


















# Quantum Chromodynamics

**HASCO Summer School, Göttingen,  
17 July - 25 July 2023**

**Daniel Reichelt**

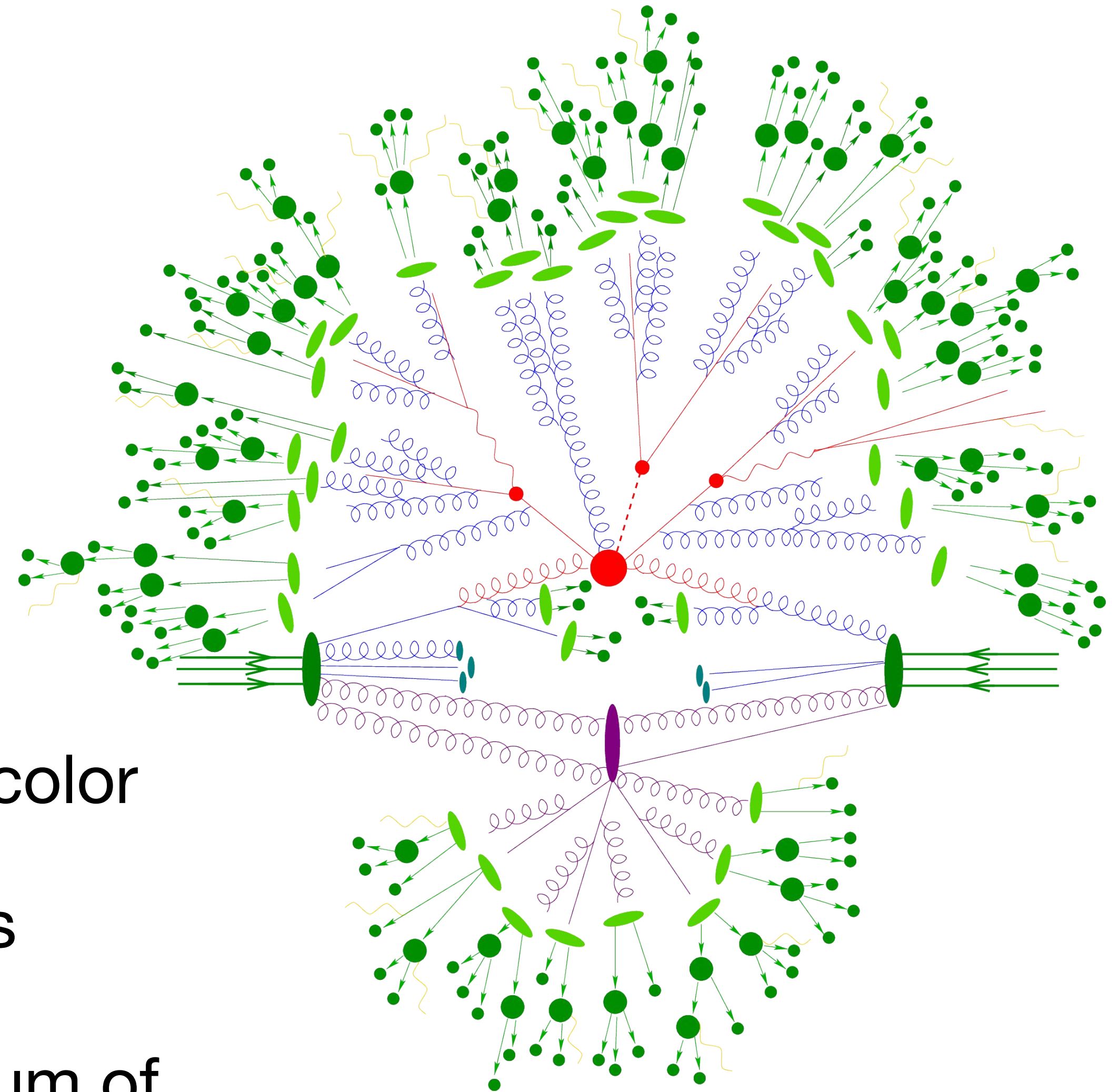
# quantum chromodynamics - overview

- strong interaction part of the standard model
- jet production
- internal structure of hadrons
- ingredients:
  - 3 families of quarks/anti-quarks, come in 3 colours
  - gluon, 8 colour states
  - coupling constant  $\alpha_s \sim 0.1$ , relatively large  $\rightarrow$  “strong” coupling

QUARKS	mass $\rightarrow$ $\approx 2.3 \text{ MeV}/c^2$ charge $\rightarrow$ $2/3$ spin $\rightarrow$ $1/2$  up	mass $\rightarrow$ $\approx 1.275 \text{ GeV}/c^2$ charge $\rightarrow$ $2/3$ spin $\rightarrow$ $1/2$  charm	mass $\rightarrow$ $\approx 173.07 \text{ GeV}/c^2$ charge $\rightarrow$ $2/3$ spin $\rightarrow$ $1/2$  top	mass $\rightarrow$ 0 charge $\rightarrow$ 0 spin $\rightarrow$ 1  gluon	mass $\rightarrow$ $\approx 126 \text{ GeV}/c^2$ charge $\rightarrow$ 0 spin $\rightarrow$ 0  Higgs boson
	mass $\rightarrow$ $\approx 4.8 \text{ MeV}/c^2$ charge $\rightarrow$ $-1/3$ spin $\rightarrow$ $1/2$  down	mass $\rightarrow$ $\approx 95 \text{ MeV}/c^2$ charge $\rightarrow$ $-1/3$ spin $\rightarrow$ $1/2$  strange	mass $\rightarrow$ $\approx 4.18 \text{ GeV}/c^2$ charge $\rightarrow$ $-1/3$ spin $\rightarrow$ $1/2$  bottom	mass $\rightarrow$ 0 charge $\rightarrow$ 0 spin $\rightarrow$ 1  photon	
	mass $\rightarrow$ $0.511 \text{ MeV}/c^2$ charge $\rightarrow$ -1 spin $\rightarrow$ $1/2$  electron	mass $\rightarrow$ $105.7 \text{ MeV}/c^2$ charge $\rightarrow$ -1 spin $\rightarrow$ $1/2$  muon	mass $\rightarrow$ $1.777 \text{ GeV}/c^2$ charge $\rightarrow$ -1 spin $\rightarrow$ $1/2$  tau	mass $\rightarrow$ $91.2 \text{ GeV}/c^2$ charge $\rightarrow$ 0 spin $\rightarrow$ 1  Z boson	GAUGE BOSONS
LEPTONS	mass $\rightarrow$ $< 2.2 \text{ eV}/c^2$ charge $\rightarrow$ 0 spin $\rightarrow$ $1/2$  electron neutrino	mass $\rightarrow$ $< 0.17 \text{ MeV}/c^2$ charge $\rightarrow$ 0 spin $\rightarrow$ $1/2$  muon neutrino	mass $\rightarrow$ $< 15.5 \text{ MeV}/c^2$ charge $\rightarrow$ 0 spin $\rightarrow$ $1/2$  tau neutrino	mass $\rightarrow$ $80.4 \text{ GeV}/c^2$ charge $\rightarrow$ $\pm 1$ spin $\rightarrow$ 1  W boson	

# QCD at colliders

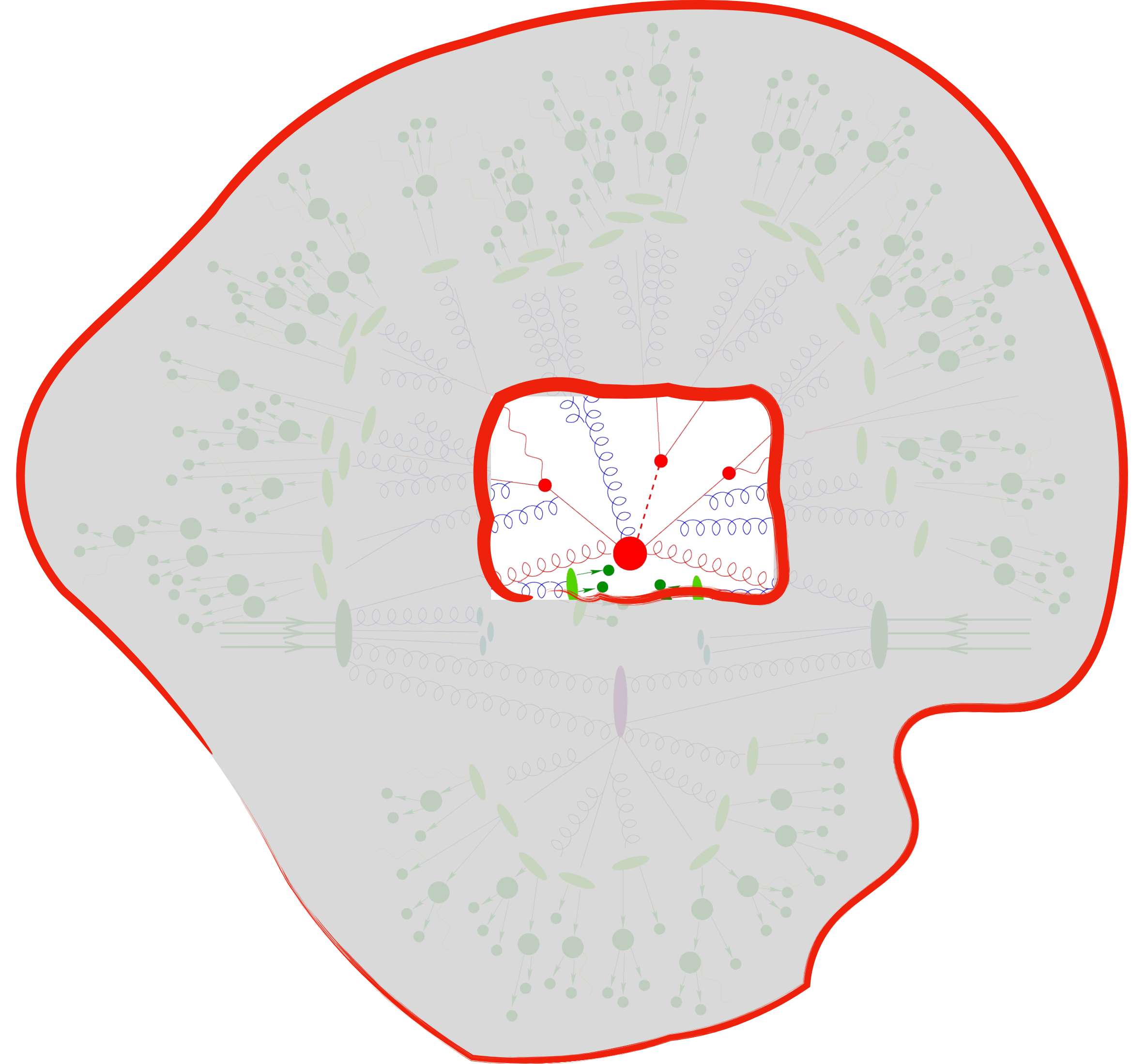
- Events factorised into
  - **Hard Process**
  - QCD radiation
  - PDFs/Beams
  - **Hadrons** ✓
    - parton model: hadrons consist of color neutral combinations of 3 quarks (Baryons) or quark anti-quark pairs (Mesons)
    - lattice QCD confirms mass spectrum of hadrons as property of QCD





# QCD at colliders

- Events factorised into
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# recap of pQCD so far

- running coupling  $\alpha_s(\mu) \rightarrow \infty$  as  $\mu \rightarrow \Lambda_{QCD}$  (confinement) and  $\alpha_s(\mu) \rightarrow 0$  as  $\mu \rightarrow \infty$  (asymptotic freedom)  $\rightarrow$  LHC energies naively within range of perturbative QCD

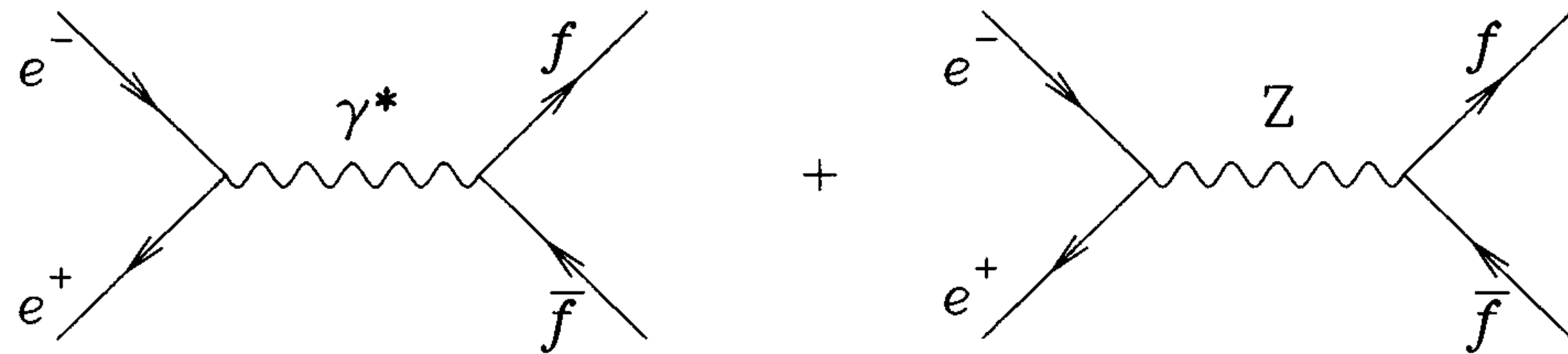
- Feynman rules derived from Lagrangian for  $SU(3)$  gauge theory

$$\mathcal{L}_{QCD} = \bar{\psi}_q^a \left( i\gamma^\mu (D_\mu)_{ab} - \delta_{ab} m_q \right) \psi_q^b - \frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu}$$

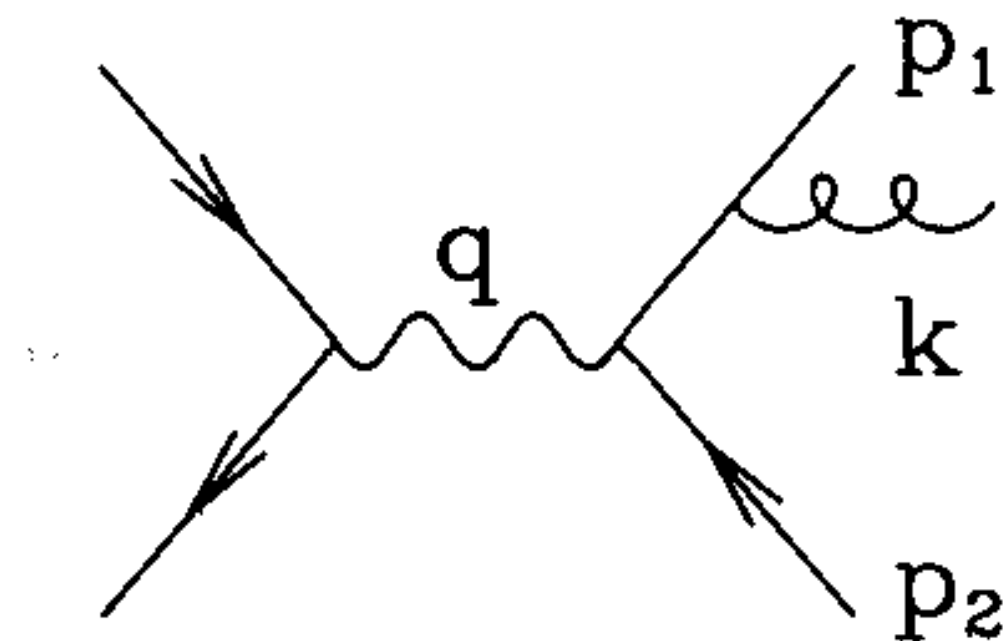
- non-abelian theory  $\rightarrow$  interaction between gluons in addition to gluon-quark vertices

# QCD calculations

- simplest process involving quarks:  $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$
- not actually involving QCD, generic fermion production (but  $N_c$  times)

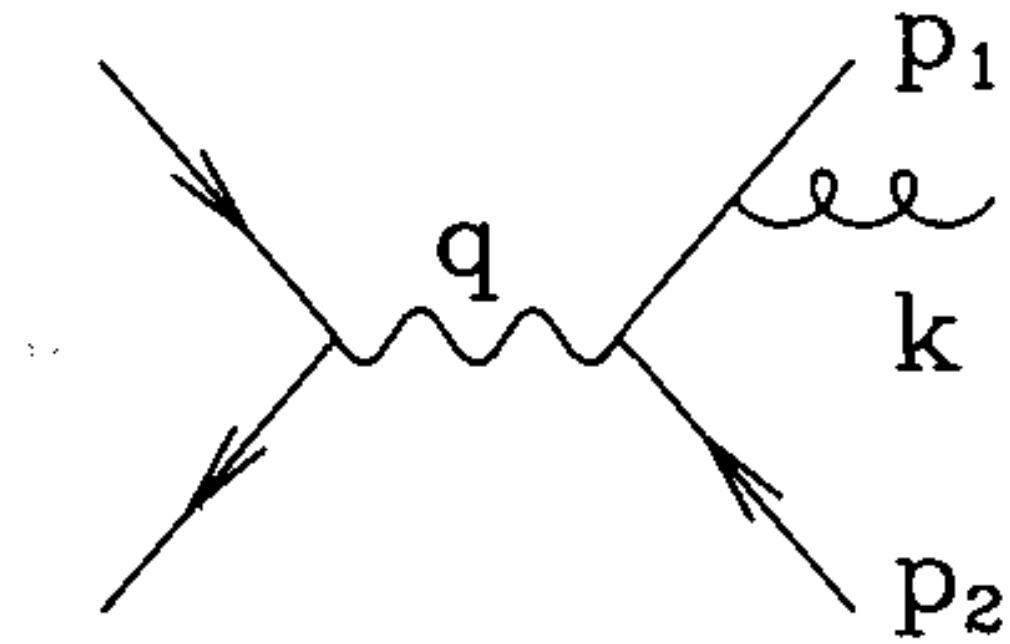


- next higher order (+ gluon attached to other quark + virtual corrections)



$$\sim \bar{u}(p_1)(-ig_s)t^A\gamma^\alpha \frac{i(\not{p}_1 + \not{k})}{(p_1 + k)^2}(-ie)\gamma^\mu v(p_2)\epsilon_\alpha$$

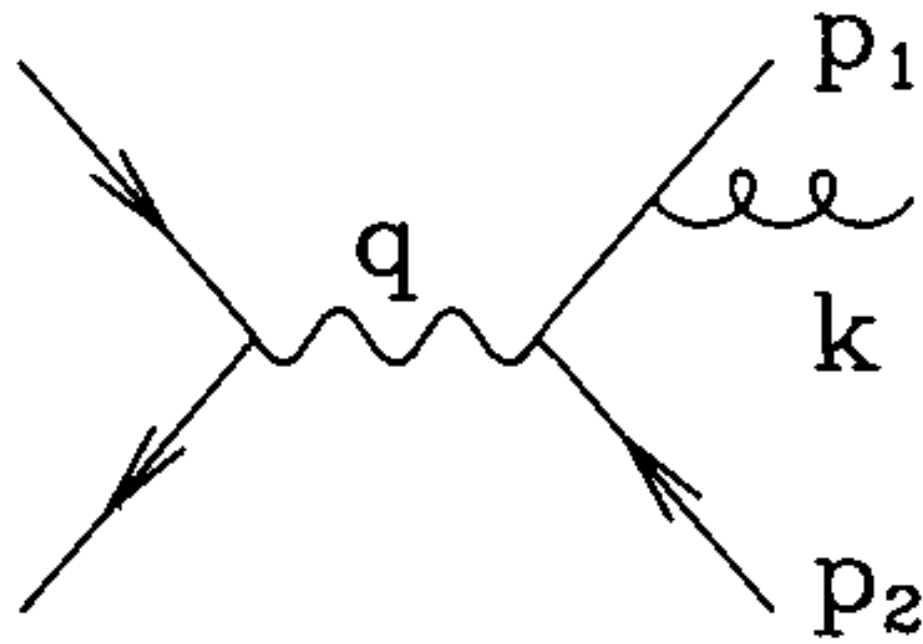
# QCD calculations — soft limit



$$\sim \bar{u}(p_1)(-ig_s)t^A\gamma^\alpha\frac{i(\not{p}_1+\not{k})}{(p_1+k)^2}(-ie)\gamma^\mu v(p_2)\epsilon_\alpha$$



# QCD calculations — soft limit

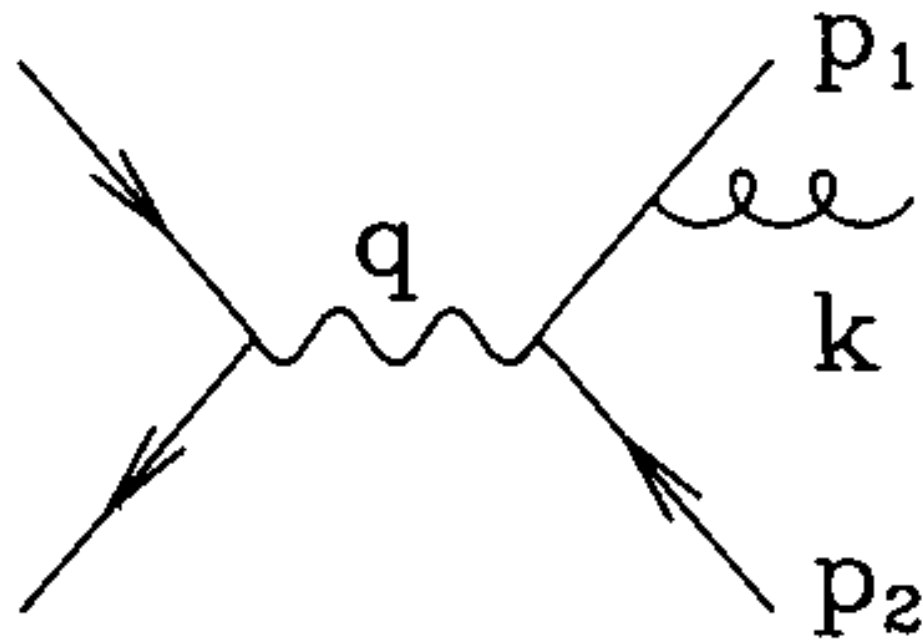


$$\sim \bar{u}(p_1)(-ig_s)t^A\gamma^\alpha\frac{i(\not{p}_1+\not{k})}{(p_1+k)^2}(-ie)\gamma^\mu v(p_2)\epsilon_\alpha$$

$$\sim \bar{u}(p_1)(-ig_s)t^A\not{\epsilon}\frac{i\not{p}_1}{2p_1\cdot k}(-ie)\gamma^\mu v(p_2)$$

assume massless partons,  $p_1^2 = 0$ ,  $k^2 = 0$  and analyse the soft gluon  $k \rightarrow 0$  limit

# QCD calculations — soft limit



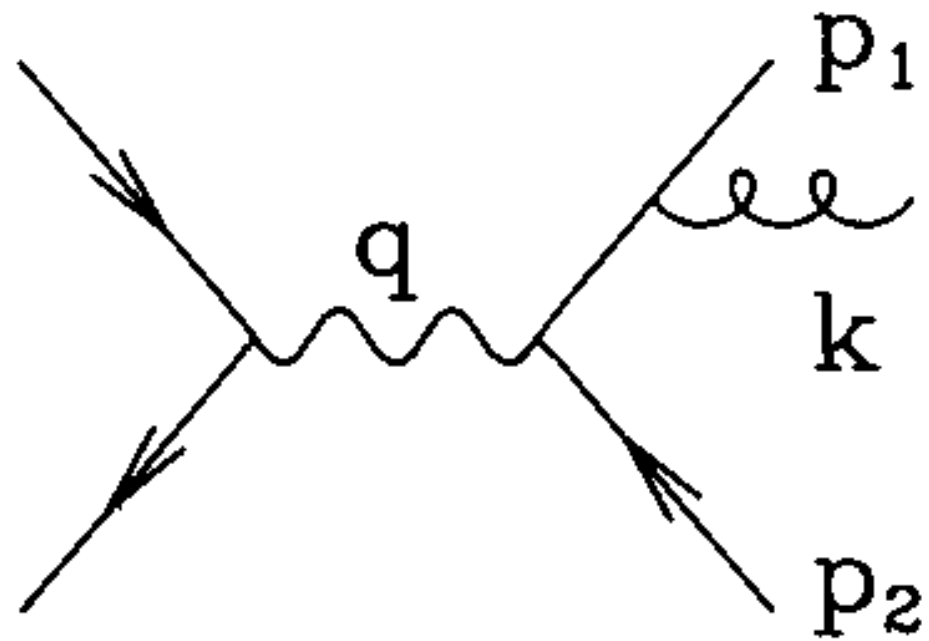
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$$\sim \bar{u}(p_1)(-ig_s)t^A\frac{i p_1 \cdot \epsilon}{p_1 \cdot k}(-ie)\gamma^\mu v(p_2)$$

use  $\not{\epsilon}\not{p}_1 = 2\epsilon \cdot p_1 - \not{p}_1\not{\epsilon}$  and the Dirac equation  $\bar{u}(p_1)\not{p}_1 = 0$

# QCD calculations — soft limit



$$\sim \bar{u}(p_1)(-ig_s)t^A\gamma^\alpha\frac{i(\not{p}_1+\not{k})}{(p_1+k)^2}(-ie)\gamma^\mu v(p_2)\epsilon_\alpha$$

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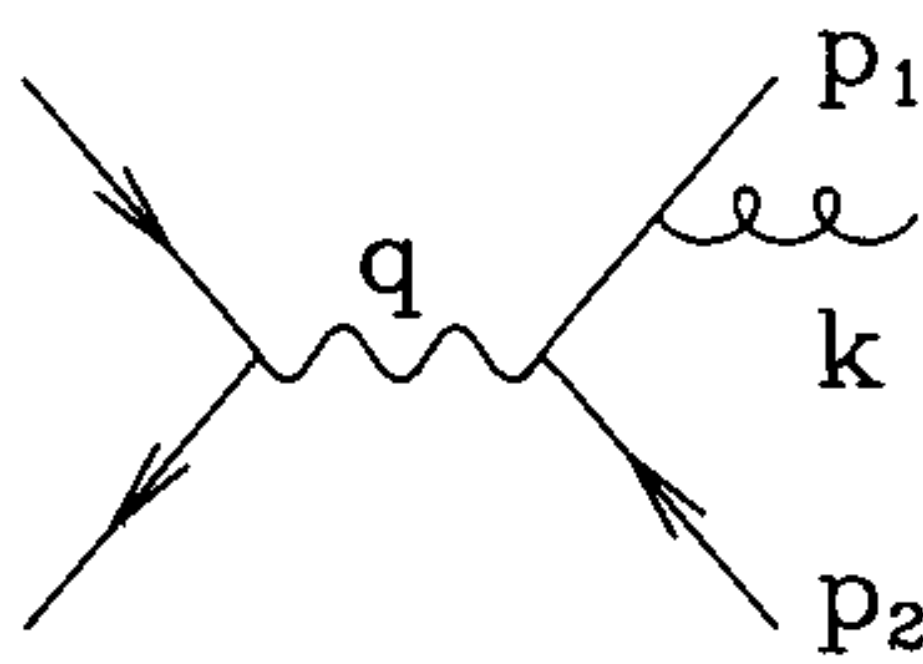
$$\sim \bar{u}(p_1)(-ig_s)t^A\frac{i p_1\cdot\epsilon}{p_1\cdot k}(-ie)\gamma^\mu v(p_2)$$

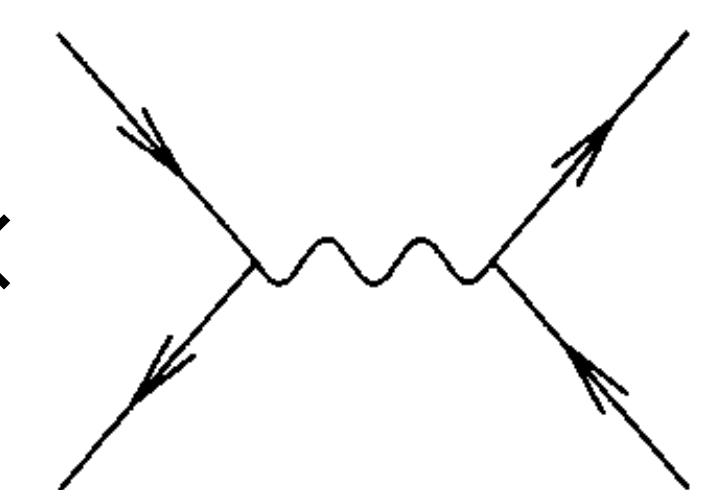
$$\sim g_s t^A \frac{p_1\cdot\epsilon}{p_1\cdot k} \times$$

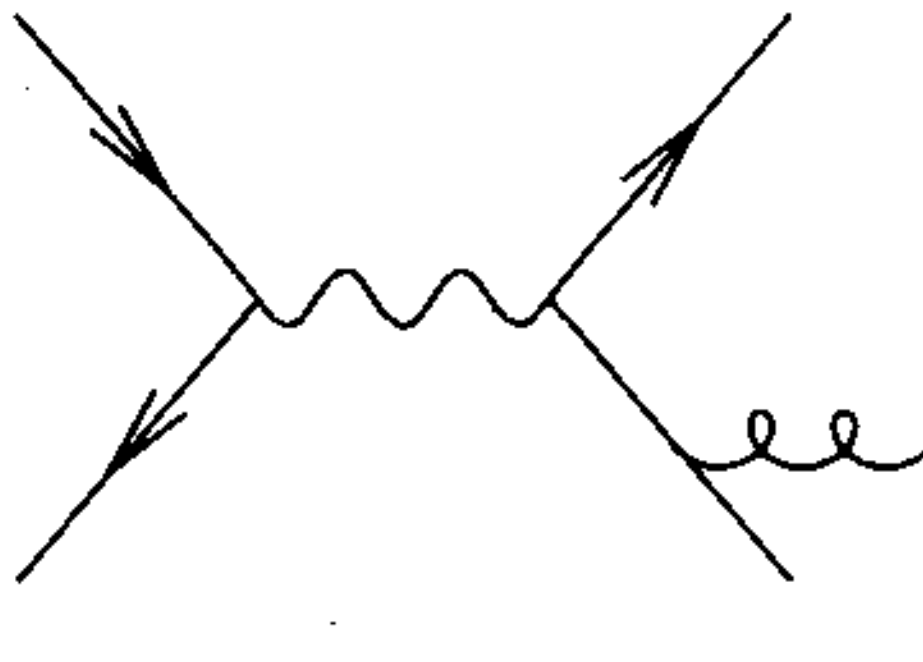
soft gluon emissions factorise!

