Comparison of Harmonic Spin Matching Schemes using Orbit Bumps in the FCC-ee

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Energy calibration in FCC-ee

- FCC-ee will operate on 4 centre-of-mass energies

  Z^0 bosons (91 GeV), WW pairs (160 GeV), Higgs bosons (240 GeV) and top quark pairs (350-365 GeV)

- High-precision centre-of-mass energy calibration
  - basis for precise measurements of the standard model particle properties
  - make it possible for the new rare process detection
  - precise measurements in FCC-ee will contribute to the measurements in FCC-hh

The current precision targets for the energy calibration: 4 keV at Z mass and 100 keV at W mass

the most promising way to achieve this target: **resonant depolarization**

FCC collaboration, "FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2", in European Physical Journal: Special Topics, 228, pp. 261-623, 2019.

Objectives

Ensure a sufficient spin polarization level (at least $5-10\%$)

1. Estimate the achievable polarization under various lattice conditions
   e.g. misalignments+field errors
2. Use special structure to improve polarization
   e.g. closed orbit bumps

- $\hat{n}_0(s)$: one-turn periodic solution of the T-BMT equation on closed orbit
- Spins on the closed orbit precess around $\hat{n}_0$ for ν_0 turns in every revolution
  $\Rightarrow \nu_0$: closed orbit spin tune
- $\nu_0 = a\gamma$ in the perfectly aligned flat ring without solenoids
  $\nu_0 \approx a\gamma$ in misaligned lattice
  $|\nu_0 - a\gamma|$ impacts the measurement precision
- ST effect + radiative depolarization $\Rightarrow$ equilibrium polarization

Effective model for error seeds creation

- Use an effective model to simulate **residual orbits after lattice correction**
- Random small errors generated from truncated Gaussian distributions (truncated at $2.5\,\sigma$)

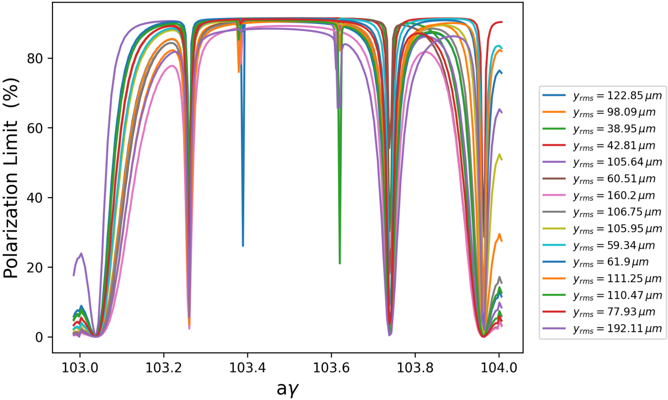
Type	$\sigma_{\Delta \mathrm{X}}$ (nm)	$\sigma_{\Delta \mathrm{Y}}$ (nm)	$\sigma_{\Delta \mathrm{Z}}$ (nm)	$\sigma_{\Delta \mathrm{PSI}}$ (μrad)	$\sigma_{\Delta \mathrm{THETA}}$ (μrad)	$\sigma_{\Delta \mathrm{PHI}}$ (μrad)
Arc quadrupole	120	120	120	2	2	2
Arc sextupole	120	120	120	2	2	2
Dipoles	120	120	120	2	0	0
IR quadrupole	120	120	120	2	2	2
IR sextupole	120	120	120	2	2	2

