# Action and phase jump analysis for local correction


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Thanks to the OMC team at CERN


Optics Tuning and Corrections for Future colliders

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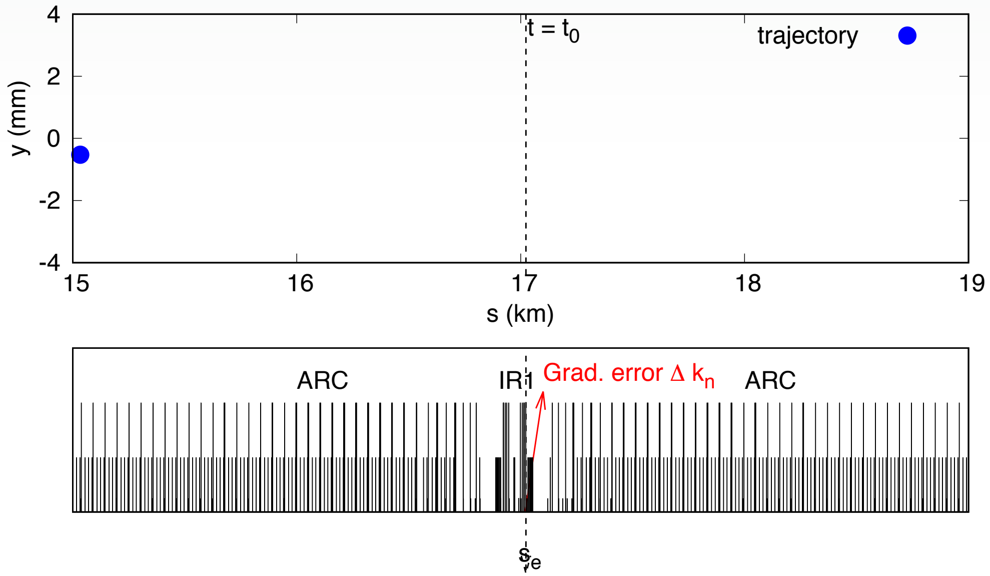
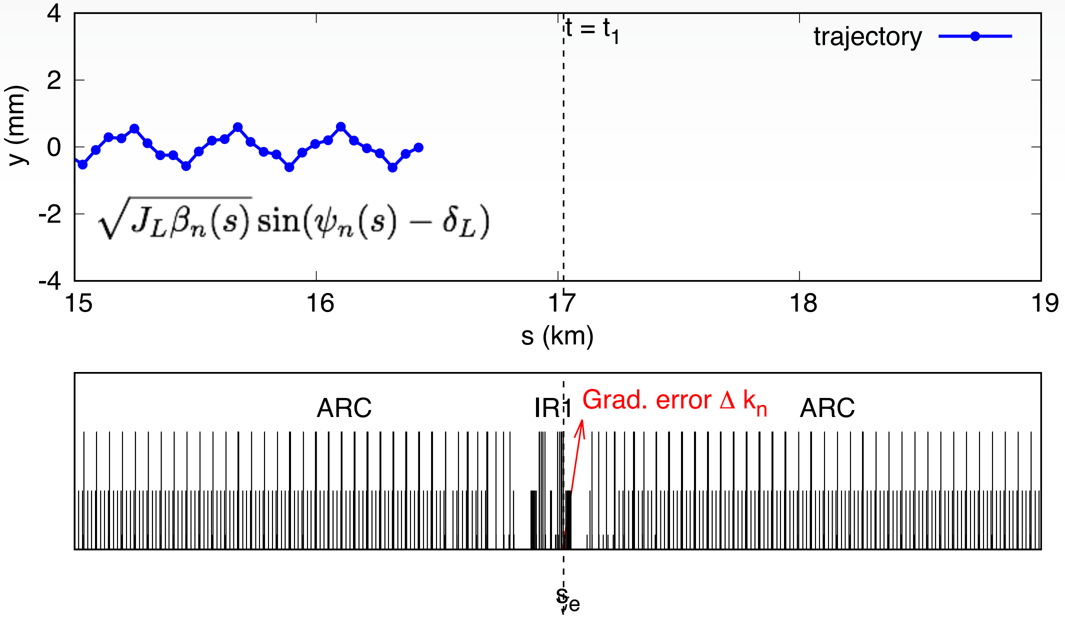
# Principle of Action and Phase Jump




- Launch a particle a the beginning of an  ideal lattice + grad error.
- The lattice can be a segment of the LHC lattice

# Principle of Action and Phase Jump

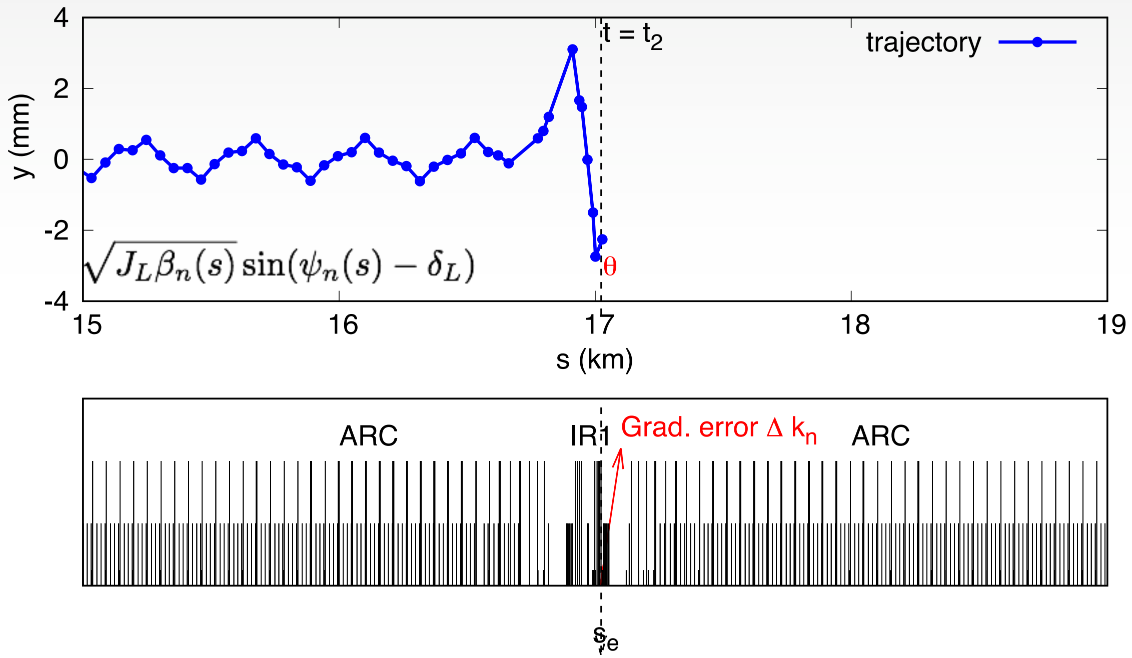



- For $s < s_e$ :

$$y(s) = \sqrt{J_L\beta_n(s)}\,\sin(\psi_n(s)-\delta_L)$$

Where $\beta_n,\ \psi_n$ are the nominal lattice functions.

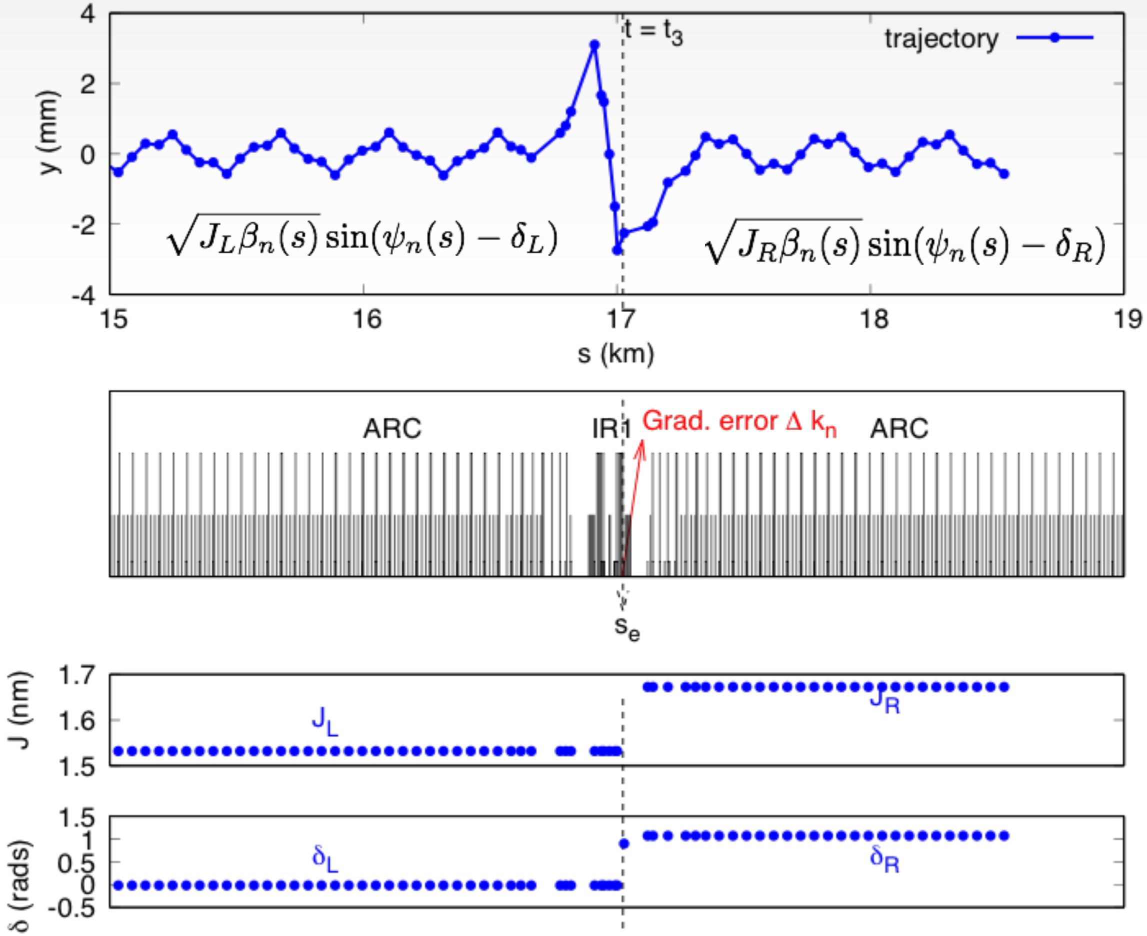
# Principle of Action and Phase Jump




- For $s = s_e$ the particle receive and aditional magnétic kick:

$$\theta_y = \Delta k_n L y(s_e)$$

# Principle of Action and Phase Jump




- For $s > s_e$

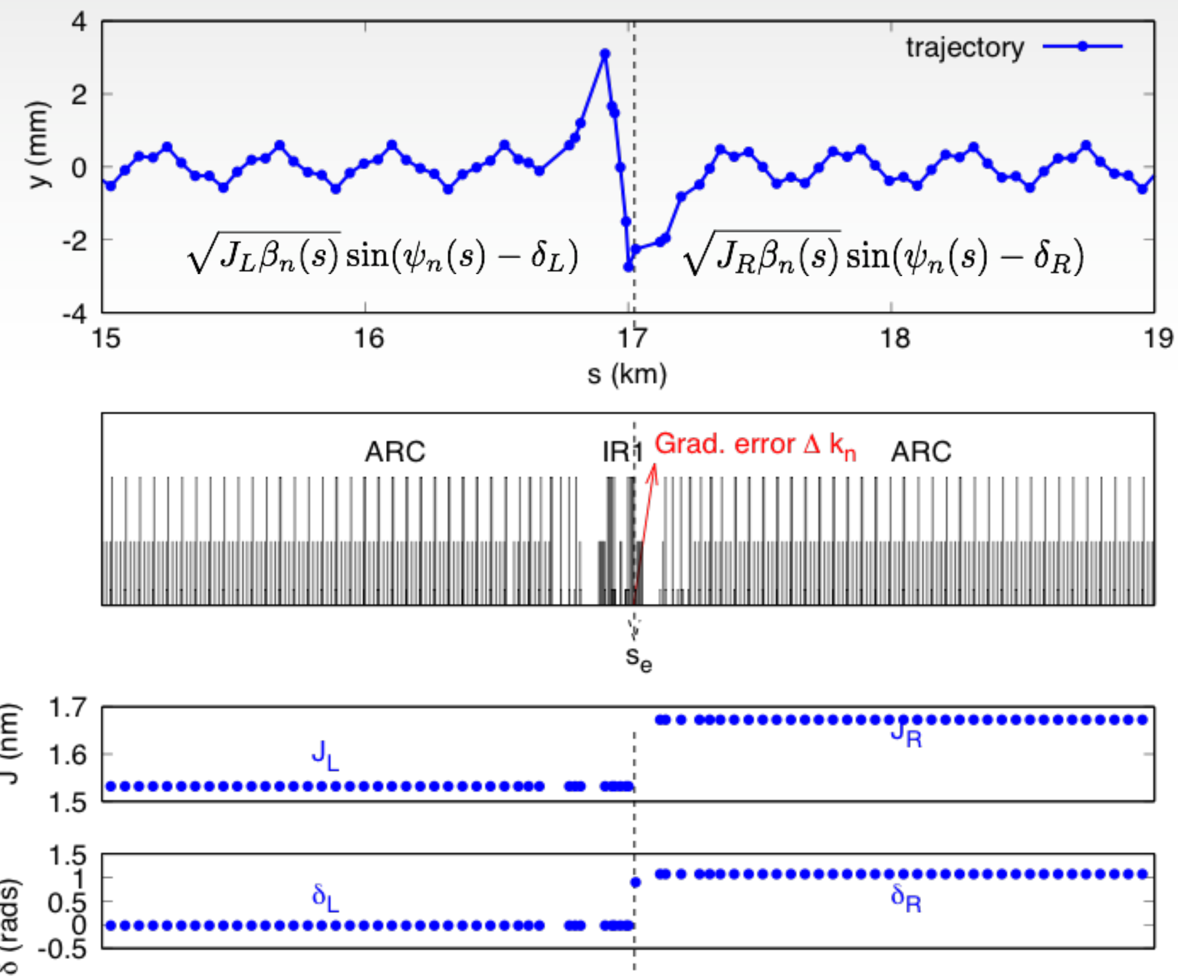
$$
\begin{aligned}
y(s) &= \sqrt{J_L\beta_n(s)}\sin\left(\psi_n(s)-\delta_L\right) \\
&\quad + A_y\sqrt{\beta_n(s)}\sin\left(\psi_n(s)-\eta\right) \\
&= \sqrt{J_R\beta_n(s)}\sin\left(\psi_n(s)-\delta_R\right)
\end{aligned}
$$

where: $A_y = \theta_y\sqrt{\beta_n(s)}$

$\eta = \psi_n(s_e)$

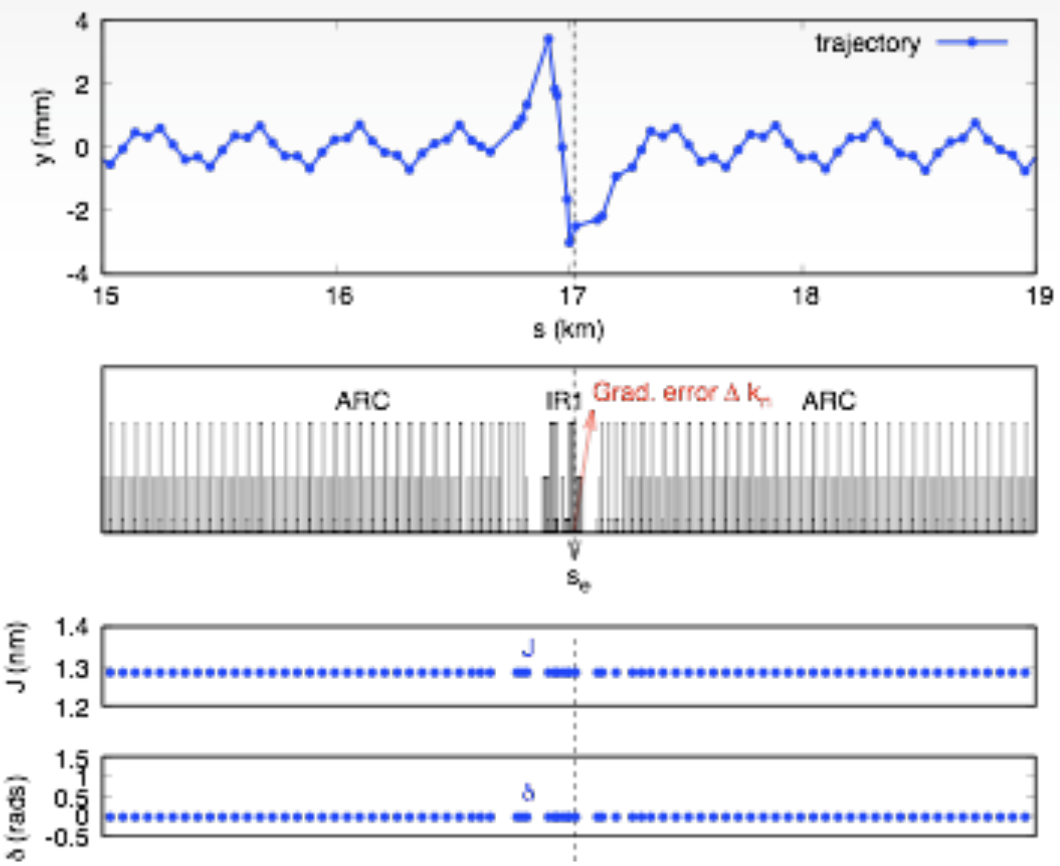
- $J_L, J_R, \delta_L, \delta_R$ can be estimated from BPM measurements.

# Principle of Action and Phase Jump




- These equations give an exact description of the simulated trajectory.
- This analysis can be extended to any number of quadrupole errors.
- This analysis is also valid for average trajectories costructed from TBT data sets (J. Cardona, et al., PRAB20.111004).

# The Conventional Betatron Equation




- It is also possible to use the conventional betatron equation:

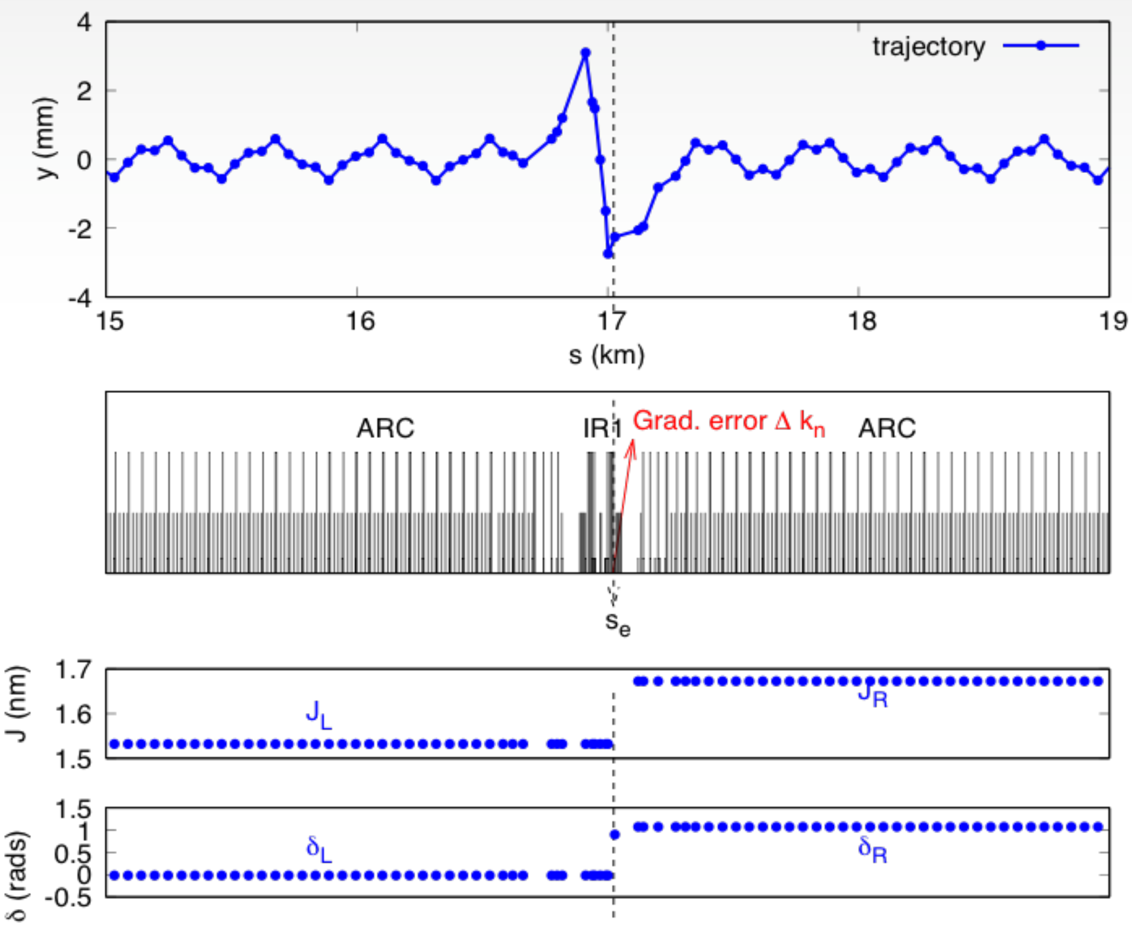
$$y(s) = \sqrt{J\beta_r(s)}\sin\left(\psi_r(s) - \delta\right)$$

where

$$\beta_r(s) = \beta_n(s) + \Delta\beta(s)$$
$$\psi_r(s) = \psi_n(s) + \Delta\psi(s)$$

and $\Delta\beta(s), \Delta\psi(s)$ are the beta and phase beating due to $\Delta k_n$.

