

Correction of linear optics and coupling with closed-orbit modulation

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FCC Optics Tuning and Correction Workshop, CERN

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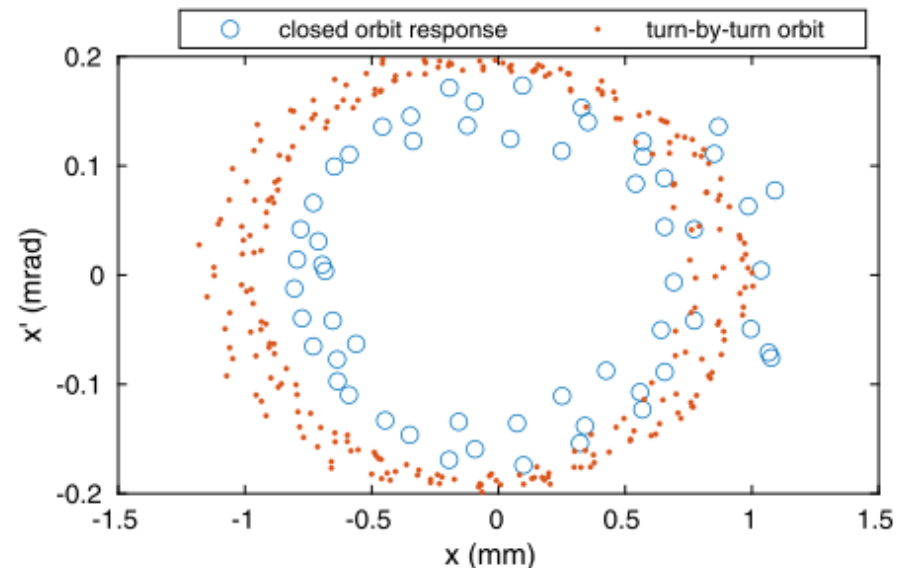
- The LOCOM method*
 - Closed-orbit modulation for sampling linear optics
 - Decomposition of orbit modulation
 - The fitting setup
- Experimental demonstration on NSLS-II**
 - Experimental setup
 - Test case 1 – one known quadrupole error
 - Test case 2 – random optics and coupling errors
- Summary

* See X. Huang, PRAB 24, 072805 (2021) and X. Huang, X. Yang PRAB 26, 052802 (2023)

**Experimental demonstration on NSLS-II was done in collaboration with Xi Yang (NSLS-II)

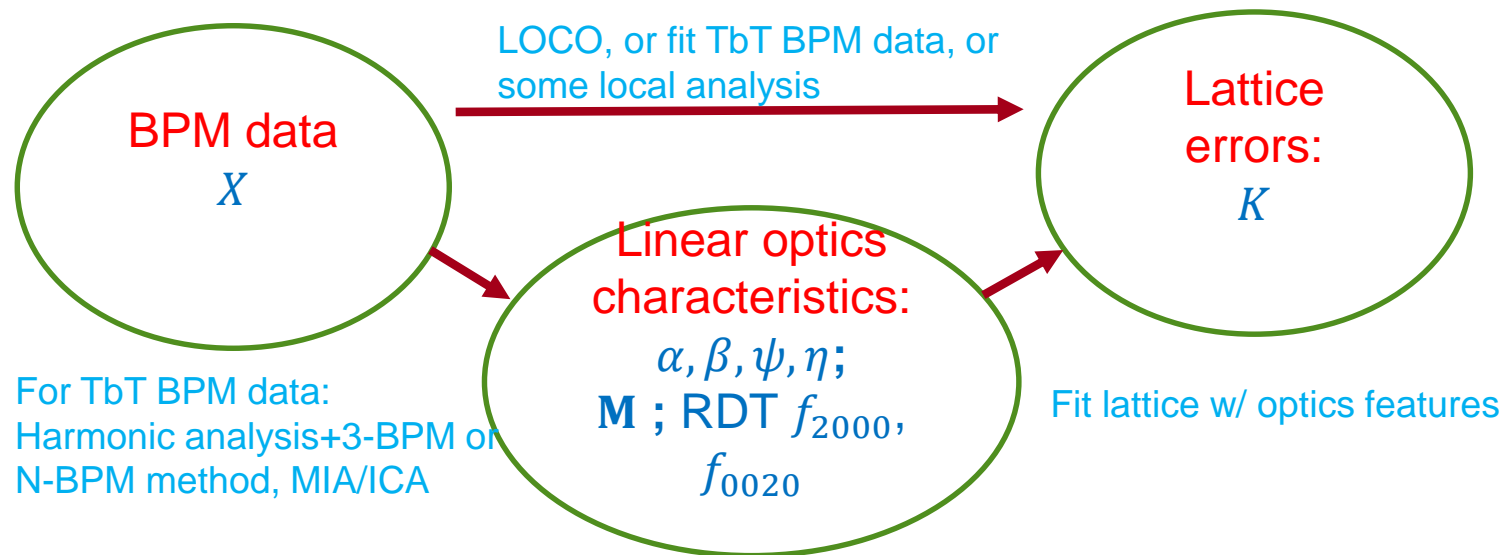
Sampling linear optics with BPM data

- Linear optics impacts the beam motion; conversely, beam motion, observed by BPMs, reflects linear optics.
 - Linear optics is characterized by Twiss functions and betatron phase advances, or equivalently, the transfer matrices
 - Beam motion is characterized by deviation from a reference orbit
 - The connection: $\mathbf{X}_j = \mathbf{M}(j|i)\mathbf{X}_i$, where $\mathbf{X} = (x, x')^T$, or $\mathbf{X} = (y, y')^T$
- Two types of BPM data
 - Closed orbit deviation,
 - e.g., orbit response matrix data
 - Static, measured with high accuracy
 - Turn-by-turn (TbT) measurement
 - Fast changing, lower accuracy
 - But can acquire lots of data



Determination and correction of lattice errors

- Lattice errors affect linear optics characteristics, which can be used to determine and correct linear optics.



Examples of local analysis

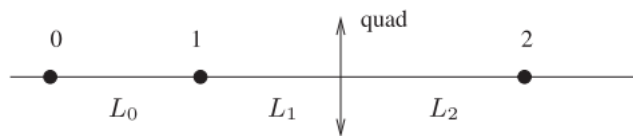
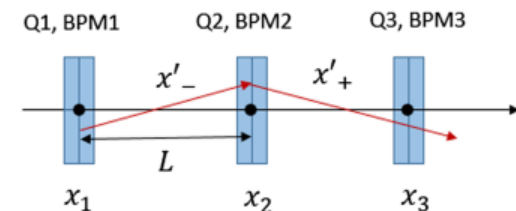


FIG. 1. Calibration of one quadrupole with three BPMs.



