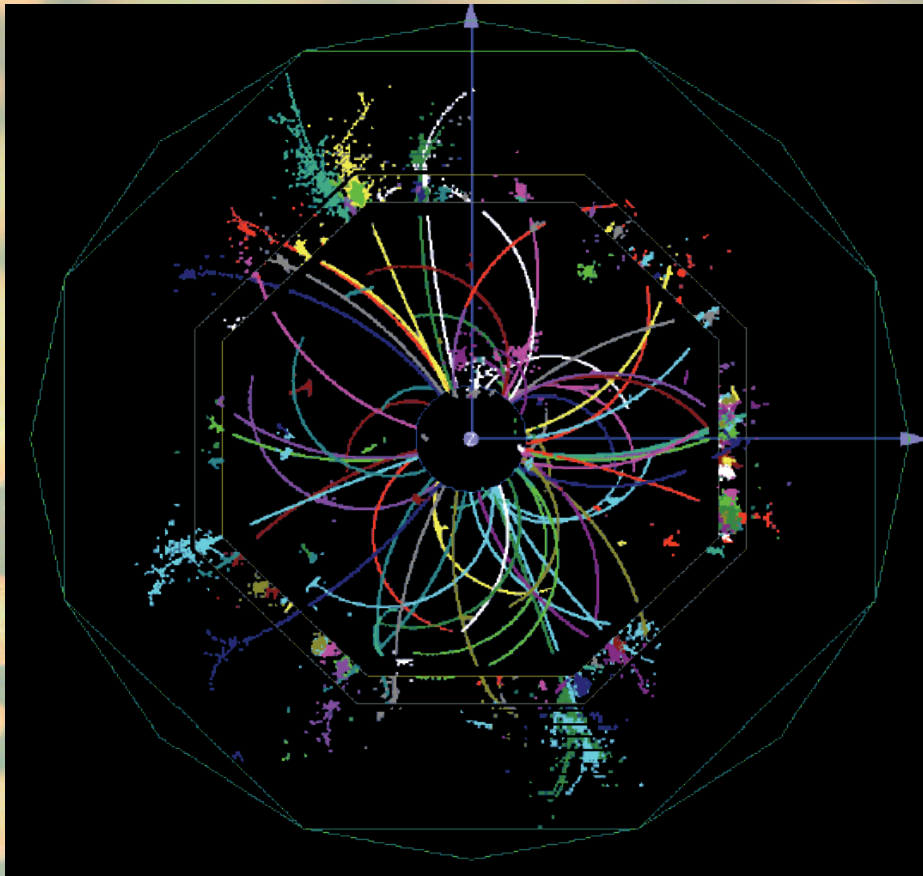


# Omega



Electronics  
in particle  
physics

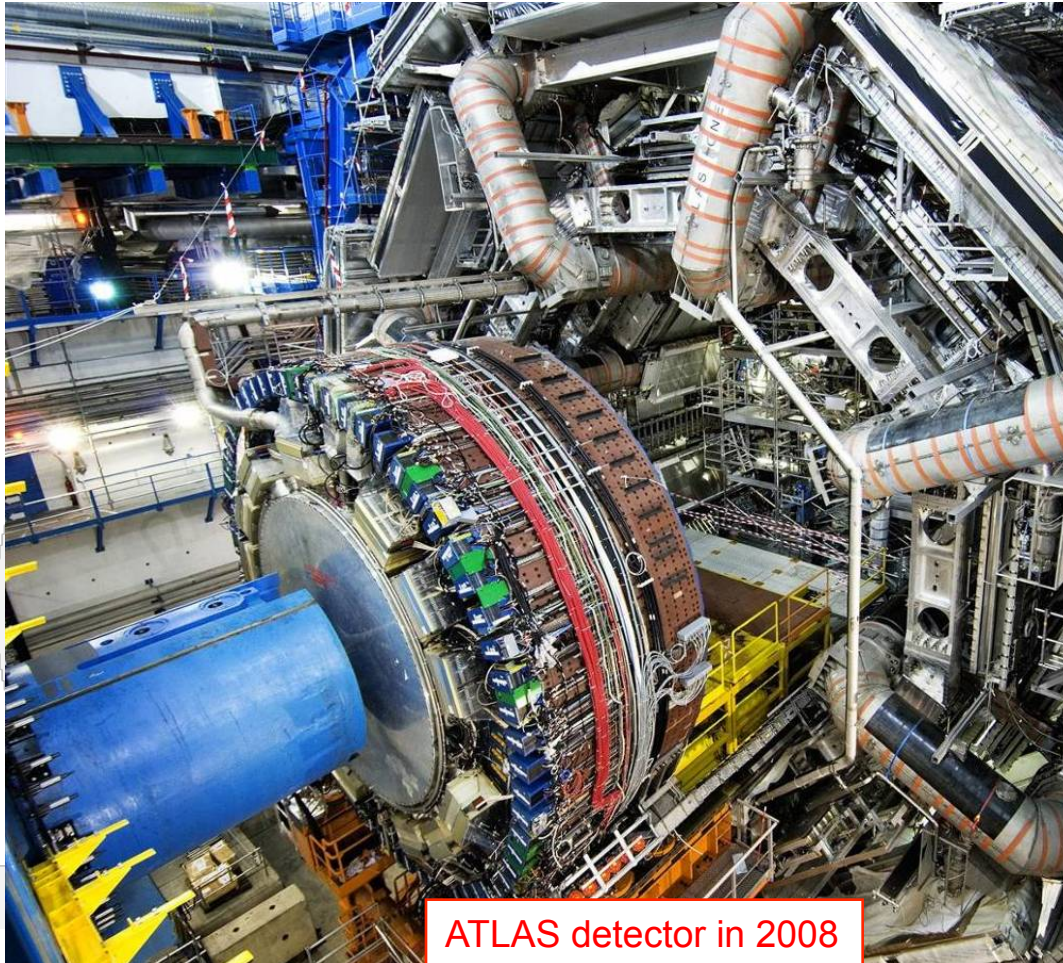
EDIT school  
CERN 2011

C. de LA TAILLE  
IN2P3  
[Taille@lal.in2p3.fr](mailto:Taille@lal.in2p3.fr)

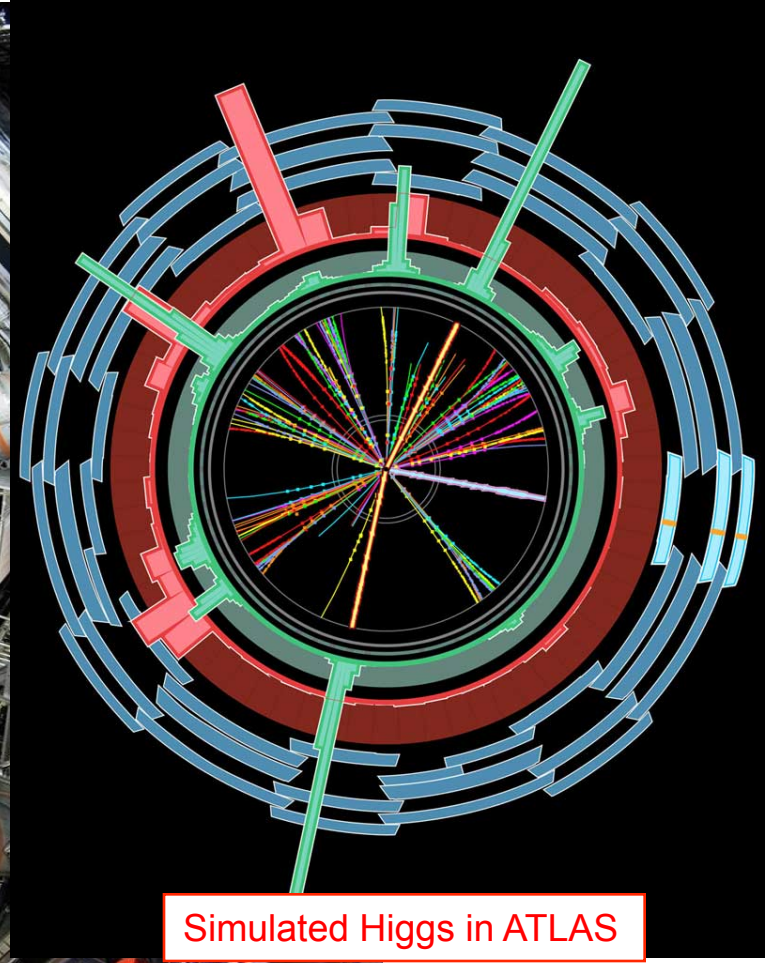


## Electronics in experiments

- A lot of electronics in the experiments...
  - The performance of electronics often impacts on the detectors
  - Analog electronics (V,A,A...) / Digital electronics (bits)

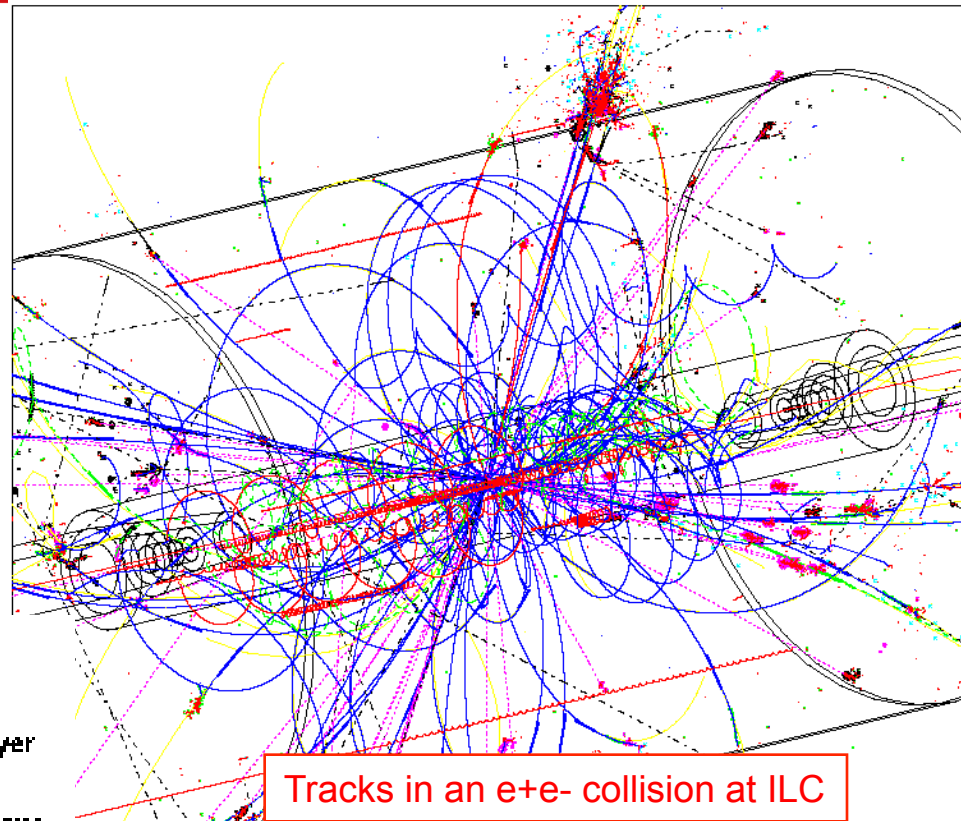


ATLAS detector in 2008

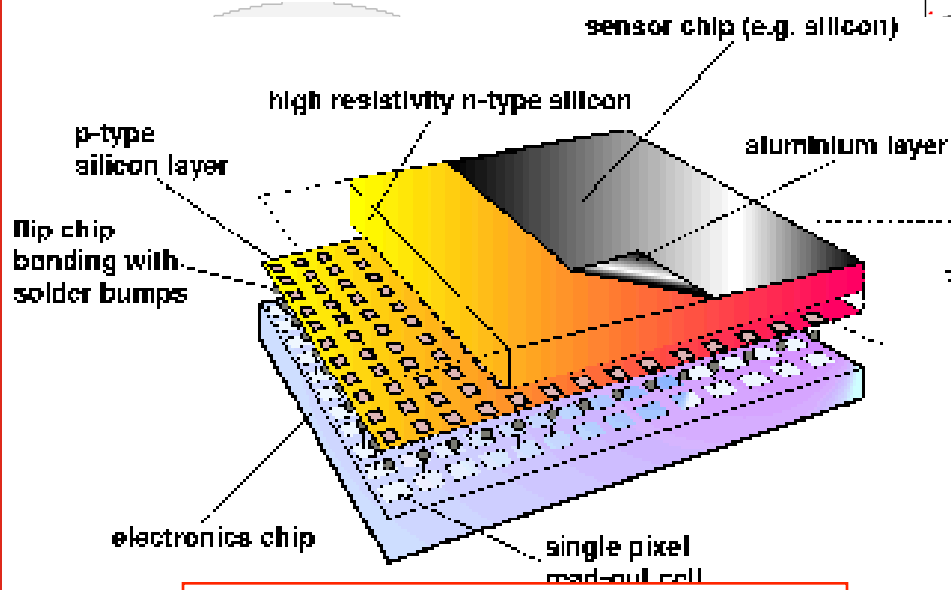


Simulated Higgs in ATLAS

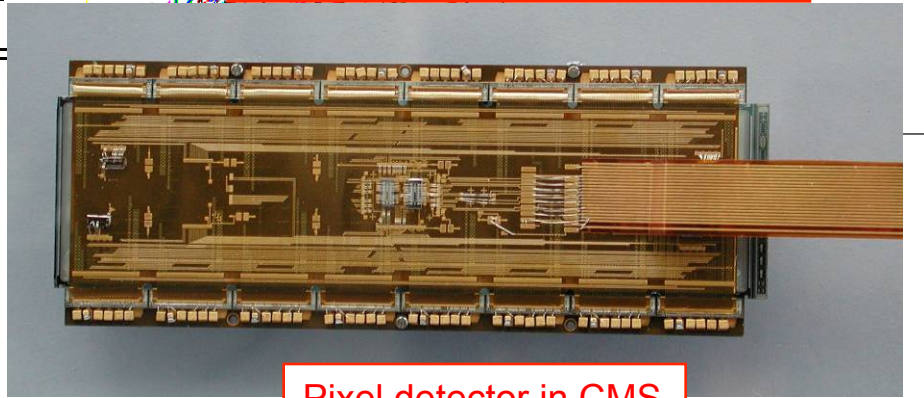
- Measurement of (charged) particle tracks
  - millions of pixels ( $\sim 100 \mu\text{m}$ )
  - binary readout at 40 MHz
  - High radiation levels
  - Made possible by ASICs



Tracks in an e+e- collision at ILC



Pixel detector and readout electronics

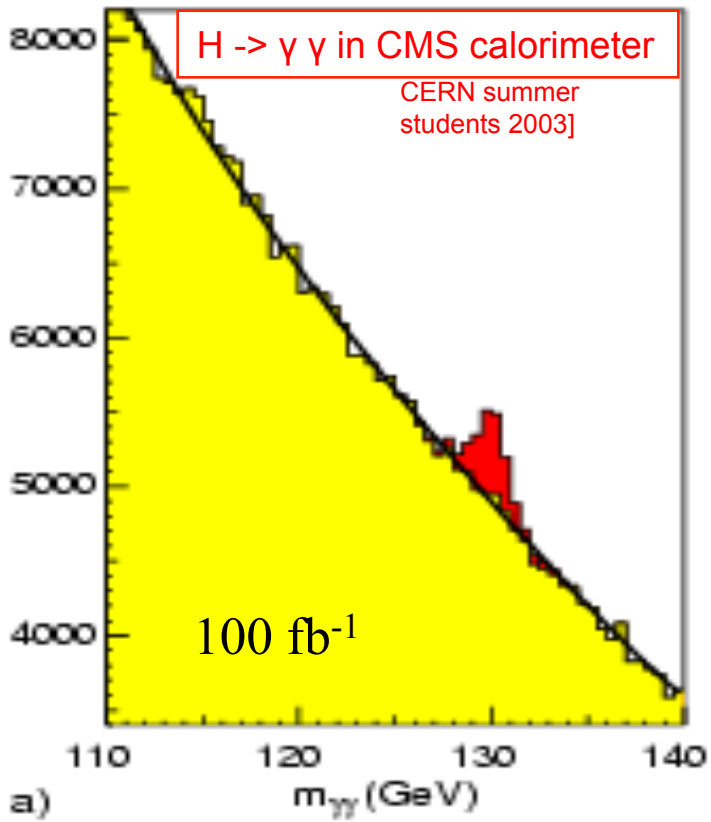
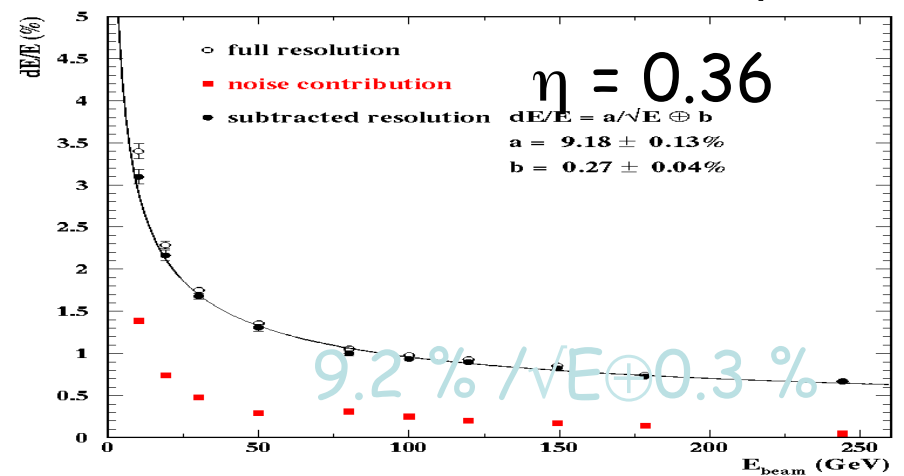


Pixel detector in CMS

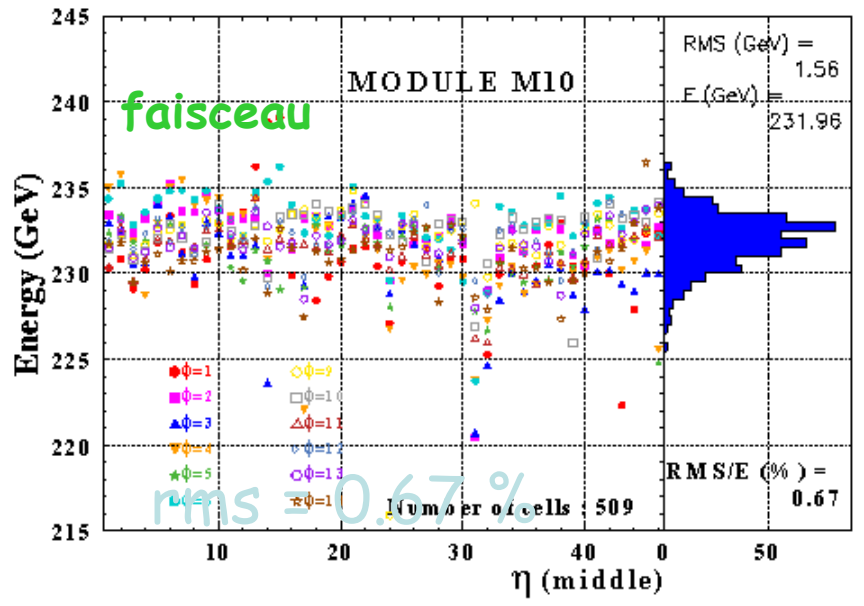
# Importance of electronics : calorimeters



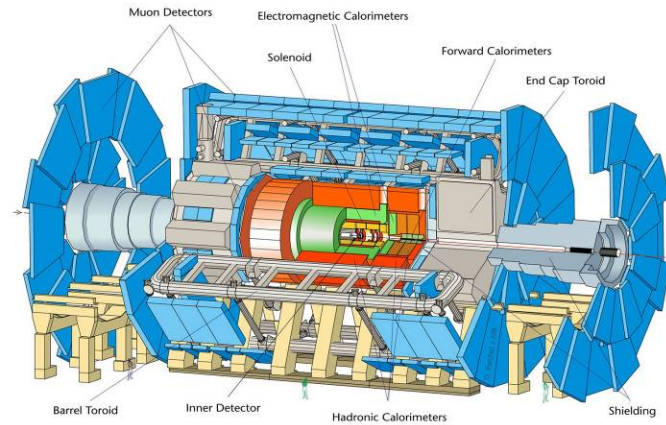
- Large dynamic range ( $10^4$ - $10^5$ )
- High Precision  $\sim 1\%$ 
  - Importance of low noise, uniformity, linearity...
  - Importance of calibration



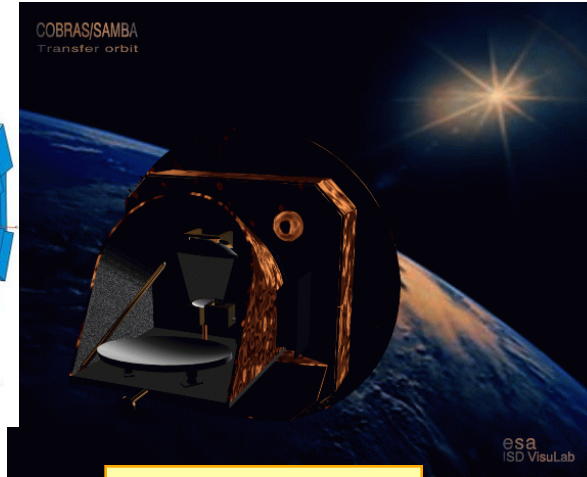
Energy resolution and uniformity in ATLAS



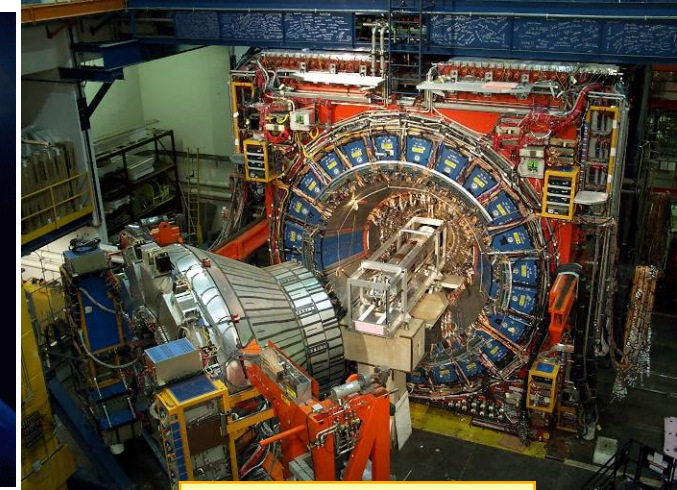
# A large variety of detectors...



