

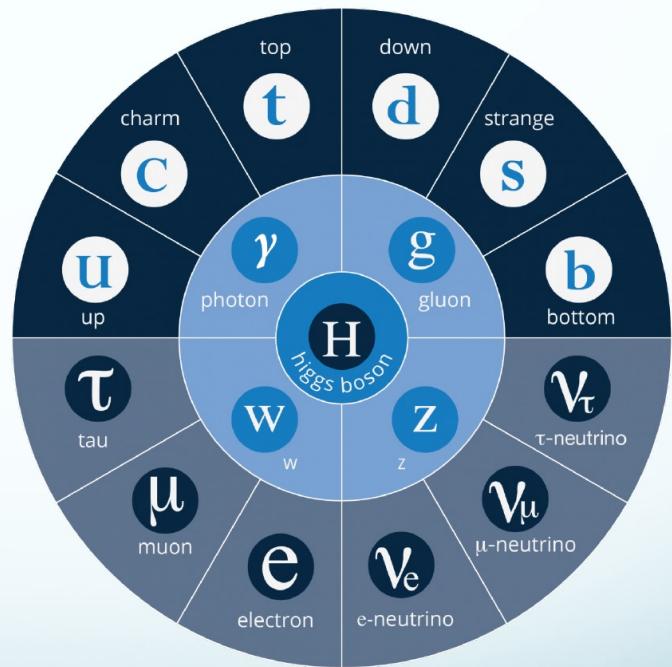
The Standard-Model

Ulla Blumenschein, Queen Mary University of London

- Introduction Standard model
- Precision tests of electroweak physics at the LHC

From time immemorial, man has desired to comprehend the complexity of nature in terms of as few elementary concepts as possible.

Abdus Salam (Nobel Price in physics 1979)



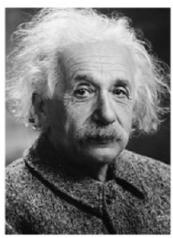
Relativistic notation:

Space-time 4-vector:

$$x^\mu = (ct, x, y, z)$$

$$x_\mu = (ct, -x, -y, -z)$$

Minkowski metric:



$$a \cdot b = a^\mu b_\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$



Albert Einstein

Herbert Minkowski,
Goettingen 1902-09

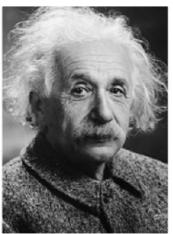
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HEP: Energy-momentum 4-vector:

$$p^\mu = m\gamma \frac{dx^\mu}{dt} = \left(\frac{E}{c}, \mathbf{p} \right)$$

$$E = \gamma mc^2$$

$$\mathbf{p} = \gamma m\mathbf{v}$$

Lorentz-
factor

$$\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-1/2}$$

In most slides, we use natural units: $\hbar = c = 1$

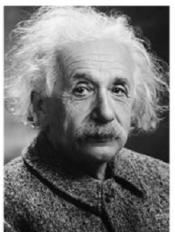
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The norm of an Energy-momentum 4-vector is its (invariant) mass

$$p^2 = \frac{E^2}{c^2} - |\mathbf{p}|^2 = m^2$$

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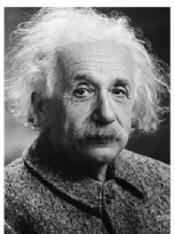
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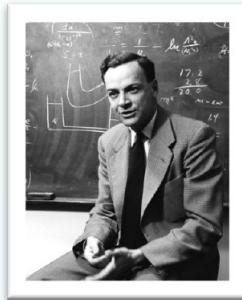
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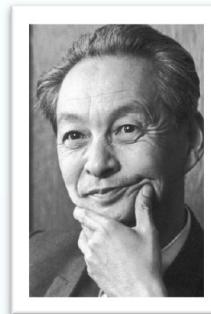
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(1) Towards QED



Richard Feynman



Shin'Ichiro Tomonaga



Julian Schwinger

“I would rather have questions that can't be answered than answers that can't be questioned.”