Energy scan with multiple error seeds

Setting 1

$\sigma = 120\,\mathrm{nm}$ for x,y,z misalignments

$\sigma = 2\,\mu\mathrm{rad}$ for angular deviations

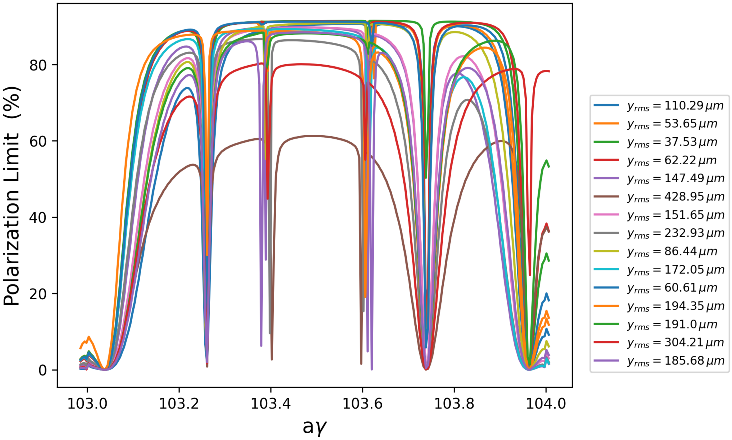


First order energy scan showing equilibrium polarization levels near Z energy

Setting 2

$\sigma = 200\,\mathrm{nm}$ for x,y,z misalignments

$\sigma = 2\,\mu\mathrm{rad}$ for angular deviations

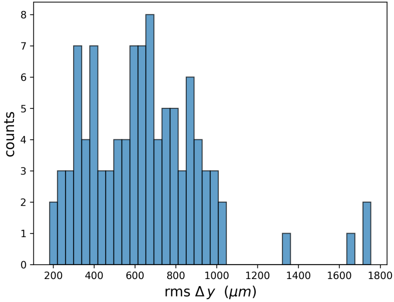


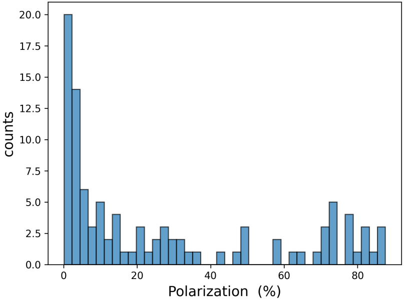
100 error seeds at Z energy

quadrupoles+dipoles:

$\sigma = 1\,\mu\mathrm{m}$ x,y,z misalignments

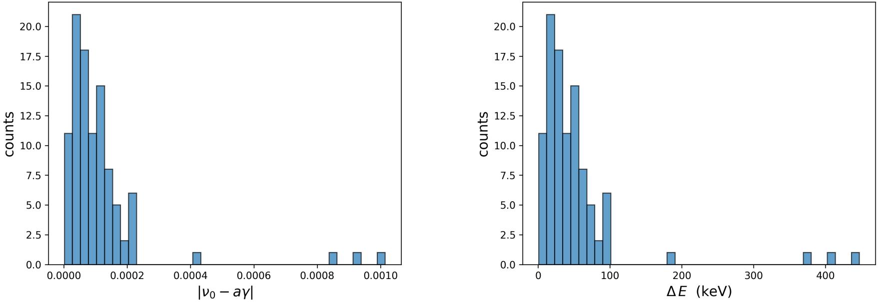
$\sigma = 1\,\mu\mathrm{rad}$ angular deviations





100 error seeds at Z energy

quadrupoles+dipoles:
$\sigma = 1\,\mu\mathrm{m}$ x,y,z misalignments
$\sigma = 1\,\mu\mathrm{rad}$ angular deviations



More realistically

Larger error seeds ($100\,\mu$m) $+$ orbit correction (correctors and BPMs)

How much polarization?

Ongoing!

Harmonic spin matching (HSM)

Conventional lattice correction + harmonic spin matching

- Misaligned ring $\rightarrow$ $\hat{n}_0(s)$ not vertical $\rightarrow$ stronger spin diffusion
- Random vertical quadrupole misalignments are difficult to control
- HSM: use multiple vertical orbit correctors to create an additional controllable $\hat{n}_0$ tilt and reduce $(\delta\hat{n}_0)_{\mathrm{rms}}$ $\rightarrow$ elevate polarization
- Use closed vertical bumps to avoid disturbing the orbits outside bumps

D. P. Barber, et al. "High spin polarization at the HERA electron storage ring." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 338.2-3 (1994): 166-184.

Harmonic bump composition

EPFL

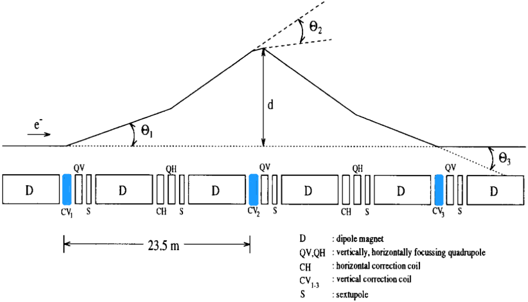
1st corrector: give a vertical kick

2nd corrector: kick y back to initial value at the 3rd corrector

3rd corrector: kick y' back to initial value

Kicks of 2nd and 3rd correctors are adjusted to make the bump **CLOSED**

one independent variable for each bump



a schematic of a bump in HERA*

*D. P. Barber, et al. "High spin polarization at the HERA electron storage ring." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 338.2-3 (1994): 166-184.

