

# Mitigation of synchro-betatron resonances for the FCC-ee booster

Optics Tuning and Correction for Future Colliders Workshop

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CERN

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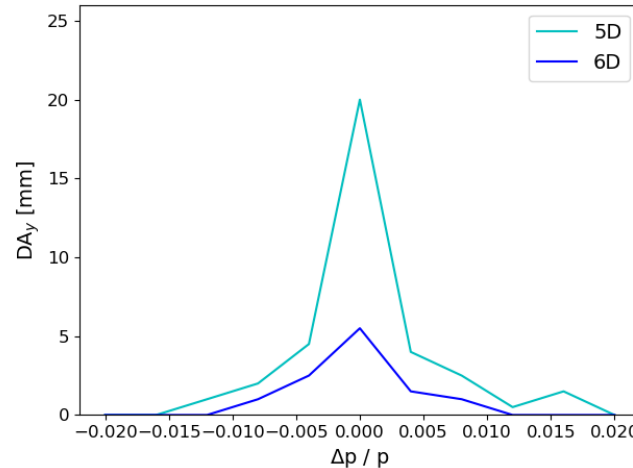
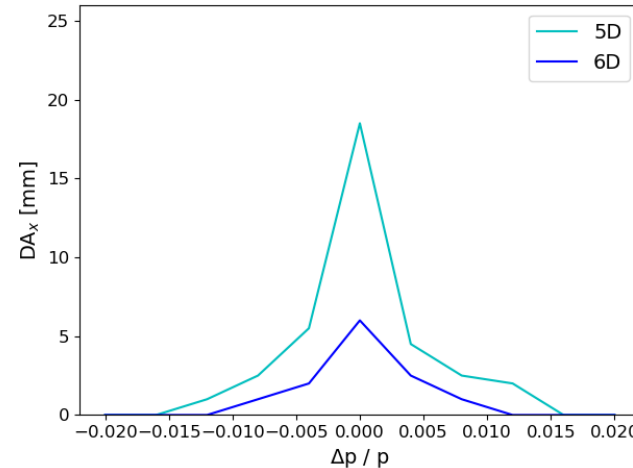
FCCIS – The Future Circular Collider Innovation Study. This INFRADEV Research and Innovation Action project receives funding from the European Union's H2020 Framework Programme under grant agreement no. 951754.



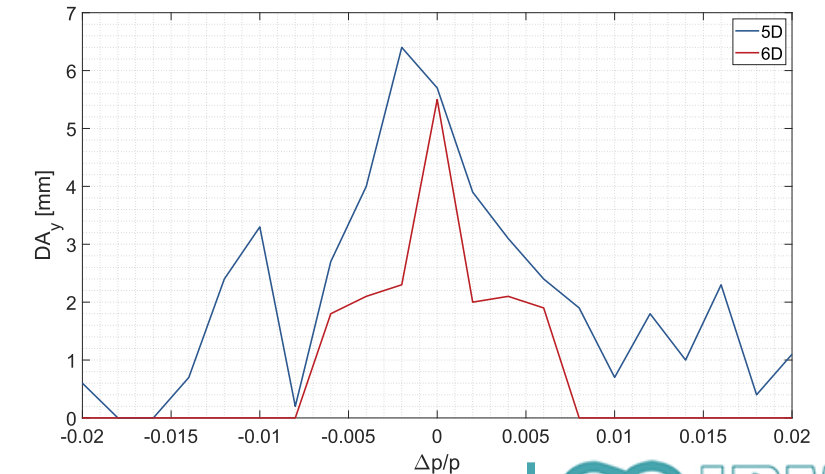
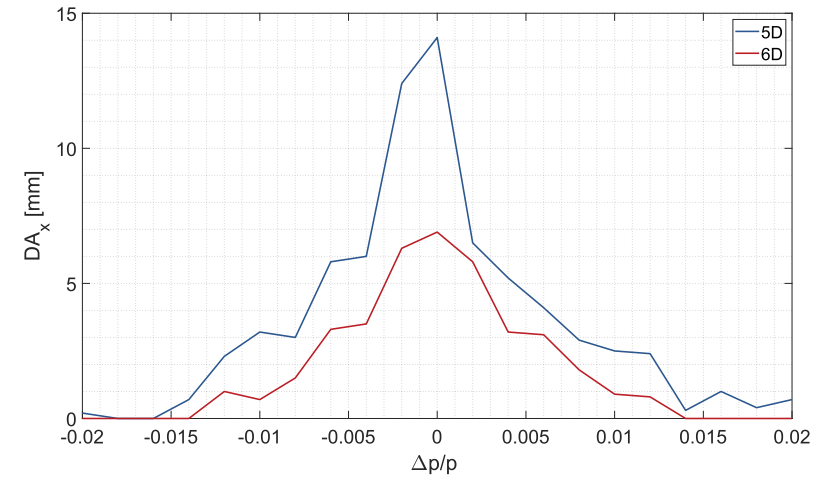
Tracking the particles in presence of RF cavities (6D tracking) dramatically diminished the size of Dynamic Aperture.  
( Lattice reference : PA31, Mode\_tt [90°/90°], @ 20 GeV)  
( RF : 400 MHz , Total Voltage : 62 MV )

- The results of tracking in PTC and AT are not exactly the same, especially for vertical plane.
- But, the aperture reduction and poor off-momentum aperture were confirmed by both of them.
- In the first step of the analysis of the 6D tracking, the study has been limited to the tracking of on-momentum particles in the positive direction of the horizontal axis.