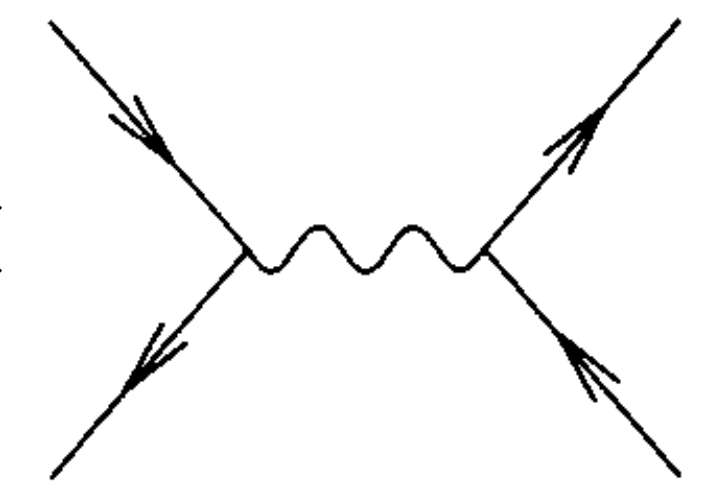


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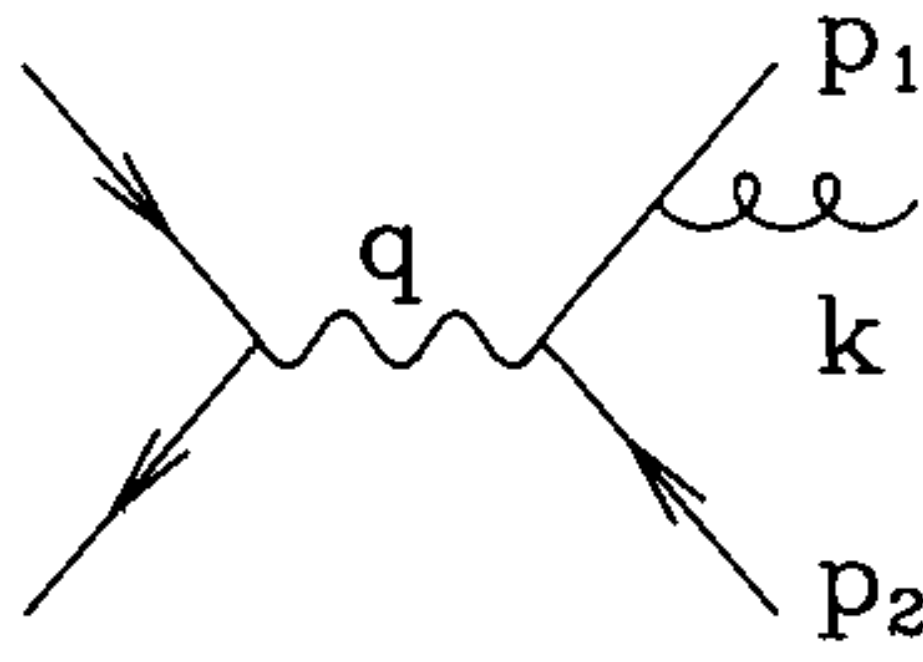


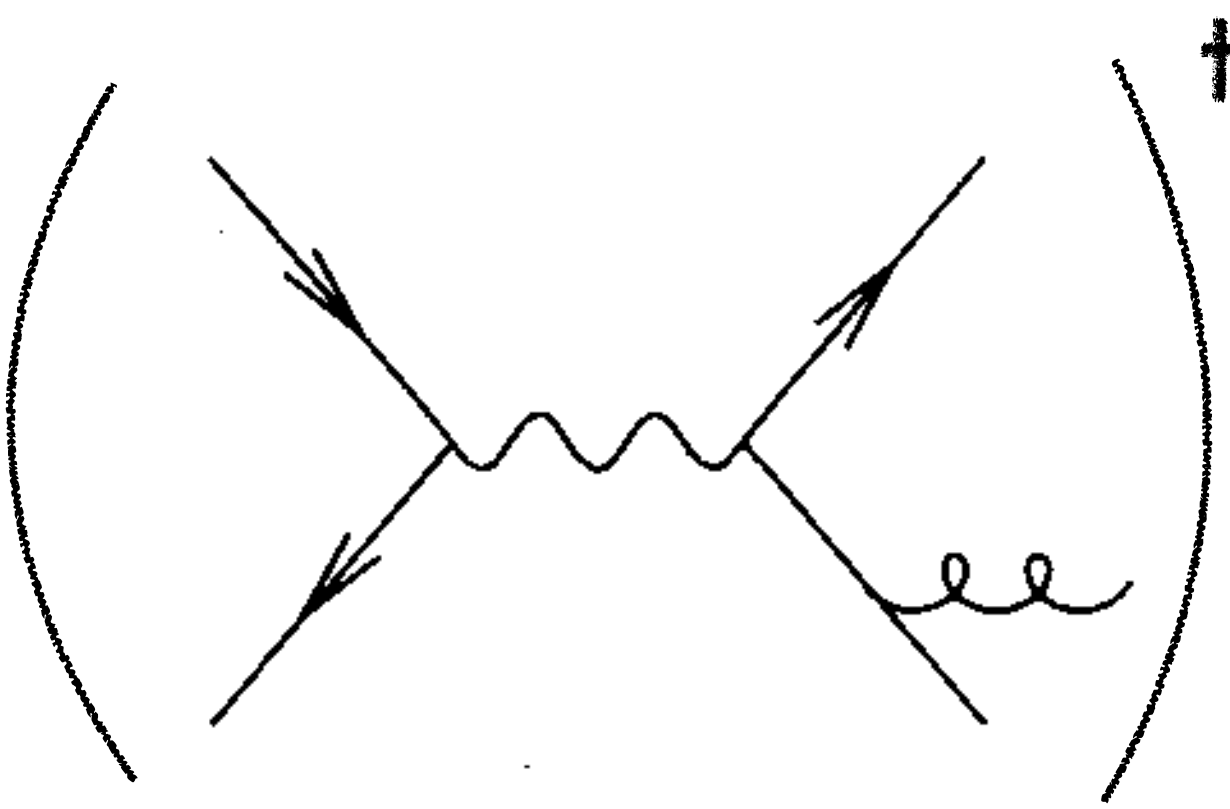
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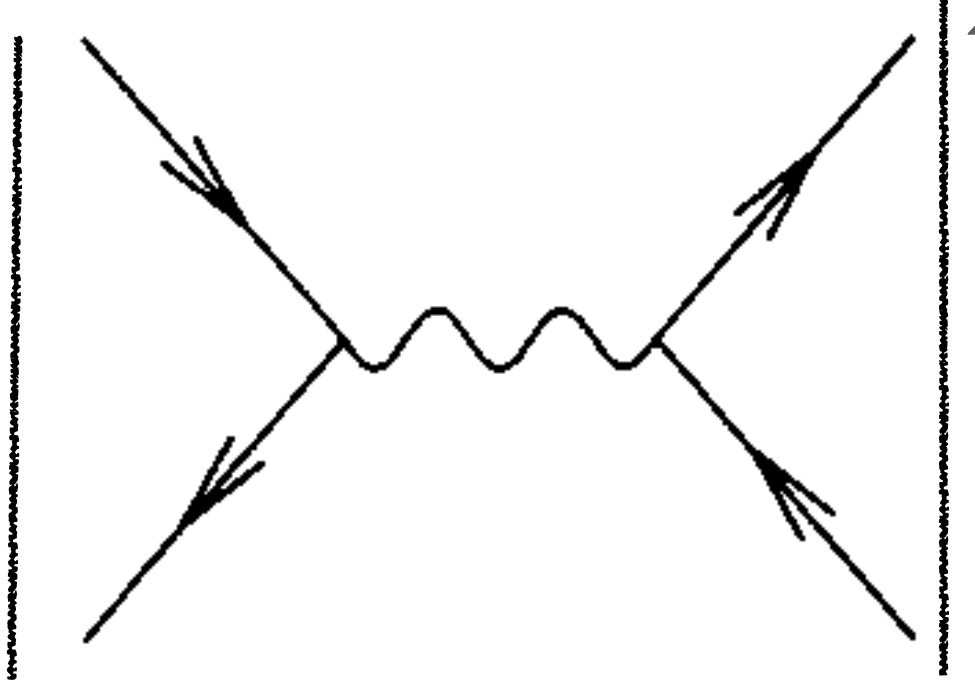


$$\sim g_s t^A \frac{p_2 \cdot \epsilon}{p_2 \cdot k} \times$$


2





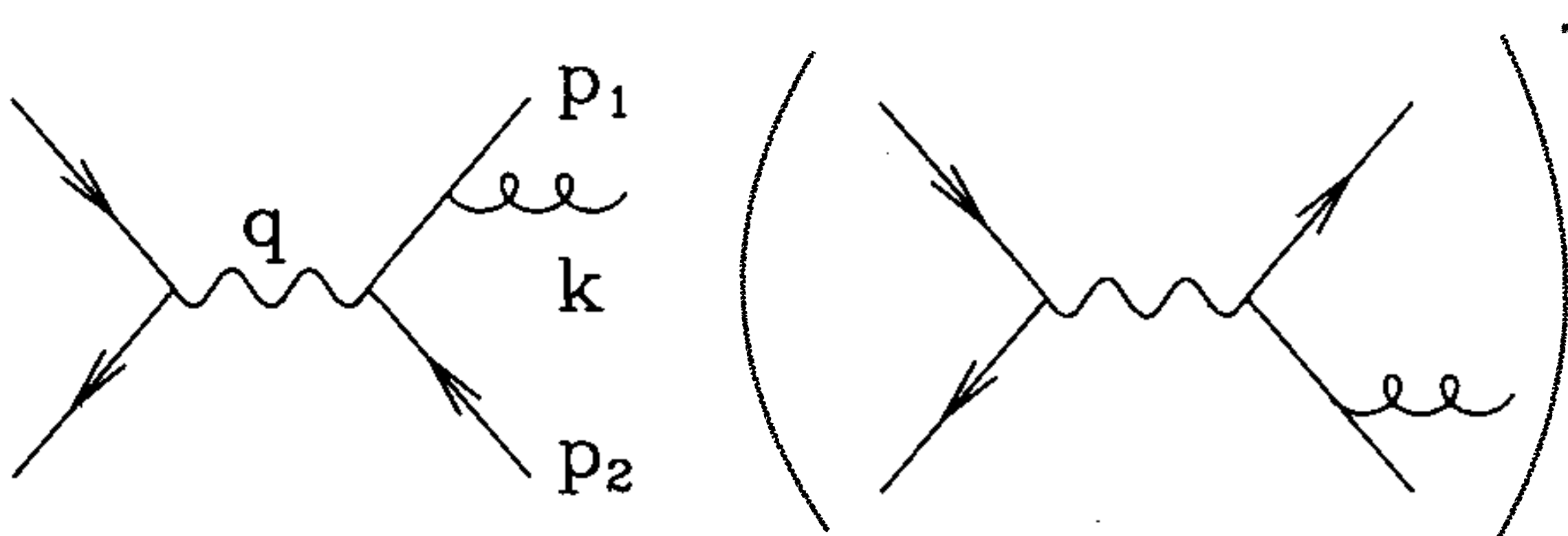
$$\sim 2 g_s^2 t^A t^B \frac{p_1 \cdot \epsilon}{p_1 \cdot k} \frac{p_2 \cdot \epsilon}{p_2 \cdot k}$$


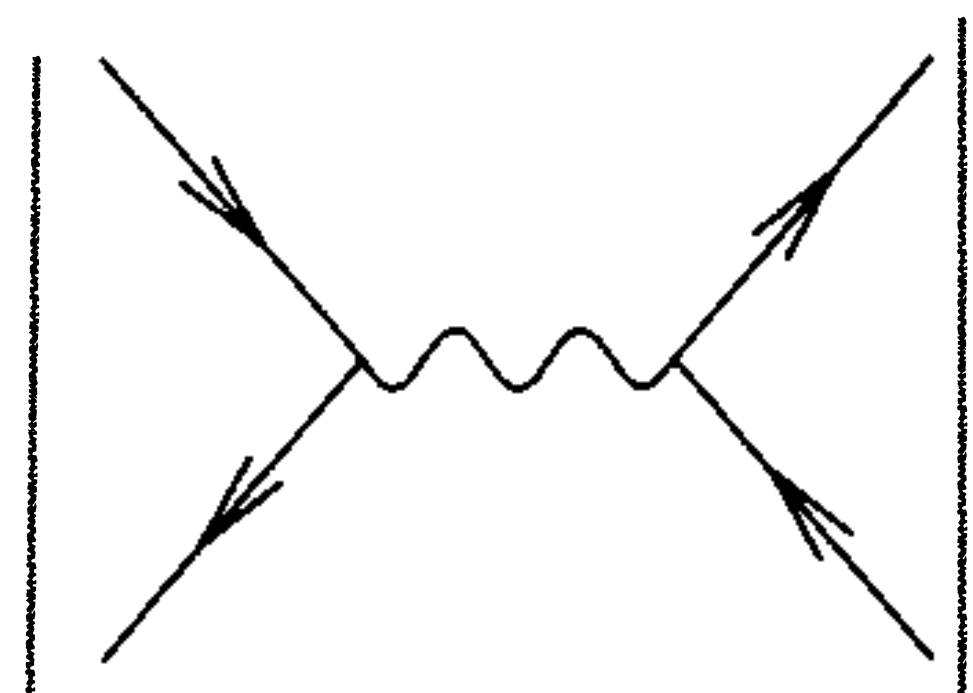
soft gluon emissions factorise!

11

# QCD calculations — soft limit

2

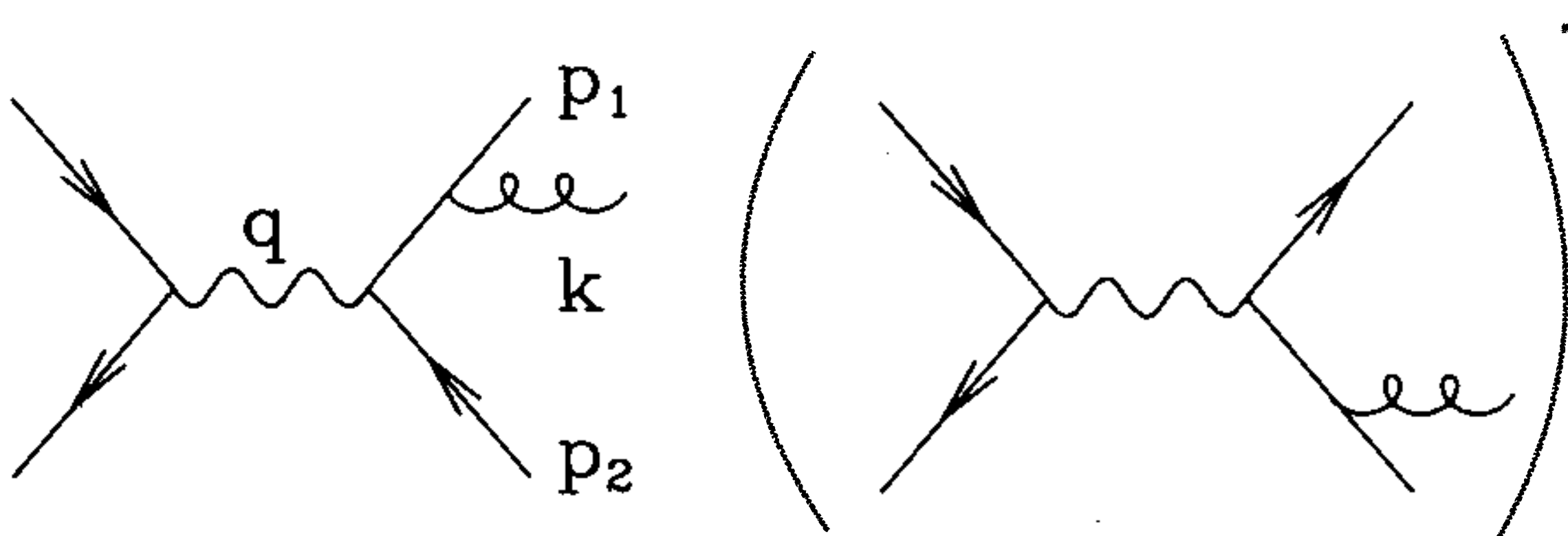


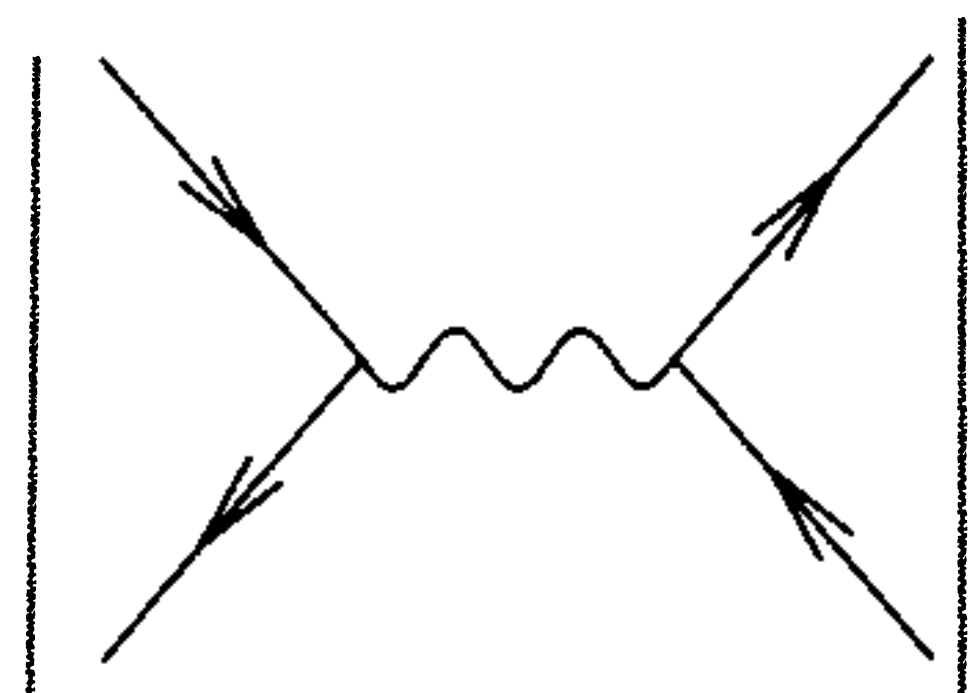
$$\sim 2g_s^2 t^A t^B \frac{p_1 \cdot \epsilon}{p_1 \cdot k} \frac{p_2 \cdot \epsilon}{p_2 \cdot k}$$


2

# QCD calculations — soft limit

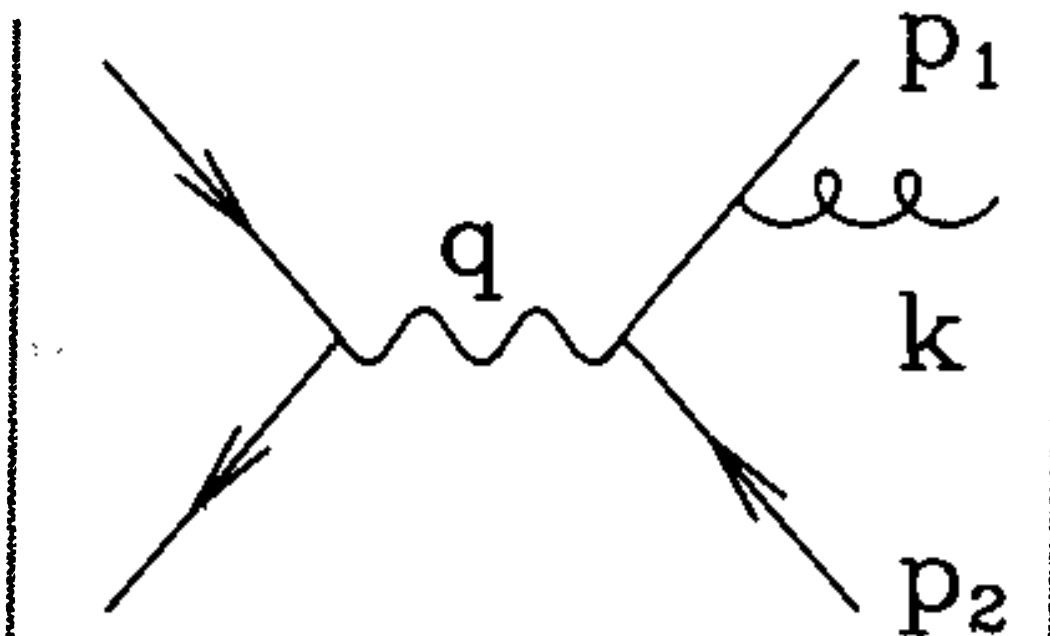
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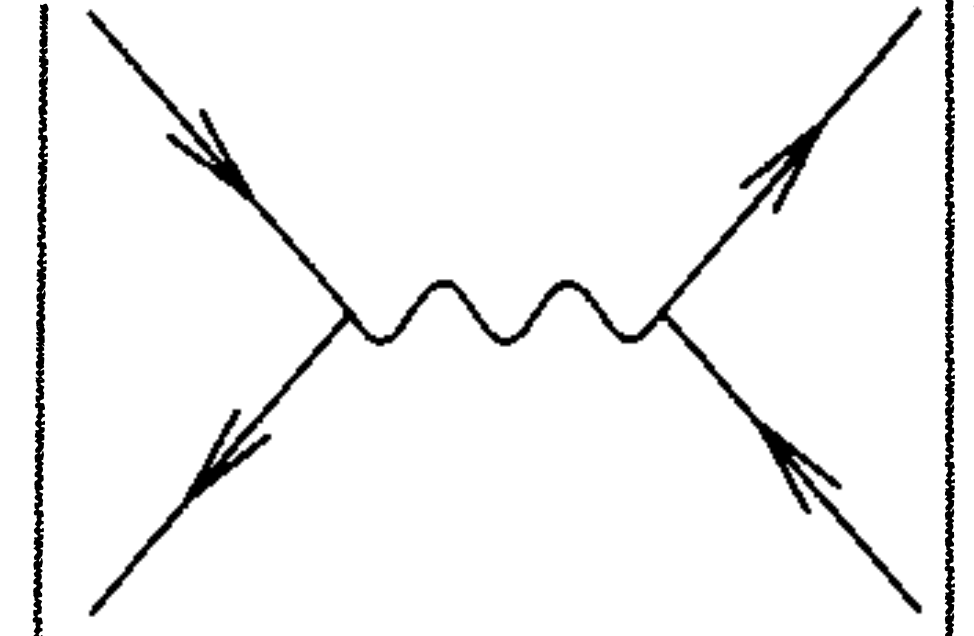


$$\sim 2g_s^2 t^A t^B \frac{p_1 \cdot \epsilon}{p_1 \cdot k} \frac{p_2 \cdot \epsilon}{p_2 \cdot k}$$


2

perform sum over gluon polarisations  $\epsilon_\mu \epsilon_\nu \rightarrow -g_{\mu\nu}$ , and colours  $t^A t^B \rightarrow C_F$



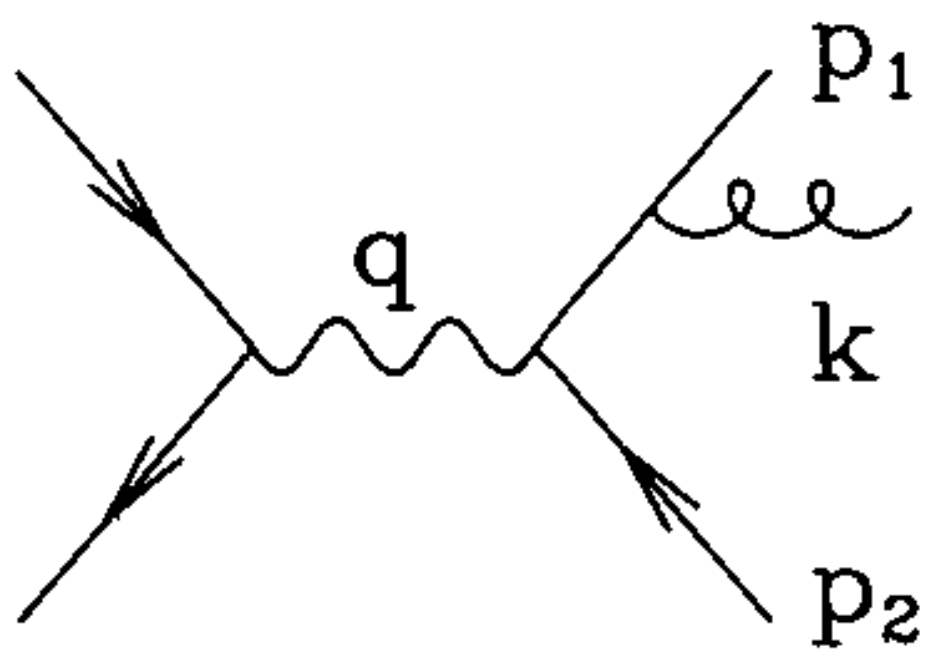
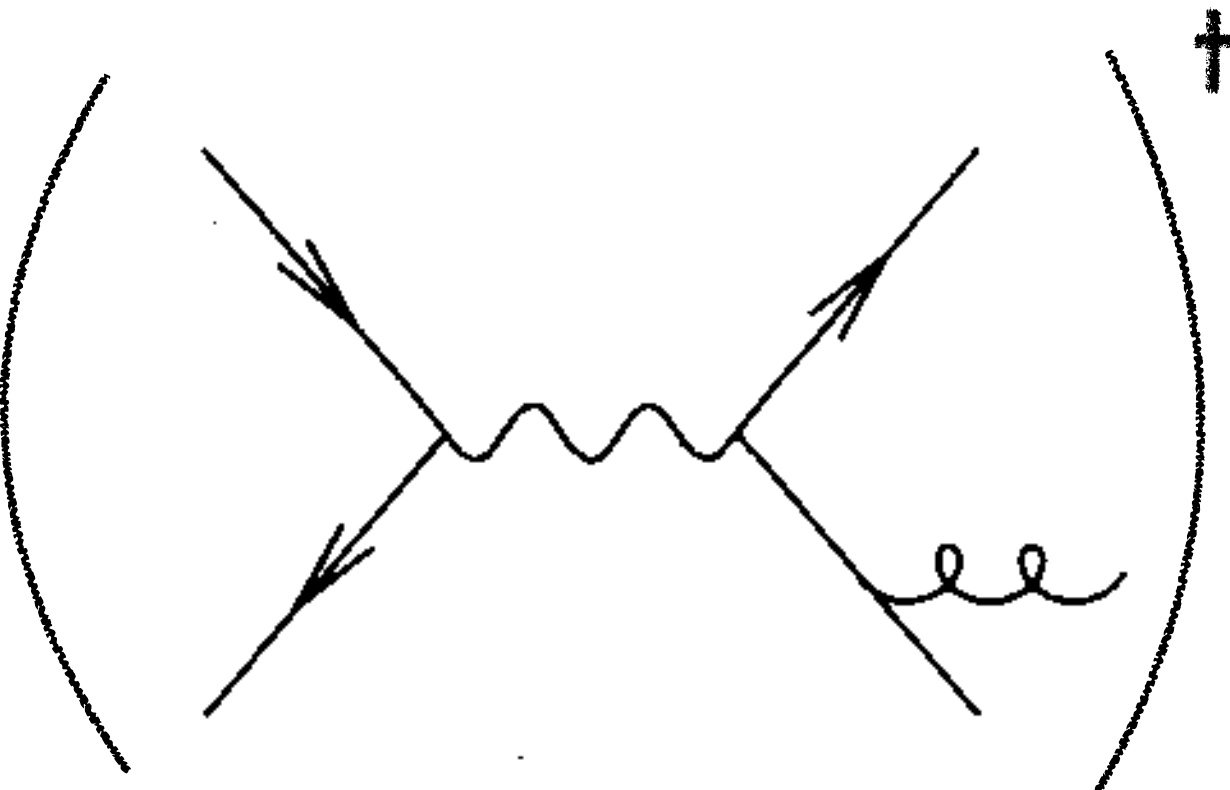
$$\sim g_s^2 C_F \frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}$$


