

I. Kodeli





# Neutron transport – Boltzmann equation

Rate of particle gain = Rate of particle loss

#### or:

Inscattering gain + Source production gain = Leakage loss + Outscattering loss

$$\begin{split} &\frac{1}{v}\frac{d\phi\left(\bar{r},E,\bar{\Omega},t\right)}{dt} + \overline{\nabla}\cdot\bar{\Omega}\phi\left(\bar{r},E,\bar{\Omega},t\right) + \Sigma_{T}\left(\bar{r},E\right)\phi\\ &= S\left(\bar{r},E,\bar{\Omega},t\right) + \int\!dE'\!\int\!\Sigma_{S}\left(\bar{r},E'\to E,\bar{\Omega}'\to\bar{\Omega}'\right)\phi\left(\bar{r},E',\bar{\Omega}',t\right)d\bar{\Omega}' \end{split}$$

Statistical approach: n = averageparticle density in a particular region of phase space at a certain time t

$$\Phi(\vec{r}, \vec{\Omega}, E, t) = vn(\vec{r}, \vec{\Omega}, E, t)$$



#### Ludwig Boltzmann



Ludwig Boltzmann

Born Ludwig Eduard Boltzmann

> February 20, 1844 Vienna, Austrian Empire (present-day Austria)

September 5, 1906

(aged 62)

Tybein near Trieste, Austria-

Hungary (present-day

Duino, Italy) Suicide

Residence Austria, Germany

Nationality Austrian Fields Physics

Institutions University of Graz

> University of Vienna University of Munich University of Leipzig

Alma mater University of Vienna

Doctoral advisor Josef Stefan

Doctoral students Paul Ehrenfest Philipp Frank Gustav Herglotz Franc Hočevar Ignacij Klemenčič

Other notable students

Lise Meitner

Known for Boltzmann constant

> Boltzmann equation Boltzmann distribution

H-theorem

#### **BOLTZMANN** transport equation: assumptions

$$\frac{1}{v}\frac{\partial}{\partial t}\Phi = B\Phi + Q = (P - L)\Phi + Q$$

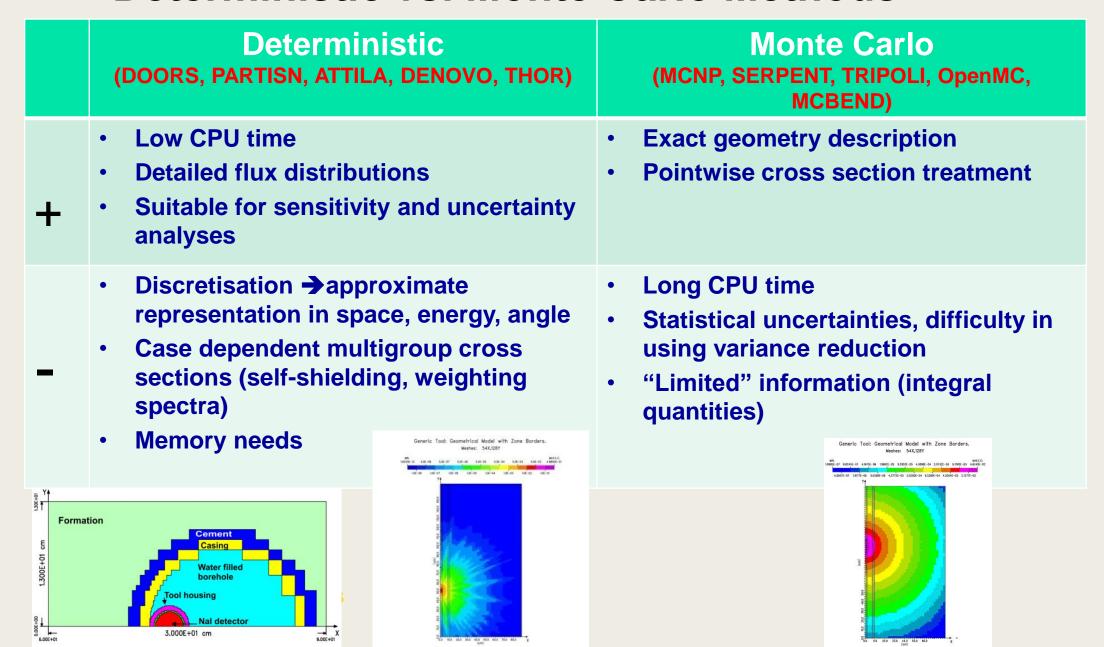
- <u>Diluted gas approximation:</u> only collisions between 2 particles are considered, 3-body collisions are assumed to be very rare
- Molecular chaos approx.: no correlation between colliding particles, so that the movement is fully chaotic;  $(W_{12}=W_1\ W_2)$
- The <u>collision time is much shorter</u> than the time that elapses between collisions; time rate of change of  $W_1$  and  $W_2$  can be neglected.
- Over the range of the forces of interaction, the <u>gradient of inhomogeneities in</u> the transport medium may be ignored. This assumption effectively makes W a function of  $(r_1-r_2)$  only, and allows gradients of W to be neglected over a distance of the range of molecular forces.