PTC

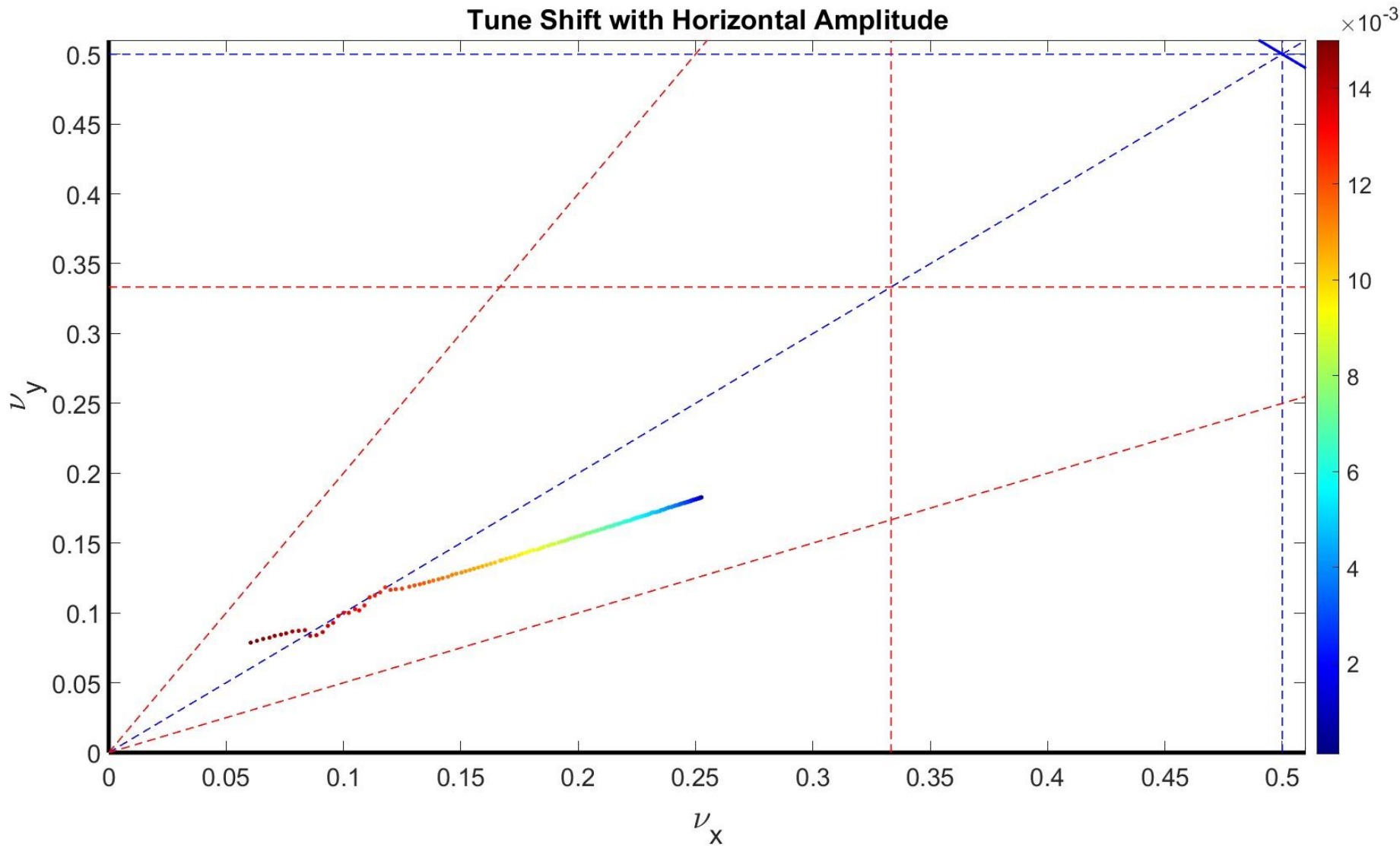


AT





## 5D vs 6D Tracking – Frequency Domain

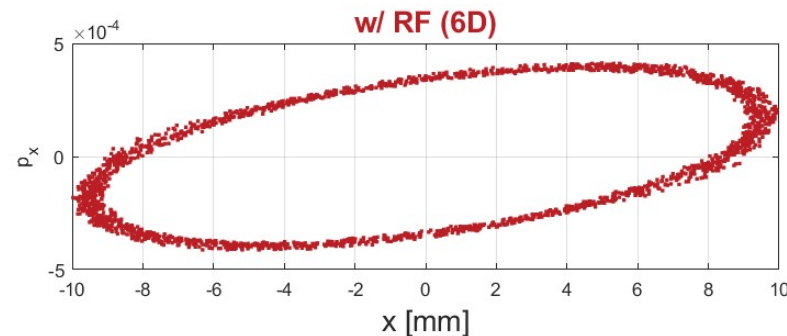
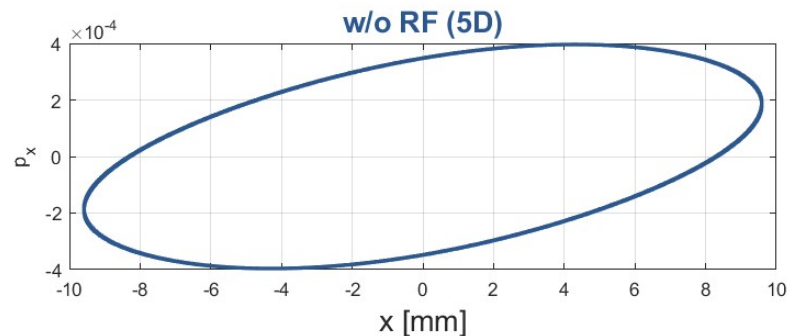


nds

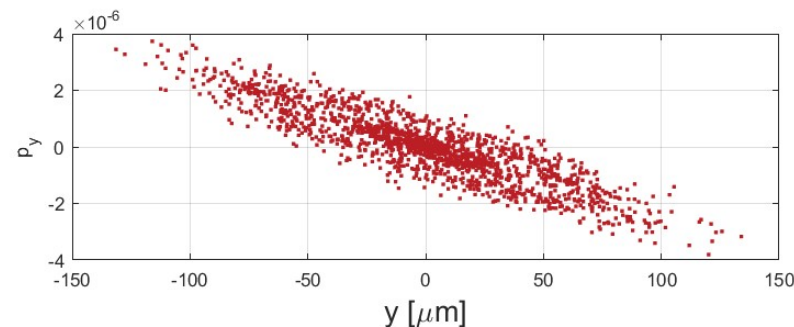
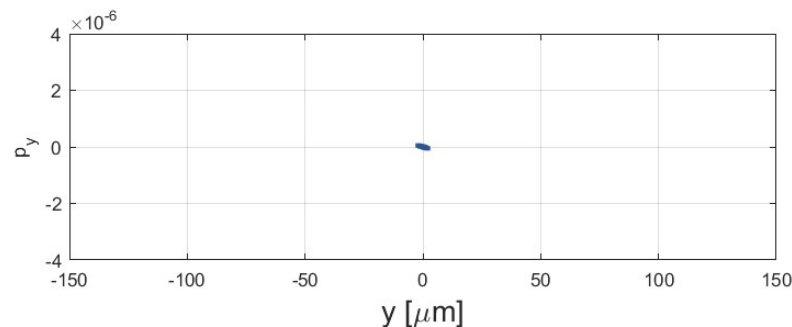


## 5D vs 6D Tracking – Phase Space

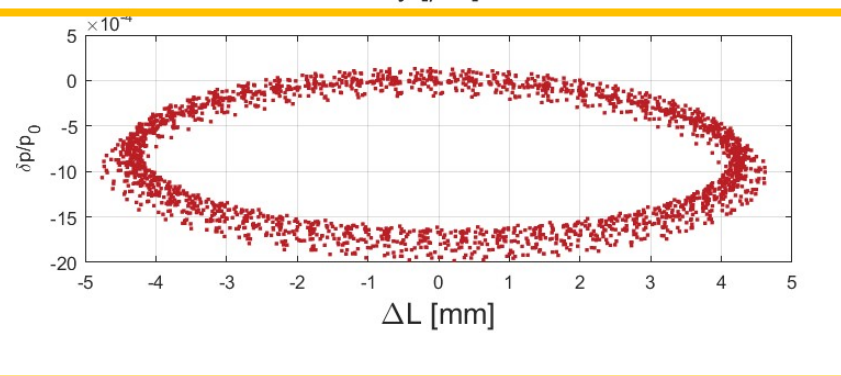
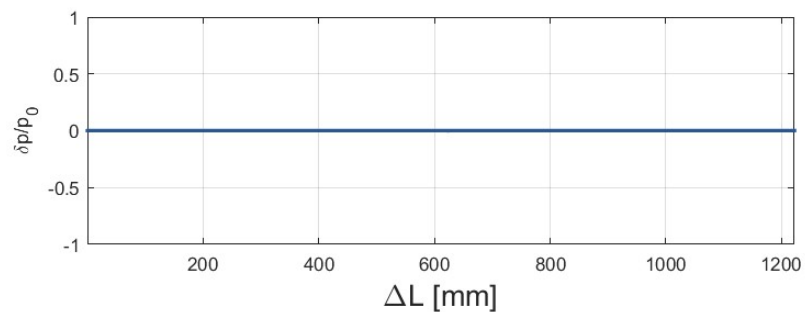
initial Condition : [8.3 mm, 0, 1  $\mu\text{m}$ , 0, 0, 0] ; No.Turn : 2250