Three correction schemes

1. HERA formalism (used in HERA)

   D. P. Barber, et al. A general harmonic spin matching formalism for the suppression of depolarisation caused by closed orbit distortion in electron storage rings. No. DESY–85-044. DESY, 1985.

2. Rossmanith-Schmidt scheme (used in PETRA)

   R. Rossmanith and R. Schmidt, Compensation of depolarizing effects in electron-positron storage rings. Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 236.2 (1985): 231-248.

3. LEP method (Deterministic) (used in LEP)

   R. W. Assmann, Optimierung der transversalen Spin-Polarisation im LEP-Speicherring und Anwendung für Präzisionsmessungen am Z-Boson. Diss. Munich U., 1994.

HERA formalism

$$\delta \hat{n}_0 = \alpha \hat{m} + \beta \hat{l}$$

$\hat{l}(s), \hat{n}_0(s), \hat{m}(s)$ are periodic spin axes that form a right-hand coordinate system. Expand α and β to Fourier series

$$(\alpha - i\beta)(s) = -i \frac{C}{2\pi} \sum_k \frac{f_k}{k - \tilde{\nu}} e^{i2\pi ks/C}$$

f_k: Fourier coefficients, related to the closed orbit and perturbing fields

Make additional orbit corrections using orbit bumps to reduce the rms tilt by minimizing the Fourier coefficients

Simplified HERA formalism

- Extract n_0 direction at the end of all elements
- Expand $n_{0x}(s) + in_{0z}(s)$ into Fourier series
- Minimize target coefficients using four bumps

$$n_{0x} + in_{0z} \approx \sum_{k=-N}^{N} c_k \cdot e^{i2\pi ks/C}$$

$$c_k = \frac{1}{C}\int (n_{0x} + in_{0z}) \cdot e^{-i2\pi ks/C} ds \approx \frac{\sum_{j=1}^{M} [n_{0x}(s_j) + in_{0z}(s_j)] \cdot e^{-i2\pi ks_j/C}}{M}$$

Response matrix

Each closed bump can be represented using a single variable, and each bump has an independent and linear contribution to the Fourier coefficients

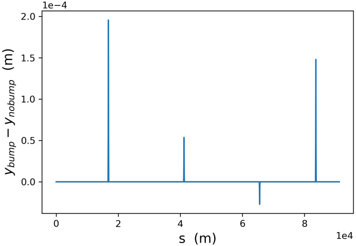
$$\mathbf{MK} = \mathbf{C}$$

K: amplitudes (the first kick value) of the bumps
C: real and imaginary parts of the required harmonics coefficients $[c_{0\text{real}}, c_{0\text{imag}}, c_{1\text{real}}, c_{1\text{imag}}]$

If the harmonics 0 and 1 of a misaligned lattice is $\mathbf{A}$, the bumps should generate $-\mathbf{A}$, and bump amplitudes can be estimated via inversing matrix

$$\mathbf{K} = \mathbf{M}^{-1}(-\mathbf{A})$$

Changes after adding bumps



- How does orbit change?
  - Orbits outside of the bumps are unaffected.
  - max 0.2 mm Δy within bumps is at the level of rms orbit of the effective lattice
- How does vertical dispersion change?
  - $(\eta_y)_{\mathrm{rms}}$ 7.602 mm $\to$ 7.675 mm, 0.96% increase
  - $[\Delta\eta_y(s)]_{\mathrm{rms}} = 0.4\,\mathrm{mm}$
- How does vertical emittance change?
  - ε_y 0.703 pm $\to$ 0.719 pm, 2.16% increase
- How does ν_0 shift?
  - $a\gamma = 103.983116$, $\nu_0 = 103.983100158 \to 103.983100156$, insignificant

