Towards an FRG study of many-body localization

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based on work with Romain Daviet (Köln) and Vincent Grison (Paris)

ND, Phys. Rev. E, 2019, 100, 030102(R)
ND & R. Daviet, Phys. Rev. E, 2020, 101, 042139
V. Grison & ND, unpublished

Outline

- a (very brief) introduction to many-body localization
- finite-temperature phase diagram of a disordered 1D Bose gas
    - perturbative RG
    - non-perturbative FRG: “LPA”
    - non-perturbative FRG: “DE2”

Many-body localization

- free electrons in a disordered potential (1D) : all wavefunctions are localized

$$\sigma(T)=0 \text{ for any } T$$

- coupling to phonons (dephasing due to inelastic processes) implies a finite conductivity

$$\sigma(T)\sim\exp(-(T_0/T)^{1/2})$$

- electron-electron interactions lead to many-body localization

$$\sigma(T)=0 \text{ for } T\leq T_c$$

[Gornyi, Mirlin, Polyakov, PRL 2005]

[Basko, Altshuler, Aleiner, Annal. Phys. 2006]

1D disordered Bose gas

[Michal, Aleiner, Altshuler, Shlyapnikov, PNAS 2016]

- Lieb-Liniger model $H = \int dx\ \psi^\dagger(x)\left(-\frac{\partial_x^2}{2m}\right)\psi(x) + \frac{g}{2}\left(\psi^\dagger(x)\psi(x)\right)^2 \qquad \gamma = \frac{mg}{\hbar^2 n}$
  - $\gamma \to \infty$: mapping onto free fermions (the Bose-gas is insulating in the presence of disorder)
  - $\gamma < \gamma_0$, the Bose gas is superfluid at T=0 (Luttinger parameter K>3/2)

  [Giamarchi-Schulz PRB 1988]

- proposed phase diagram of the disordered Lieb-Liniger model

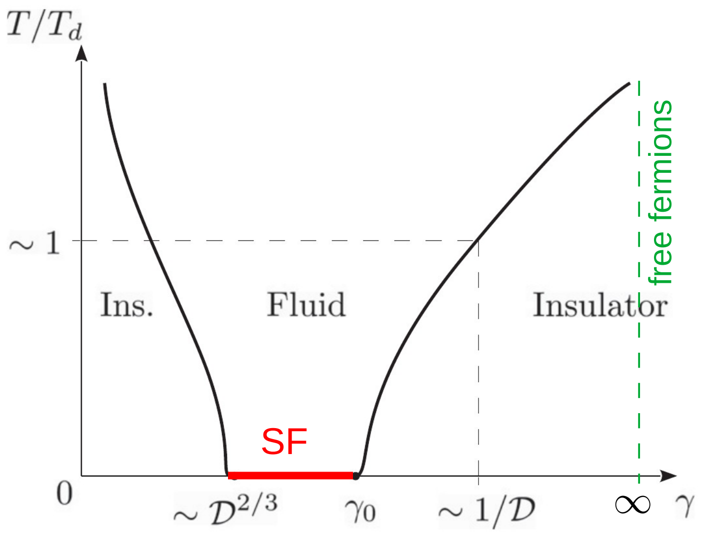


Fig. 2. Fluid–insulator transition of finite temperature repulsive bosons in a weak disorder, $\mathcal{D} \ll 1$ (see text).

1D Bose gas

- Hamiltonian $$H = \int dx \ \psi^\dagger(x) \left(-\frac{\partial_x^2}{2m}\right) \psi(x) + \frac{g}{2} \left(\psi^\dagger(x)\psi(x)\right)^2$$
- bosonization [Haldane 1981]

$$\psi(x) = e^{i\theta(x)} \sqrt{\rho(x)}$$
$$\rho(x) = \rho_0 - \frac{1}{\pi}\partial_x \varphi(x) + 2\rho_2 \cos(2\pi\rho_0 x + 2\varphi(x)) + \cdots$$

$$[\theta(x), \partial_y \varphi(y)] = i\pi\delta(x-y)$$

- Luttinger liquid: superfluid state (*T*=0) without broken U(1) symmetry

$$H_{\text{LL}} = \int dx \frac{v}{2\pi} \left\{ K(\partial_x \theta)^2 + \frac{1}{K} (\partial_x \varphi)^2 \right\}$$