ATLAS : Higgs boson



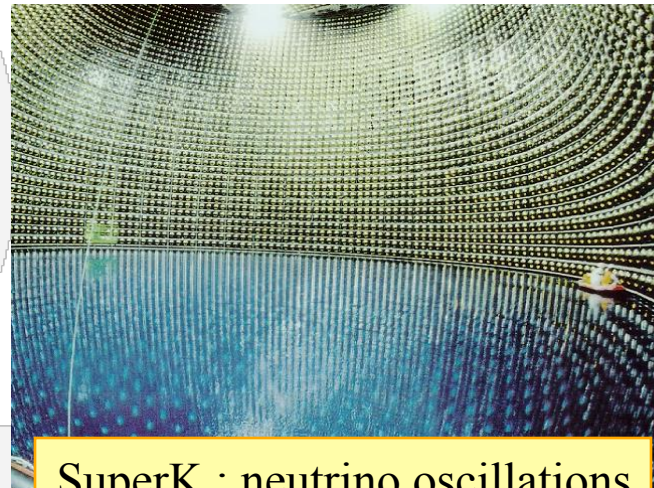
Planck : CMB



CDF : top quark



Edelweiss : dark matter



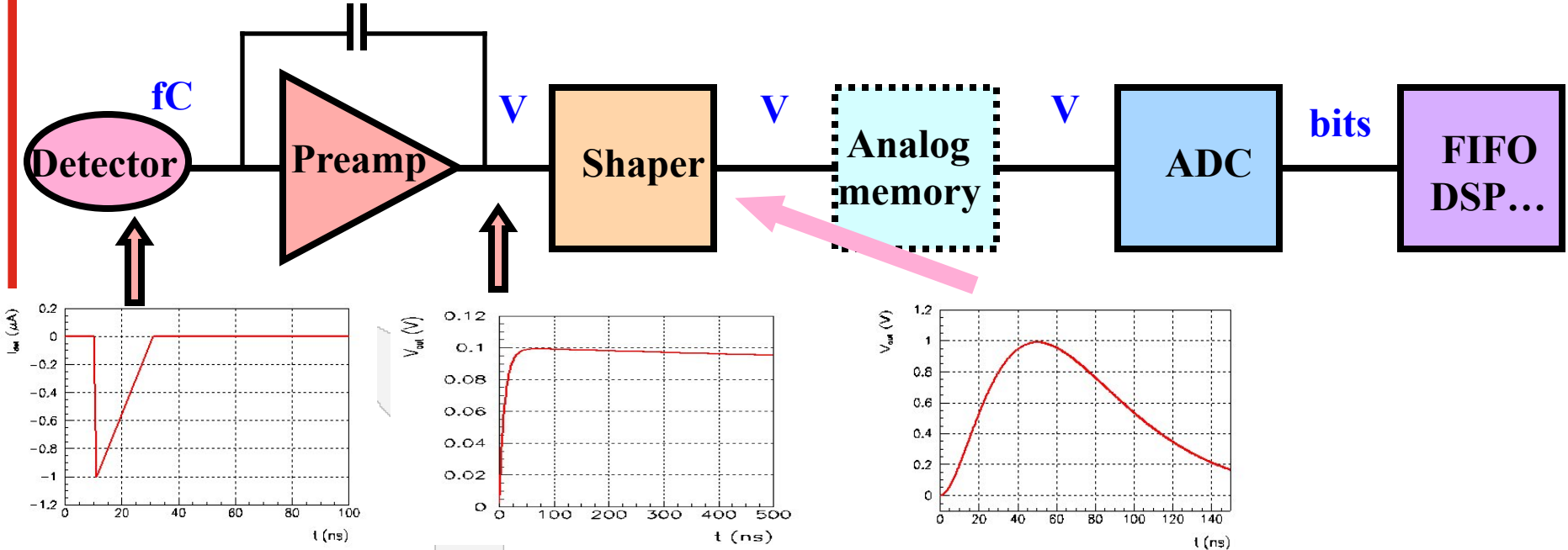
SuperK : neutrino oscillations



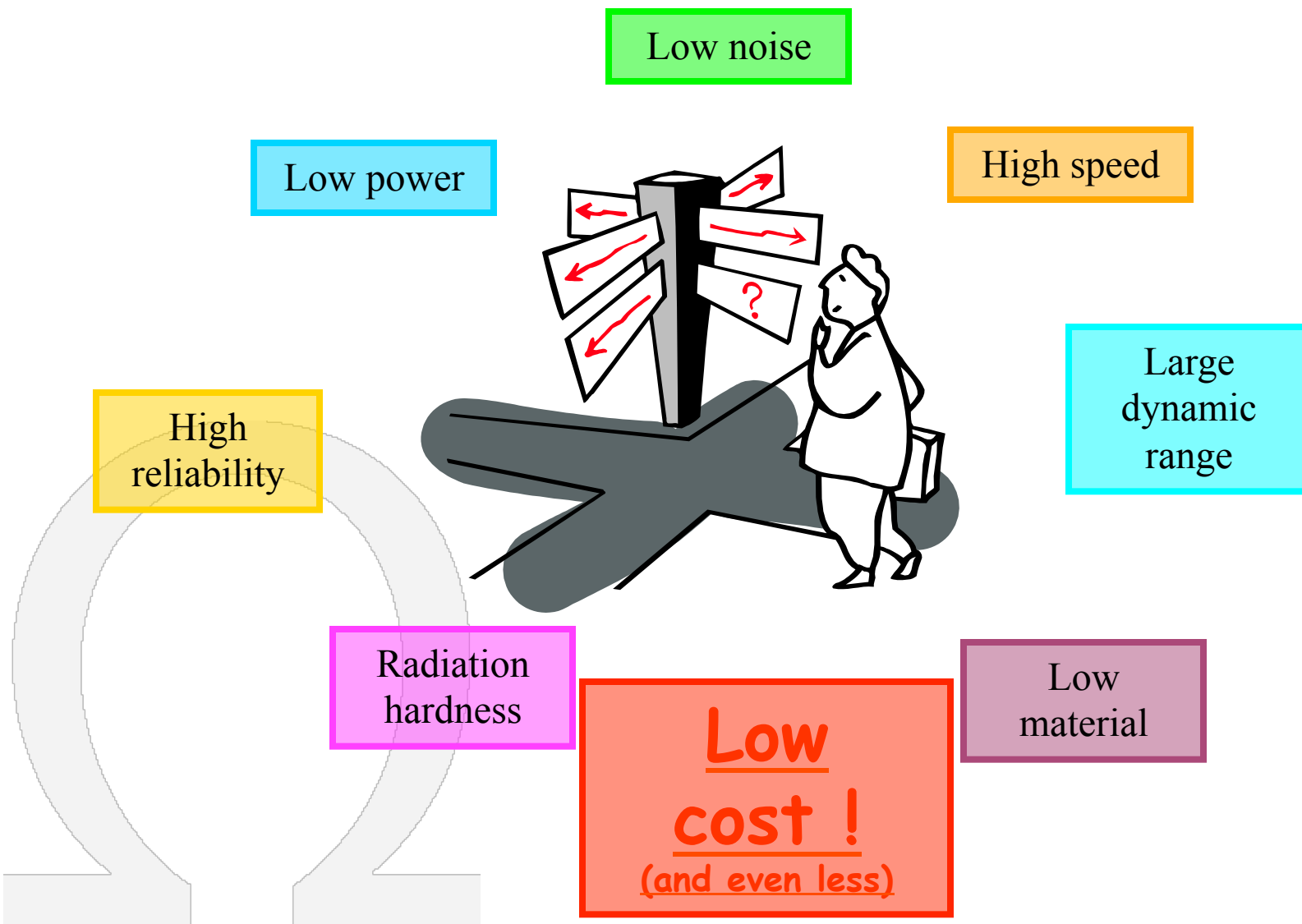
AUGER : cosmic rays  $10^{20}$ eV

# Overview of readout electronics

- Most front-ends follow a similar architecture

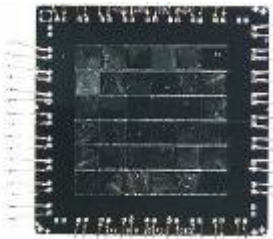


- Very small signals (fC) -> need **amplification**
- Measurement of **amplitude** and/or **time** (**ADCs**, **discris**, **TDCs**)
- Several thousands to millions of channels

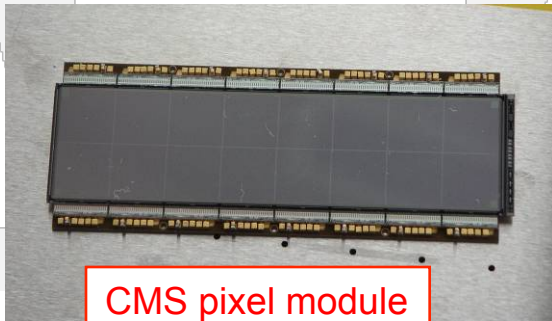


## Detector(s)

- A large variety
- A similar modelization



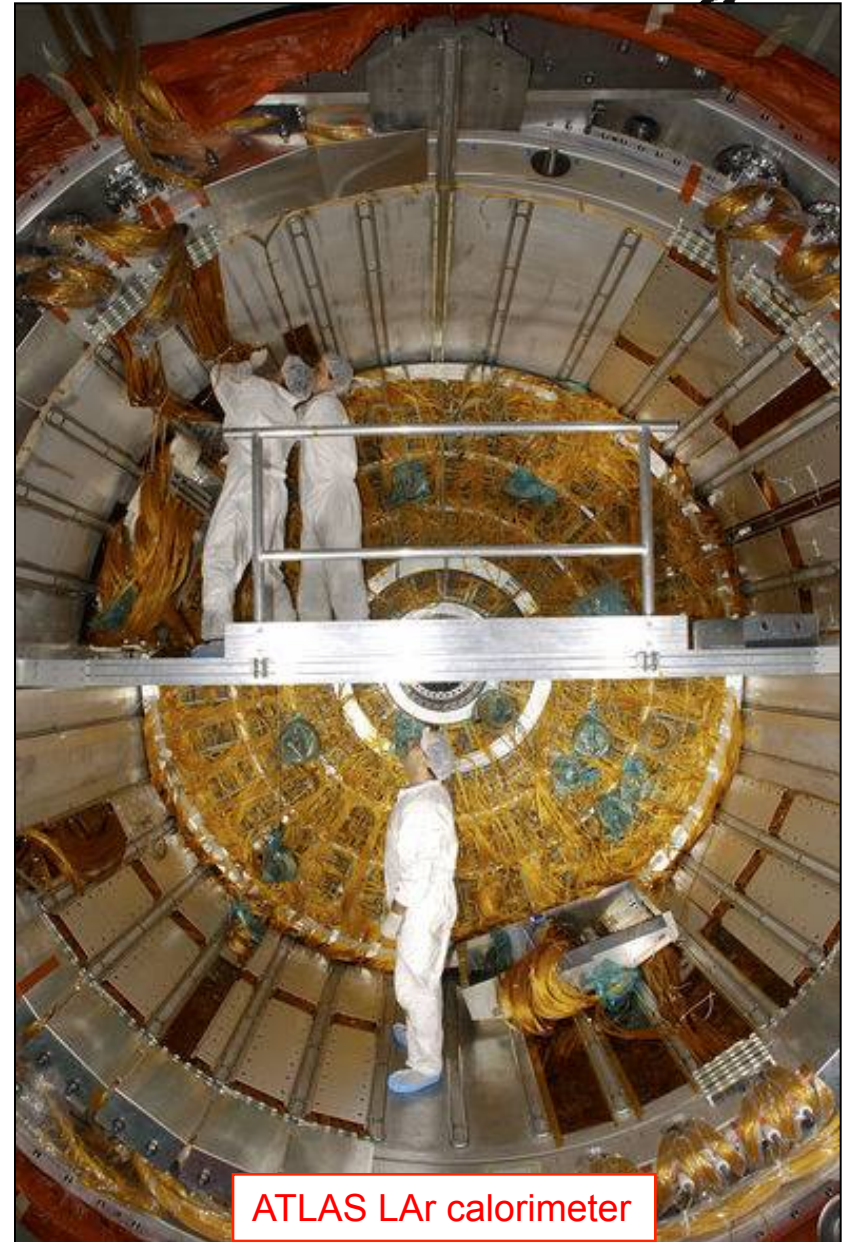
6x6 pixels, 4x4 mm<sup>2</sup>  
HgTe absorbers, 65 mK  
12 eV @ 6 keV



CMS pixel module



PMT in ANTARES

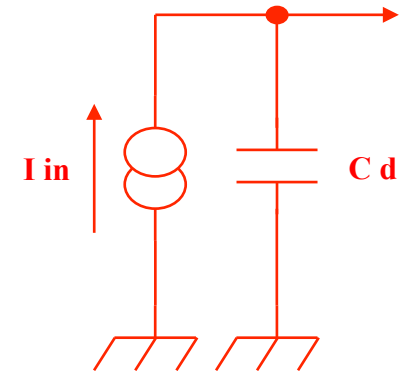


ATLAS LAr calorimeter

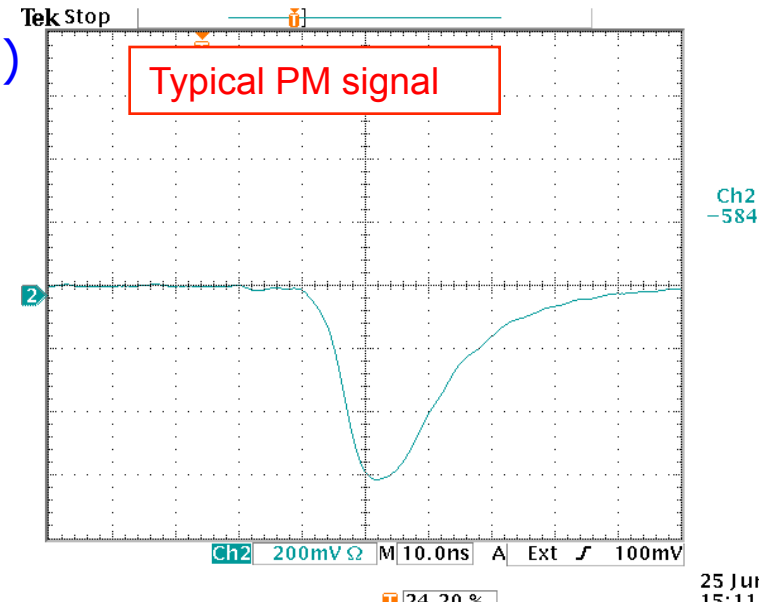


## Detector modelization

- Detector = capacitance  $C_d$ 
  - Pixels : 0.1-10 pF
  - PMs : 3-30pF
  - Ionization chambers 10-1000 pF
  - Sometimes effect of transmission line
- Signal : current source
  - Pixels :  $\sim 100e^-/\mu m$
  - PMs : 1 photoelectron  $\rightarrow 10^5-10^7 e^-$
  - Modelized as an impulse (Dirac) :  $i(t)$
- Missing :
  - High Voltage bias
  - Connections, grounding
  - Neighbours
  - Calibration...

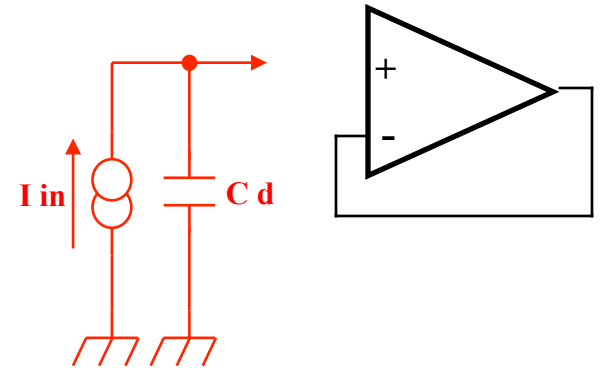


Detector modelization



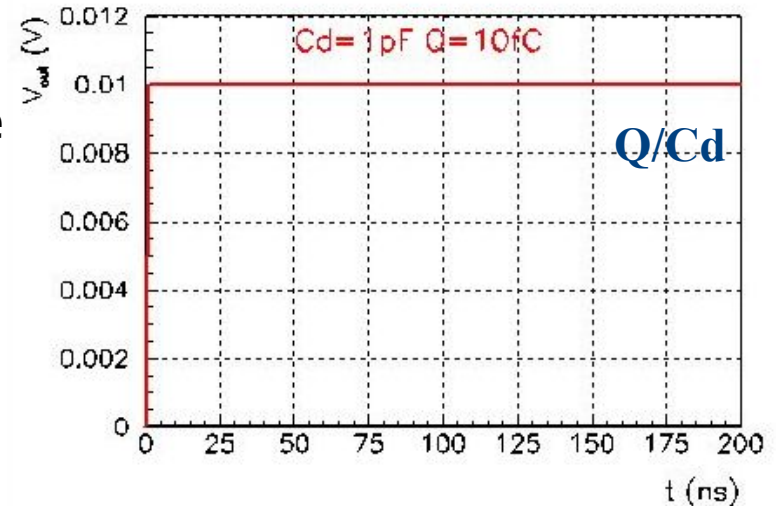
# Reading the signal

- Signal
  - Signal = current source
  - Detector = capacitance  $C_d$
  - Quantity to measure
    - Charge => integrator needed
    - Time => discriminator + TDC



Voltage readout

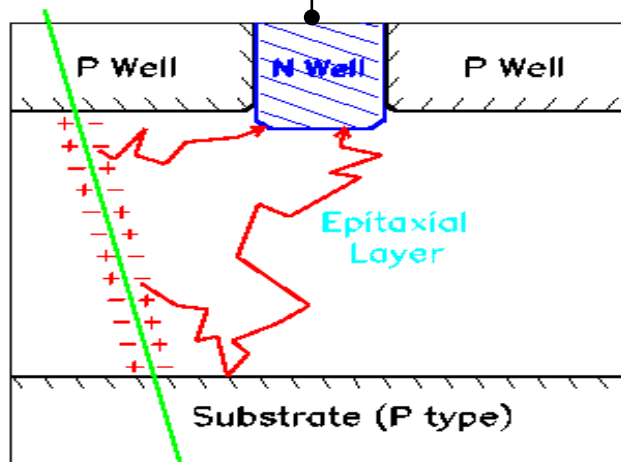
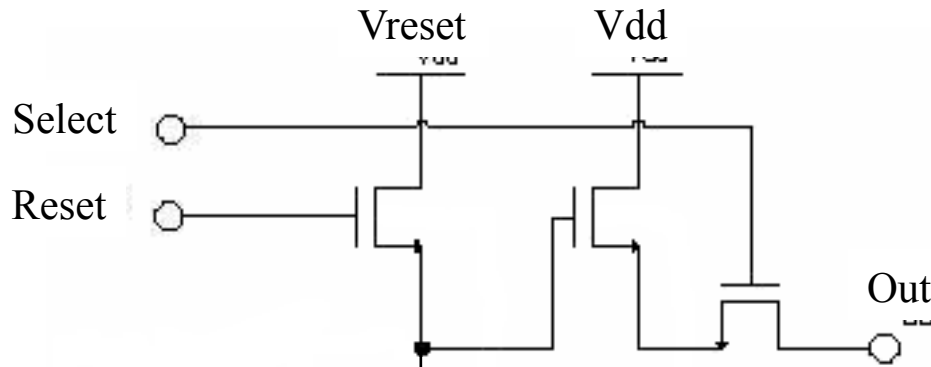
- Integrating on  $C_d$ 
  - Simple :  $V = Q/C_d$
  - « Gain » :  $1/C_d$  : 1 pF -> 1 mV/fC
  - Need a follower to buffer the voltage => parasitic capacitance
  - Gain loss, possible non-linearities
  - crosstalk
  - Need to empty  $C_d$ ...