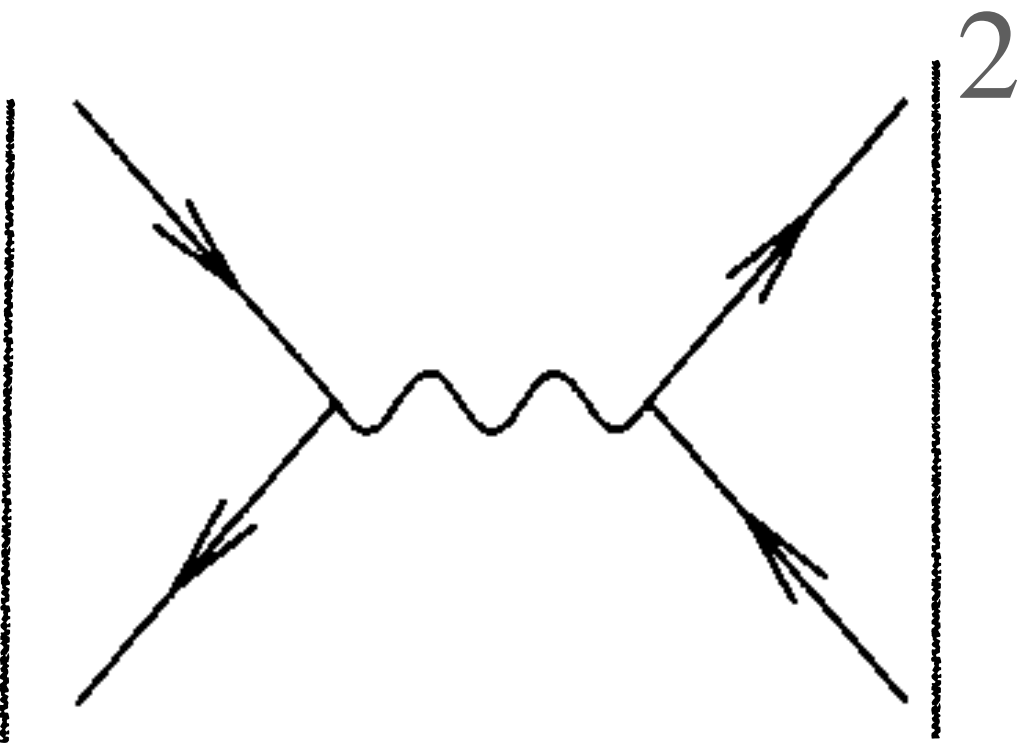
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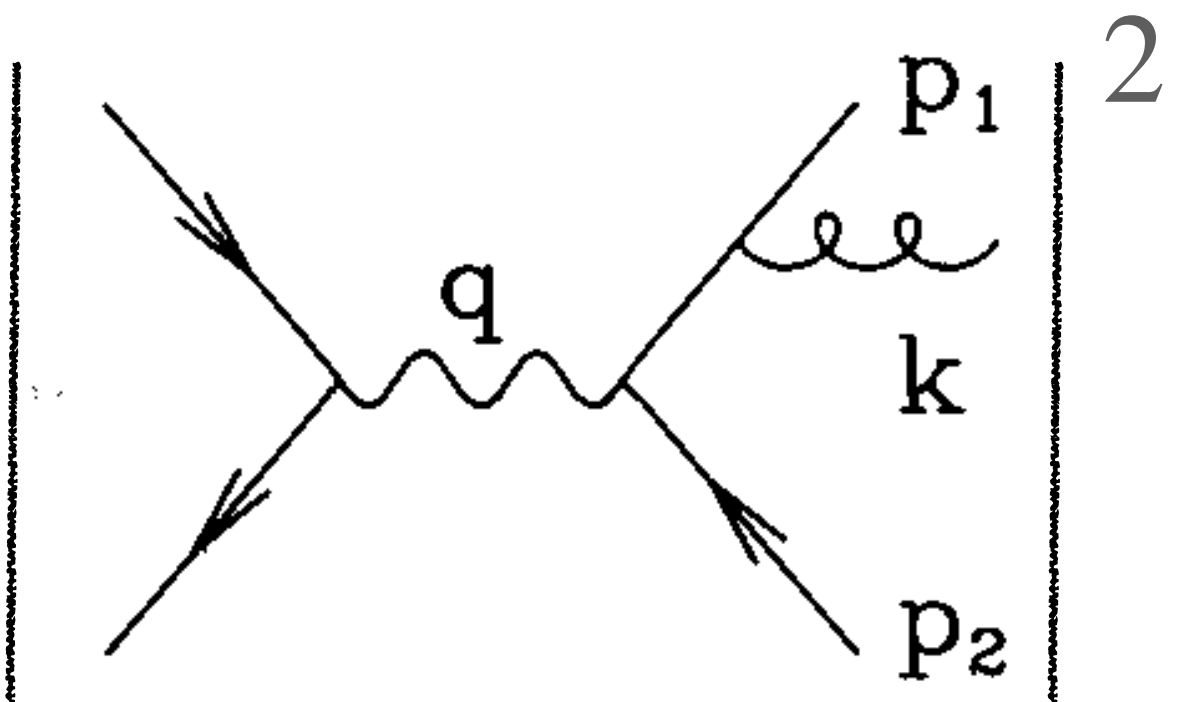
# QCD calculations — soft limit

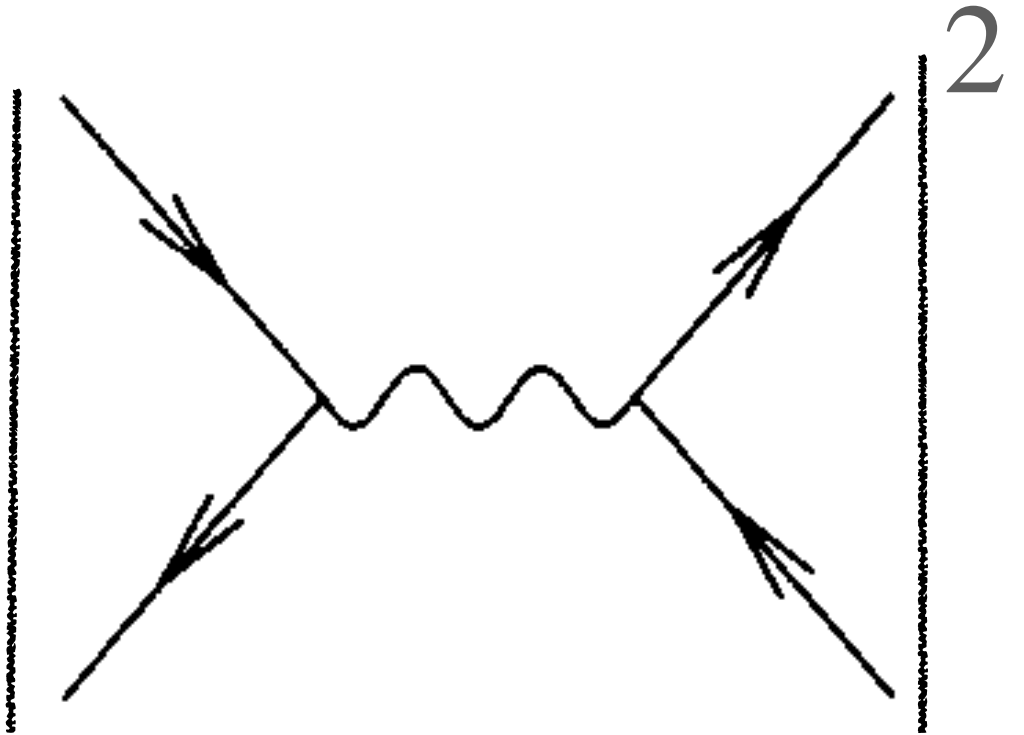
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$$\sim 2g_s^2 t^A t^B \frac{p_1 \cdot \epsilon}{p_1 \cdot k} \frac{p_2 \cdot \epsilon}{p_2 \cdot k}$$


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$$\sim g_s^2 C_F \frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}$$


Note: phase space factorises as well

$$d\phi_{q\bar{q}g} = d\phi_{q\bar{q}} d\phi_{+1}$$

Factorisation with “eikonal” factor!

# eikonal

$$\frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)}$$

observation: divergent if  $k \parallel p_1$  or  $k \parallel p_2$  or  $k \rightarrow 0$

$\Rightarrow$  collinear and soft/infrared limits

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Explicitly in some reference frame, use  $p_i \cdot k = E_i E_k (1 - \cos \theta_{ik})$

$$\frac{2p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} \sim \frac{1}{E_k^2} \frac{1}{(1 - \cos \theta_{1k})(1 - \cos \theta_{2k})}$$

$\Rightarrow$  divergencies visible for  $\theta_{ik} \rightarrow 0$  (collinear) and  $E_k \rightarrow 0$  (soft)



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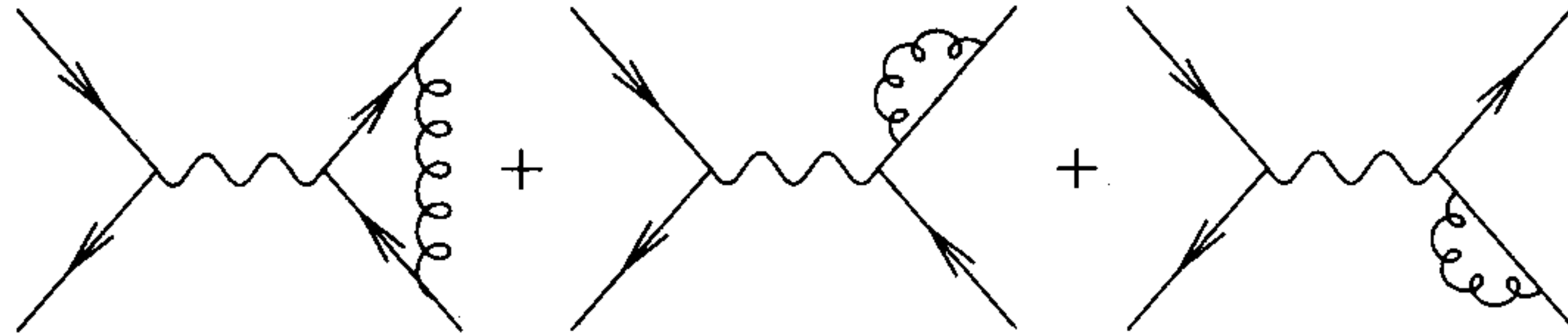
General structure, divergencies and factorisation in the soft and collinear limits is a universal property of QCD amplitudes!

# divergencies ... ?

- If amplitudes in QCD are divergent in the infrared, how can we ever calculate meaningful results?

# divergencies ... ?

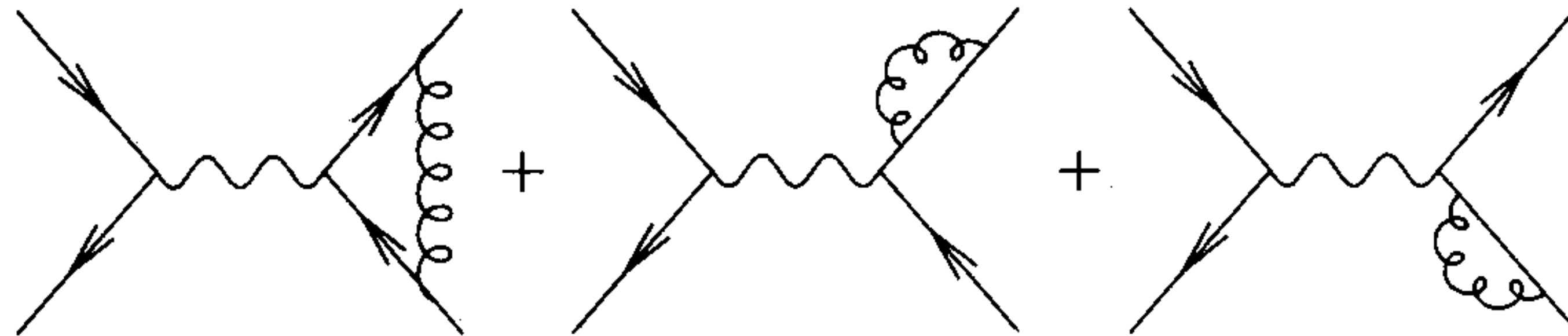
- If amplitudes in QCD are divergent in the infrared, how can we ever calculate meaningful results?
- Answer: we did not yet consider the full  $\mathcal{O}(\alpha_s)$  correction, also have to take into account **virtual terms**



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- Answer: we did not yet consider the full  $\mathcal{O}(\alpha_s)$  correction, also have to take into account **virtual terms**



- on their own divergent as well  $\Rightarrow$  sum turns out to be finite!
- $\Rightarrow$  we can calculate at least inclusive (enough) cross sections (e.g.  $e^+e^- \rightarrow$  hadrons, but not  $e^+e^- \rightarrow$  exactly 2 quarks)

# What is “inclusive enough”?

- We have seen that not all observables are well defined in QCD, since we must not disturb the cancellation of real and virtual singularities
- We must exclude anything that is sensitive to arbitrarily soft and/or collinear emissions
  - typical example: multiplicities
- Observables that are **not** affected by a soft/collinear emission are called **infrared-collinear (IRC) safe**



# IRC safe observables: Jets

- A typical example for the construction of IRC safe quantities are sequential recombination algorithms used to define jets
  1. compute distance measure  $d_{ij}$  for each pair of final-state particles and the beam distance  $d_{iB}$  for each particle
  2. determine minimum of all  $d_{ij}$ ,  $d_{iB}$ 
    - A. if one of the  $d_{ij}$  is smallest, combine those particles  $i, j$
    - B. if one of the beam distances  $d_{iB}$  is smallest,  $i$  is a jet and removed from the procedure
  3. go back to 1, repeat until all objects are clustered

# IRC safe observables: Jets

- distance measures are a matter of choice
  - only formal requirement is IRC safety, i.e. a soft/collinear emission should not change the jets obtained from the algorithm

$k_T$  - algorithm/Durham - algorithm:

$$d_{ij} = \min \left( k_{T,i}^2, k_{T,j}^2 \right) \frac{\Delta R^2}{R^2}, \quad d_{iB} = \min \left( k_{T,i}^2, k_{T,j}^2 \right)$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

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anti- $k_T$  - algorithm:

$$d_{ij} = \min \left( k_{T,i}^{-2}, k_{T,j}^{-2} \right) \frac{\Delta R^2}{R^2}, \quad d_{iB} = \min \left( k_{T,i}^{-2}, k_{T,j}^{-2} \right)$$

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# IRC safe observables: Jets

- distance measures are a matter of choice
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Cambridge/Aachen - algorithm:

$$d_{ij} = \frac{\Delta R^2}{R^2}, \quad d_{iB} = 1$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

# IRC safe observables: Jets

- Standard reference for implementations: FastJet program
- Generalised  $k_t$  - algorithm

$$d_{ij} = \min \left( k_{T,i}^{2p}, k_{T,j}^{2p} \right) \frac{\Delta R^2}{R^2}, \quad d_{iB} = \min \left( k_{T,i}^2, k_{T,j}^2 \right)$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

Durham

$$p = 1$$

closest match to  
structure of QCD  
matrix elements,  
theoretical interest

Cambridge/Aachen

$$p = 0$$

angular ordered  
splitting sequence,  
close match to QCD  
coherence

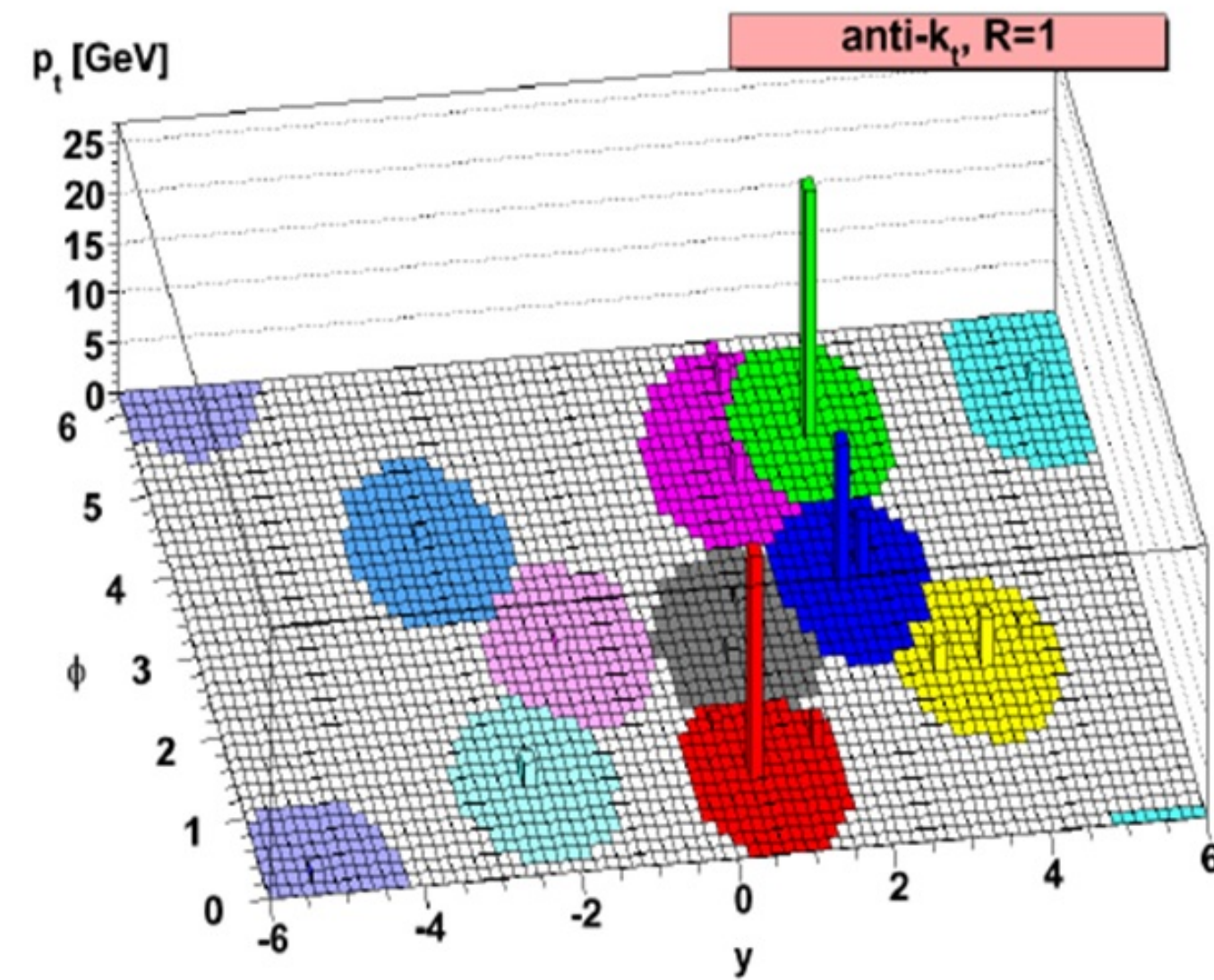
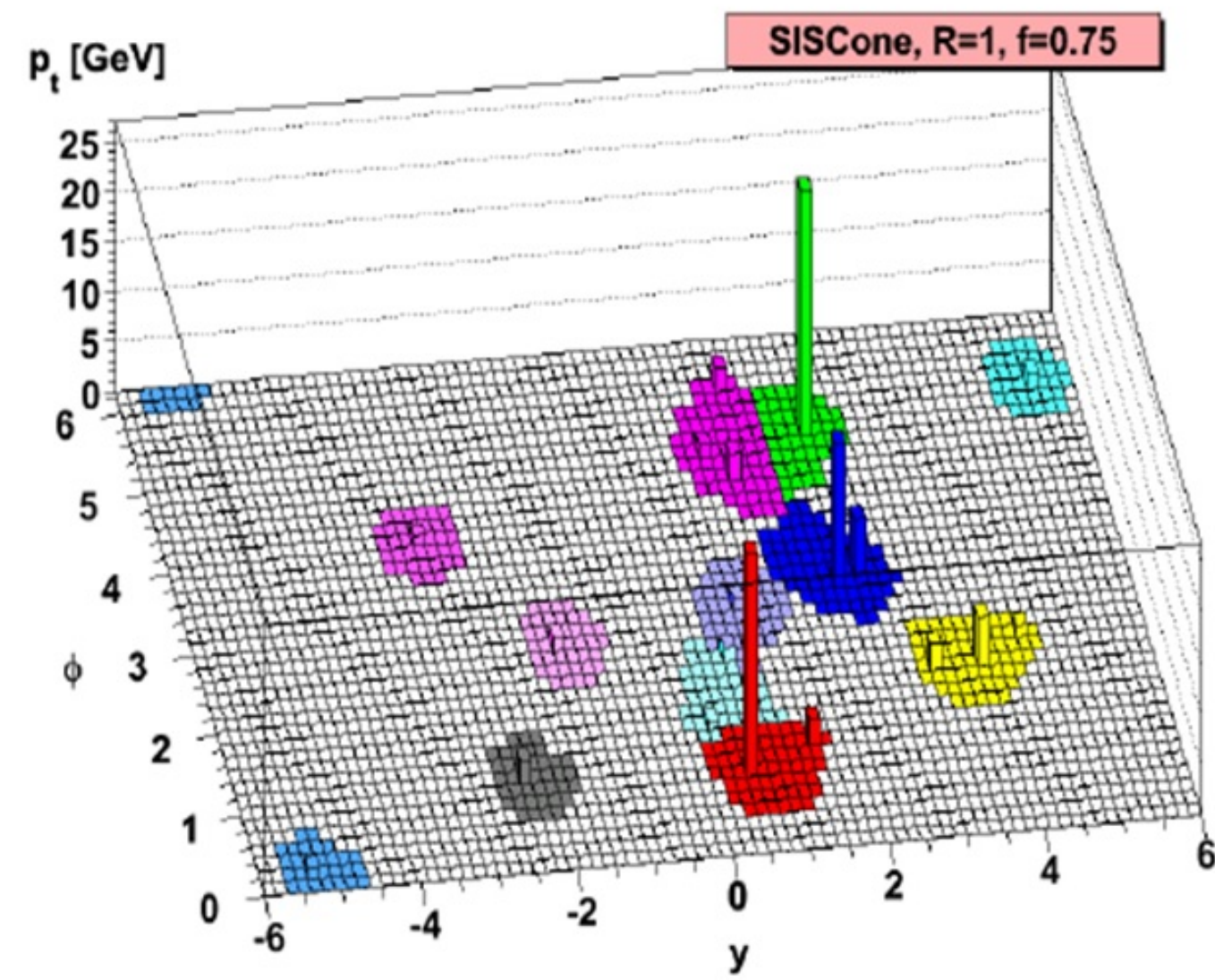
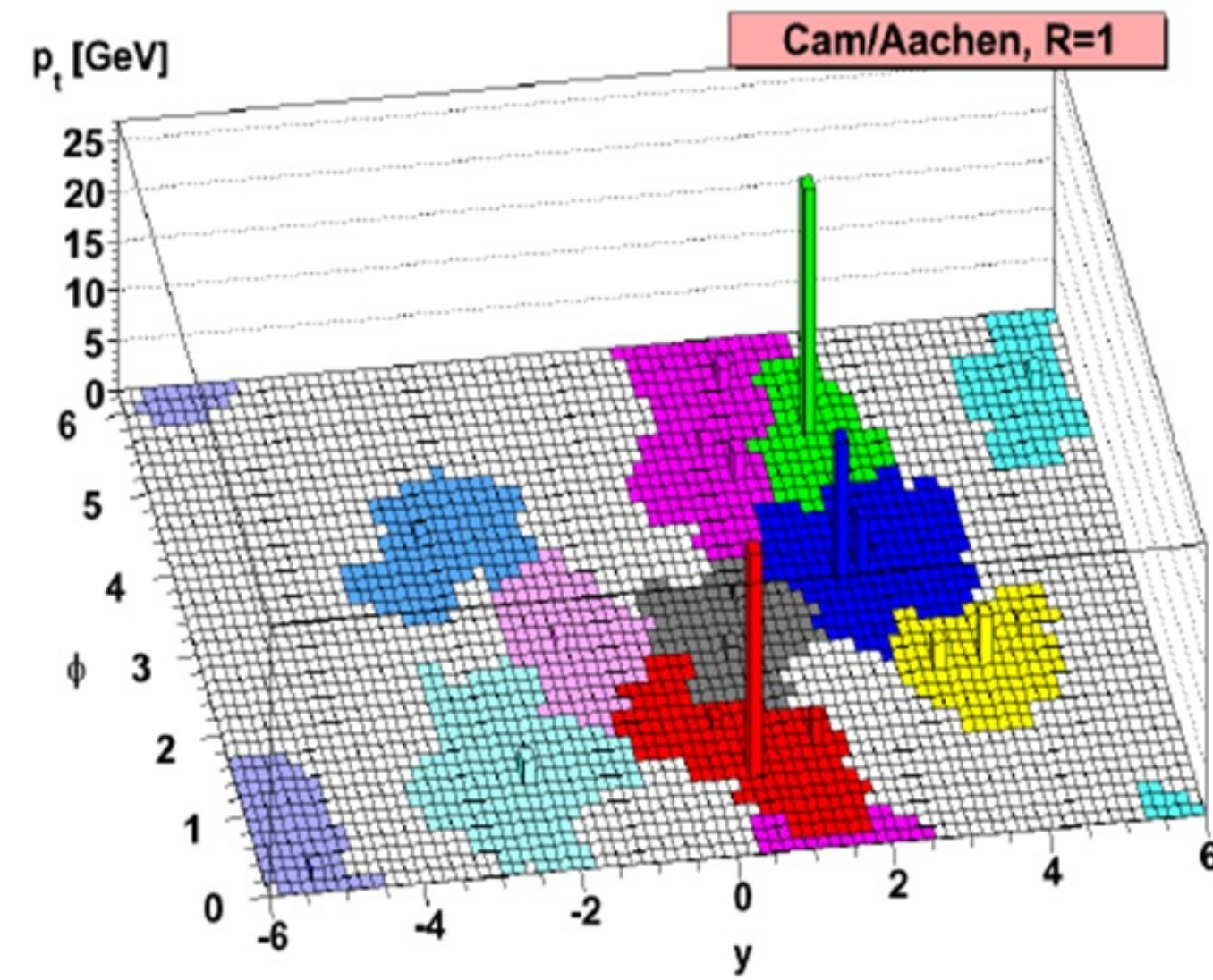
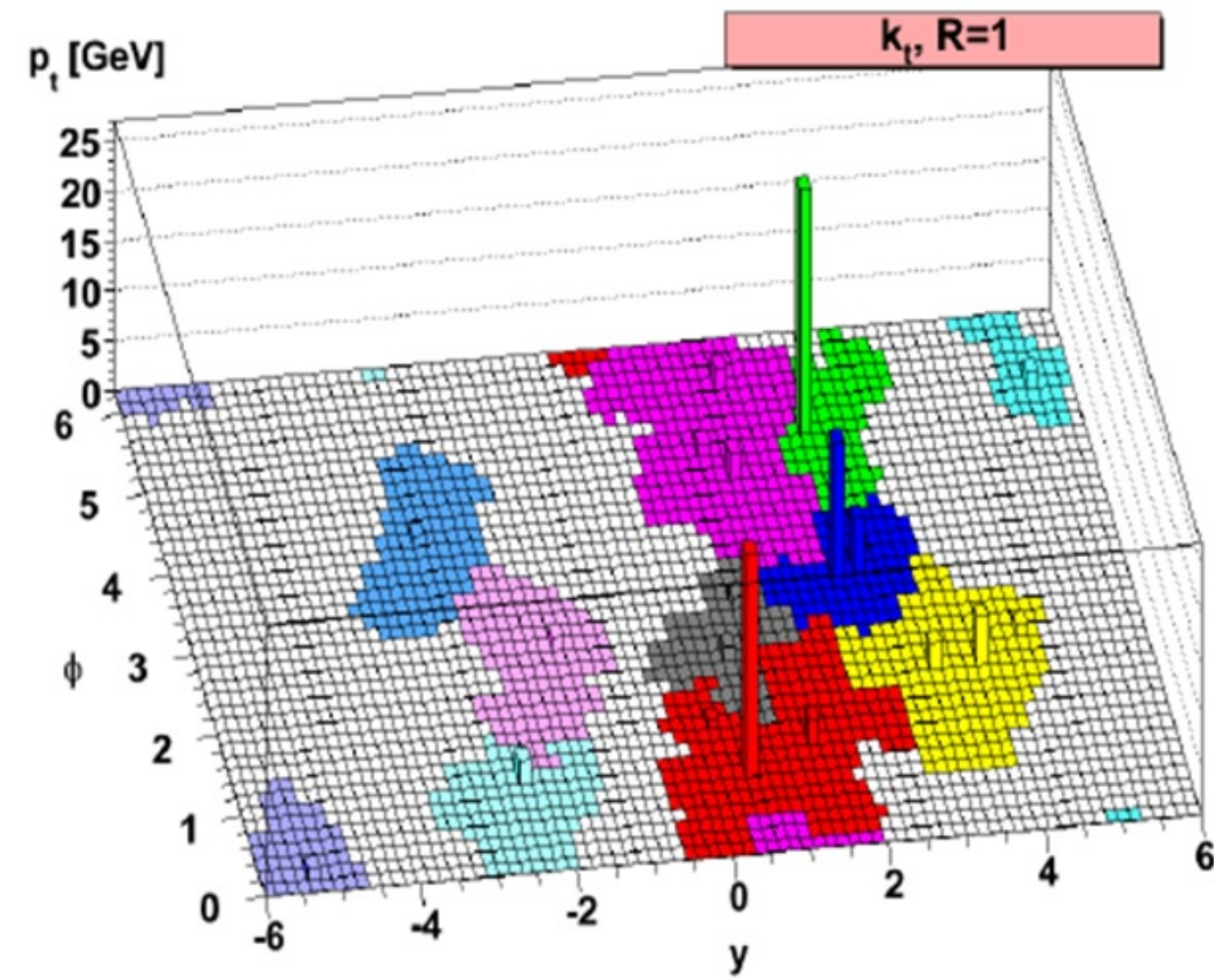
anti- $k_t$

$$p = -1$$

closest to defining jets as  
“cones” with radius  $R$  around  
hard particles, default choice  
in LHC experiments