# Estimating Quadrupole errors with APJ




- $A_y$ can be estimated from measured action and phases

$$A_y = \sqrt{2J_L + 2J_R - 4J_L J_R \cos(\delta_L - \delta_R)}$$

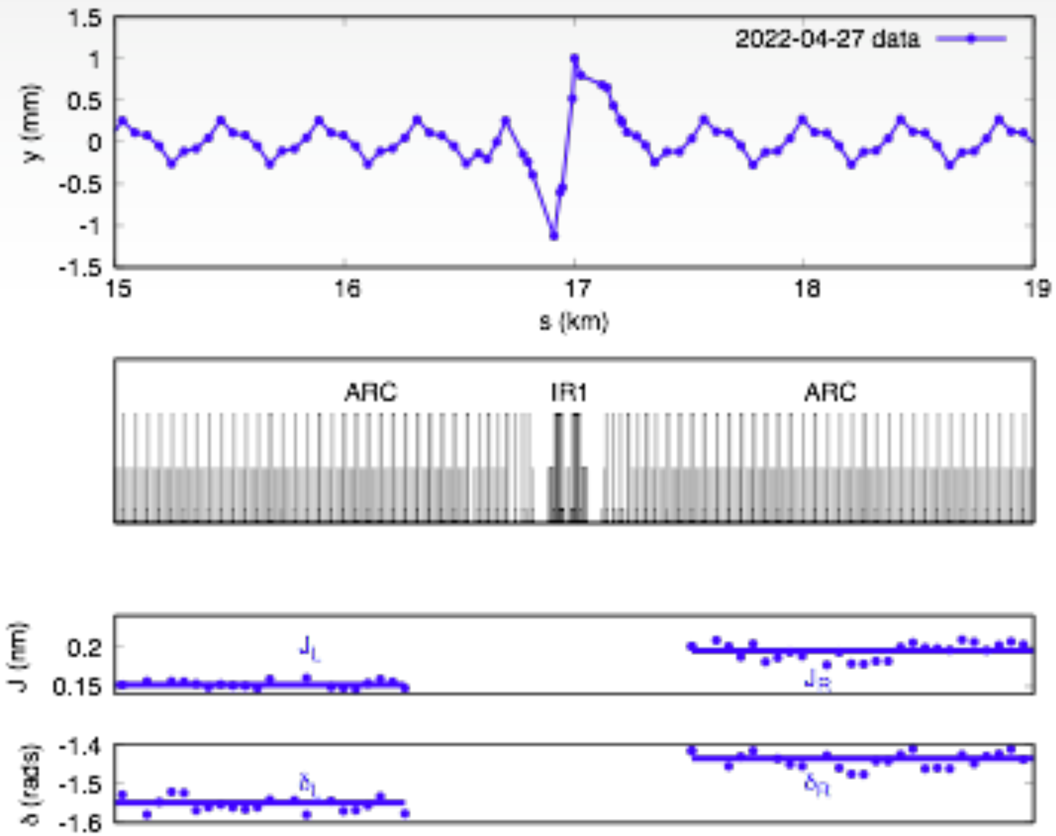
- $A_y$ can be related to grad. errors

$$A_y = \frac{y(s_e)}{\sqrt{\beta_{ny}(s_e)}} \Delta k_n \beta_{ny}(s_e) L$$

- Also to skew errors

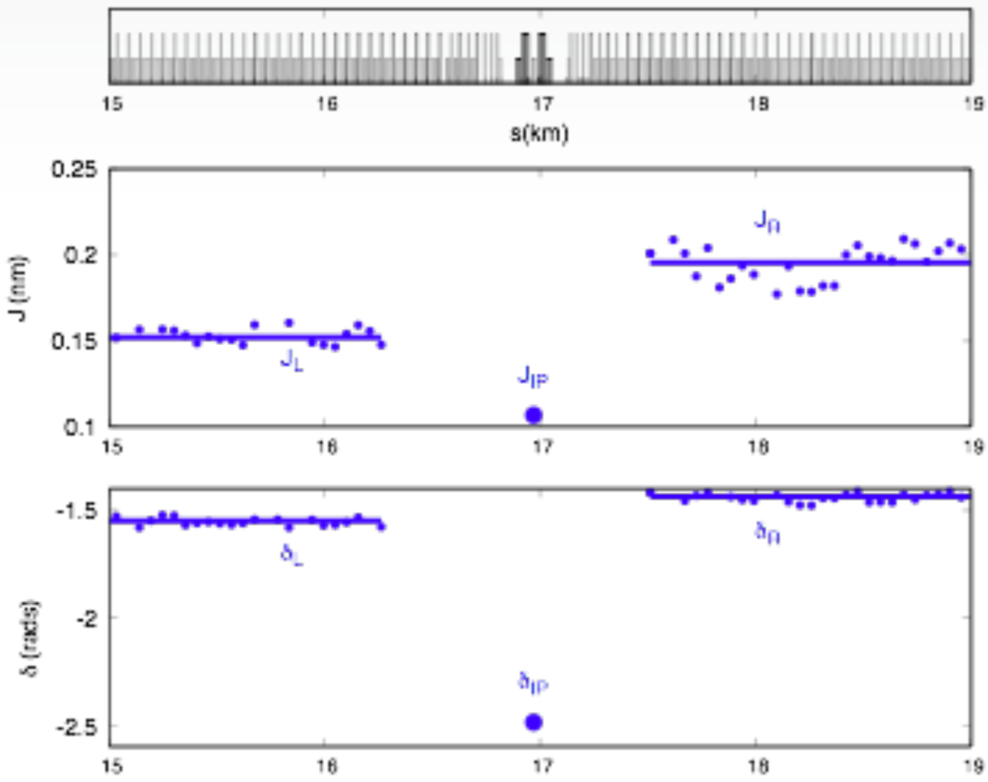
$$A_y = \frac{x(s_e)}{\sqrt{\beta_{nx}(s_e)}} \Delta k_s \sqrt{\beta_{nx}(s_e)\beta_{ny}(s_e)} L$$

# Errors in the Interaction Regions




- Action and phase jumps can be observed around high luminosity IRs.
- It also possible to estimate actions and phases in the IP using outputs of k-modulation experiments.

# Errors in the Interaction Regions




- $A_y$ can be estimated from action and phase plots.

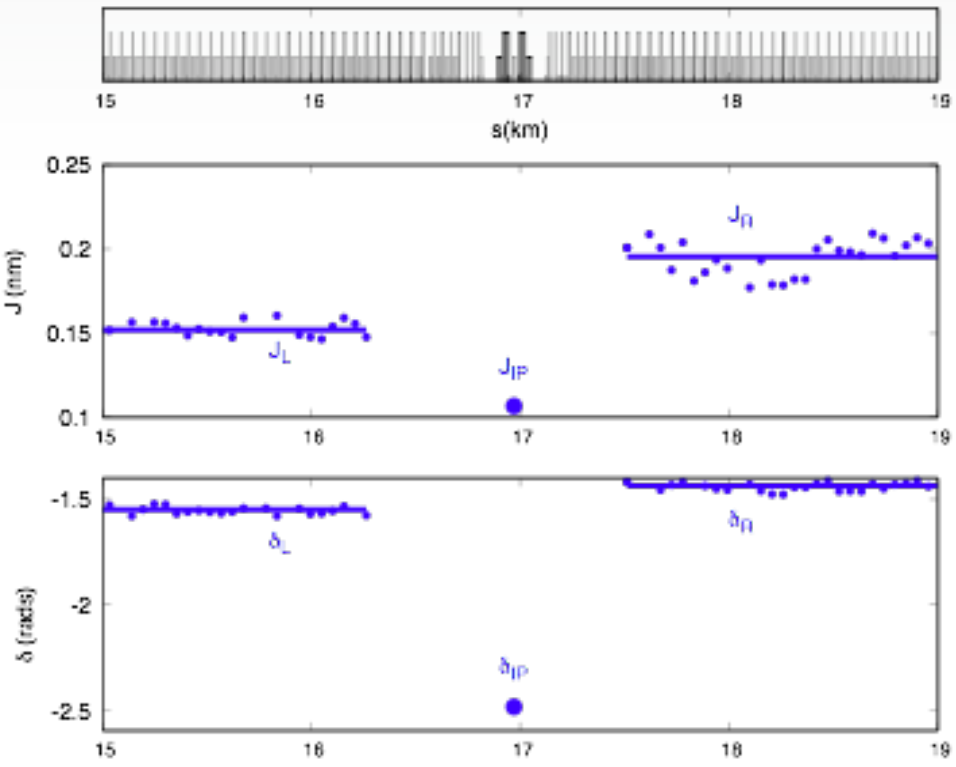
- For the left triplet:

$$A_y = \sqrt{2J_L + 2J_{IP} - 4\sqrt{J_L J_{IP}} cos(\delta_L - \delta_{IP})}$$

- For the right triplet:

$$A_y = \sqrt{2J_R + 2J_{IP} - 4\sqrt{J_R J_{IP}} cos(\delta_R - \delta_{IP})}$$

# Errors in the Interaction Regions




- $A_y$ is mainly produced by grad. errors in the triplet quadrupoles Q1, Q2 and Q3
- Assuming equal betatron phases in all 3 quads:

$$A_y = \frac{y(s_0)}{\sqrt{\beta_{ny}(s_0)}} \left(\Delta k_1 I_{y_1} + \Delta k_2 I_{y_3} + \Delta k_3 I_{y_3}\right)$$

where

$$I_{y_i} = \int_0^{L_i} \beta_{ny}(s) ds$$

- And similarly for x-plane

$$A_x = -\frac{x(s_0)}{\sqrt{\beta_{nx}(s_0)}} \left(\Delta k_1 I_{x1} + \Delta k_2 I_{x2} + \Delta k_3 I_{x3}\right)$$

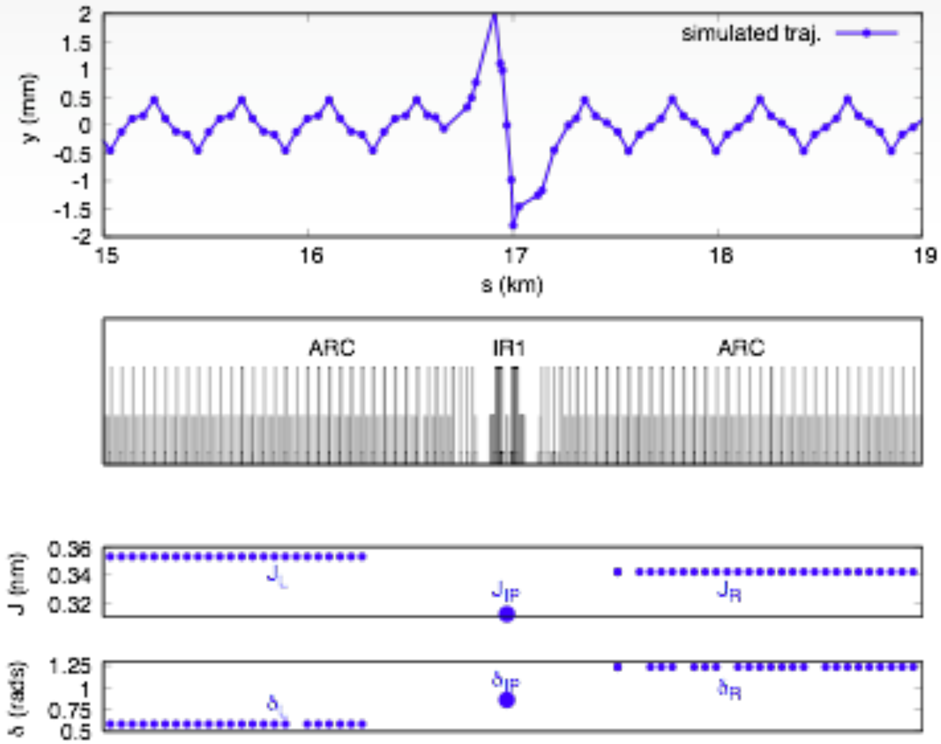
# Corrections in the IRs

- It is not possible to find the 3 errors with the above equations.
- However, it is possible to find how much the strength of Q2 and Q3 should be changed to compensate for the errors.

$$A_x + A_{x\,corr} = 0$$
$$A_y + A_{y\,corr} = 0$$

$$A_x = \frac{x(s_0)}{\sqrt{\beta_{nx}(s_0)}}\left(\Delta k_{2\,corr} I_{x2} + \Delta k_{3\,corr} I_{x3}\right)$$
$$A_y = -\frac{y(s_0)}{\sqrt{\beta_{ny}(s_0)}}\left(\Delta k_{2\,corr} I_{y2} + \Delta k_{3\,corr} I_{y3}\right)$$

# Simulation of the Correction



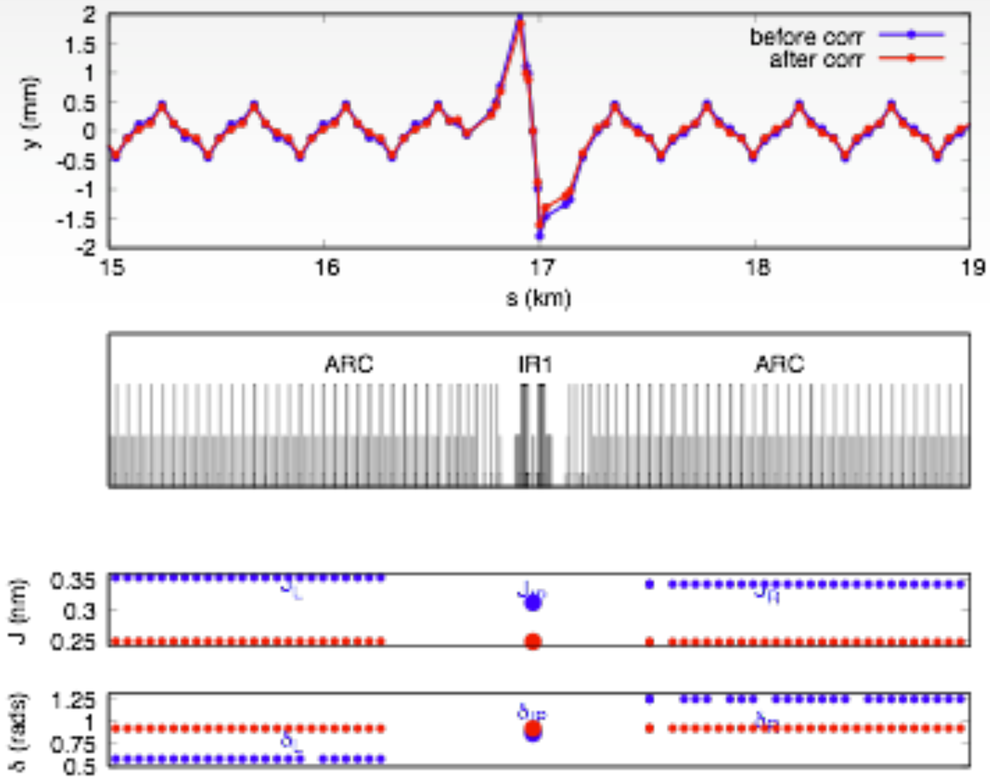

- In the left triplet.$(10^{-5}m^{-2})$:

$$\Delta k_1 = +1$$
$$\Delta k_2 = -1$$
$$\Delta k_3 = +1$$

- In  the right triplet  $(10^{-5}m^{-2})$ :

$$\Delta k_1 = -1$$
$$\Delta k_2 = +1$$
$$\Delta k_3 = -1$$

# Simulation of the Correction




- In the left triplet $(10^{-5}m^{-2})$ :

$$\Delta k_1 = +1$$
$$\Delta k_2 = -1$$
$$\Delta k_3 = +1$$
$$\Delta k_{2\,corr} = +0.86$$
$$\Delta k_{3\,corr} = -1.22$$