X. Huang, et al, PRSTAB 13, 114002 (2010)

X. Huang (SLAC), June 27, 2023, CERN

T. Zhang, et al, PRSTAB 21, 092801 (2018) 4

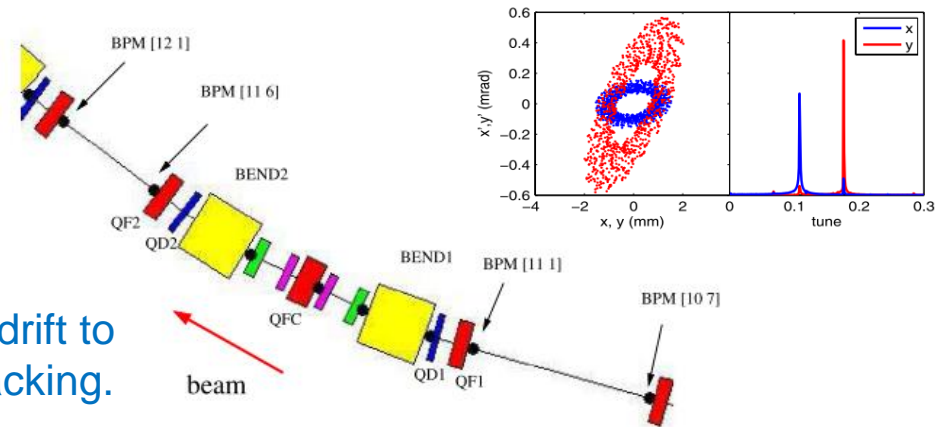
Fitting TbT or shot-by-shot BPM data for lattice errors

- Directly fitting TbT BPM data for lattice errors was tested on SPEAR3

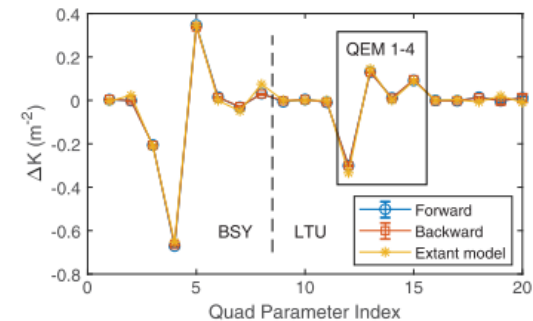
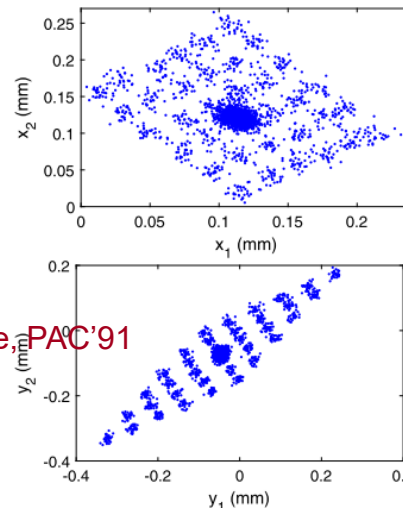
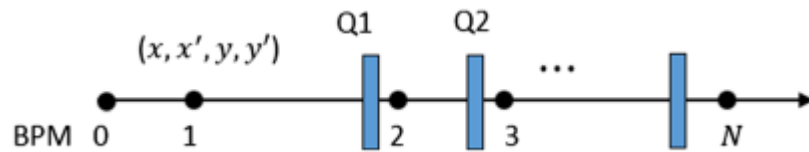
$$\chi^2 = \sum_{n=1}^N \sum_{i=2}^{M+1} \left[\left(\frac{x_i(n) - \tilde{x}_i[\mathbf{p}; \mathbf{X}_1(\mathbf{n})]}{\sigma_{xi}} \right)^2 + \left(\frac{y_i(n) - \tilde{y}_i[\mathbf{p}; \mathbf{X}_1(\mathbf{n})]}{\sigma_{yi}} \right)^2 \right],$$

The key is to use two BPMS separated by a drift to calculate angle coordinates, to be used in tracking.

X. Huang, et al, PRSTAB 13, 114002 (2010)



- This has been extended to shot-by-shot BPM data on LCLS



T. Zhang, et al, PRSTAB 21, 092801 (2018)

- Two upstream correctors were proposed to scan the phase space

But grid scan was already in use, see P. Emma and W. Spence, PAC'91

- Both forward or backward tracking can be used in fitting.

X. Huang (SLAC), June 27, 2023, CERN

Closed-orbit modulation: a new way to sample linear optics

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- Scanning along the phase space ellipse would be better
 - Our original plan for LCLS was to scan the ellipse
 - In simulation we found it is more efficient than grid scan
- Natural extension in rings: to use a pair of correctors to scan the phase space ellipse and to use the orbits to fit for lattice errors

Coordinate at P1 with the two kicks

$$\mathbf{m} \begin{pmatrix} 0 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \theta_1 \end{pmatrix} = \begin{pmatrix} \sqrt{\beta_1 \beta_2} \theta_2 \sin \psi_{12} \\ \theta_1 + \sqrt{\frac{\beta_2}{\beta_1}} \theta_2 (\cos \psi_{12} - \alpha_1 \sin \psi_{12}) \end{pmatrix}$$

We want the normalized coordinates to scan a circle

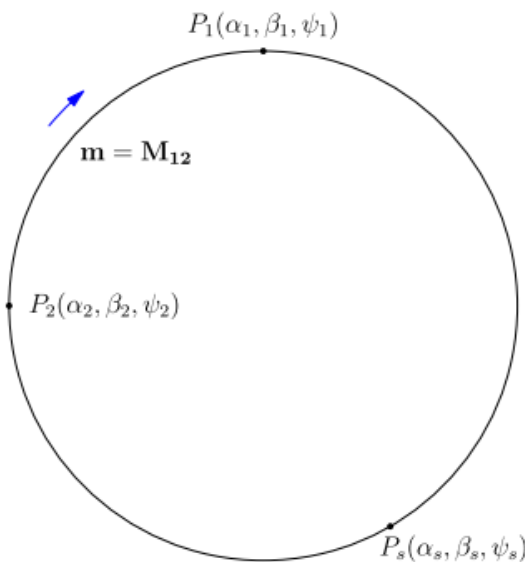
$$\begin{pmatrix} x \\ \alpha_1 x + \beta_1 x' \end{pmatrix} = \sqrt{\beta_1} \begin{pmatrix} \sqrt{\beta_2} \theta_2 \sin \psi_{12} \\ \sqrt{\beta_1} \theta_1 + \sqrt{\beta_2} \theta_2 \cos \psi_{12} \end{pmatrix}$$