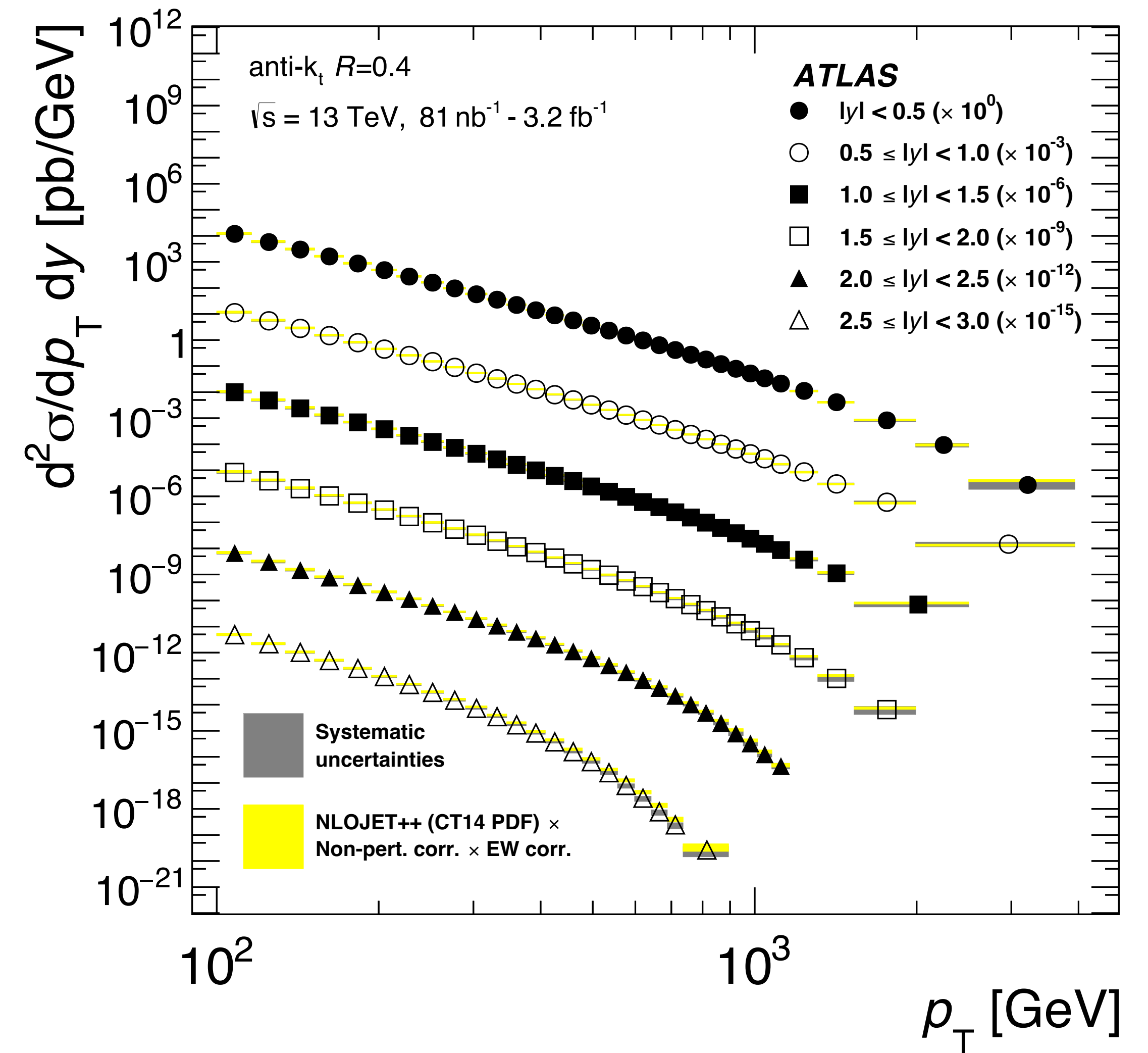
# jet algorithms





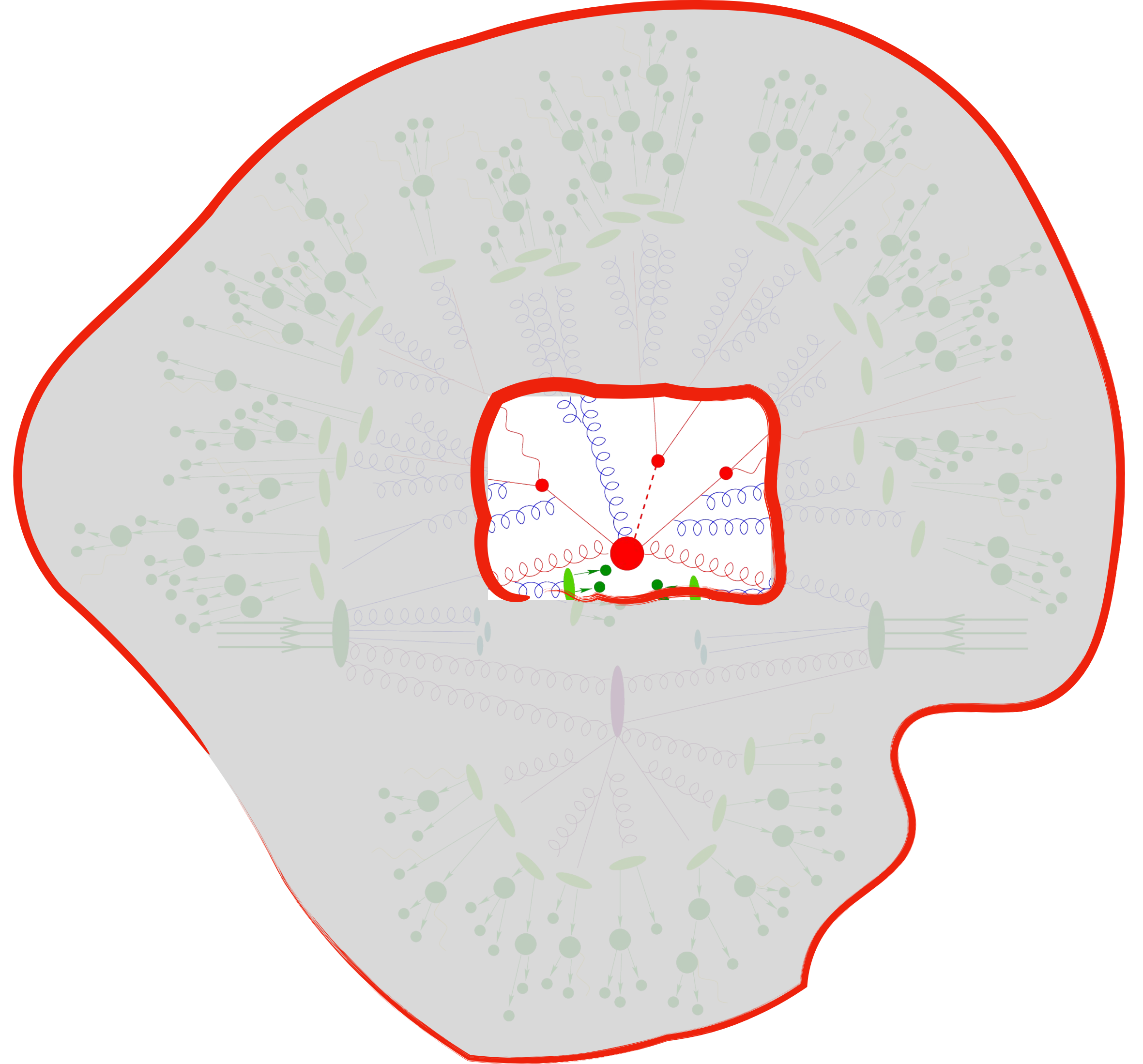
# Jet cross sections at the LHC

- jets are infrared safe, so we can compute their production cross section in perturbation theory
- powerful tests of perturbative QCD
- here: compared to measurement by ATLAS at  $\sqrt{s} = 13$  TeV



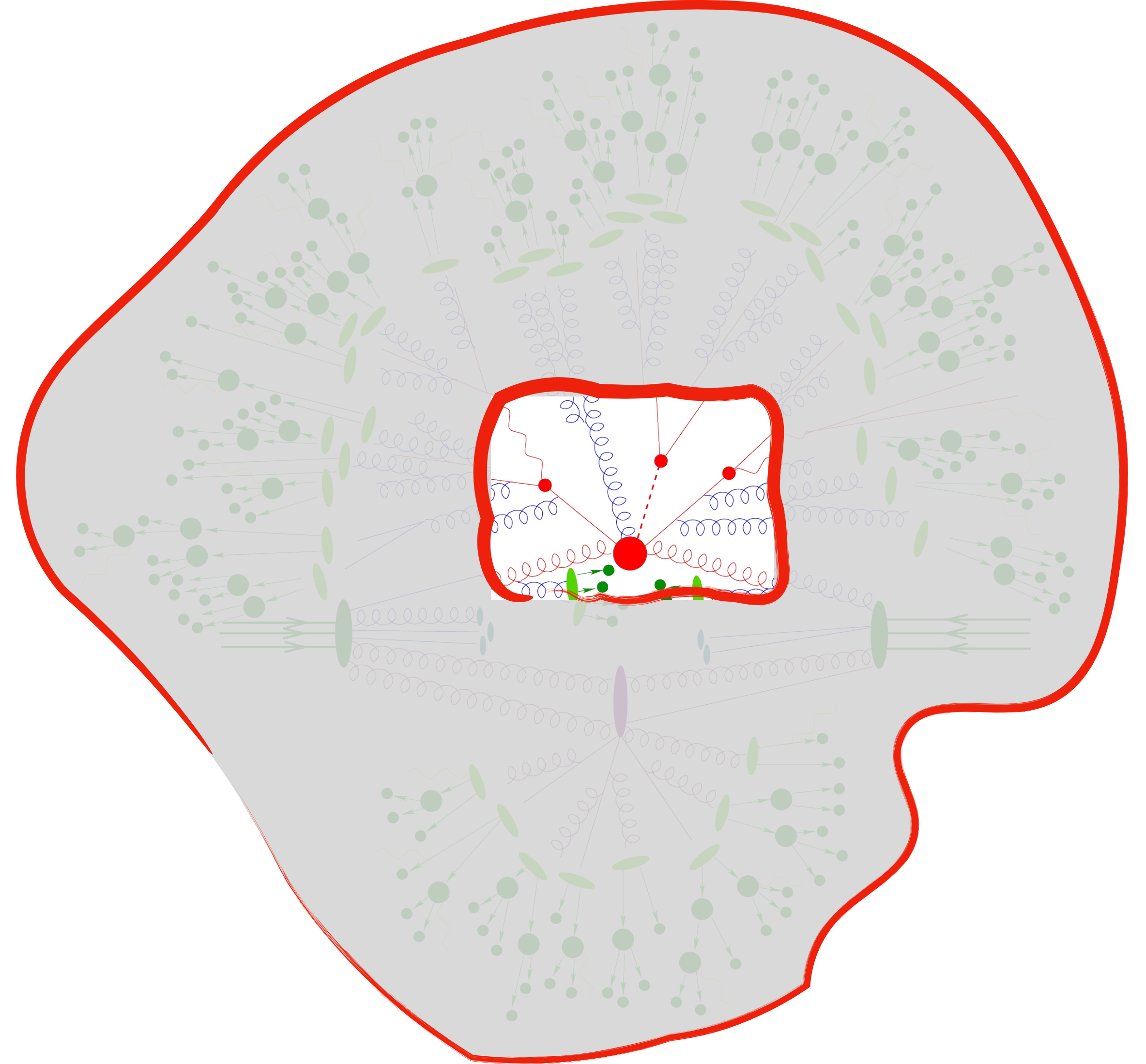
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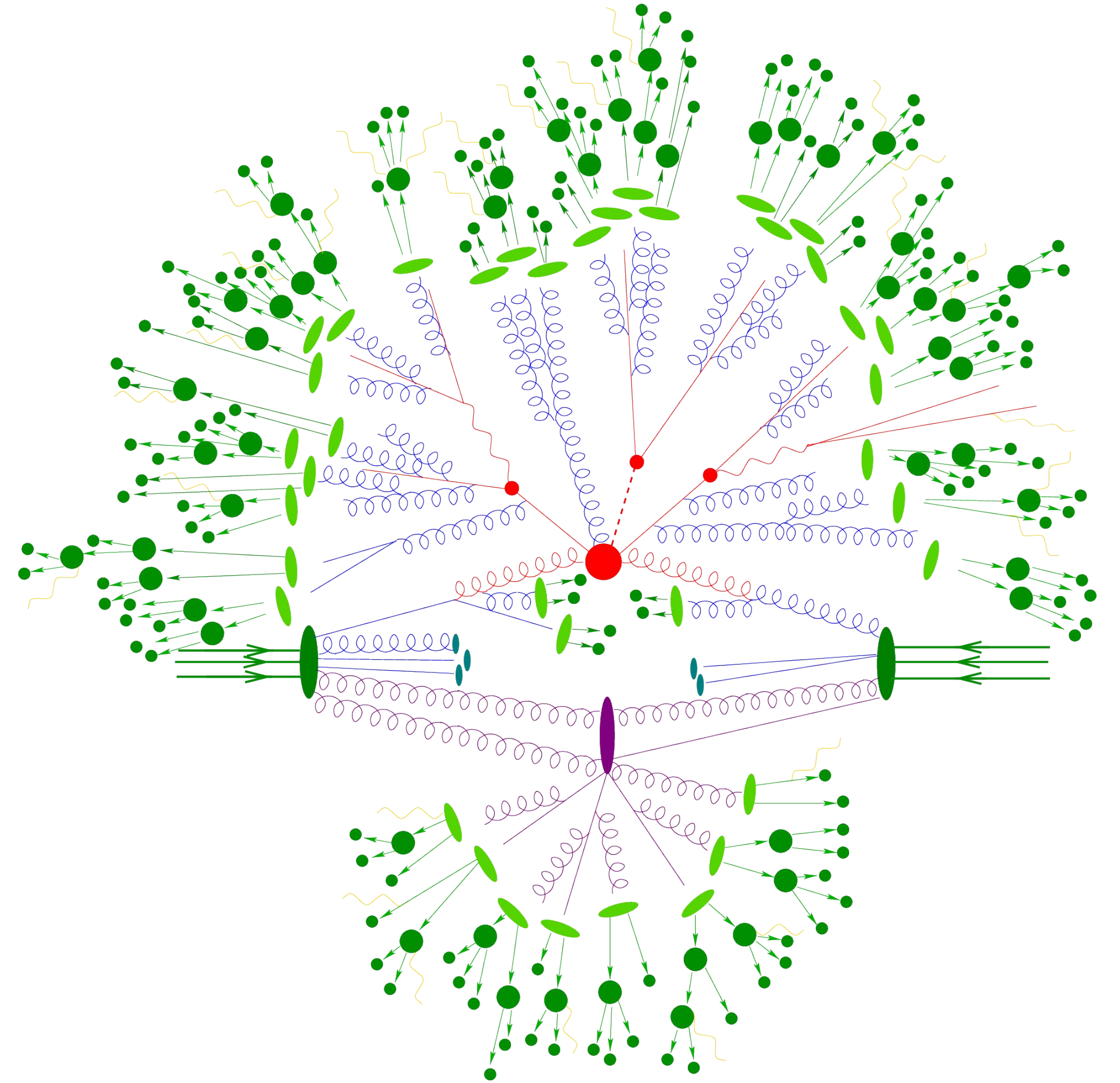
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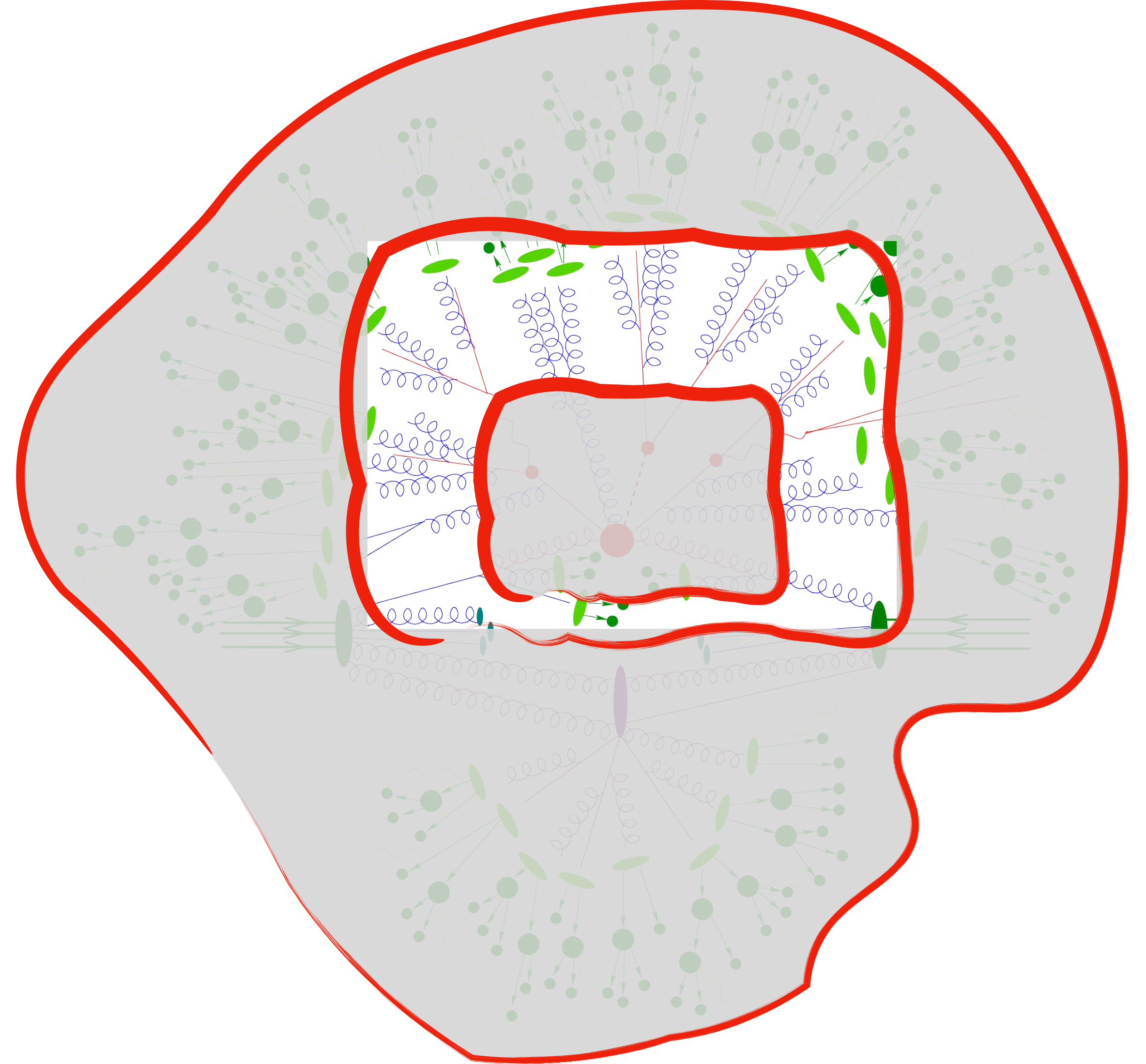
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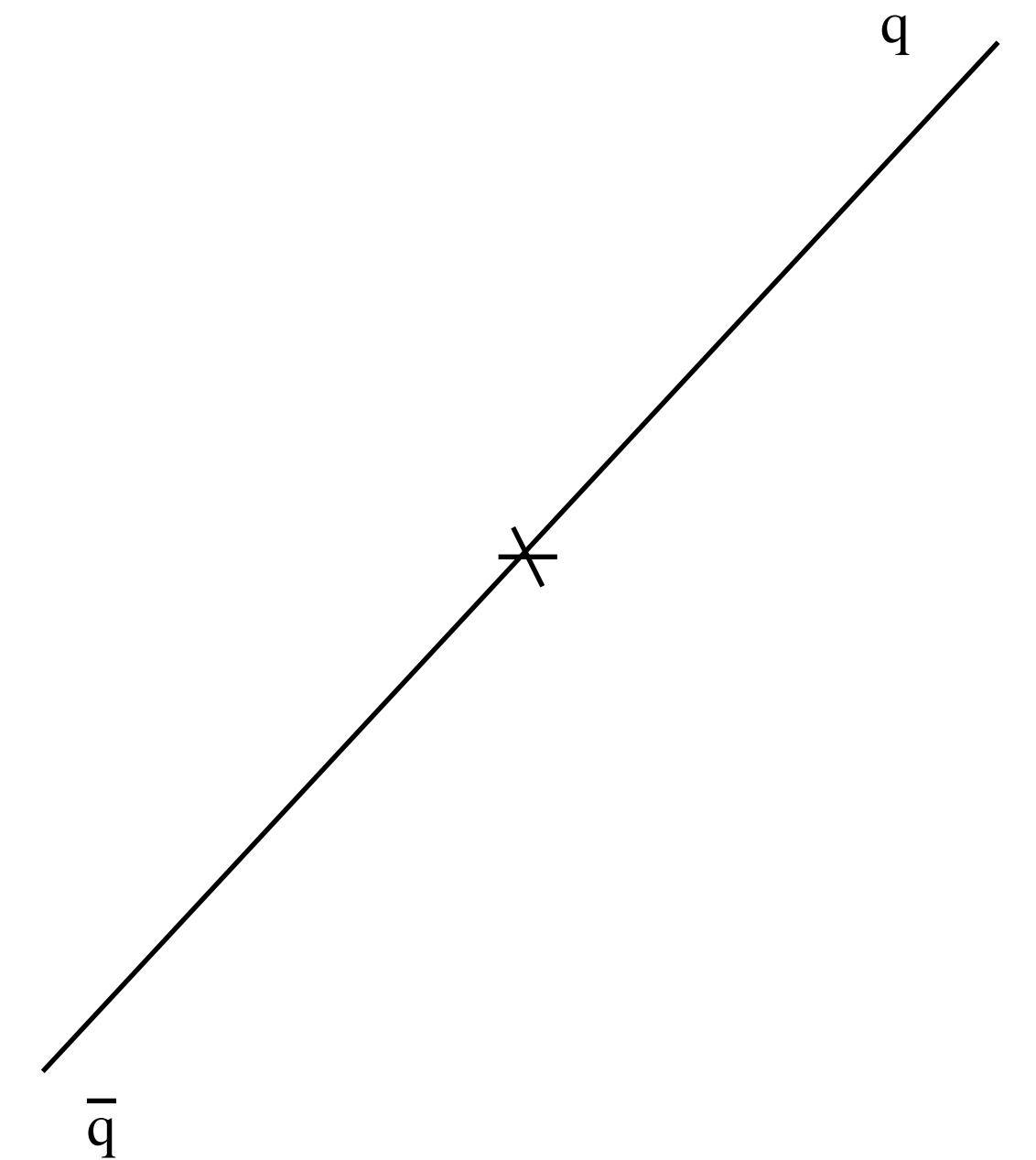
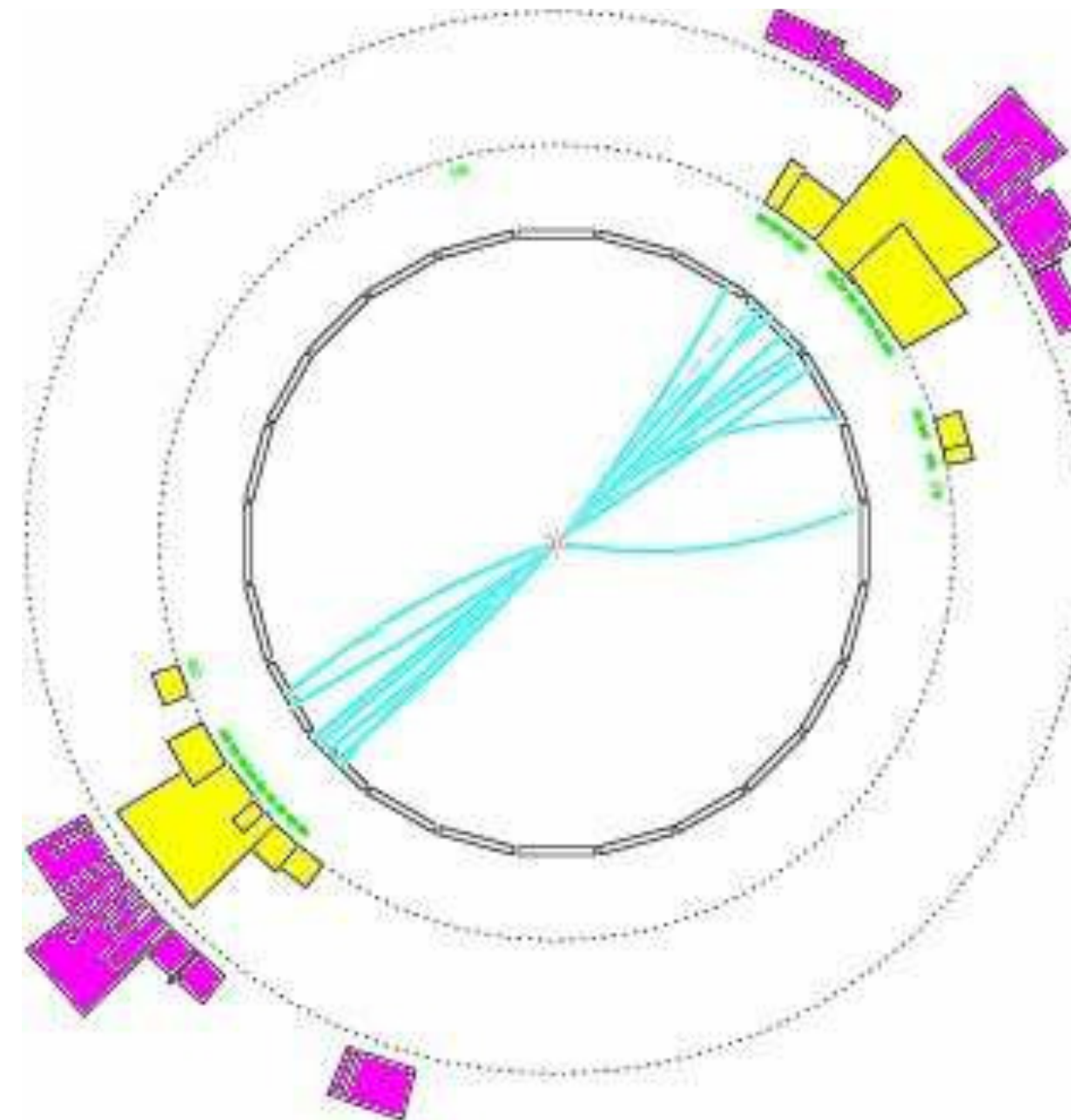
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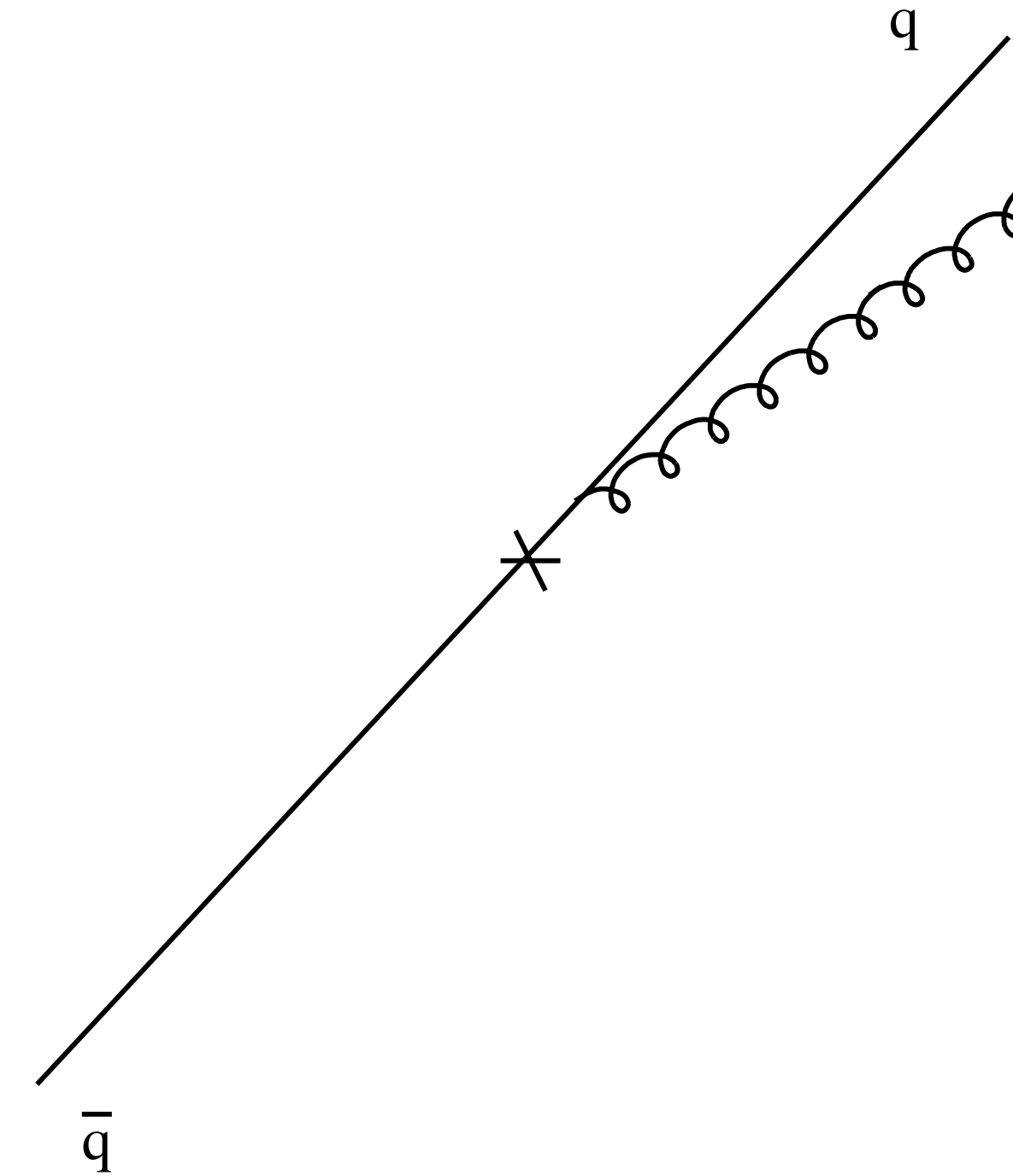
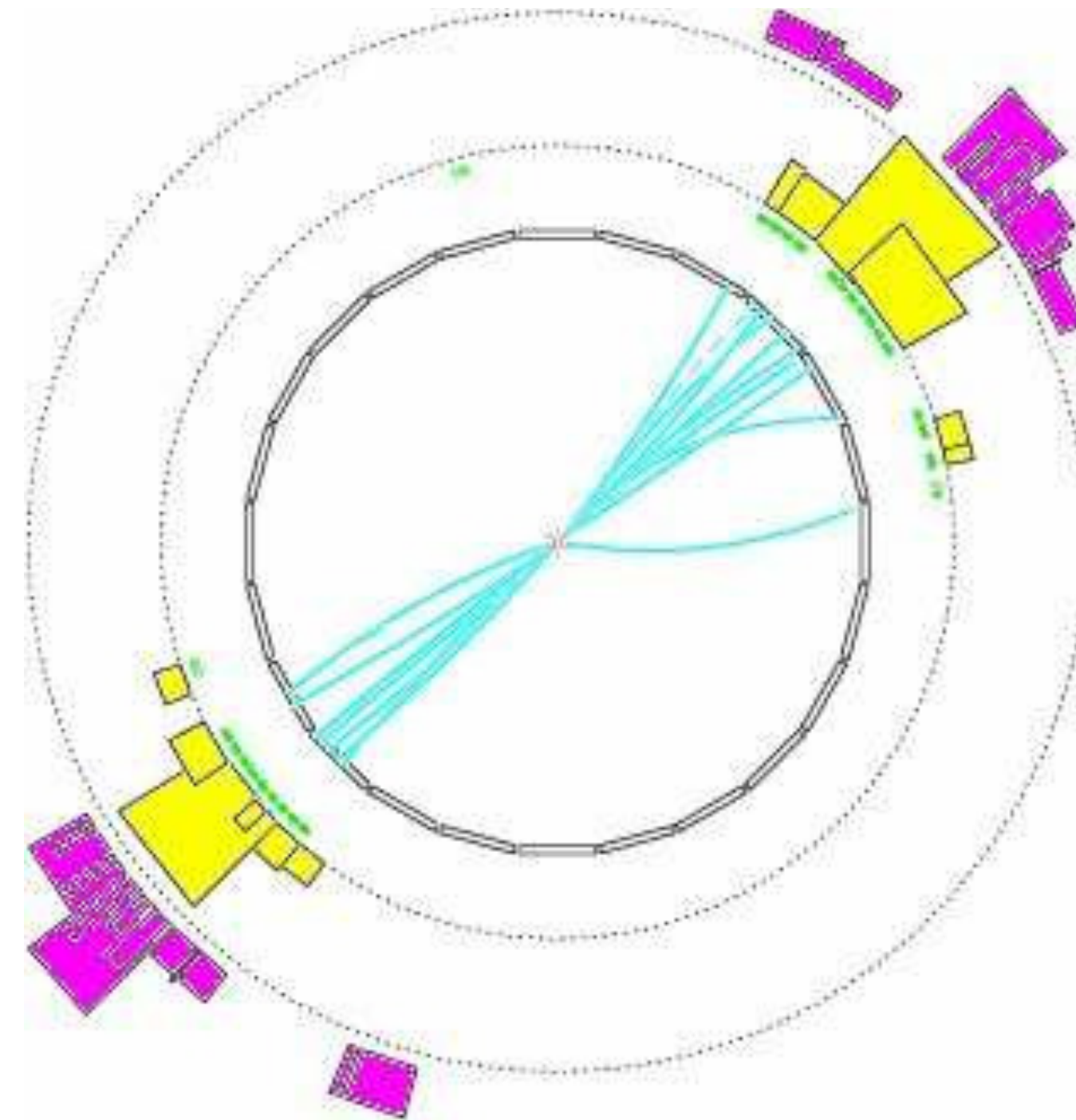
# jet evolution