#### **Boltzmann – Louville equations**

$$\frac{1}{v}\frac{\partial}{\partial t}\Phi = B\Phi + Q = (P - L)\Phi + Q$$

 $f_1(r,v,t)$ , as defined in Louville equation, is the probability (averaged over phase space) that particle number one is at position r with velocity v at time t. The Boltzmann equation, however, is a balance equation for f(r,v,t), where  $f(r,v,t)dr\ dv$  is the average number of particles in the volume element dr with velocity lying between v and v+dv at time t.

#### **Deterministic vs. Monte Carlo Methods**



#### **Evaluated Cross Section Files Data and Formats**

#### **General purpose lib.**

- BROND-3.1
- **CENDL-3.1**
- ENDF/B-VIII
- JEFF-3.3
- **JENDL-5.0**
- RUSFOND-2010
- TENDL-2021

 $E \sim 10^{-5} \text{ eV} - 150 \text{ MeV}$ 

- MF=1: description text
- MF=2: resonances parameters
- MF=3: energy cross-sections
- MF=4: angular Distributions
- **MF=5**: energy Distributions
- **MF=6**: energy-angle distributions
- MF=7: thermal scattering data
- MF=8: radioactive data
- MF=9-10: nuclide production data
- MF=12-15: photon production data
- MF=30-40: covariance data

# Main uncertainties in Neutronics Calculations

- Mathematical methods and simplifications: e.g. M/C statistics, S<sub>N</sub> space/energy/angular discretization, anisotropic scattering order, convergence criteria
- <u>Nuclear data uncertainties</u>: nuclear cross-sections, fission spectra (U, Pu), standards, response functions (kerma, dpa)
- Radiation source description (space/energy distribution)
- Geometry modelling, material compositions, conditions (1D/2D/3D, temperature, exact locations...)
- "Human factor"
- Validation & Verification
  - Benchmark experiments
  - Sensitivity and uncertainty analysis

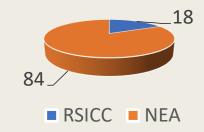
- **■** Benchmark experiments
- ND Sensitivity/uncertainty analysis & SUSD3D/XSUN-2023

## Benchmark experiments

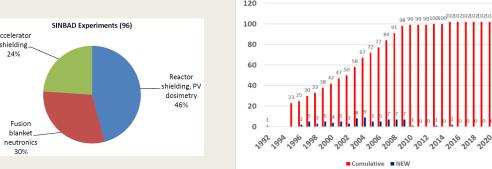
- Preserve results from expensive experiments in an user-friendly format
- Evaluate the quality, consistency and completeness of experimental information
- Validation of radiation transport codes, nuclear data performance, reactor design concepts / components
- Developing an approach of global method and data validation, using integral experiments
- Education & training
- OECD/NEA NSC sponsored projects on integral experiments:
  - SINBAD Radiation Shielding Experiments
  - ICSBEP International Handbook of Evaluated Criticality Safety
     Benchmark Experiments
  - IRPhEP Reactor Physics Experiments



#### **SINBAD - Radiation Shielding Experiments**



- Objective: Compilation of high-quality experiments for validation and benchmarking of computer codes and nuclear data used for radiation transport and shielding problems encompassing:
  - reactor shielding, PV dosimetry (48)
  - fusion blanket neutronics (31)
  - accelerator shielding (23)
- Quality review 51 benchmarks Simplicity of use; easy & low-cost maintenance



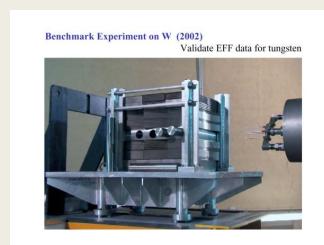
- OECD/NEA Working Party on Scientific Issues of Reactor Systems (WPRS) Expert Group on Physics of Reactor Systems (EGPRS): monitor, steer and support the continued development of SINBAD (Chair R. Grove, ORNL)
- Working Party on Evaluation Cooperation (WPEC) SG47 "Use of SINBAD for Nuclear Data Validation" 2019 – 2022 (I. Kodeli)
- NEA Task Force on SINBAD development (>2022) (T. Miller, ORNL, O. Buss, NEA)