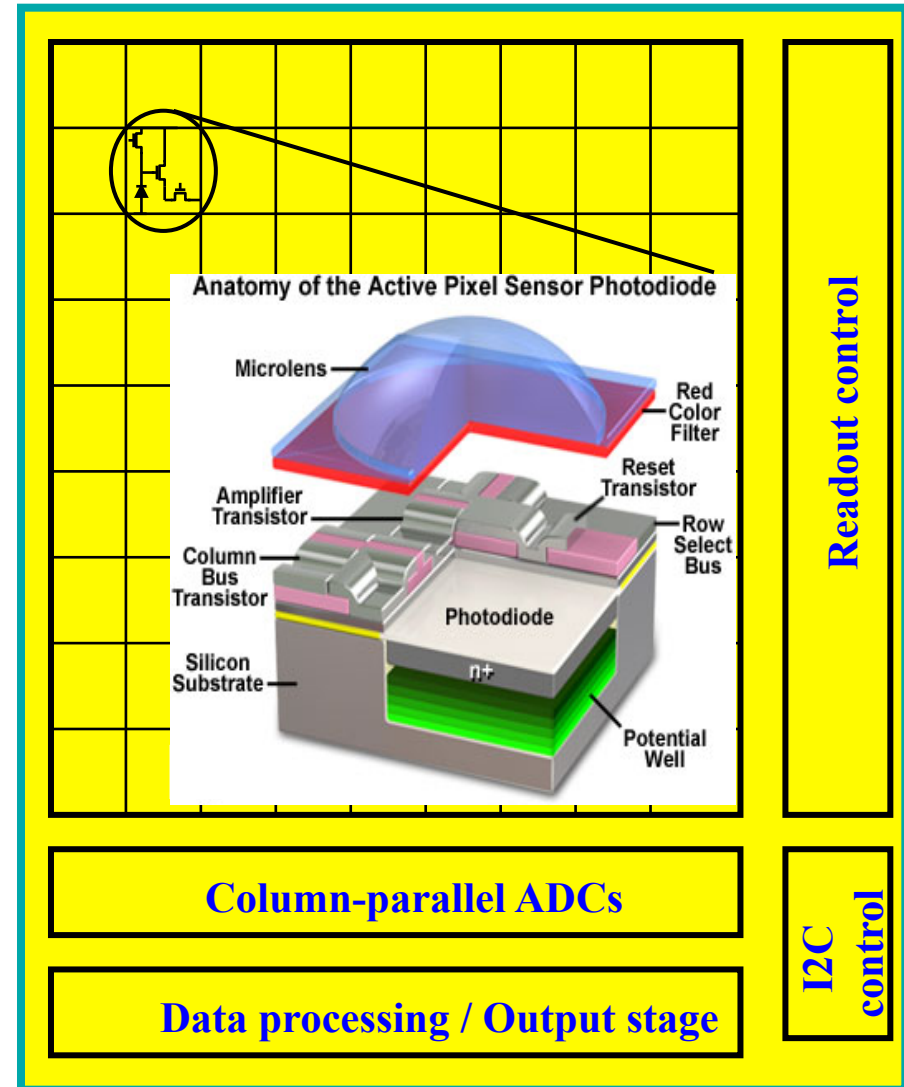
Impulse response

# Monolithic active pixels

- Collect charge by diffusion
- Read  $\sim 100 e^-$  on  $C_d \sim 10 \text{ fF} = \text{few mV}$



MAPS readout



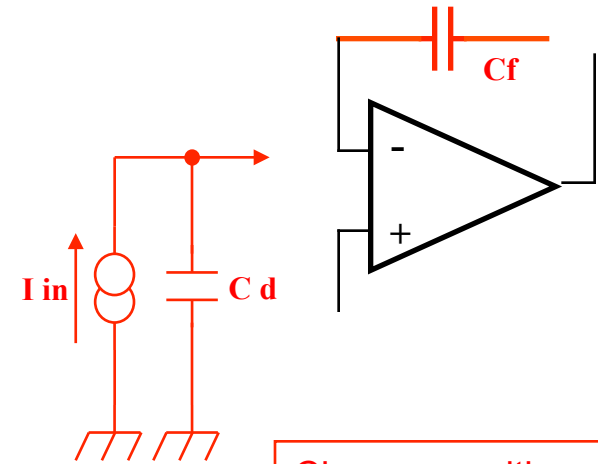
## Ideal charge preamplifier

- ideal opamp in transimpedance
  - Shunt-shunt feedback
  - transimpedance :  $v_{out}/i_{in}$
  - $V_{in}=0 \Rightarrow v_{out}(\omega)/i_{in}(\omega) = -Z_f = -1/j\omega C_f$
  - **Integrator** :  $v_{out}(t) = -1/C_f \int i_{in}(t)dt$

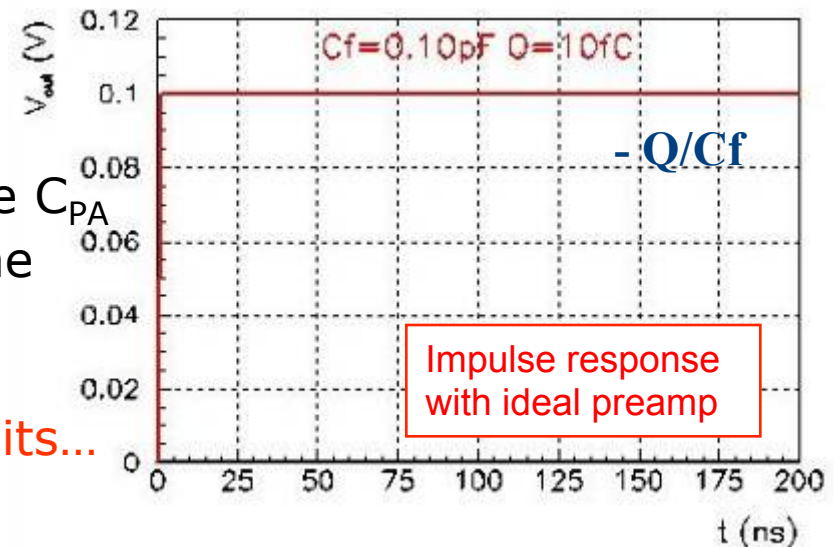
$$v_{out}(t) = -Q/C_f$$

- « Gain » :  $1/C_f$  : 0.1 pF  $\rightarrow$  10 mV/fC
- $C_f$  determined by maximum signal

- Integration on  $C_f$ 
  - Simple :  $V = -Q/C_f$
  - Unsensitive to preamp capacitance  $C_{PA}$
  - Turns a short signal into a long one
  - **The front-end of 90% of particle physics detectors...**
  - **But always built with custom circuits...**

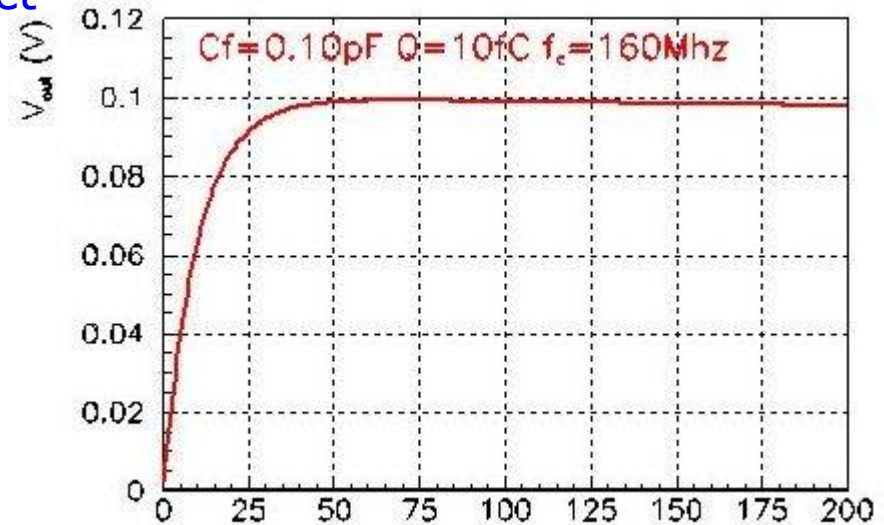
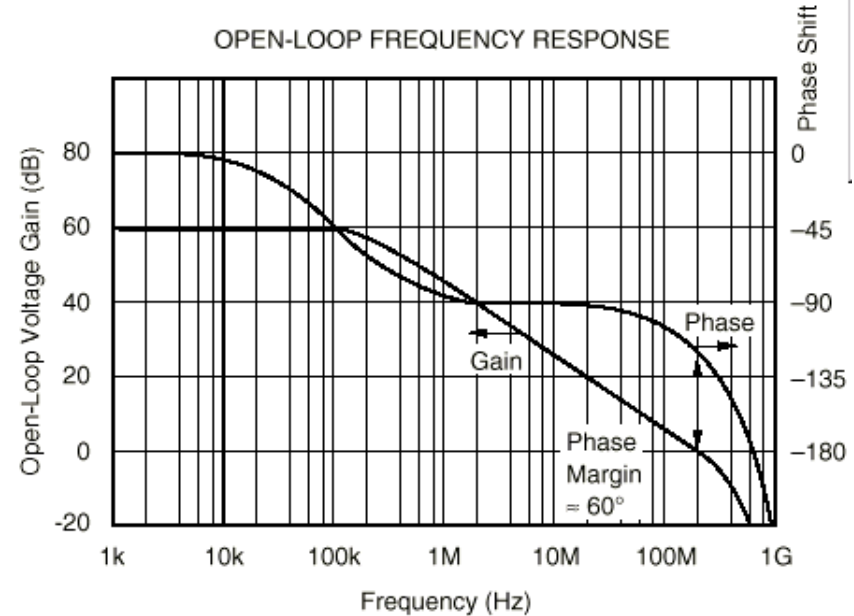


Charge sensitive preamp



# Non-ideal charge preamplifier

- Finite opamp gain
  - $V_{out}(\omega)/i_{in}(\omega) = -Z_f / (1 + C_d / G_0 C_f)$
  - Small signal loss in  $C_d/G_0 C_f \ll 1$  (ballistic deficit)
- Finite opamp bandwidth
  - First order open-loop gain
  - $G(\omega) = G_0/(1 + j \omega/\omega_0)$ 
    - $G_0$  : low frequency gain
    - $G_0\omega_0$  : gain bandwidth product
- Preamp risetime
  - Due to gain variation with  $\omega$
  - Time constant :  $\tau$  (*tau*)
  - $\tau = C_d/G_0\omega_0 C_f$
  - Rise-time :  $t_{10-90\%} = 2.2 \tau$
  - Rise-time optimised with  $w_C$  or  $C_f$



Impulse response with non-ideal preamp

## Charge preamp seen from the input

- Input impedance with ideal opamp
  - $Z_{in} = Z_f / G + 1$
  - $Z_{in} \rightarrow 0$  for ideal opamp
  - « Virtual ground » :  $V_{in} = 0$
  - Minimizes sensitivity to detector impedance
  - Minimizes crosstalk

- Input impedance with real opamp

- $Z_{in} = 1/j\omega G_0 C_f + 1/G_0 \omega_0 C_f$
- Resistive term :  $R_{in} = 1/G_0 \omega_0 C_f$ 
  - Exemple :  $\omega_C = 10^{10}$  rad/s  $C_f = 1$  pF  
 $\Rightarrow R_{in} = 100 \Omega$

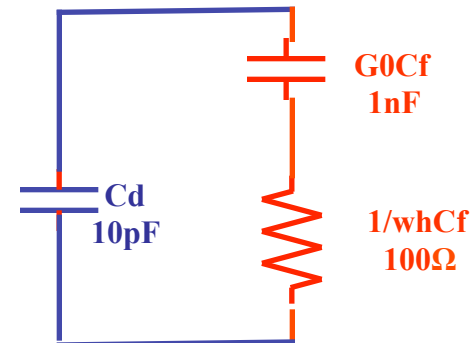
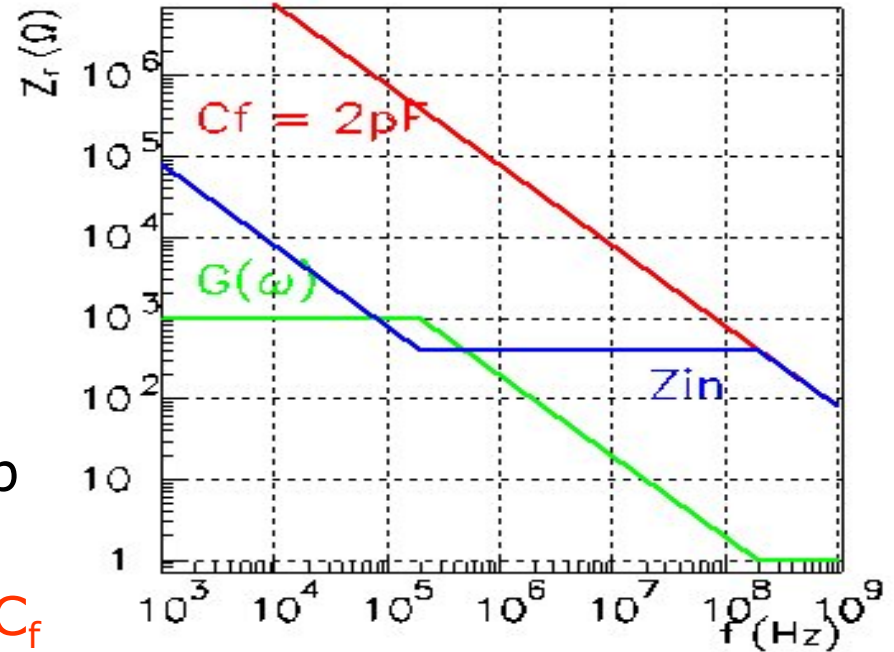
- Determines the input time constant :

$$t = R_{eq} C_d$$

- Good stability = (...!)

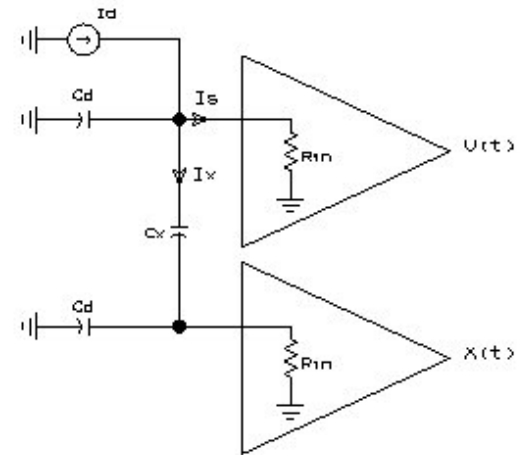
- Equivalent circuit :

Input impedance or charge preamp



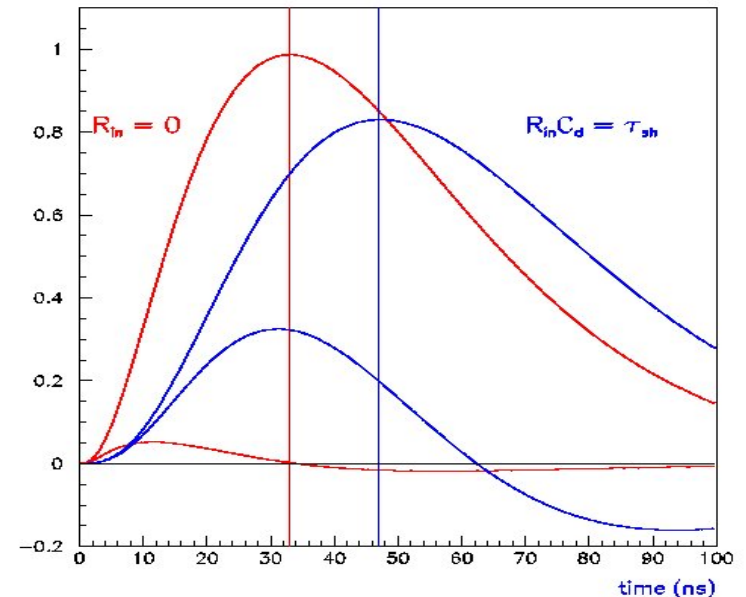
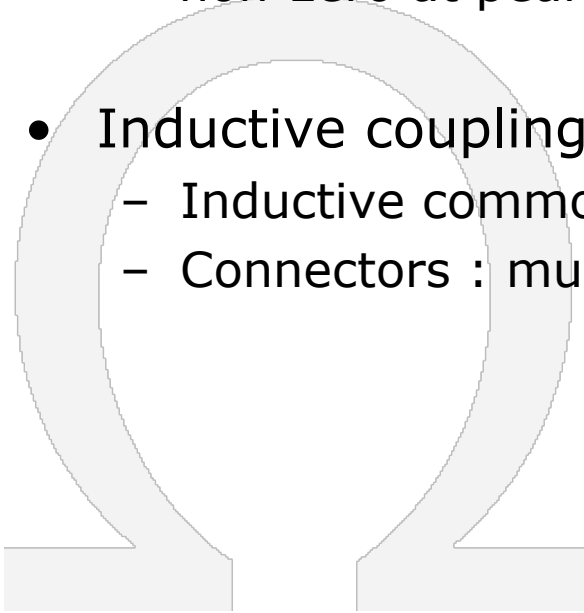
# Crosstalk

- Capacitive coupling between neighbours
  - Crosstalk signal is **differentiated and with same polarity**
  - Small contribution at signal peak
  - Proportionnal to  $C_x/C_d$  and preamp input impedance
  - Slowed derivative if  $R_{in}C_d \sim t_p \Rightarrow$  non-zero at peak



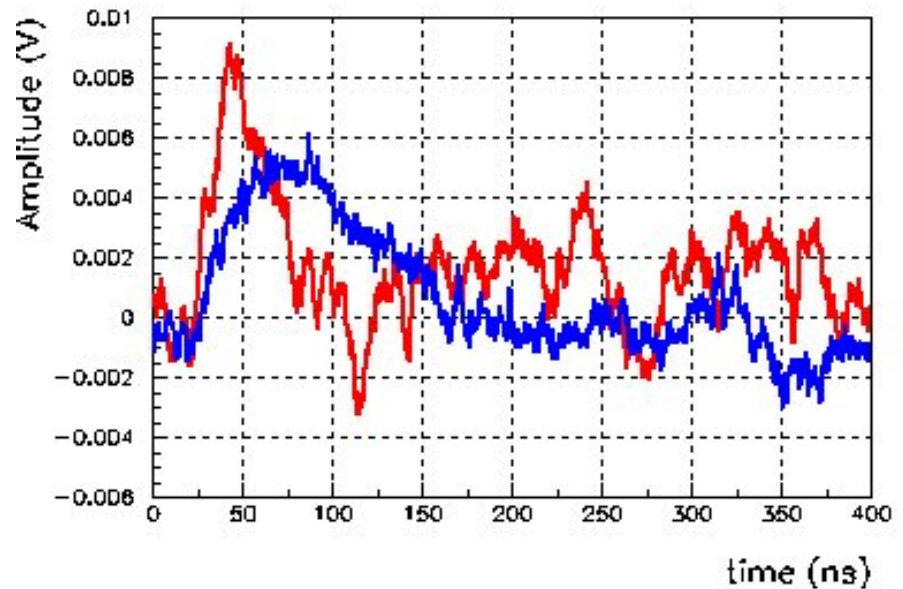
Crosstalk electrical modelization

- Inductive coupling
  - Inductive common ground return
  - Connectors : mutual inductance



# Electronics noise

- Definition of Noise
  - Random fluctuation superposed to interesting signal
  - Statistical treatment
  
- Three types of noise
  - Fundamental noise (Thermal noise, shot noise)
  - Excess noise ( $1/f$  ...)
  - Parasitics -> EMC/EMI (pickup noise, ground loops...)