: identical



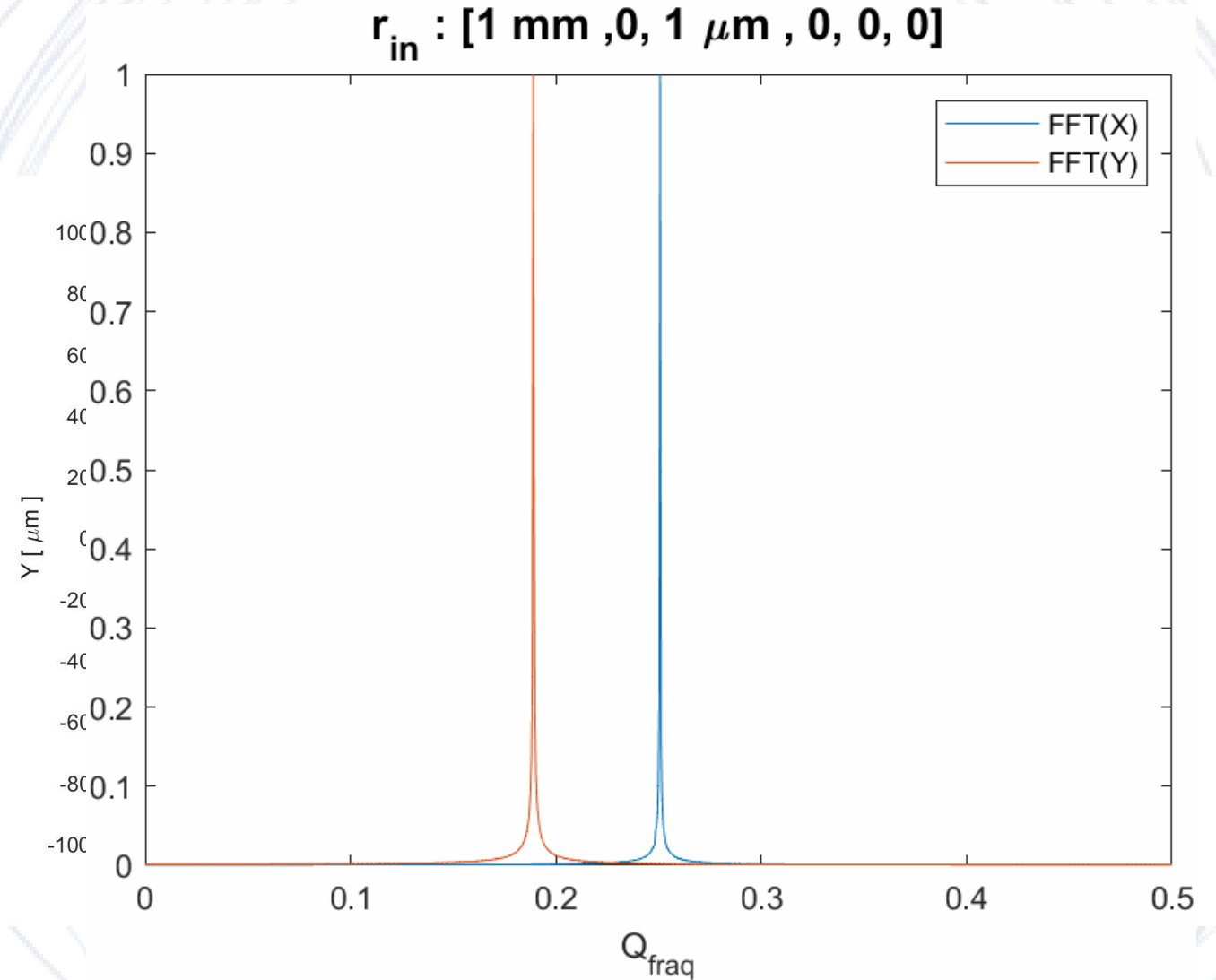
Emittance  
to grow from  
amplitude.



Shift in the  
reference  
momentum

The growth of vertical amplitude starts when the contribution of sideband resonance increased and the 1<sup>st</sup> order sidebands get close to each other.

It's like the emittance exchange due to crossing coupling resonance but for the sidebands!



## Shift in the Reference Momentum

- Shift in the reference momentum could be well predicted by study the higher order longitudinal phase space.
- Polynomial expansion of path length variation up to second order:

$$\frac{\Delta L}{L_0} = \alpha_c \delta + \alpha_1 \delta^2 + \xi + \mathcal{O}(3) \quad , \quad \delta = \frac{\Delta p}{p_0}$$

All three parameters vary by initial amplitude:

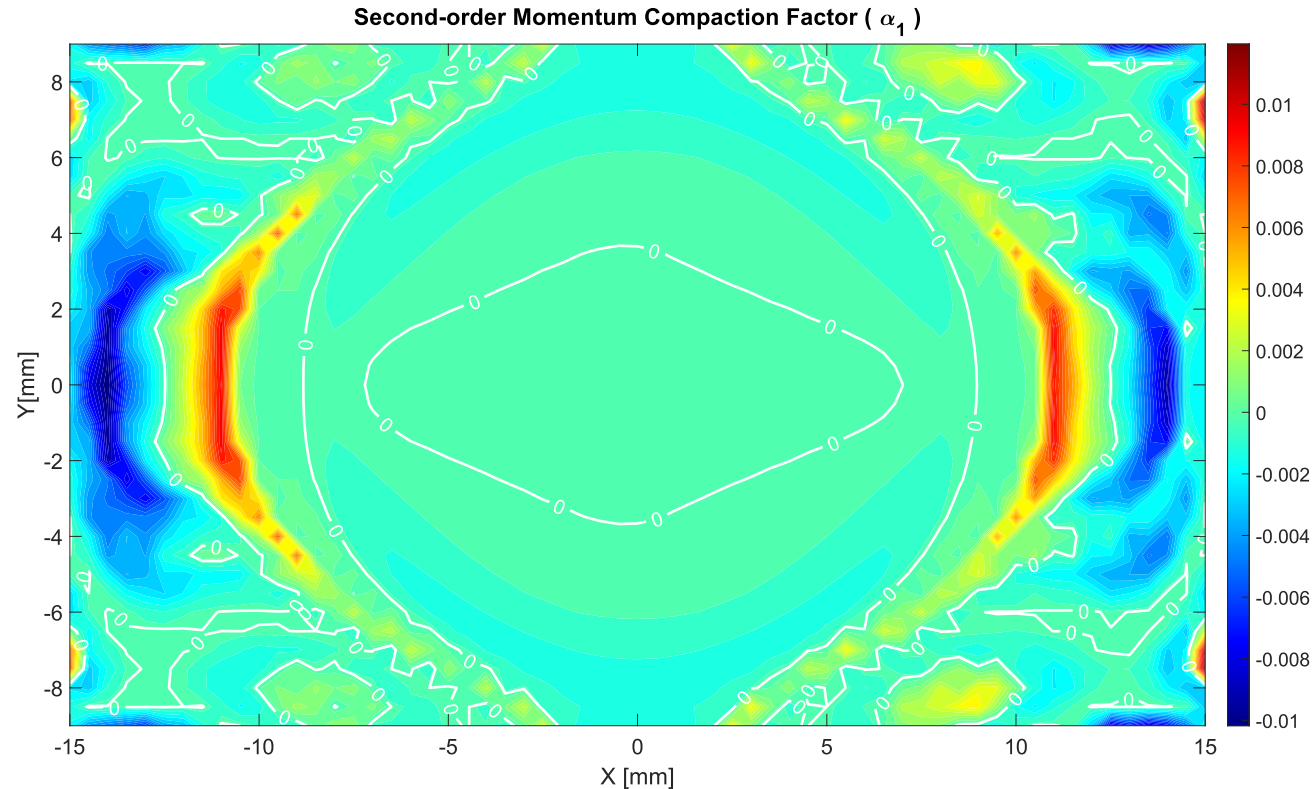




Fig. 6.22. Higher order longitudinal phase space diagrams for  $\psi_s = 0.7$ ,  $2\xi/\eta_c = -0.125$ , and a weak perturbation  $\alpha_1/\eta_c = -3.0$