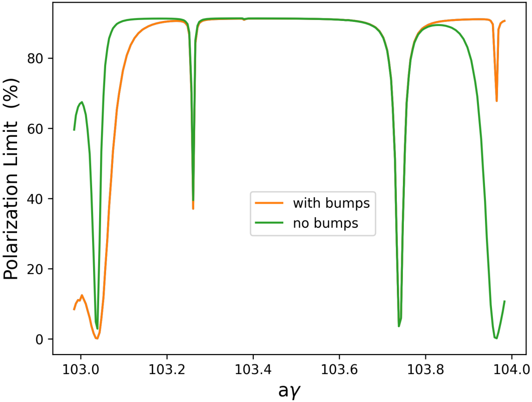
At 45.82 GeV ($a\gamma = 103.983$)

	$(\delta n_0)_{\mathrm{rms}}$ (mrad)	Polarization (%)
no correction	2.28	10.68
a set of four random bumps	0.912	90.58
optimized four bumps	0.9	90.96
a set of eight random bumps	0.903	90.79

Not necessary to use optimized locations, but better to have a symmetric layout

HERA formalism

Using 4 bumps which are optimized at 45.82 GeV ($a\gamma = 103.983$)



Rossmanith-Schmidt scheme

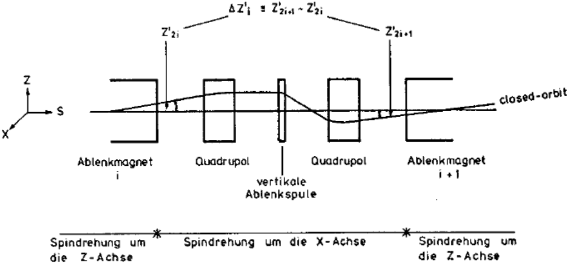
Assume that spin precessions around vertical direction only happen in bending magnets, and the radial perturbing fields on the closed orbit only exist between bending magnets

$$|\delta\vec{n}_0(s)| = \frac{1/c^2}{2(1-\cos 2\pi\nu)}\left[\left(\int_s^{s+L}\delta\Omega_x\cos\phi\,\mathrm{d}s\right)^2 + \left(\int_s^{s+L}\delta\Omega_x\sin\phi\,\mathrm{d}s\right)^2\right]$$

$$\delta\Omega_x = \frac{e}{m_0 c\gamma}(1+a\gamma)B_x \text{ and } \phi = \gamma a\alpha$$

$$\text{also } \frac{e}{m_0 c\gamma}\int_{s_{2i}}^{s_{2i+1}} B_x(s)\mathrm{d}s = -\Delta y_i^{'}$$

$$|\delta\vec{n}_0| = \frac{1/c^2}{2(1-\cos 2\pi\nu)}(1+\gamma a)\left[\left(\sum_{i=1}^{N}\sin(\gamma a\alpha_i)\Delta y_i^{'}\right)^2 + \left(\sum_{i=1}^{N}\cos(\gamma a\alpha_i)\Delta y_i^{'}\right)^2\right]$$



Rossmanith-Schmidt scheme

Expand $\Delta y^{'}(\alpha)$ into Fourier series

$$\Delta y^{'}(\alpha) = \sum_{k=1}^{\infty}(a_k \cos k\alpha + b_k \sin k\alpha)$$

$$\begin{matrix} a_k \\ b_k \end{matrix} = \frac{1}{N}\sum \Delta y_i^{'}(\alpha_i) \begin{matrix} \cos k\alpha_i \\ \sin k\alpha_i \end{matrix}$$

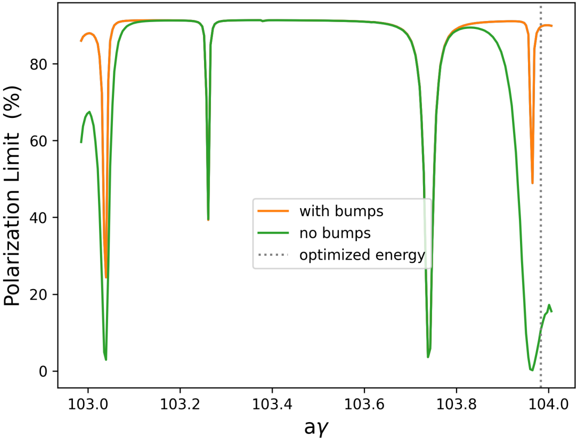
The harmonics which are adjacent to $a\gamma$ contribute most to the sum. FCC-ee (Z) operates between $a\gamma$ 103 and 104, so that a/b_{103} and a/b_{104} are to be suppressed using four closed bumps.