$K =$ Luttinger parameter
$v =$ sound-mode velocity

LL action $$S_{\text{LL}} = \int dx \int_0^\beta d\tau \frac{v}{2\pi K} \left\{ (\partial_x \varphi)^2 + \frac{(\partial_\tau \varphi)^2}{v^2} \right\}$$

1D Bose gas in a random potential

$$H = H_{\mathrm{LL}} + \int dx\, V(x)\rho(x)$$

Disorder-average partition function of *n* replicas

$$\overline{Z^n} = \overline{\prod_{a=1}^{n} Z} = \int \mathcal{D}[\{\varphi_a\}]\, e^{-S[\{\varphi_a\}]}$$

with replicated action (sine-Gordon-like model)

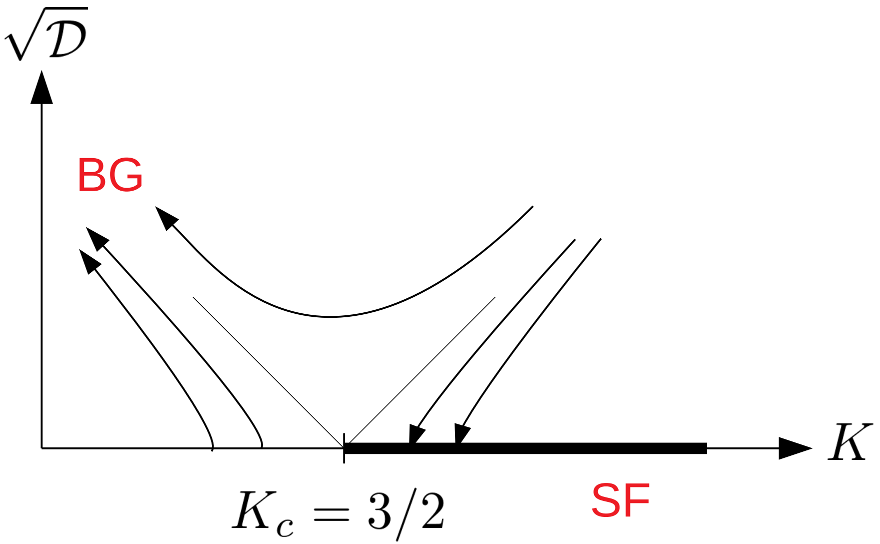
$$S[\{\varphi_a\}] = \sum_{a=1}^{n} \int dx \int_0^{\beta} d\tau \frac{v}{2\pi K} \left\{ (\partial_x \varphi_a)^2 + \frac{(\partial_\tau \varphi_a)^2}{v^2} \right\} - \mathcal{D} \sum_{a,b=1}^{n} \int dx \int_0^{\beta} d\tau\, d\tau' \cos[2\varphi_a(x,\tau) - 2\varphi_b(x,\tau')]$$

where *D* is the variance of the random potential: $\overline{V(x)V(x')} = D\delta(x - x')$

Perturbative RG T=0 [Giamarchi, Schulz PRB 1988, Ristivojevic *et al.* PRB 2012]

- phase diagram

$$\frac{dK}{dl} = -K^2 \frac{\mathcal{D}}{\pi v^2}$$
$$\frac{d\mathcal{D}}{dl} = (3 - 2K)\mathcal{D}$$
$$\frac{d}{dl}\left(\frac{v}{K}\right) = 0$$
(BKT flow)



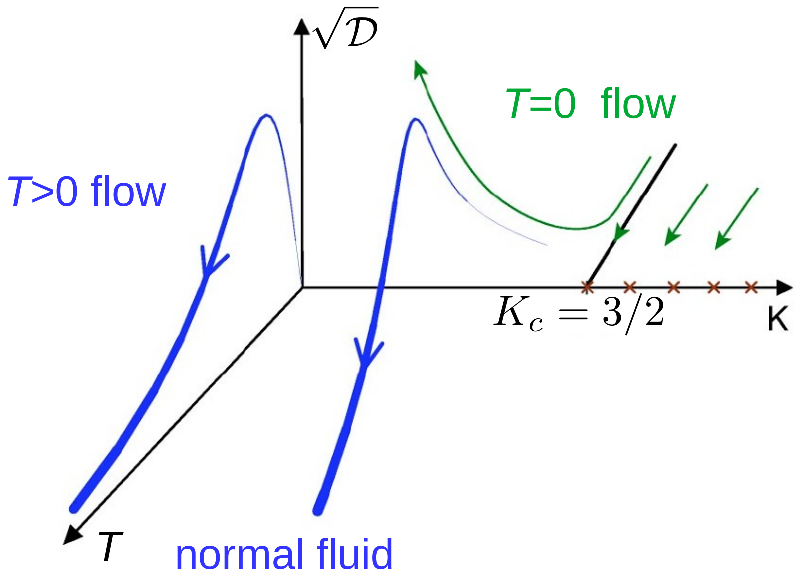
- Bose-glass phase [Fisher *et al.* 1989]