For sinusoidal waveforms

$$\theta_1 = \theta_{\text{amp}} \sin \phi, \quad \theta_2 = \sqrt{\frac{\beta_1}{\beta_2}} \theta_{\text{amp}} \cos(\phi + \chi),$$

The condition for \tilde{x} and \tilde{p}_x to be orthogonal

$$\chi = \frac{\pi}{2} - \psi_{12}$$

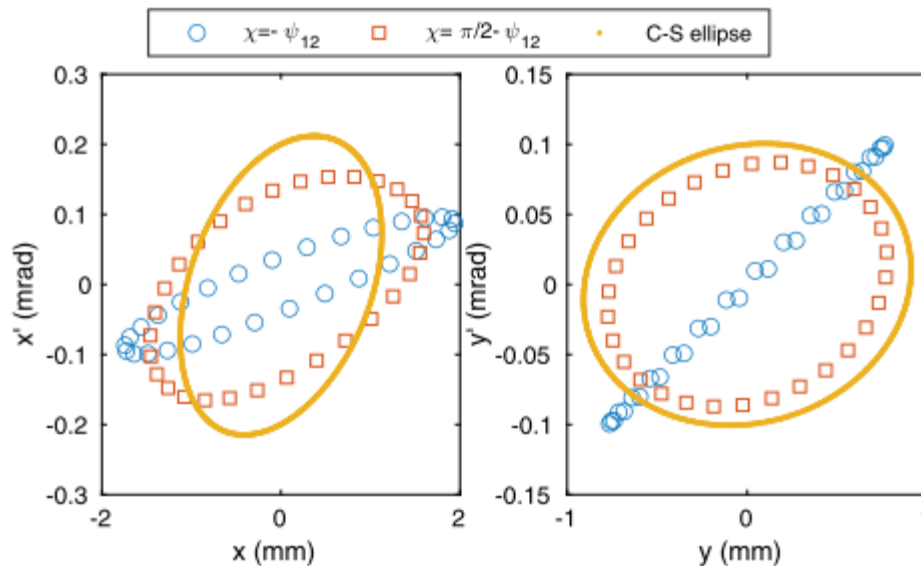


Waveform phase difference: good choice vs. bad choice

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- A good choice of waveform phase difference avoids degeneracy

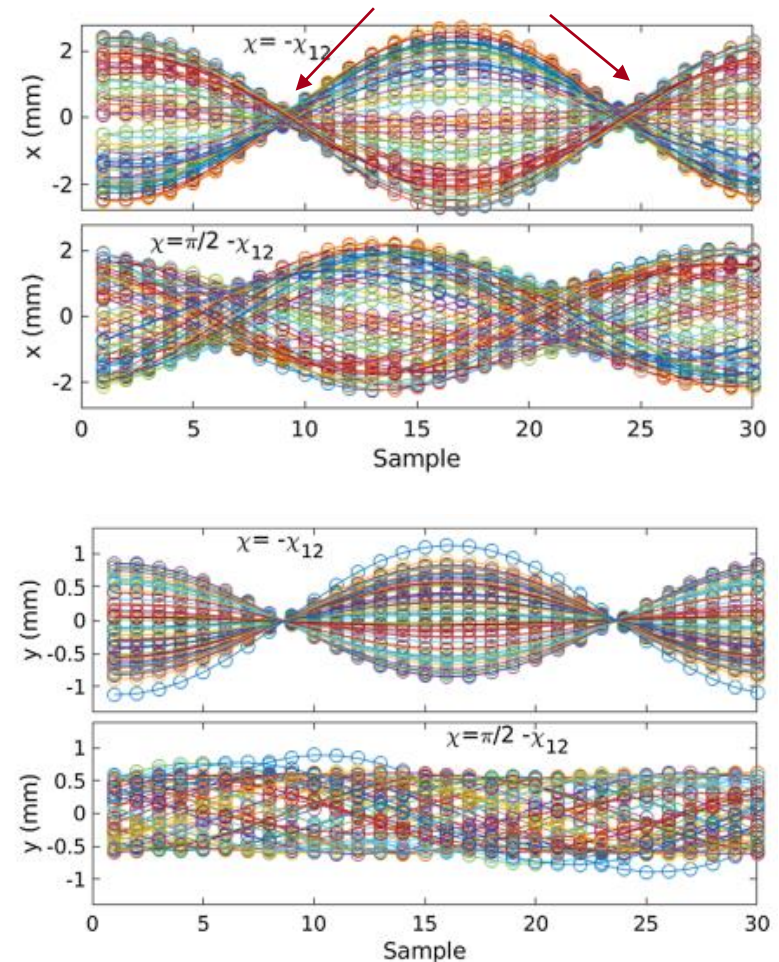
When it is degenerate, there are nodes in this plot



For this SPEAR3 example (simulation):