- In the right triplet $(10^{-5}m^{-2})$:

$$\Delta k_1 = -1$$
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$$\Delta k_3 = -1$$
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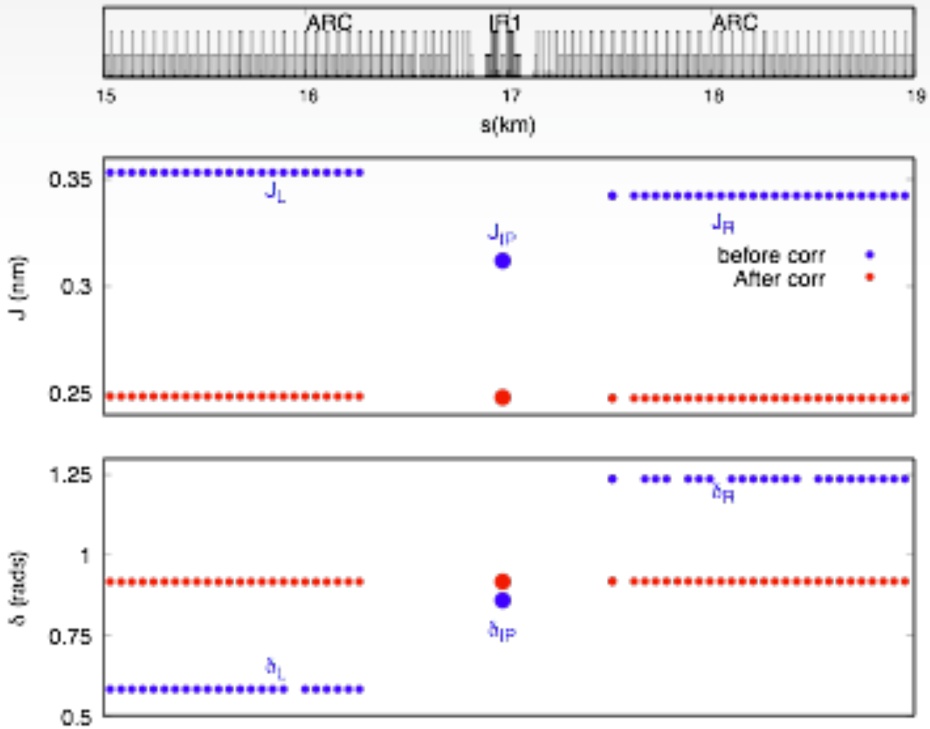
# Simulation of the Correction




The correction supresses the jump

- In the left triplet $(10^{-5}m^{-2})$ :

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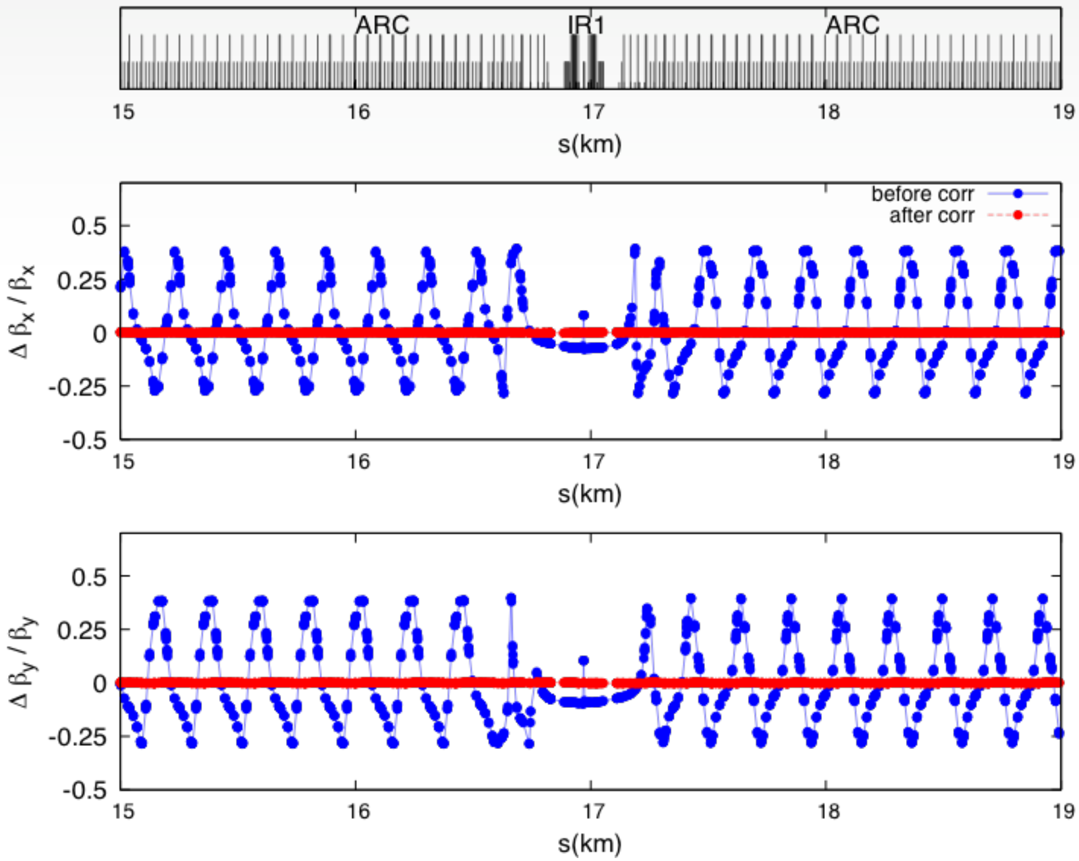



The correction also supresses the beta-beat

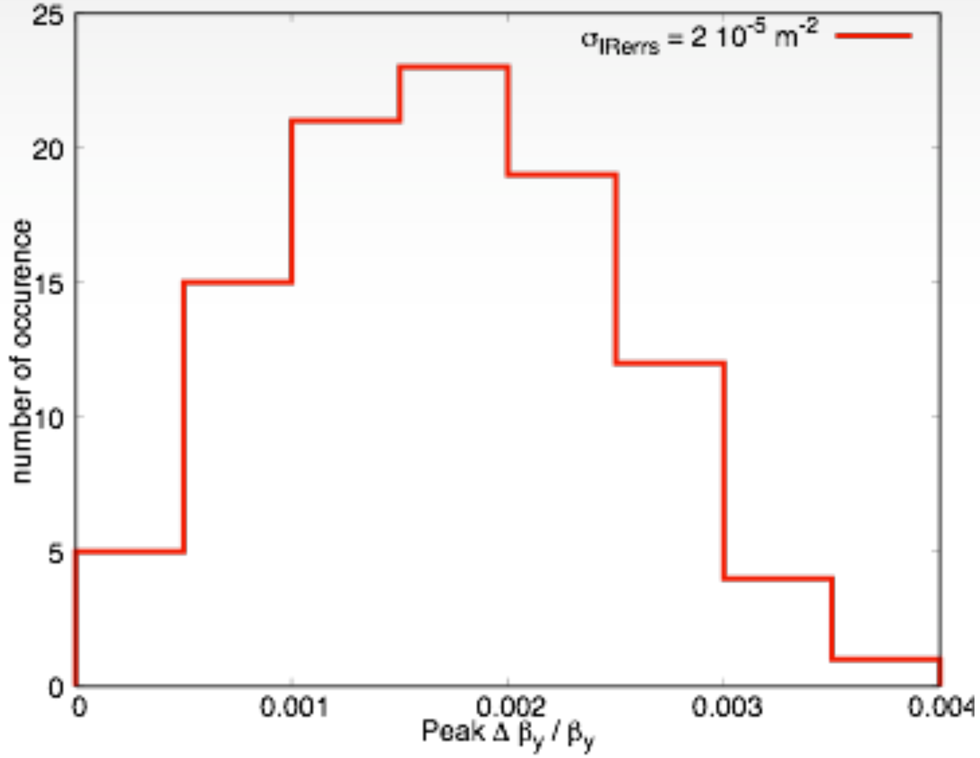
- In the left triplet $(10^{-5}m^{-2})$ :

$$\Delta k_1 = +1$$
$$\Delta k_2 = -1$$
$$\Delta k_3 = +1$$
$$\Delta k_{2\,corr} = +0.86$$
$$\Delta k_{3\,corr} = -1.22$$

- In the right triplet $(10^{-5}m^{-2})$:

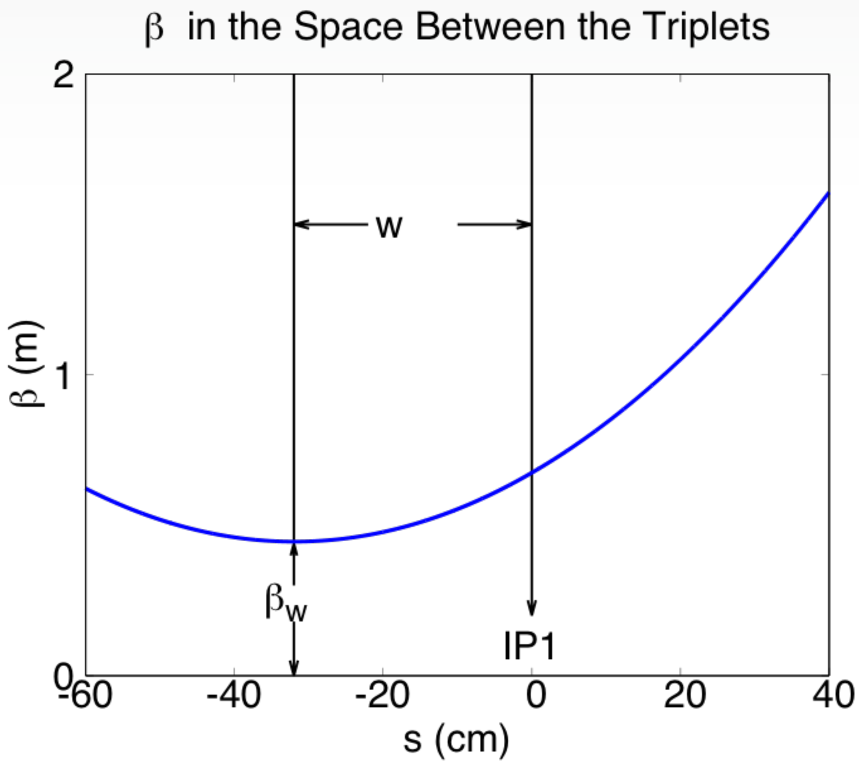
$$\Delta k_1 = -1$$
$$\Delta k_2 = +1$$
$$\Delta k_3 = -1$$
$$\Delta k_{2\,corr} = -0.86$$
$$\Delta k_{3\,corr} = +1.22$$

# Residual Beta-Beat after Correction




- The correction procedure is applied to 100 different error distributions.
- Histograms show the residual Peak beta-beat after corrections were applied.
- The peak beta-beat is below 0.4%.