compressibility: $d\kappa/dl = 0, \quad \kappa > 0$

localized phase: $\xi_{\mathrm{loc}} \sim \mathcal{D}^{-\frac{1}{3-2K}}$

gapless conductivity: $\sigma(\omega) \sim \omega^2$

Perturbative RG $T>0$ [Glatz & Nattermann PRB 2004]



The fluid is in the normal phase at finite T: $\sigma(T)>0$

Functional renormalization group

$$S[\{\varphi_a\}] = \sum_{a=1}^{n} \int dx \int_0^\beta d\tau \frac{v}{2\pi K}\left\{(\partial_x \varphi_a)^2 + \frac{(\partial_\tau \varphi_a)^2}{v^2}\right\}$$
$$- \int dx \int_0^\beta d\tau\, d\tau' \sum_{a,b=1}^{n} \underbrace{\mathcal{D} \cos[2\varphi_a(x,\tau) - 2\varphi_b(x,\tau')]}$$

Renormalized disorder correlator $V(\varphi_a(x,\tau) - \varphi_b(x,\tau'))$

- classical disordered systems: the functional disorder correlator $V(\varphi_a - \varphi_b)$ may assume a non-analytic "cuspy" form that encodes the metastable states of the system and the ensuing glassy properties: pinning, "shocks" and "avalanches", chaotic behavior, aging, etc.

- long history in classical disordered systems… Fisher 1985, Narayan, Balents, Nattermann, Chauve, Le Doussal, Wiese, etc.

- non-perturbative (Wetterich's) formulation: Tissier & Tarjus 2004- (RFIM)

- truncation of the effective action $\Gamma_k[\{\phi_a\}] = \sum_a \Gamma_{1,k}[\phi_a] - \sum_{a,b} \Gamma_{2,k}[\phi_a, \phi_b] + \dots$

$$\Gamma_{1,k}[\phi_a] = \int dx \int_0^\beta d\tau \frac{v_k}{2\pi K_k} \left\{ (\partial_x \phi_a)^2 + \frac{(\partial_\tau \phi_a)^2}{{v_k}^2} \right\},$$

$$\begin{aligned}\Gamma_{2,k}[\phi_a, \phi_b] = \int dx \int_0^\beta d\tau\, d\tau' \Big\{ & V_k(\phi_a(x,\tau) - \phi_b(x,\tau')) \\ & + W_{1,k}(\phi_a(x,\tau) - \phi_b(x,\tau'))\partial_x\phi_a(x,\tau)\partial_x\phi_b(x,\tau') \\ & + \frac{1}{2} W_{2,k}(\phi_a(x,\tau) - \phi_b(x,\tau'))[(\partial_x\phi_a(x,\tau))^2 + (\partial_x\phi_b(x,\tau'))^2] \\ & + \frac{1}{2} W_{3,k}(\phi_a(x,\tau) - \phi_b(x,\tau'))[(\partial_\tau\phi_a(x,\tau))^2 + (\partial_{\tau'}\phi_b(x,\tau'))^2] \Big\}\end{aligned}$$

LPA approximation

DE_2 approximation

with initial conditions: $v_\Lambda = v, \quad K_\Lambda = K, \quad V_\Lambda(u) = 2\mathcal{D}\cos(2u), \quad W_{1,\Lambda} = W_{2,\Lambda} = W_{3,\Lambda} = 0,$

and $\frac{v_k}{K_k} = \frac{v}{K}$ [statistical tilt symmetry: $S_{\text{dis}}[\{\varphi_a\}]$ invariant in $\varphi_a(x,\tau) \to \varphi_a(x,\tau) + \alpha(x)$]