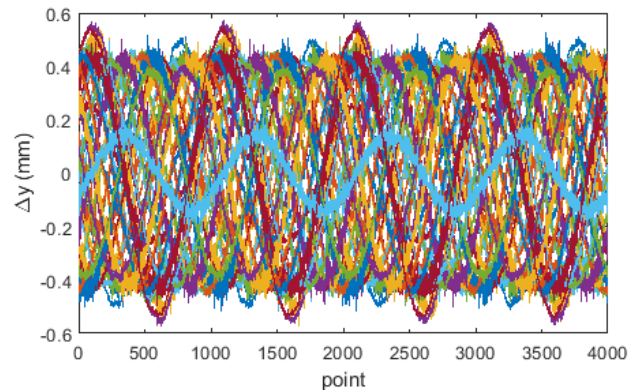
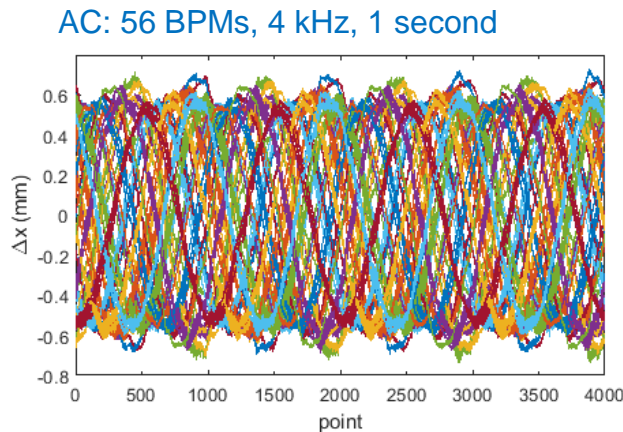
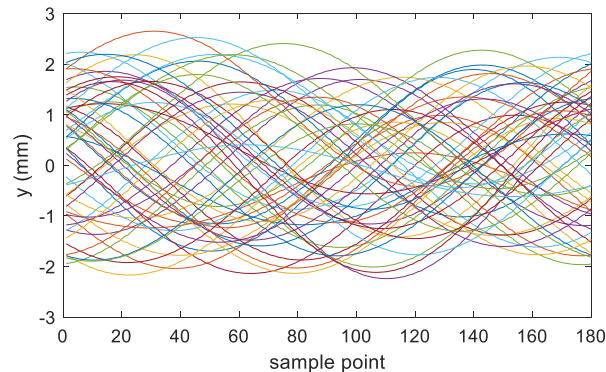
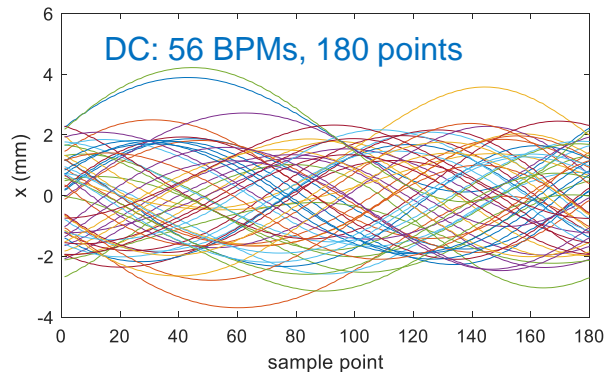
$$\psi_{x,12} = 1.60\pi$$

$$\psi_{y,12} = 0.52\pi$$

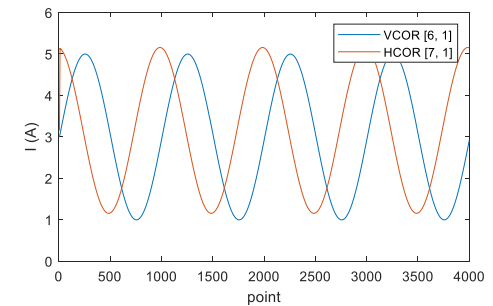
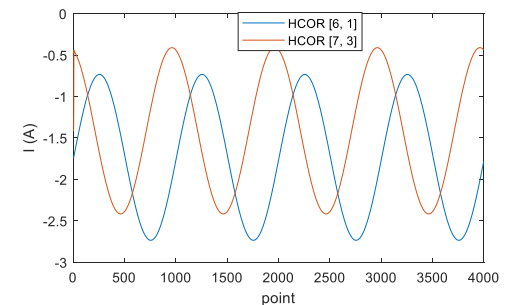


Experimental data from SPEAR3

- Two ways to measure orbit modulation data
 - DC: step the correctors according to the waveforms (or not)
 - AC: drive the correctors synchronously with sinusoidal waveforms



Corrector waveforms for SPEAR3 AC-LOCOM



A recent improvement in data processing

- The original scheme is to fit the modulated orbit to the lattice
 - Similar to LOCO, but with fewer corrector gains (now only 2)
- The recent improvement is to extract features from modulated orbits for fitting
 - Reduces the number of data points in fitting and thus the size of Jacobian matrix
 - Initially tried to extract Twiss functions and phase advances by fitting
 - Settled on decomposing the orbit waveforms in orthogonal modes and using mode amplitudes as features

$$\begin{aligned}x_i(n) &= A_i \sin(2\pi\nu n + \phi_i) \\ &= A_{si} \sin 2\pi\nu n + A_{ci} \cos 2\pi\nu n,\end{aligned}$$

If we choose modulation frequency, $\nu = \frac{1}{N}$

$$\begin{aligned}A_{si} &= \frac{2}{N} \sum_{n=1}^N x_i(n) \sin \frac{2\pi n}{N}, \\ A_{ci} &= \frac{2}{N} \sum_{n=1}^N x_i(n) \cos \frac{2\pi n}{N}.\end{aligned}$$

Mode amplitudes are connected with transfer matrices

- The orbit waveforms can be calculated from the corrector waveforms

Corrector waveforms

$$\theta_1(n) = \theta_{1m} \sin(2\pi\nu n + \phi_1),$$

$$\theta_2(n) = \theta_{2m} \sin(2\pi\nu n + \phi_2),$$

The coordinates at downstream of corrector 1

$$\mathbf{y}_1(n) \equiv \begin{pmatrix} y_1 \\ y'_1 \end{pmatrix} = \mathbf{Q}_1 \begin{pmatrix} \cos 2\pi\nu n \\ \sin 2\pi\nu n \end{pmatrix}$$

$$Q_{1,11} = m_{12}\theta_{2m} \sin \phi_2,$$

$$Q_{1,12} = m_{12}\theta_{2m} \cos \phi_2,$$

$$Q_{1,21} = m_{22}\theta_{2m} \sin \phi_2 + \theta_{1m} \sin \phi_1,$$

$$Q_{1,22} = m_{22}\theta_{2m} \cos \phi_2 + \theta_{1m} \cos \phi_1.$$

The closed orbit at a BPM downstream of corrector 1

$$\mathbf{Y}_P(n) = \mathbf{M}_{P1} (\mathbf{I} - \mathbf{M}_1)^{-1} \mathbf{Q}_1 \begin{pmatrix} \cos 2\pi\nu n \\ \sin 2\pi\nu n \end{pmatrix}$$

$$y_P(n) = (A_{yc} \ A_{ys}) \begin{pmatrix} \cos 2\pi\nu n \\ \sin 2\pi\nu n \end{pmatrix}$$

Thus the mode amplitudes of the position coordinate, A_{yc} and A_{ys} , are equal to the (1,1) and (1,2) elements of matrix

$$\mathbf{M}_{P1} (\mathbf{I} - \mathbf{M}_1)^{-1} \mathbf{Q}_1$$

Therefore, the mode amplitudes are features of the linear optics (in part also dependent on the corrector waveforms), which can be used to fit the lattice.