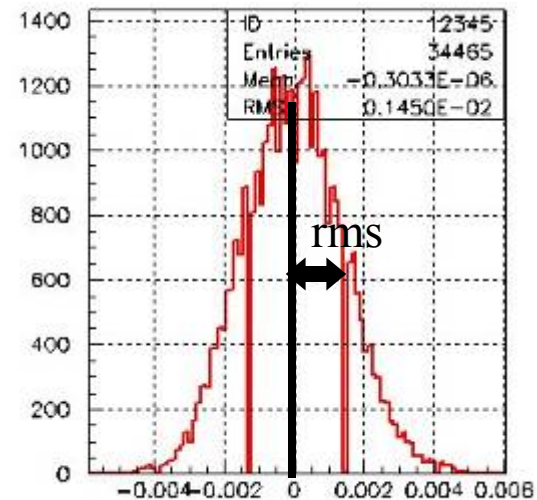
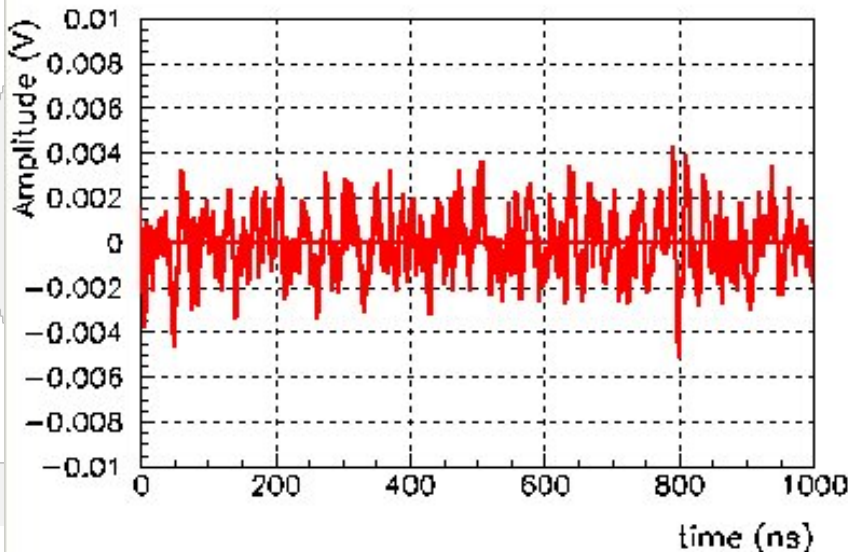
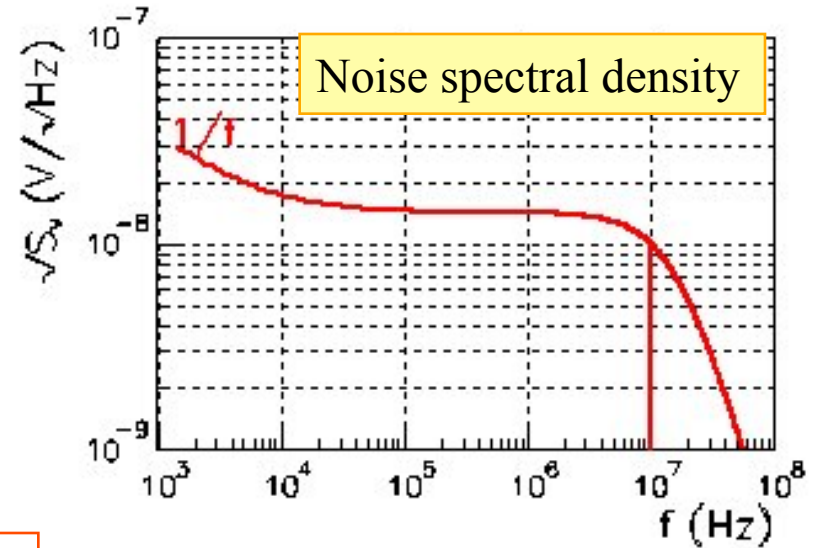
# Electronics noise

- Modelization

- Noise generators :  $e_n, i_n$
- Noise spectral density of  $e_n$  &  $i_n$   
 $S_v(f)$  &  $S_i(f)$
- $S_v(f) = | \mathcal{F}(e_n) |^2$  ( $V^2/Hz$ )

- Rms noise  $V_n$

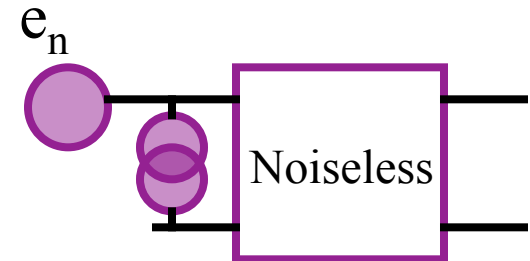
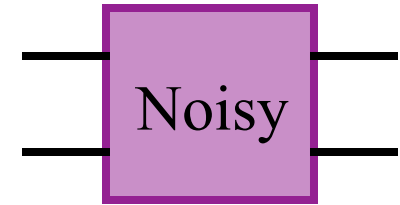
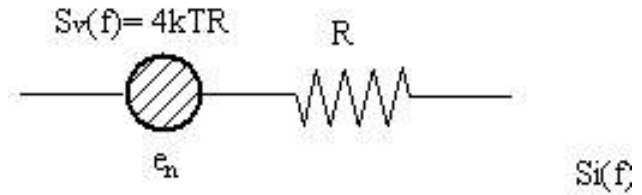
- $V_n^2 = \int e_n^2(t) dt = \int S_v(f) df$
- White noise ( $e_n$ ) :  $v_n = e_n \sqrt{\frac{1}{2} \pi f_{-3dB}}$



Rms noise  $v_n$

# Calculating electronics noise

- Fundamental noise
  - Thermal noise (**resistors**) :  $S_v(f) = 4kTR$
  - Shot noise (**junctions**) :  $S_i(f) = 2qI$
- Noise referred to the input
  - All noise generators can be referred to the input as **2** noise generators :
  - A voltage one  $e_n$  in series : **series noise**
  - A current one  $i_n$  in parallel : **parallel noise**
  - Two generators : no more, no less...



Noise generators referred to the input

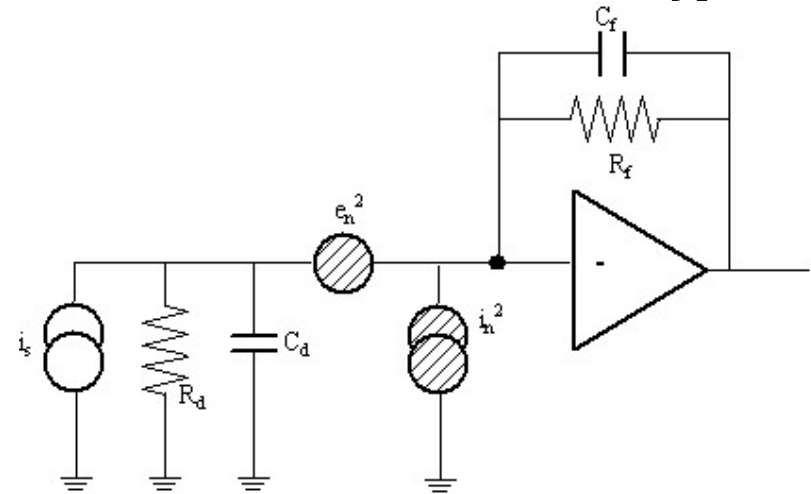
■ **To take into account the Source impedance**

## Golden rule

- **Always calculate the signal before the noise**  
**what counts is the signal to noise ratio**

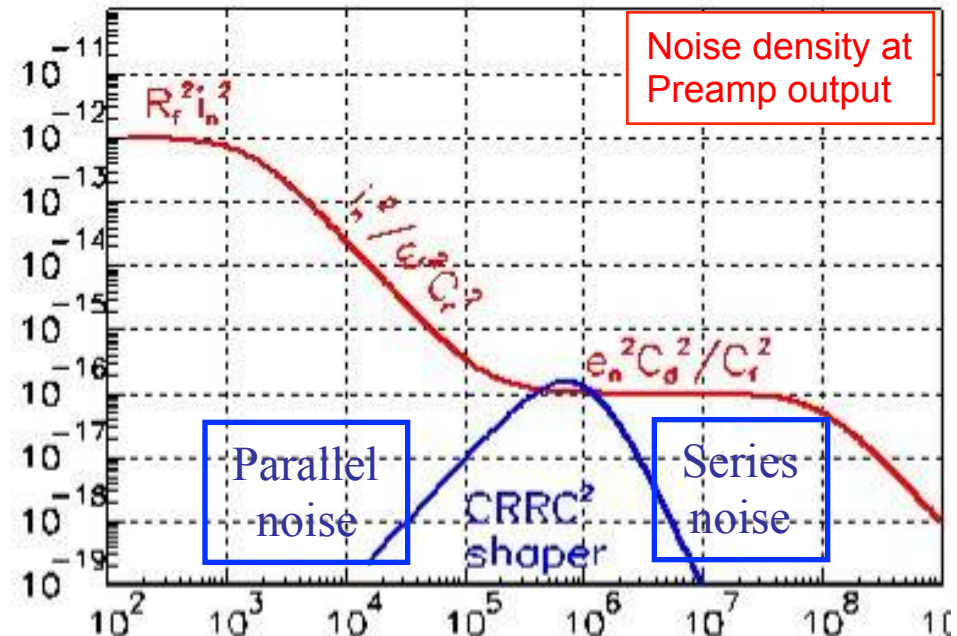
## Noise in charge pre-amplifiers

- 2 noise generators at the input
  - Parallel noise : ( $i_n^2$ ) (leakage currents)
  - Series noise : ( $e_n^2$ ) (preamp)
- Output noise spectral density :
  - $S_v(\omega) = (i_n^2 + e_n^2/|Z_d|^2) / \omega^2 C_f^2$   
 $= i_n^2 / \omega^2 C_f^2 + e_n^2 C_d^2 / C_f^2$
  - Parallel noise in  $1/\omega^2$
  - Series noise is flat, with a « noise gain » of  $C_d/C_f$
- rms noise  $V_n$ 
  - $V_n^2 = \int S_v(\omega) d\omega / 2\pi \rightarrow \infty$  (!)
  - Benefit of shaping...



Noise generators in charge preamp

$S_v (V^2/Hz)$



Noise density at Preamp output

Parallel noise

CRRC<sup>2</sup> shaper

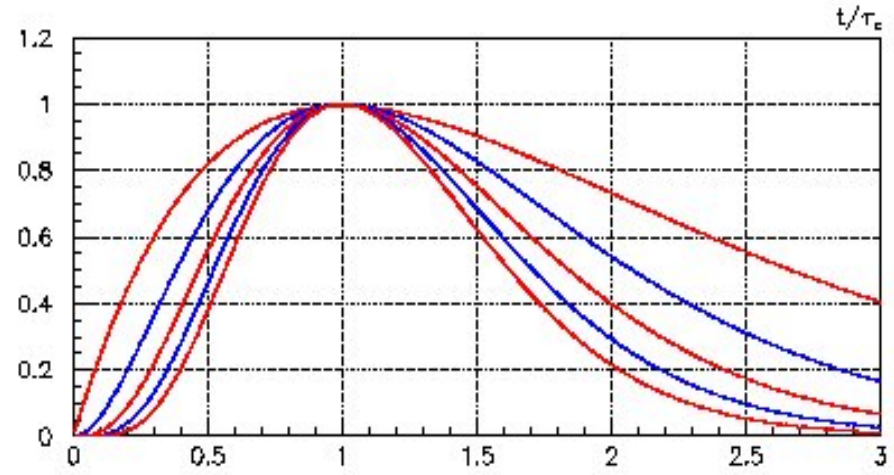
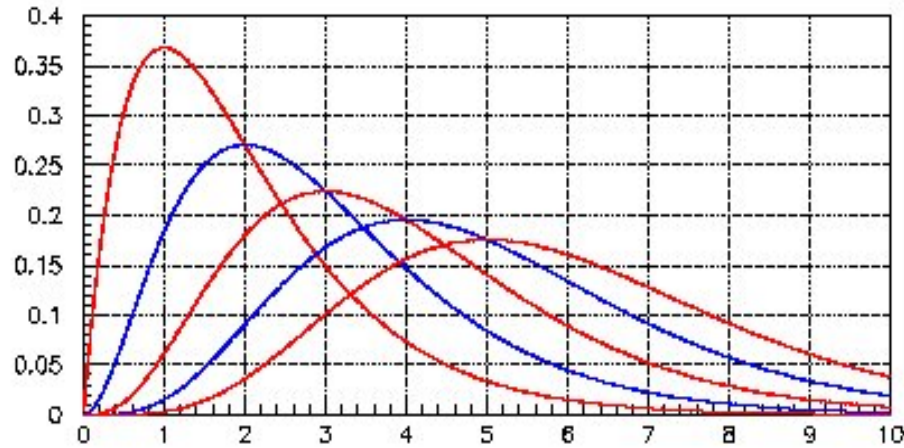
Series noise

# Equivalent Noise Charge (ENC) after CRRC<sup>n</sup> *Omega*

- Noise reduction by optimising useful bandwidth
  - Low-pass filters (**RC<sup>n</sup>**) to cut-off high frequency noise
  - High-pass filter (**CR**) to cut-off parallel noise
  - -> pass-band filter CRRC<sup>n</sup>

## • Equivalent Noise Charge : ENC

- Noise referred to the input in electrons
- $ENC = I_a(n) e_n C_t / \sqrt{\tau} \oplus I_b(n) i_n^* \sqrt{\tau}$
- Series noise in  $1/\sqrt{\tau}$
- Parallel noise in  $\sqrt{\tau}$
- 1/f noise independant of  $\tau$
- Optimum shaping time  $\tau_{opt} = \tau_c / \sqrt{2n-1}$

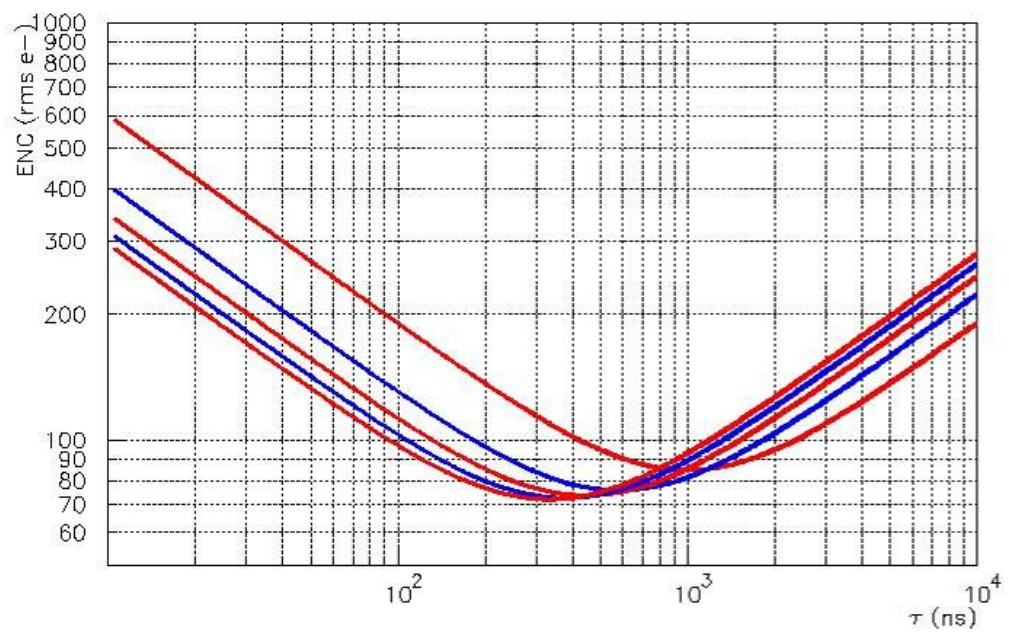
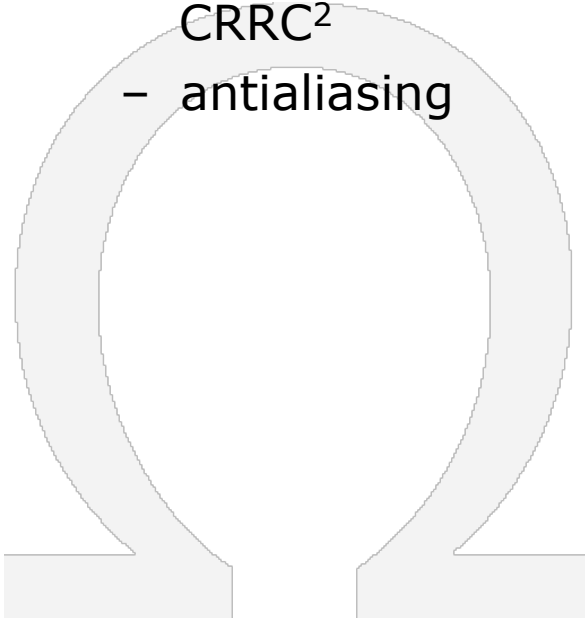


Step response of CR RC<sup>n</sup> shapers

# Equivalent Noise Charge (ENC) after CRRC<sup>n</sup> *Omega*

- Peaking time  $t_p$  (5-100%)
  - ENC(**tp**) independent of n
  - Also includes preamp risetime

- Complex shapers are **obsolete** :
  - Power of **digital filtering**
  - Analog filter = CRRC ou CRRC<sup>2</sup>
  - antialiasing



ENC vs tau for CR RCn shapers

## Equivalent Noise Charge (ENC) after CRRC<sup>n</sup>

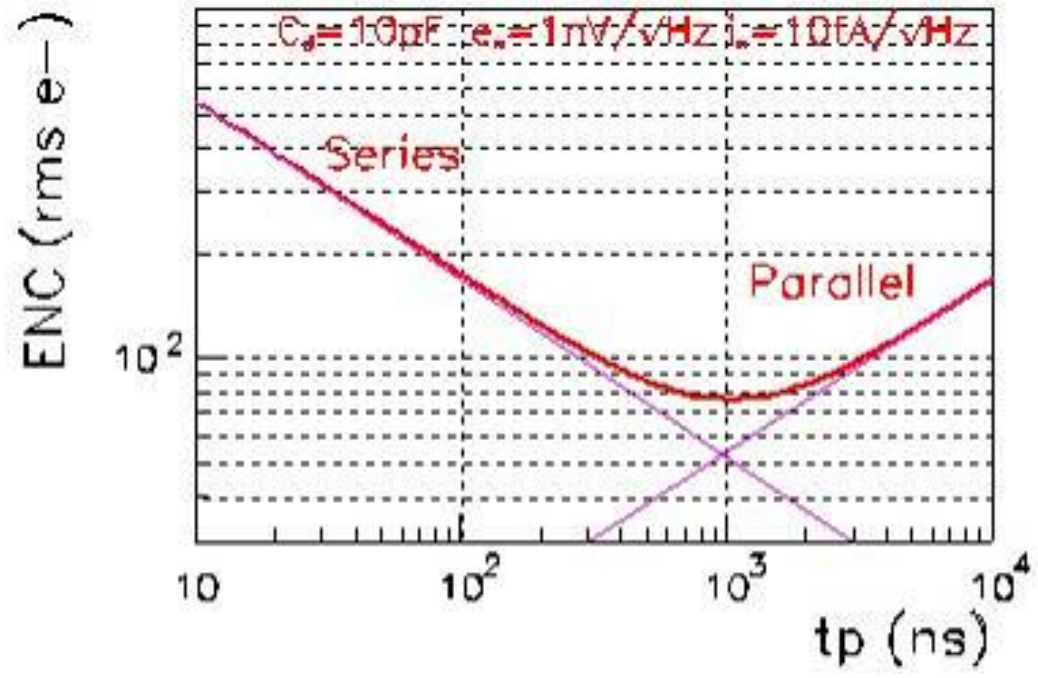
• A useful formula : **ENC (e- rms) after a CRRC<sup>2</sup> shaper :**

$$ENC = 174 e_n C_{tot} / \sqrt{t_p} (\delta) \oplus 166 i_n \sqrt{t_p} (\delta)$$

- $e_n$  in nV/  $\sqrt{\text{Hz}}$ ,  $i_n$  in pA/  $\sqrt{\text{Hz}}$  are the **preamp** noise spectral densities
- $C_{tot}$  (in pF) is dominated by the detector ( $C_d$ ) + input preamp capacitance ( $C_{PA}$ )
- $t_p$  (in ns) is the shaper peaking time (5-100%)

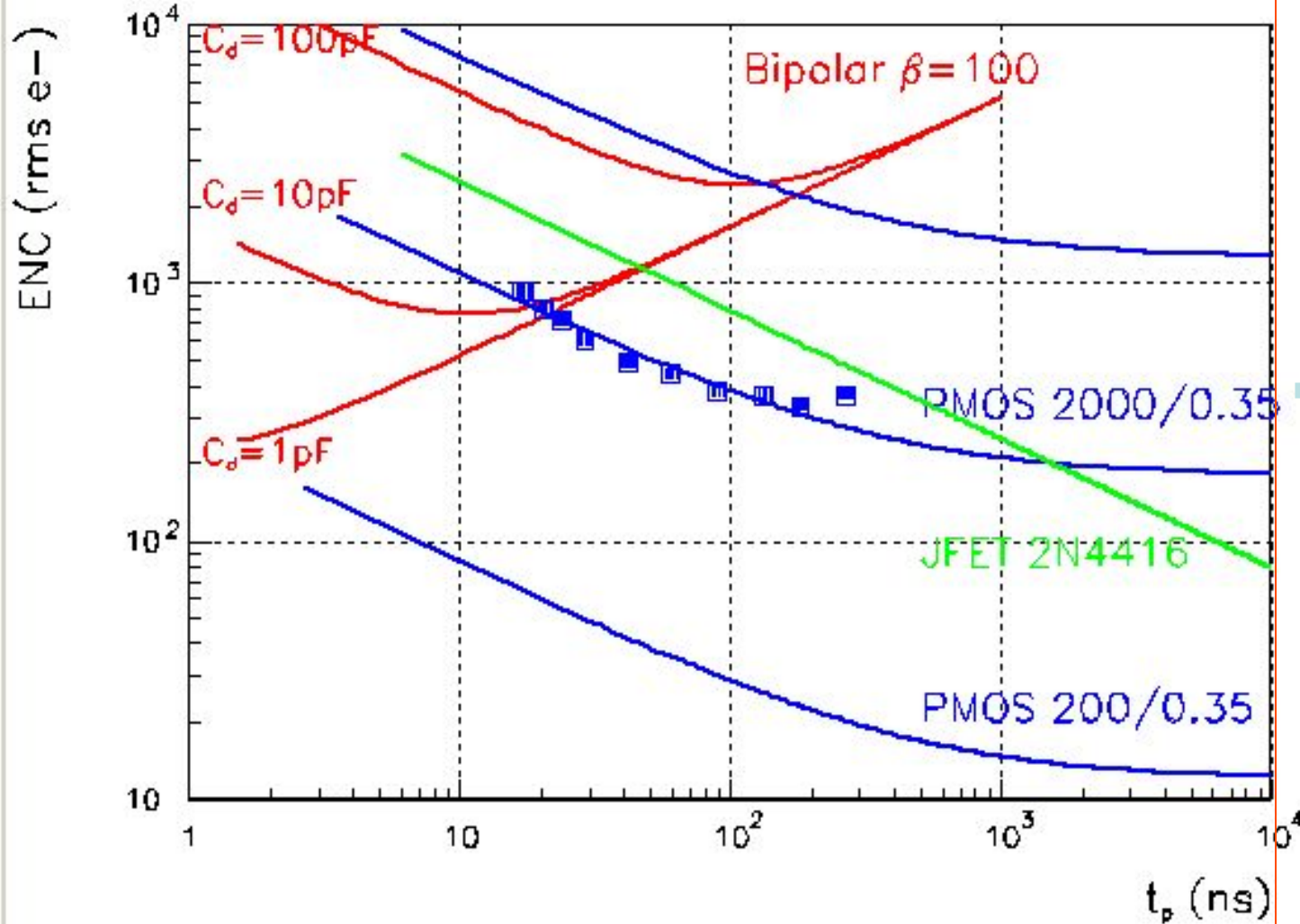
### Noise minimization

- Minimize source capacitance
- Operate at optimum shaping time
- Preamp series noise ( $e_n$ ) best with high transconductance ( $g_m$ ) in input transistor  
=> large current, optimal size