- so far jets look rather empty, consist of usually 1 parton at LO, at most 2 at NLO
- but we saw matrix elements for additional soft emissions diverging  $\Rightarrow$  no suppression of higher orders, should expect arbitrary number of soft/collinear gluons



# jet evolution

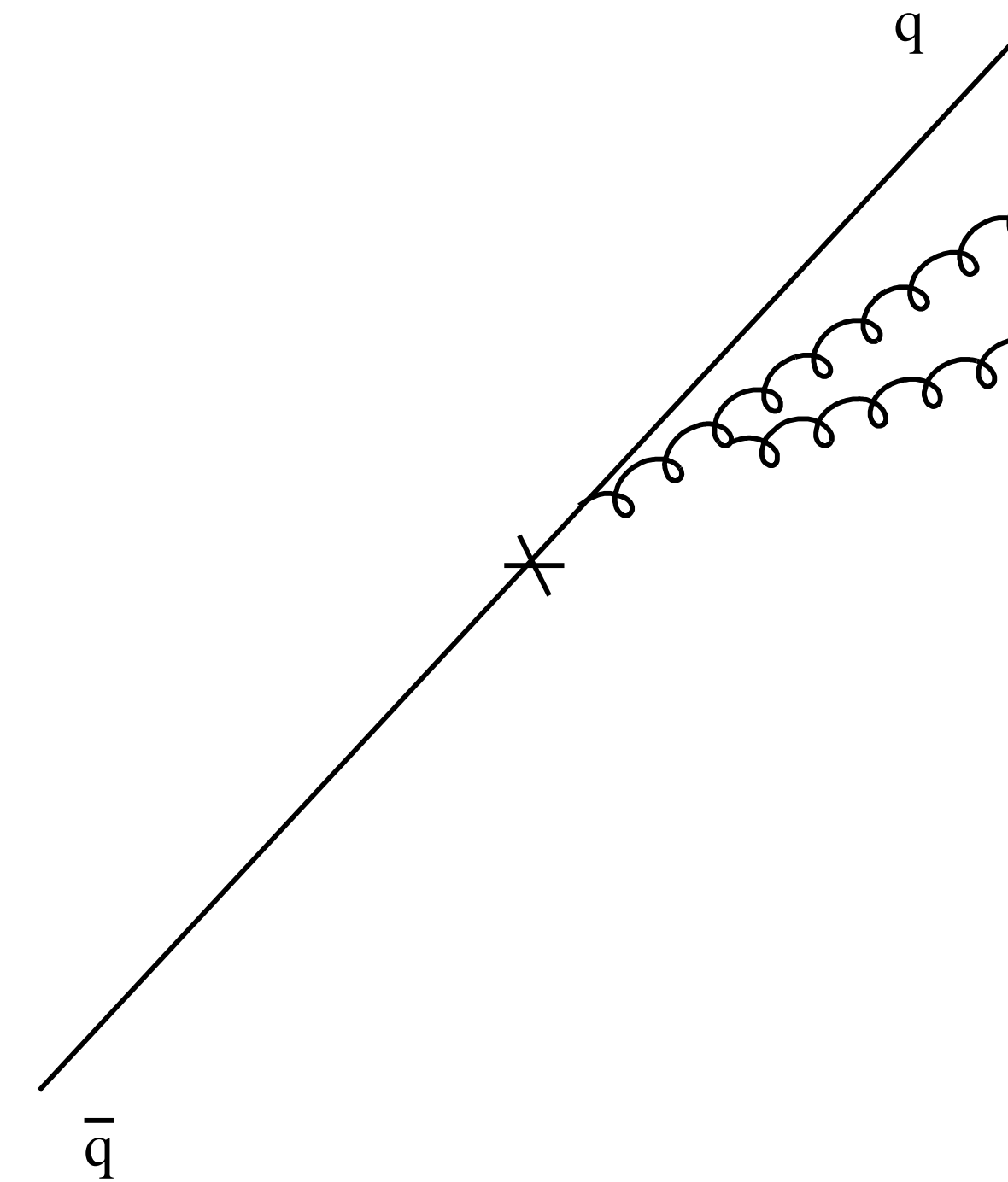
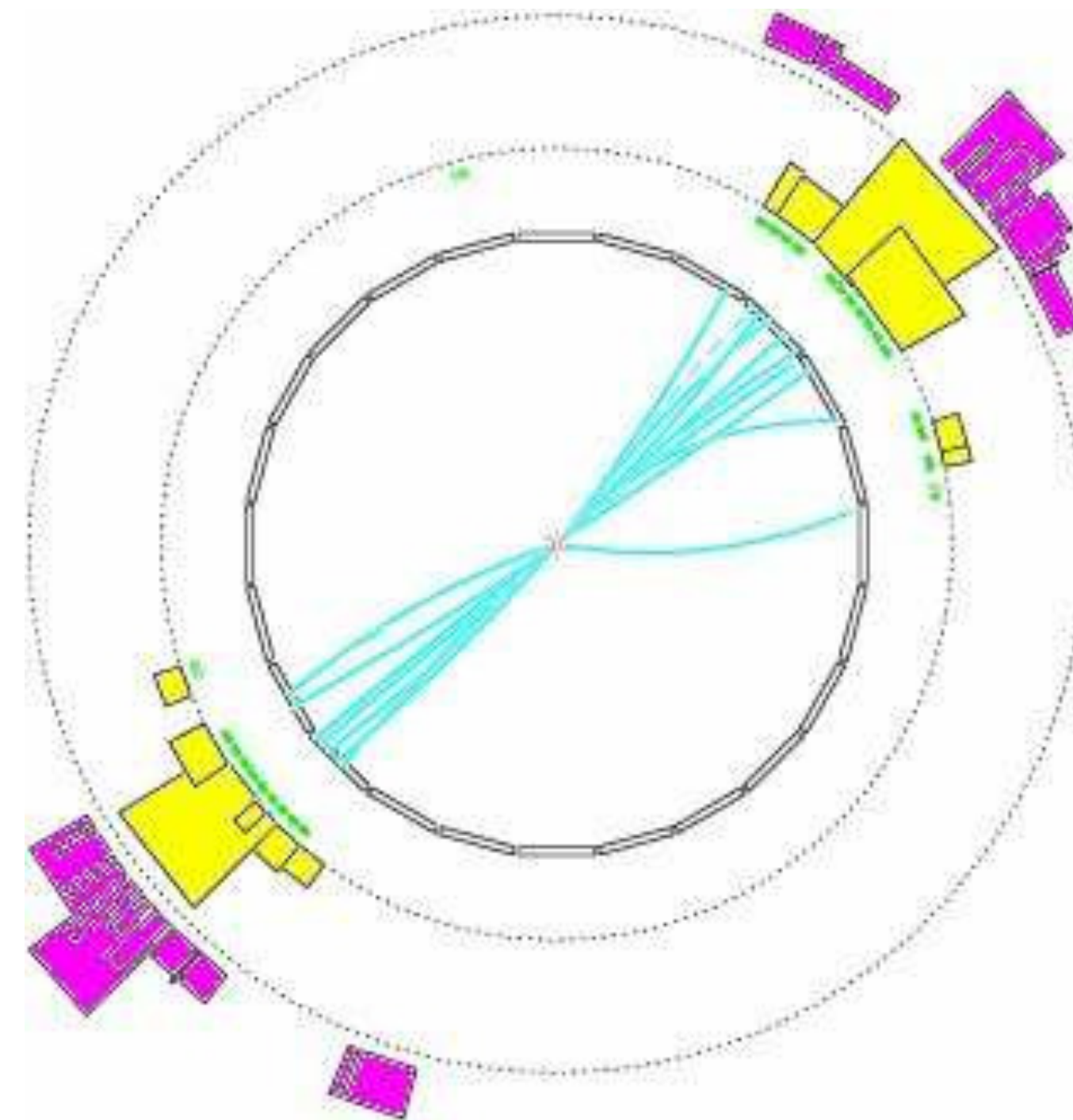
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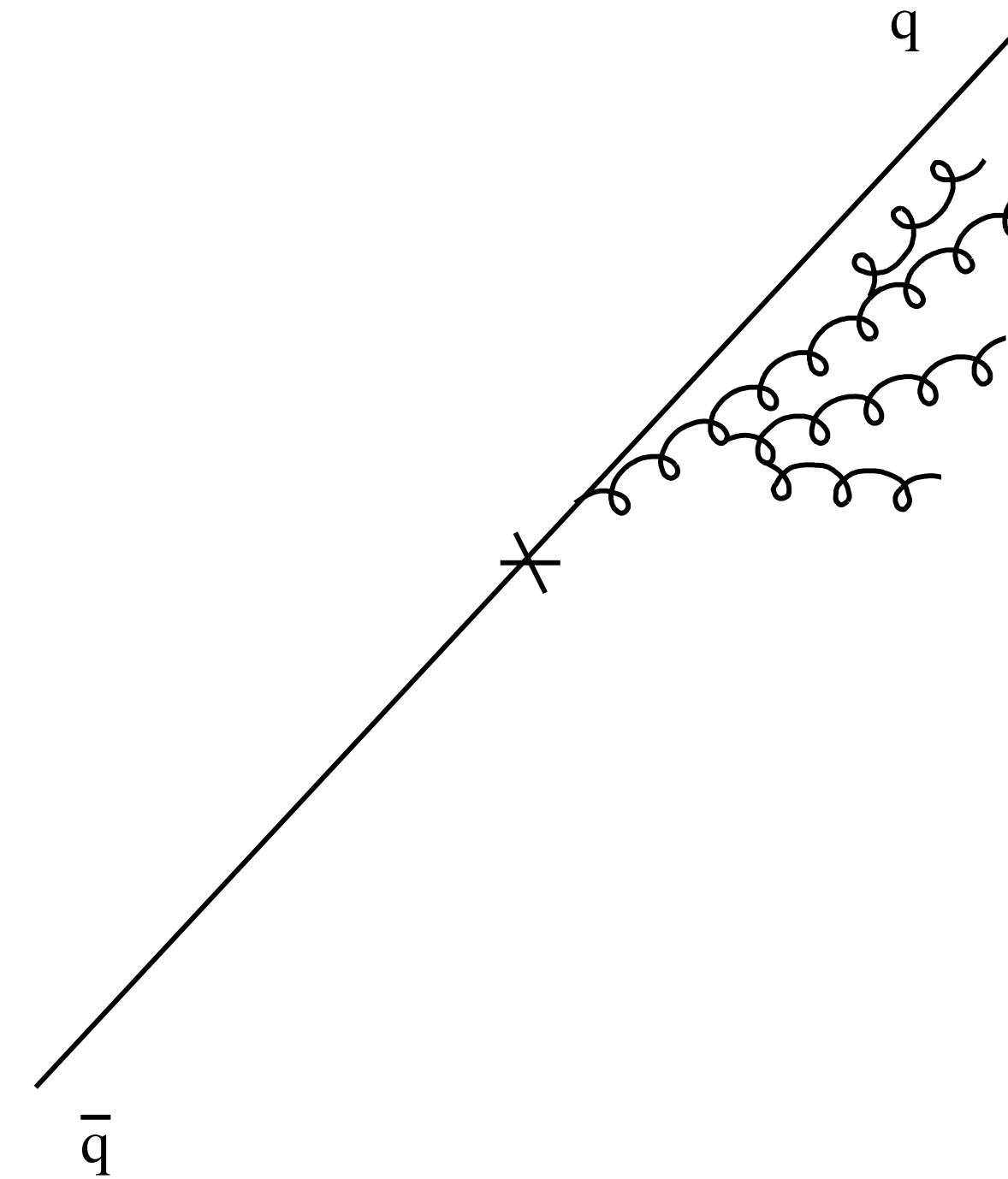
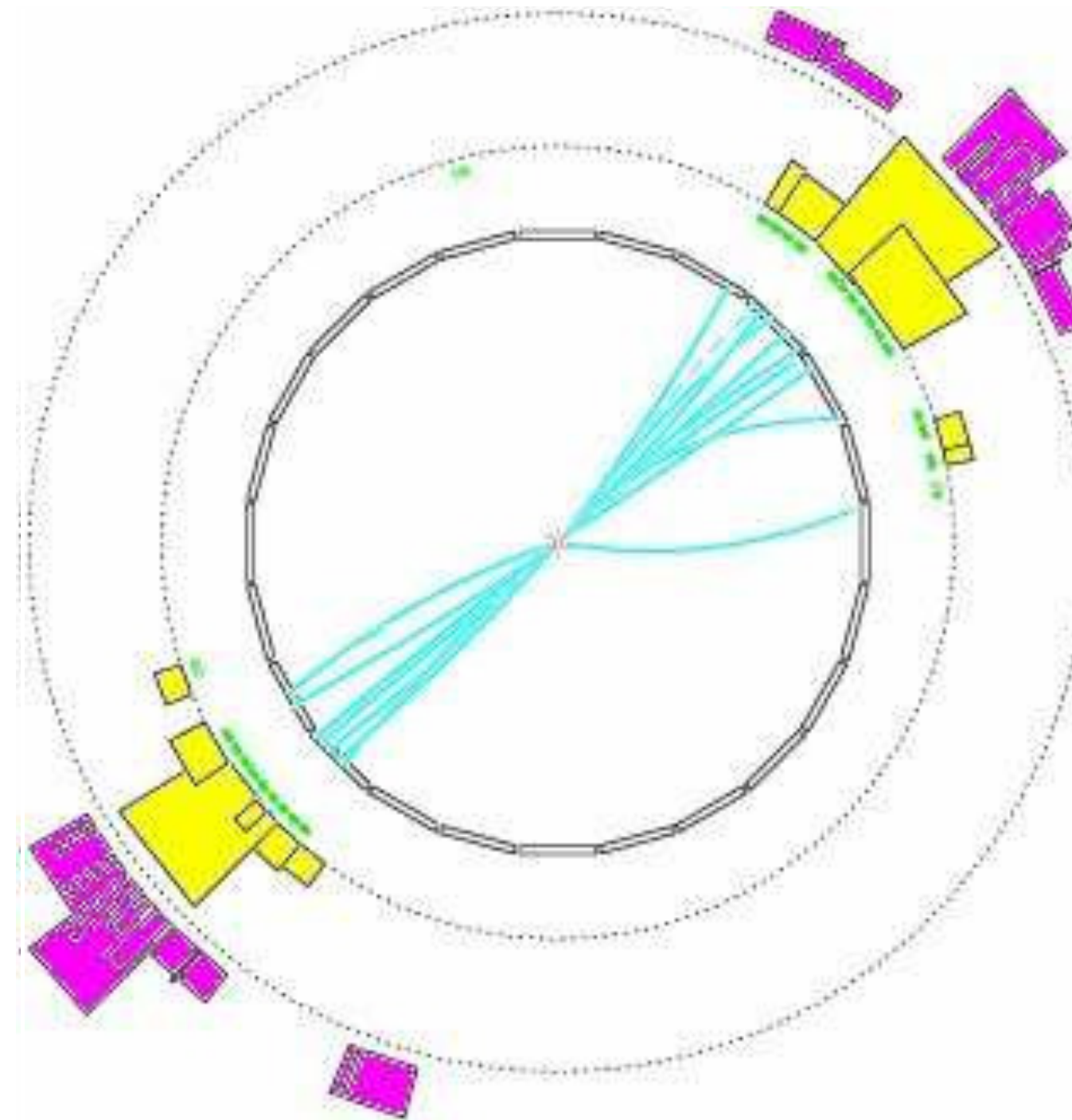
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# jet evolution