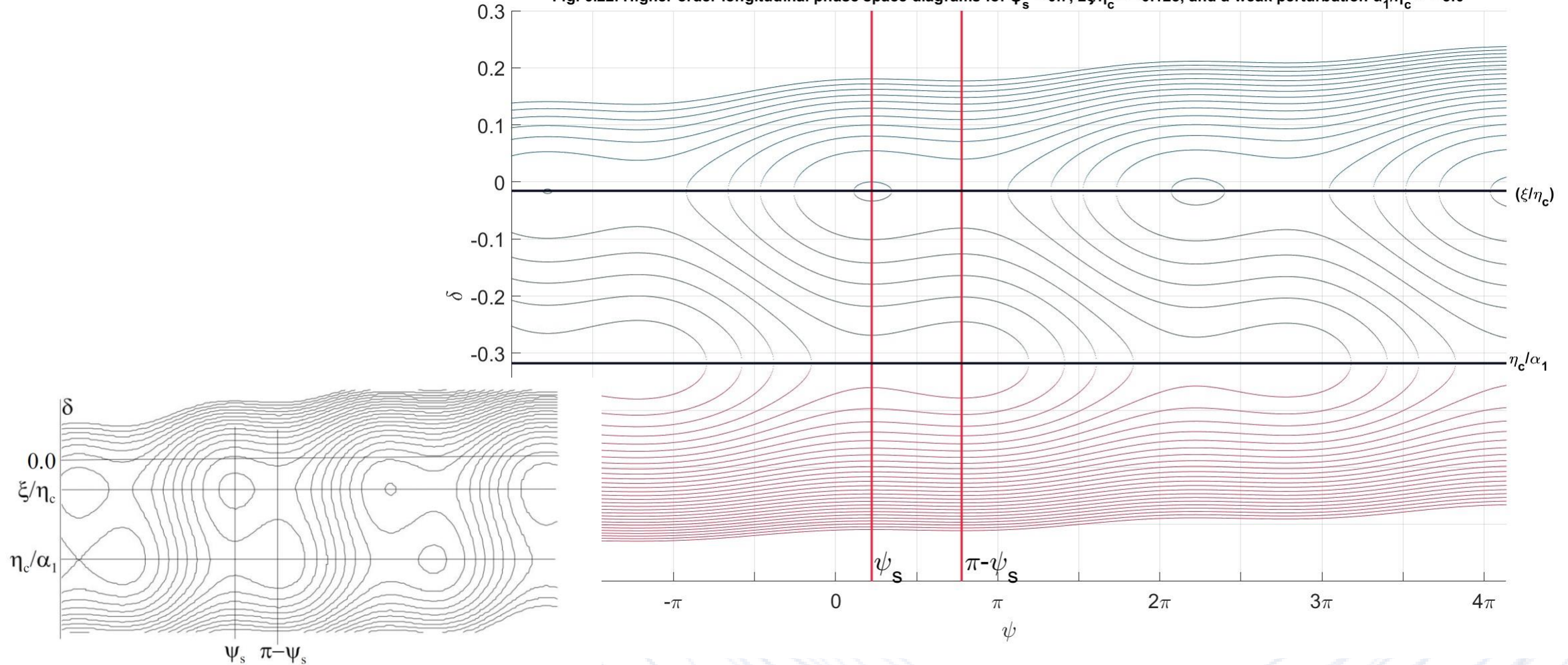


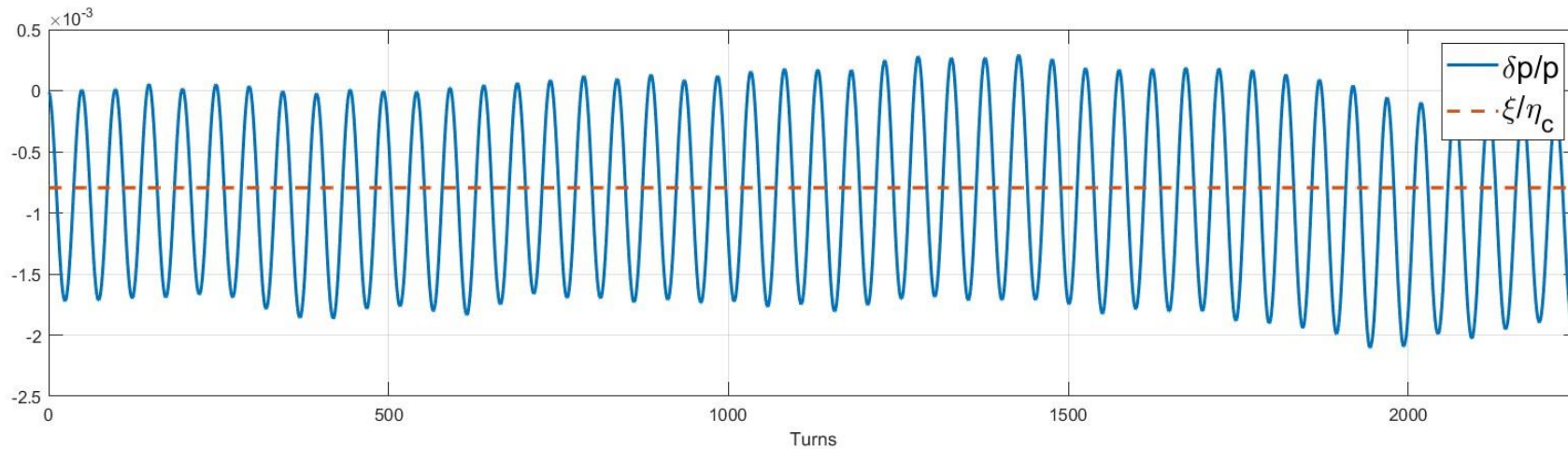
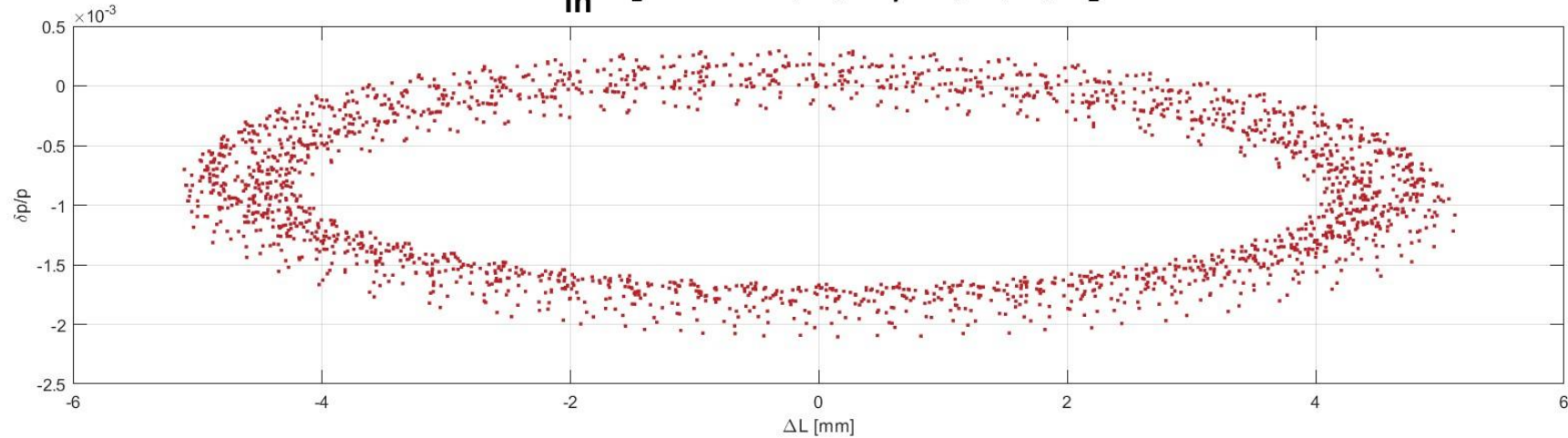
Fig. 6.22. Higher order longitudinal phase space diagrams for  $\psi_s = 0.7$ ,  $2\xi/\eta_c = -0.125$ , and a weak perturbation  $\alpha_1/\eta_c = -3.0$