LPA for the two-replica effective action

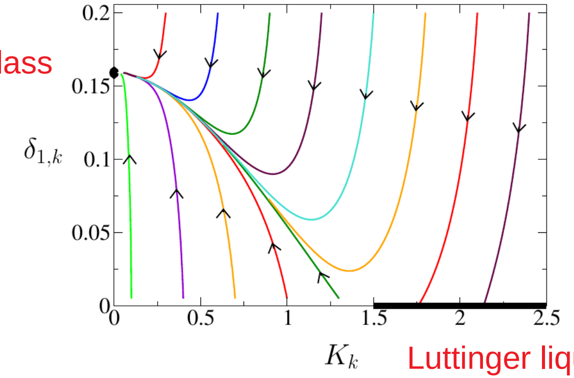
[ND & R. Daviet, Phys. Rev. E, 2020, 101, 042139]

$$\Gamma_{2,k}[\phi_a,\phi_b] = \int dx \int_0^\beta d\tau\, d\tau'\, V_k(\phi_a(x,\tau)-\phi_b(x,\tau'))$$

- flow diagram

$$\delta_k(u) = -\frac{K^2}{v^2}\frac{V_k''(u)}{k^3} = \sum_{n=1}^{\infty} \delta_{n,k}\cos(2nu)$$

Bose glass



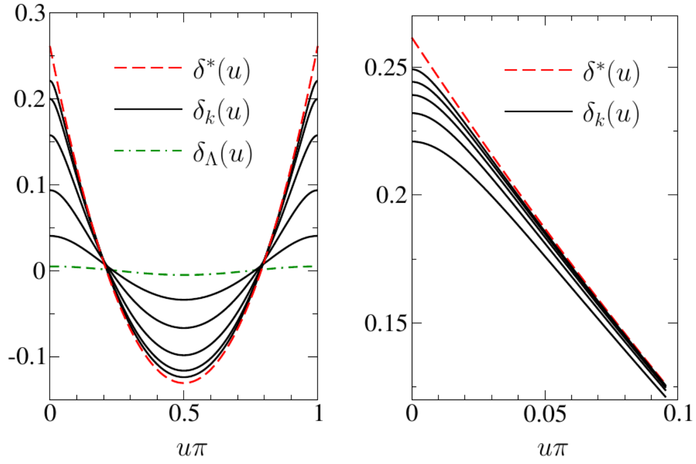
Luttinger liquid

- Bose-glass fixed point

$$K^* = 0, \quad K_k \sim k^\theta \quad \text{no quantum fluctuations, hence pinning}$$

$$\rho_{s,k} = \frac{v_k K_k}{\pi} \sim k^{2\theta} \quad \text{vanishing superfluid stiffness}$$

$$\delta^*(u) = \frac{1}{2a_2}\left[\left(u-\frac{\pi}{2}\right)^2 - \frac{\pi^2}{12}\right] \quad \text{for} \quad u \in [0,\pi]$$



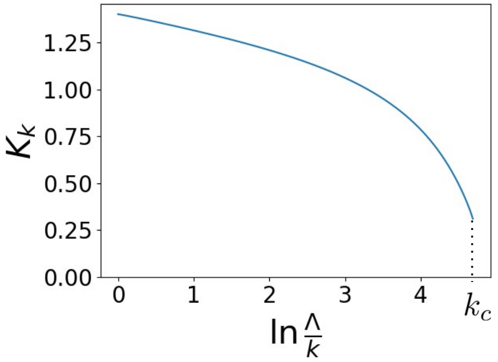
cusp and quantum boundary layer (controlled by $K_k \sim k^\theta$)

$\sigma(\omega) \sim \omega^2$

- normal fluid at T>0 (as in perturbative RG)