# Action and phase in the IP

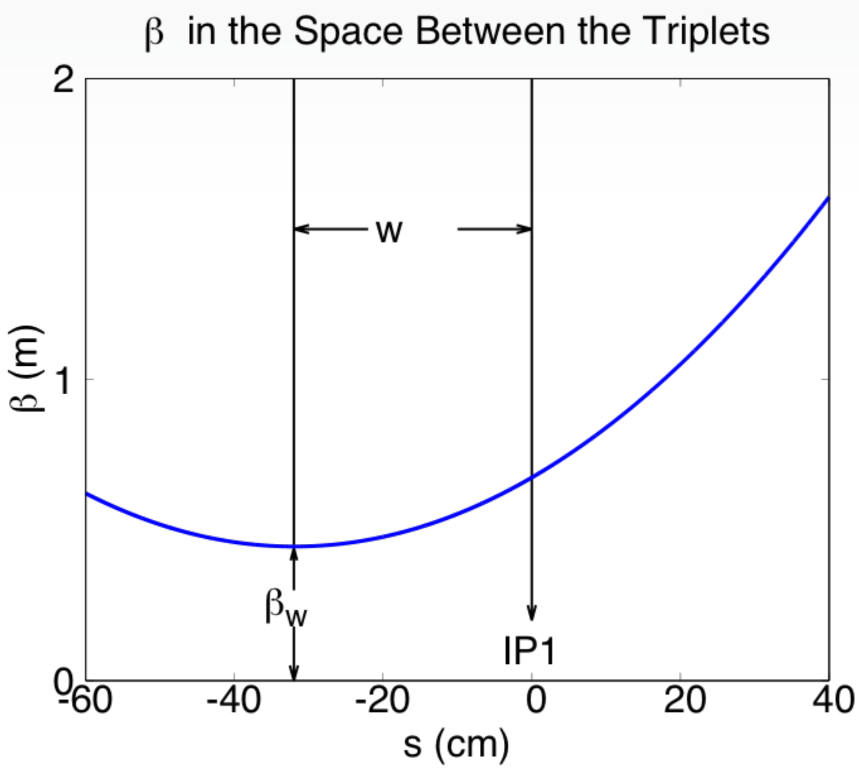



- If the space between the triplets is considered as a free space:

$$y(s) = \sqrt{2J_{IP}\beta_n(s)}\sin(\psi_n(s) - \delta_{IP}) \\ = \sqrt{2J\beta_m(s)}\sin(\psi_m(s) - \delta)$$

$$\beta(s) = \beta_w + \frac{w^2}{\beta_w}$$

# Action and phase in the IP




$$J_{IP} = J\frac{\beta_{wn}}{\beta_{wr}}\gamma_c\left(1+\tan^2\gamma_t\right)$$

$$\delta_{IP} = \psi_n(s_0) + \arctan\left(\frac{L+w_n}{\beta_{wn}}\right) - \gamma_t$$

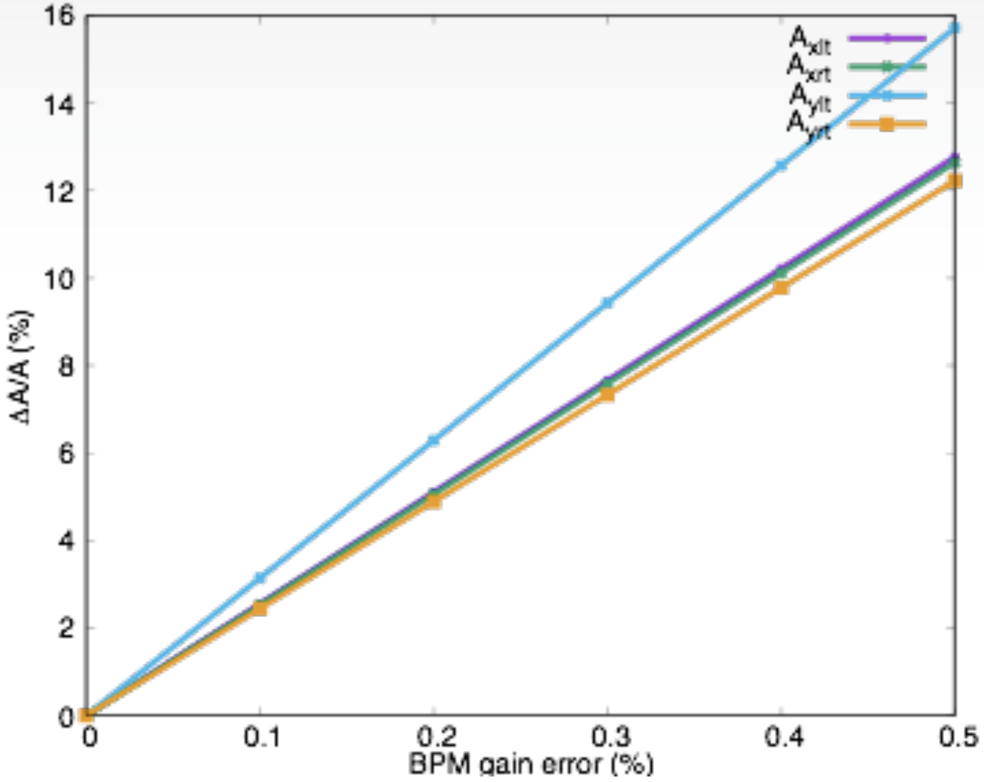
Where

$$\gamma_c = \psi_r(s_0) + \arctan\left(\frac{L+w_r}{\beta_{wr}}\right) - \delta$$

$$\gamma_t = \arctan\left(\frac{w_n - w_r + \beta_{wr}\tan\gamma_c}{\beta_{wn}}\right)$$

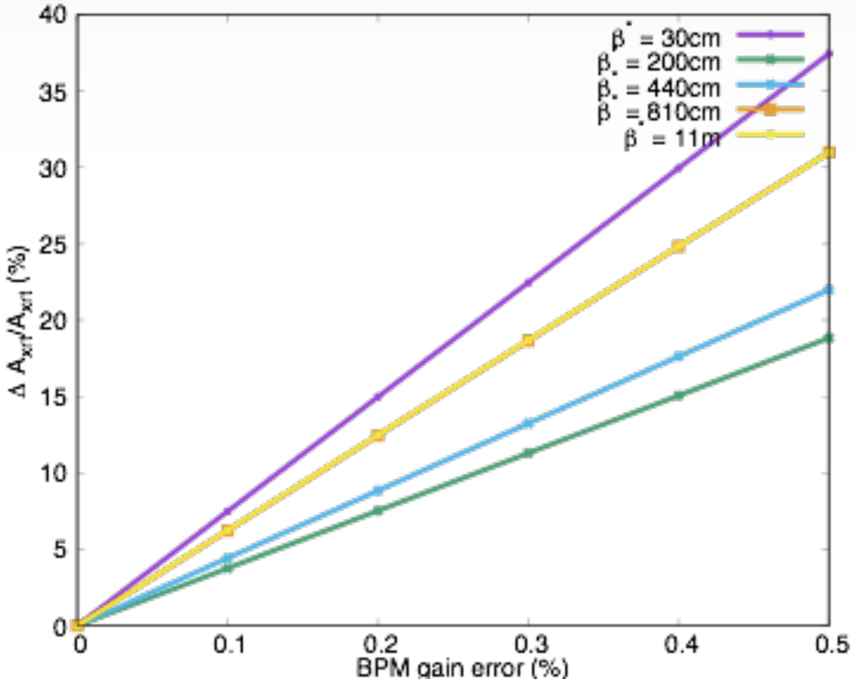
$w_r$ and $\beta_{wr}$ are measured with k-modulation.

# Action and phase in the IP




- Action and phase in the IP can also be estimated with the two closests BPM to the IP, the BPMSWs.
- This estimate is very sensitive to BPM calibration.
- Plots are generated with a gain error in BPMSWL.
- These plots depend on the distribution of errors.

# Action and phase in the IP

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- This estimate is very sensitive to BPM calibration.
- Plots are generated with a gain error in BPMSWL.
- These plots depend on the distribution of errors.

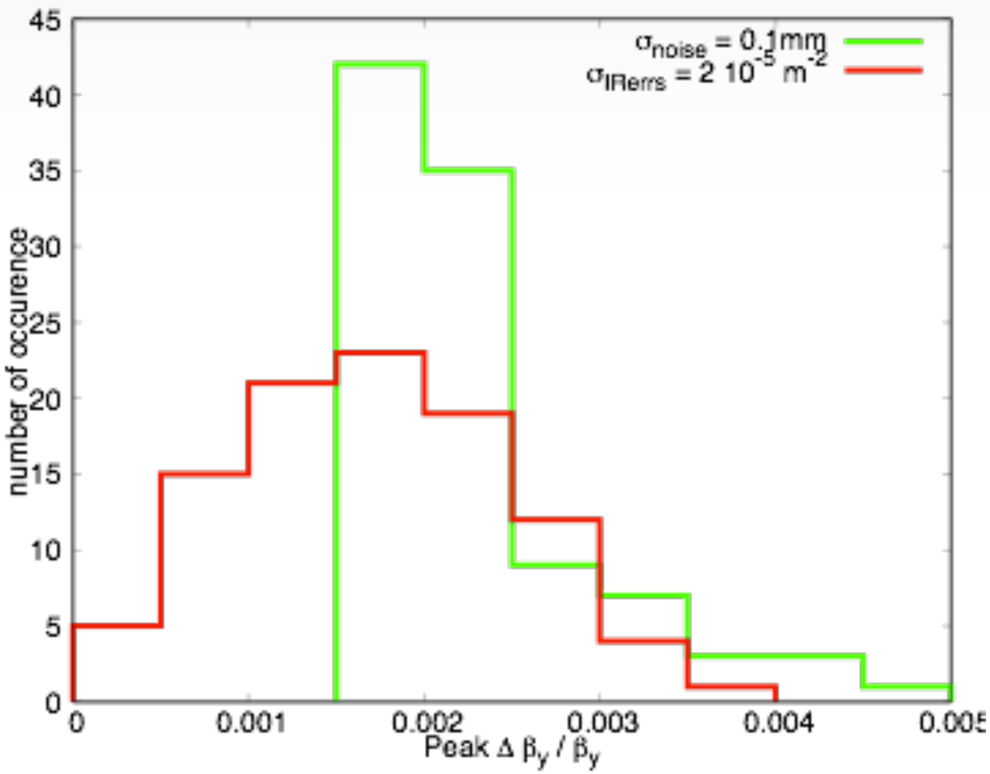
# Effect of the uncertainties in the correction

- Input data: TBT data, k-modulation, lattice functions.
- Uncertainties from TBT data: BPM Noise and gain errors.
- Uncertainties from k-mod:  waist shift uncertainty.
- Uncertainties from model: Gradient errors in the arcs.