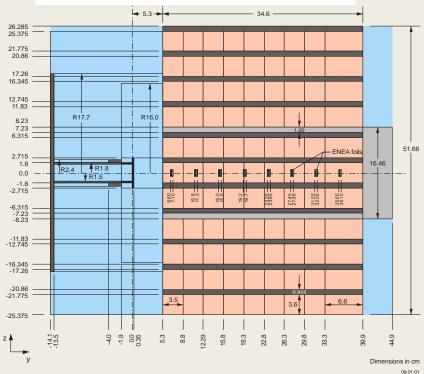
- Distribution on CD-ROM by the NEA DB and the RSICC.
   <a href="https://www.oecd-nea.org/science/wprs/shielding/sinbad/">https://www.oecd-nea.org/science/wprs/shielding/sinbad/</a>
   <a href="https://www-rsicc.ornl.gov/BENCHMARKS.aspx">https://www-rsicc.ornl.gov/BENCHMARKS.aspx</a>

# SINBAD (Radiation Shielding Experiments Data Base)

#### **Shielding Materials**

- **■** Fe/SS (27 benchmarks)
- $\blacksquare$  H<sub>2</sub>O (11)
- Air (9)
- Na (6)
- Concrete (5)
- **Pb, W, Si/SiC (4)**
- Graphite, Al (3)
- O, V, Cu (2)
- Li, Mn, Ni, N, Nb, Be, Th, Bi,  $(CH_2)_{2n}$  (1)
- Multiple materials (18)





- **■** Benchmark experiments
- ND Sensitivity/uncertainty analysis & SUSD3D/XSUN-2023

#### Propagation of uncertainty – "sandwich" formula

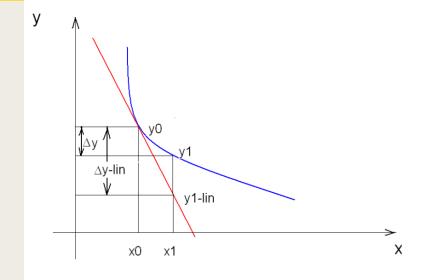
Often a quantity (y) is not measured directly, but is obtained from other quantities (x). In first order approximation:

$$y_{j} = y_{j}(x_{i})$$

$$\vec{y} = (y_{i}^{t}), \vec{x} = (x_{i}^{t})$$

$$y = ax$$

$$\delta y = a\delta x + x\delta a + \delta a\delta x$$



$$\frac{\delta y_{j}}{y_{j}} = \sum_{i} \frac{x_{i} \partial y_{j}}{y_{j} \partial x_{i}} \bigg|_{x = \langle x \rangle} \frac{\delta x_{i}}{x_{i}} + \dots$$

$$V(\vec{y}) = S^{t} V(\vec{y}) S$$

$$V_{ij} = \frac{\langle \delta x_i \delta x_j \rangle}{\langle x_i \rangle \langle x_j \rangle} = \text{relative covariance matrix}$$

$$S_{ij} = \frac{x_j}{y_i} \frac{\partial y_i}{\partial x_j} = \frac{\partial \ln y_i}{\partial \ln x_j} = \text{relative sensitivity matrix}$$

**Derivatives are evaluated at 1**<sup>st</sup> **order**, i.e. assumed constant for the variations around  $\langle x \rangle$  within the interval described by the corresponding uncertainties ( $\delta x$ ) valid for linear relations between quantities within few  $\sigma$  (or small uncertainties).

### **Methods for Cross-section Sensitivity Analysis**

- Analytical method
- <u>Several independent calculations (brute force)</u> time consuming but possible with todays computers (total M/C- TENDL, XSUSA)
- <u>Perturbation methods:</u> sensitivity of a small number of responses to a very large number of uncertain inputs;
  - forward and adjoint (deterministic, Monte Carlo) flux calculations; 1<sup>st</sup> order perturbations;
  - Monte Carlo differential operator perturbation method, correlated sampling;
- Examples of available computer codes:
  - ERANOS, TSUNAMI, SUSD3D, SERPENT, MCSEN, MCNP-6, XSUSA, TENDL based code systems

Eugène P. WIGNER « Effect of Small Perturbations on Pile Period » classified document of Manhattan Project (CP 3048, 13 june 1945).