**H.Wiedemann, Particle Accelerator Physics, 3rd edition**



## Shift in Reference Momentum

$$\mathbf{R}_{in} : [ 8.3 \text{ mm}, 0, 1 \text{ } \mu\text{m}, 0, 0, 0 ]$$

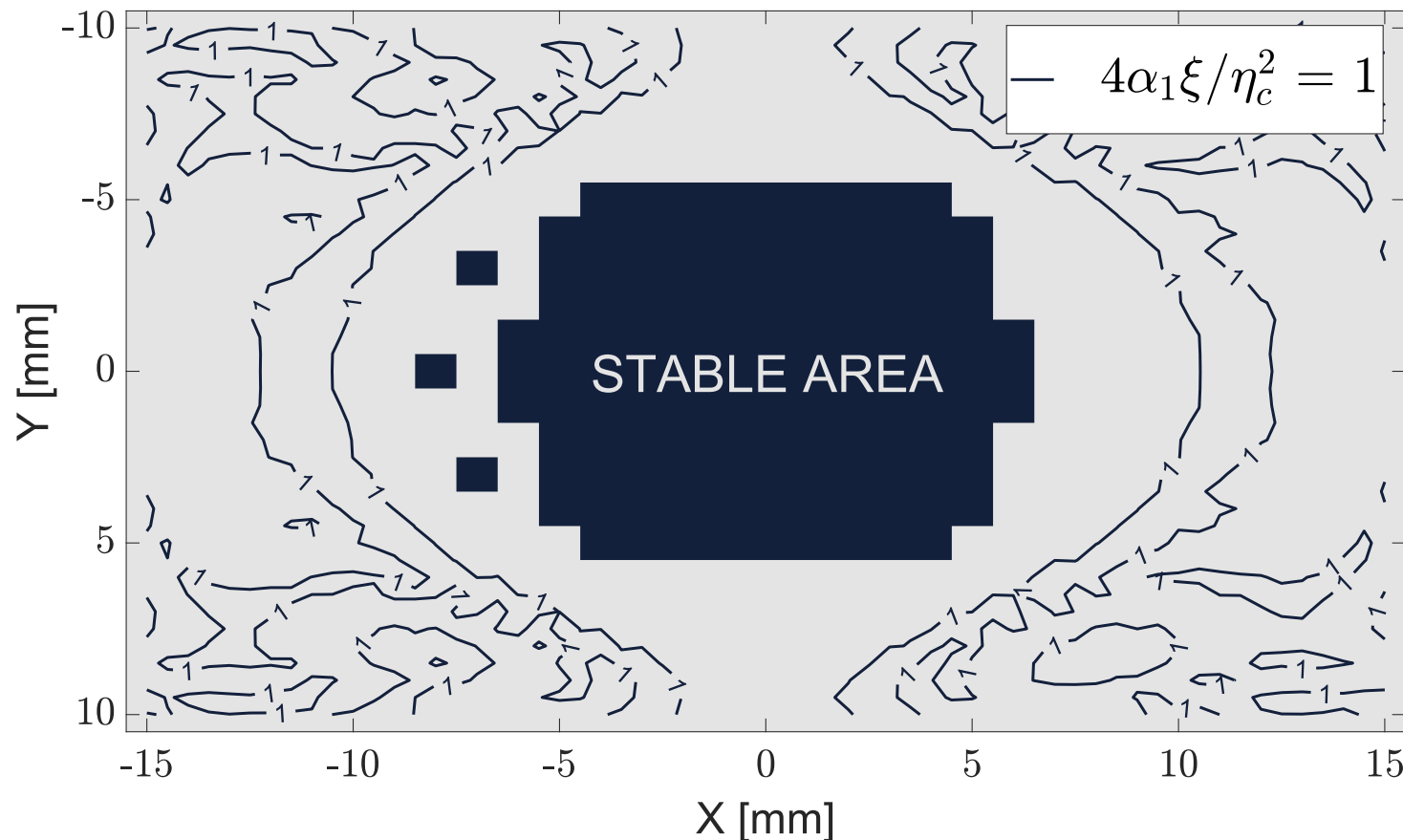




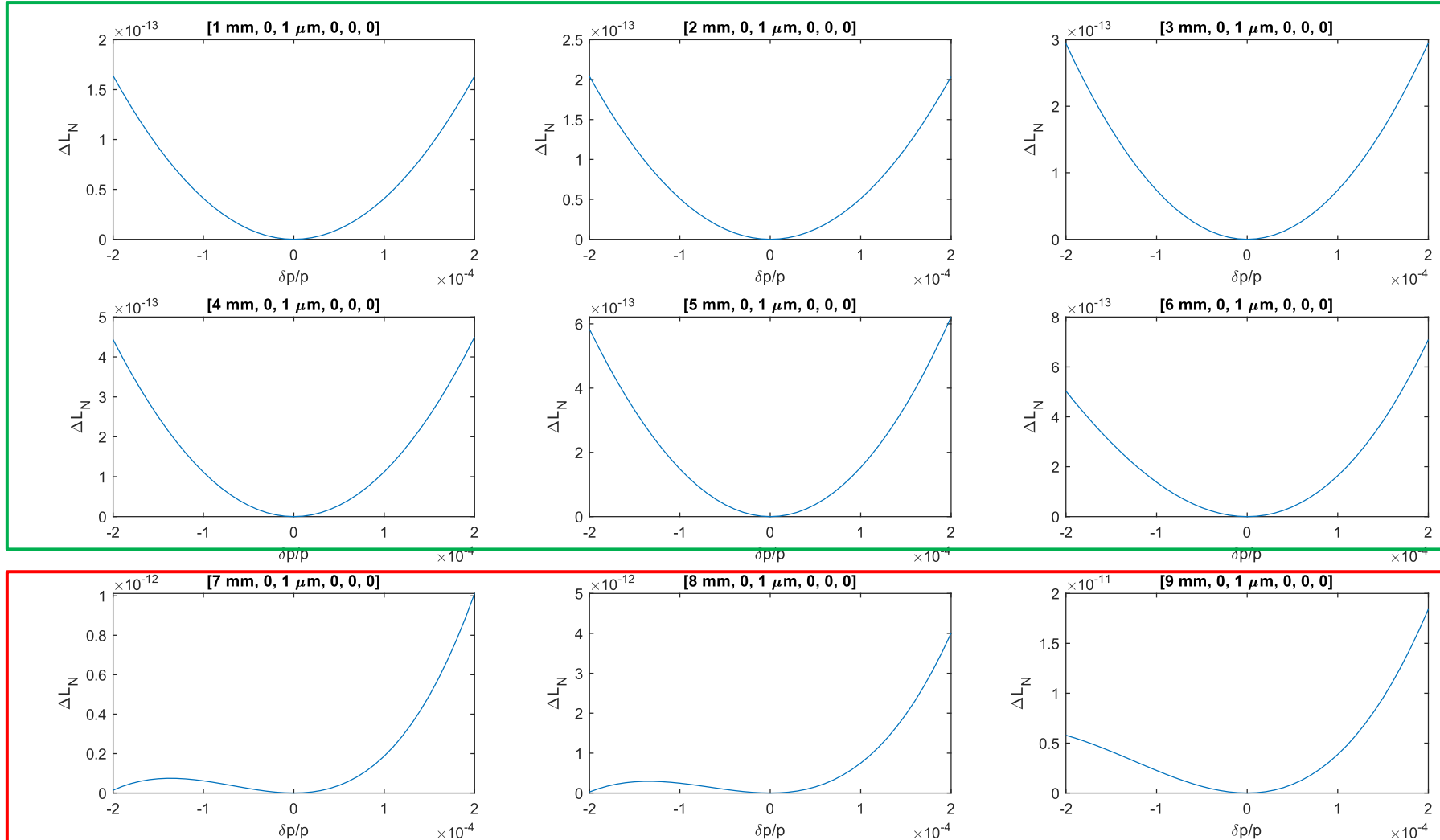
- The longitudinal phase space of particles at HEB is under the influence of the higher-order momentum compaction factor terms.
- The stability criterion for stable synchrotron motion is defined by:

$$\frac{4\xi\alpha_1}{\eta_c^2} < 1$$

A rough estimate of the DA could be achieved from this criterion



- Is the second-order polynomial expansion of the path length variation valid for all initial transverse amplitudes?



$$\Delta L_N = \frac{\Delta L}{L_0} - \alpha_c \delta - \xi$$

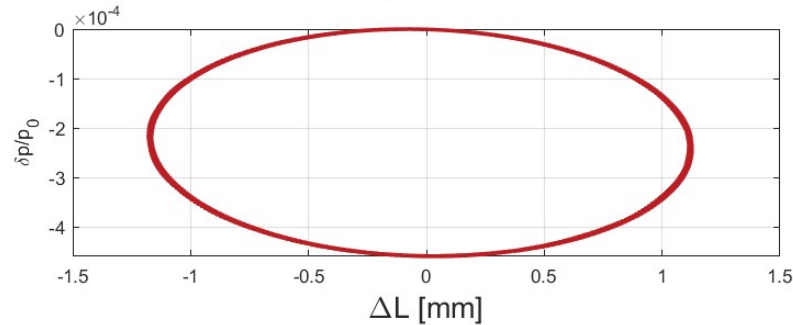