Richard Feynman (Nobel Price in physics 1965)

QM description of Fermions

Applying quantum substitution

$$\vec{p} \rightarrow i\hbar\vec{\nabla}$$

$$E \rightarrow i\hbar\frac{\partial}{\partial t}$$

(1) to **classic energy-momentum relation**

$$E = p^2/2m + V$$



E. Schrödinger

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→ Schroedinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r}, t) + V(\vec{r}, t)\psi(\vec{r}, t)$$

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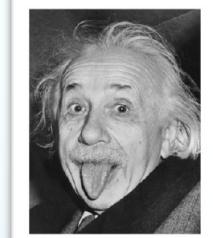
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$$p_\mu \rightarrow i\hbar\partial_\mu$$

$$\partial_\mu = \frac{\partial}{\partial X^\mu}$$



(2) to **relativistic energy-momentum relation:**

$$E^2 = p^2c^2 + m^2c^4$$

A. Einstein

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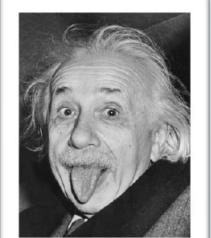
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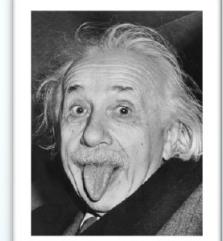
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describes scalar particles but not fermions. → need equation first order in time



The Dirac equation

→ 4D matrices: γ -matrices

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}$$

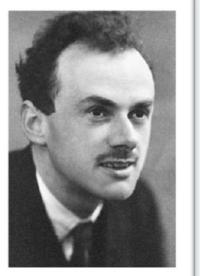
$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

Paul Dirac

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



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$$\psi = (\psi_0, \psi_1, \psi_2, \psi_3)$$

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Implicit sum convention: Sum over all indices: $\mu = 0,1,2,3$

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Solutions:

$$\psi^{(i)} = u^{(i)}(E, \mathbf{p}) e^{-\frac{1}{\hbar}(Et - \mathbf{p}\mathbf{x})}$$

spinor

wave

The Dirac equation



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Electrons:



Positrons



spin component
in direction of motion

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ p_z/(E+m) \\ (p_x + ip_y)/(E+m) \end{bmatrix} \quad u_2 = \begin{bmatrix} 0 \\ 1 \\ (p_x - ip_y)/(E+m) \\ -p_z/(E+m) \end{bmatrix}$$



$$v_1 = \begin{bmatrix} (p_x - ip_y)/(E+m) \\ -p_z/(E+m) \\ 0 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} p_z/(E+m) \\ (p_x + ip_y)/(E+m) \\ 1 \\ 0 \end{bmatrix}$$



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probability density/current:

$$j^\mu = \bar{\psi} \gamma^\mu \psi = (\bar{\psi} \gamma_0 \psi + \bar{\psi} \vec{\gamma} \psi) = (\rho, \vec{j})$$

Electrons:



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The EM force

Classic Maxwell equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$



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→ **Homogeneous MWE:** B and E as derivative of potentials V and \vec{A} \Rightarrow

$$\vec{B}(\vec{x}, t) = \nabla \times \vec{A}(\vec{x}, t)$$

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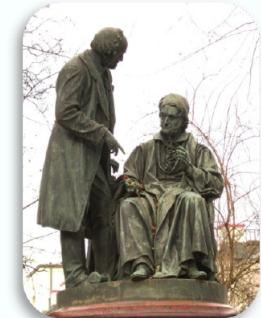
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→ relativistic description: 4-potential
4-current

$$A^\mu = (V, \vec{A})$$

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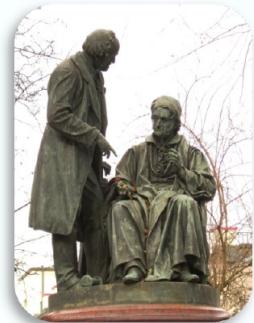
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Field strength tensor:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix}$$

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→ **Inhomogeneous MWE:**

$$\partial_\mu F^{\mu\nu} = J^\nu$$

The Lagrangian density

Classical mechanics: Lagrange function: $L = T - V$

Fundamental law of motion: Euler-Lagrange equation:

$$\frac{\partial L}{\partial q_j} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_j} = 0$$

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Field theory: Lagrangian is a function
of fields and their 4-derivatives

$$\mathcal{L}(\phi, \partial_\mu \phi)$$

→ Euler-Lagrange equation:

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial_\mu \phi)} = \frac{\delta \mathcal{L}}{\delta \phi}$$