Lattice fitting, with coupling included

- Fitting data: 8 mode amplitude per BPM, plus H/V dispersion
 - 4 in-plane (2 H + 2 V) amplitudes, 4 cross-plane amplitudes
 - 10 N_P data points total
- Fitting parameters:
 - N_Q , Quadrupoles in lattice
 - N_{SQ} , Skew quadrupoles
 - $4N_P$, BPM gains and coupling coefficients
 - 8, Corrector gains and coupling coefficients
- Fitting method: same as LOCO and other optics correction methods
 - Levenberg-Marquadt with penalty term to slow down divergence on under-constrained directions*

*X. Huang, et al, ICFA Newsletter 44, 60 (2007)

Experimental tests on NSLS-II

- Systematic tests were done at NSLS-II to demonstrate the method
 - TBT BPM and pinger at NSLS-II provide independent verification of correction

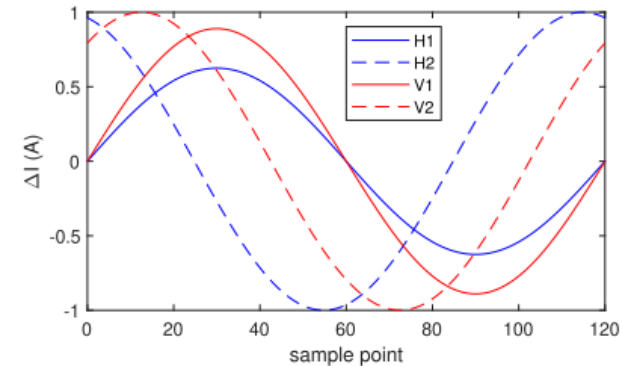
Corrector waveforms

180 BPMs,

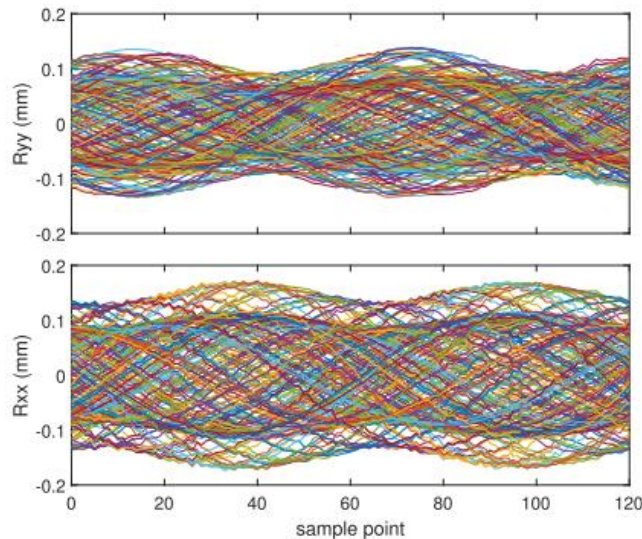
DC modulation for 120 points

(AC modulation of correctors not synchronized with BPM data)

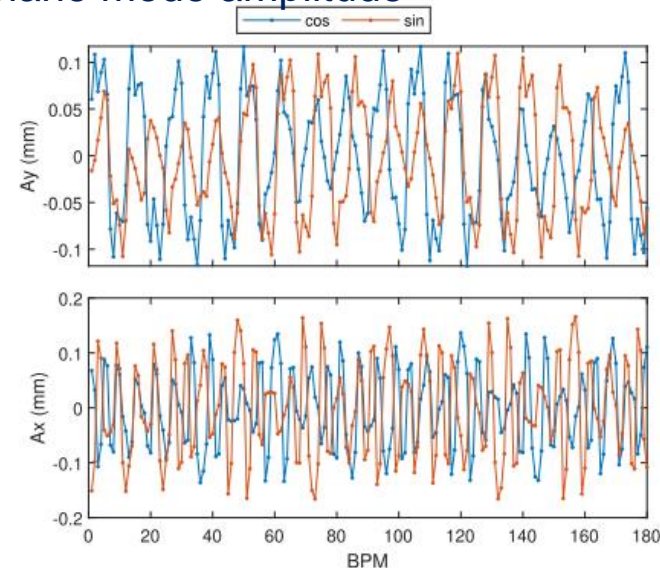
Fit 180 quadrupole parameters, 30 skew quads



Raw modulation data



In-plane mode amplitude



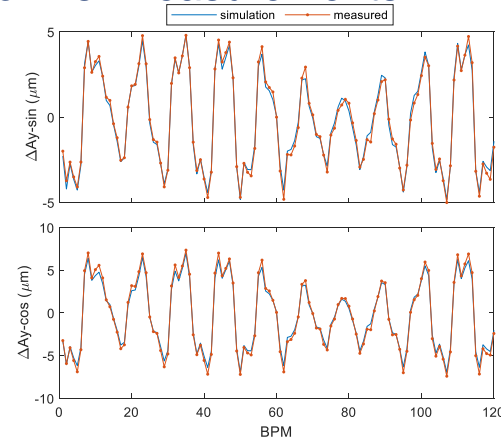
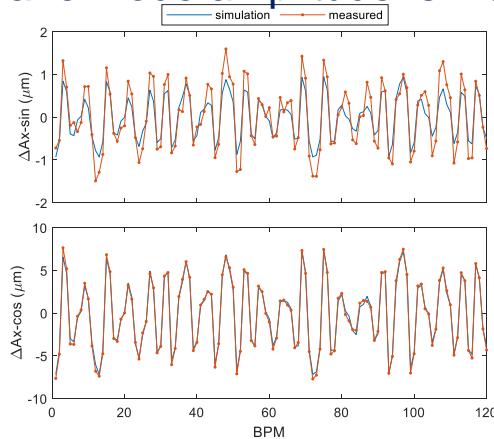
X. Huang, X. Yang, PRAB 26, 052802 (2023)

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Test case 1 – change one quadrupole