# Effect of the uncertainties in the correction

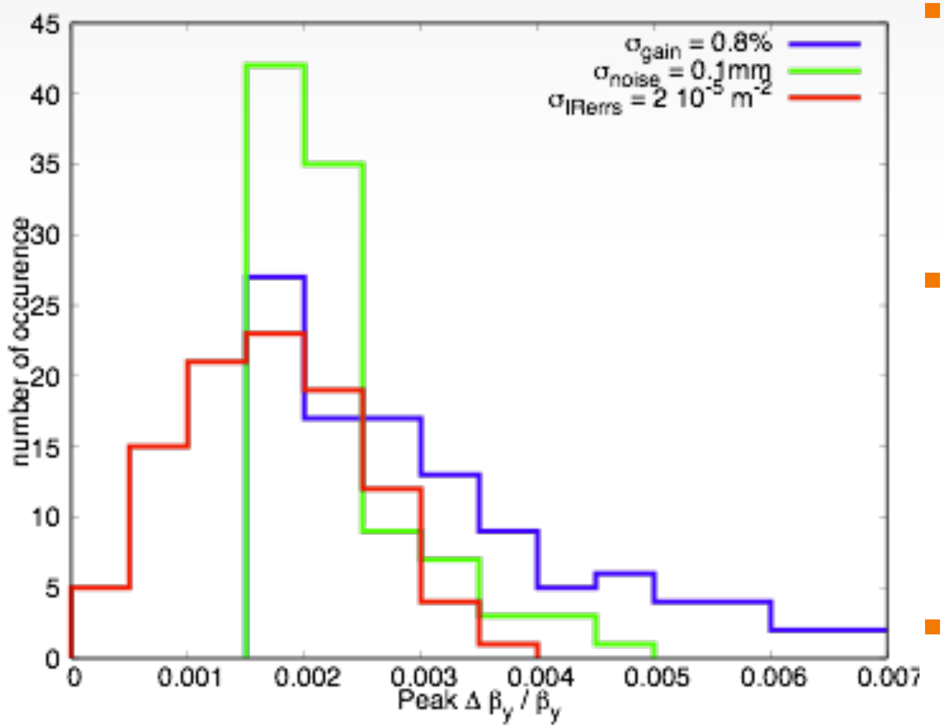
- BPM noise around 0.1 mm (L. Malina, OMC BI meeting 07/06/2019).
- BPM gain errors around 3% but they can be calibrated to 0.8% (J. Cardona, arXiv:2103.03964).
- Waist uncertainty of 4.4cm (extrapolated from fig 8 of PRAB20.011005, F. Carlier).
- Gradient errors in the arcs of 3*10-6 m^-2 (E. Fol, THPRB077, IPAC 2019).

# Effect of BPM Noise in Residual Beta-Beat



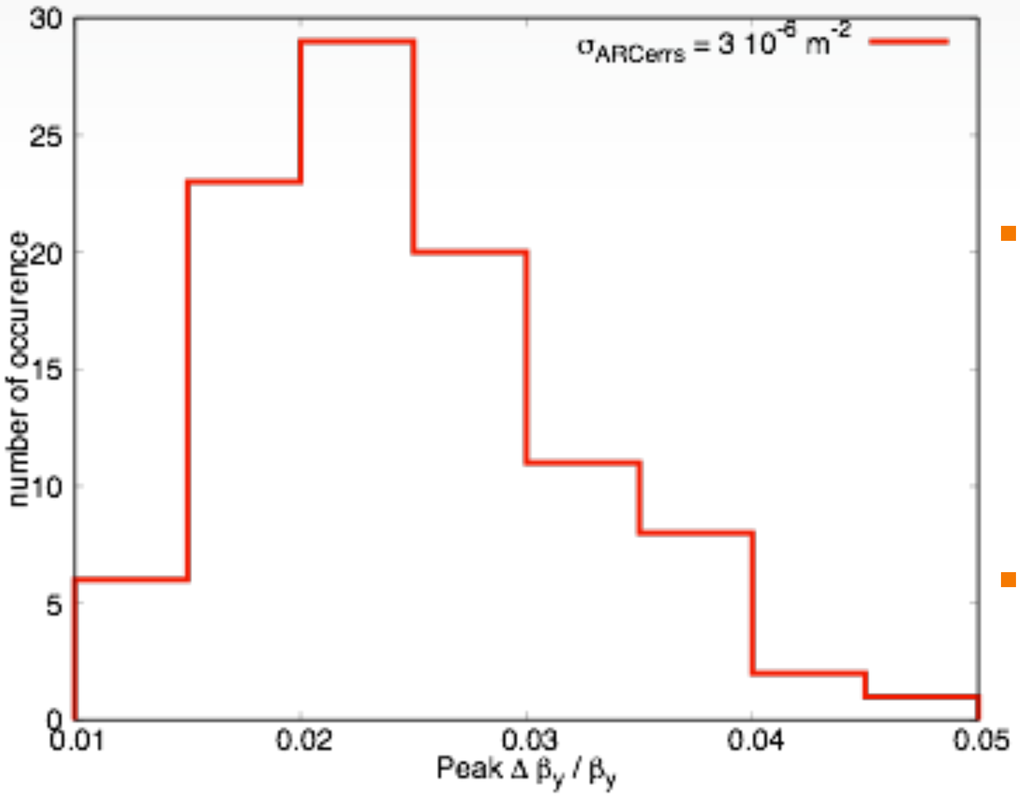

- 100 simulated TBT data sets are generated with BPM noise equal to 0.1mm.
- The correction procedure is applied to each TBT data set and its peak residual beta-beat is recorded.
- Peak residual beta-beat is just above the corresponding value for IR errors.

# Effect of BPM Gain Errors in Residual Beta-Beat



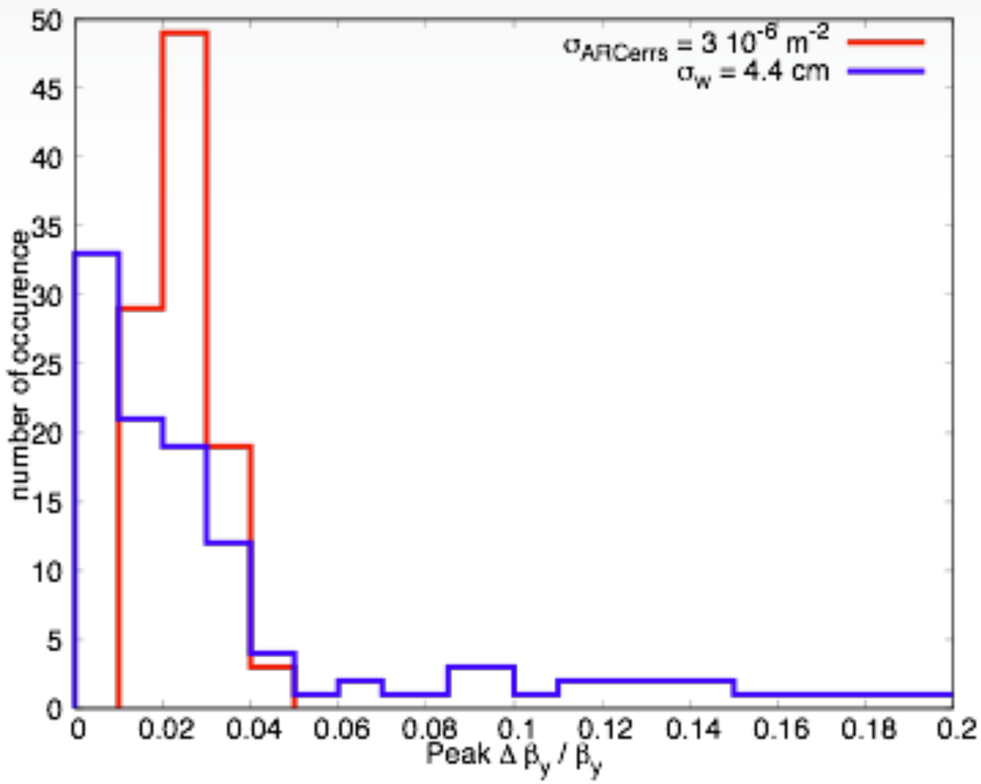

- 100 simulated TBT data sets are generated with BPM gain errors equal to 0.8%
- The correction procedure is app lied to each TBT data set and its peak residual beta-beat is recorded.
- Peak residual beta-beat is similar to other uncertainties.

# Effect of arc grad errors in Residual Beta-Beat




- 100 simulated TBT data sets are generated with grad errors of 3*10^-6 m^-2.
- The correction procedure is applied to each TBT data set and their peak residual beta-beat is recorded.
- Peak residual beta-beat is now one order of magnitude larger but around 2.5 %.

# Effect of waist uncertainty in Residual Beta-Beat




- 100 simulated TBT data sets are generated with waist uncertainties of 4.4 cm
- The correction procedure is applied to each TBT data set and their peak residual beta-beat is recorded.
- Peak residual beta-beat is comparable with the case of arc grad errors.

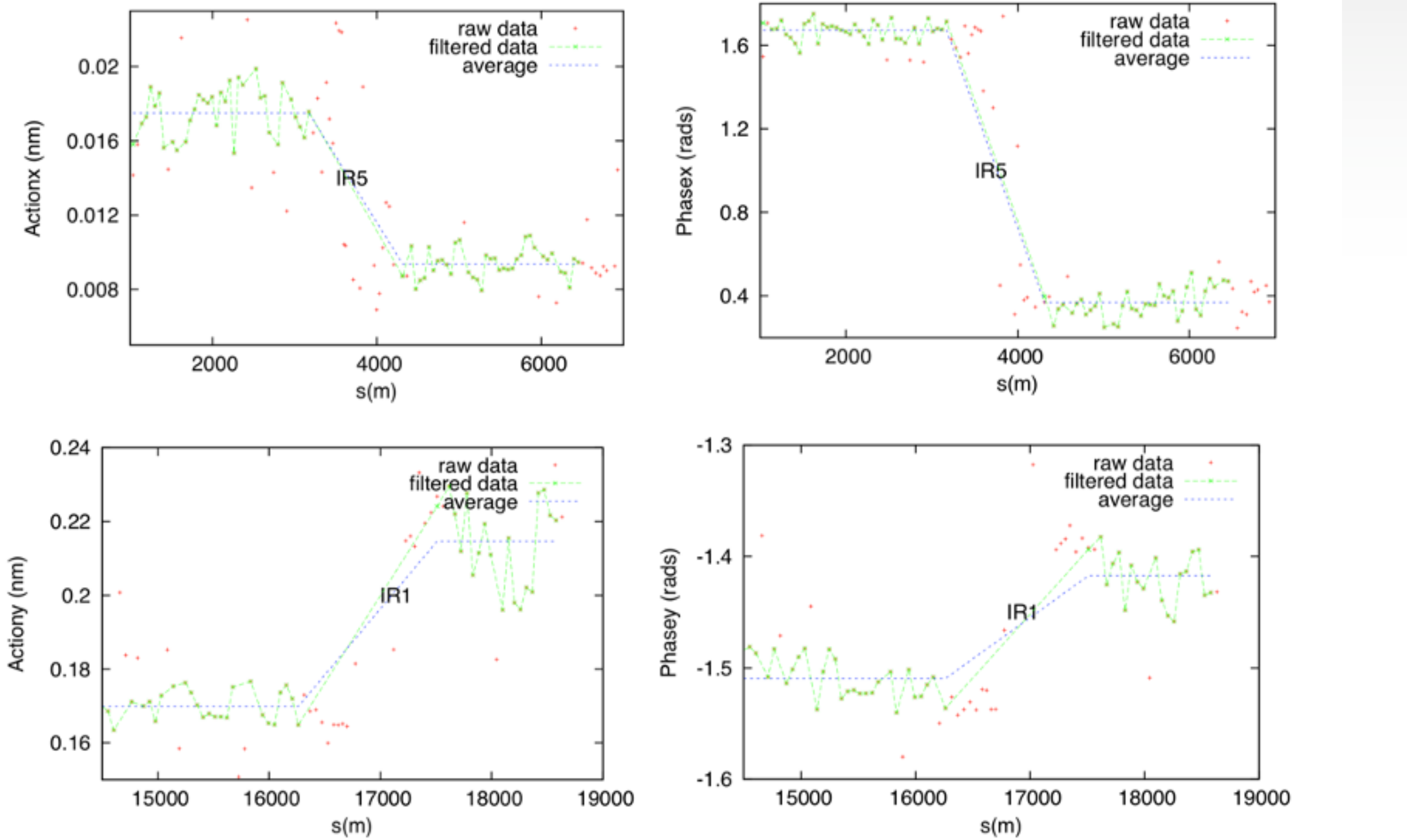
# IR Corrections during LHC run 3

- Local corrections for several IRs were estimated and applied during run3 of the LHC.

- Two methods were used: SbS and APJ.

- The two methods used the same input data: TBT data sets and K-modulation outputs.