L. N. USACHEV: Equations for the Importance of neutrons, reactor kinetics and the theory of perturbations, Int. Conf. Peaceful Uses of Atomic Energy 5, 503-510, UN, N.Y. (1955).

**L.N. USACHEV,** « Perturbations theory for the breeding ratio and for other number ratios pertaining to various reactor processes », J. Nucl. Energy, 18 p.571 (1964)

**A.GANDINI,** « A generalized perturbation method for bilinear fuctionals of the real and adjoint fluxes » J. Nucl. Energy, 21, 735-745, 1967.

#### M. L. Williams, Perturbation Theory for Nuclear Reactor Analysis, CRC Press, 1986

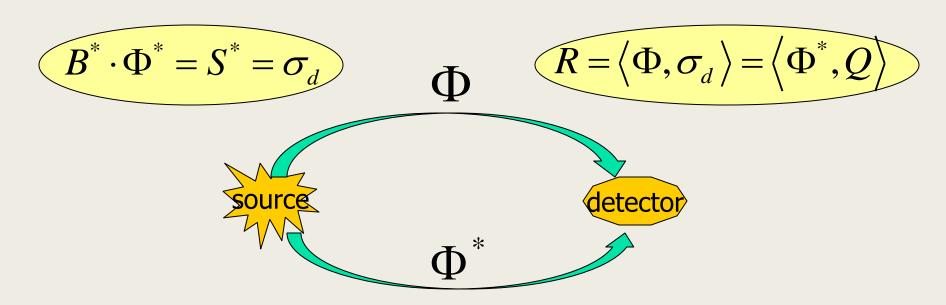
- Multiplication factor
- BOC peak to average power
- Control rod worth
- Breeding ratio
- Moderator void coefficient
- Absorbed radiation dose
- Material damage
- Detector reading, ex core
- Peak transient power
- EOC fuel inventory
- EOC reactivity and EOC neutron flux

#### Direct and adjoint transport equation – importance fonction

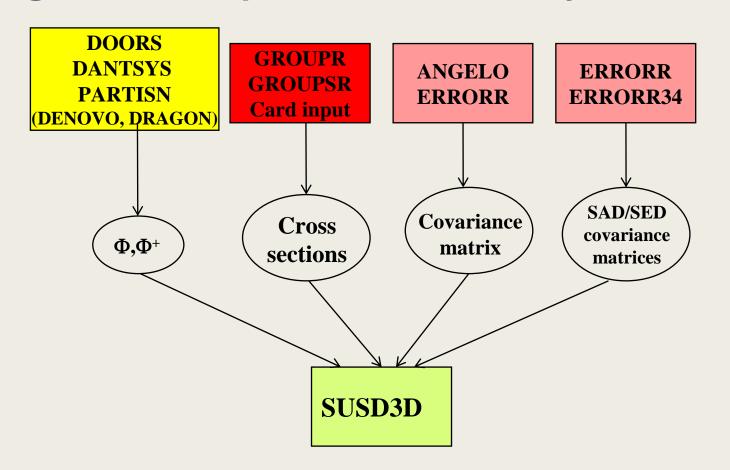
#### Calculation of the reaction rates:

$$R = \left\langle \sigma_d \cdot \Phi \right\rangle = \int_{V}^{E_{\text{max}}} \int_{0}^{E_{\text{max}}} \Phi\left(\vec{r}, \vec{\Omega}, E\right) \sigma_d\left(\vec{r}, \vec{\Omega}, E\right) d\vec{r} dE d\vec{\Omega}$$

Corresponding adjoint operator can be defined as:



# SUSD3D: Nuclear data sensitivity & uncertainty based on generalized perturbation theory

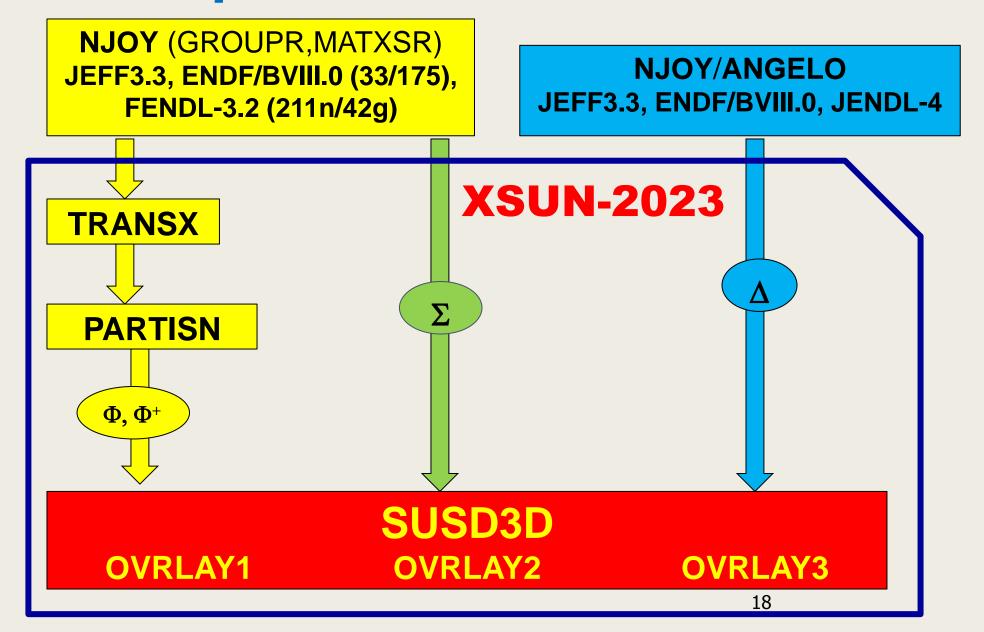


#### SUSD3D

- Complex geometries 1D, 2D, 3D
- Sensitivity-uncertainty to distributions in (E,⊕) of emitted neutrons
- Shielding, criticality (fusion, fission)
- PC DOS, LINUX
- written Fortran 95

- First use for the PV surveillance (REP 900), 3D version developed in the scope of the EFF project in the early 1990-ies., Used for the FNG benchmark pre- and post-analyses.
- Available from NEA Data Bank and RSICC since 2000.
- XSUN-2013 & 2017 released at NEADB, XSUN-2023 in preparation

# **Computational scheme**



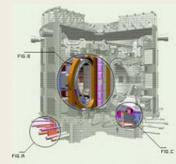
#### SUSD3D, A FEW MAIN / NOVEL FEATURES

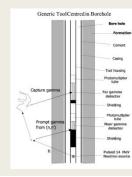
- Sensitivity to different cross-section types: MF=3 (gain term), MF=6 (loss term), MF=2 (direct term-response function), MF=4 (SAD uncertainty), MF=5 (SED uncertainty).
- Covariance terms MF=33, 34, 35 can be used to propagate these uncertainties using the « sandwich » equation;
- Nuclear data for up to up to 440 nuclides from JEFF-3.3 and ENDF/B-VIII.0 in 175- and 33-groups, FENDL-3.2 (211n/42g)
- Sensitivity calculations make use of **angular moments** rather than angular fluxes (considerable computer space savings **3D**).
- SAD/SED distributions: Re-normalisation of **prompt/delayed fission spectra** covariance matrices can be applied using the **constrained sensitivity method**. This option is useful in case if the fission spectra covariances used do not comply with the ENDF-6 Format Manual rules.
- $\blacksquare$  Sensitivity of the effective delayed neutron fraction ( $\beta_{eff}$ )
- Sensitivity to gamma-ray quantities (gamma flux, heating, ...)
- Available from NEA-DB and RSICC since 2000: Present packages available in SUSD3D (NEA 1628/01-/03) and XSUN (NEA-1882)

# **Examples of SUSD3D Analysis**

- Reactor pressure vessel surveillance dosimetry: uncertainty in predicted dosimeter reaction rates and PV exposition, determination of safety margins → reactor lifetime predictions,
- Critical & shielding benchmarks (fission):
  - thermal (KRITZ-2, VENUS-2) and fast (SNEAK-7A/7B, ....);
  - PV surveillance: ASPIS-Fe, -FE88 (2D), VENUS-3 (3D)
- Shielding experiments for fusion reactors for ITER:
  - Preparation and analysis of 9 benchmarks at FNG, Frascati: validation of cross sections and of design of some components; XS of Fe, SiC, W, Be, Pb, Cu and tritium production in HCPB, HCLL & WCLL modules;
- Validation of M/C sensitivity method (MCSEN, SERPENT) against Discrete Ordinates
- Beta-eff SU analysis (several fast benchmarks, MYRRHA ADS reactor): reactor safety, XS data adjustment.
- Activation analysis: FISPACT II
- Gamma-ray S/U analysys: gamma spectra benchmark studies, oil well logging C/O logging (S/U of C/O gamma ratio)