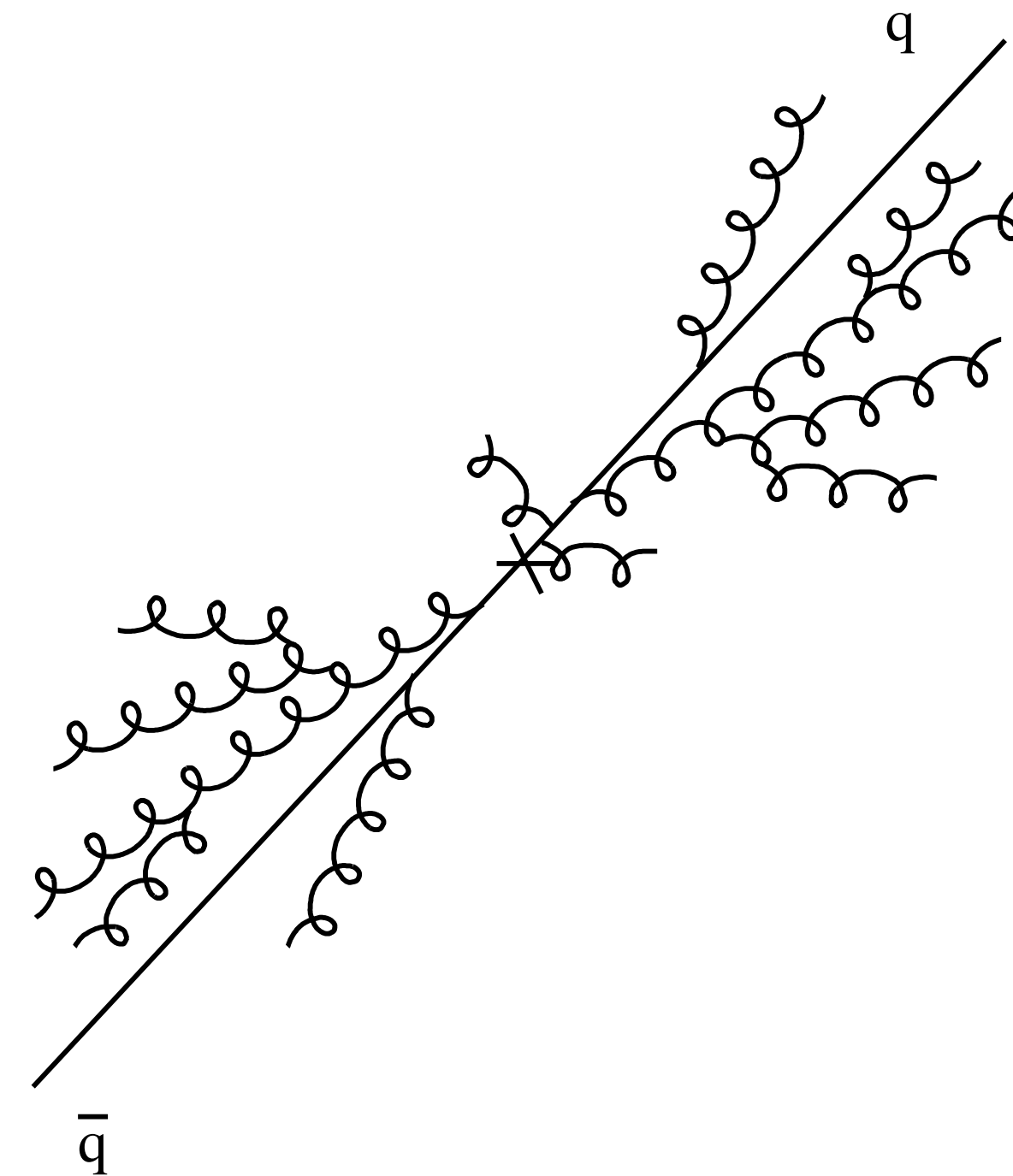
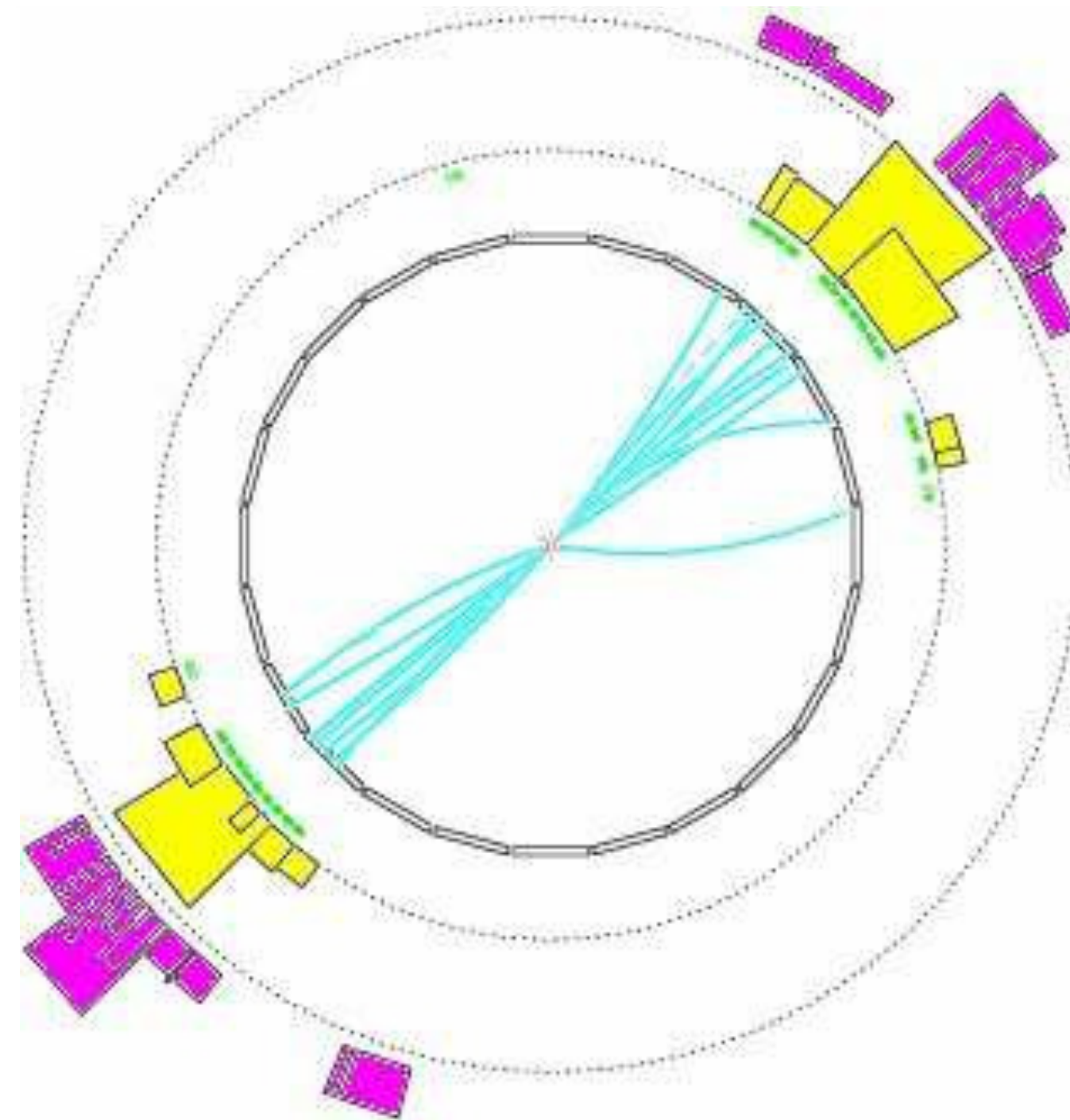
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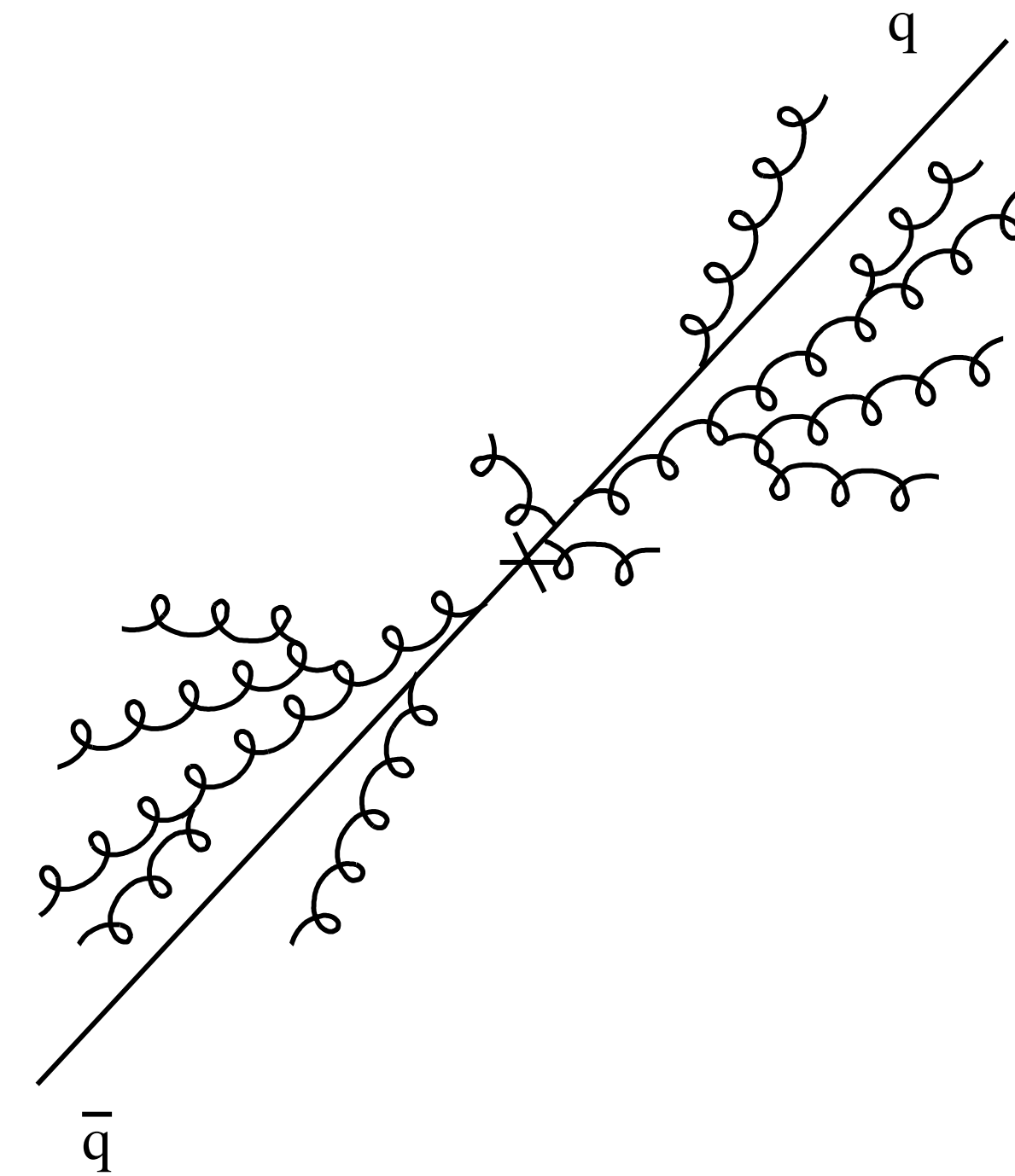
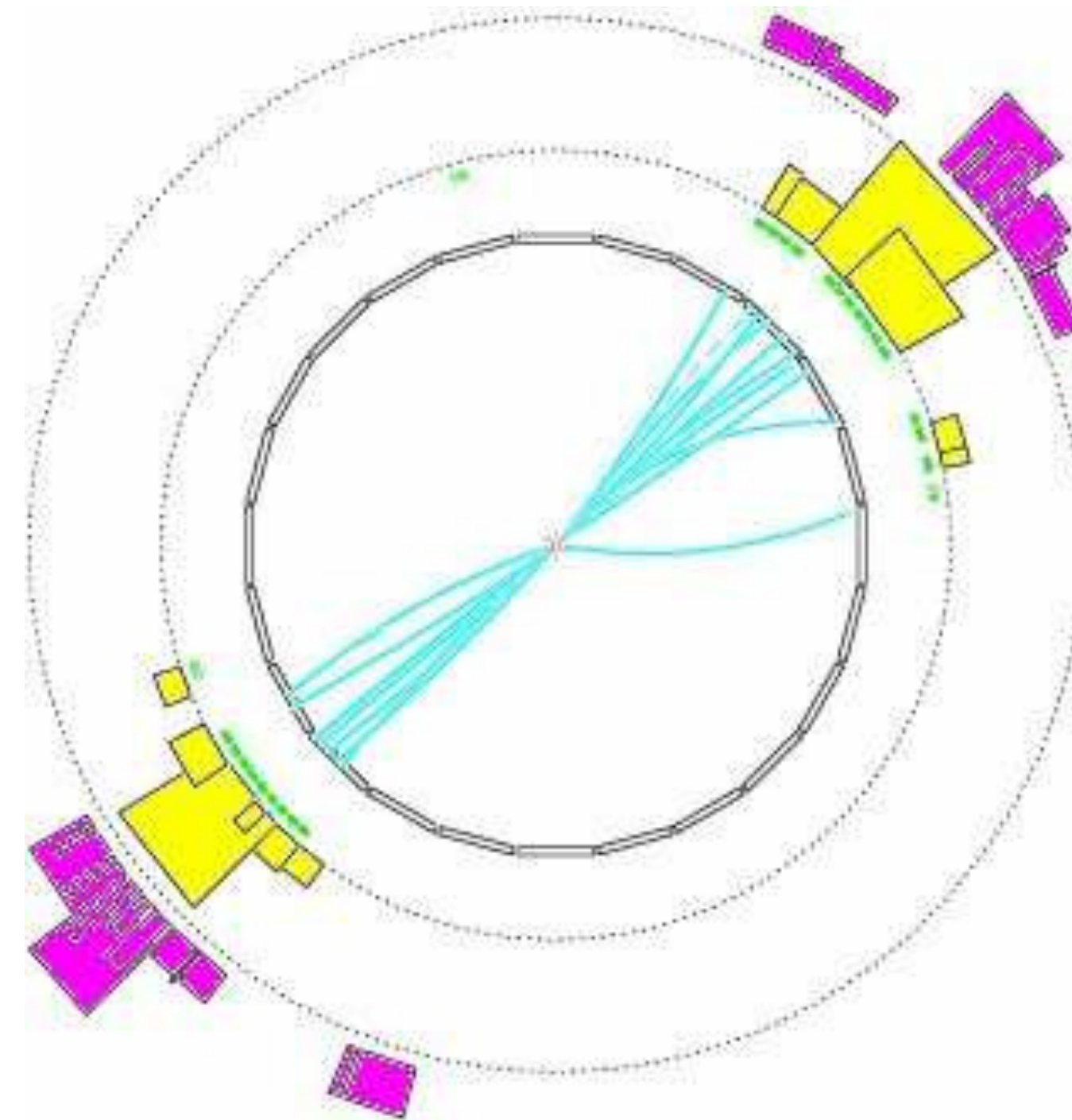
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Nice pictures... Can we be quantitative about this?

# average number of gluons

- we already saw how to calculate approximate higher orders via factorisation:

$$\begin{aligned} & \approx \frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\theta} \\ & \approx \frac{2\alpha_s C_A}{\pi} \frac{dE}{E} \frac{d\theta}{\theta} \end{aligned}$$

- picture: emission of additional gluons from emitter, same divergence structure, only different colour factor
- repeats at all orders  $\rightarrow$  we can iteratively generate emissions from the hard process + from subsequent emissions



# average number of gluons

- quark with energy  $Q$ , on average we expect

$$\langle N_g \rangle = \frac{2\alpha_s C_F}{\pi} \int^Q \frac{dE}{E} \int^{\pi/2} \frac{d\theta}{\theta} \Theta(E\theta > Q_0)$$

- note we cut  $E\theta \sim k_t > Q_0$  to avoid soft and collinear divergencies

$$\langle N_g \rangle \sim \frac{\alpha_s C_F}{\pi} \ln^2 \frac{Q}{Q_0}$$

- reasonable values  $Q = 200 \text{ GeV}$ ,  $Q_0 = 1 \text{ GeV} \rightarrow \ln^2 \frac{Q}{Q_0} \approx 30$

$\rightarrow \langle N_g \rangle > 1 \Rightarrow$  adding single orders in perturbation theory not sufficient

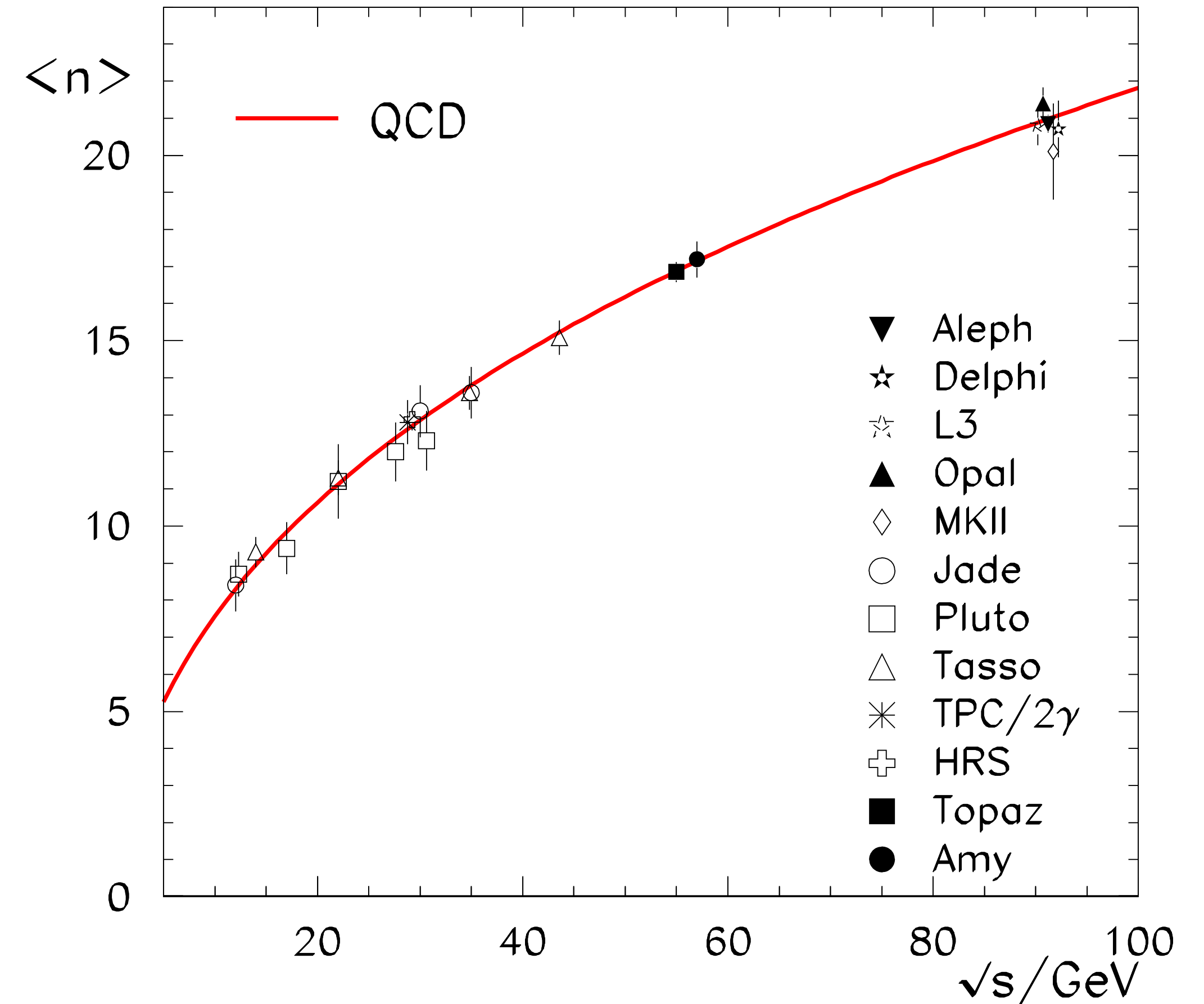
# average number of gluons

- all orders result for gluon multiplicity:

$$\langle N_g \rangle \sim \frac{C_F}{C_A} \sum_{n=1}^{\infty} \frac{1}{(n!)^2} \left( \frac{C_A}{2\pi\beta_0\alpha_s(Q)} \right)^n$$

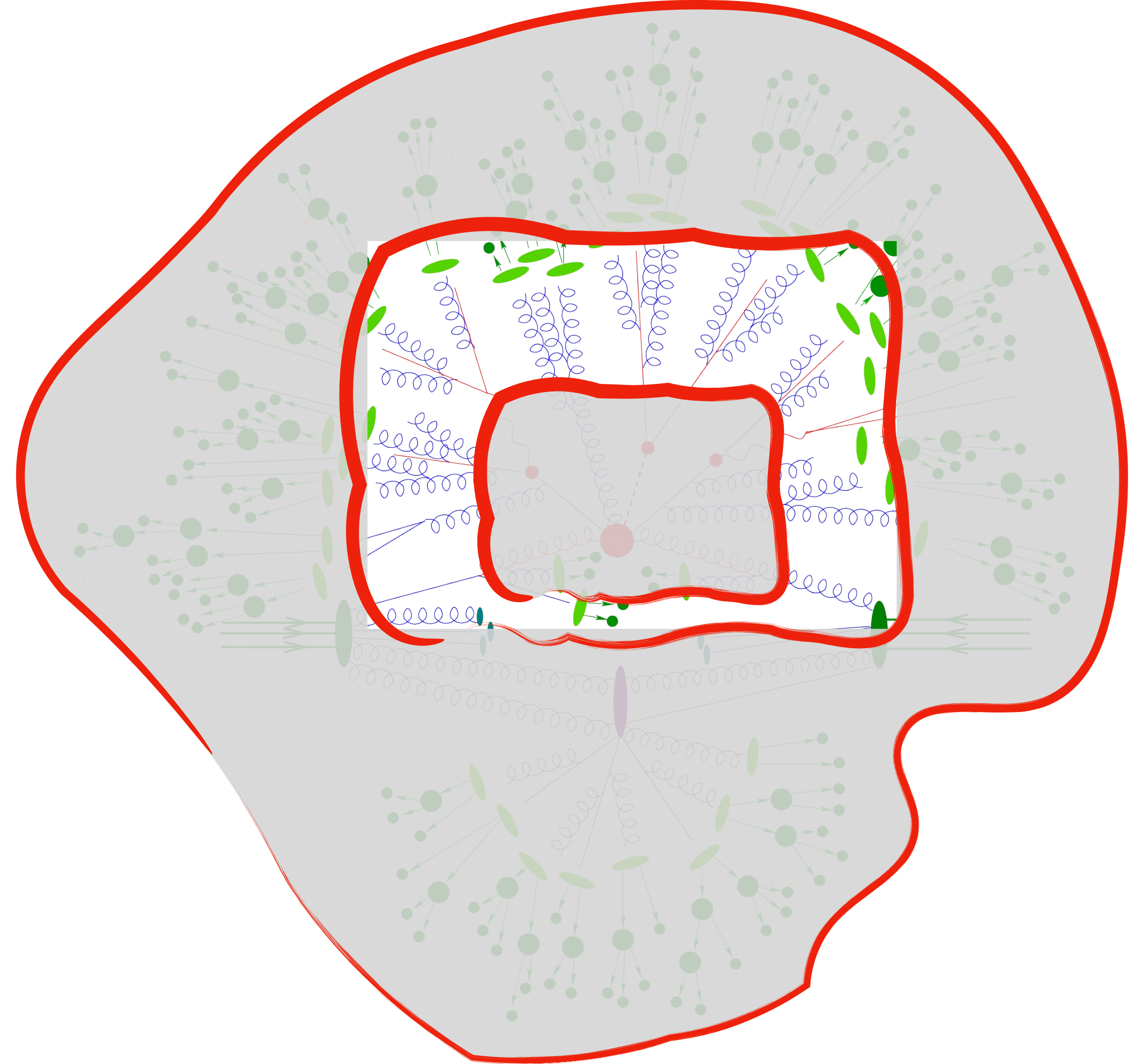
$$\langle N_g \rangle \sim \frac{C_F}{C_A} \exp \left( \sqrt{\frac{2C_A}{\pi\beta_0\alpha_s(Q)}} \right)$$

- comparison to data, assume  $\langle N_g \rangle \sim \langle N_{had} \rangle$



# QCD at colliders

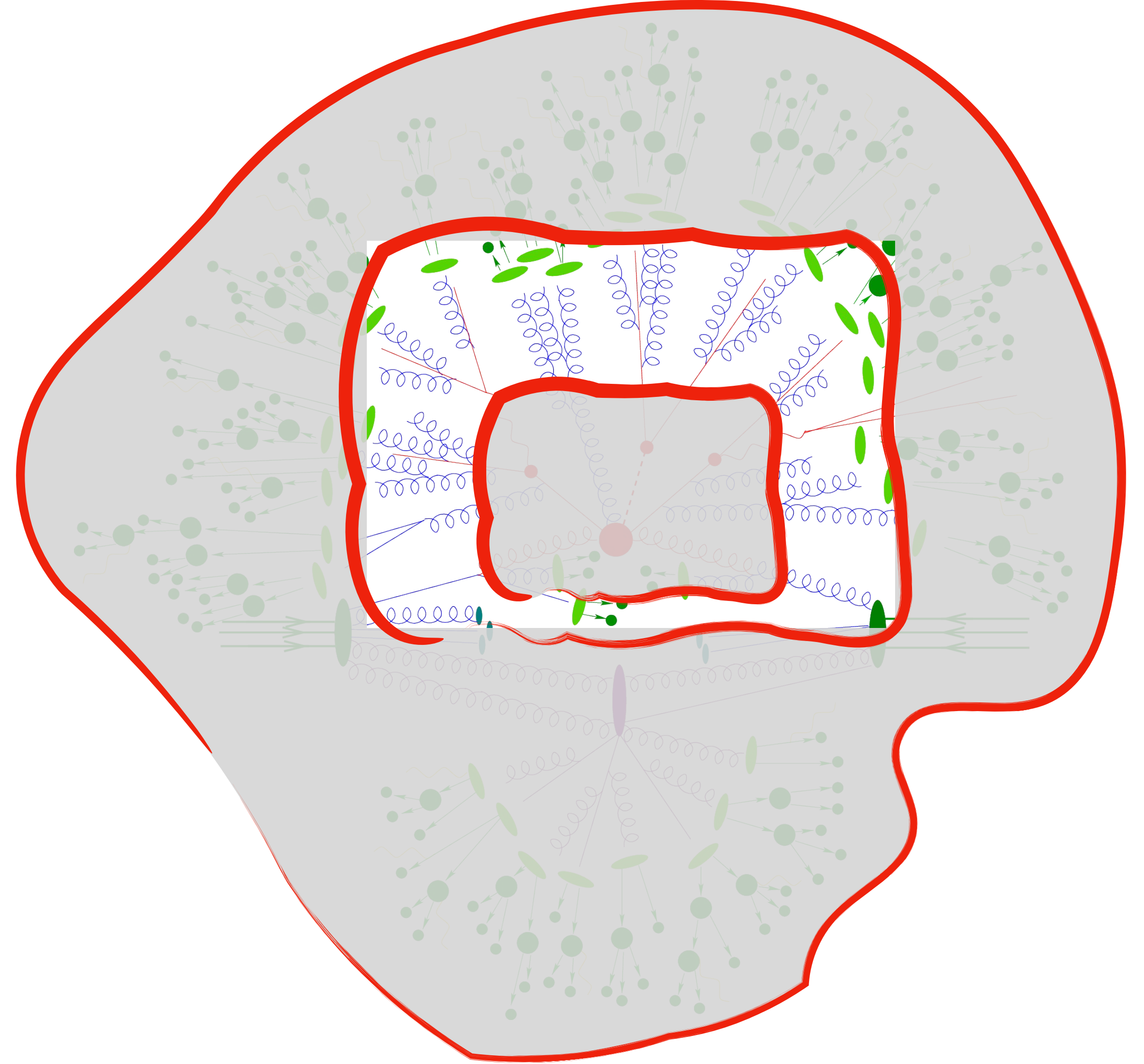
- Events factorised into
  - Hard Process ✓
  - QCD radiation
  - PDFs/Beams
  - Hadrons ✓





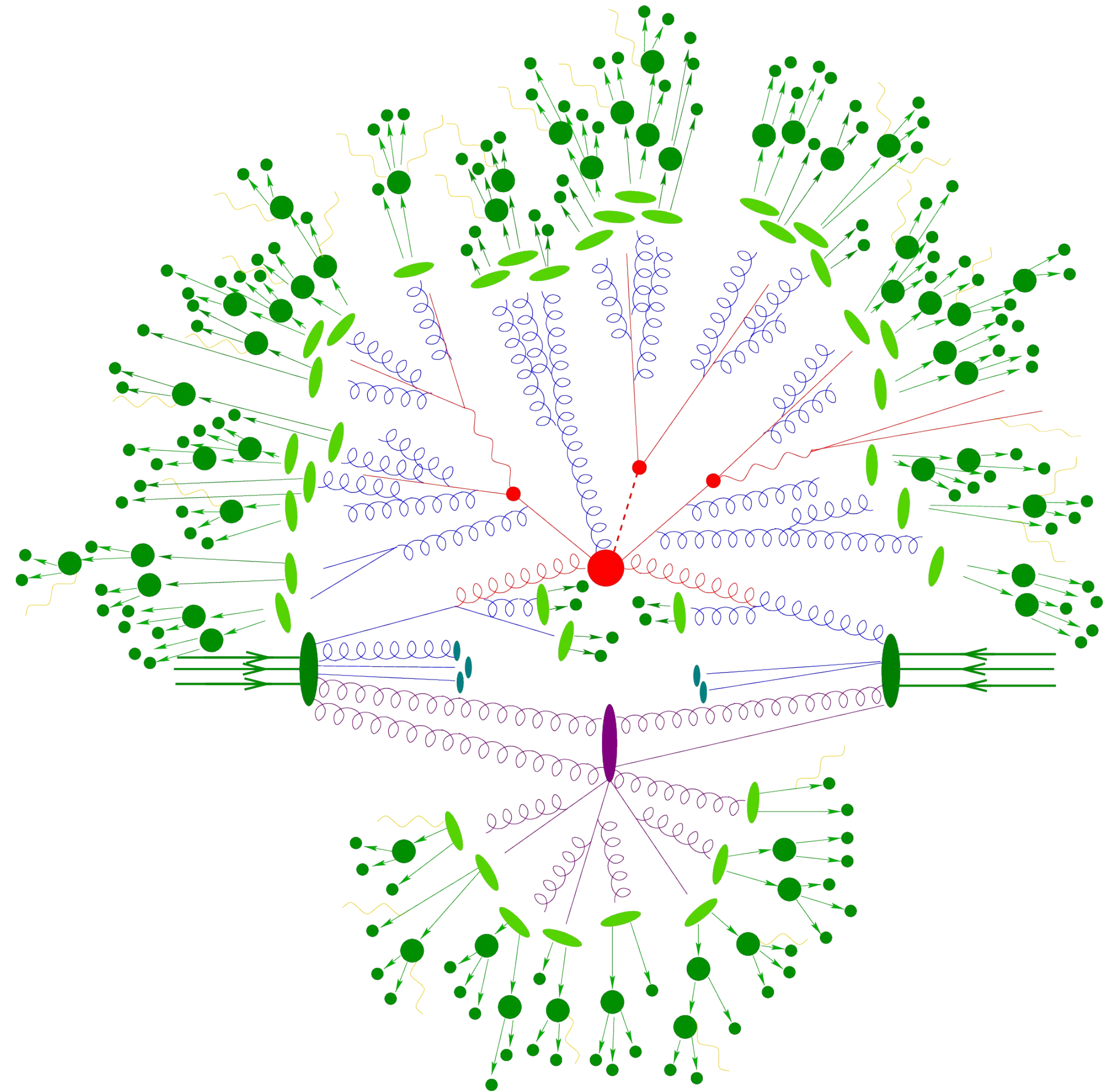
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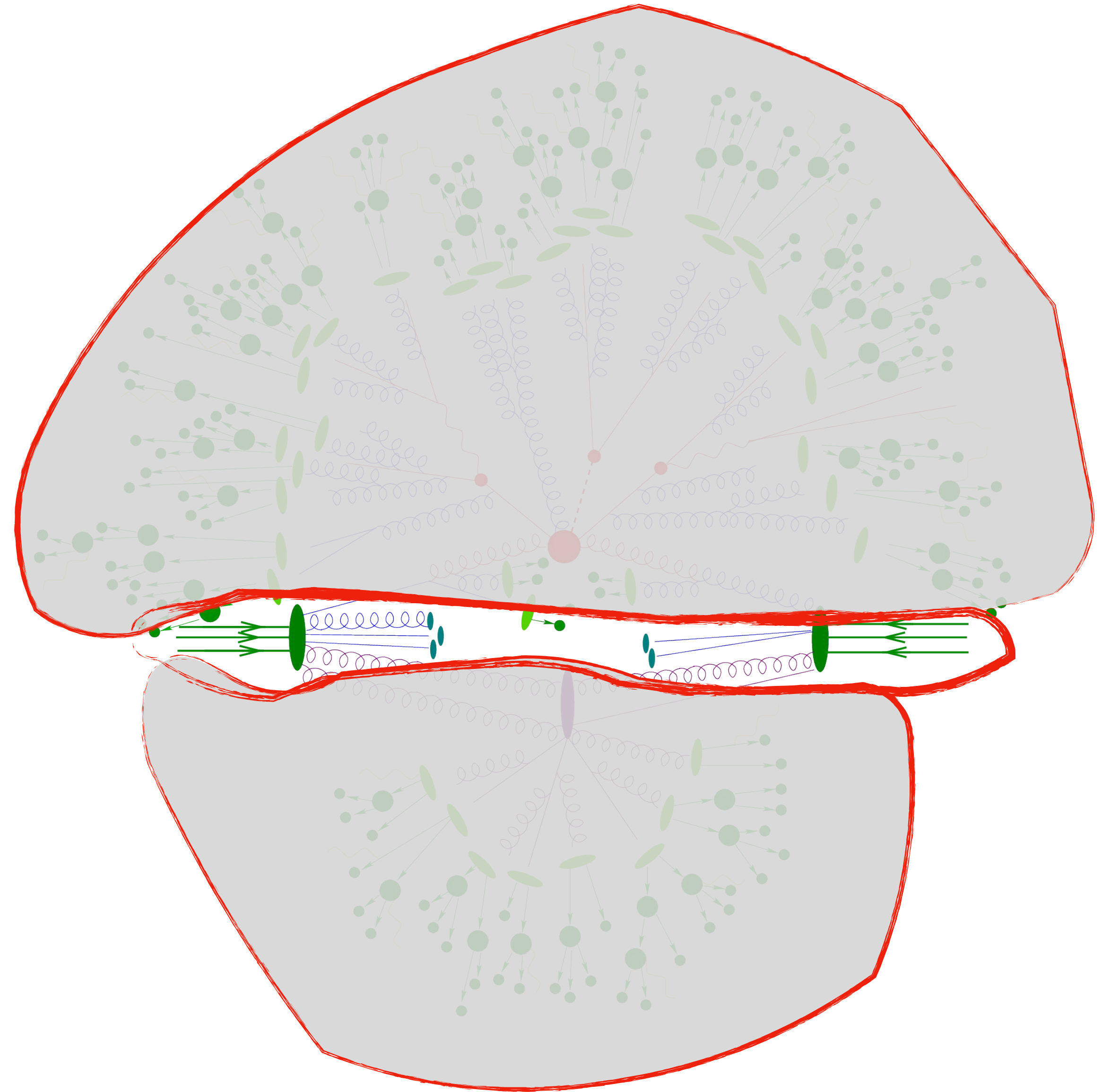
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# QCD at colliders