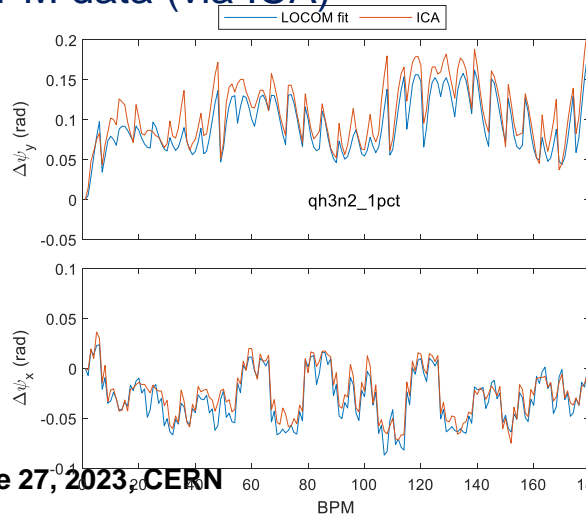
- One quadrupole was changed by 1%

The change of in-plane mode amplitude: simulation vs. measurements



Contribution of
baseline optics is
subtracted

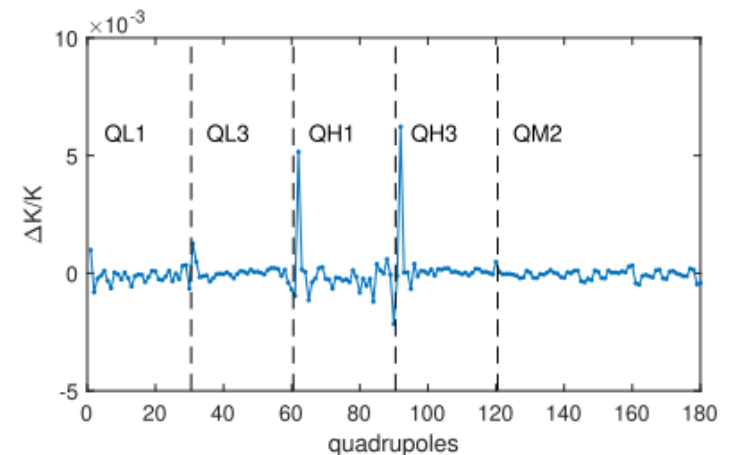
Phase advance errors obtained by fitting LOCOM
data or by TBT BPM data (via ICA)



Including optics errors
in the baseline optics

Fitted quadrupole errors by LOCOM

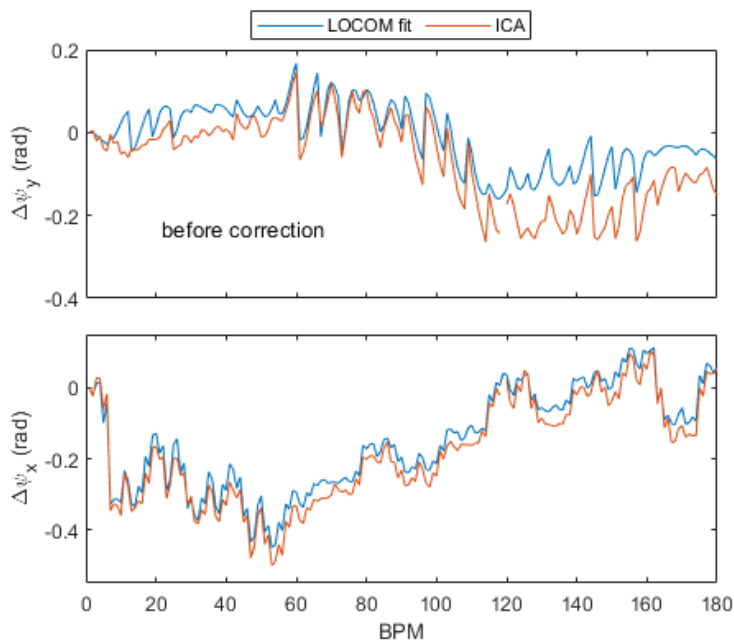
Contribution of baseline optics is subtracted



Test case 2 – random errors

- Initial machine condition: operation lattice plus random errors to 300 quadrupoles, with 30 skew quads turned off
 - Rms $\frac{\Delta K}{K}$ was 1% for the random changes

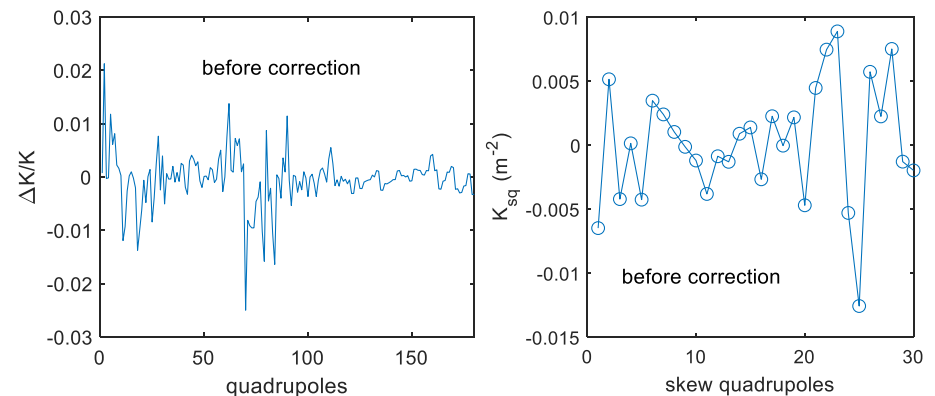
Phase advance errors obtained by fitting LOCOM data or by TBT BPM data (via ICA)



One BPM (#119) was not giving valid TBT data.

