

Silicon Detectors: from Hits to Tracking to Vertexing

Andrei Nomerotski (Oxford University)

Outline

- Why vertexing with silicon detectors
- Limits on the hit resolution
- Alignment of silicon systems
- Pattern recognition and track fitting

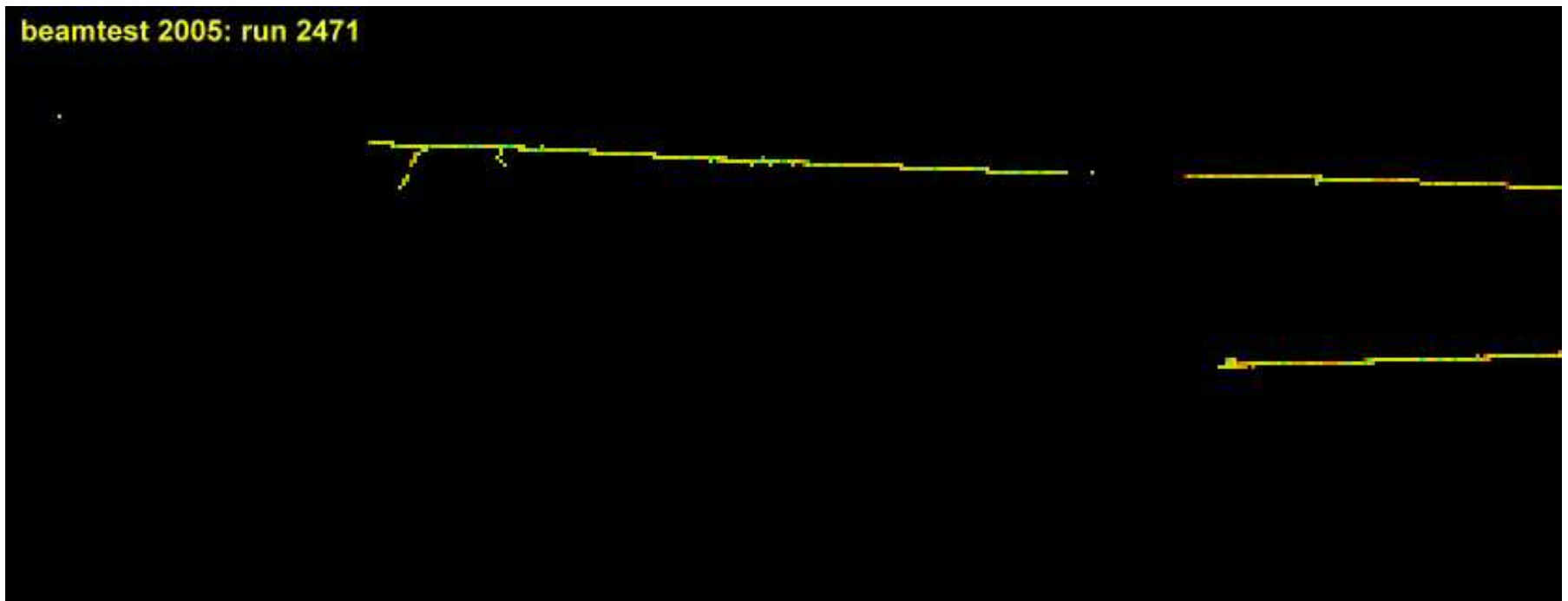
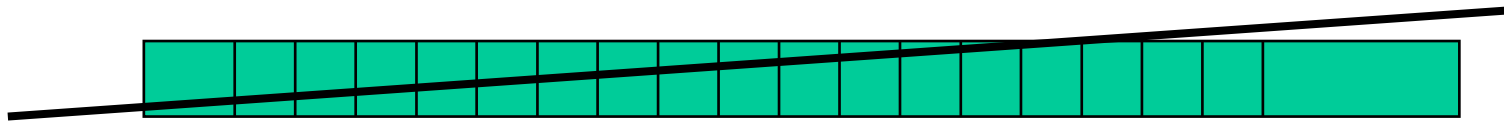
- Applications of this precision detectors outside HEP

Solid State Tracking Detectors

- Why Silicon?
 - ◆ Crystalline silicon band gap is 1.1 eV (small)
 - ▲ yields 80 electron-hole pairs/ μm for minimum-ionizing track
 - (1 e-h pair per 3.6 eV of deposited energy)
 - ▲ 99.9% of ejected electrons have less than $1\mu\text{m}$ path length
 - fine-granularity devices can easily be made
 - ◆ Integrated Circuit manufacturing techniques make just about anything possible, and at industrial prices
 - ▲ no real need to “home-grow” these detectors
- ⇒ detector performance could be as good as bubble chamber

Silicon Pixel Detector

200 MeV protons hitting CMS pixel module at shallow angle (R.Horisberger)

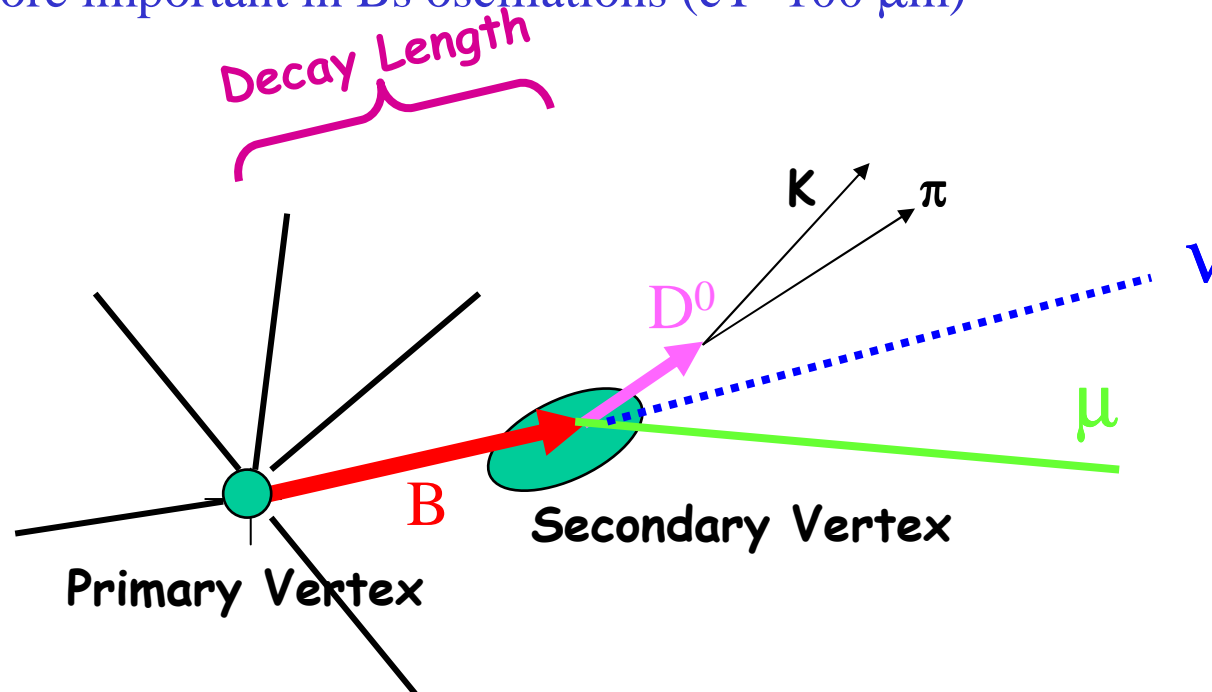


Physics Motivation

- Exclusive reconstruction of decays with secondary vertices
 - ◆ Physics of b-quark: lifetime, oscillations, CP violation
- B-tagging
 - ◆ Physics of top quark, Higgs and SUSY searches etc
 - ◆ More inclusive approach to keep efficiency high

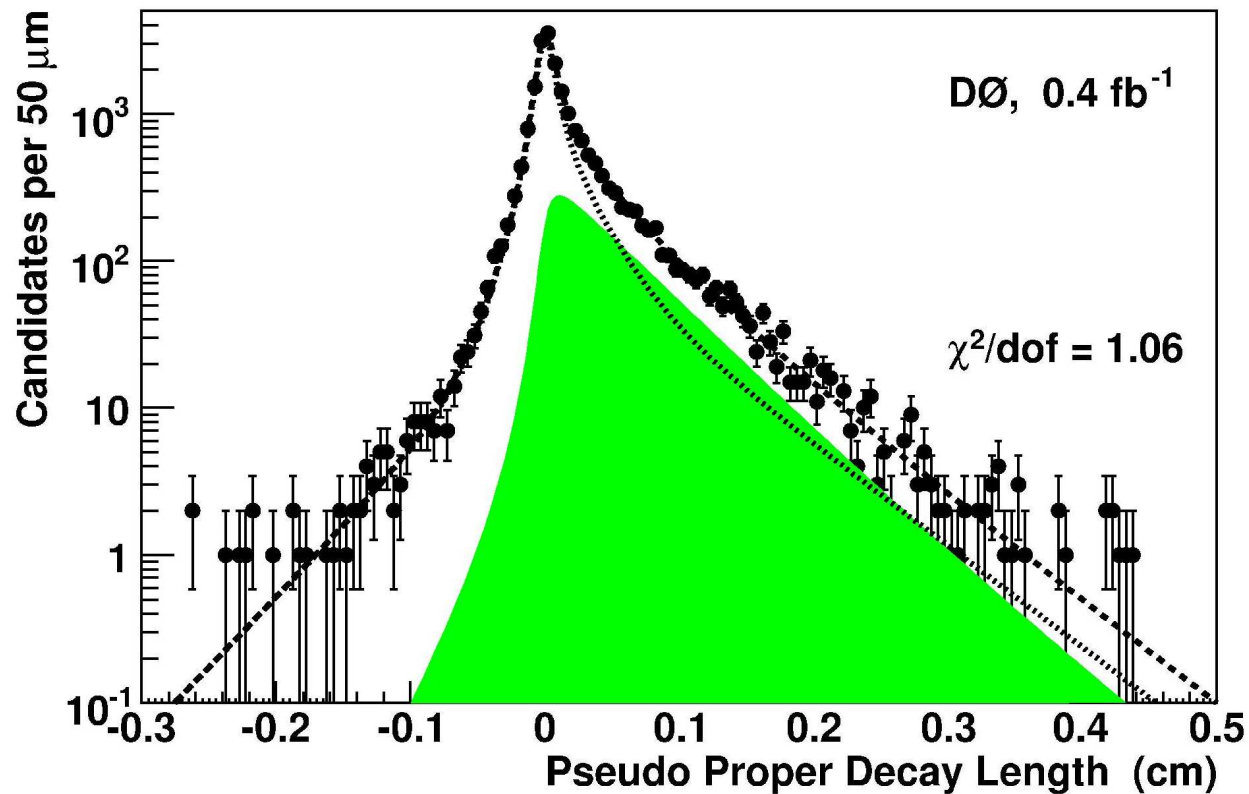
Example: Measurement of B Meson Lifetime

- Look for B vertex and measure decay length - distance between primary and secondary vertices
- Most of decays of B mesons happen within 1-2 mm of interaction point ($c\tau \sim 0.5$ mm, stretched by relativistic time dilation)
- Need vertex detectors with excellent position resolution $\sim 10 \mu\text{m}$
 - ◆ Even more important in B_s oscillations ($cT \sim 100 \mu\text{m}$)