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$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

→ Euler Lagrange equation
= Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

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Example: **Lagrangian of free photon:**

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(B^2 - E^2)$$

Gauge symmetries

Noether theorem:

Continuous symmetries → corresponding conserved quantities

*Emmy Noether,
Goettingen 1915-33*



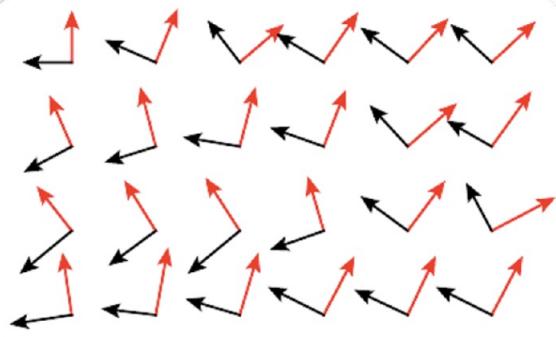
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Dirac Lagrangian is invariant
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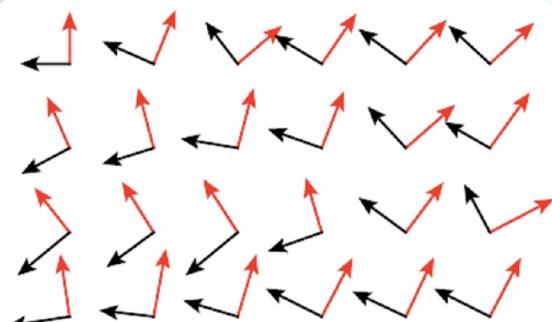
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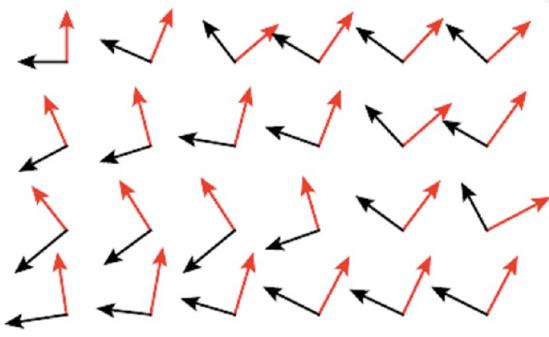
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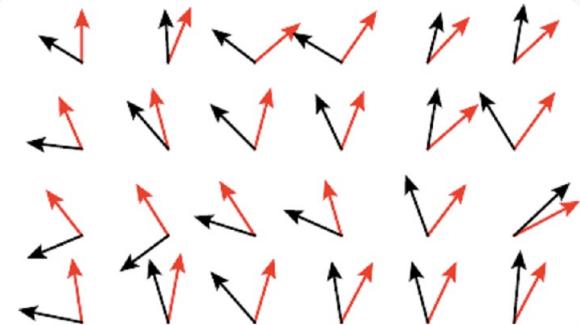
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$$\psi \rightarrow \psi' = e^{-iq\alpha} \psi$$

Now require also invariance under
local phase transitions



Objects are only influenced by their
immediate surroundings
(principle of locality)

Gauge symmetries

Noether theorem:

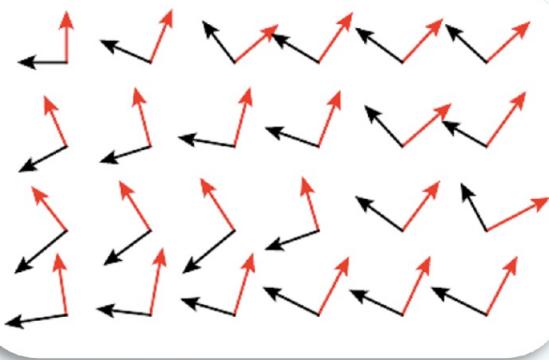
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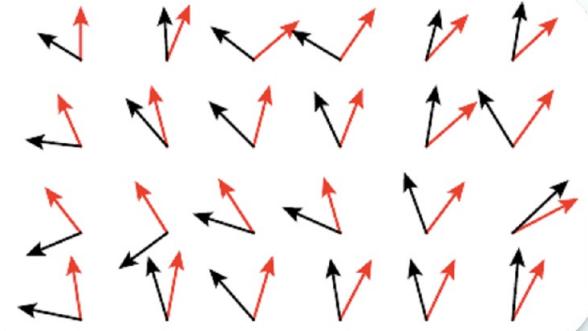
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$$\psi \rightarrow \psi' = e^{-iq\alpha(x_\mu)} \psi$$

Gauge invariance

When calculating the QED Lagrangian

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

with the transformed fermion field

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Gauge invariance

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We obtain a spurious inner derivative $\partial_\mu \alpha(x)$ destroying the gauge invariance

Gauge invariance

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We obtain a spurious inner derivative

$$\partial_\mu \alpha(x)$$

destroying the gauge invariance

But what about the principle of locality

?