R. Schmidt, Polarisationsuntersuchungen am Speicherring PETRA. No. DESY-M-82-22. DESY, 1982.

Rossmanith-Schmidt scheme

$$45.82\,\text{GeV}\ (a\gamma = 103.983)$$

$$\delta n_0 : 2.28\,\mathrm{mrad} \Rightarrow 0.90\,\mathrm{mrad}\ ,\ P_{DK} : 10.68\% \Rightarrow 89.65\%$$



Modified Rossmanith-Schmidt scheme

EPFL

$$\int B_x(s)\mathrm{d}s \propto -\Delta y' \approx \sum_{i=1}^{N_{\mathrm{quad}}} k_{1i} y_i L_i + \sum_{j=1}^{N_{\mathrm{Vkicker}}} \mathrm{kick}_j$$

- Use y_{eff} to avoid the errors from thin lens approximation
  $y_{\mathrm{eff}} \approx \frac{1-\cos\sqrt{k}L}{\sqrt{k}L\sin\sqrt{k}L}(y_1 + y_2)$ $(k < 0)$ or $y_{\mathrm{eff}} \approx \frac{\cosh\sqrt{k}L-1}{\sqrt{k}L\sinh\sqrt{k}L}(y_1 + y_2)$ $(k > 0)$
- If two BPMs are installed at both ends of each quadrupole
  Polarization 10.68% $\rightarrow$ 84.71%
- 3056 dipoles, 1856 quadrupoles $\rightarrow$ fewer BPMs required

LEP method

Assume radial fields in quadrupoles and proportional to the beam position. Analyze unweighted vertical BPM readings and minimize critical harmonics

$$a_k = \frac{1}{\pi} \sum_{i=1}^{N_{BPM}} y_i \cdot \Delta\theta_i \cdot \cos\left(k \cdot \theta_i\right)$$

$$b_k = \frac{1}{\pi} \sum_{i=1}^{N_{BPM}} y_i \cdot \Delta\theta_i \cdot \sin\left(k \cdot \theta_i\right)$$

If there is a BPM next to each quadrupole and functions properly

$$45.82\,\text{GeV}\ (a\gamma = 103.983)$$

$$\delta n_0 : 2.28\,\mathrm{mrad} \Rightarrow 2.03\,\mathrm{mrad}\ ,\ P_{DK} : 10.68\% \Rightarrow 13.72\%$$

F. Sonnemann, Increase of spin polarization for energy calibration at LEP. Diss. Aachen, Tech. Hochsch., 1998.

Comparison

At 45.82 GeV ($a\gamma = 103.983$)

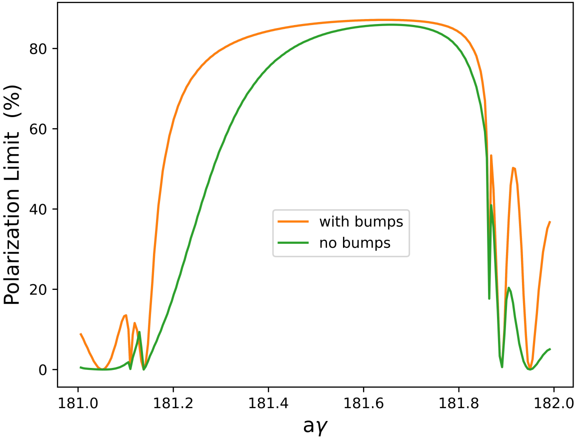
Method	$(\delta n_0)_{rms}$ (mrad)	Polarization (%)
no correction	2.28	10.68
HERA formalism	0.90	90.96
Rossmanith-Schmidt scheme	0.90	89.65
Modified R-S scheme	1.01	84.71
LEP method	2.03	13.72

Many questions remained regarding all three schemes

W energy using R-S scheme

$$80.192\,\text{GeV}\ (a\gamma = 181.986),\ (\Delta y)_{\rm rms} = 117.44\,\mu\text{m}$$

$$\delta n_0 : 5.87\,\mathrm{mrad} \Rightarrow 2.88\,\mathrm{mrad}\ ,\ P_{DK} : 4.69\% \Rightarrow 34.65\%$$