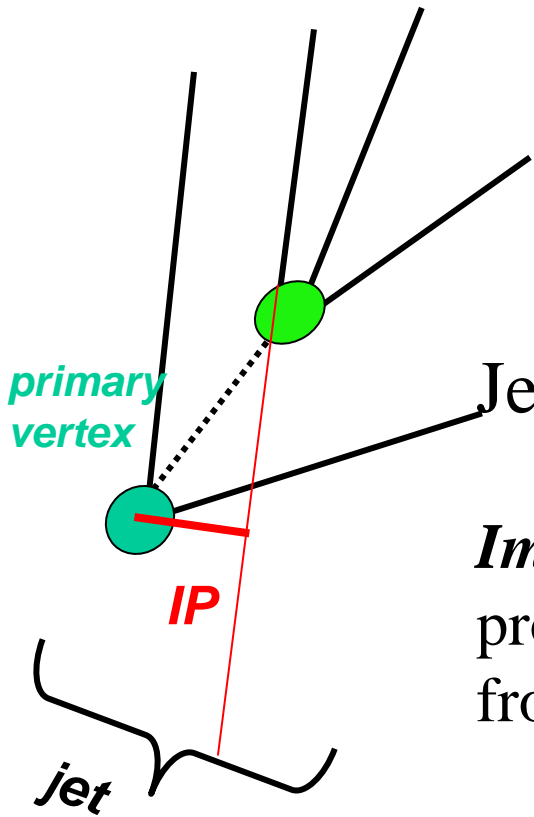


Bs Meson Lifetime

- Proper lifetime : corrected for relativistic time dilation

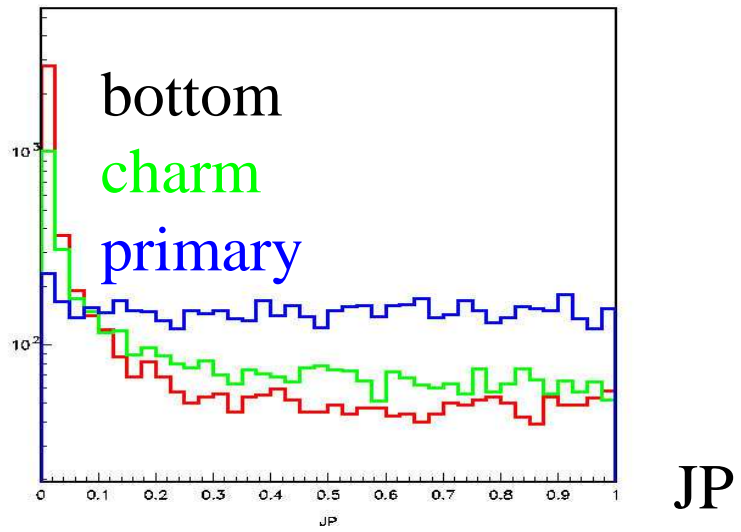


Example: b-tagging



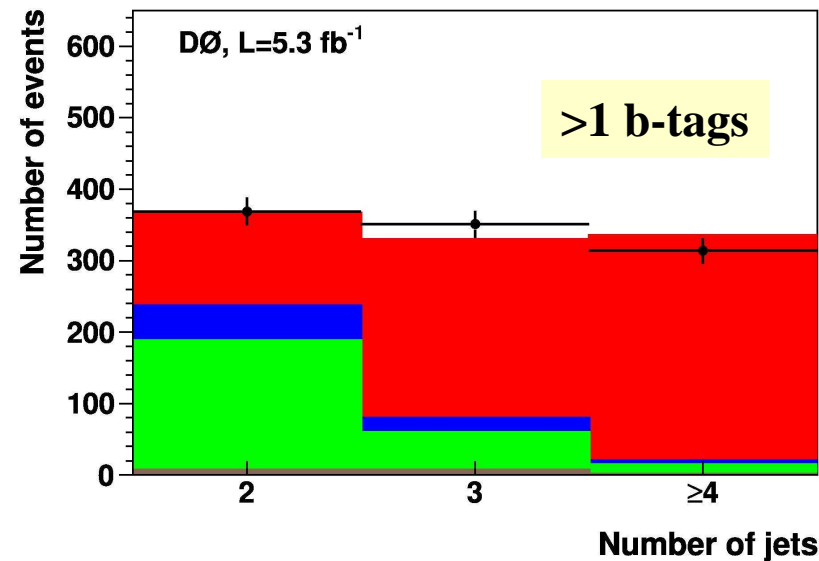
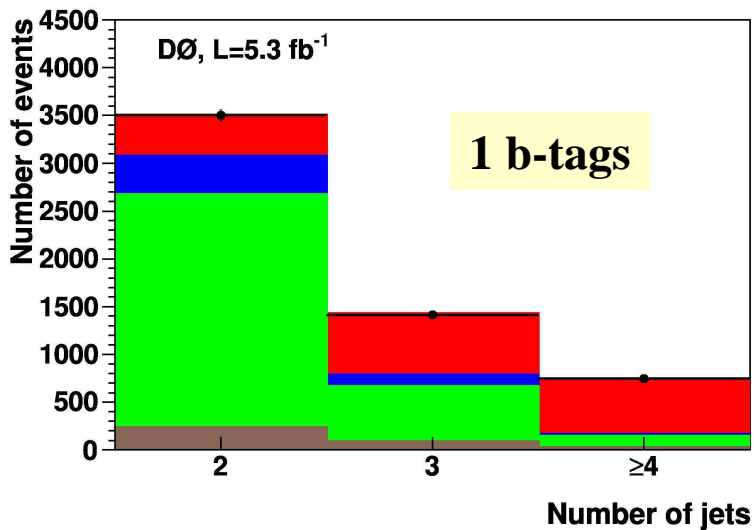
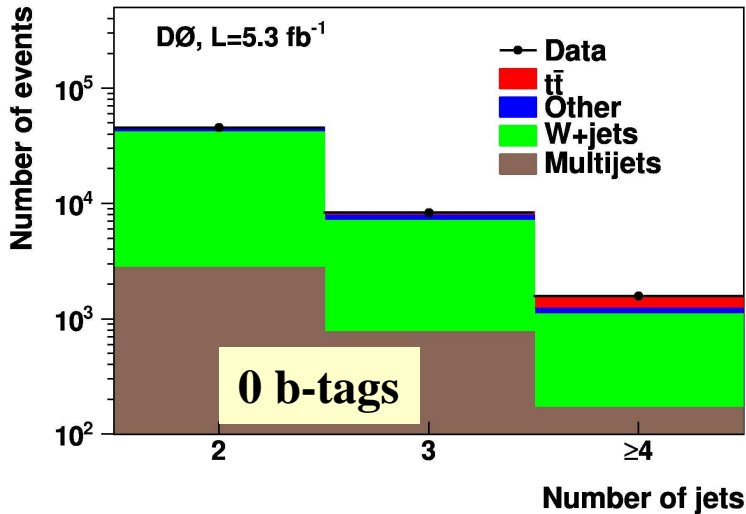
Jet Probability (JP) tagging algorithm

Impact parameter \Rightarrow *Track probability*
probability that track is consistent with coming from primary vertex.



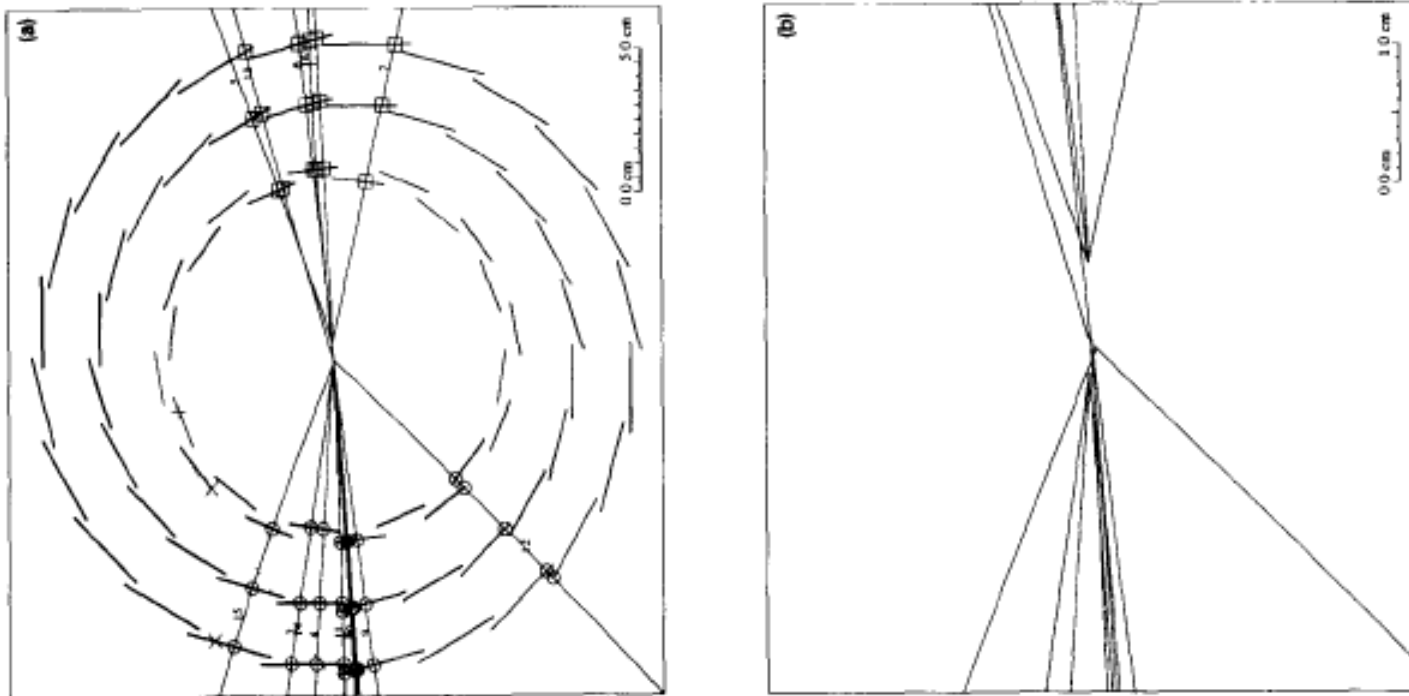
Example: b-tagging

- Top sample at DZero, Tevatron
- $tt \rightarrow bbWW \rightarrow \text{lepton} + \text{jets}$
- Pure signal after two tags!



Vertexing

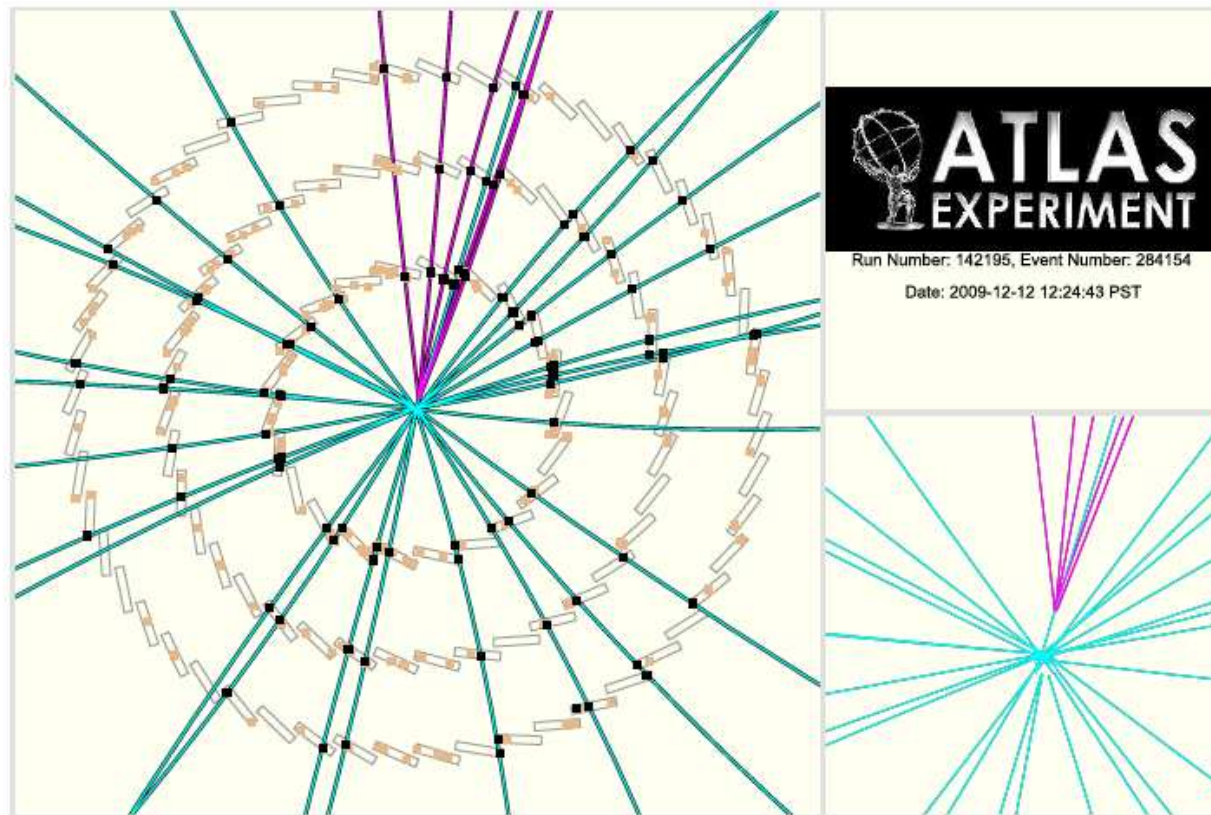
- DELPHI (e^+e^- collisions producing Z^0 bosons)



- Need precision for separation of vertices

Vertexing

- ATLAS (pp collisions)



- Silicon is viable *and* crucial at hadron colliders as well

Vertex Detectors

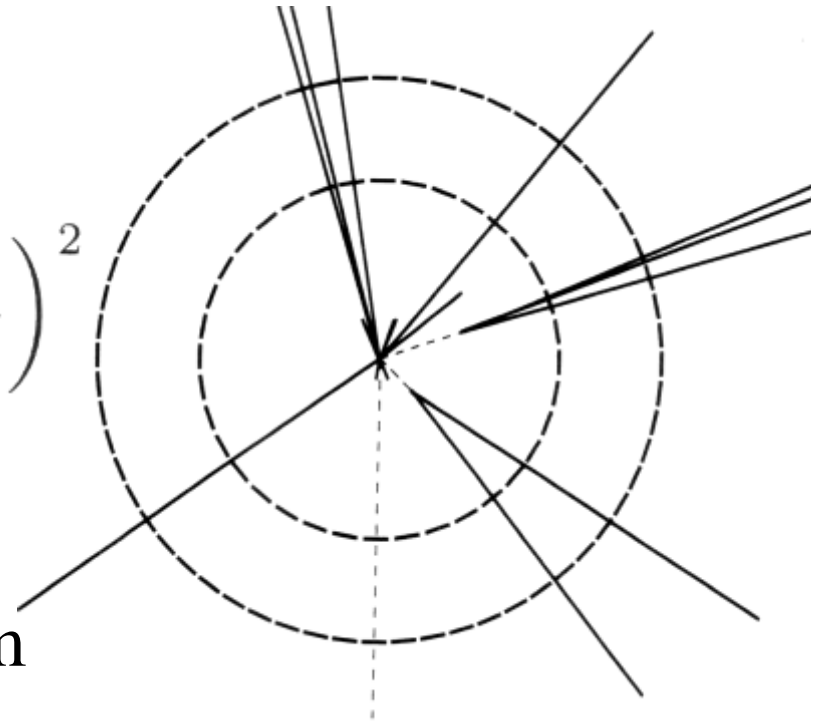
- For two layers: Error propagated to interaction point

$$\sigma_b^2 \approx \left(\frac{\sigma_1 r_2}{r_2 - r_1} \right)^2 + \left(\frac{\sigma_2 r_1}{r_2 - r_1} \right)^2 = \frac{1}{(r_2 - r_1)^2} [(\sigma_1 r_2)^2 + (\sigma_2 r_1)^2]$$

- Assuming equal resolutions

$$\left(\frac{\sigma_b}{\sigma} \right)^2 \approx \left(\frac{1}{1 - r_1/r_2} \right)^2 + \left(\frac{1}{r_2/r_1 - 1} \right)^2$$

- r_1/r_2 should be as small as possible
- for $\sigma=10 \mu\text{m}$, $r_1/r_2=0.5$, $\sigma_b = 22 \mu\text{m}$



Some figures and examples here and later from

Helmuth Spieler “Semiconductor Detector Systems”, 2005 Oxford University Press

Multiple Scattering

- In the above cannot make r_2 too large – need to account for multiple scattering
- For ex. Be beam pipe (ϕ 5 cm, thickness 1 mm)
 - ◆ $X_0=35.3$ cm; $x/X_0=0.0028$
 - ◆ Corresponds to 28 μm at IP for $P = 1$ GeV

Conclusions

- Measure hits as precisely as possible
- First layer as close as possible to Interaction Point
- First layer as thin as possible

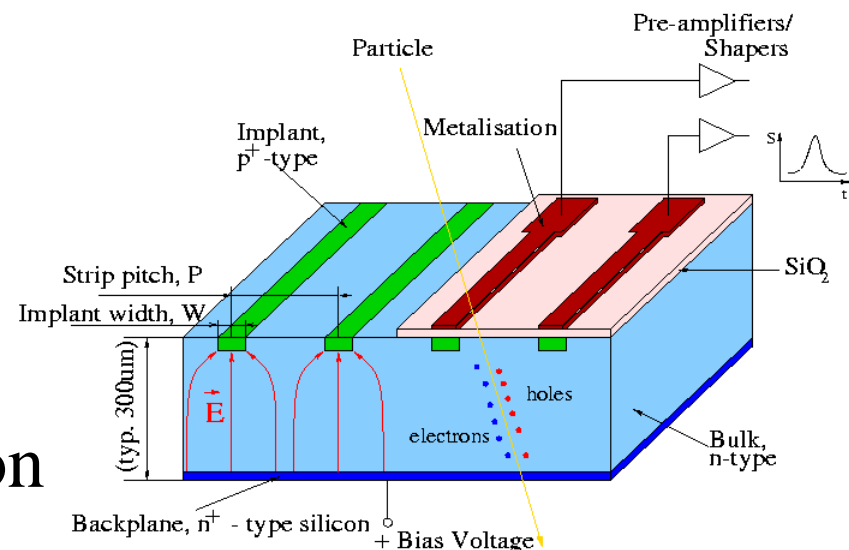
Position Resolution: Geometry

- Strip detectors are 100% efficient despite of gaps between strips – all field lines end on electrodes
→ electrical segmentation determined by pitch
- If tracks are distributed uniformly and every strip is readout:

$$\sigma^2 = \int_{-p/2}^{p/2} \frac{x^2}{p} dx = \frac{p^2}{12}$$

- If signal split across strips charge sharing can improve on this resolution

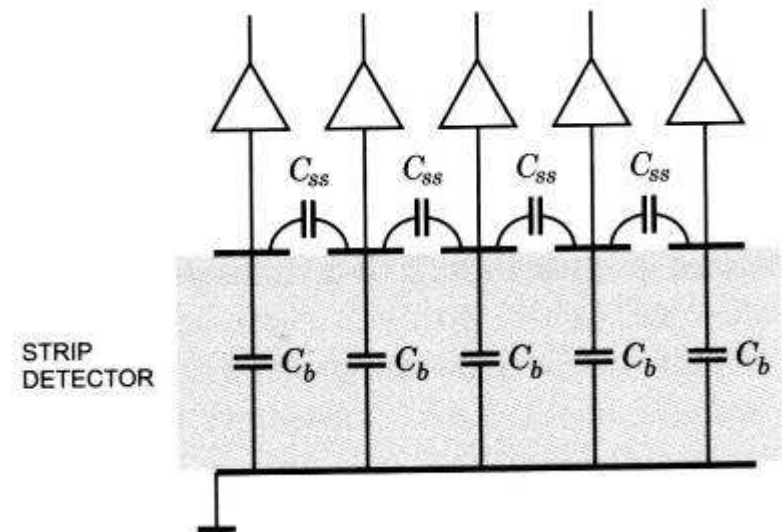
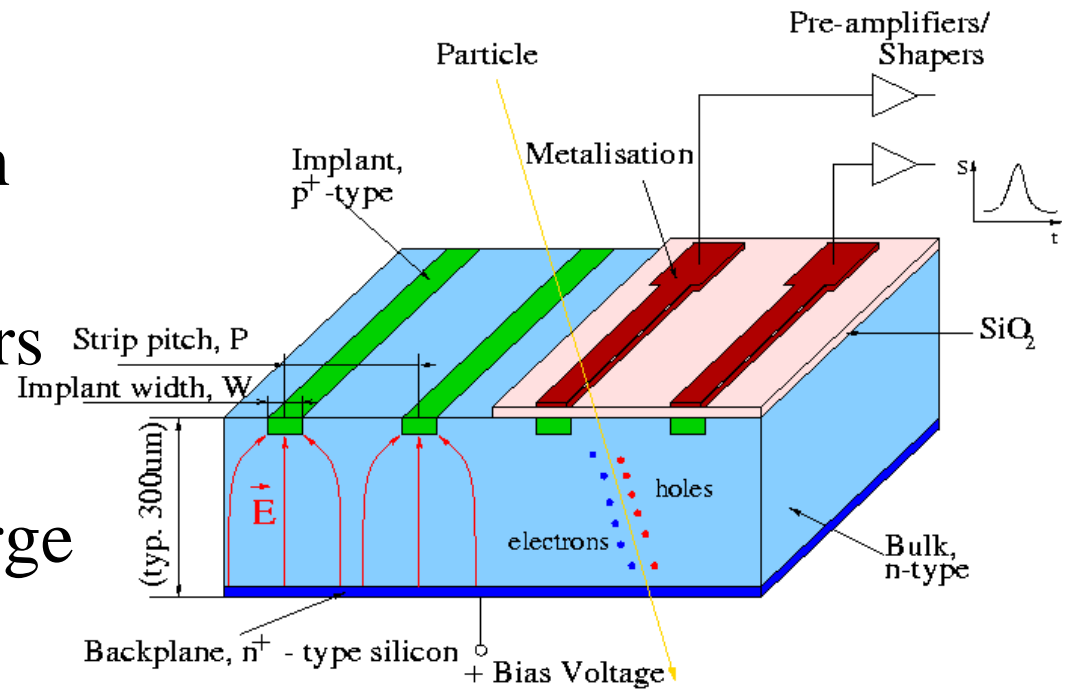
Principles of operation



Signals in Silicon

- In a silicon detector each strip has capacitance to backplane and neighbours
- If amplifier input capacitance high all charge is collected
- If input capacitance low charge flows to neighbours
→ deteriorating position resolution

Principles of operation



Position Resolution: Diffusion

- Diffusion spreads charge transversely

$$\sigma_y = \sqrt{2Dt} \approx \sqrt{2 \frac{kT}{e} \frac{d^2}{V_b}}$$

- Collection time

$$t_c \approx \frac{d}{v} = \frac{d}{\mu E} = \frac{d^2}{\mu V}$$

25 ns in typical silicon sensors

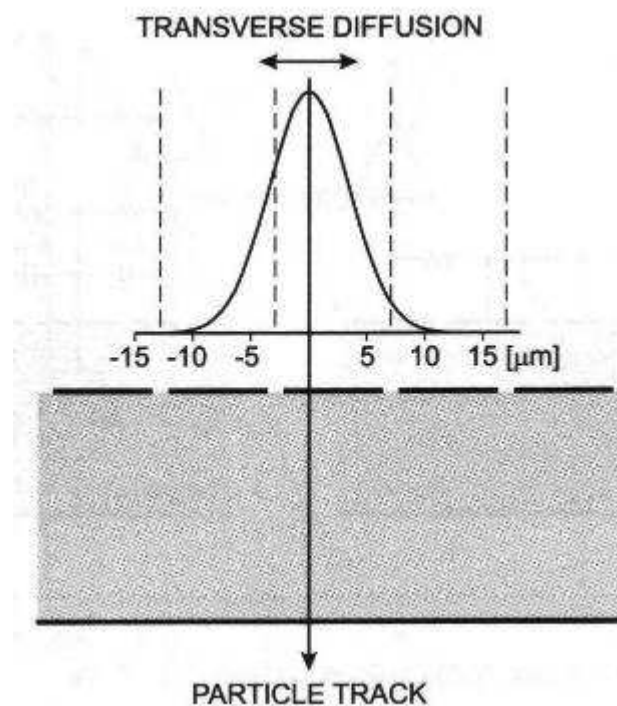
- Diffusion constant is linked to mobility as well

$$D = \frac{kT}{e} \mu$$

- Leads to diffusion of $\sim 7 \mu\text{m}$

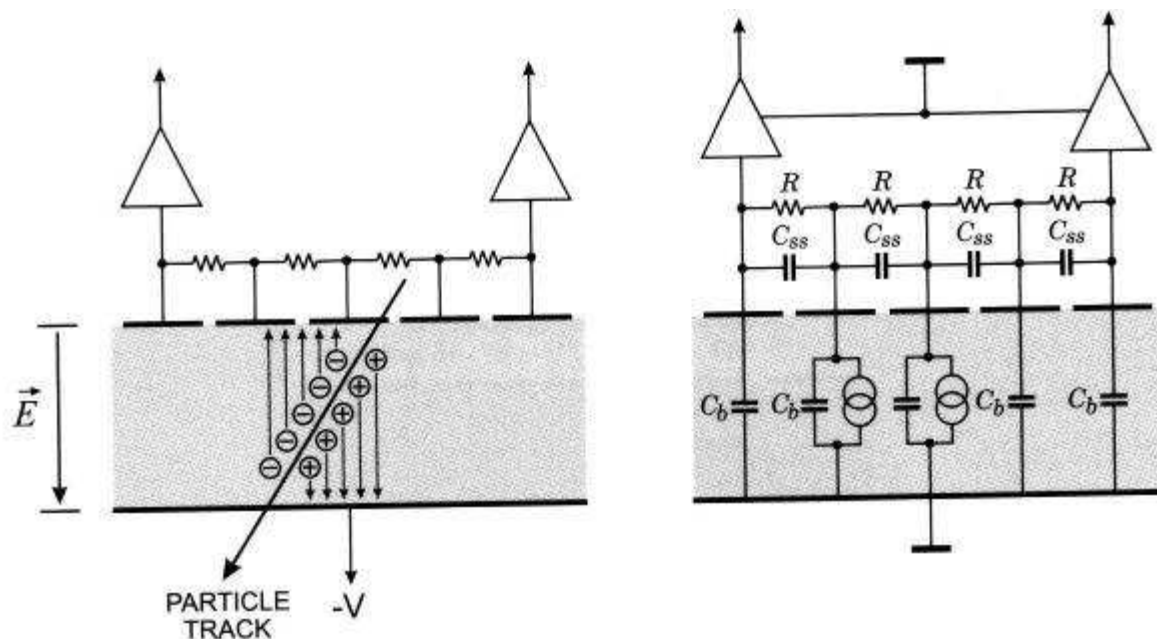
Charge Sharing

- Charge spreading improves resolution!
 - ◆ Centre of gravity interpolation
 - ◆ Resolution proportional to S/N
- Allows to beat $\sqrt{12}$ rule
 - ◆ Achieved resolutions $1.2 \mu\text{m}$ for $25 \mu\text{m}$ pitch ($25/\sqrt{12}=7 \mu\text{m}$)
 - ◆ Requires $S/N > 50$ to achieve this
- Strip pitch should be comparable with diffusion



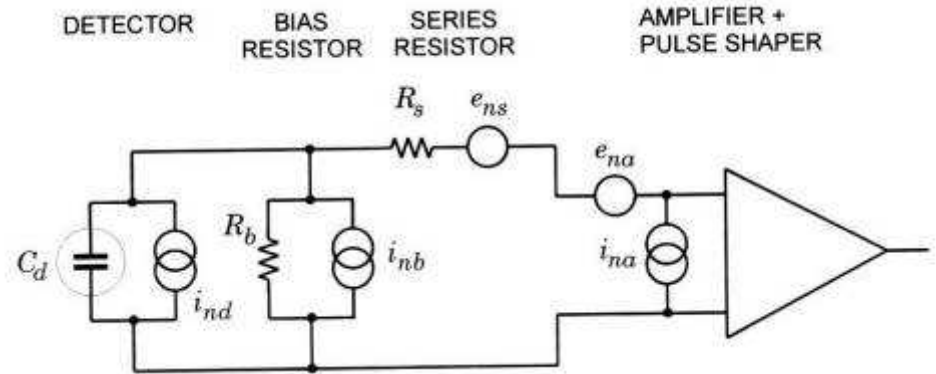
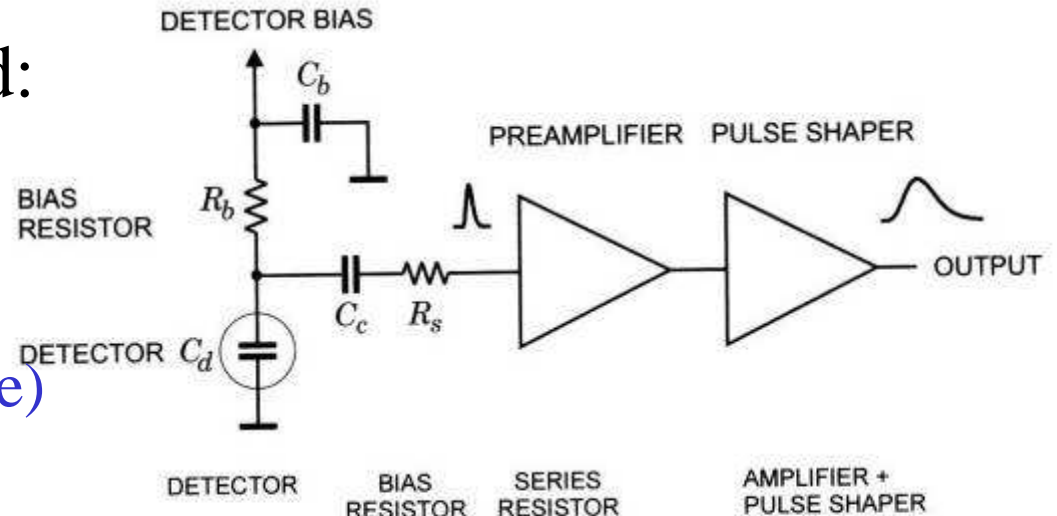
Intermediate Strips

- Charge division can be extended by introducing intermediate strips
- Strips are coupled capacitively to neighbours
- Signal loss to backplane $C_b/C_{ss}=0.1$
→ ~20% loss



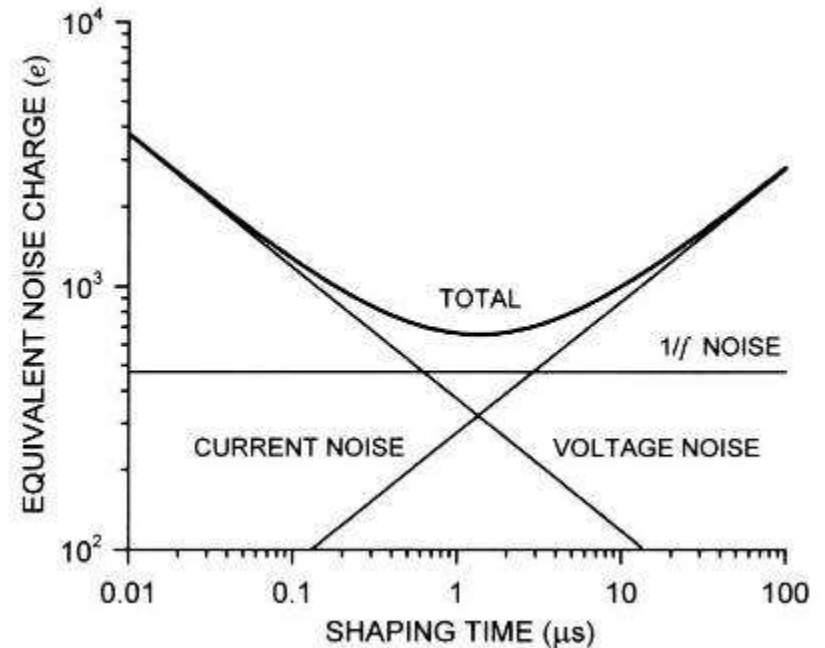
Readout Electronics: Noise (I)

- Typical detector front-end:
- Sources of noise
 - ◆ Leakage current (shot noise)
 - ◆ Noise in resistors: thermal fluctuations of carrier velocities
 - ◆ Parallel (to input) resistors act as current sources, resistors in series act as voltage sources



Readout Electronics: Noise (II)

- Equivalent Noise Charge (ENC) = Signal that yields $S/N = 1$
- After filtering by a shaper (integrator + differentiator with same shaping time: T_s)

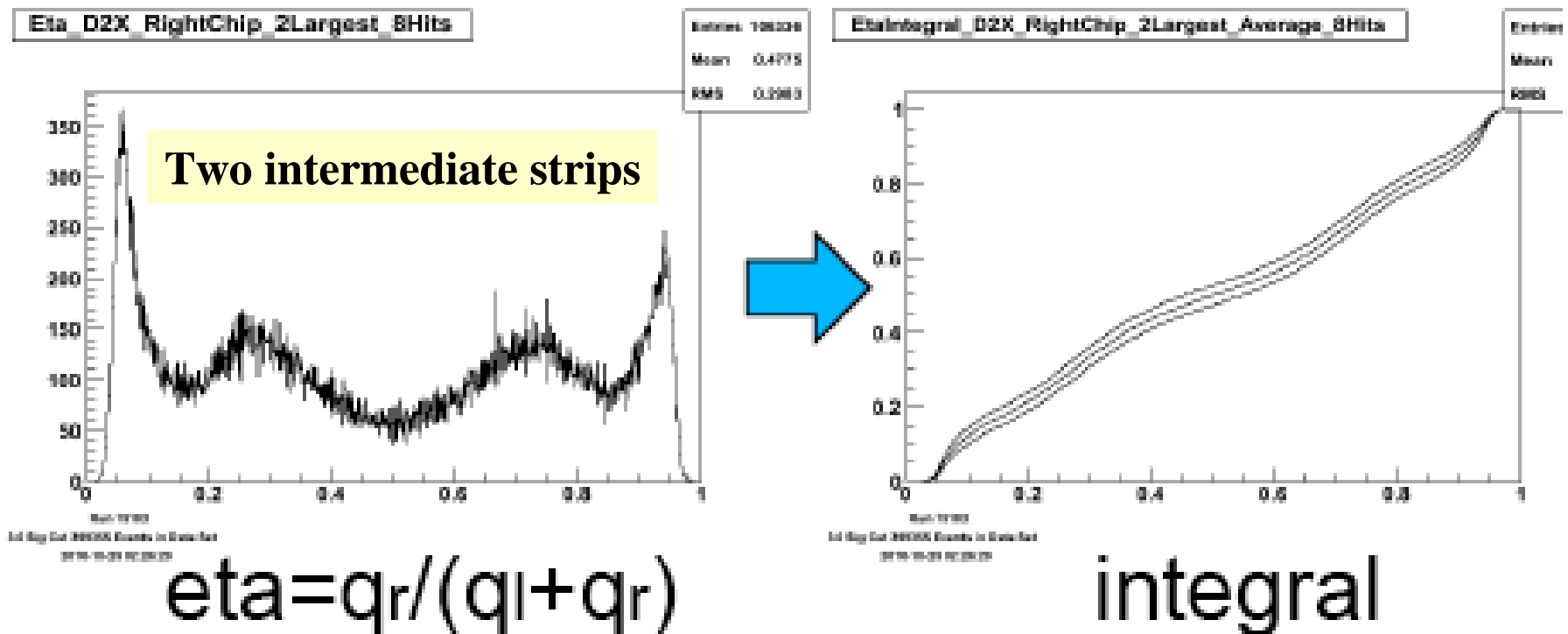


$$Q_n^2 = \left(2eI_d + \frac{4kT}{R_b} + i_{na}^2 \right) F_i T_S + (4kTR_s + e_{na}^2) F_v \frac{C_d^2}{T_S} + F_{vf} A_f C_d^2$$