Gauge invariance

When calculating the QED Lagrangian with the transformed fermion field

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

$$\psi \rightarrow \psi' = e^{-iq\alpha(x_\mu)}\psi$$

We obtain a spurious inner derivative $\partial_\mu \alpha(x)$ destroying the gauge invariance

→ Replace the 4-derivative by the covariant derivative D_μ by adding interaction with photon field A_μ

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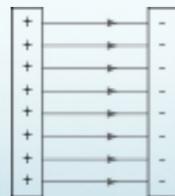
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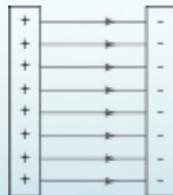
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using the gauge freedom of the photon field

→ inner derivative is cancelled → gauge invariance restored



Gauge invariance

→ New Lagrangian:

$$\mathcal{L}_{QED} = i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi$$

Adding kinematic term for free photon:

$$D_\mu = \partial_\mu + iqA_\mu$$

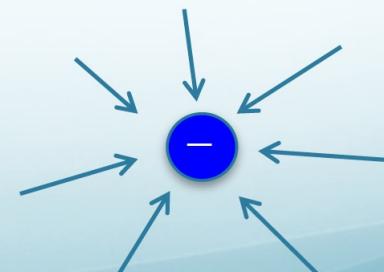
$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi$$

.. Yields the Maxwell-equations for current density

$$j_{EM}^\mu = e\bar{\psi}\gamma^\mu\psi$$

Needed to introduce photon field by requiring local U(1) gauge invariance of the Lagrangian

Charged particles are always accompanied by EM field



Perturbation theory

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi - e\bar{\psi}\gamma^\mu A_\mu\psi$$

→ Scattering matrix

$$S_{\alpha\beta} = \langle \alpha_{in} | S | \beta_{out} \rangle$$

$$S = T \exp \left(-i \int_{-\infty}^{\infty} dt V(t) \right)$$

$$V = e \int d^3x \bar{\psi}\gamma^\mu\psi A_\mu$$

Perturbation Hamiltonian
in the Interaction picture

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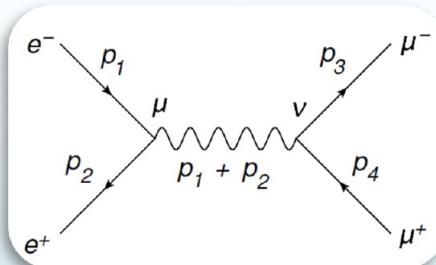
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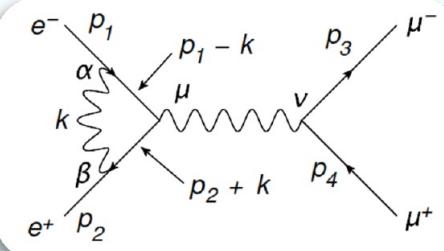
Perturbation Hamiltonian
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→ Perturbation expansion: terms can be graphically represented as
example: $e^-e^+ \rightarrow \mu^-\mu^+$ Feynman diagrams