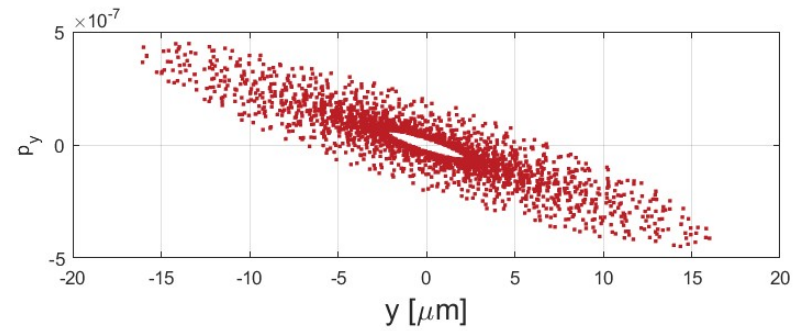
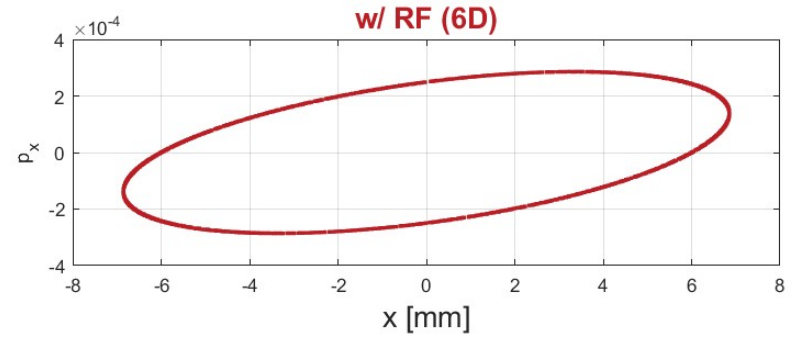
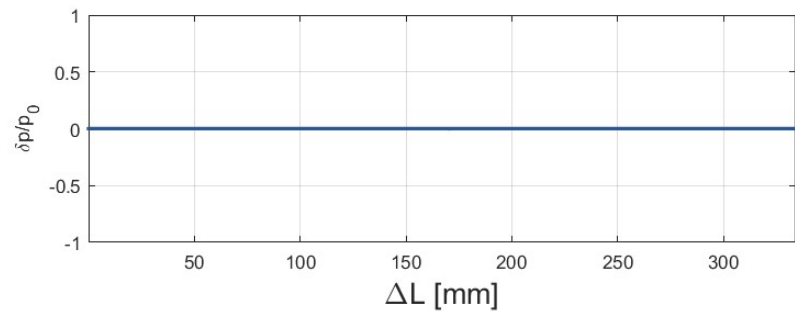
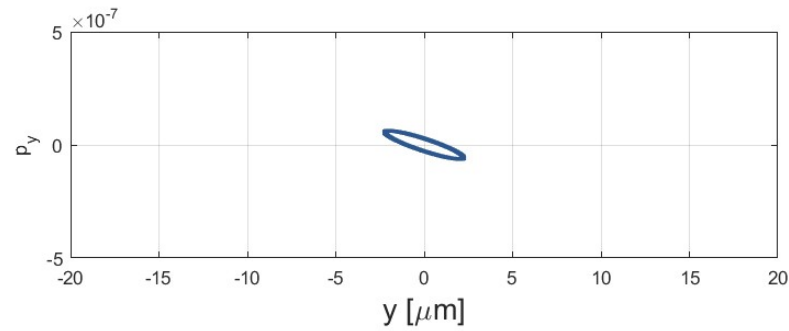
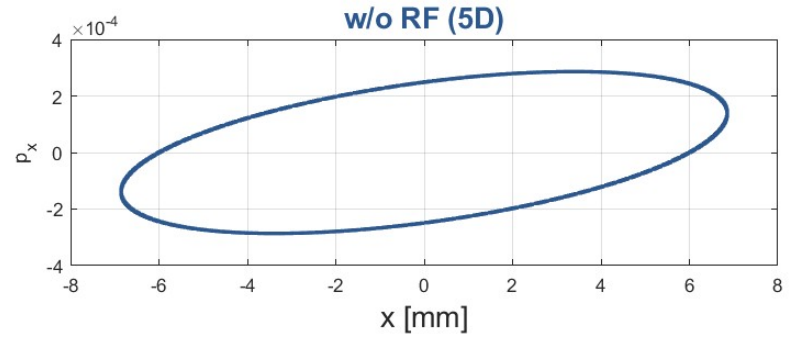
The third-order polynomial could accurately be fitted to these curves

- In order to compensate for the destructive effects of RF cavities presence and restore the 5D dynamic aperture the following adjustments seem to be beneficial:
  - Minimizing the momentum-independent path length variation to control the synchro-betatron resonance.
  - At the same time, control the higher-order momentum compaction factor with respect to the synchrotron oscillation stability criteria.
  - Modifying the stability criteria by including at least a third-order momentum compaction factor to cover the higher amplitudes.