- As T_s changes noise goes through minimum
 - Optimization possible

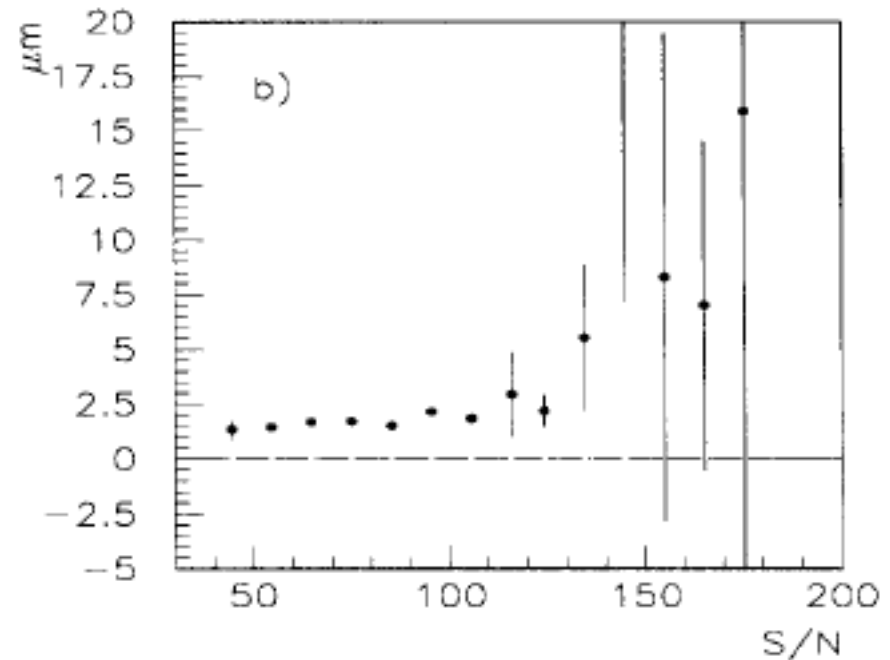
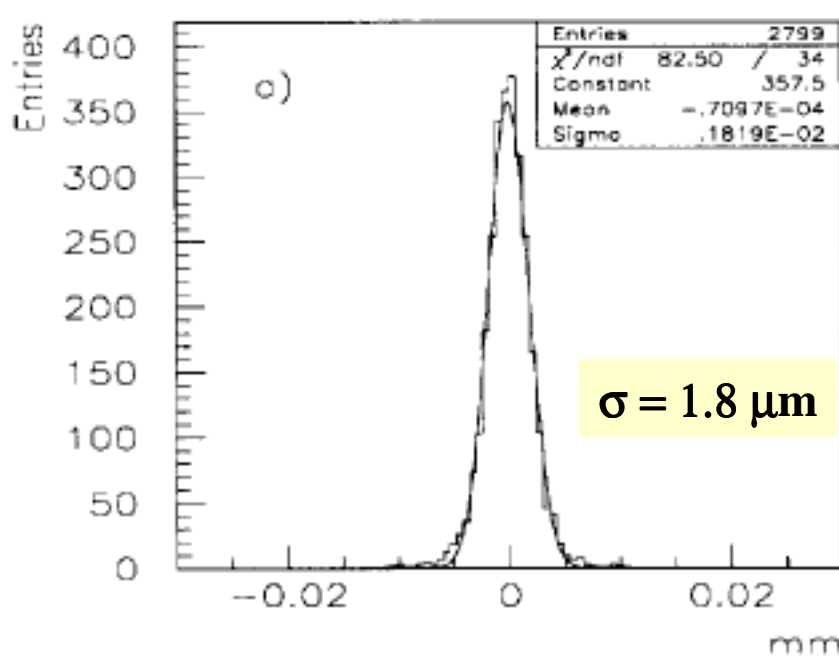
Eta Algorithm

- Define η as $PH_r / (PH_l + PH_r)$
 - ◆ Diffusion biases response to uniform illumination
- Determine charged particle position by un-folding



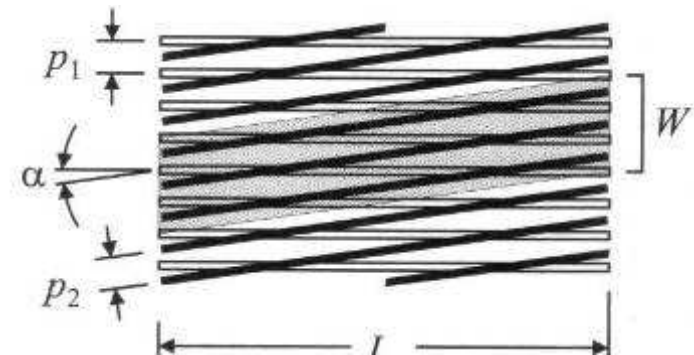
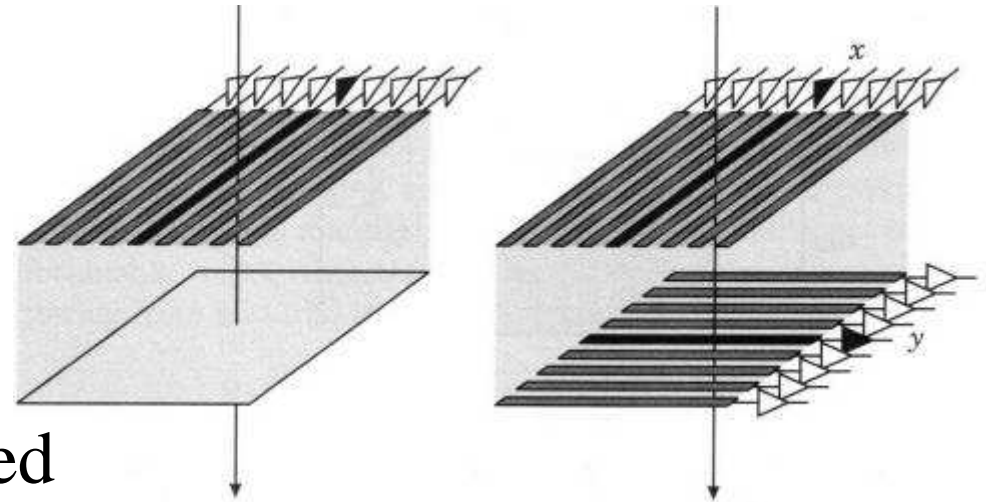
Ultimate Position Resolution

- Push all handles to the extreme
 - ◆ Minimise readout pitch (25 μm)
 - ◆ Shaping time to several μs (S/N \rightarrow 50, 70 or more)
 - ◆ Minimise diffusion/limit charge deposition (no δ -rays)
 - ◆ Use η algorithm



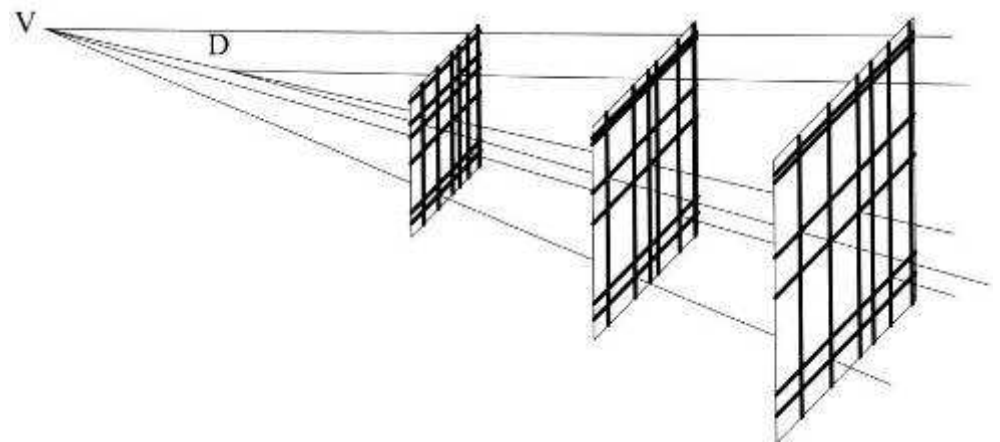
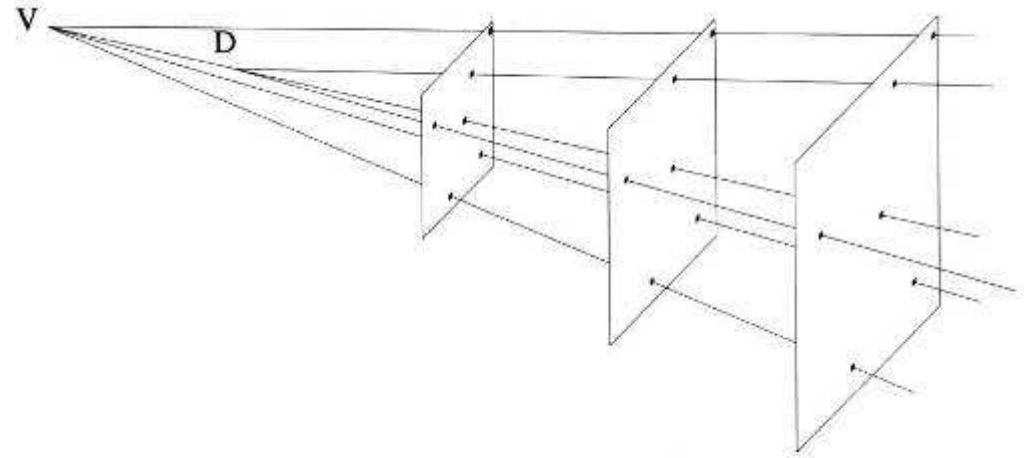
Two Dimensional Information

- 2D information allows to reconstruct 3D points – advantageous for track reconstruction
 - ◆ Good for both precision and pattern recognition
- Pixel detector vs double sided strip detectors
- Segment other side of the sensor in orthogonal direction
 - ◆ Gives best resolution
- Small angle stereo
 - ◆ Resolution in orthogonal direction $\sim \text{pitch} / \sin \alpha$



Ghosts in Tracking

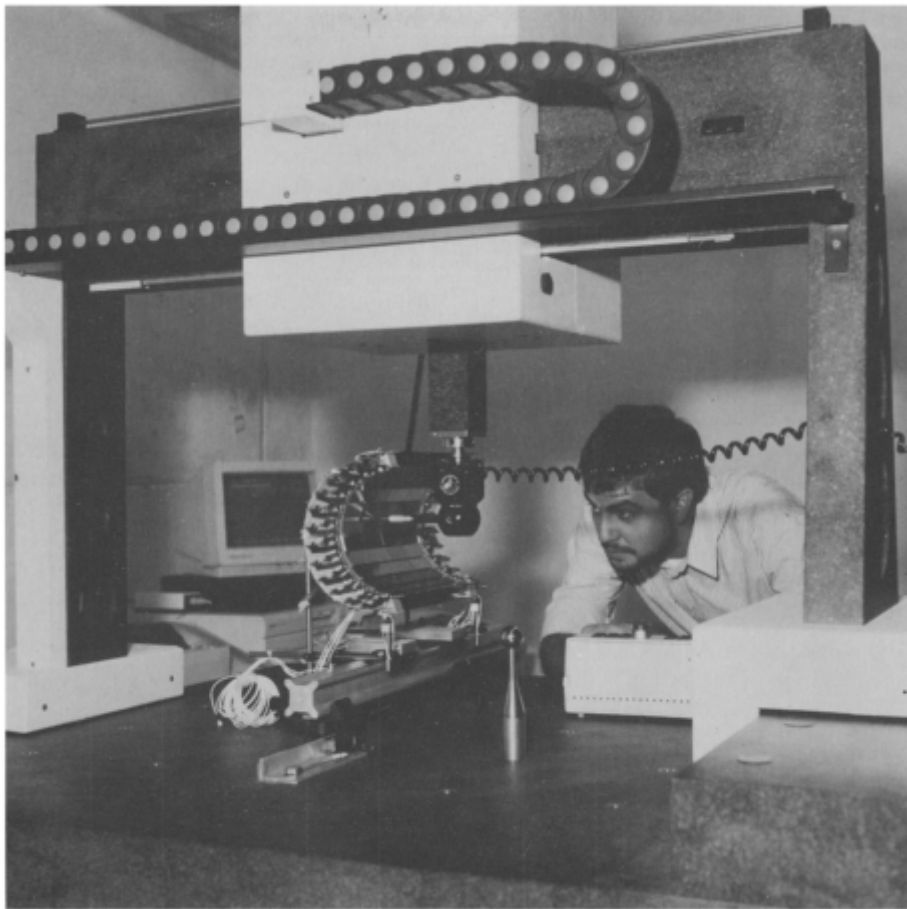
- Ghosts appear in multi-track environment when more than one particle hit the sensor
- N^2-N ghost tracks for strip detectors with orthogonal strips



Alignment

Mechanical Survey During Construction

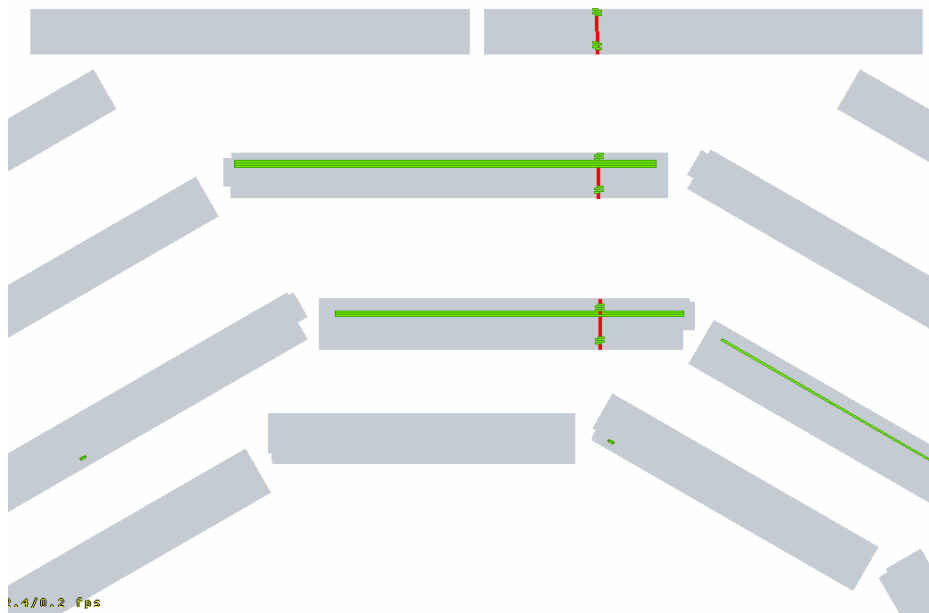
- Constrain sub-assembly alignment during fabrication
- Survey whole tracker prior to installation