LO

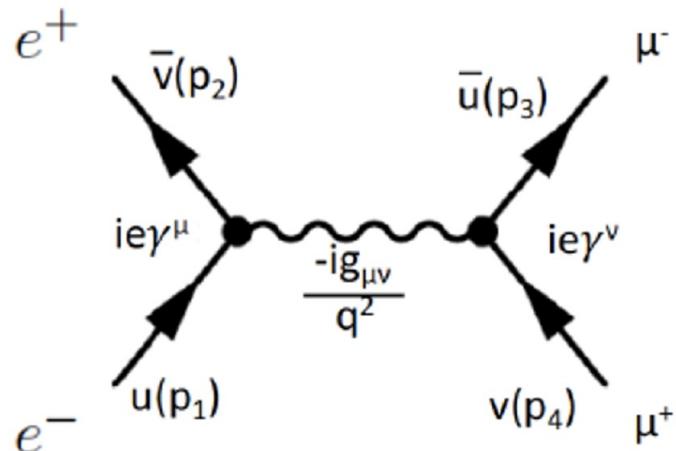
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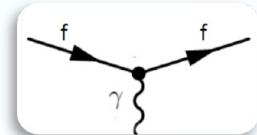
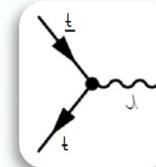
NLO

+

Feynman diagrams



EM vertex:



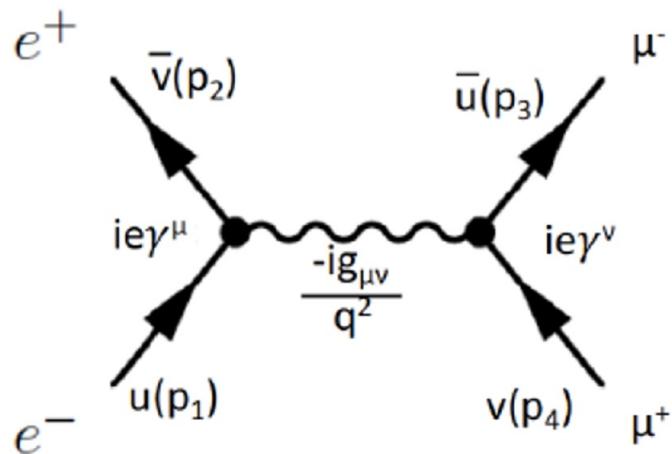
Each line and each vertex represents a mathematical term

$$\bar{\psi} \equiv \psi^\dagger \gamma^0$$

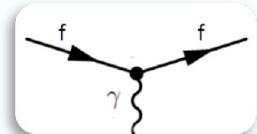
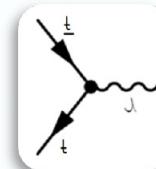
$$g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

A set of Feynman Rules allows to translate any diagram to an amplitude without explicitly carrying out the perturbative expansion of the S-matrix

Feynman diagrams



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 → Resulting matrix element:

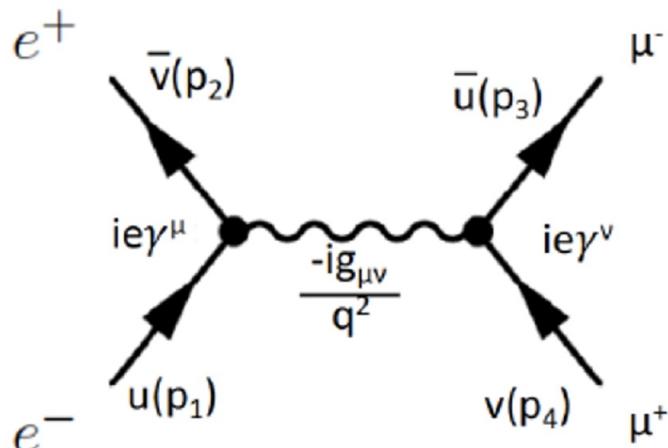
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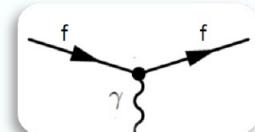
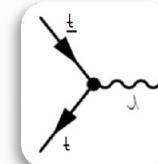
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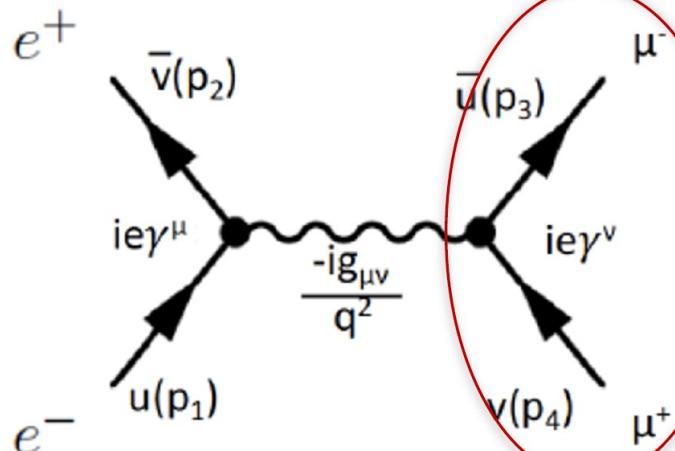
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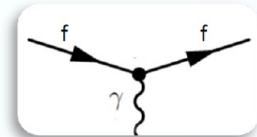
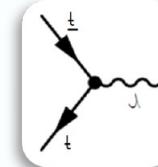
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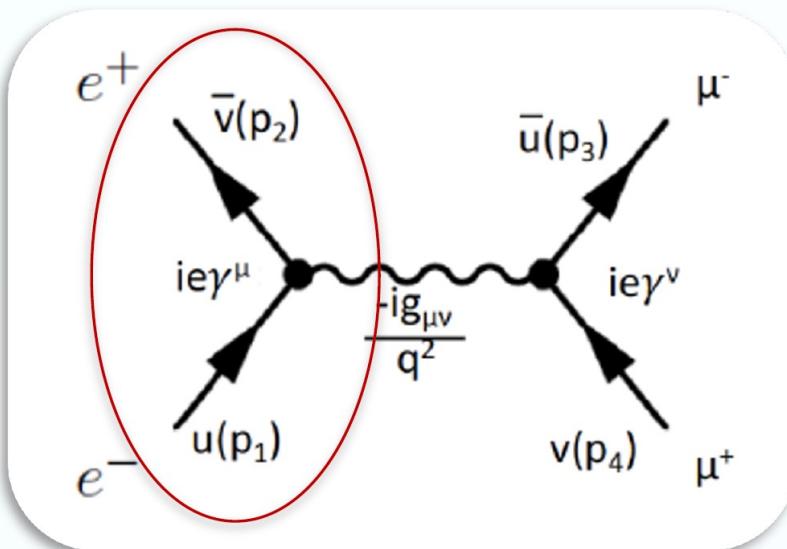
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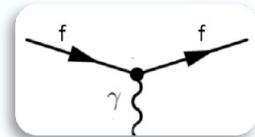
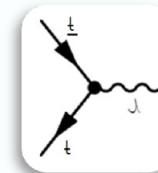
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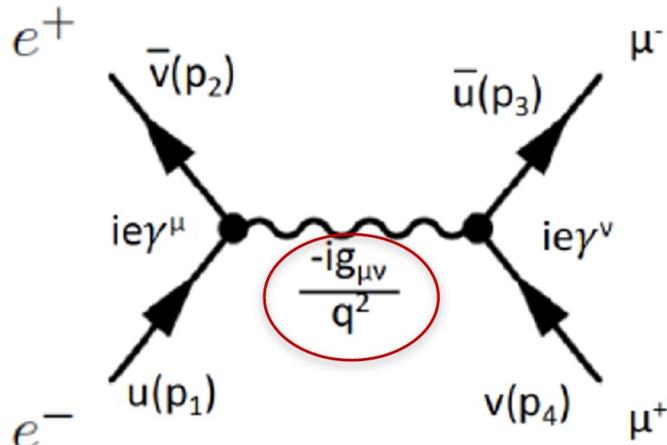
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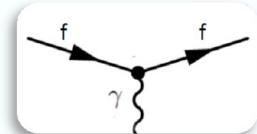
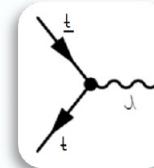
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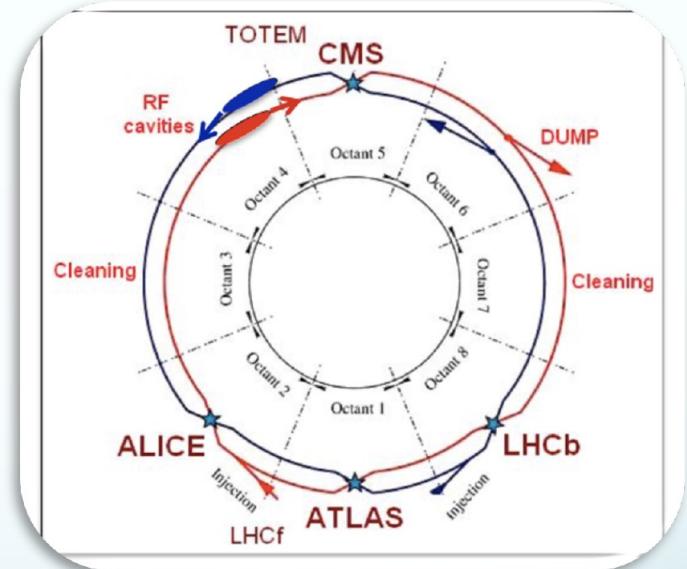
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From amplitudes to cross sections