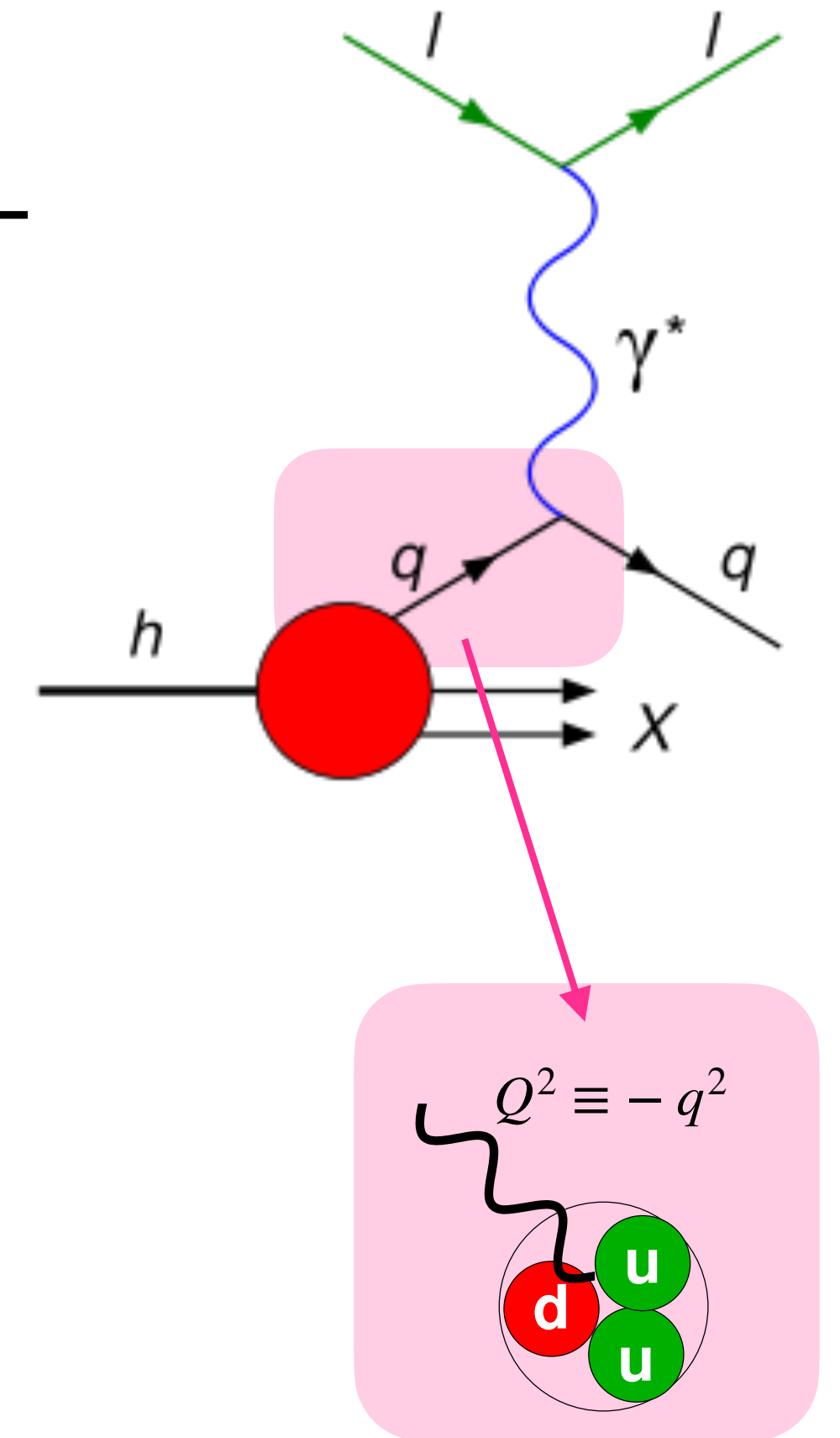
- Events factorised into
  - Hard Process ✓
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  - PDFs/Beams
  - Hadrons ✓





# parton distribution functions

- initial states are not partons but hadrons made up of them
- in pQCD, we can only calculate matrix element with incoming partons - information about **internal structure of hadrons**
- parton model picture:
  - quarks bound inside proton
  - in collision, single parton is involved, carrying momentum fraction  $x$  of the proton momentum
  - which parton?
    - select according to probability distributions  $f_p(x)$  for each parton species



# parton distribution functions

- hadronic cross sections factorise

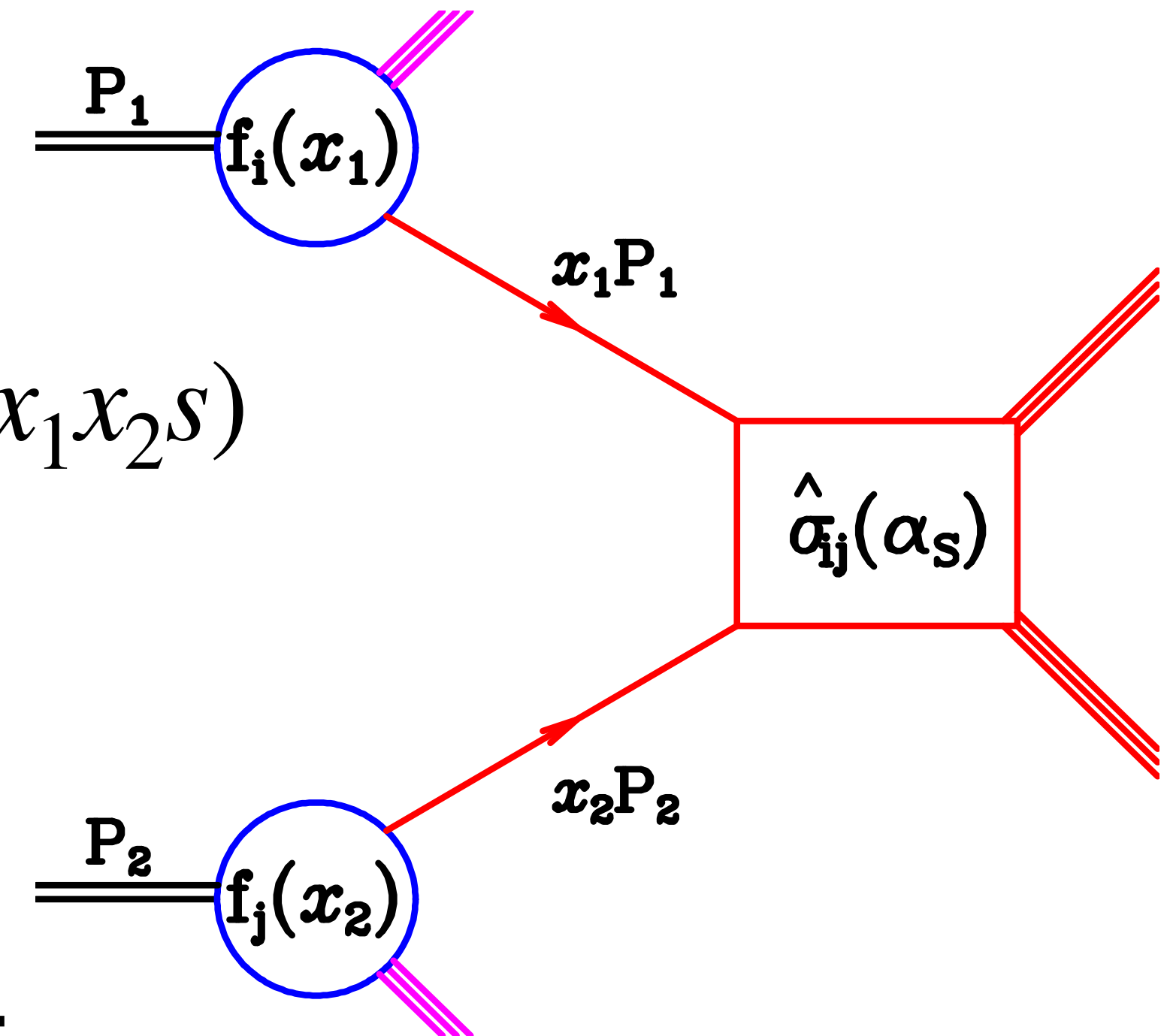
$$\sigma_{hh \rightarrow X}(s) = \sum_{p_1, p_2} \int dx_1 f_{p_1}(x_1) \int dx_2 f_{p_2}(x_2) \sigma_{p_1 p_2 \rightarrow X}(x_1 x_2 s)$$

- note:

- partons assumed collinear with hadron/proton:

$$\vec{p}_i = x_i \vec{p}_h$$

- effective parton collision with reduced center of mass energy  $\hat{s} = x_1 x_2 s$



# parton distribution functions — sum rules

- can we say anything about the PDFs?
- we know overall make-up of hadrons  $\Rightarrow$  sum rules, e.g. for proton  $|uud\rangle$ :

$$\int_0^1 dx \left( f_u(x) - f_{\bar{u}}(x) \right) = 2 \quad \text{and}$$

$$\int_0^1 dx \left( f_d(x) - f_{\bar{d}}(x) \right) = 1$$

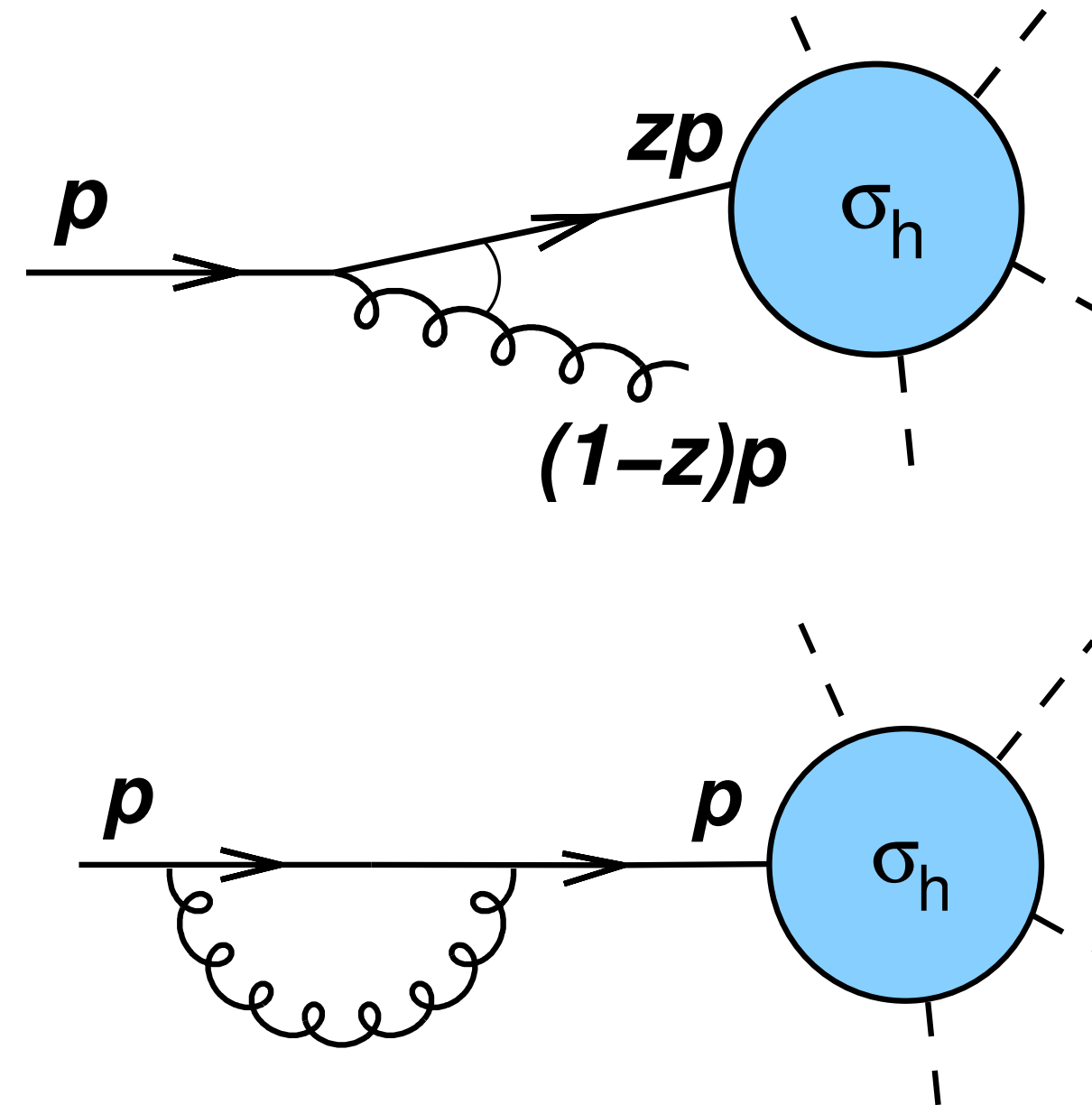


# pdf and factorisation scale

- lets have a closer look at real - virtual cancellation:

$$\sigma_{g+h}(p) \simeq \sigma_h(zp) \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

$$\sigma_{V+h}(p) \simeq -\sigma_h(p) \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$



$$\sigma_{g+h} + \sigma_{V+h} \simeq \underbrace{\frac{\alpha_s C_F}{\pi} \int_0^{Q^2} \frac{dk_t^2}{k_t^2}}_{\text{infinite}} \underbrace{\int_0^1 \frac{dz}{1-z} [\sigma_h(zp) - \sigma_h(p)]}_{\text{finite}}$$

sum finite for  $z \rightarrow 1$  (soft),  
but not  $k_t \rightarrow 0$  (collinear)

additional divergence  
regulated by factorisation  
scale  $\mu_F$  and “absorbed”  
into pdf

# pdf and factorisation scale

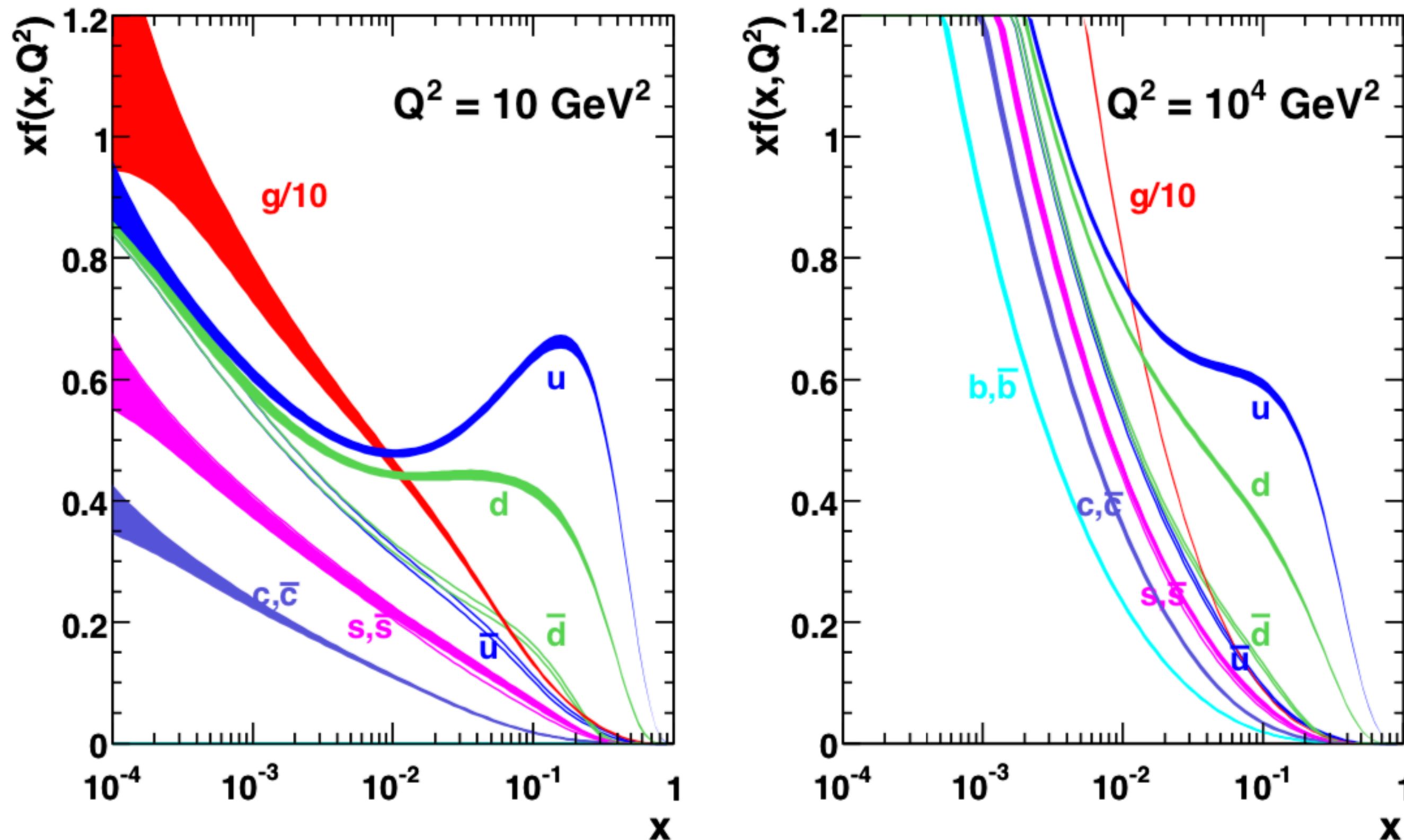
- full factorisation theorems include dependence on factorisation scale

$$\sigma_{hh \rightarrow X}(s) = \sum_{p_1, p_2} \int dx_1 f_{p_1}(x_1, \mu_F^2) \int dx_2 f_{p_2}(x_2, \mu_F^2) \sigma_{p_1 p_2 \rightarrow X}(\hat{s}, \mu_R^2, \mu_F^2)$$

- emission with  $k_t < \mu_F$  part of pdf
- emission with  $k_t > \mu_F$  part of the hard process
- we can calculate the change of pdf with energy scale  $\mu_F$  in perturbative QCD (collinear emissions, again use factorisation)
- typically  $\mu_F \sim Q$  energy of the hard process

# parton distribution functions — measured

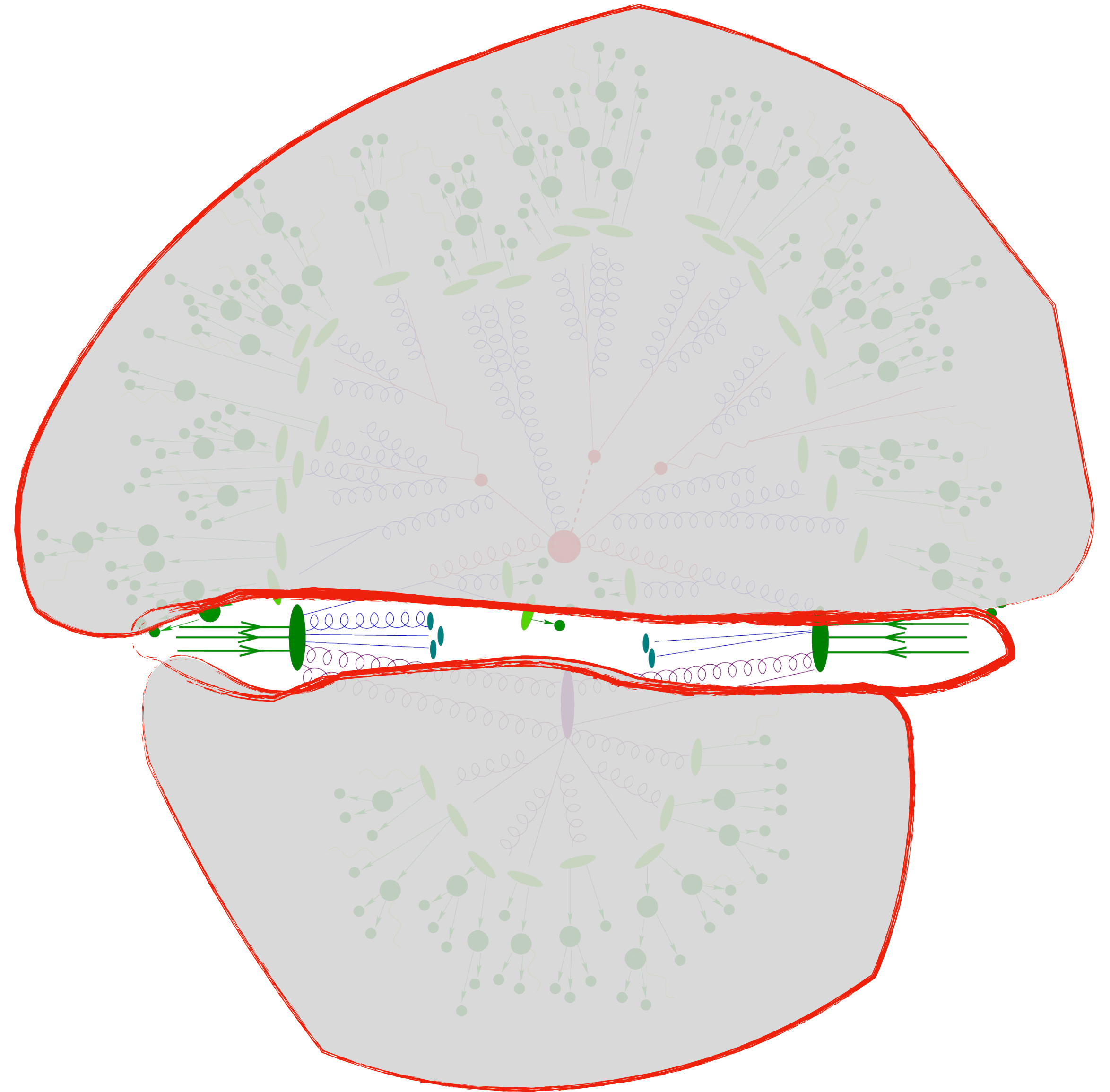
MSTW 2008 NLO PDFs (68% C.L.)



- PDFs fitted to cross sections various processes, in ranges where fixed order QCD calculations are applicable
- note  $g$  is scaled by a factor  $1/10 \rightarrow$  gluons by far dominating at the LHC

# QCD at colliders

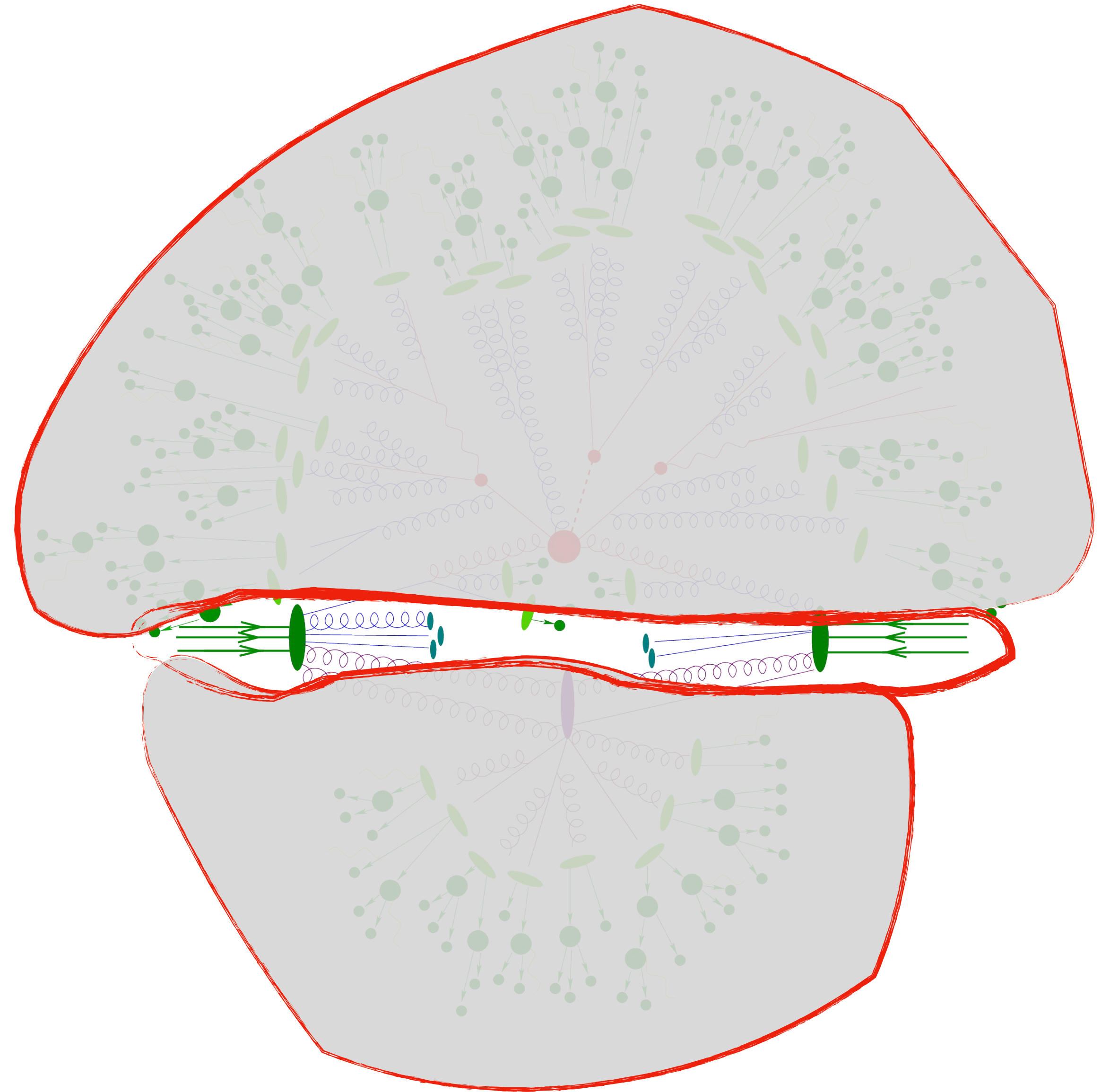
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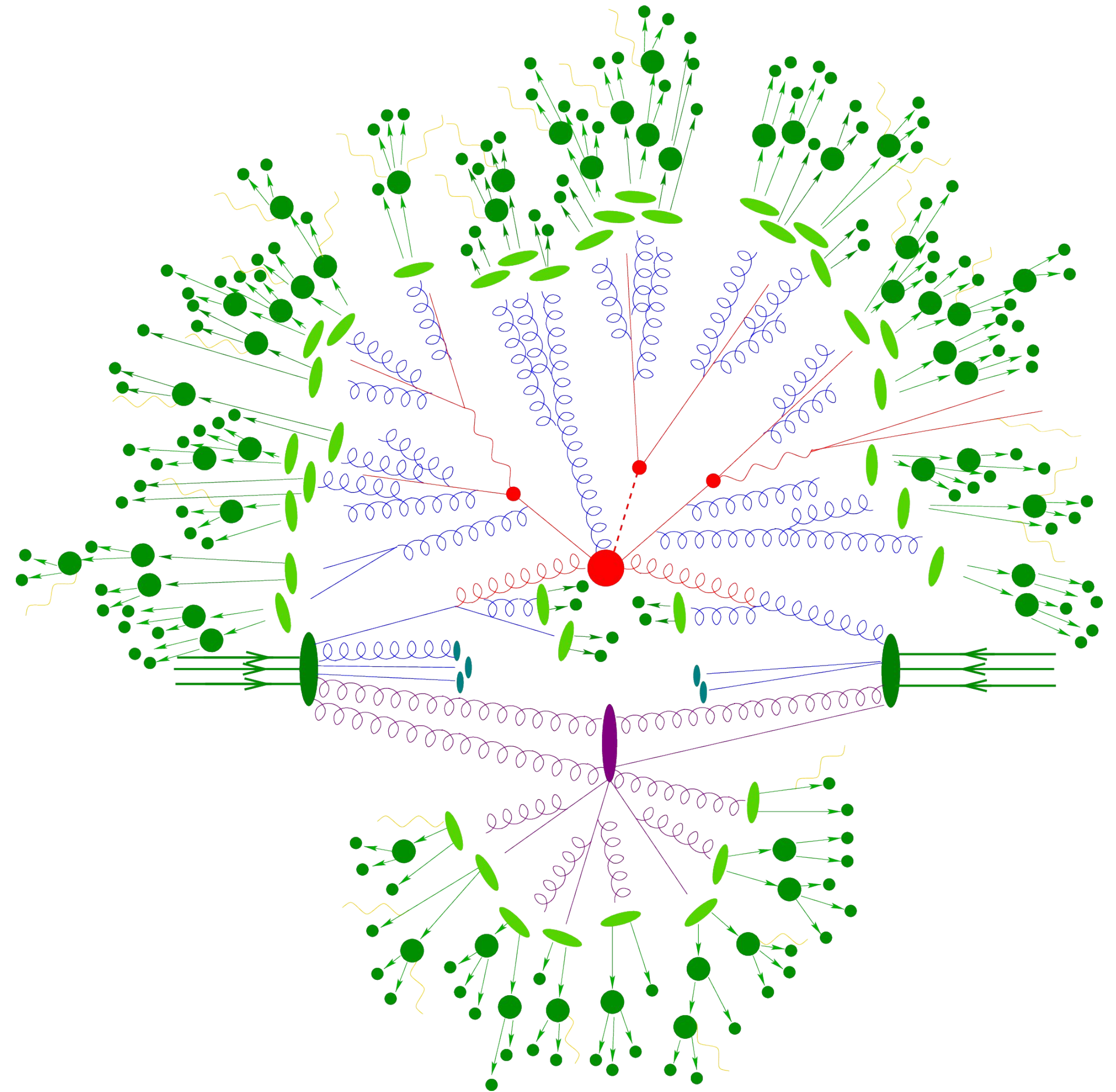
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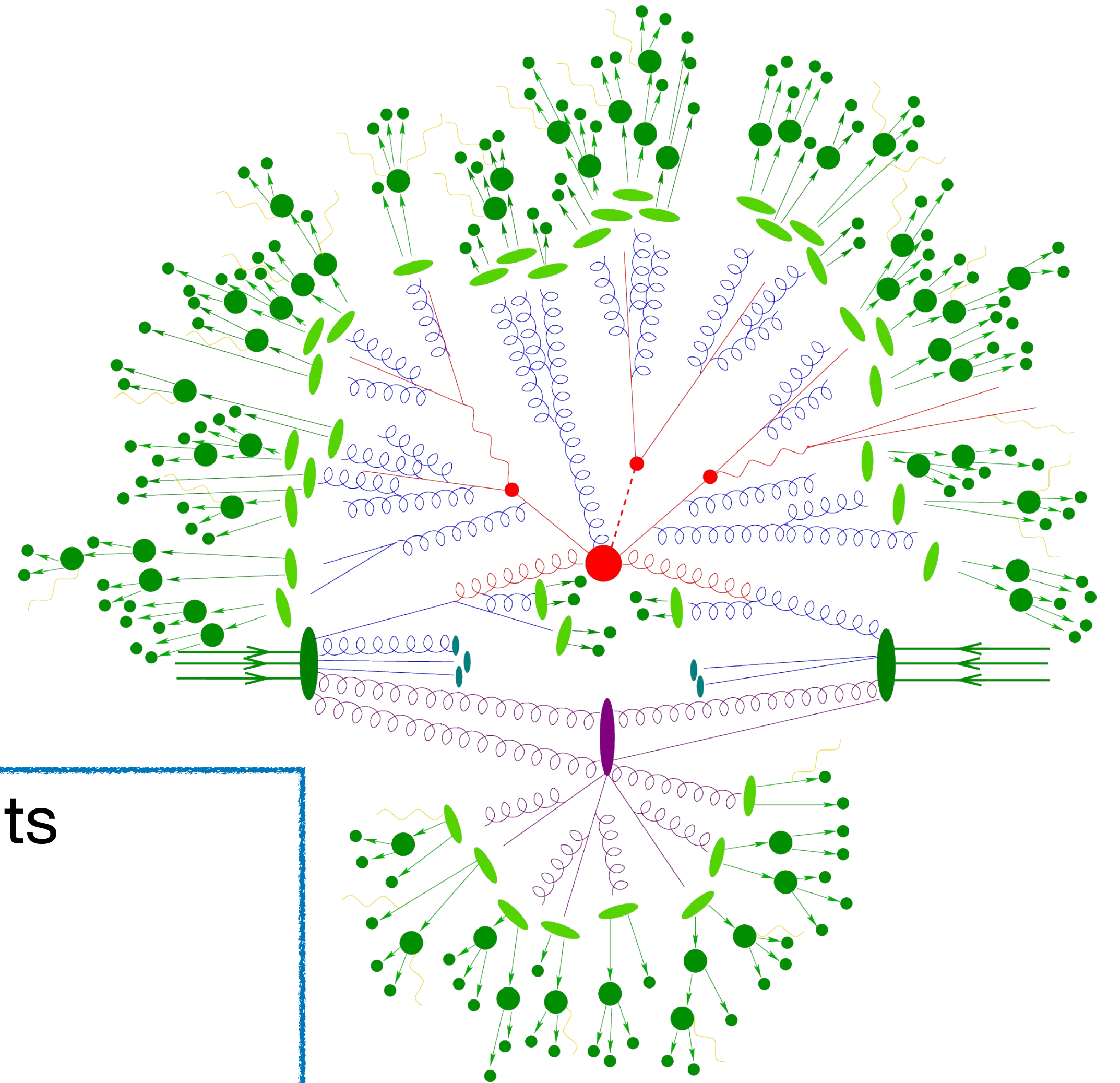
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Left out so far: non-perturbative components