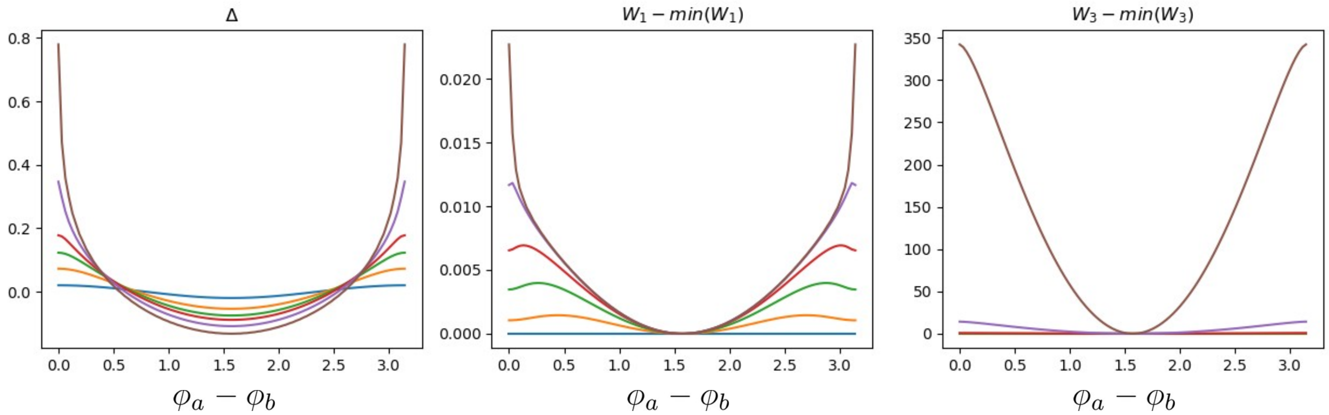
DE$_2$ for the two-replica effective action

- singularity in the flow at k_c



$$\xi = k_c^{-1} \qquad \text{localization length}$$

dimensionless potentials



reminiscent of the T=0 LPA fixed point but the cusp is more pronounced

- breakdown of the DE: $$\Gamma^{(2)}_{1,k<k_c}(q,i\omega) = \frac{v}{\pi K}\left(q^2 + \frac{\omega^2}{v_k^2}\right) + C_k|\omega|^\alpha \qquad \text{with} \qquad \alpha < 2$$
  $$\omega \sim |q|^{2/\alpha} \quad \Rightarrow \quad \text{non-superfluid gas (from Landau's criterion)}$$

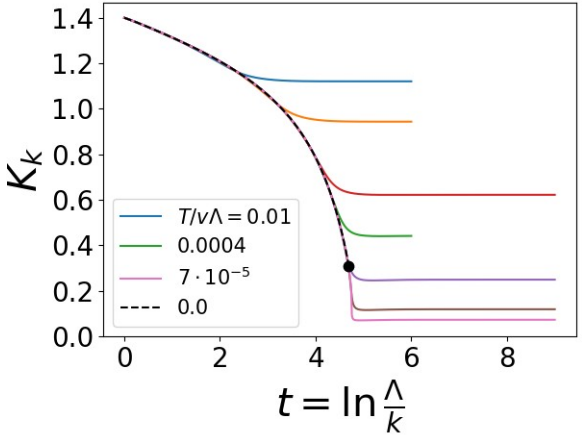
- flow for $k<k_c$? Blaizot—Méndez-Galain--Wschebor (BMW) approximation required

$$\Gamma^{(2)}_{1,k}(q,i\omega) = \frac{v}{\pi K} q^2 + \Sigma_k(i\omega)$$

- if we set K=0 for $k<k_c$, the flow reaches a cuspy fixed point at k=0 similar to the LPA fixed point