Pros and cons

1. HERA formalism
   - Pro: systematic and rigorous mathematical derivation
   - Con: empirically setting the bumps will be inevitable
2. Rossmanith-Schmidt scheme
   - Pro: based on the acquisition of a more measurable quantity
   - Con: BPMs at both ends of each dipole/quadrupole $\rightarrow$ extra cost
   - Con: restricted by BPM misalignments and calibration errors
3. LEP method
   - Pro: based on the real observables
   - Con: restricted by BPM misalignments and calibration errors

Outlooks

- Model the lattice using multiple **larger** error seeds, and estimate the maximum acceptable orbits that guarantee a sufficient polarization
- Complete the harmonic spin matching schemes
  - solve the remaining questions regarding the three schemes
  - find an effective method that relies on the analysis of real observables
  - test its effectiveness under different lattice conditions
- Possible polarization at Higgs and $\mathrm{t\bar{t}}$ energies in FCC-ee
  (1% at H in CEPC simulation)

Thank you!

Remained problems of HSM

- What's the harmonics that should be corrected in the simplified HERA formalism
- Whether there is a way to extract $\Delta y'$ information from vertical BPM readings in quadrupoles
- How to make LEP method work
- If it's possible to correct vertical resonance (not HSM)
- What will happen if the errors are much larger
- What will happen near other integers besides 103 and 104

Backup

	τ_{ST}	τ_{BKS}	τ_{dep}	
FCC-ee $(\Delta y)_{\rm rms} = 72\,\mu\rm m$	11 779 min	11 773 min	$4.26 \times 10^6\,\rm min$	90 min for 10% with wigglers (CDR)
HERA (26.7 GeV)	$\sim 43\,\rm min$	$\sim 40\,\rm min$	$\sim 10\,\rm min$	$\tau_{dk} \sim 8\,\rm min$
LEP	$\sim 310\,\rm min$ 46 GeV	—	$\sim 24\,\rm min$ 46.5 GeV	30 min for 10% no wigglers

in LEP $(\Delta y)_{\rm rms} = 530\,\mu\rm m$, $(\eta_y)_{\rm rms} = 13\,\rm cm$

Backup

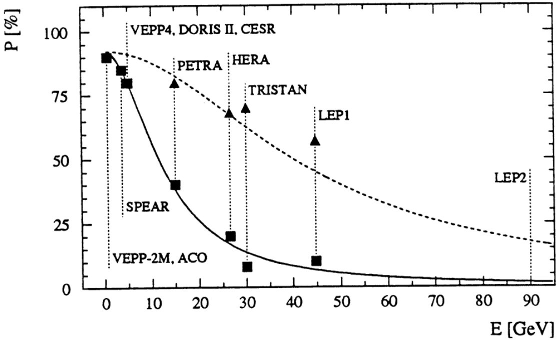
$$\mathrm{P(t)}{=}\mathrm{P}_{dk}\left[1-e^{-t/\tau_{dk}}\right]+P_0e^{-t/\tau_{dk}}\simeq P_0e^{-t/\tau_{dep}}$$

Backup

Systematic errors of the average beam energy determination

- Energy dependent momentum compaction
- Vertical orbit distortions (radial fields)
- Longitudinal fields

Backup



Maximum measured polarization in different storage rings with HSM (triangles) and without HSM (squares)

R. W. Assmann, et al. Polarization Studies at LEP in 1993. No. CERN-ALEPH-PUB-94-135. CM-P00061204, 1994.

Possible problems with LEP method

- radial fields not only exist in quads
- the radial field seen by the particle is not fully proportional to the y position
- even if it's proportional, each quad has different strength (ky)
- how much spin rotates is an integration of radial field within the element (kyL)

Computing the response matrix

- Search for all possible locations in a perfectly aligned lattice
- Add one bump in one possible location
- Adjust 2nd and 3rd kicks to make y and yp be all 0 outside the bump
- Analyze its contribution to the target harmonics
- Change the bump position and redo the analysis
- Select four bumps that build a matrix with the largest determinant
- Add four bumps into a misaligned machine with estimated bump amplitudes, and match the orbits to make bumps closed