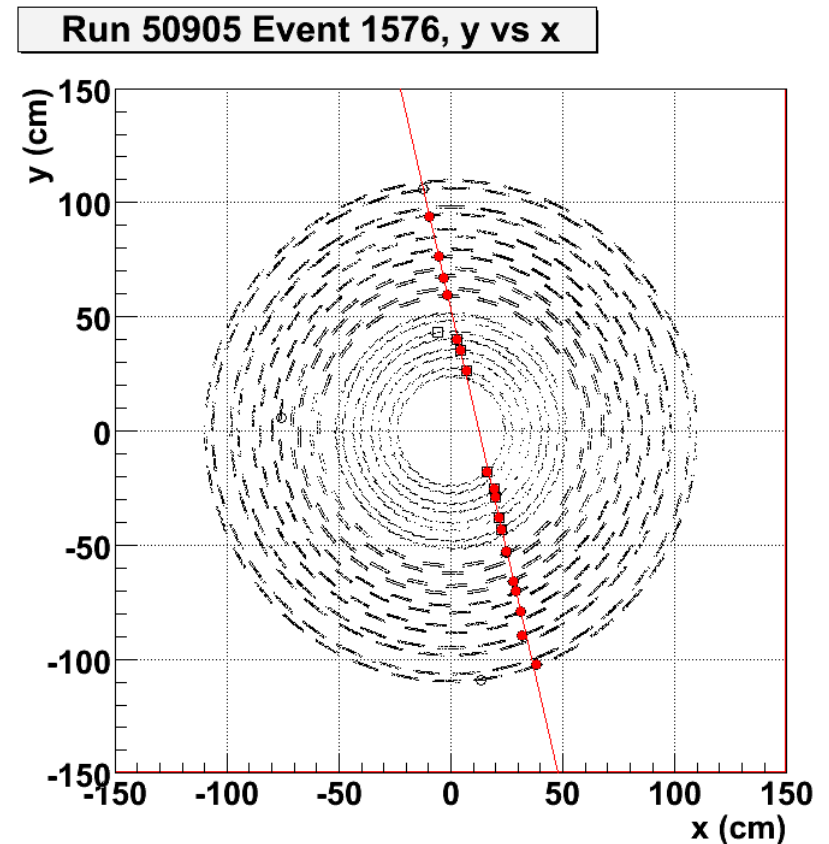
- 3D coordinate measurement
 - Few μm precision over 1m^3 volumes
 - Lots of systematics to understand before this data is useful

Early Alignment with Cosmics

- After tracker is installed, have two sources of particles to use for calibration: **cosmics** and **collisions**
 - ◆ movies from CMS: Cosmics muon spectrometer and hits in silicon tracker

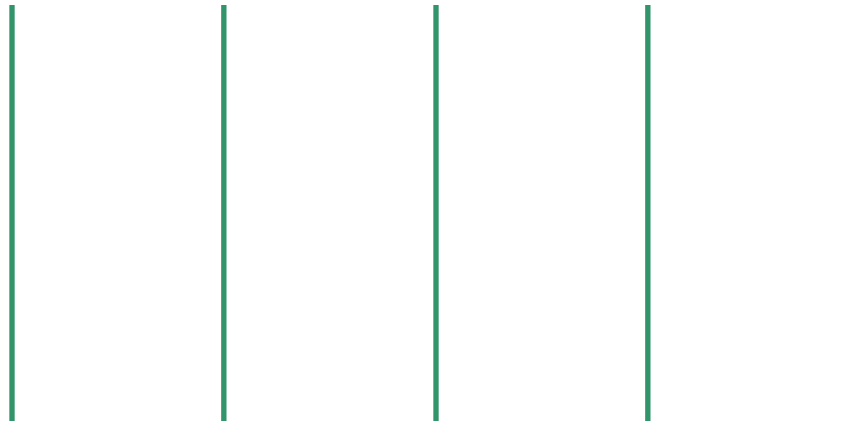


(movies in .ppt version)



Tracker Alignment

How do you fix this?

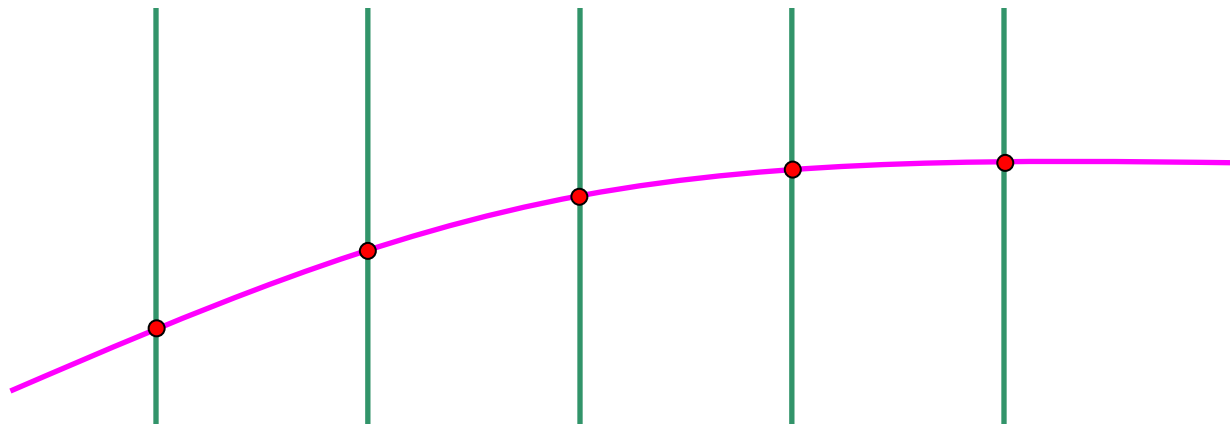


Consider a five-layer tracker

borrowed from F. Meier

Tracker Alignment

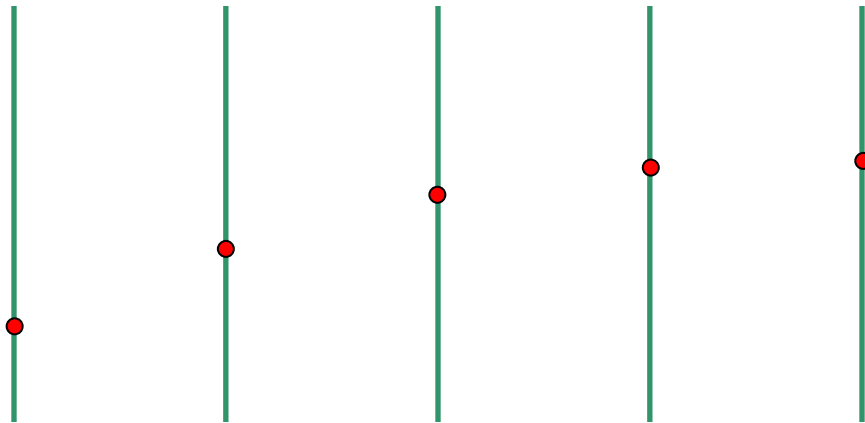
How do you fix this?



A track goes through, leaving hits

Tracker Alignment

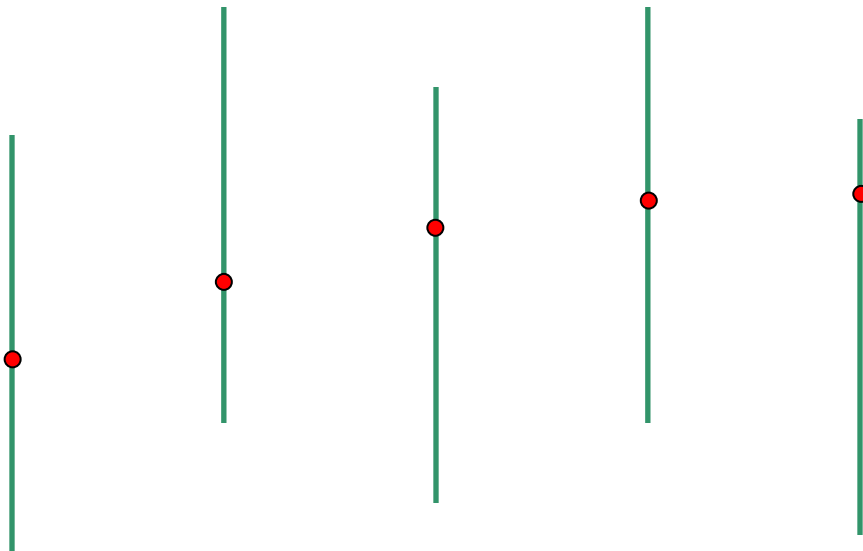
How do you fix this?



All you really see are the hits, actually

Tracker Alignment

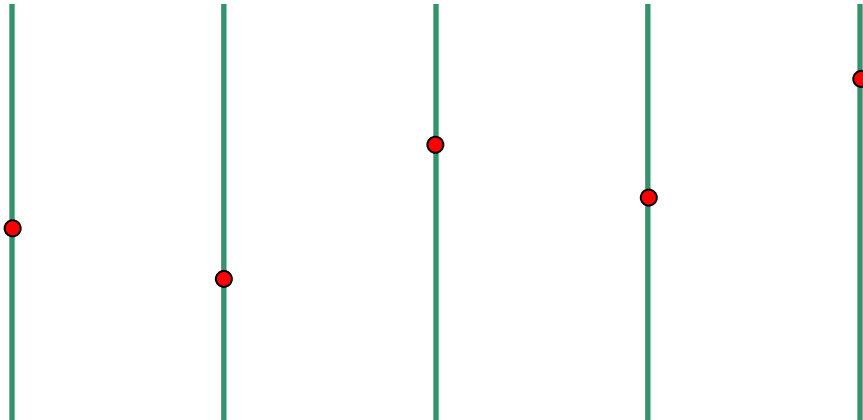
How do you fix this?



Now, if your tracker is misaligned, the hits positions really look like this

Tracker Alignment

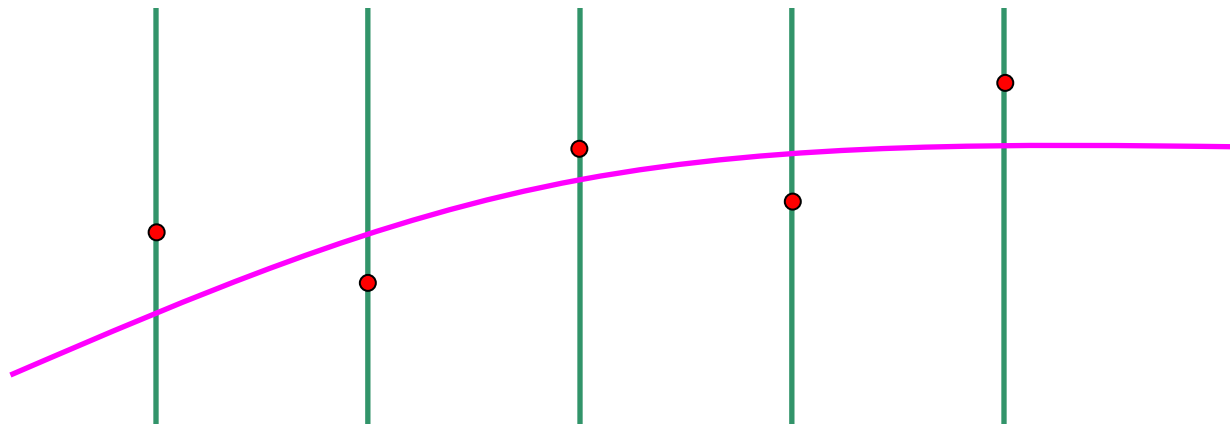
How do you fix this?



If you assume the module positions are “ideal”, you see this

Tracker Alignment

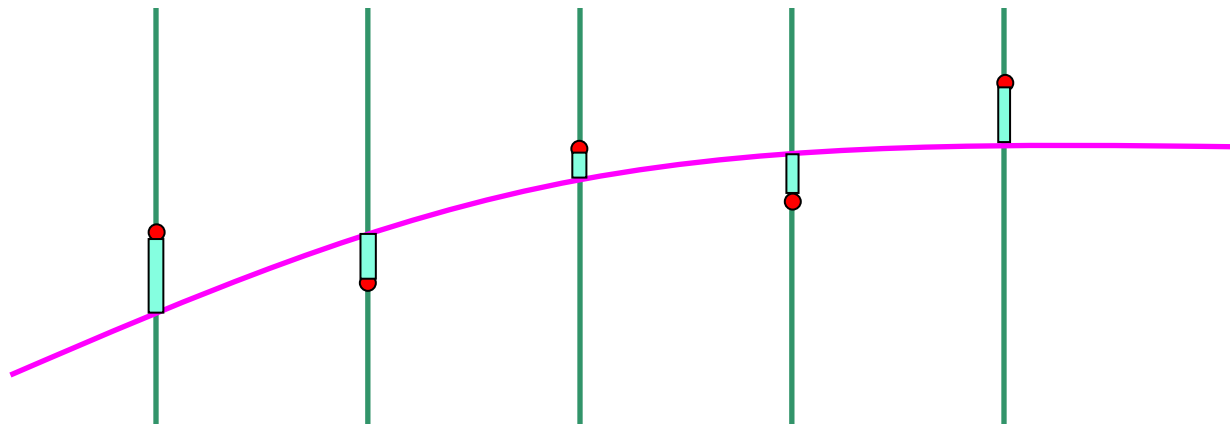
How do you fix this?

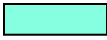


So your track really looks like this

Tracker Alignment

How do you fix this?



To “align”, we keep track of the “residuals” between the hits and the projected track positions (shown as ) for many tracks, then adjust the positions of the actual detectors to minimize the residuals across the whole tracker.

Tracker Alignment: In 3D

χ^2 minimization:
$$\chi^2(\mathbf{p}, \mathbf{q}) = \sum_j^{\text{tracks}} \sum_i^{\text{hits}} \mathbf{r}_{ij}^T(\mathbf{p}, \mathbf{q}_j) \mathbf{V}_{ij}^{-1} \mathbf{r}_{ij}(\mathbf{p}, \mathbf{q}_j)$$

where \mathbf{p} parametrize the tracker geometry, \mathbf{q}_j are the track parameters, and \mathbf{r}_{ij} are the residuals: $\mathbf{r}_{ij} = \mathbf{m}_{ij} - \mathbf{f}_{ij}(\mathbf{p}, \mathbf{q}_j)$, \mathbf{m} are measured hits and \mathbf{f} are predicted hits.