Particle Collider: Interaction rate depends on luminosity and cross section

$$N_{\text{events}} = \sigma(e^+e^- \rightarrow \mu^+\mu^-) \cdot \mathcal{L}dt$$

Luminosity \mathcal{L} depends on number of particles per bunch, bunch frequency and beam profile: “flux”



From amplitudes to cross sections

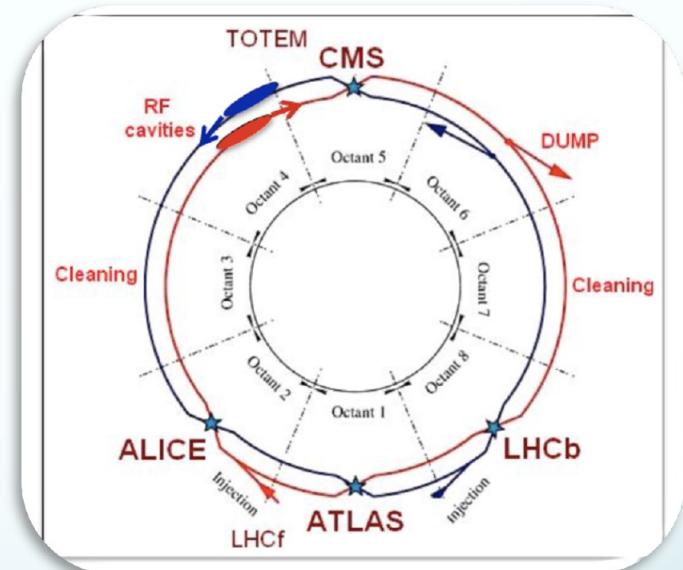
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depends on squared matrix element

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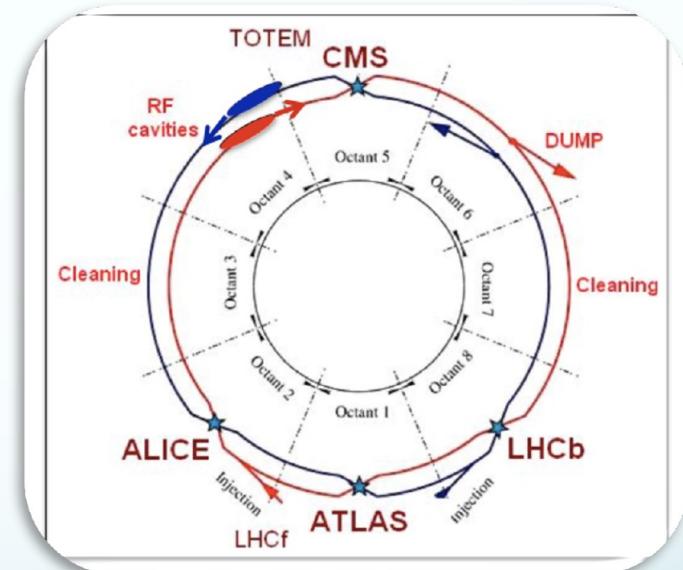
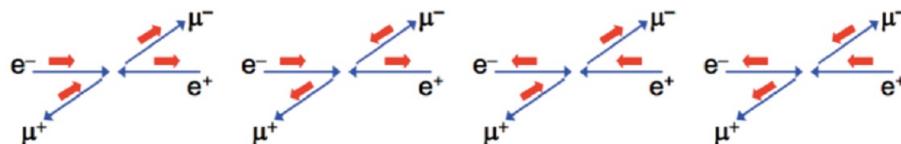
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Add matrix elements for the various spin combinations, example: $e^+e^- \rightarrow \mu^+\mu^-$