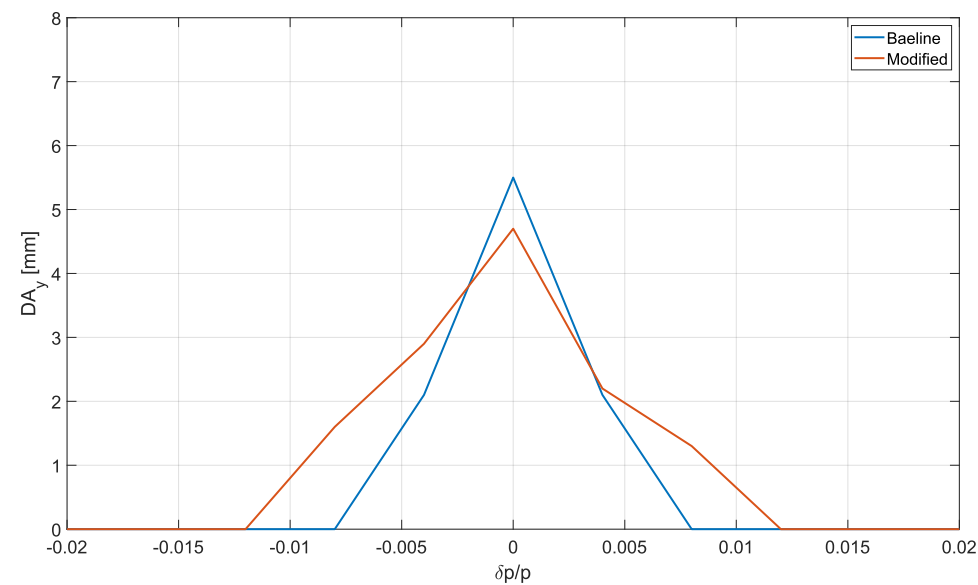
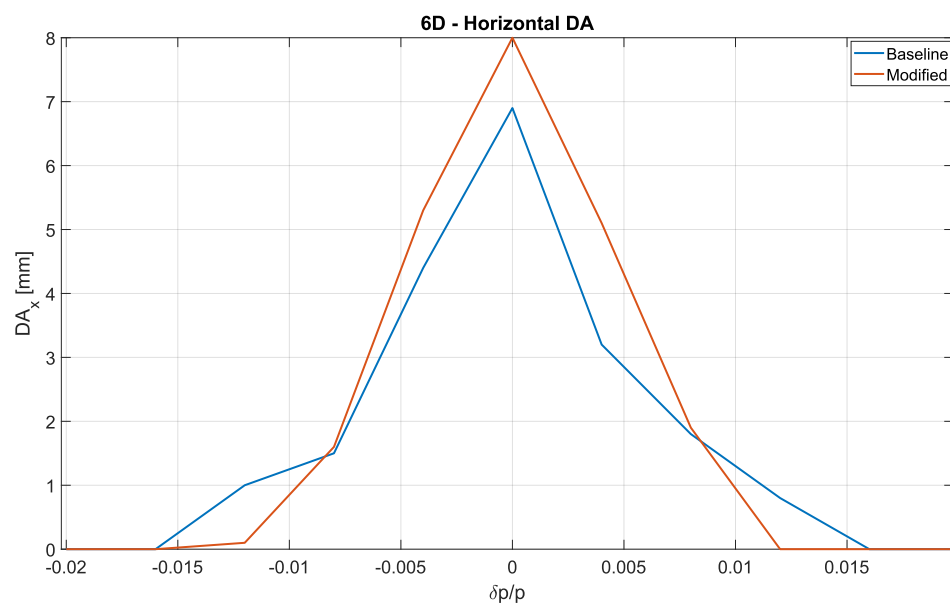
# Thank you for your attention

PA31

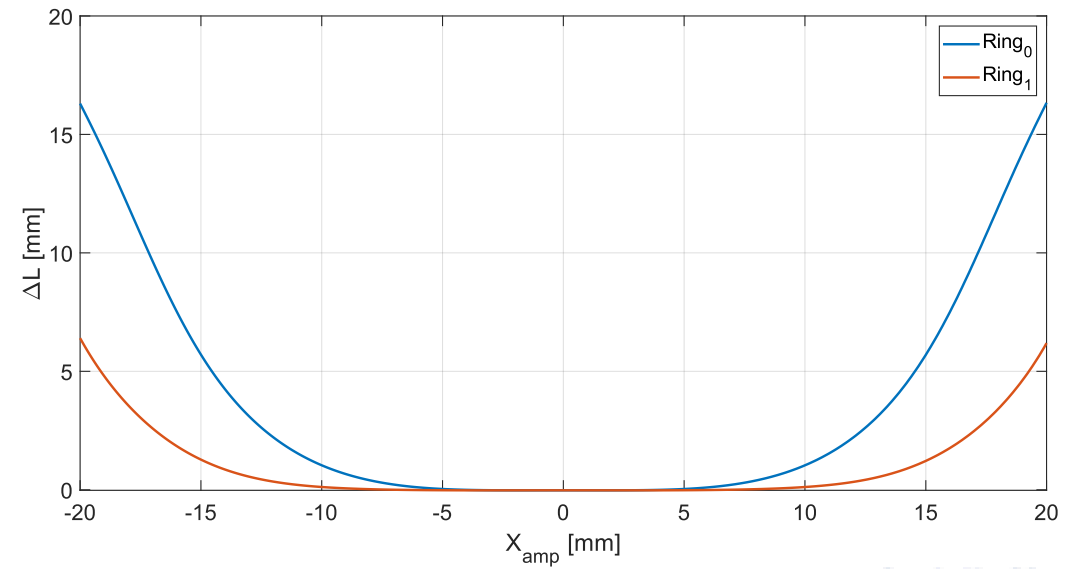
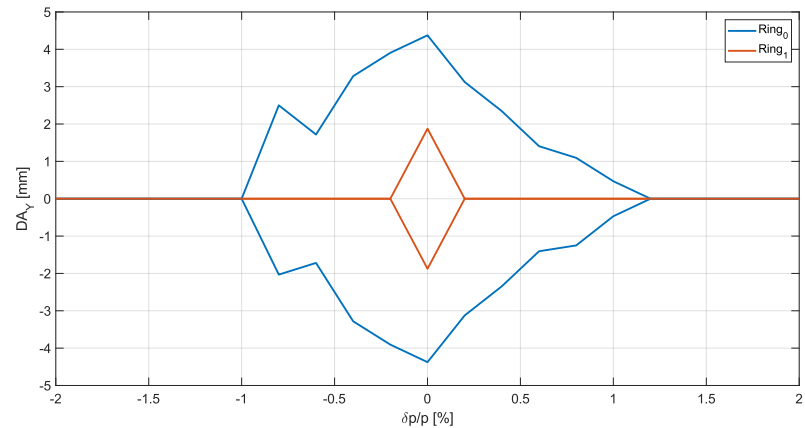
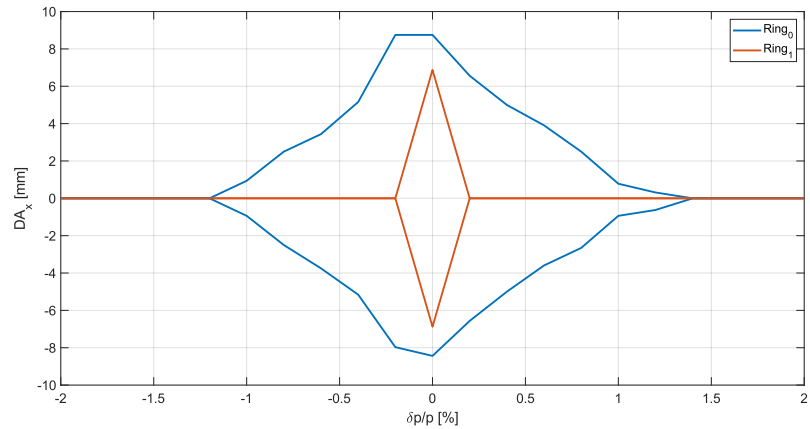
initial Condition : [ 6 mm, 0, 1  $\mu$ m, 0, 0, 0 ] ; No.Turn : 2250