Scale of Problem: (CMS Tracker)

- Each module: 6 degrees of freedom:
 - ◆ 16588 modules x 6 = $\sim 10^5$ parameters
- Each track has 5 degrees of freedom,
 - need 10^6 tracks or more
 - $\Rightarrow \sim 10^7$ parameters to deal with
 - \Rightarrow Not easy!

Alignment Techniques

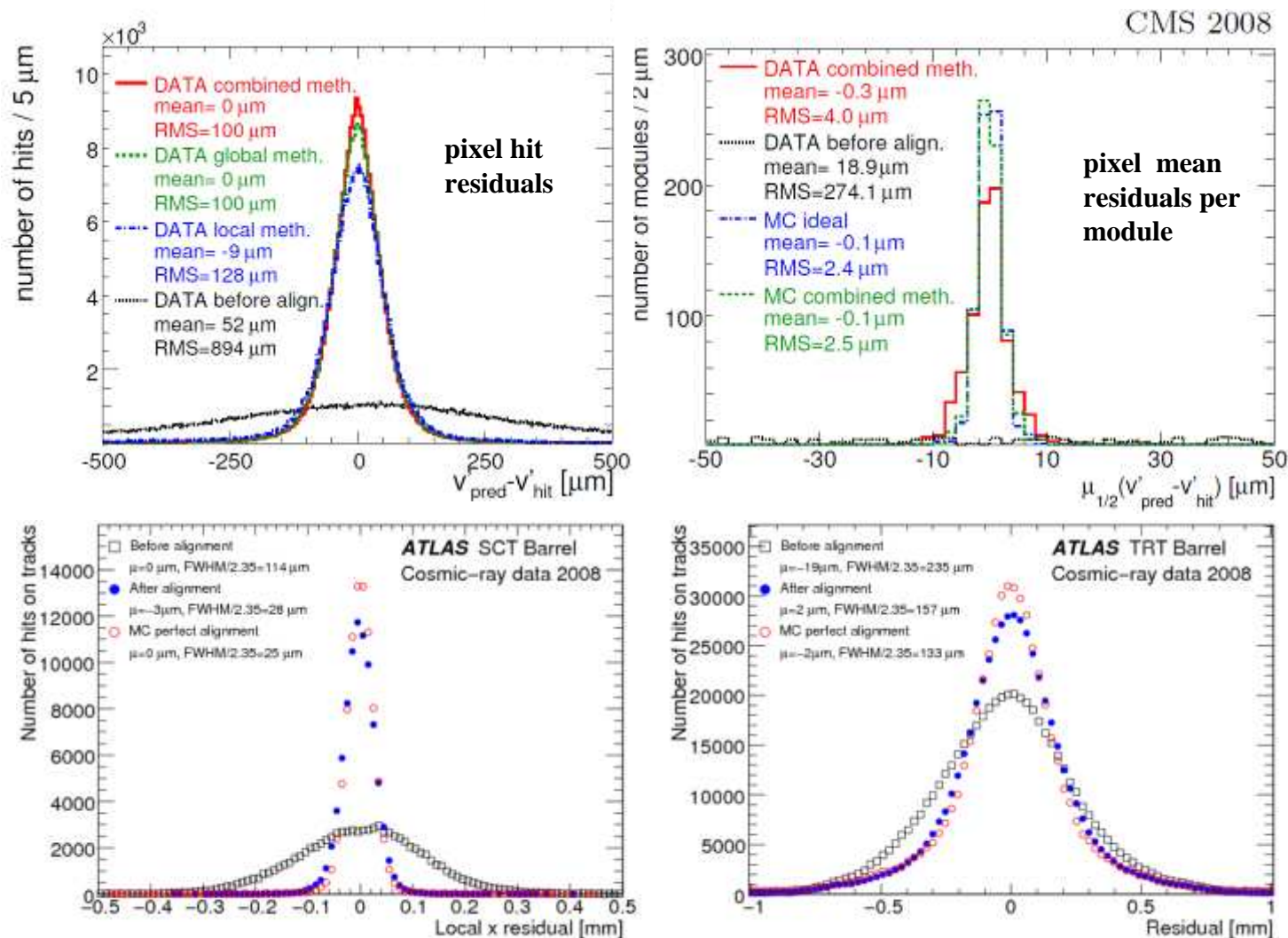
1. Global (e.g. “Millepede-II” for CMS)

- ◆ Matrix inversion determines module parameters only:
 - ▲ $\sim 10^5 \times 10^5$ matrix
 - ▲ Correlations between modules included
 - ▲ simplified tracking parameterization: no E_{loss} , Multiple Scattering
 - ▲ few iterations

2. Local

- ◆ Local minimization of residuals: ~ 10 parameters at a time
- ◆ Incorporate survey data as a constraint
- ◆ Full track extrapolation with Scattering and E_{loss}
- ◆ Includes local correlations between adjacent modules

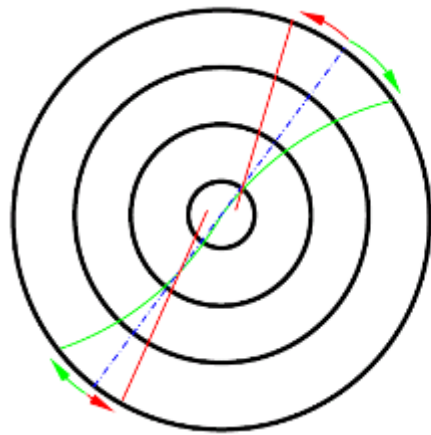
Alignment Results (cosmics)



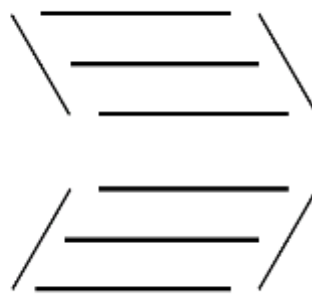
⇒ Basically, all detectors reached near-optimal alignment before collisions

Alignment Pitfalls

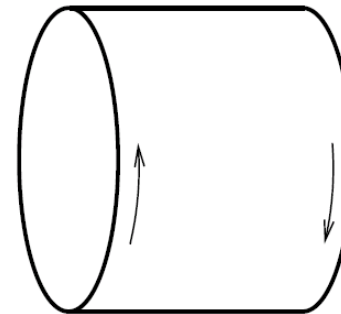
- Exist modes of detector deformation with no change in total χ^2 , yet physical locations not “ideal”



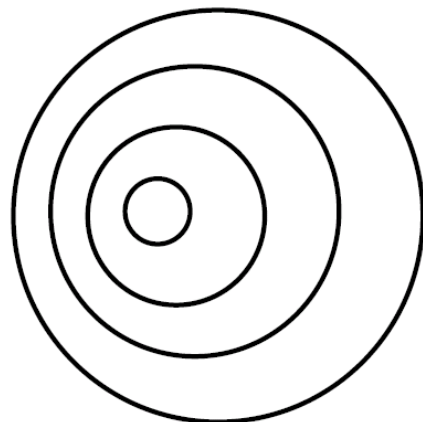
z shear



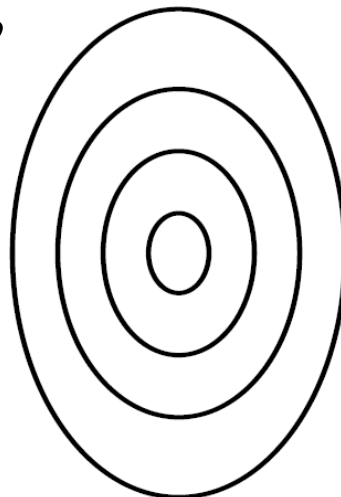
z twist



shear (red) or bend (green) in $r-\phi$



$r-r\phi$ mode 1



$r-r\phi$ mode 2

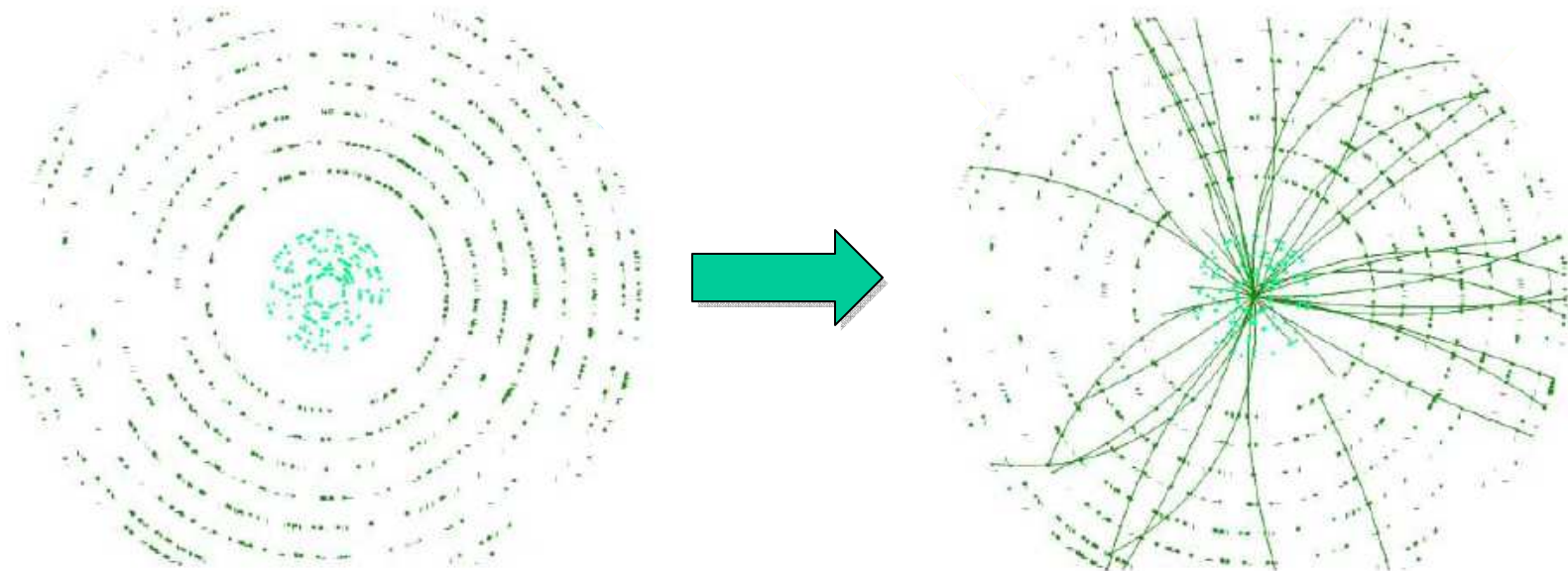
This is tricky...

Need orthogonal sets of tracks to constrain these modes:

- cosmics, which don't pass through the tracker origin
- collision tracks
- collision tracks with $B=0$

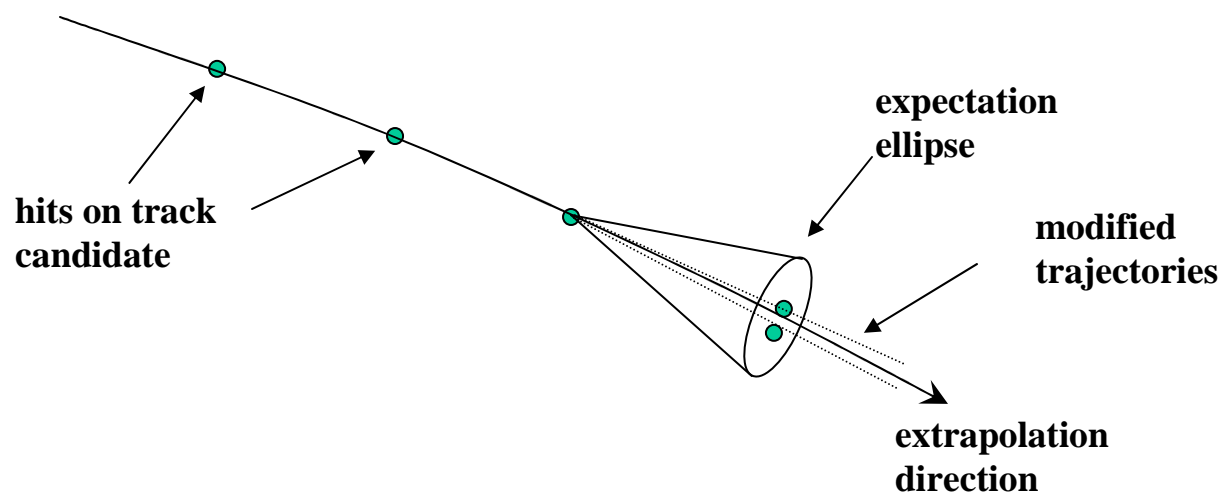
Putting it All Together: Tracking

- First, find track candidates:
 - ◆ “Pattern Recognition”
- Then (or simultaneously) estimate the track parameters
 - ◆ “Fitting”
- **The Trick:**



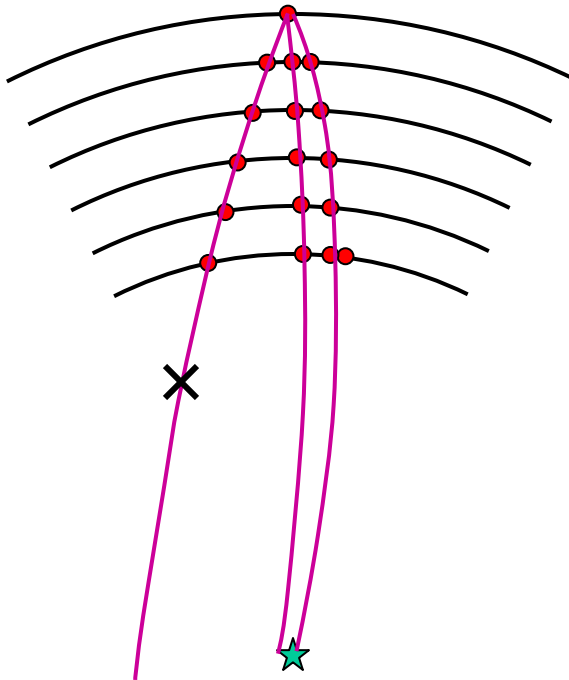
Pattern Recognition: Road-Following

- Simplest to understand, not optimal in some cases
- Subset of well-separated hits (and possibly a beam spot) are used to create initial track hypotheses
- Candidate tracks extrapolated to next layers to add potential new hits, refine track parameters, continue