# ENC for various technologies

- ENC for  $C_d=1, 10$  and  $100$  pF at  $I_D = 500$   $\mu$ A
  - MOS transistors best between  $20$  ns –  $2$   $\mu$ s



## Parameters

### Bipolar :

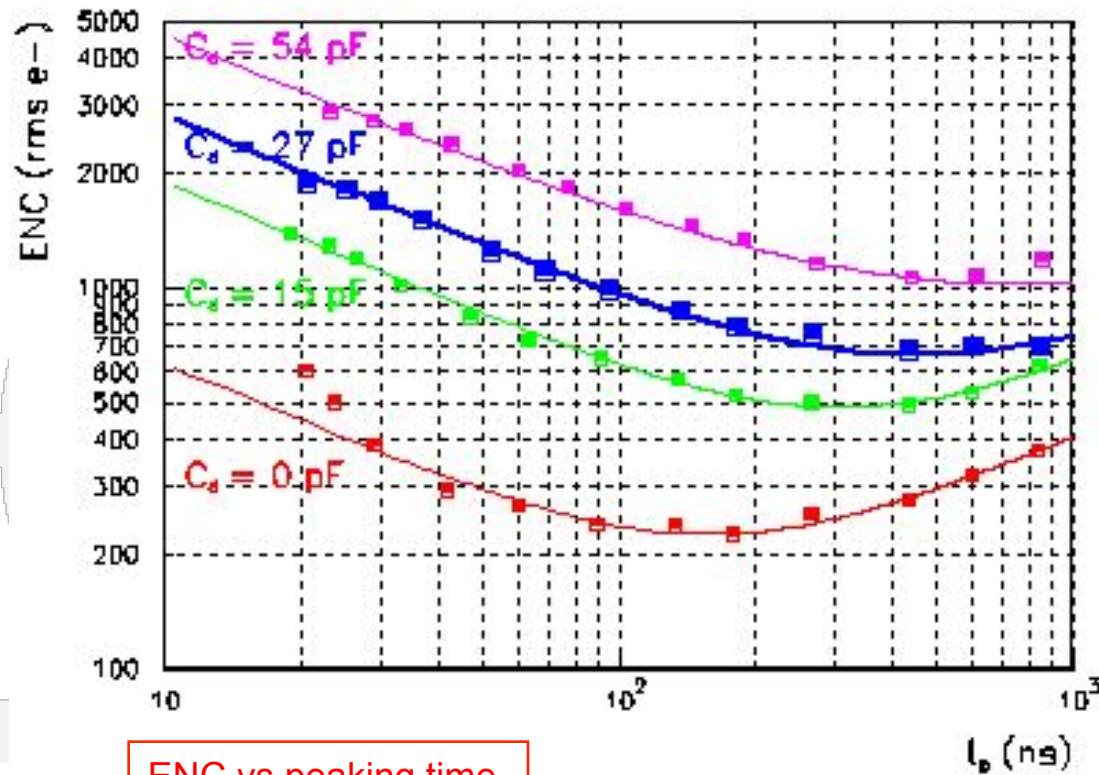
- $g_m = 20$  mA/V
- $R_{BB'} = 25$   $\Omega$
- $e_n = 1$  nV/ $\sqrt{\text{Hz}}$
- $I_B = 5$   $\mu$ A
- $i_n = 1$  pA/ $\sqrt{\text{Hz}}$
- $C_{PA} = 100$  fF

### PMOS 2000/0.35

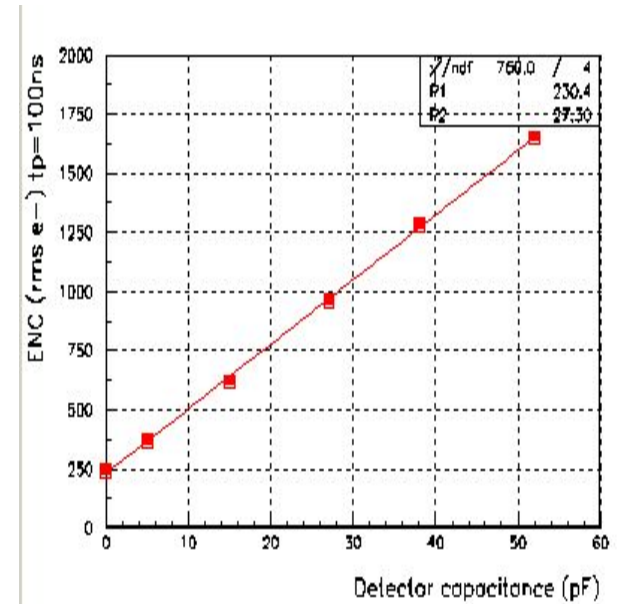
- $g_m = 10$  mA/V
- $e_n = 1.4$  nV/ $\sqrt{\text{Hz}}$
- $C_{PA} = 5$  pF
- $1/f$  :

# Example of ENC measurement

- SKIROC ASIC (ILC readout) : 0.35 $\mu$ m SiGe
  - Series :  $e_n = 1.4$  nV/ $\sqrt{\text{Hz}}$ ,  $C_{pA} = 7$  pF
  - 1/f noise : 12 e-/pF
  - Parallel :  $i_n = 40$  fA/ $\sqrt{\text{Hz}}$



ENC vs peaking time



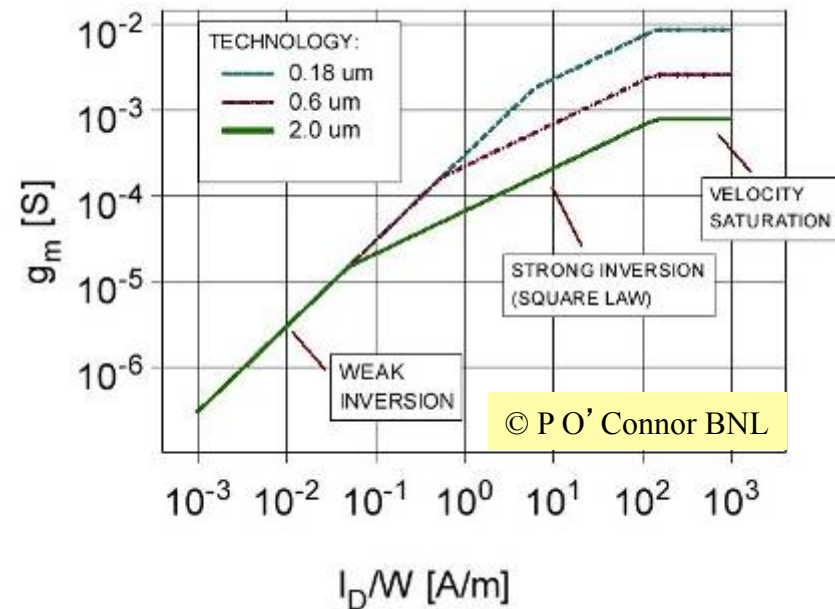
ENC vs Capacitance  $t_p=100$ ns



# MOS input transistor sizing

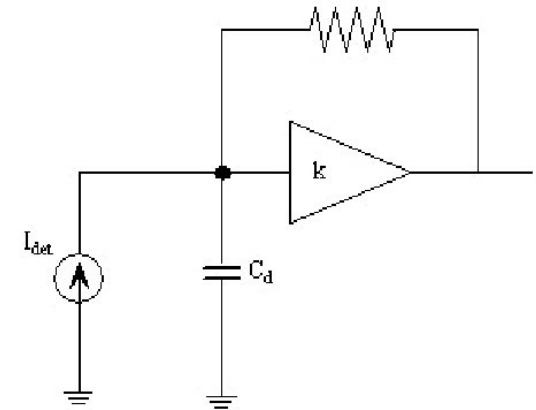
- Capacitive matching : strong inversion
  - $g_m$  proportionnal to  $W/L \sqrt{I_D}$
  - $C_{GS}$  proportionnal to  $W \cdot L$
  - ENC propotionnal to  $(C_{det} + C_{GS}) / \sqrt{g_m}$
  - Optimum  $W/L$  :  $C_{GS} = 1/3 C_{det}$
  - Large transistors are easily in moderate or weak inversion at small current

- Optimum size in weak inversion
  - $g_m$  proportionnal to  $I_D$  (indep of  $W, L$ )
  - ENC minimal for  $C_{GS}$  minimal, provided the transistor remains in weak inversion



# Current preamplifiers :

- Transimpedance configuration
  - $V_{out}(\omega)/i_{in}(\omega) = -R_f / (1 + Z_f/GZ_d)$
  - Gain =  $R_f$
  - High counting rate
  - Typically optical link receivers

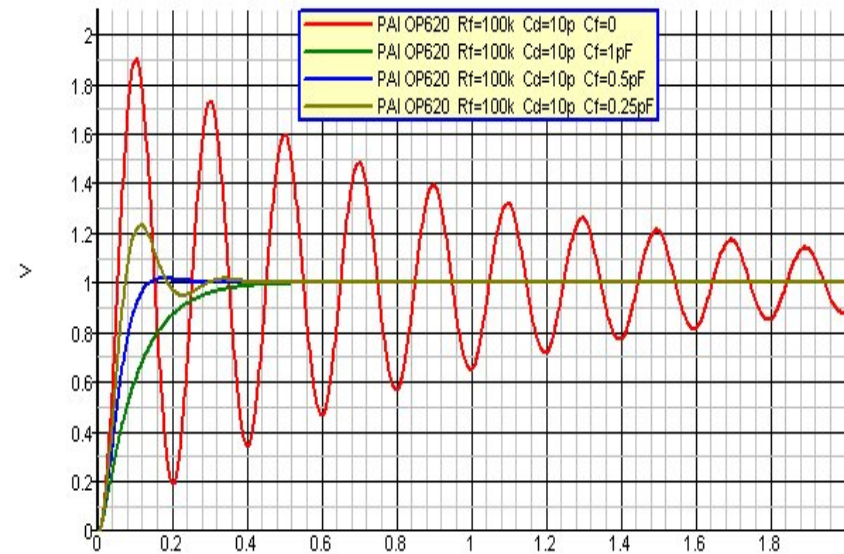


Current sensitive preamp

- Easily oscillatory
  - Unstable with capacitive detector
  - Inductive input impedance

$$L_{eq} = R_f / \omega_C$$

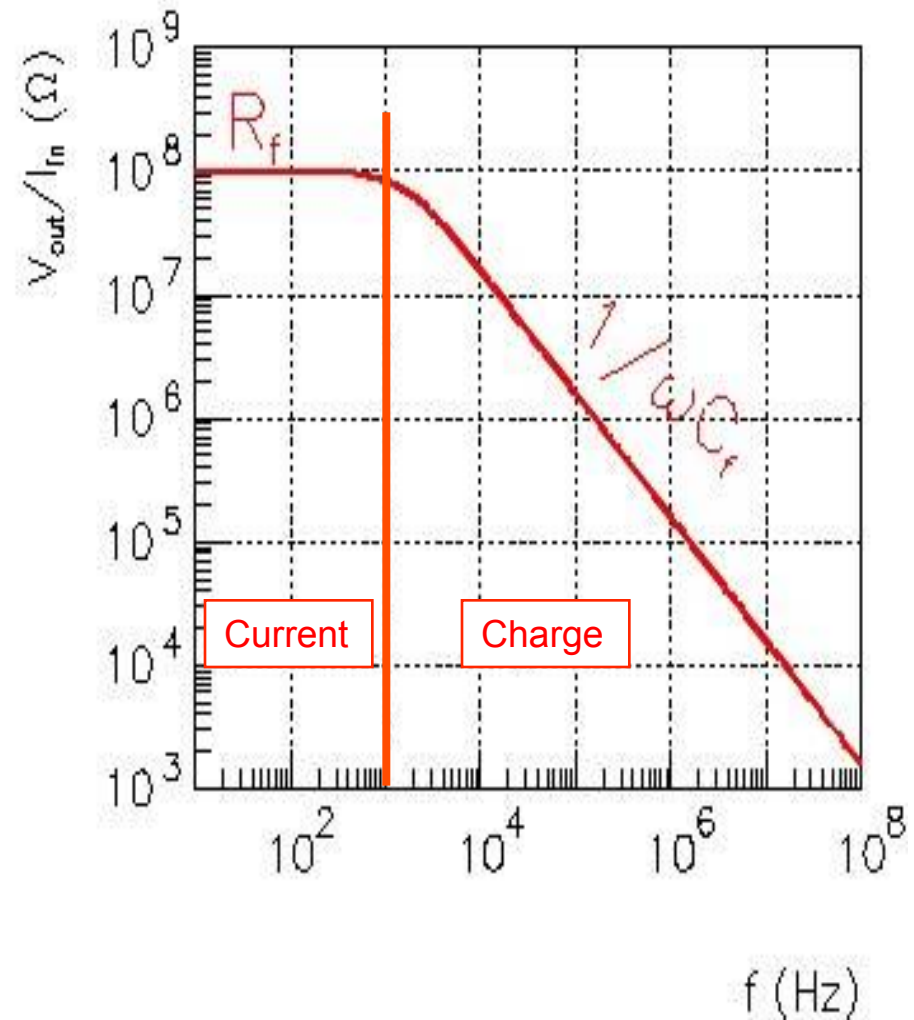
- Resonance at :  $f_{res} = 1/2\pi \sqrt{L_{eq}C_d}$
- Quality factor :  $Q = R / \sqrt{L_{eq}/C_d}$ 
  - $Q > 1/2 \rightarrow$  ringing
- Damping with capacitance  $C_f$ 
  - $C_f = 2 \sqrt{(C_d/R_f G_0 \omega_0)}$
  - Easier with fast amplifiers



Step response of current sensitive preamp

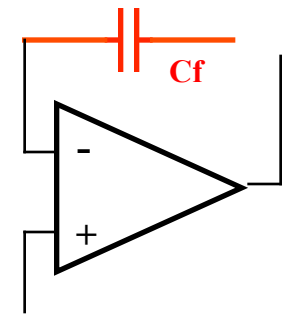
# Charge vs Current preamps

- Charge preamps
    - Best noise performance
    - Best with short signals
    - Best with small capacitance
  - Current preamps
    - Best for long signals
    - Best for high counting rate
    - Significant parallel noise
- Charge preamps are not slow, they are long  
• Current preamps are not faster, they are shorter (but easily unstable)

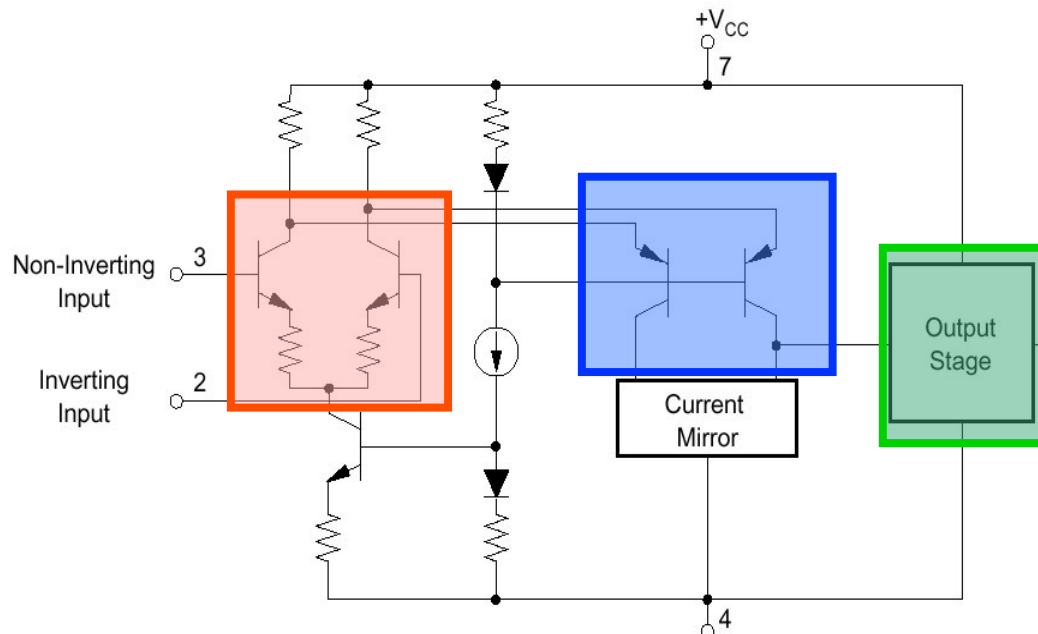


## Charge preamp design

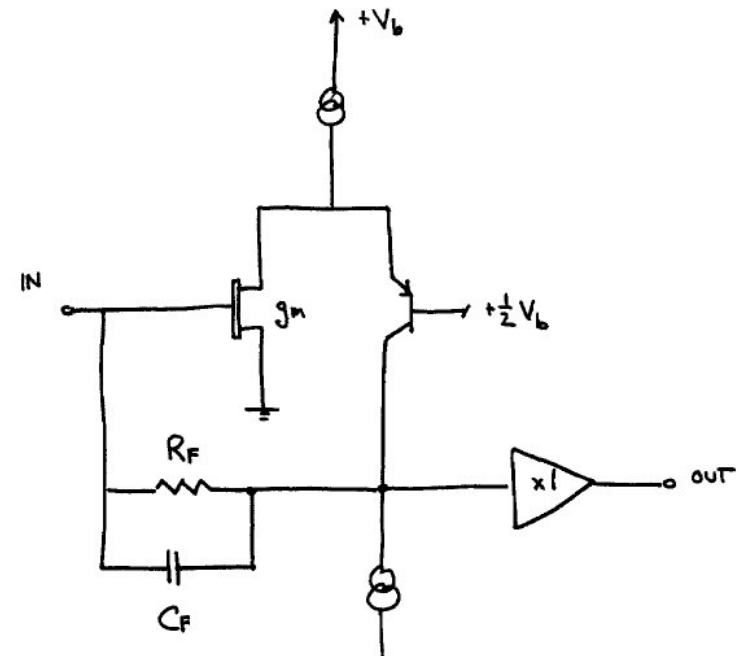
- From the schematic of principle
  - Using of a fast opamp (OP620)
  - Removing unnecessary components...
  - Similar to the traditional schematic «Radeka 68 »
  - Optimising transistors and currents



Charge preamp



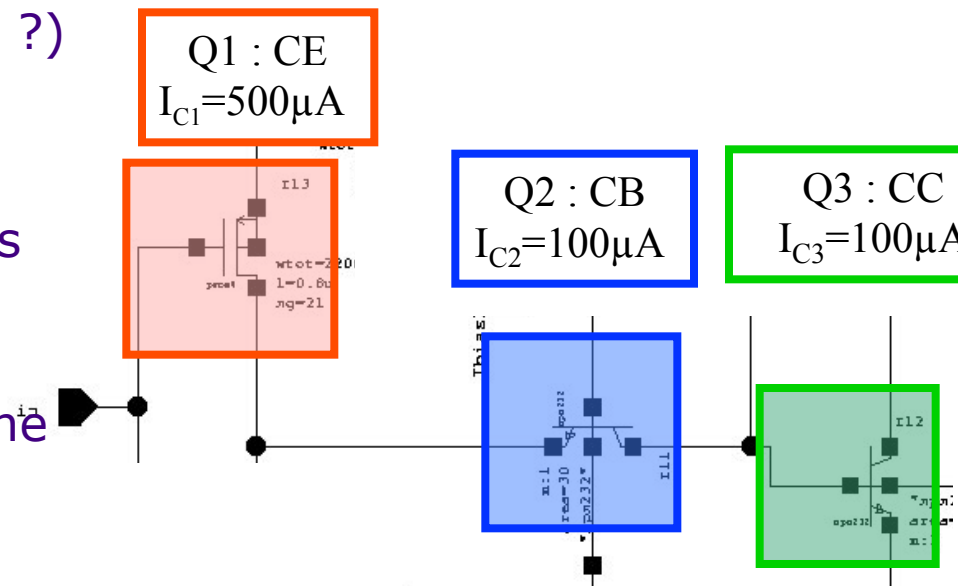
Schematic of an OP620 opamp ©BurrBrown



Original charge preamp ©Radeka 1968

# Example : designing a charge preamp (2) *Omega*

- Simplified schematic
- Optimising components
  - What transistors (PMOS, NPN ?)
  - What bias current ?
  - What transistor size ?
  - What is the noise contributions of each component, how to minimize it ?
  - What parameters determine the stability ?
  - What is the saturation behaviour ?
  - How vary signal and noise with input capacitance ?
  - How to maximise the output voltage swing ?
  - What the sensitivity to power supplies, temperature...

