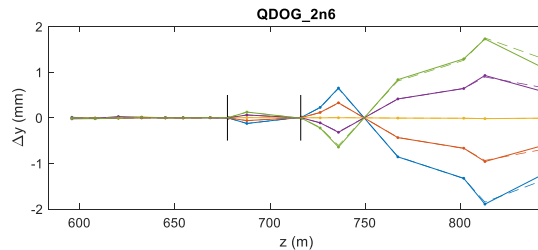
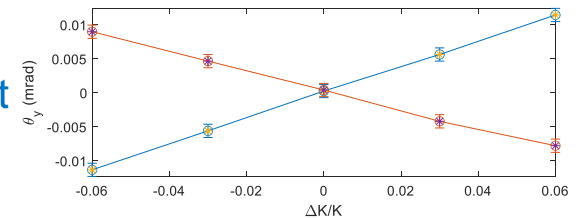
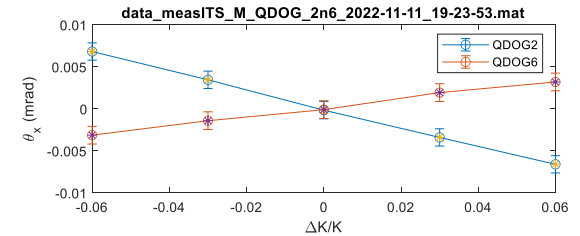
The largest difference are from QFC+QFY group (18 quads, marked in squares), as QFC cannot alternate signs and thus modulation level is low (only 1%), and also because of the large optics distortion (more in horizontal plane).

# Application to LCLS-II: Dogleg quadrupoles

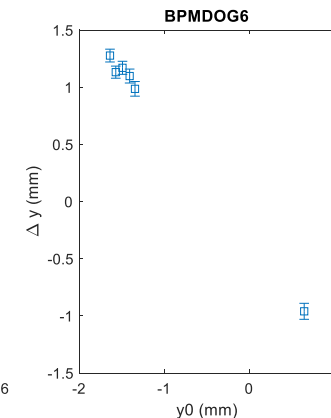
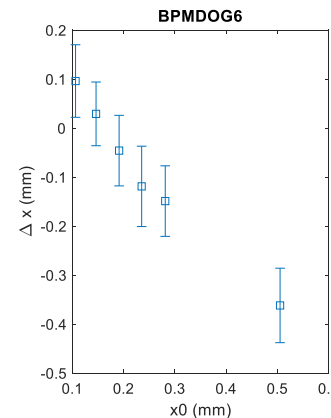
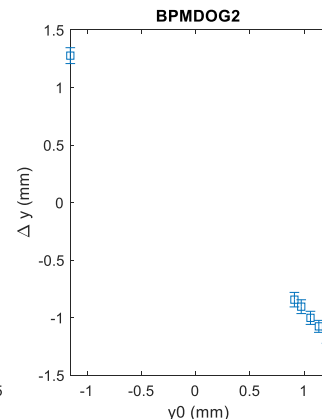
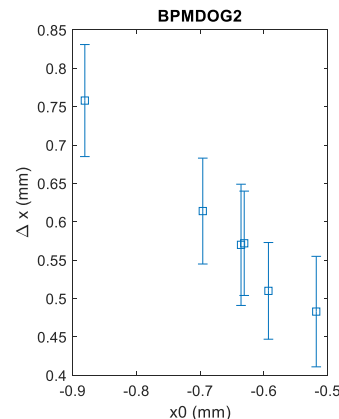
- Quadrupole pairs in Dogleg area



Measured IOS  
For a pair of quads



Calculate kicks w/ quad-  
to-BPM response, then fit  
kicks vs. modulation for  
the offset



Repeat the process with steered beam orbit, find the  
zero-crossing.

- We proposed and tested two methods for parallel BBA
  - PBBA and P-QMS
- Method 1 (PBBA): correction or minimization of induced orbit shifts
- Method 2 (P-QMS): resolve kicks by quadrupoles (or sextupoles) with model calculated response matrix; scan the orbits to find zero-crossing
- Both methods were applied to storage ring (SPEAR3) and one-pass system (LCLS and LCLS-II) successfully
- Both method can be extended to nonlinear magnets