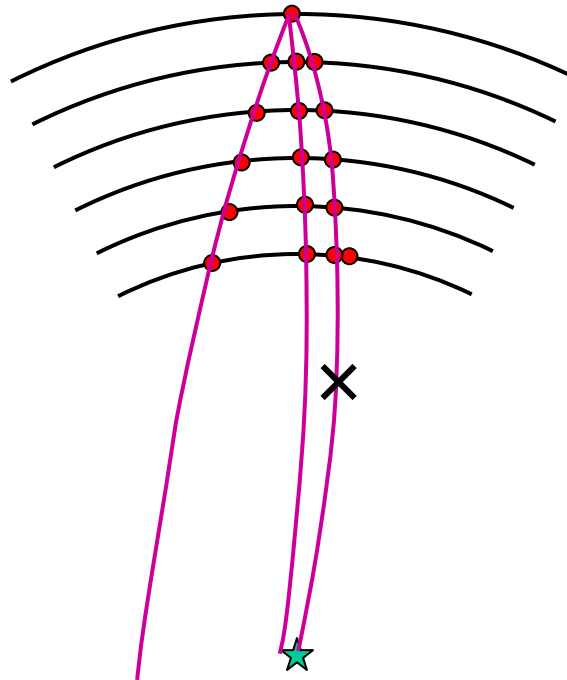


Pattern Recognition: Simplifications

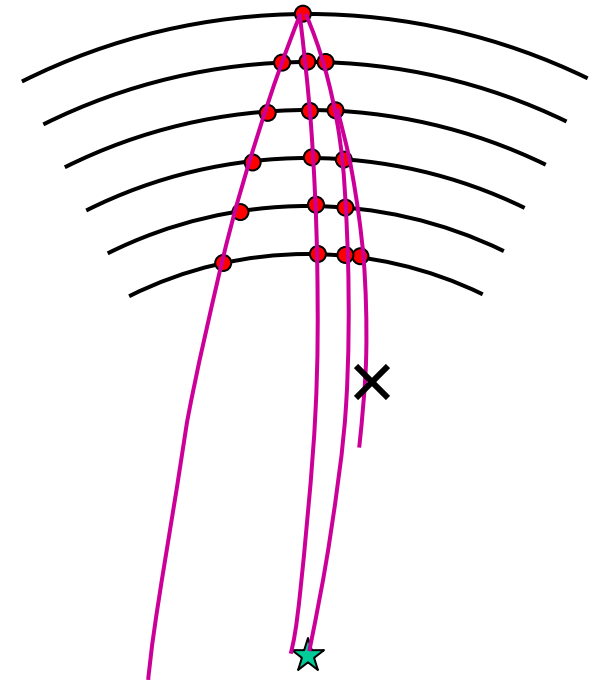
- Track finding struggles in high-occupancy environments
 - ◆ too many fakes, or takes way too long...
- Compromises to efficiency necessary to speed things up:



Accept tracks that originate near the IP



Prefer higher momentum tracks (min p_T cut)



Limit number of misses or extrapolation residual

EDIT2011 Nomerotski/Trischuk

Track Fitting: Least Squares (I)

following P. Avery

- Once you've determined a set of measurements y_l use them to estimate track parameters α such that $y_l = f_l(\alpha)$.
- If we take an initial guess α_A at the parameters and make a linear expansion around that solution, we get


$$y_l = f_l(\alpha_A) + (\partial f_l / \partial \alpha_i)(\alpha_i - \alpha_{A i})$$

- This allows us to define a χ^2 measure

$$\begin{aligned}\chi^2 &= \sum_l (y_l - f_l(\alpha_A) - A_{li}(\alpha_i - \alpha_{A i}))^2 / \sigma_l^2 \quad \leftarrow \text{individual measurement errors} \\ &= (\mathbf{y} - \mathbf{f}(\alpha_A) - \mathbf{A}(\alpha - \alpha_A))^T \mathbf{V}_y^{-1} (\mathbf{y} - \mathbf{f}(\alpha_A) - \mathbf{A}(\alpha_A - \alpha)) \\ &\equiv (\Delta \mathbf{y} - \mathbf{A}(\alpha - \alpha_A))^T \mathbf{V}_y^{-1} (\Delta \mathbf{y} - \mathbf{A}(\alpha - \alpha_A))\end{aligned}$$

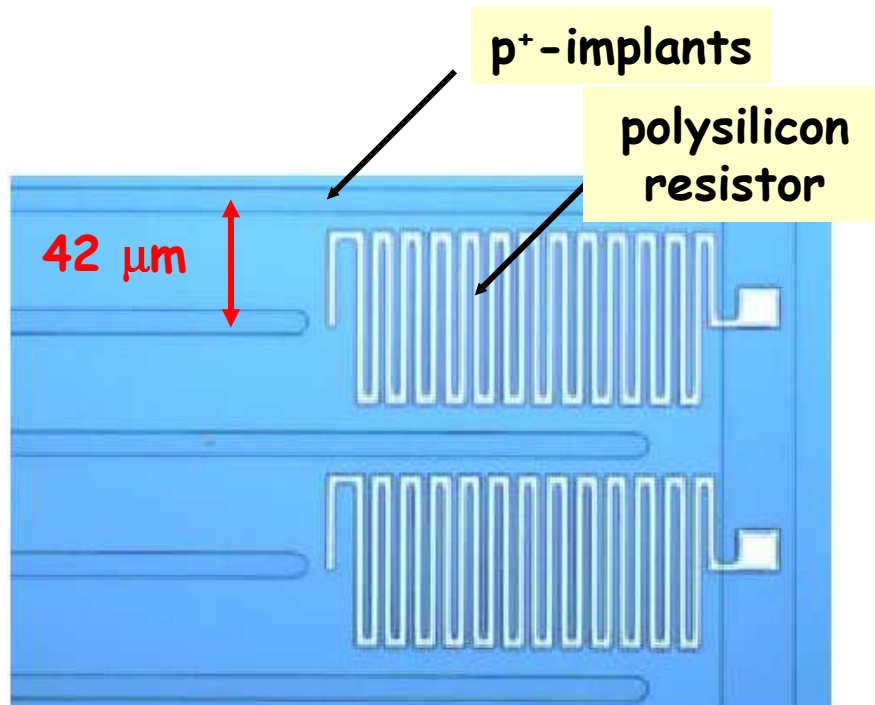
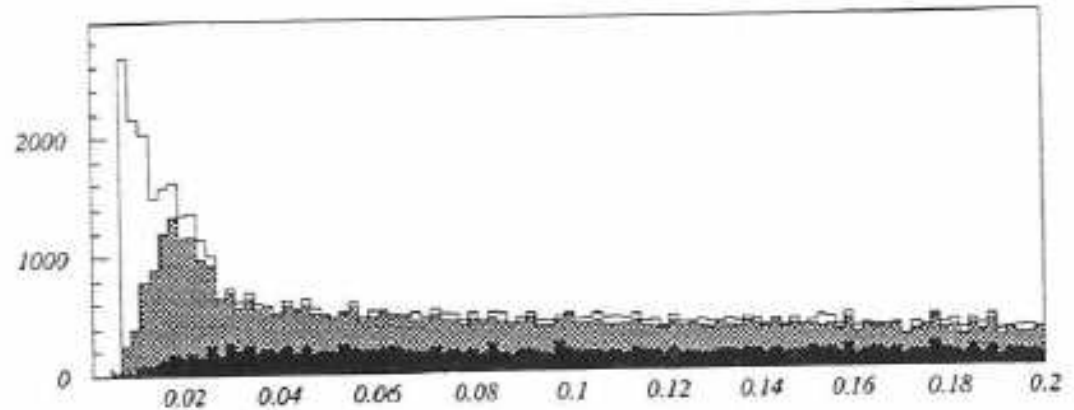
where $\Delta \mathbf{y} = \mathbf{y} - \mathbf{f}(\alpha_A)$ and $A_{li} = \partial f_l(\alpha) / \partial \alpha_i |_{\alpha_A}$ is a matrix of constant derivatives. \mathbf{V}_y is covariance matrix of the measurements.

Track Fitting: Least Squares (II)

- We want the parameter estimation that minimizes the distance between the measured points and the fitted track, so we set $\partial\chi^2/\partial\alpha_i = 0$ which gives us the solution $\alpha = \alpha_A + \mathbf{V}_A \mathbf{A}^T \mathbf{V}_y^{-1} \Delta \mathbf{y}$ where $\mathbf{V}_A = (\mathbf{A}^T \mathbf{V}_y^{-1} \mathbf{A})^{-1}$
covariance matrix of A
- Ideally, iterate to get best estimate of the parameters α
- This method has several problems:
 - ◆ Only works well if all of the points are independent
 - ◆ All of the points have equal weight
- More sophisticated techniques exist (Kalman filters...)

A Detail: Cluster Splitting

- Clusters can be split by dead strips
- Also by design features:

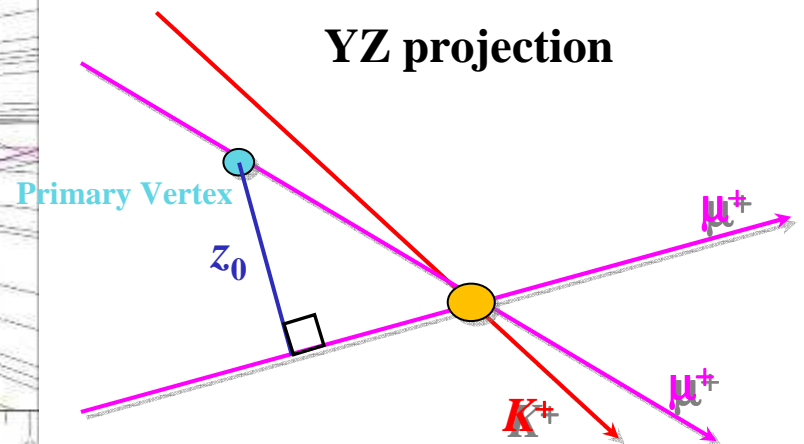
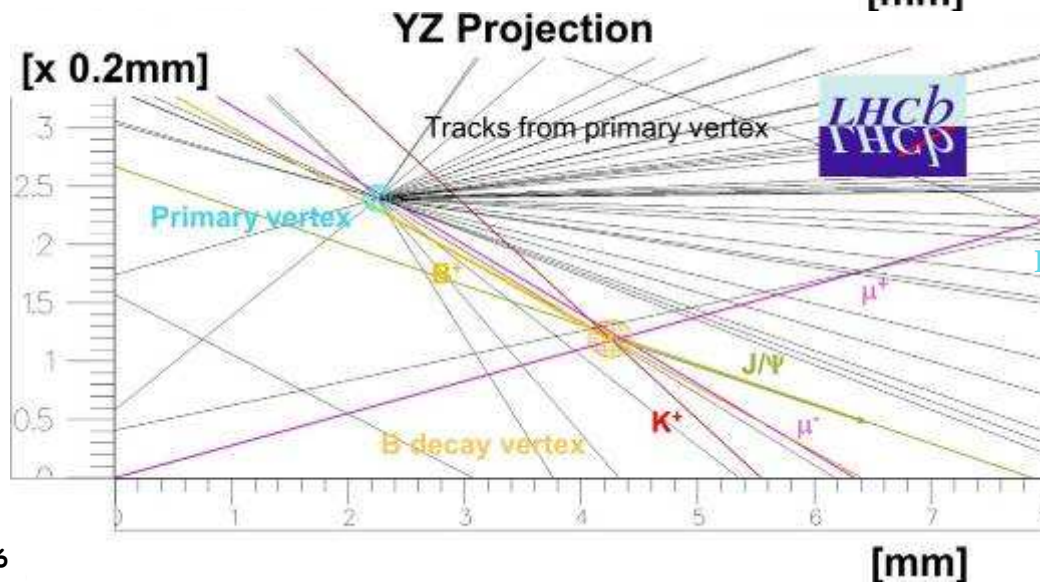
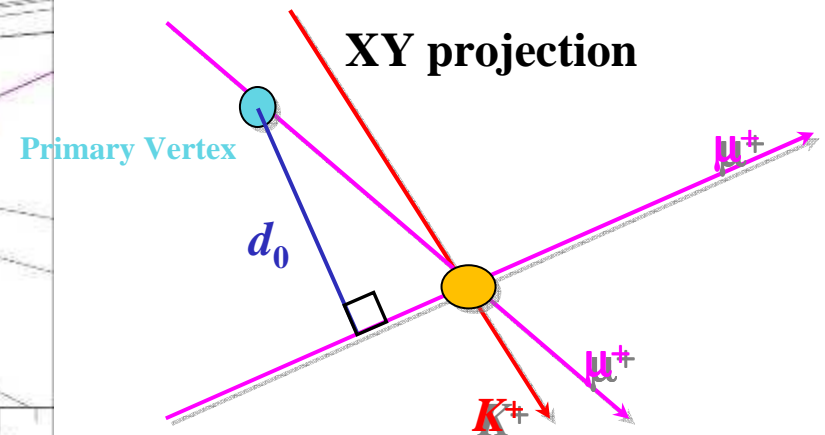
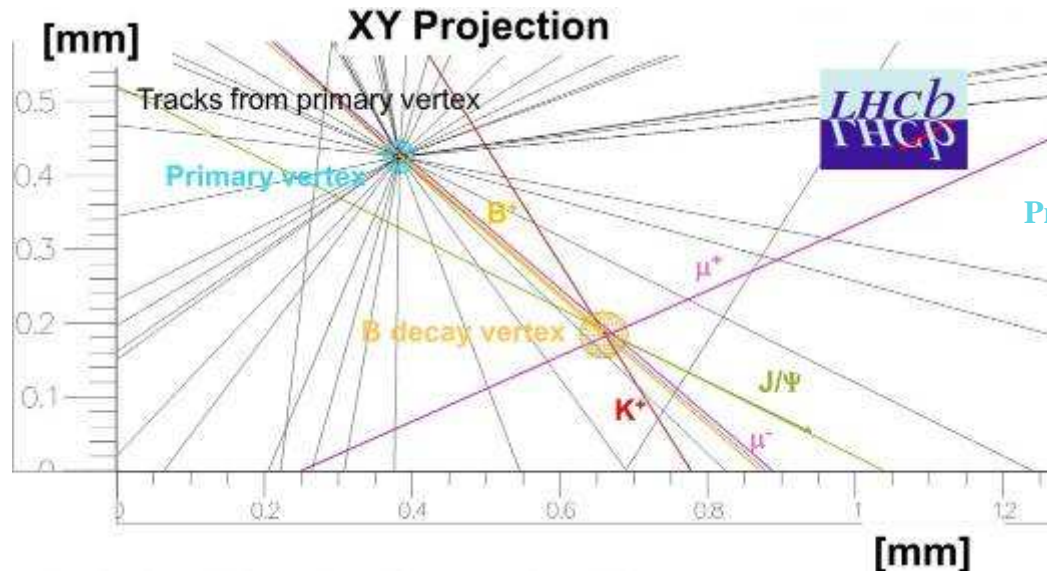


Distance to the closest cluster before/after cluster repair

- Double strip pitch to accommodate polySi resistors caused split clusters (DELPHI VD)
- Fixed by a software algorithm