- parton - hadron transition
- underlying event

no first principle QCD calculation, modelling required

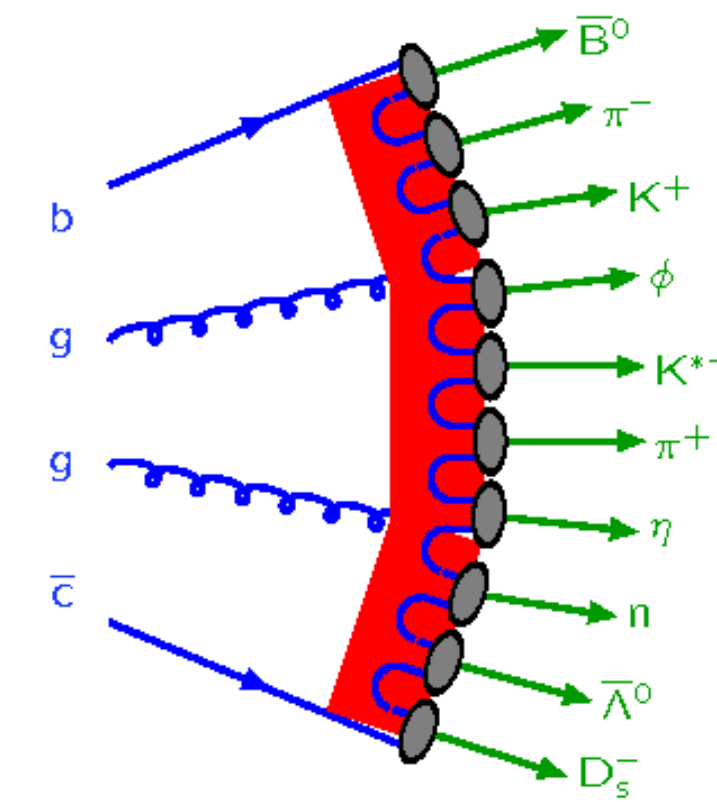


# hadronisation models

- capture main non-perturbative aspects of QCD
- universality:
  - should not depend on process/energy (typical: tune/measure at LEP, then apply at the LHC)
- work in large  $N_c$  limit  $\Rightarrow$  each quark uniquely connected to another one, form string/cluster

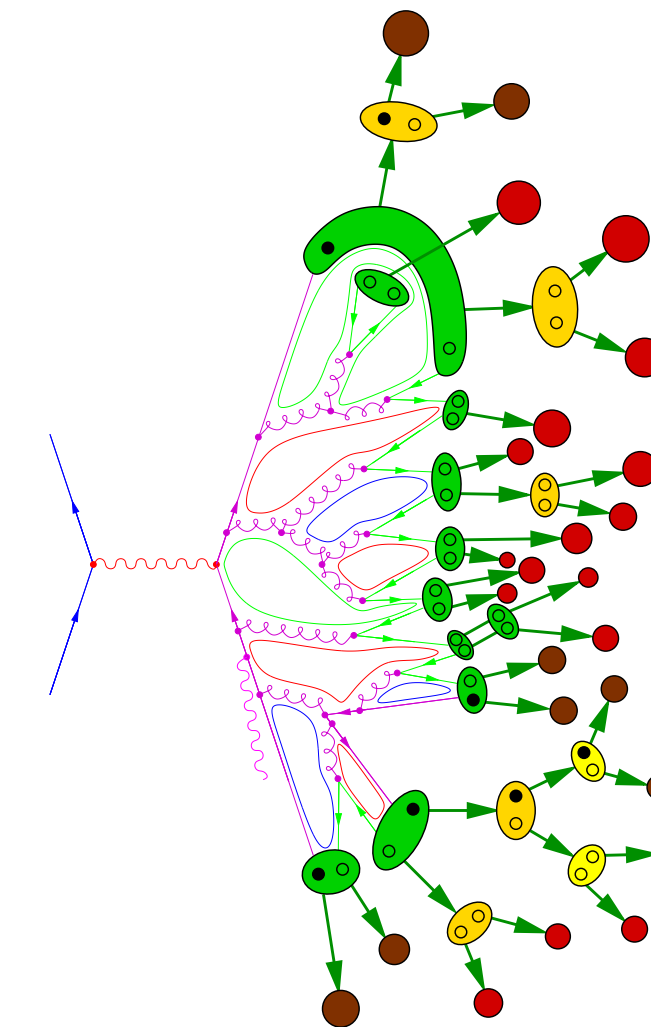
## Lund-string fragmentation

implemented in PYTHIA



## cluster-hadronisation model

implemented in HERWIG & SHERPA





# underlying event modelling

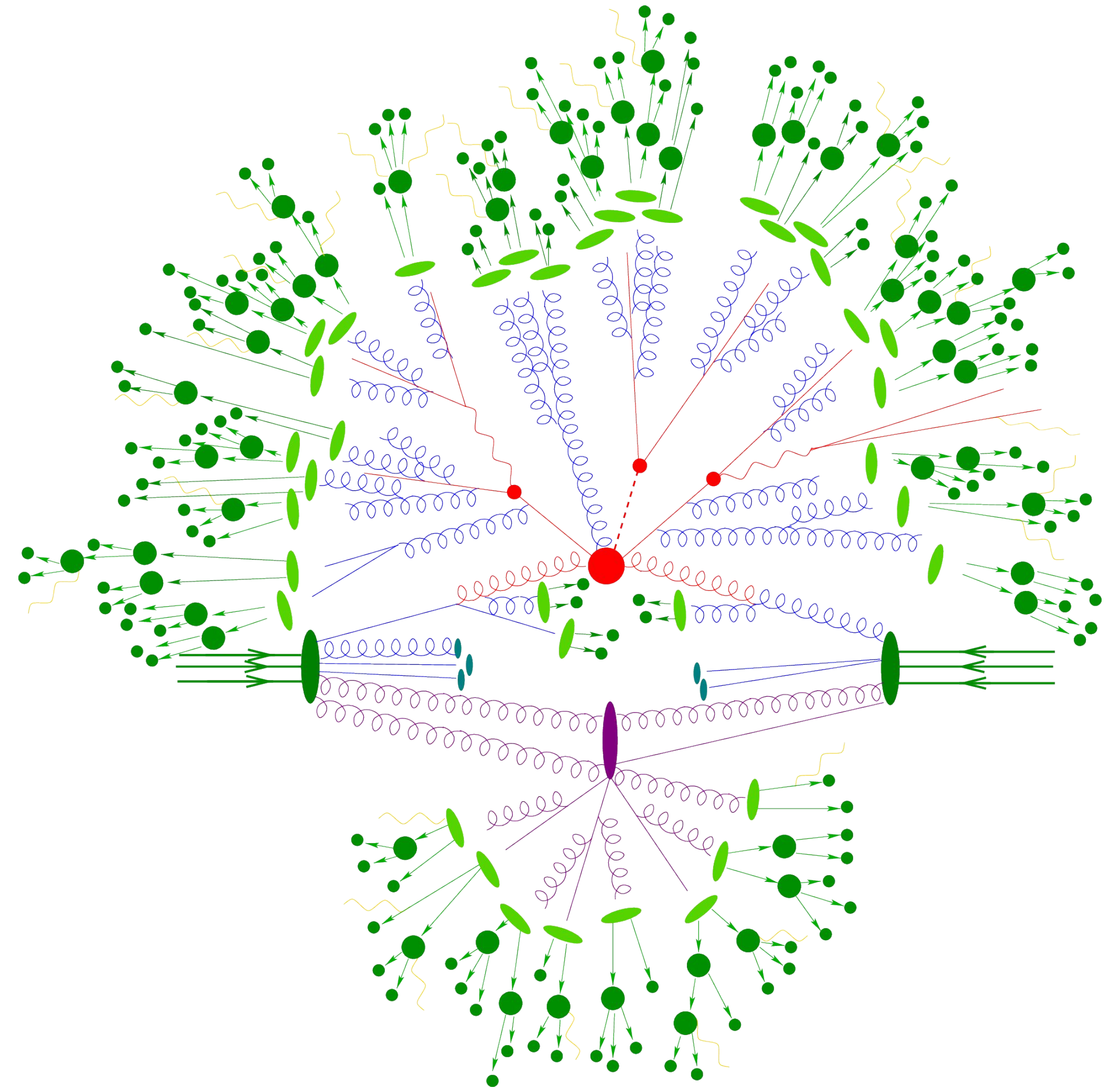
- primary parton collision  $\rightarrow$  leaves energetic remnants, can in principle interact again
- simple model [Sjöstrand, Zijl Phys. Rev. D 36 (1987) 2019]:
  - hard process at scale  $p_{t,hard}$
  - generate sequence of additional  $2 \rightarrow 2$  scatterings ordered in  $p_t$

$$\mathcal{P}(p_t) = \frac{1}{\sigma_{ND}} \frac{d\sigma_{QCD}^{2 \rightarrow 2}}{dp_t^2} \exp \left[ - \int_{p_t^2}^{p_{t,hard}^2} \frac{1}{\sigma_{ND}} \frac{d\sigma_{QCD}^{2 \rightarrow 2}}{dp_t'^2} dp_t'^2 \right]$$



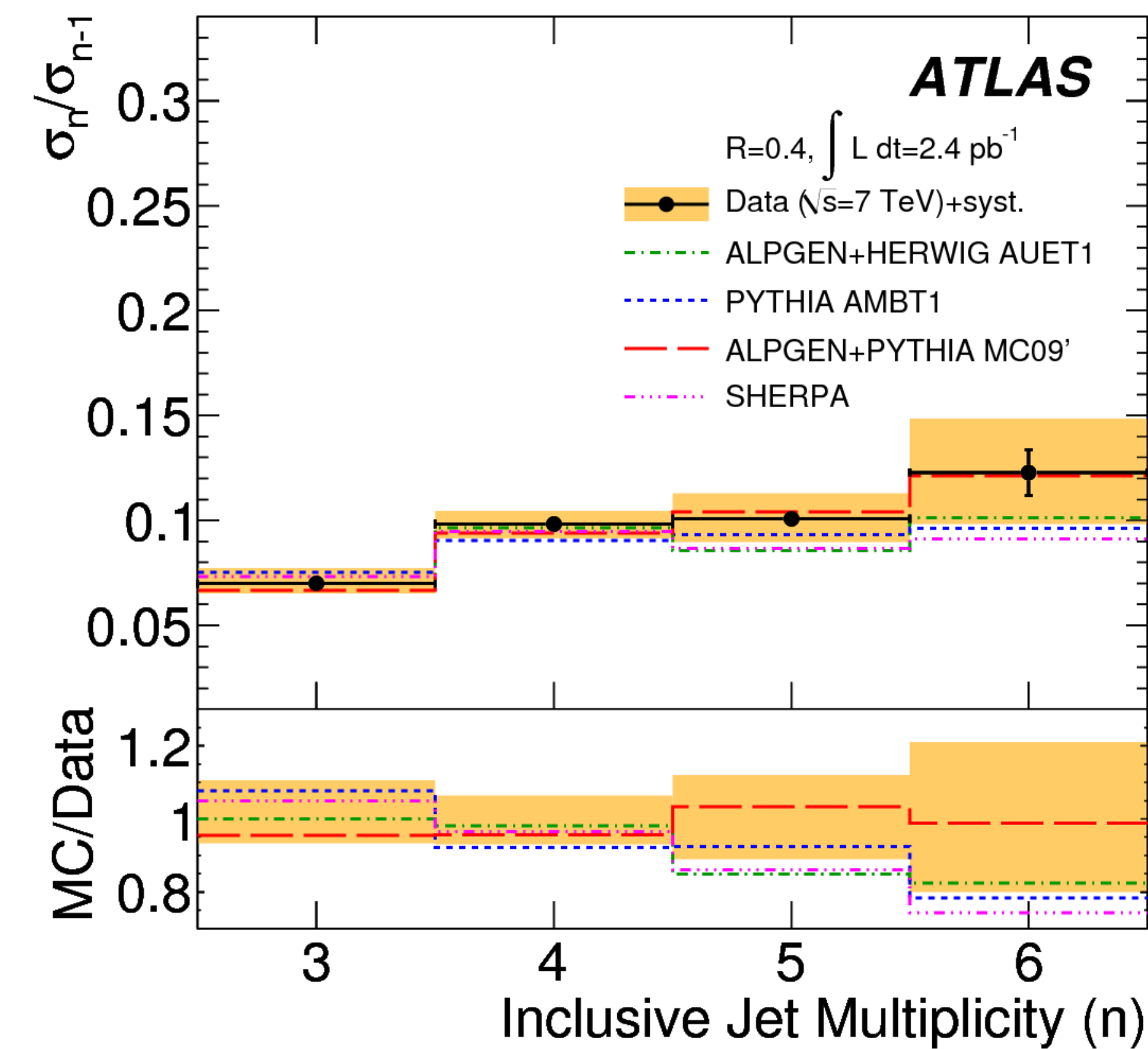
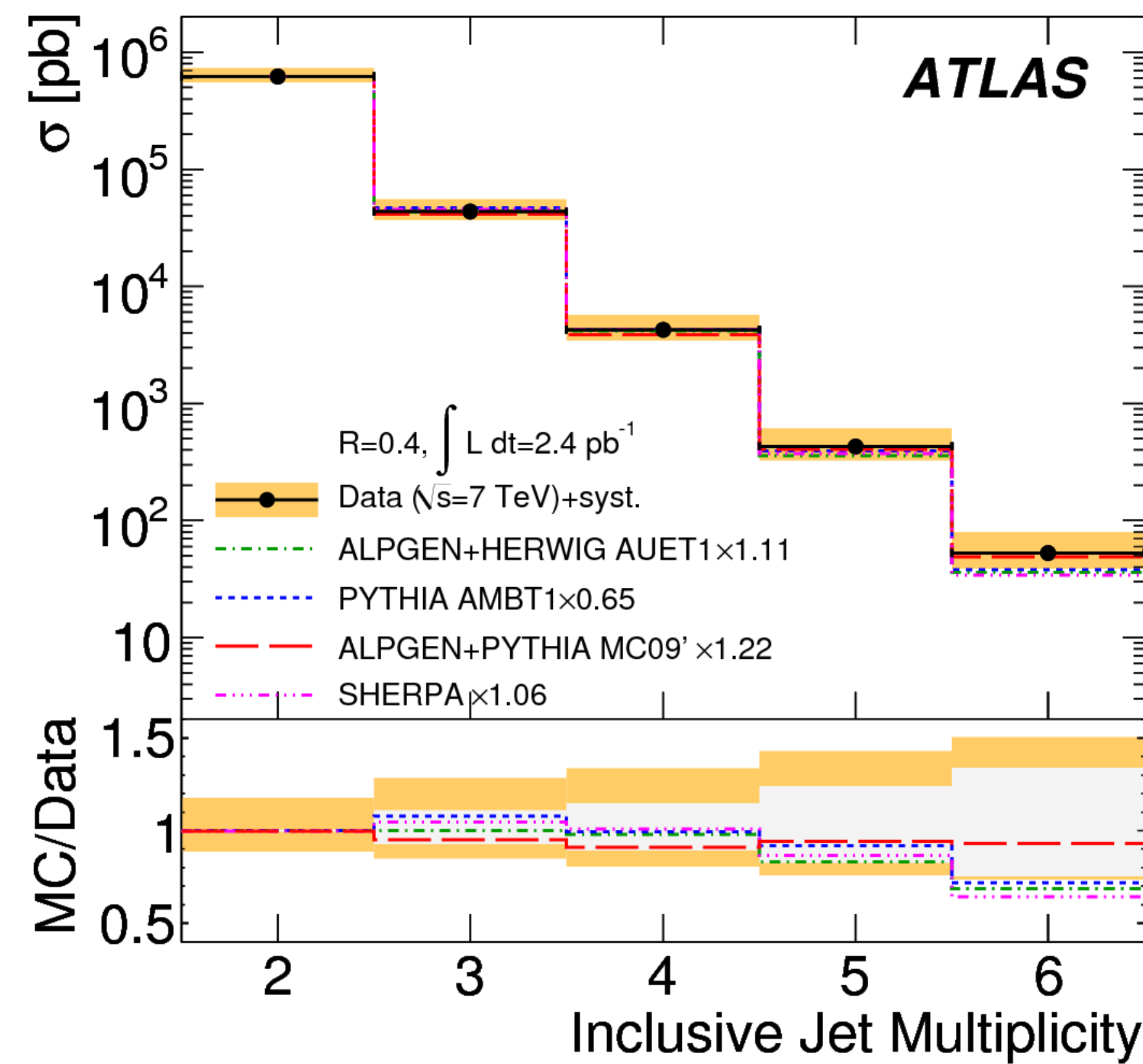
# putting it all together - event generators

- Events factorised into
  - **Hard Process**
    - automated generation of up to **NLO** **matrix elements**
  - **QCD radiation**
    - Markov-Chain Monte Carlo in **parton shower** algorithms
    - **matching** to NLO calculations
  - **PDFs/Beams/Underlying event**
    - **modelling** of beam remnants / underlying event / multiple parton interactions
  - **Hadrons**
    - modelling of **hadronisation**
    - simulation of **hadron decays**



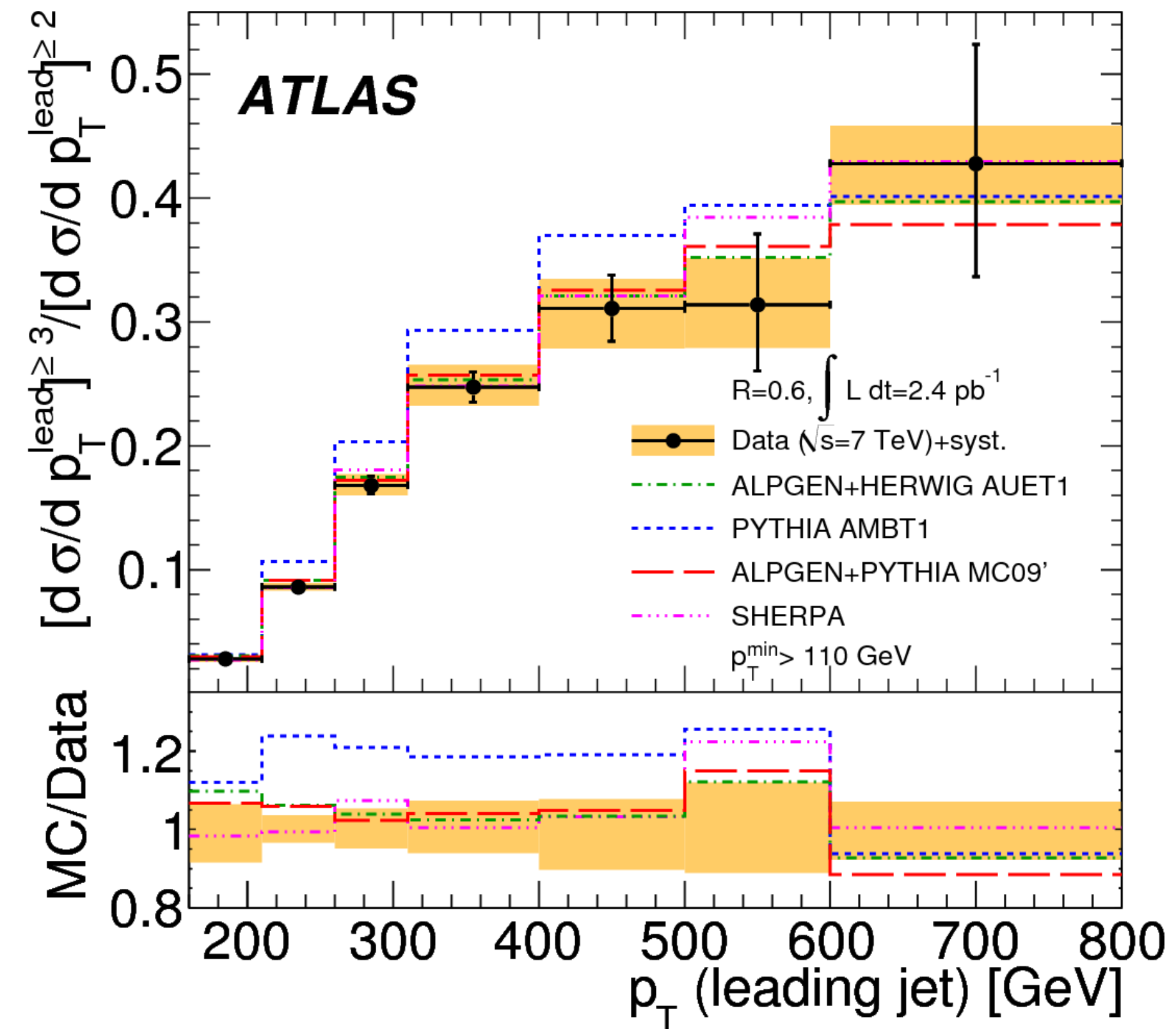
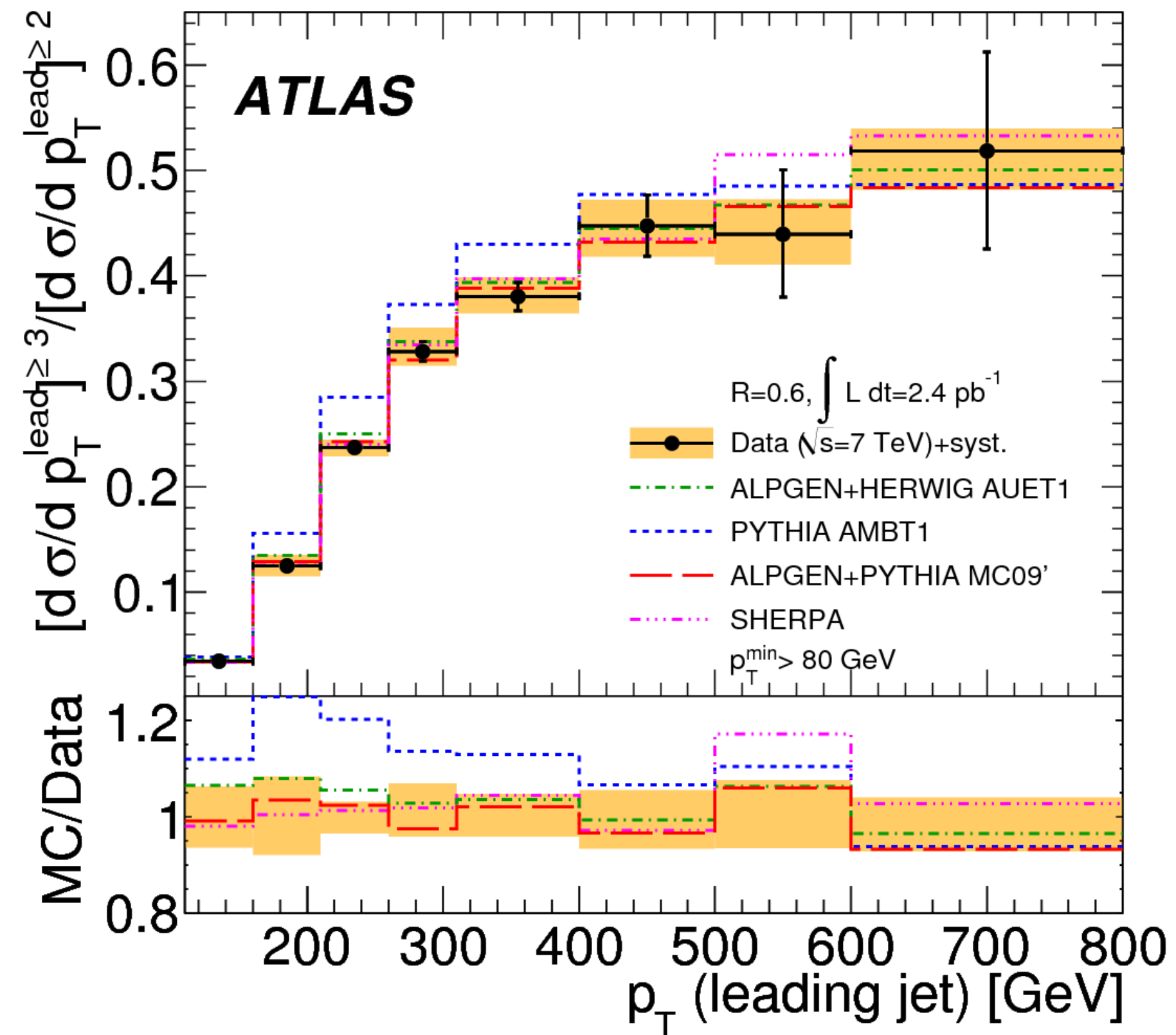
# QCD in event generators

## ATLAS pure jets analysis [G. Aad et al. Eur. Phys. J. C **71** (2011) 1763]



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# QCD in event generators

ATLAS  $Z(\rightarrow e^+e^-/\mu^+\mu^-) + \text{jets}$  analysis [G. Aad et al. Phys. Rev. D **85** (2012) 032009]