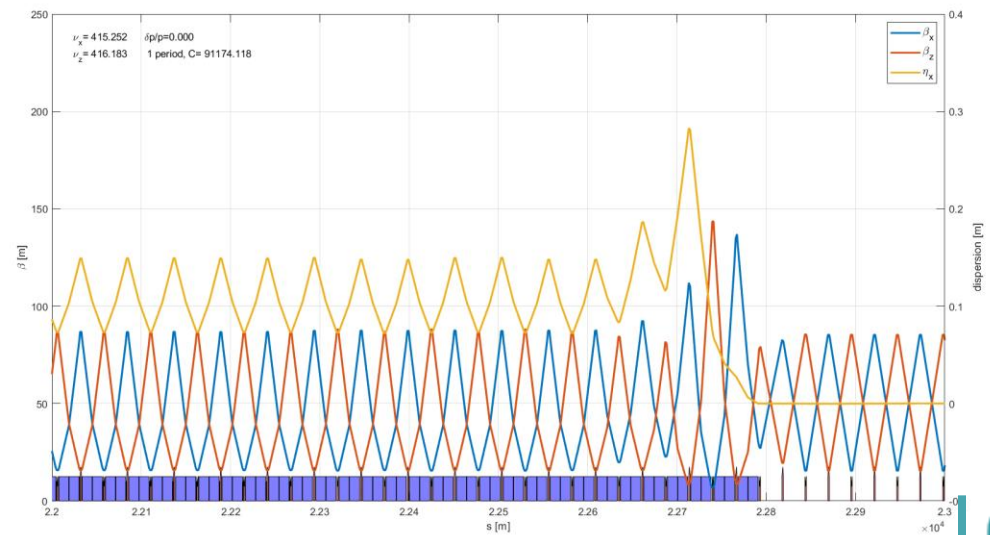
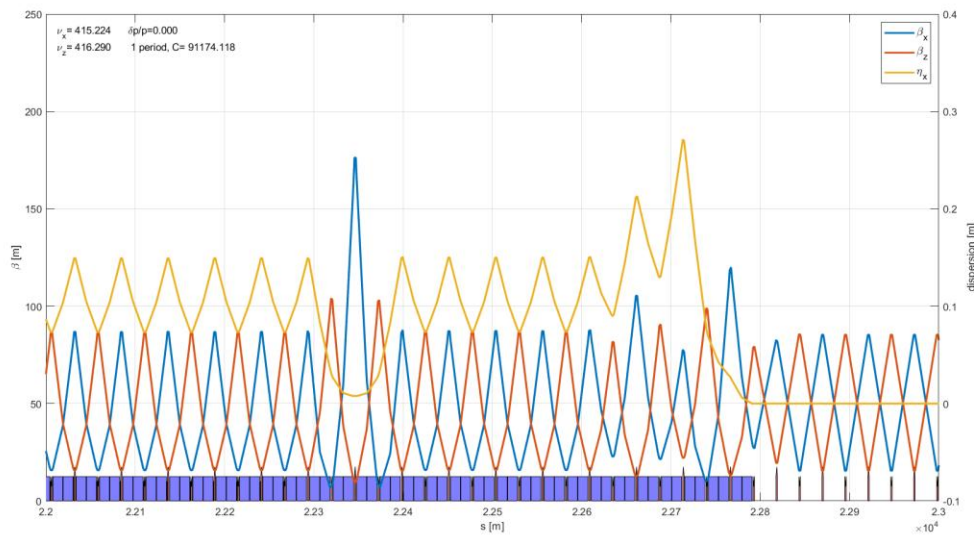
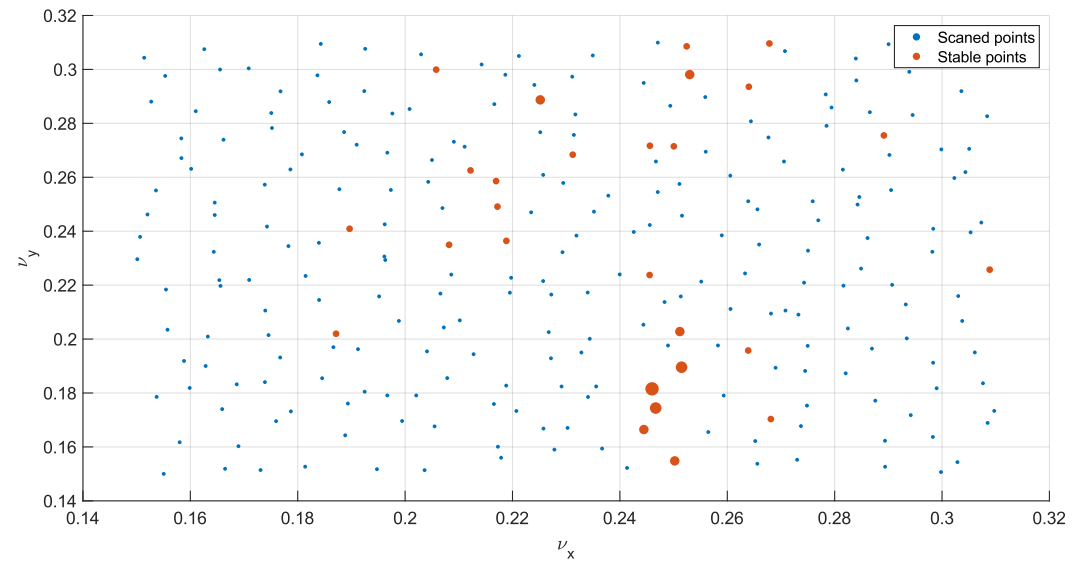




## Path Length Optimization (one turn)



## Ongoing lattice modification



## Ongoing lattice modification