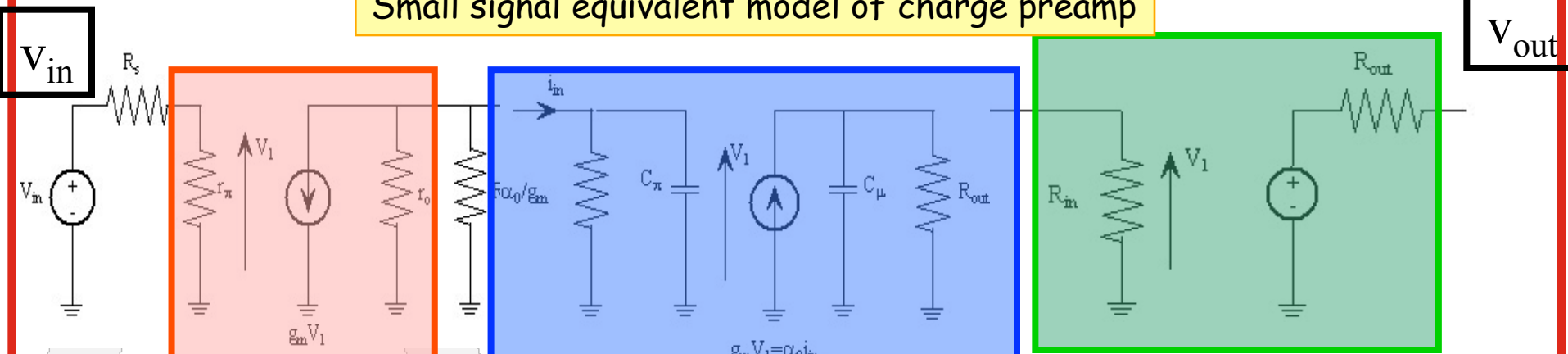


Simplified schematic of charge preamp

# Example : designing a charge preamp (3) *Omega*

- Small signal equivalent model
  - Transistors are replaced by hybrid  $\pi$  model
  - Allows to calculate open loop gain

Small signal equivalent model of charge preamp



$g_{m1}$

$R_0 C_0$   
 $R_0 = R_{out2} // R_{in3} // r_{o4}$

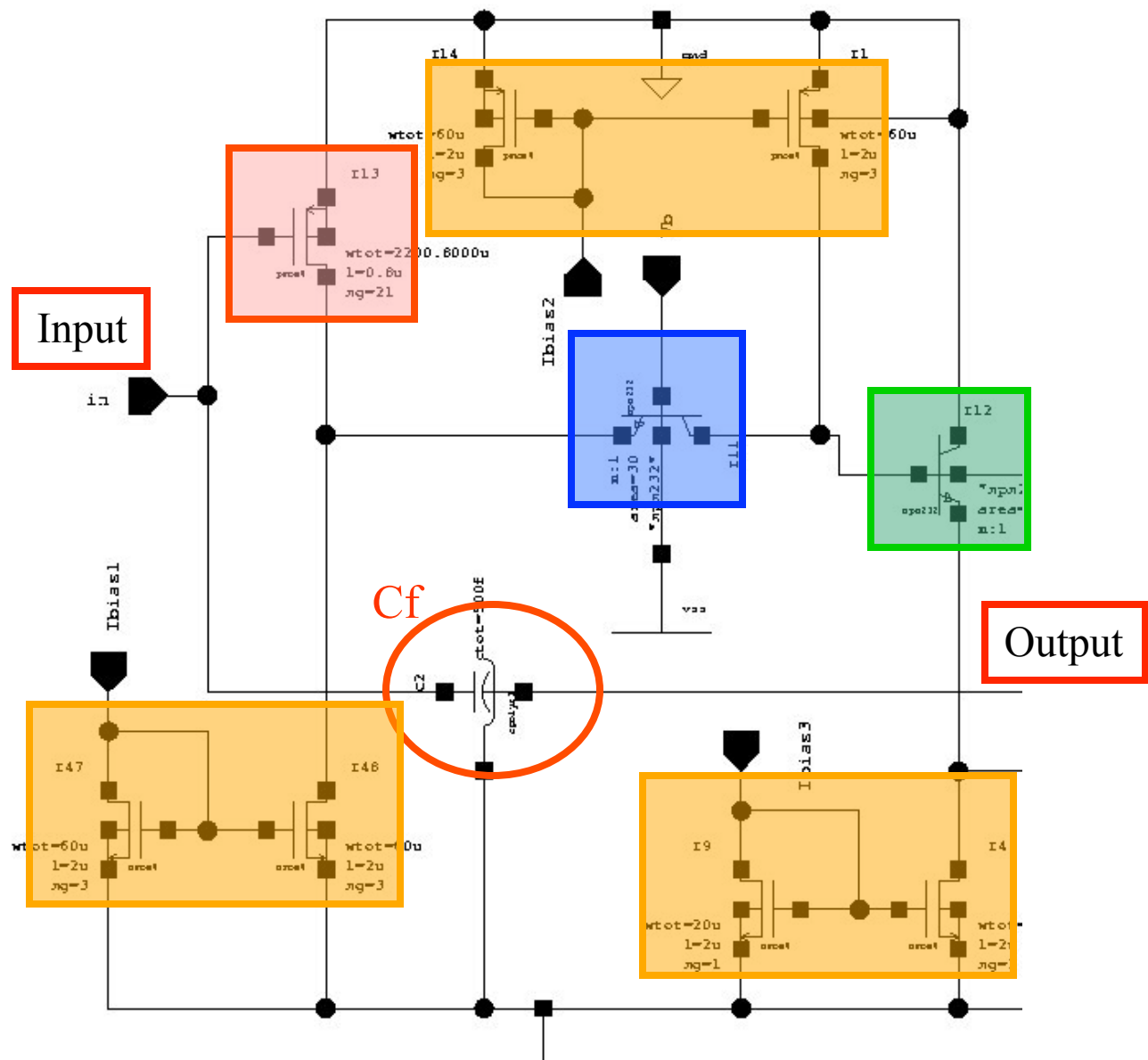
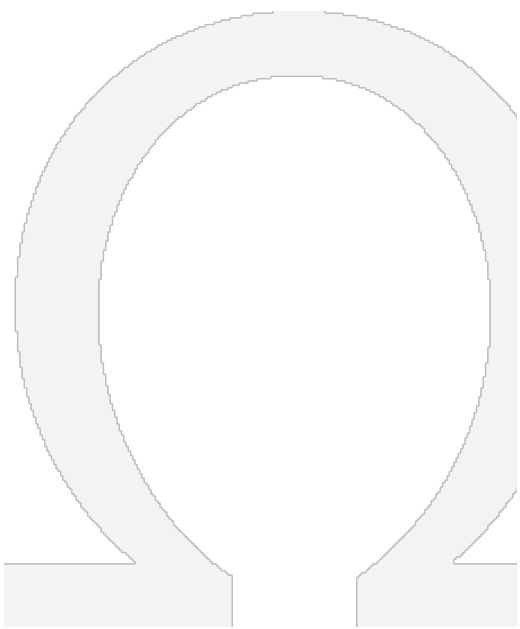
■ Gain (open loop) :

$$v_{out}/v_{in} = - g_{m1} R_0 / (1 + j\omega R_0 C_0)$$

■ Ex :  $g_{m1} = 20 \text{ mA/V}$ ,  $R_0 = 500 \text{ k}\Omega$ ,  $C_0 = 1 \text{ pF} \Rightarrow G_0 = 10^4$   $\omega_0 = 210^6$   $G_0 \omega_0 = 2 \cdot 10^{10} = 3 \text{ GHz}!$

# Example : designing a charge preamp (4) *Omega*

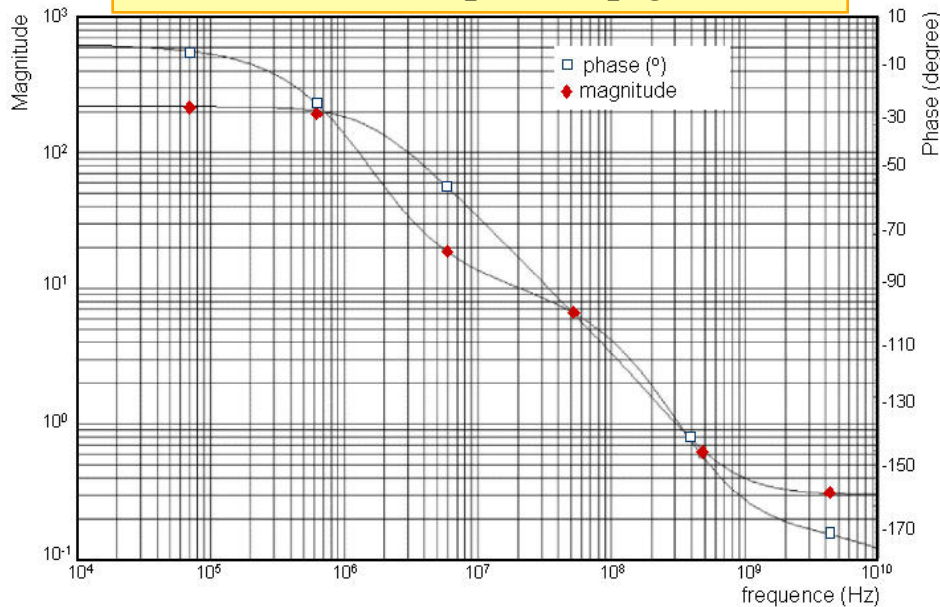
- Complete schematic
  - Adding bias elements



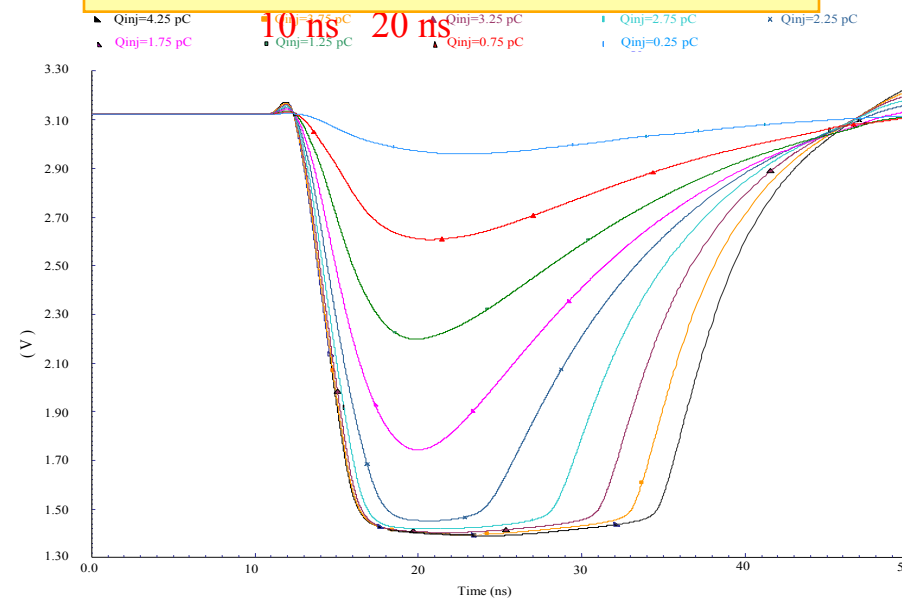
# Example : designing a charge preamp (5) *Omega*

- Complete simulation
  - Checking hand calculations against 2<sup>nd</sup> order effects
  - Testing extreme process parameters (« corner simulations »)
  - Testing robustness (to power supplies, temperature...)

Simulated open loop gain



Saturation behaviour

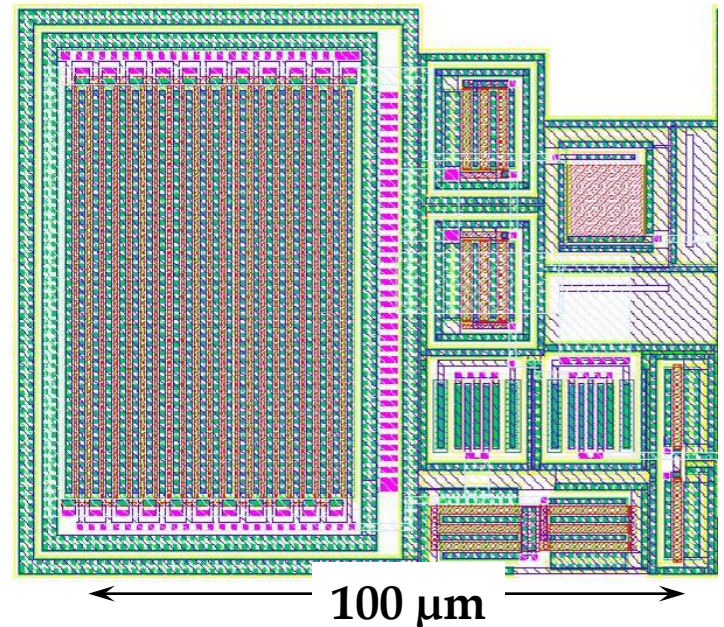


1 MHz



# Example : designing a charge preamp (6) *Omega*

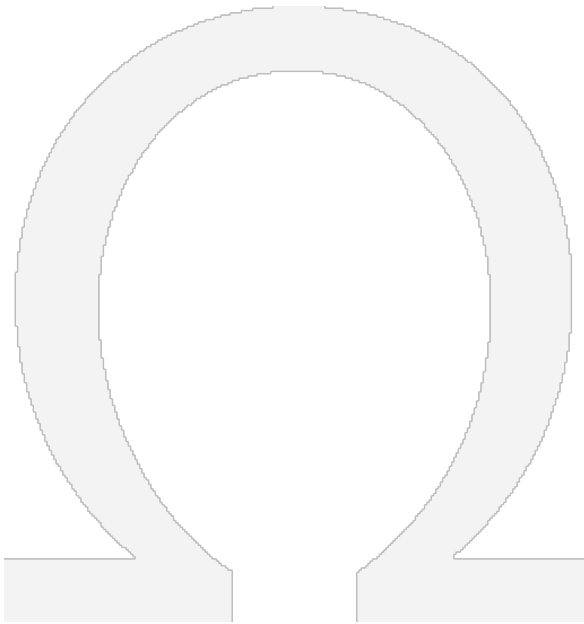
- Layout
  - Each component is drawn
  - They are interconnected by metal layers
- Checks
  - DRC : checking drawing rules (isolation, minimal dimensions...)
  - ERC : extracting the corresponding electrical schematic
  - LVS (layout vs schematic) : comparing extracted schematic and original design
  - Simulating extracted schematic with parasitic elements
- Generating GDS2 file
  - Fabrication masks : « reticule »



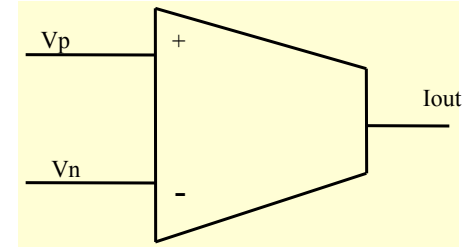
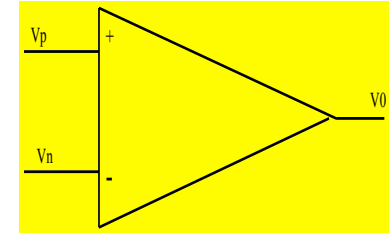
- Coexistence analog-digital
  - Capacitive, inductive and common-impedance couplings
  - A full lecture !
  - A good summary : there is no such thing as « ground », pay attention to current return



extension slides



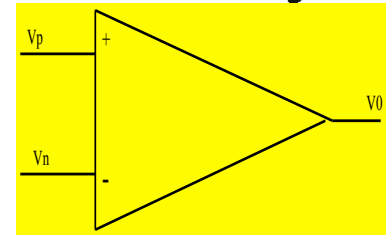
- Voltage feedback operationnal amplifier (VFOA)
- Voltage amplifiers, RF amplifiers (VA,LNA)
- Current feedback operationnal amplifiers (CFOA)
- Current conveyors (CCI, CCII +/-)
- Current (pre)amplifiers (ISA,PAI)
- Charge (pre)amplifiers (CPA,CSA,PAC)
- Transconductance amplifiers (OTA)
- Transimpedance amplifiers (TZA,OTZ)
- Mixing up open loop (OL) and closed loop (CL) configurations !



## Only 4 open-loop configurations

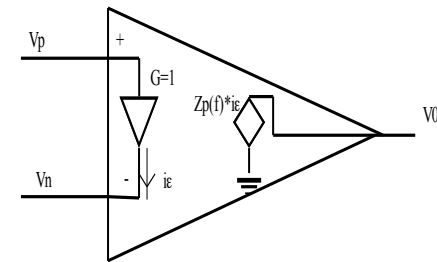
- Voltage operational amplifiers (OA, VFOA)

- $V_{out} = G(\omega) V_{in\ diff}$
- $Z_{in+} = Z_{in-} = \infty$   $Z_{out} = 0$



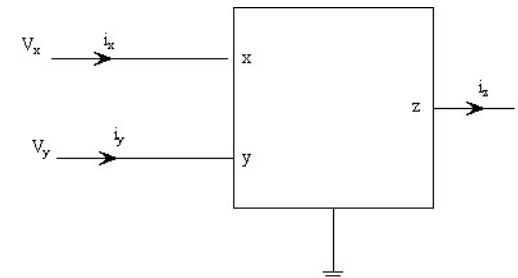
- Transimpedance operational amplifier (CFOA !)