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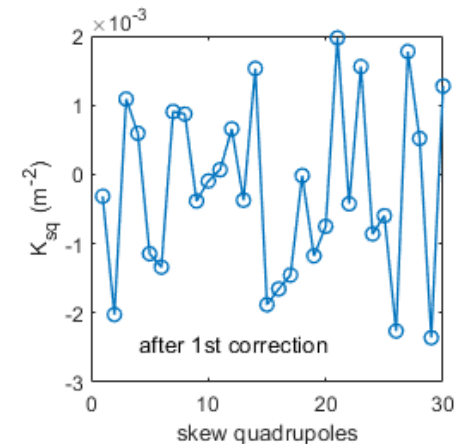
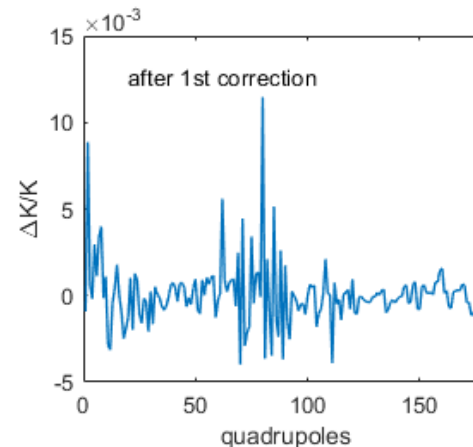
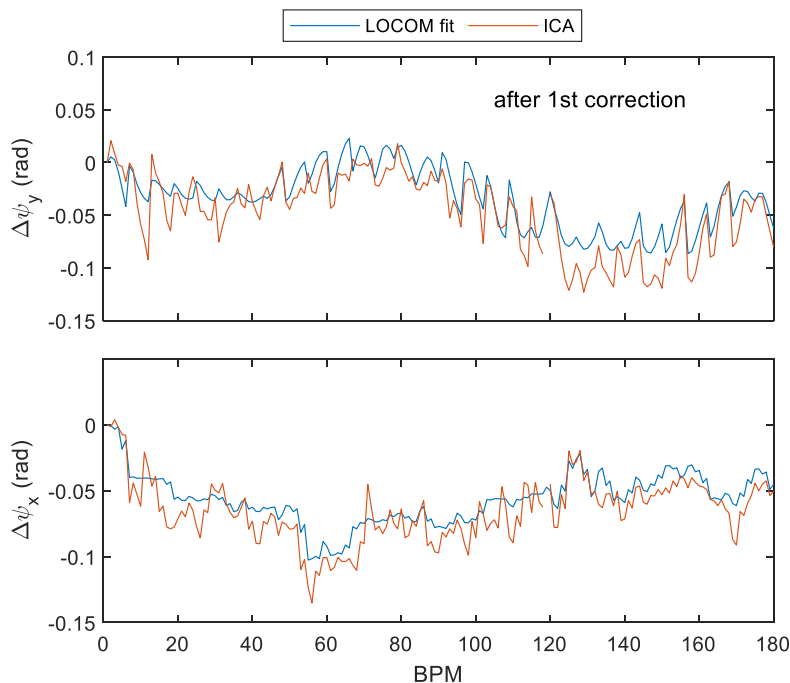
Fitted quadrupole and skew quadrupole errors by LOCOM



The results were applied on the machine for correction.

After the 1st correction

- The 1st correction reduced beta beating and coupling substantially
 - Initial rms beta beating is 11.5% (H) and 10.1% (V), respectively
 - After the 1st correction, it becomes 1.7% (H) and 3.9% (V)

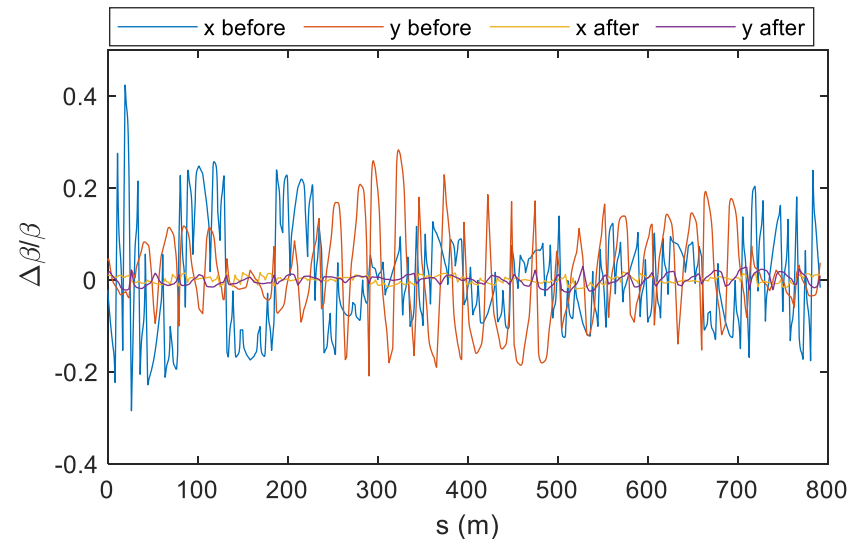
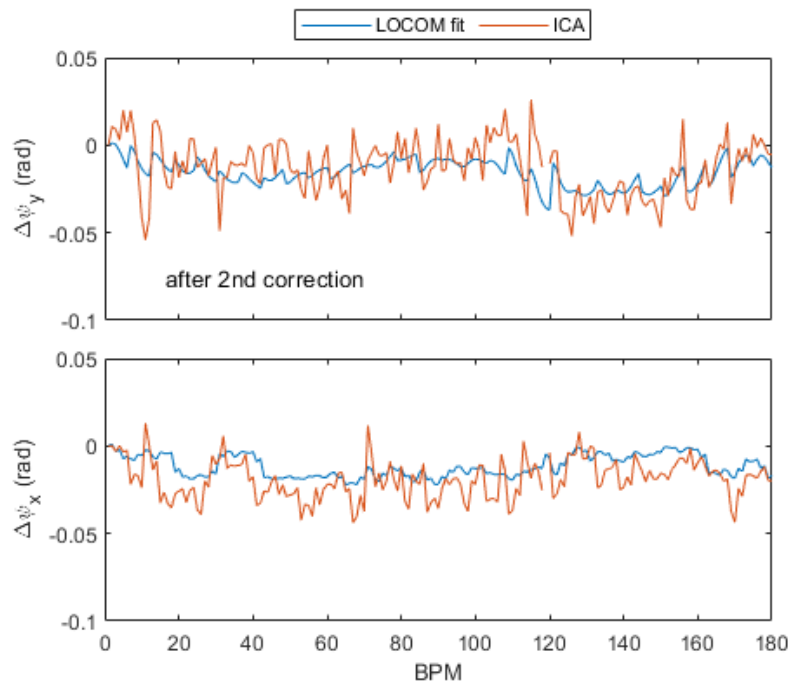


The fitting results were applied on the machine for a second correction.