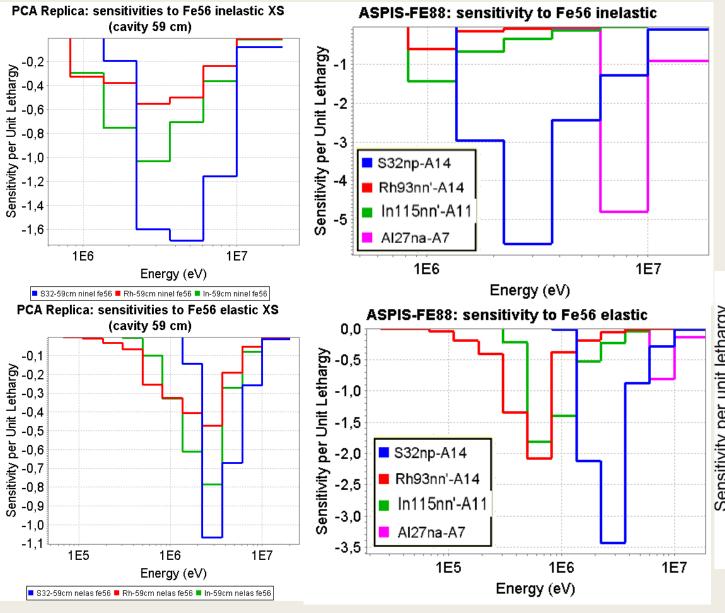




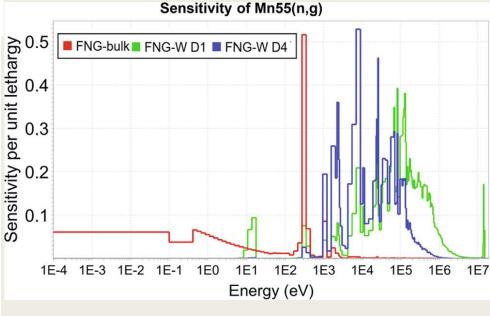


$$i=1 \\
\Delta t = \frac{ln(1 + \frac{\Delta R}{R})}{\lambda}$$

## **Sensitivity profiles**



- S/U provide uncertainty in the result
- Ranking key parameters (most sensitive, contributing the most to the uncertainty)
- Gives confidence in the simulation results
- sensitivity profiles available for: ASPIS Fe88, PCA Replica, FNG (Cu, W, HCPB, HCLL, ...), VENUS-3, LLNL.

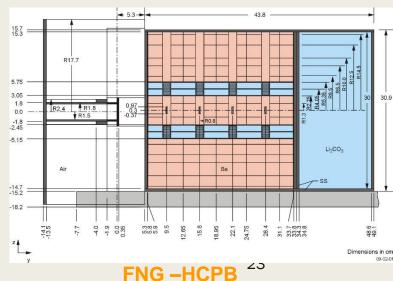


#### **Conclusions**

- Reactor design and safety parameters are biased due to uncertainties in nuclear data. Improved safety and better cost efficiency can be achieved by the reduction of uncertainties in design parameters.
- Monte Carlo and deterministic codes and nuclear data used for n/g transport need to be validated
- Uncertainties of calculated results can be estimated by sensitivity & uncertainty analysis and by comparison with experiments.
- Sensitivity and uncertainty analysis can also identify areas of weakness in data files and guide further evaluations.
- Powerful computational tools and databases needed for such analysis are available.

# Benchmarking activities at UKAEA

- Focus on nuclear fusion: analyses typically involve large and complex geometries, neutron & gamma source terms, higher energy neutrons than fission.
- Involved in fusion reactor projects such as JET, ITER, STEP, MAST-U, DEMO;
- Licencing needs: V&V, understand and eventually reduce uncertainties
- Fusion research needs (W, Cu, Fe, V, Mo, Cr, Y, Ti, C, Zr, Li, Pb, Be, Si,...)
- Predominantly MCNP M/C calculations using acceleration methods, but use of alternative codes is actively studied: Serpent2, OpenMC, GEANT4, TRIPOLI,
- SINBAD database serves for validation of transport codes and nuclear data,
- CAD based computational workflows
- Project on automation of benchmark analysis.
- Nuclear data sensitivity and uncertainty analysis
- Activation & nuclear waste studies (Fispact II)
- Open to international collaboration: with NEA (SINBAD Task Force), IAEA (Conderc, FENDL), F4E (JADE)



# THANK YOU FOR YOUR ATTENTION