- $V_{out} = Z(\omega) i_{in}$
- $Z_{in-} = 0$   $Z_{out} = 0$



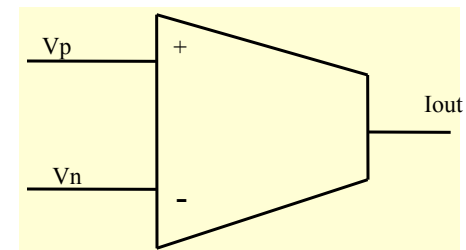
- Current conveyor (CCI, CCII)

- $I_{out} = G(\omega) I_{in}$
- $Z_{in} = 0$   $Z_{out} = \infty$



- Transconductance amplifier (OTA)

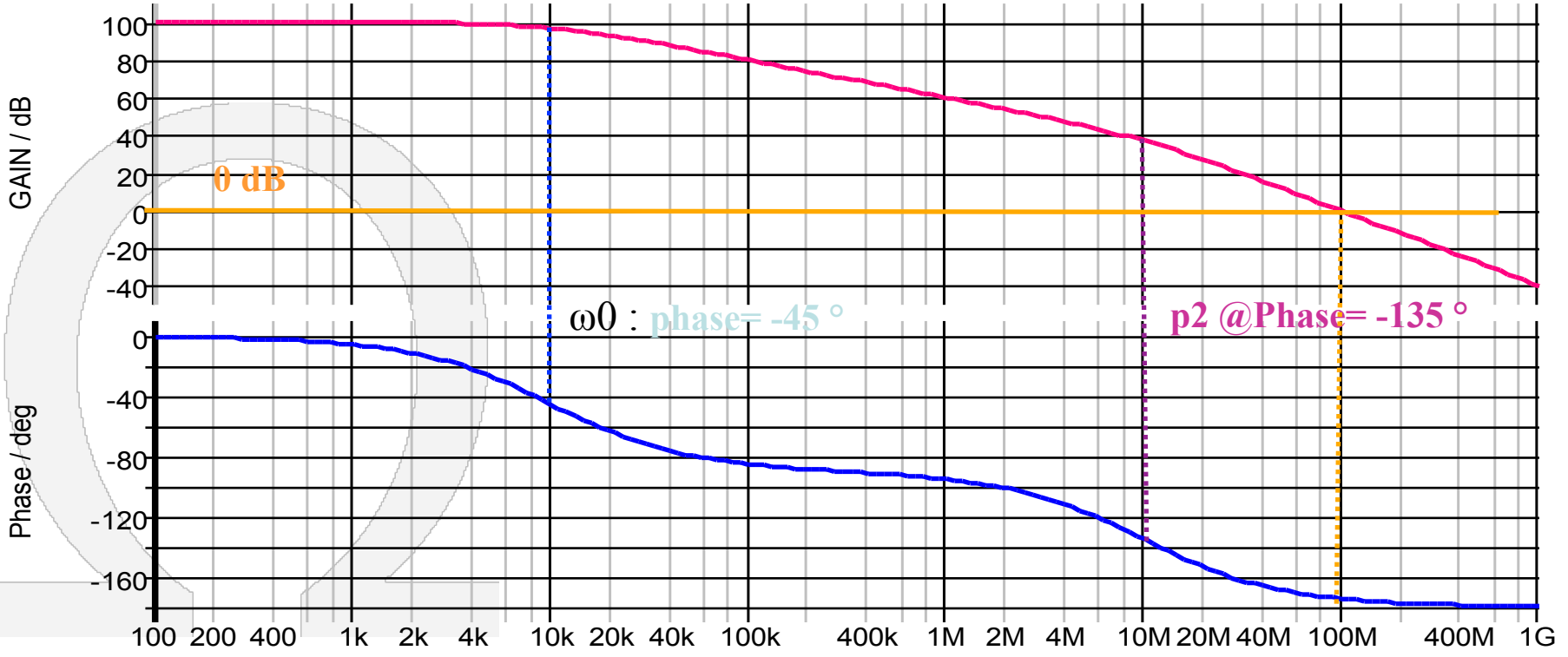
- $I_{out} = G_m(\omega) V_{in\ diff}$
- $Z_{in+} = Z_{in-} = \infty$   $Z_{out} = \infty$



# Open loop gain variation with frequency

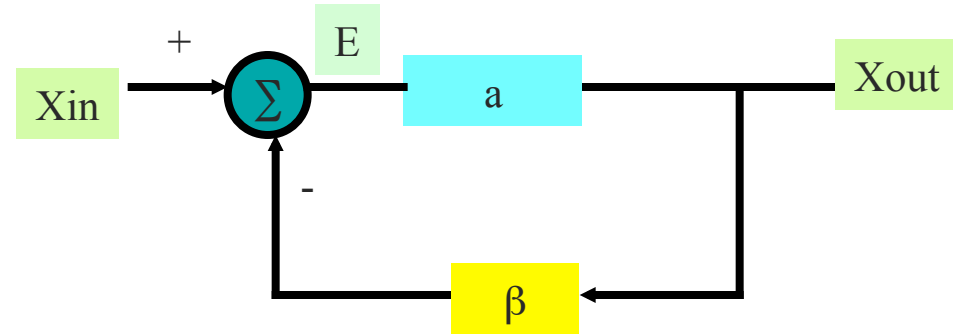


- Define exactly what is « gain »  $v_{out}/v_{in}$ ,  $v_{out}/i_{in}$ ...
- « Gain » varies with frequency :  $G(j\omega) = G_0 / (1 + j \omega / \omega_0)$ 
  - $G_0$  low frequency gain
  - $\omega_0$  dominant pole
  - $\omega_c = G_0 \omega_0$  Gain-Bandwidth product (sometimes referred to as unity gain frequency)

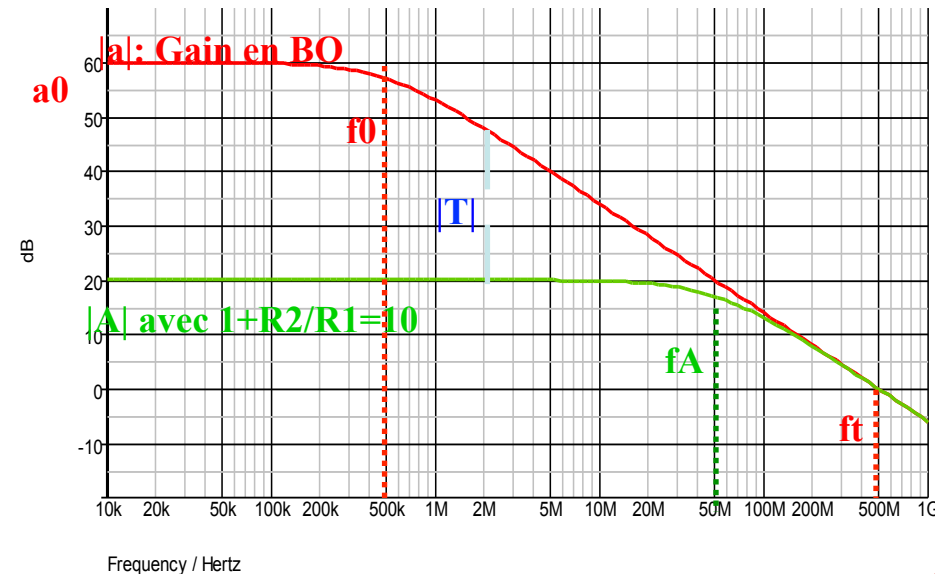


## Feedback : an essential tool

- Improves gain performance
  - Less sensitivity to open loop gain ( $a$ )
  - Better linearity
- Essential in low power design
- Potentially unstable
- Feedback constant :  $\beta = E/X_{out}$
- Open loop gain :  $a = X_{out}/E$
- Closed loop gain :  $X_{out}/X_{in} \rightarrow 1/\beta$

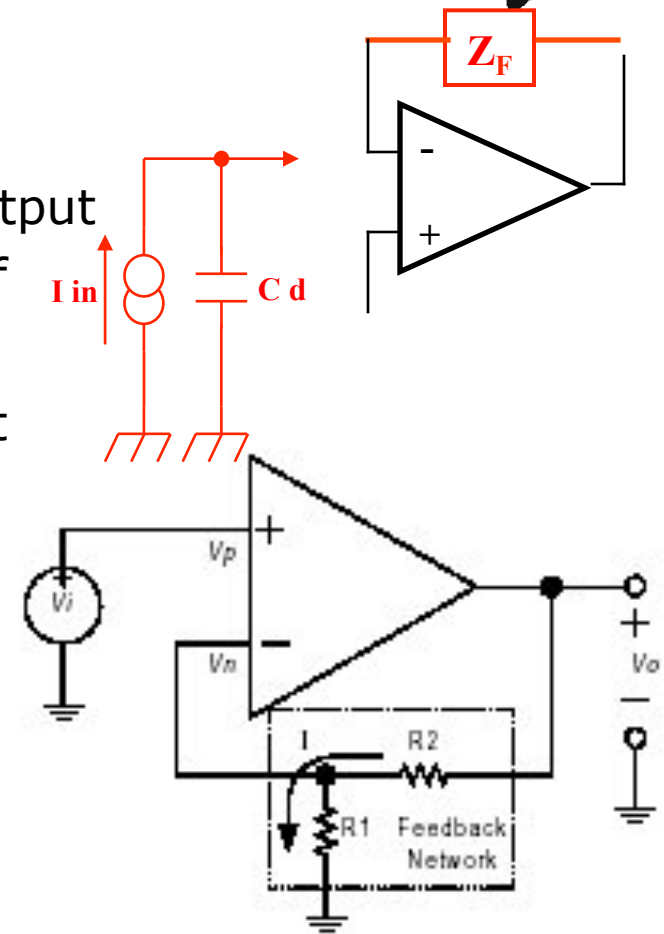


$$\frac{X_{out}}{X_{in}} = \frac{a}{1 + a\beta} = \frac{1/\beta}{1 + 1/a\beta}$$



## Only 4 feedback configurations

- Shunt-shunt = transimpedance
  - Small  $Z_{in}$  ( $= Z_{in}(OL)/T$ ) -> current input
  - small  $Z_{out}$  ( $= Z_{out}(OL)/T$ ) -> voltage output
  - De-sensitizes transimpedance  $= 1/\beta = Z_f$
- Series-shunt
  - Large  $Z_{in}$  ( $= Z_{in}(OL)*T$ ) -> voltage input
  - Small  $Z_{out}$  ( $= Z_{out}(OL)/T$ ) -> voltage
  - Optimizes voltage gain ( $= 1/\beta$ )
- Shunt series
  - Small  $Z_{in}$  ( $= Z_{in}(OL)/T$ ) -> current inp
  - Large  $Z_{out}$  ( $= Z_{out}(OL)*T$ ) -> current
  - Current conveyor
- Series-series
  - Large  $Z_{in}$  ( $= Z_{in}(OL)*T$ ) -> voltage input
  - Large  $Z_{out}$  ( $= Z_{out}(OL)*T$ ) -> current output
  - Transconductance
  - Ex : common emitter with emitter degeneration



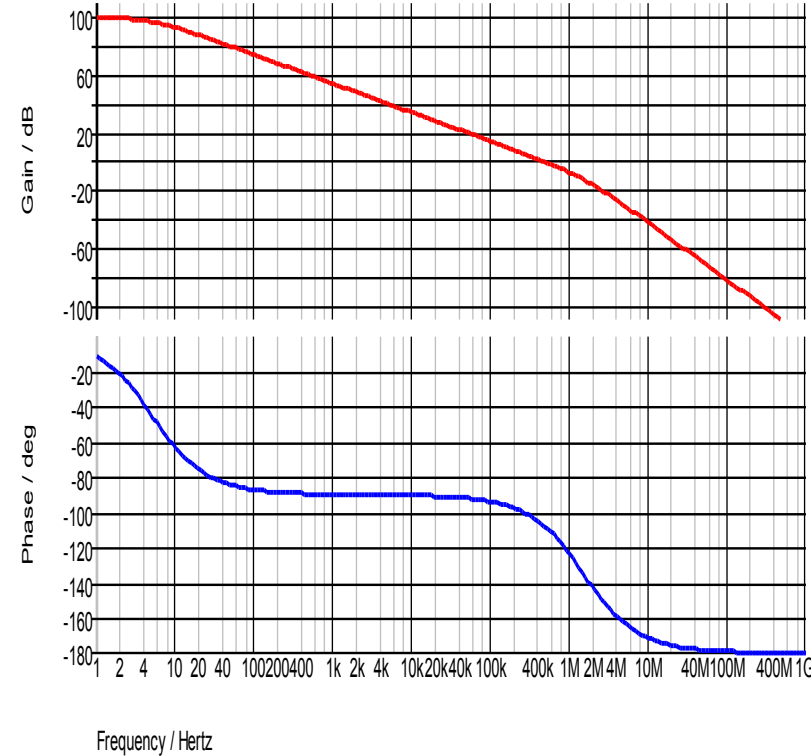
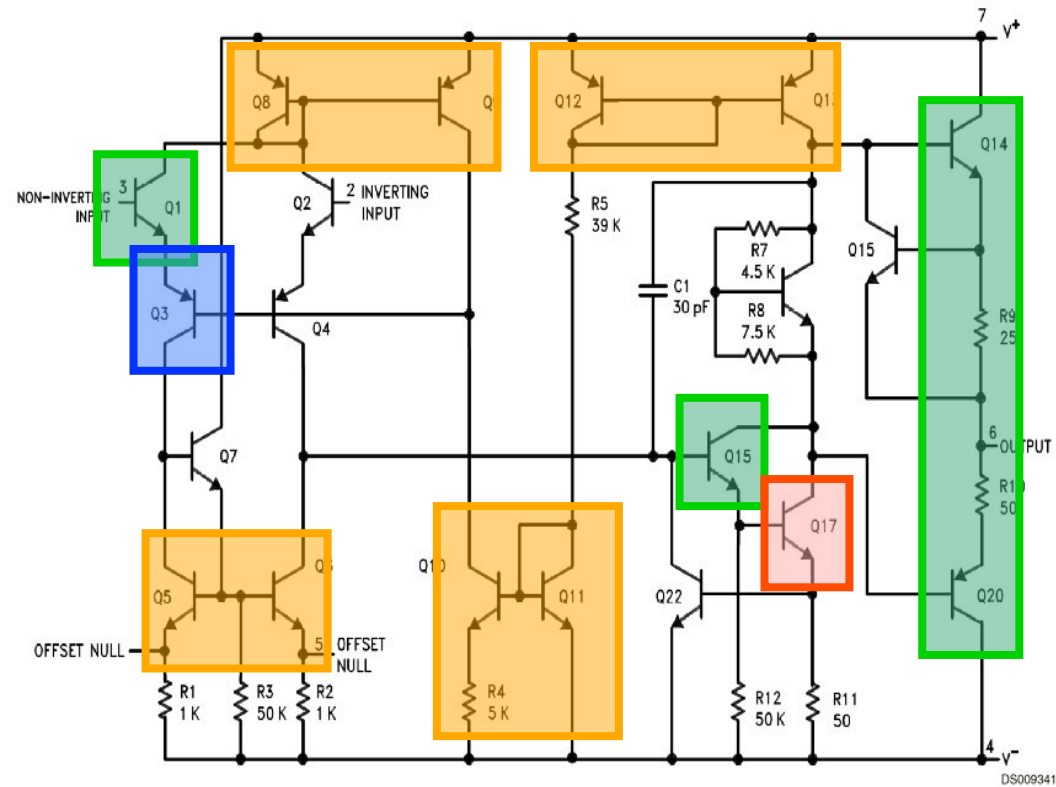
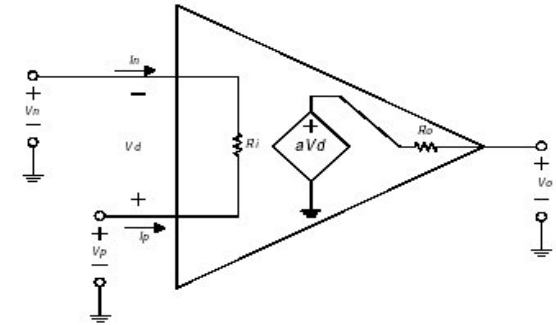


# Voltage Feedback Operational Amplifiers



(1)

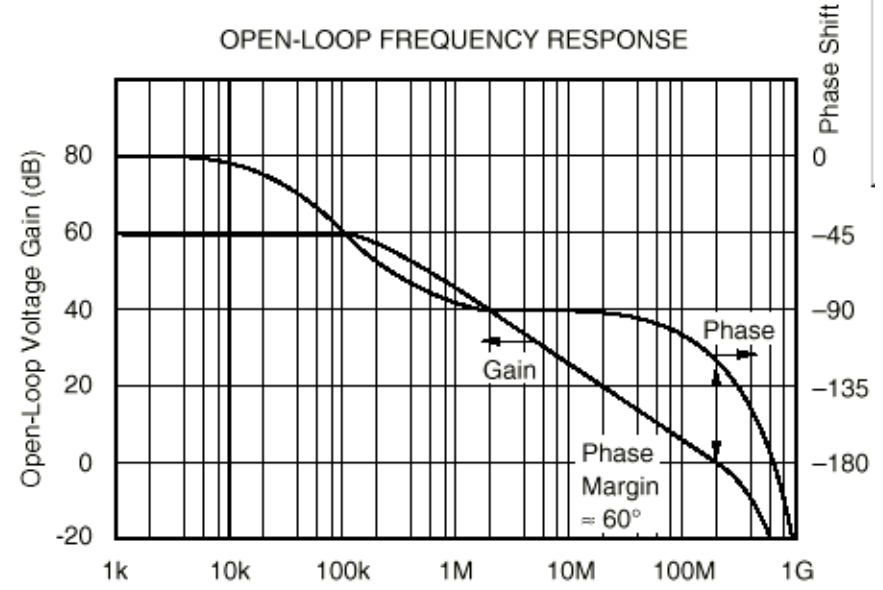
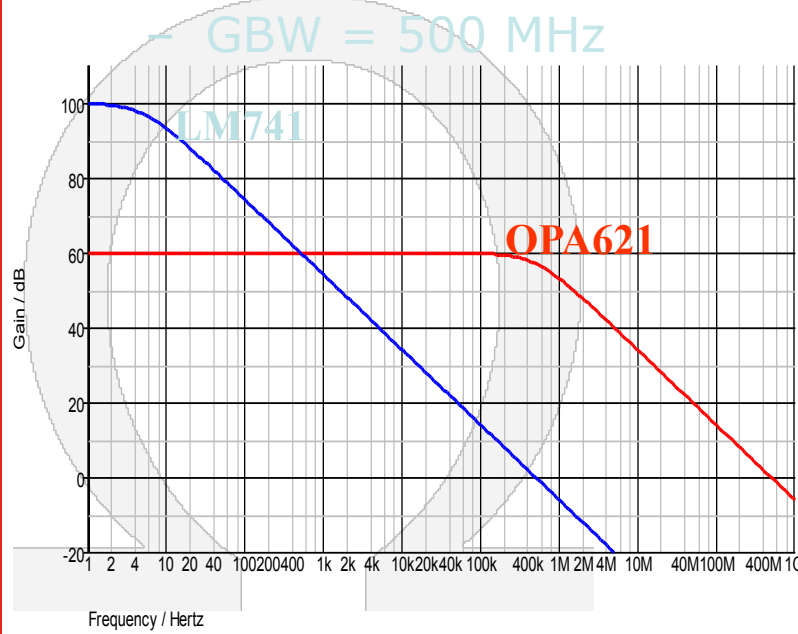
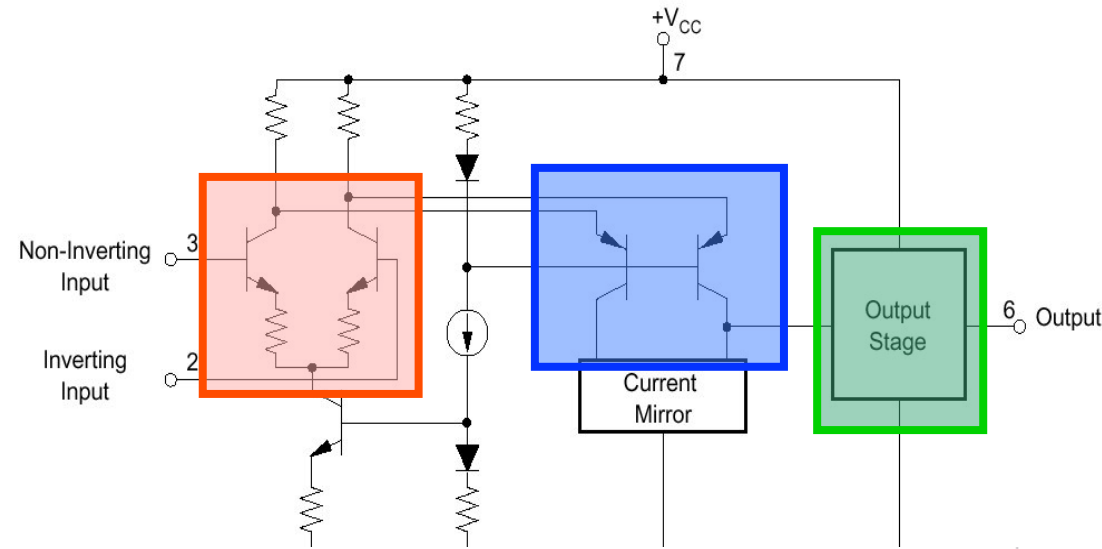
- Back to the 70's : LM741
  - 3 stages : Paraphase=CE, Darlington=CE,
  - $G_0 = 200\ 000$ ,  $f_0 = 5\text{Hz}$ ,  $\text{GBW} = 1\ \text{MHz}$ ,  $F$



Schematic diagram of a LM741 (1970) ©National Semiconductors

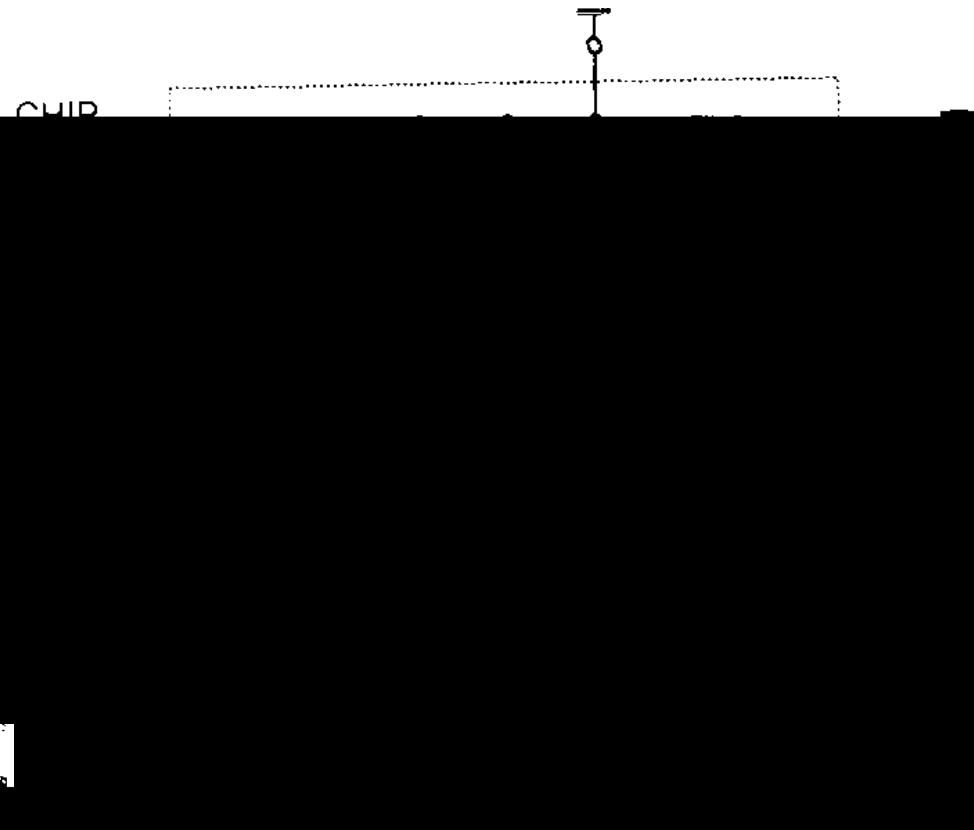
(2)