After the 2nd correction

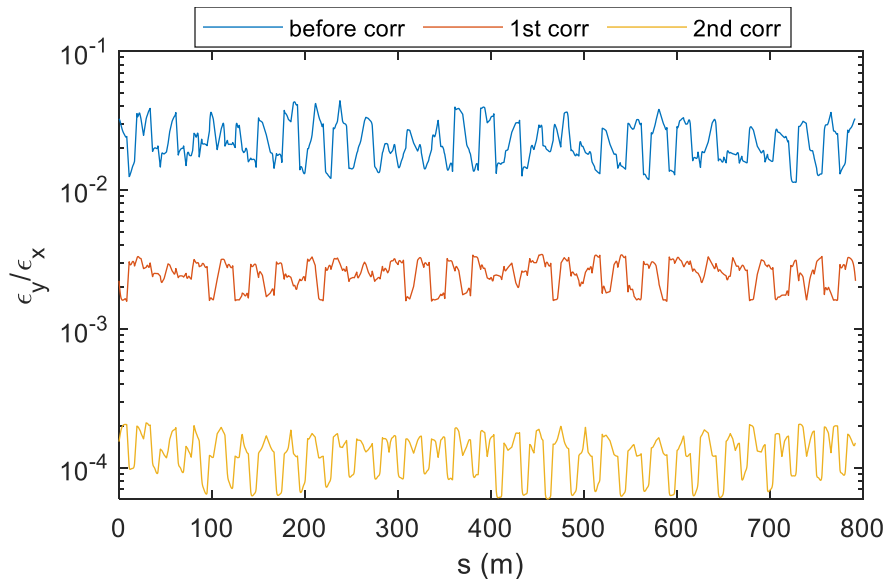
- After the 2nd correction, beta beating and coupling are reduced to a low level
 - After the 2nd correction, rms beta beating becomes 0.8% (H) and 1.1% (V)

Beta beating from the fitted lattice (by LOCOM), before correction, and after the 2nd correction



Emittance ratio after coupling correction

- Emittance ratio by the fitted lattice (by LOCOM)
 - Initial emittance ratio is 2.7% before correction
 - It was corrected to 0.25% (1st corr) and subsequently to 0.012% (2nd corr)

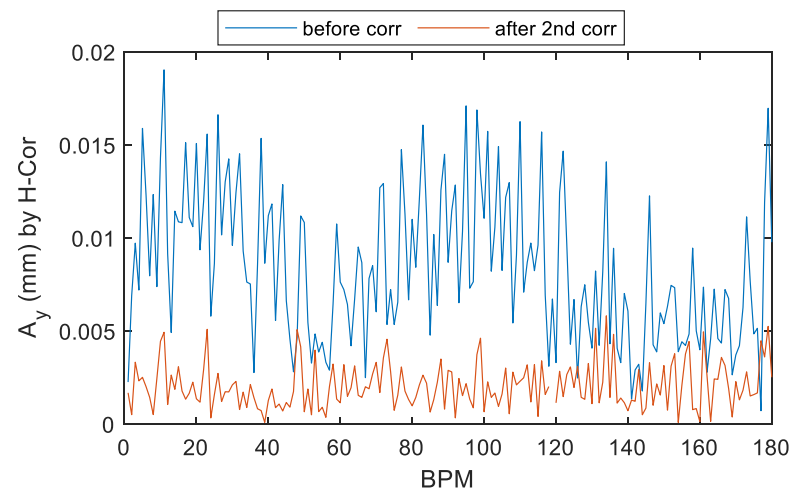
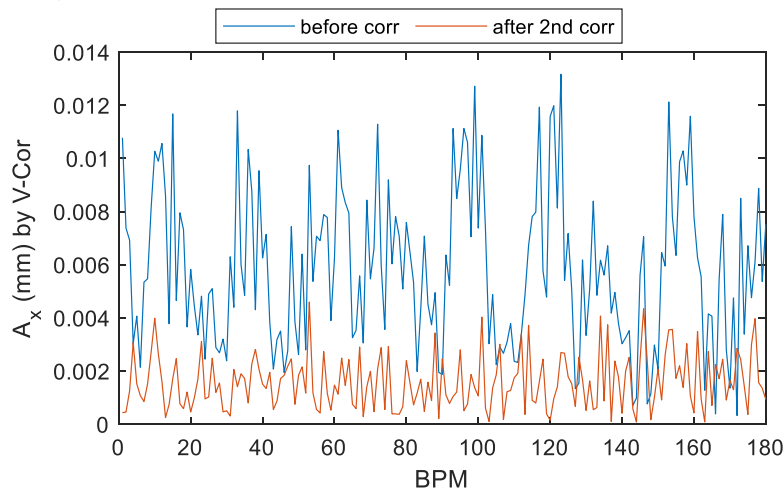


Ratio of projected emittance
calculated by Ohmi envelope at
all locations in the ring

Direct evidence of coupling reduction

- The cross-plane orbit modulation amplitude is a model-independent measure of the level of coupling

The cross-plane amplitude ($A_y = \sqrt{A_{ys}^2 + A_{yc}^2}$ by H-correctors, and similarly for A_x by V-correctors)



The mean cross-plane amplitude was reduced from 8.4 μm to 2.1 μm for A_y , and from 5.9 μm to 1.6 μm for A_x (a factor of 4).

Roughly speaking, the emittance ratio should have gone down by a factor of 16.

LOCOM – what is it good for?

- Compared to LOCO, the advantages of LOCOM include
 - It uses less correctors; it can have more data points
 - It takes data faster (sweeps with small steps)
 - AC-LOCOM is faster than AC-LOCO, and data processing is easier
 - Only 1 shot is needed; no mixing of different frequencies
 - Sinusoidal waveforms help eliminate certain measurement errors
- Compared to TBT BPM data, LOCOM
 - Does not need pinger/kicker for the other plane (other than the injection plane)
 - Uses high-accuracy closed orbit data
 - Does not suffer from decoherence from kicked beam motion
 - It is big issues for high chromaticity, high amplitude dependent detuning rings (which are becoming common)
 - Does not suffer from signal mixing, contamination, or deterioration (resulting bad TBT data)

Discussion – why two correctors per plane are enough?

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- Fundamentally, this is because the phase space is 2D (in one plane)
 - Two orbits are sufficient to capture the optics distortion effects (four orbits for both planes), as long as they are not degenerate

For $4 \times N$ matrix \mathbf{X} , with phase space coordinate $[x, x', y, y']^T$ in each column

$$\mathbf{M}_{21}\mathbf{X}_1 = \mathbf{X}_2 \quad \rightarrow \quad \mathbf{M}_{21} = \mathbf{X}_2\mathbf{X}_1^T(\mathbf{X}_1\mathbf{X}_1^T)^{-1}$$

Only need $N \geq 4$

- Additional orbits (or correctors) serve to improve accuracy and reduce degeneracy

- The LOCOM method was proposed for linear optics and coupling correction
- It has many advantages compared to LOCO and TBT BPM based methods
 - It is fast, easy to do, and does not suffer from many issues common to TBT BPM data.
- The method has been demonstrated on NSLS-II
 - High accuracy correction of optics and coupling errors, verified independently by TBT BPM data