# Samples of Action and Phase plots at IR1 and IR5




(J. Cardona et al, WEPOPT010, IPAC 2022)

# Correction estimates obtained for APJ and SbS

| | Circuit | $\Delta k\,(10^{-5}\mathrm{m}^{-2})$ | | | Polarity |
|---|---|---|---|---|---|
| | | **Run 2** | **APJ** | **SbS** | **LSA** |
| IR1 | **ktqx1.l1** | 1.23 | 0 | 1.23 | - |
| | **ktqx1.r1** | −1.23 | 0 | −1.23 | + |
| | **ktqx2.l1** | 0.65 | 1.15 | 0.41 | + |
| | **ktqx2.r1** | −1.0 | −0.87 | −0.70 | - |
| | **ktqx3.l1** | 1.22 | 1.94 | 1.22 | - |
| | **ktqx3.r1** | −1.22 | −2.88 | −1.22 | + |
| IR5 | **ktqx1.l5** | 2.0 | 0 | 2.25 | - |
| | **ktqx1.r5** | −2.0 | 0 | −2.10 | + |
| | **ktqx2.l5** | 0.26 | 0.38 | 0.16 | + |
| | **ktqx2.r5** | 1.48 | 0.93 | 1.35 | - |
| | **ktqx3.l5** | 1.49 | 3.40 | 2.25 | - |
| | **ktqx3.r5** | −1.49 | −2.46 | −2.10 | + |

(T. Persson et al, WEPOST008, IPAC 2022)

# Effect of corrections in Beta-Beat

- Either APJ or SbS Corrections reduced rms beta-beat from 150% to 20%.

| | IP 1 $\frac{\Delta\beta^*}{\beta^*}$ [%] | | | | IP 5 $\frac{\Delta\beta^*}{\beta^*}$ [%] | | | |
|---|---|---|---|---|---|---|---|---|
| | Beam 1 | | Beam 2 | | Beam 1 | | Beam 2 | |
| | H | V | H | V | H | V | H | V |
| No corr. | 122 | 55.3 | 34.0 | 331 | 130 | 65,3 | 16.0 | 76.3 |
| APJ corr. | 5.3 | 3.0 | -0.3 | -3.7 | 42.0 | 18.0 | -1.7 | 7.3 |
| SbS corr. | 15.0 | 3.3 | 12.0 | 46.7 | 32.7 | 2.7 | -2.3 | 1.7 |

- For normal operation in the LHC, APJ corrections are used for IR1 and SbS corrections are used for IR5.

# Thank you to all Members of OMC Team

- T. Persson, R. Tomás, H. García Morales, M. Hofer, J. Keintzel, F. Soubelet, F. Carlier, E. Fol, A. Wegscheider, J. Dilly, E. H. Maclean, M. Le Garrec, A. Costa Ojeda.
- Thanks for allowing an space to test the APJ method.

# Finding the Errors Rather than the Corrections

- For 1 error:

$$\eta = \psi_n(s_e)$$

- For the triplets, under the approximation of phases:

$$\eta = \psi_n(Q1) = \psi_n(Q2) = \psi_n(Q3)$$

- In general  quad phases are different, and

  $\eta$ is between these phases.

# Finding the Errors Rather than the Corrections

- For this more general condition:

$$A_z \cos\left(\eta - \psi_n(s_0)\right) = \pm \frac{z(s_e)}{\sqrt{\beta_{n\,z}(s_e)}} \left(Ic_{z\,1}\Delta k_1 + Ic_{z\,2}\Delta k_2 + Ic_{z\,3}\Delta k_3\right)$$

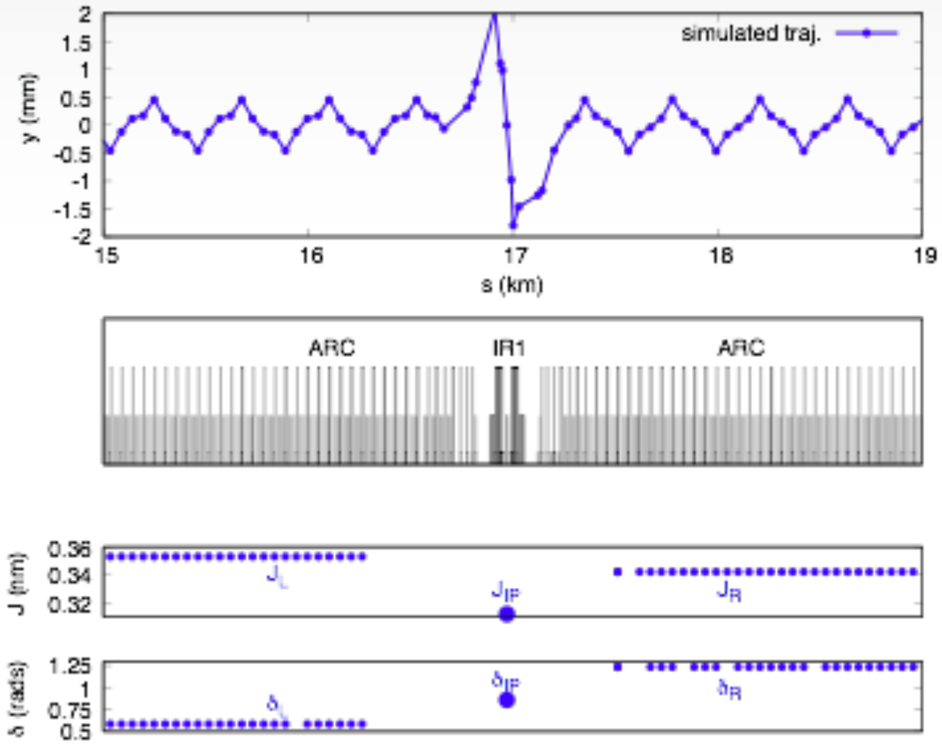
$$A_z \sin\left(\eta - \psi_n(s_0)\right) = \pm \frac{z(s_e)}{\sqrt{\beta_{n\,z}(s_e)}} \left(Is_{z\,1}\Delta k_1 + Is_{z\,2}\Delta k_2 + Is_{z\,3}\Delta k_3\right)$$

Where z is x or y and

$$Ic = \int_0^L \beta_n(s) \frac{\cos\left(\psi_n(s) - \delta_L\right)}{\sin(\psi_n(s_0) - \delta_L)} ds$$

$$Is = \int_0^L \beta_n(s) \frac{\sin\left(\psi_n(s) - \delta_L\right)}{\sin(\psi_n(s_0) - \delta_L)} ds$$

# Simulation with Errors in IR1




- In the left triplet $(10^{-5}m^{-2})$ :

$$\Delta k_1 = +1$$
$$\Delta k_2 = -1$$
$$\Delta k_3 = +1$$

- In the right triplet $(10^{-5}m^{-2})$ :

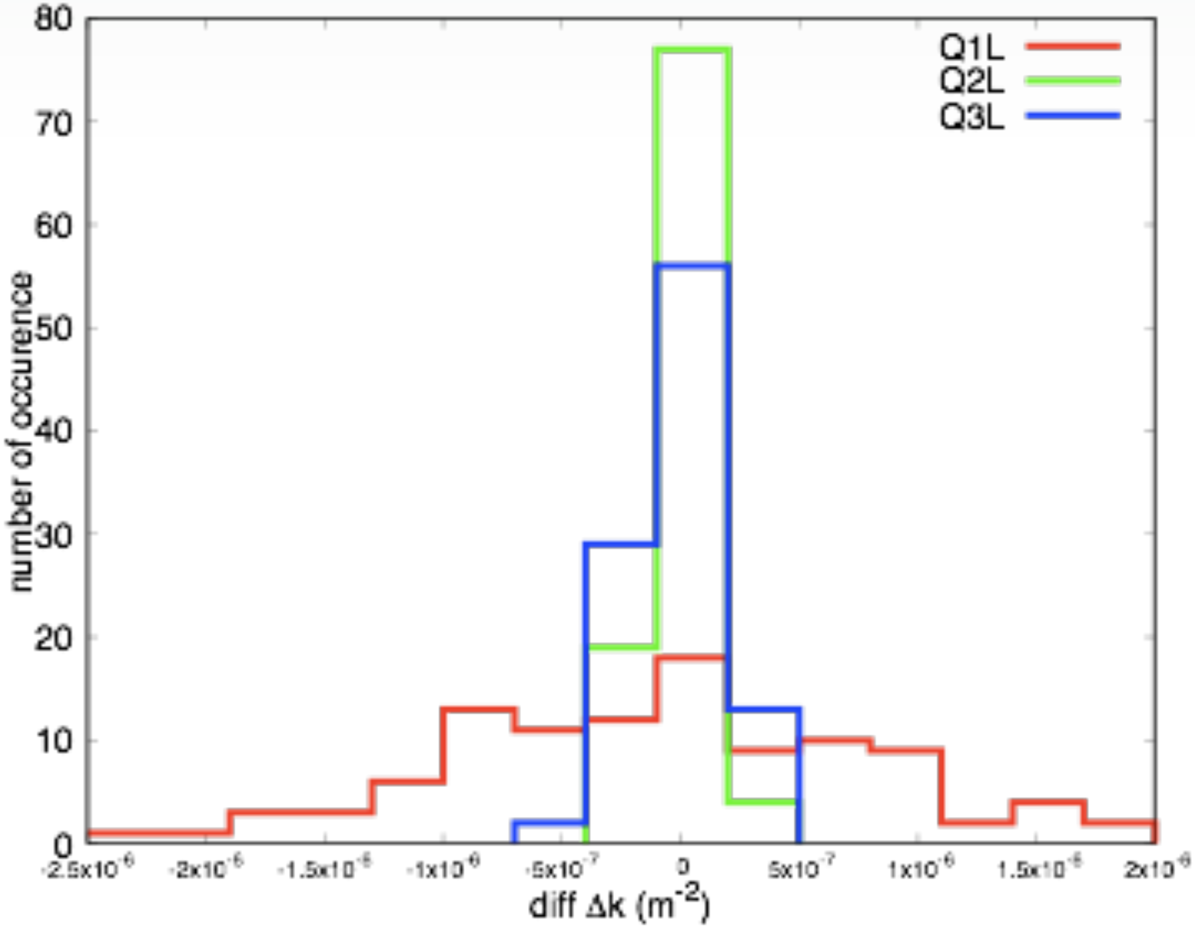
$$\Delta k_1 = -1$$
$$\Delta k_2 = +1$$
$$\Delta k_3 = -1$$

# Simulation with Errors in IR1

| | Grad. Error $(10^{-5}m^{-2})$ | Value recovered with APJ $(10^{-5}m^{-2})$ | Difference $(10^{-5}m^{-2})$ |
|---|---|---|---|
| Q3L | +1 | +1.004 | 0.004 |
| Q2L | -1 | -0.979 | 0.002 |
| Q1L | +1 | +0.984 | -0.015 |
| Q1R | -1 | -0.947 | 0.005 |
| Q2R | +1 | +0.996 | 0.000 |
| Q3R | -1 | -1.001 | -0.001 |