Bringing It All Together

- Physics Example: $B^+ \rightarrow J/\Psi K^+$

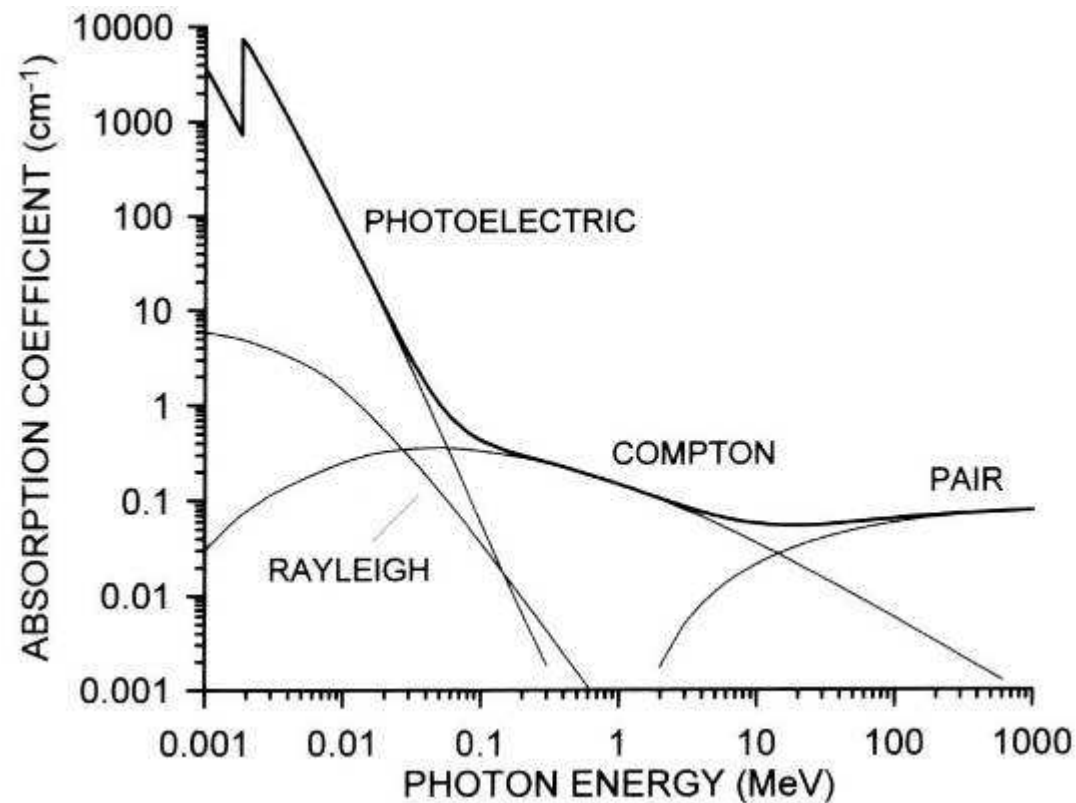


Applications Outside HEP

- Very broad area, a lot of overlap with fast and medical imaging
- Cover only a couple of examples
 - ◆ Fast radiography
 - ◆ Sound preservation

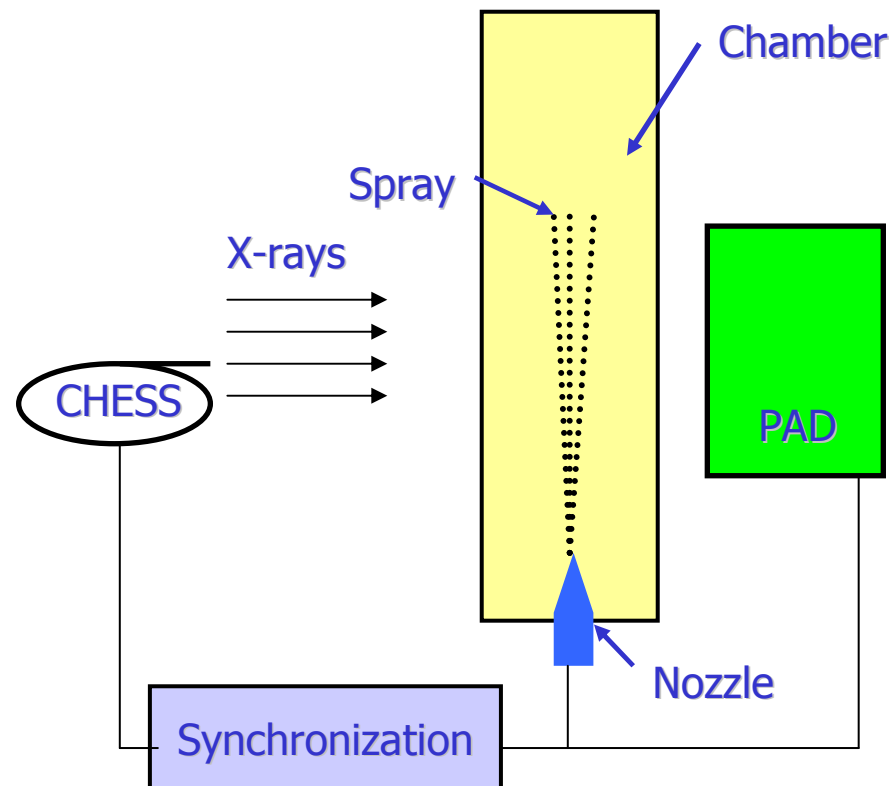
X-Rays in Silicon

- Visible photon range $\sim \mu\text{m}$
- 20 keV X-ray range $5 \mu\text{m}$
- 100 keV X-ray range $80 \mu\text{m}$

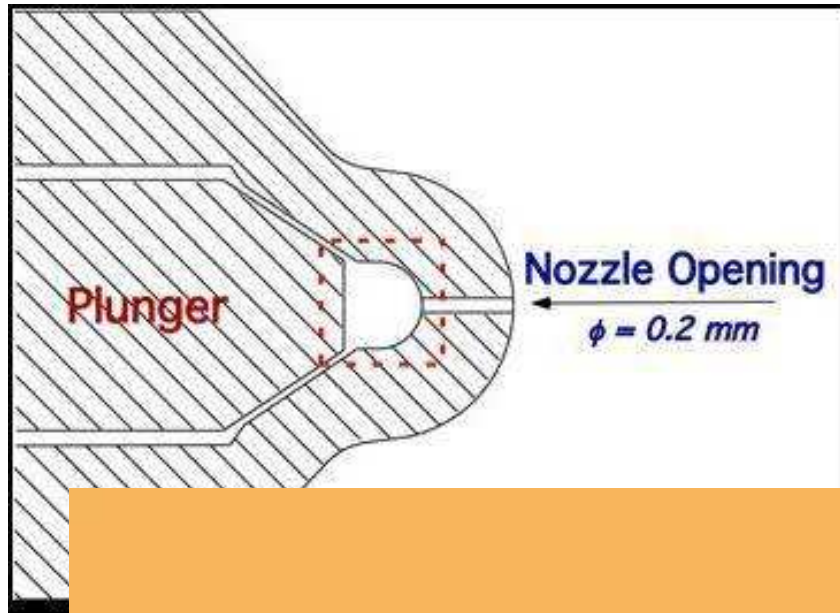


High Speed Radiography

- Supersonic spray from Diesel Fuel Injection System
 - ◆ Impossible to observe in visible light
- 6 keV X-ray beam recorded by fast silicon pixel detector



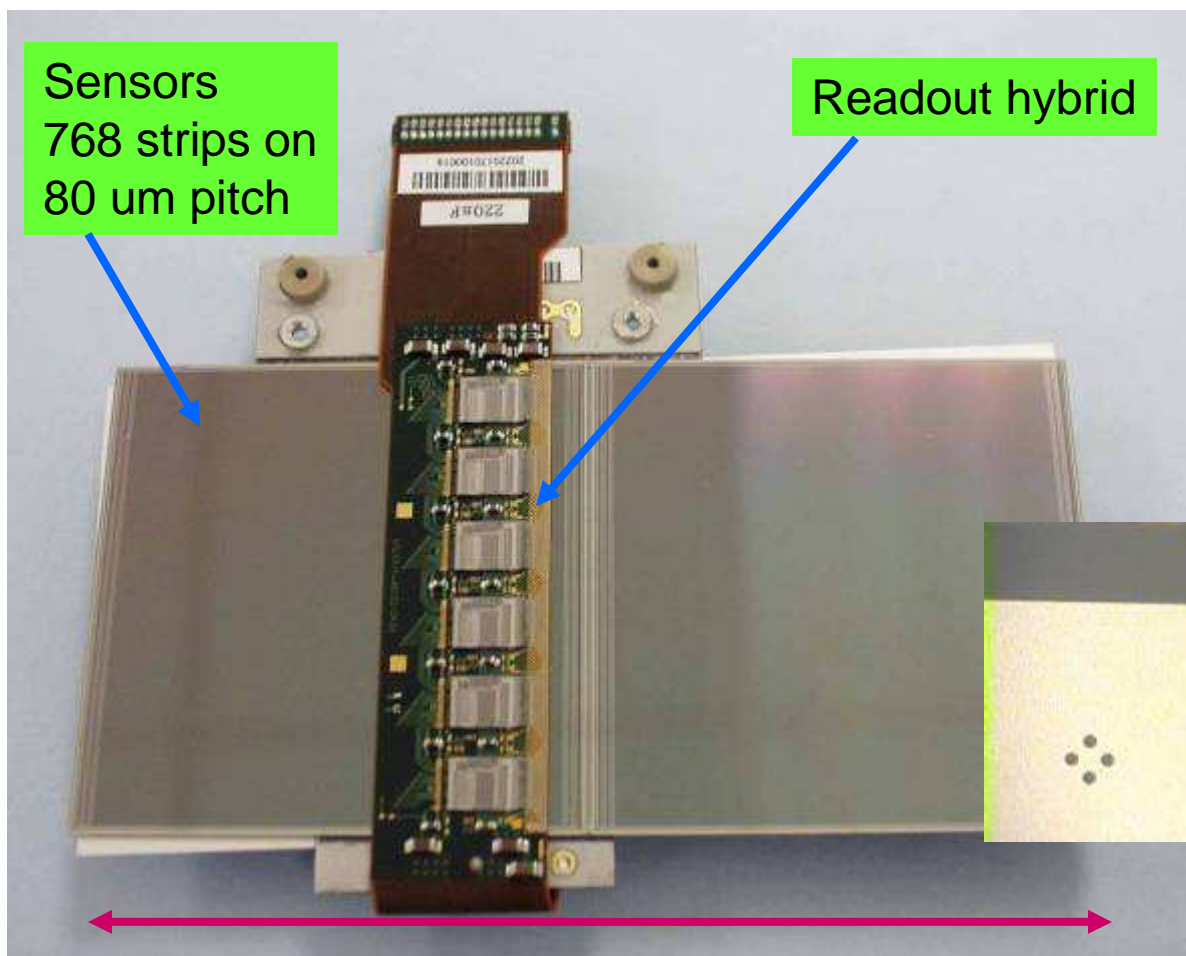
Diesel Fuel Injector Spray



- Total exposure time 1.3 ms



Optical Metrology of ATLAS Modules



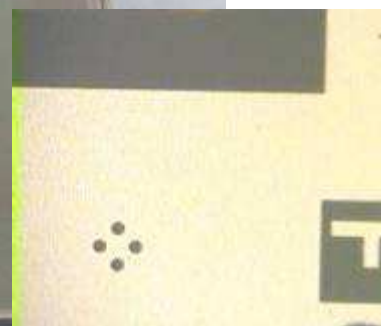
Sensors
768 strips on
80 μm pitch

Readout hybrid

12 cm



SmartScope



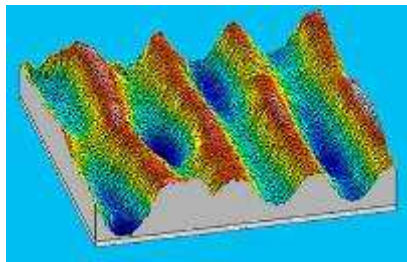
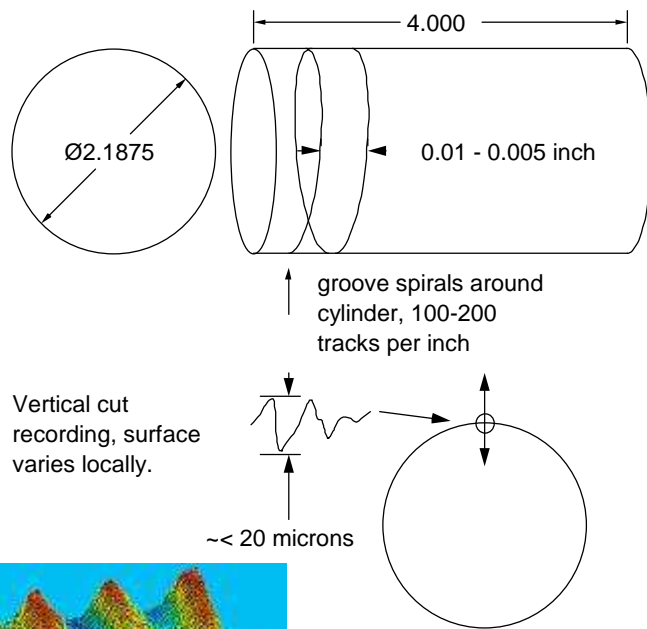
Corner
fiducial mark

Can locate detector
position with \sim micron
precision

Preservation of Mechanical Recording

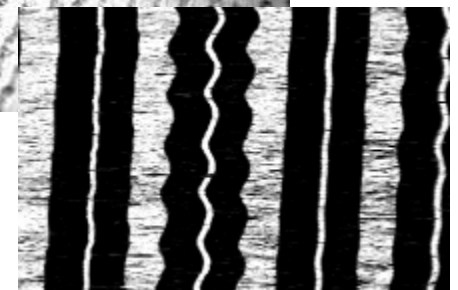


Cylinder: groove varies in depth (Vertical Cut)



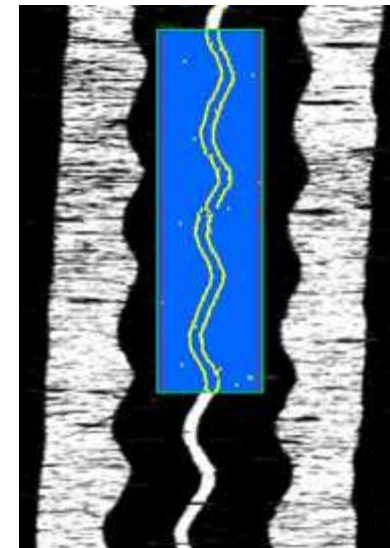
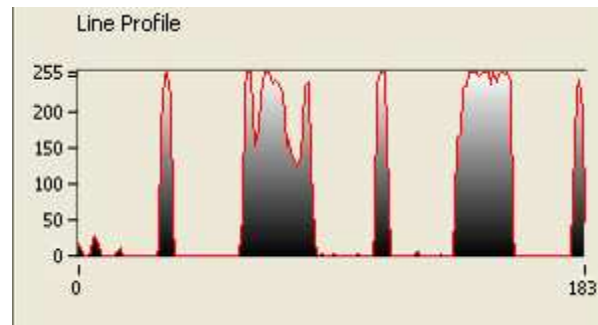
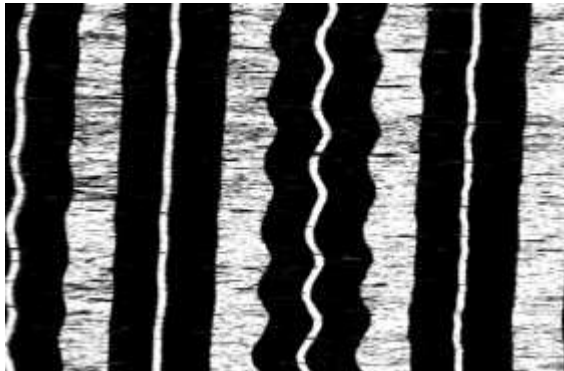
Audio is encoded in micron scale features which are >100 meters long

Disc: groove moves from side to side (Lateral Cut)



Sound Preservation: Image Analysis

Used ATLAS silicon module survey camera for scanning
(Carl Haber and co-authors)



Now being used to generate digital record of all recordings
in Smithsonian collection in Washington DC

Summary

- Silicon detectors offer un-paralleled hit precision
- Critical for B physics and ID of long-lived particles
- Need combination of
 - ◆ Well understood silicon detector
 - ◆ Low noise readout electronics
 - ◆ Clever alignment algorithms
 - ◆ Pattern recognition and track fitting

to realise the ultimate precision of these systems

- Silicon (pixel) detectors are finding lots of applications beyond Particle Physics