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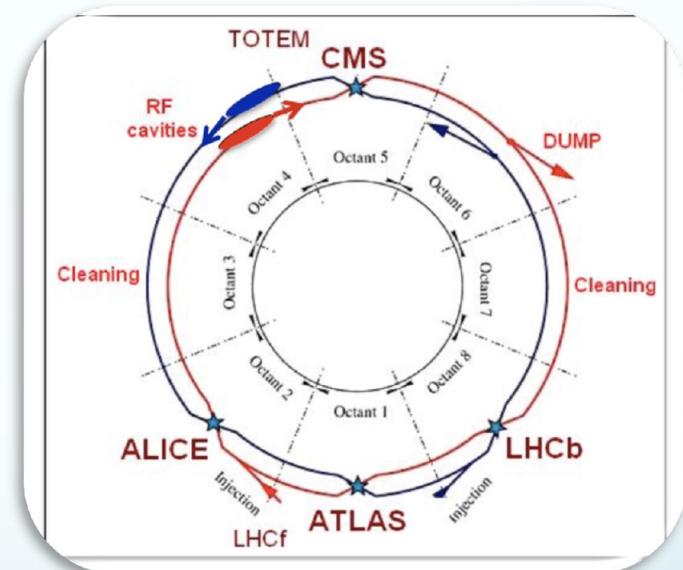
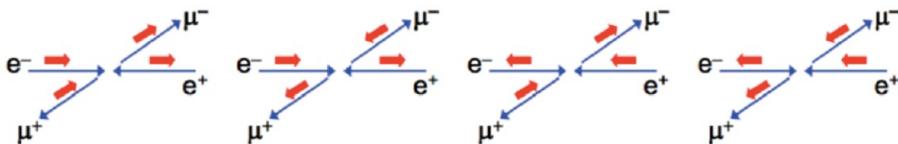
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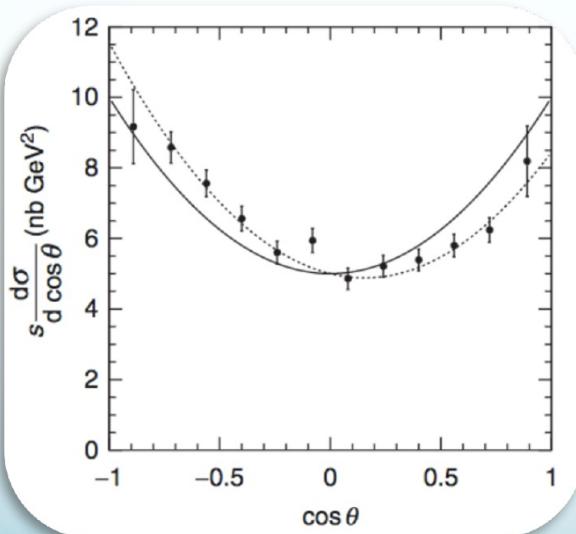
Average over *initial-state* spins, sum over *final-state* spins

From amplitudes to cross sections

Integration over phase space → total cross section

Example: $e^+ e^- \rightarrow \mu^+ \mu^-$

$$\frac{d\sigma}{\sin\theta d\theta d\phi} = \frac{\alpha^2}{4s} (1 + \cos^2\theta)$$



$$s = (E_1 + E_2)^2$$

From amplitudes to cross sections

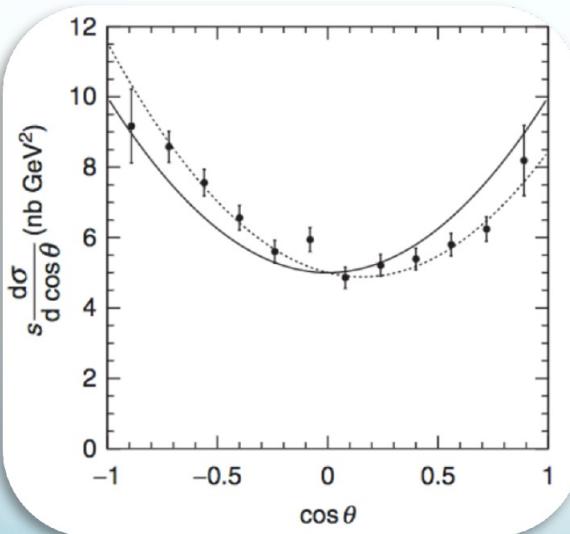
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