# Differences between Set errors and Recovered errors in IR1

## (100 different error distributions)

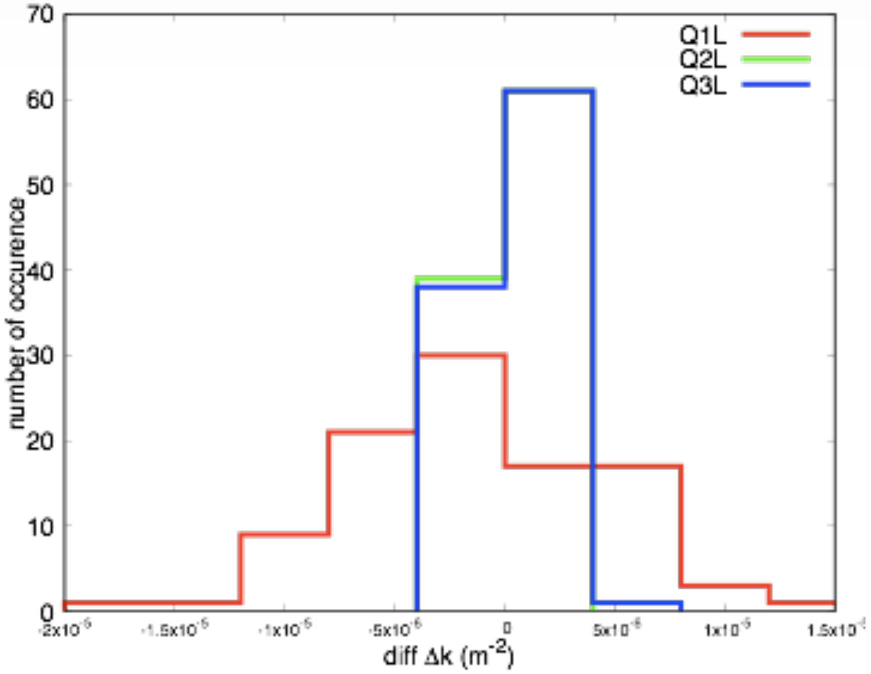



$$\sigma(\mathrm{QL1}) = 7.8 * 10^{-7} m^{-2}$$

$$\sigma(\mathrm{QL2}) = 1.0 * 10^{-7} m^{-2}$$

$$\sigma(\mathrm{QL3}) = 1.8 * 10^{-7} m^{-2}$$

# Sensitivity to Arc Errors (3e-6 m^-2)

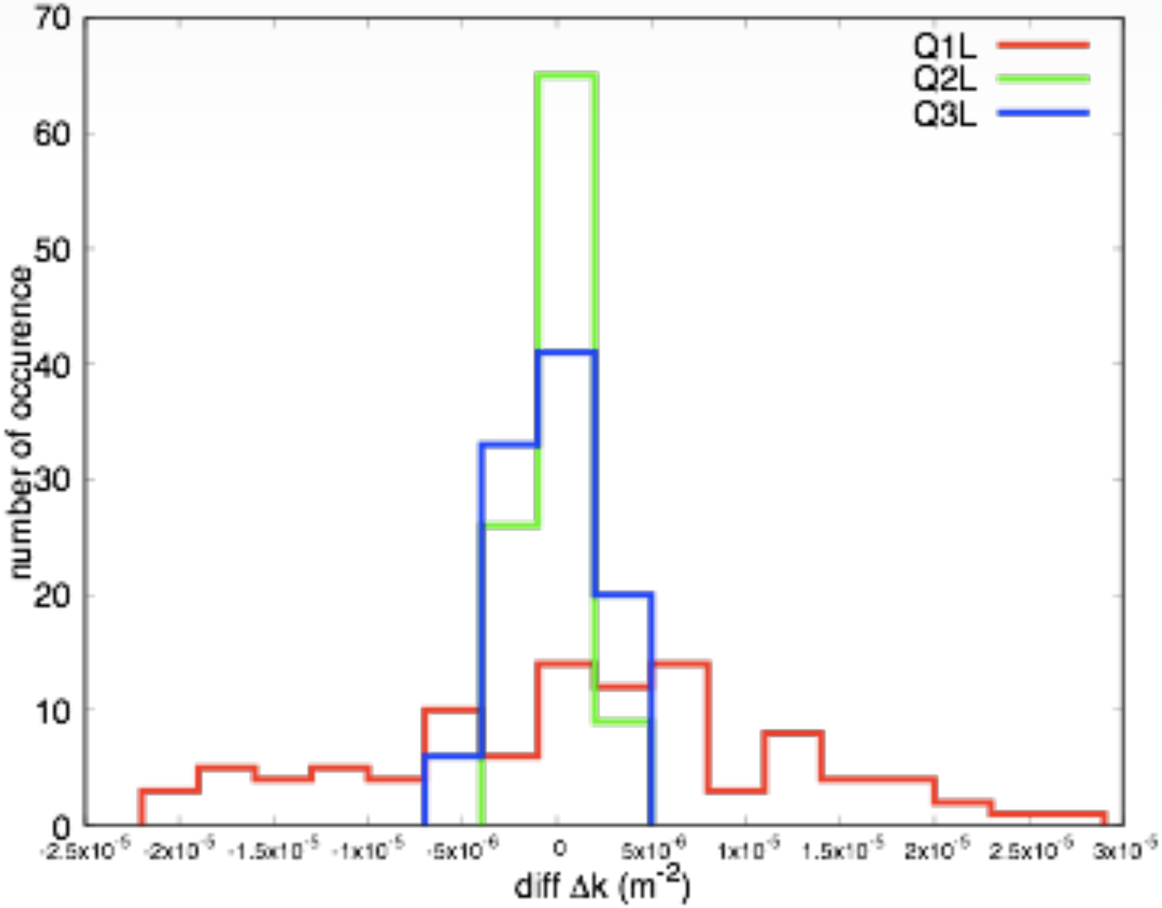



$$\sigma(\mathrm{QL1}) = 5.5 * 10^{-6} m^{-2}$$

$$\sigma(\mathrm{QL2}) = 7.1 * 10^{-7} m^{-2}$$

$$\sigma(\mathrm{QL3}) = 1.3 * 10^{-6} m^{-2}$$

# Sensitivity to Noise (0.1 mm)

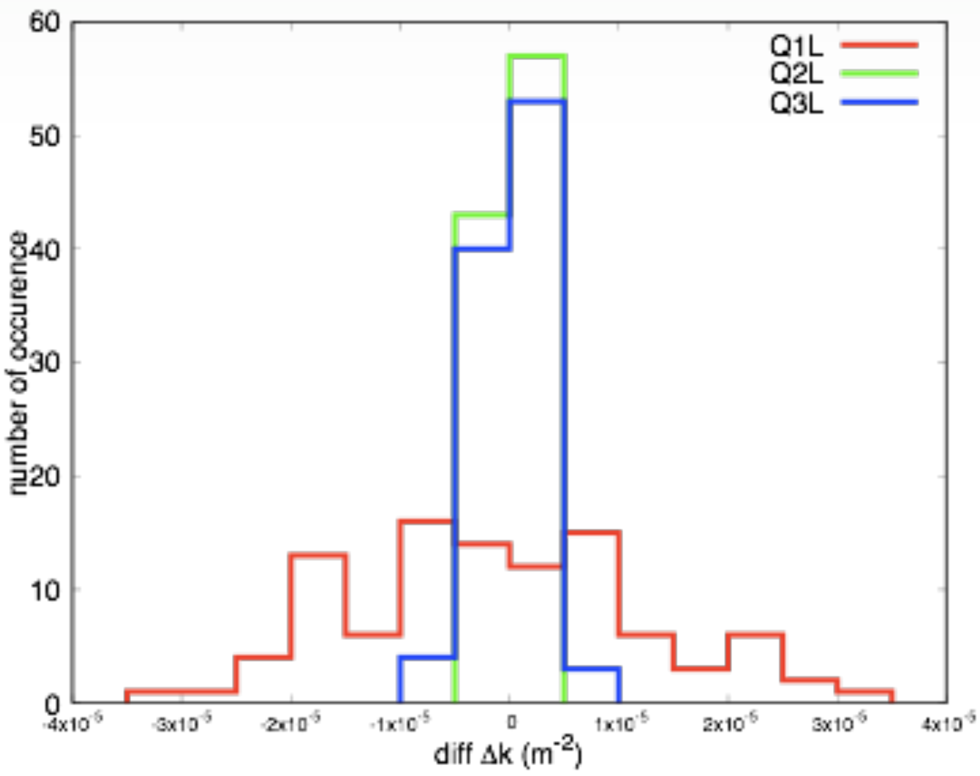



$$\sigma(\mathrm{QL1}) = 1.1 * 10^{-5} m^{-2}$$

$$\sigma(\mathrm{QL2}) = 1.4 * 10^{-6} m^{-2}$$

$$\sigma(\mathrm{QL3}) = 2.4 * 10^{-6} m^{-2}$$

# Sensitivity to Gain Errors (0.8%)

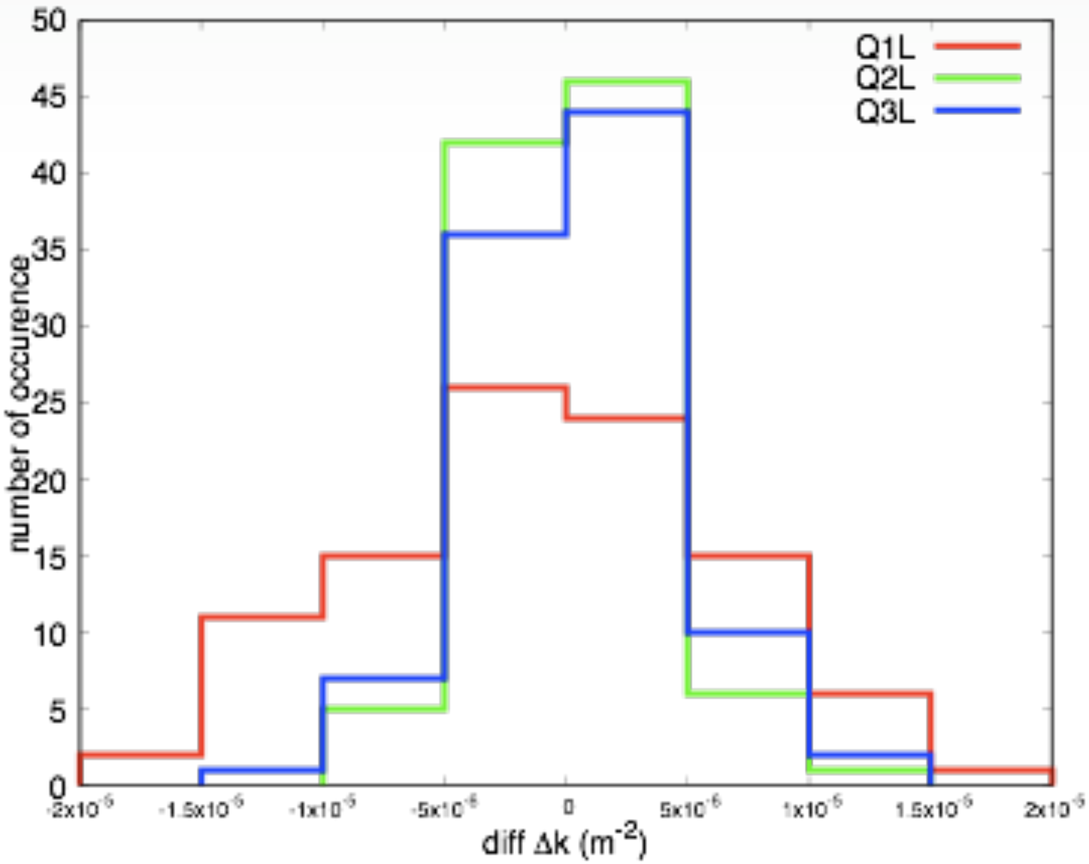



$$\sigma(\mathrm{QL1}) = 1.3 * 10^{-5} m^{-2}$$

$$\sigma(\mathrm{QL2}) = 1.7 * 10^{-6} m^{-2}$$

$$\sigma(\mathrm{QL3}) = 2.9 * 10^{-6} m^{-2}$$

# Sensitivity to waist uncertainty (4.4cm)




$$\sigma(\mathrm{QL1}) = 7.2 * 10^{-5} m^{-2}$$
$$\sigma(\mathrm{QL2}) = 3.3 * 10^{-6} m^{-2}$$
$$\sigma(\mathrm{QL3}) = 4.2 * 10^{-6} m^{-2}$$

# $\sigma$ all sources

| | Q1L $(10^{-5}m^{-2})$ | Q2L $(10^{-5}m^{-2})$ | Q3L $(10^{-5}m^{-2})$ |
|---|---|---|---|
| IR errors | 0.08 | 0.01 | 0.02 |
| Arc errors | 0.55 | 0.07 | 0.13 |
| BPM Noise | 1.10 | 0.14 | 0.24 |
| BPM gains | 1.30 | 0.17 | 0.29 |
| Waist uncert. | 7.20 | 0.33 | 0.42 |

# Alternative to find errors in the triplets

- Main reason for higher uncertainties in Q1 than in the others: Integrals in Q2 and Q3 are several times larger than integrals in Q1.
- Another source of uncertainty is $\eta$ :
  Its measurement needs to be very precise.
- Alternative: turn off Q2 and Q3 and measured error in Q1 with original method.
- Once Q1 is known, Q2 and Q3 can be found with all quadrupoles ON and the original method.

# Conclusions

- APJ equations can be used to evaluate magnetic errors and/or corrections at high luminosity interaction regions.
- Evaluation of individual magnetic errors from TBT data is limited by different sources of uncertainties. Experiments where magnets Q2 and Q3 are turn off can help to evaluate individual magnetic errors in the triplets.