- Breakthrough in the 90's : OP620-621
  - 2 stages :  
Cascode=CE, Push-pull = CC
  - Pd = 250 mW
  - G0 = 1 000
  - f0 = 500kHz

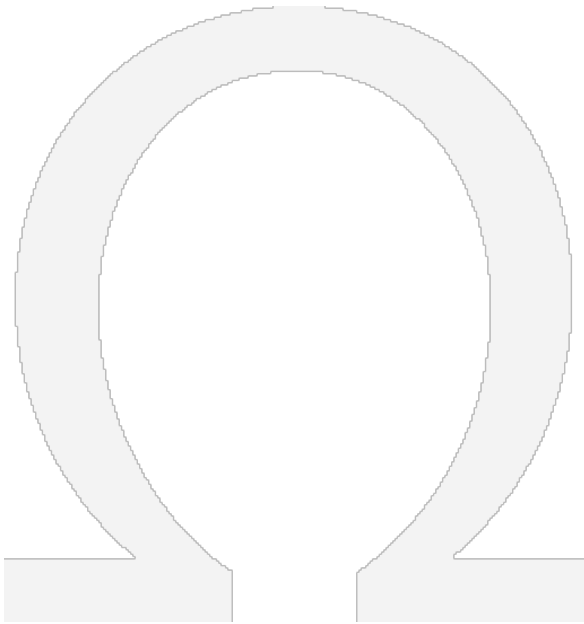


**Open loop frequency response of OP620**

- « Simple architecture »
  - CE + CC configuration
  - SiGe bipolar transi
  - CC outside feedback
  - « pole splitting »

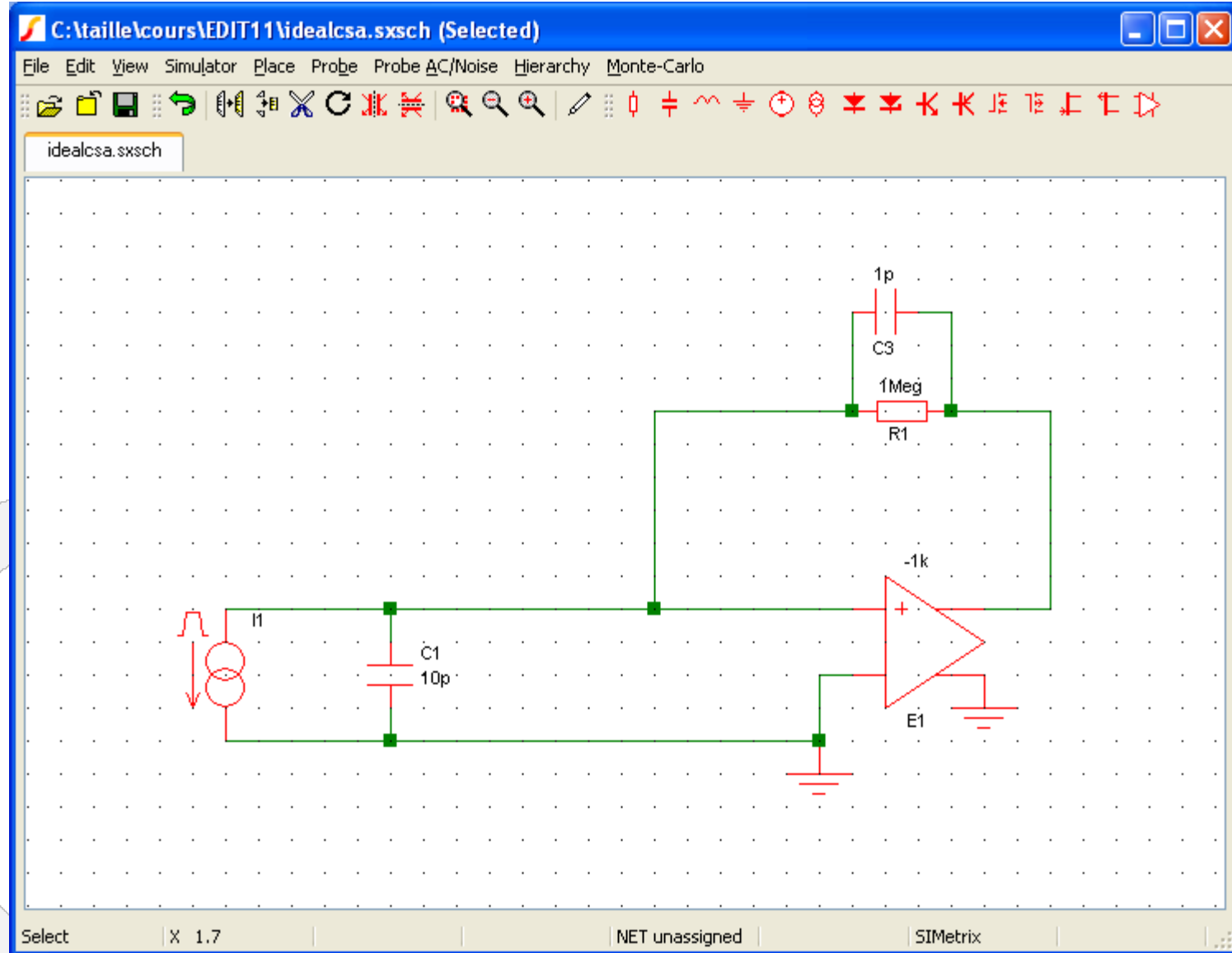


# Exercice slides

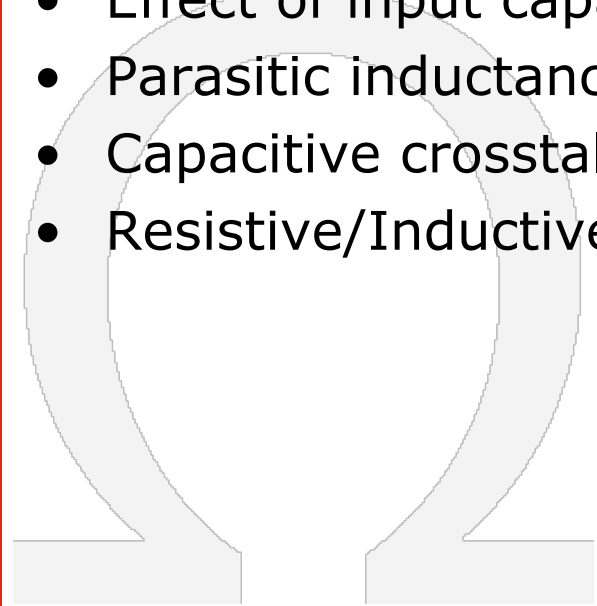


# Ideal charge preamp

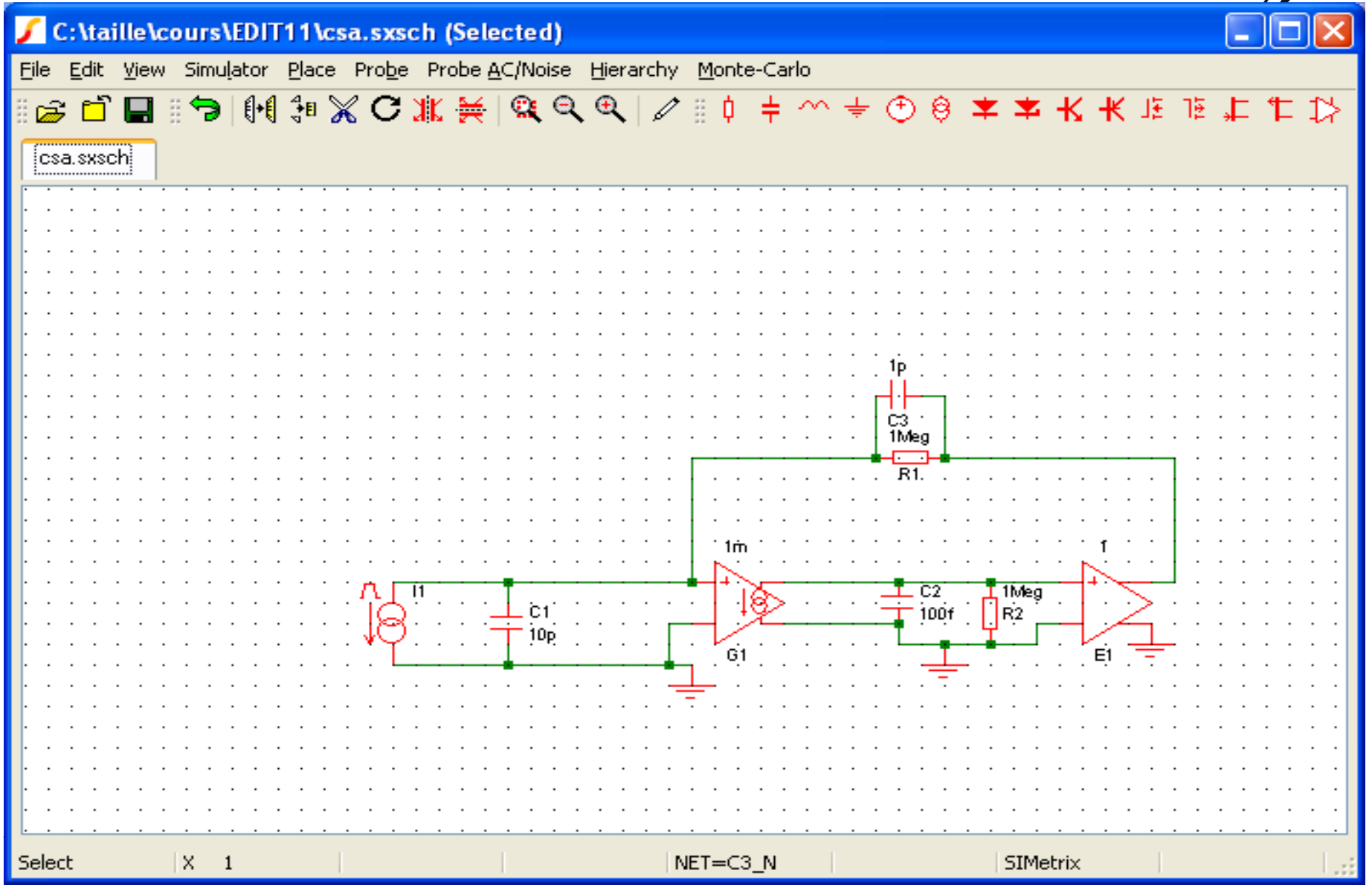
Omega



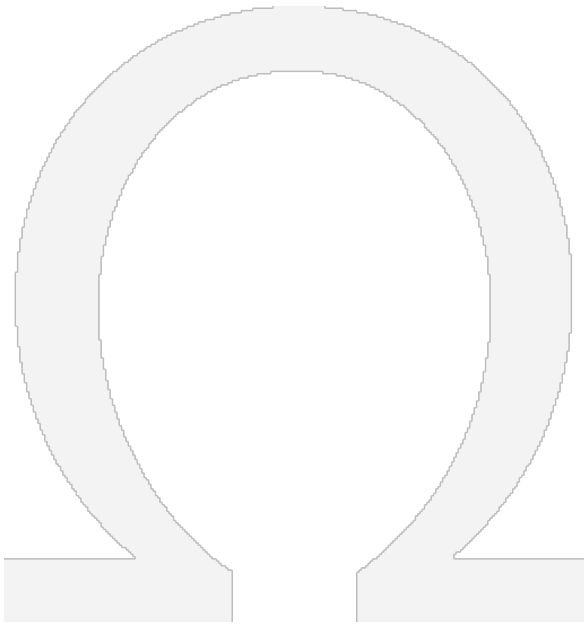
- Simulate impulse response
- Frequency response
- Input impedance
- Ballistic deficit
- Effect of amplifier gain
- Effect of resistive feedback
- Test pulse injection
- Effect of input capacitance
- Parasitic inductance
- Capacitive crosstalk
- Resistive/Inductive ground return



# Non ideal charge preamp



backup slides





- Voltage generators or source

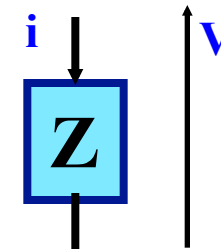
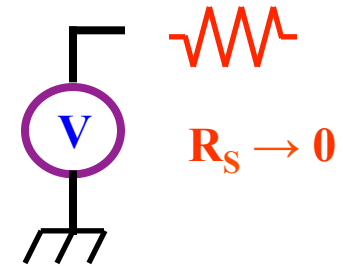
- Ideal source : constant voltage, independent of current (or load)
- In reality : non-zero source impedance  $R_S$

- Current generators

- Ideal source : constant current, independent of voltage (or load)
- In reality : finite output source impedance  $R_S$

- Ohms' law

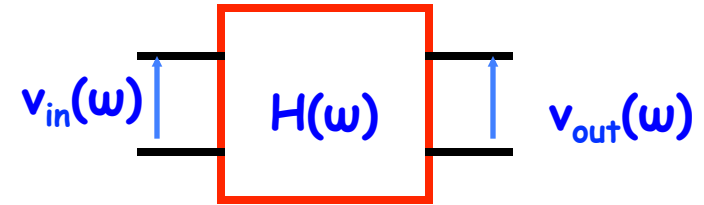
- $Z = R, 1/j\omega C, j\omega L$
- Note the **sign** convention



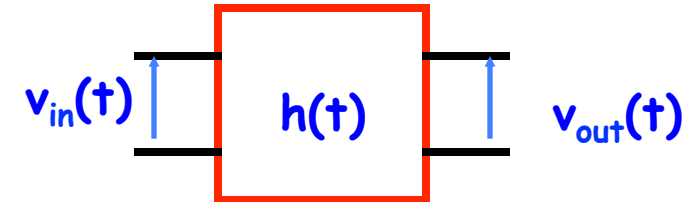
## Frequency domain & time domain

- Frequency domain :

- $V(\omega, t) = A \sin(\omega t + \varphi)$ 
  - Described by **amplitude** and **phase** ( $A, \varphi$ )
- **Transfer function** :  $H(\omega)$  [or  $H(s)$ ]
- = The ratio of output signal to input signal in the frequency domain assuming **linear** electronics



$\mathcal{F}^{-1}$



**Time domain**  $V_{out}(\omega) = H(\omega) V_{in}(\omega)$

- **Impulse response** :  $h(t)$

- = the output signal for an **impulse** (delta) input in the time domain
- The output signal for **any** input signal  $v_{in}(t)$  is obtained by **convolution** : «\*» :
- $V_{out}(t) = v_{in}(t) * h(t) = \int v_{in}(u) * h(t-u) du$

- **Correspondance through Fourier transforms**

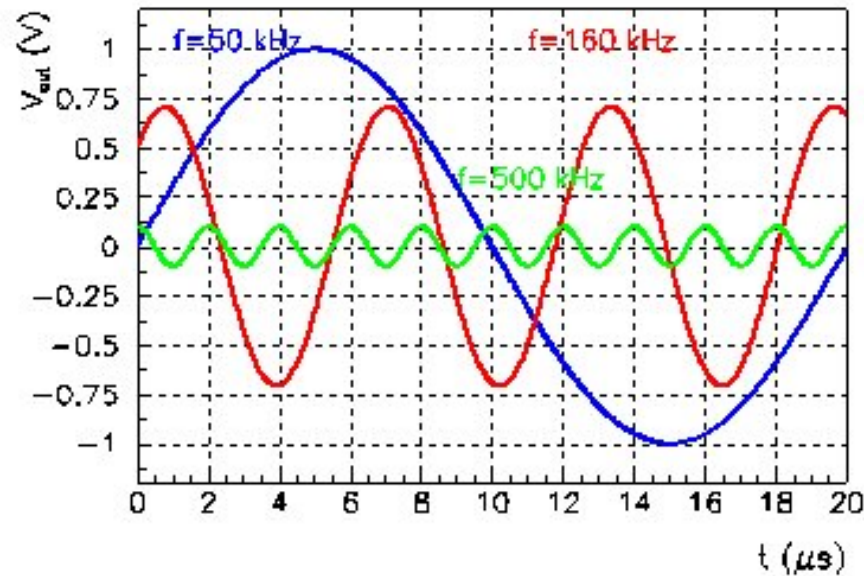
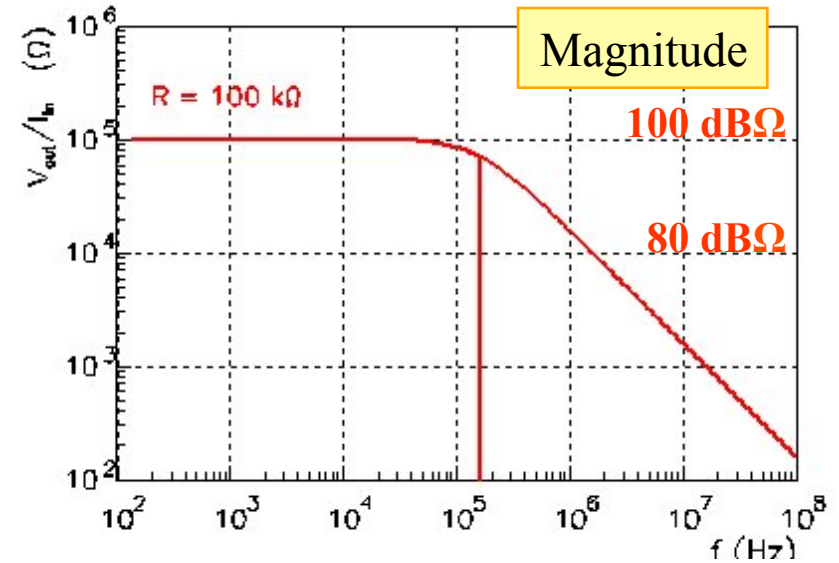
- $X(\omega) = \mathcal{F}\{x(t)\} = \int x(t) \exp(j\omega t) dt$
- a few useful Fourier transforms in appendix

- $H(\omega) = 1 \rightarrow h(t) = \delta(t)$  (impulse)
- $H(\omega) = 1/j\omega \rightarrow h(t) = S(t)$  (step)
- $H(\omega) = 1/j\omega (1+j\omega T) \rightarrow h(t) = 1 - \exp(-t/T)$
- $H(\omega) = 1/(1+j\omega T) \rightarrow h(t) = \exp(-t/T)$

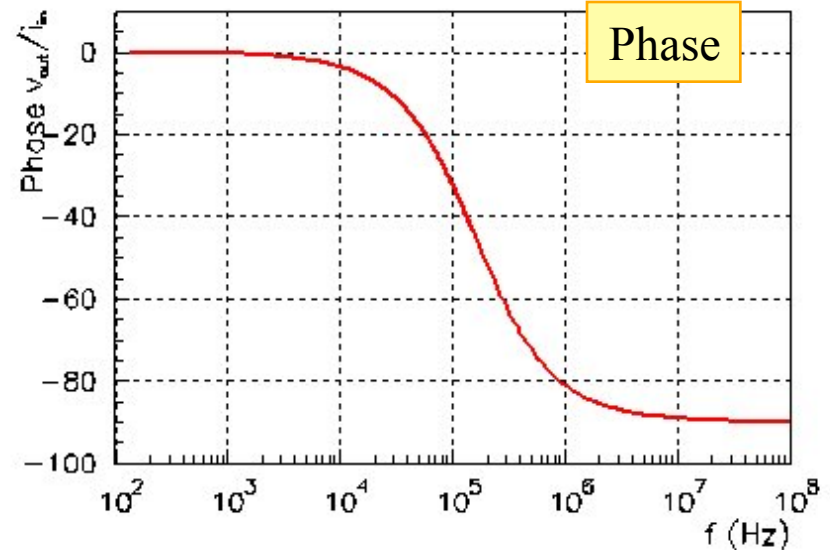
$H(\omega) = 1/(1+j\omega T)^n \rightarrow h(t) = 1/t^n$

## Frequency response

- Bode plot
  - Magnitude (dB) =  $20 \log |H(j\omega)|$
  - -3dB bandwidth :  $f_{-3dB} = 1/2\pi RC$ 
    - $R=10^5\Omega$ ,  $C=10\text{pF} \Rightarrow f_{-3dB}=160 \text{ kHz}$
    - At  $f_{-3dB}$  the signal is attenuated by  $3\text{dB} = \sqrt{2}$ , the phase is  $-45^\circ$



5dB/



## Impulse response

- $h(t) = \mathcal{F}^{-1} \{ R/(1+j\omega RC) \}$   
=  $R/\tau \exp(-t/\tau)$
- $\tau$  ( $\tau$ ) =  $RC = 1 \mu\text{s}$  : time constant

- Step response : rising exponential

- $H(t) = \mathcal{F}^{-1} \{ 1/j\omega R/(1+j\omega RC) \}$   
=  $R [ 1 - \exp(-t/\tau) ]$
- Rise time :  $t_{10-90\%} = 2.2 \tau$
- « eye diagram »