Finite-temperature flow

- regular flow for $T>T_c$



- singularity at $k\sim k_c$ for $T<T_c$: $\xi(T_c)$ remains finite

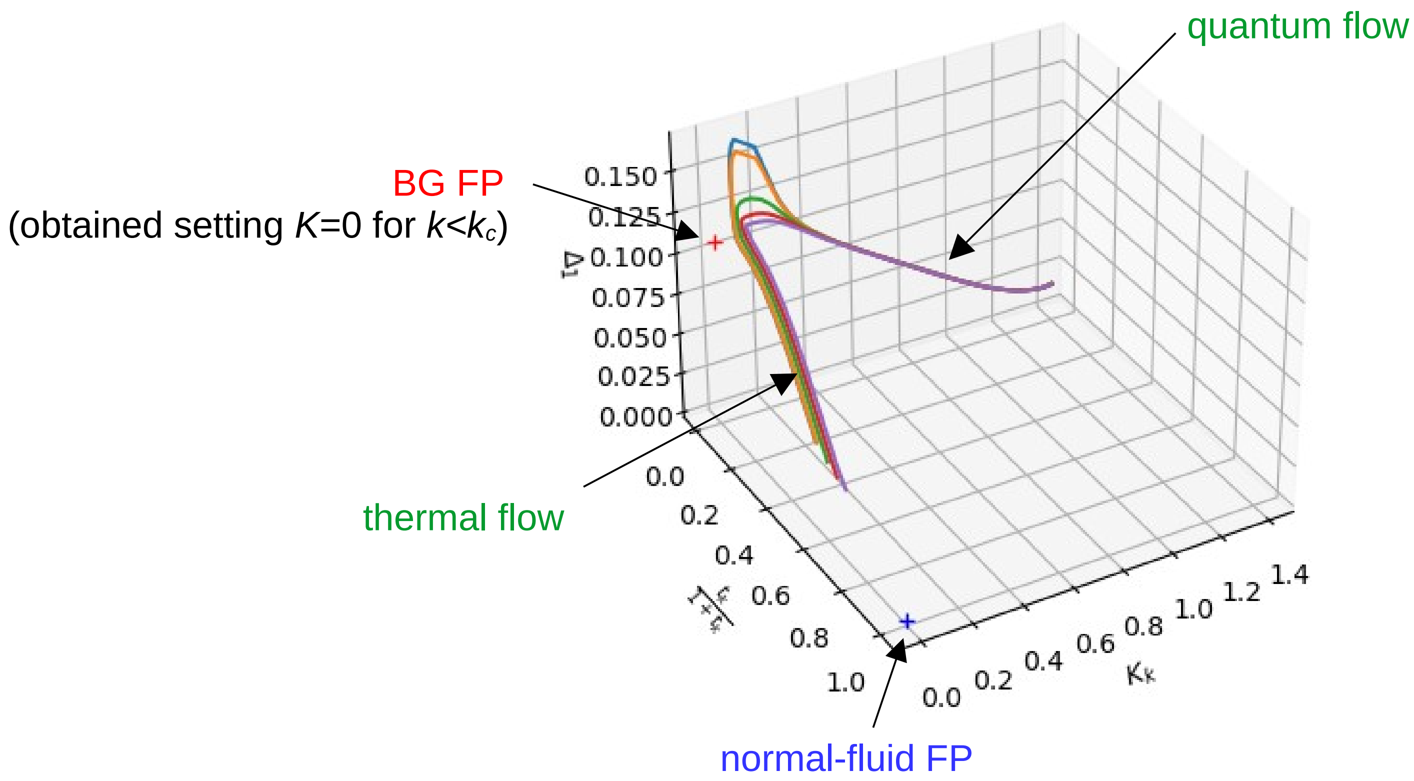
- quantum-classical crossover controlled by dimensionless temperature $\tilde{T}_k = \dfrac{T}{v_k k}$

  - quantum (T=0) flow: $\tilde{T}_k \lesssim 1$
  - classical flow: $\tilde{T}_k \gtrsim 1$
  - "phase transition" at $\tilde{T}_{k_c} \sim 1$ i.e. $\dfrac{T_c}{v_{k_c} k_c} = \dfrac{K}{v}\dfrac{T_c}{K_{k_c} k_c} \sim 1$ $\quad$ (with $k_c = \xi_{T=0}^{-1}$)

- coupling constants

$$K_k \,:\, \text{controls quantum fluctuations}$$

$$t_k = K_k \tilde{T}_k \sim \frac{1}{k} \,:\, \text{controls classical (thermal) fluctuations}$$



Flow diagram for T>Tc

- the “phase transition” at Tc exists only because $K(T=0,k\to k_c)$ has a finite value

- two possible scenarii

  - The singularity at k_c is a genuine property of the flow: there is a MBL transition at T_c in agreement with the scenario proposed by Michal *et al.* (PNAS 2016).
  - The singularity does not survive in a more elaborate treatment (e.g. BMW): T_c is a crossover temperature below which localization effect are strong.

Conclusion

- The full DE2 approximation to the disordered 1D Bose gas changes the picture radically.
- The prediction of an MBL transition (i.e. a finite-temperature fluid-insulator transition) by FRG is an open question that requires a more elaborate treatment (e.g. BMW).
- For further discussions, see poster by Vincent Grison