Interacting UV fixed points and conformal windows of 4D Super-Yang-Mills theories with matter

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Outline

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Conformal windows on semi-simple YM with matter

Non-supersymmetric case:

- NLO analysis of FPs and phase diagrams: [Bond and Litim, 2018].
- Valuable as model-building templates for BSM physics, e.g. [Bause et al., 2022; Hiller et al., 2019].
- Can we go beyond the large-N Veneziano limit?

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Supersymmetric case:

- First perturbative results ensuring AS: [Bond and Litim, 2017].
- AS requires gauge sector with AF! [Hiller et al., 2022a]
- First exact conformal windows (GY_1): [Bond and Litim, 2022].
- Steps towards UV-completing extensions of the MSSM: [Hiller et al., 2022b]

Advantages of supersymmetry

[Intriligator and Wecht, 2003; Novikov et al., 1983; Seiberg, 1995]

- Powerful non-perturbative tools, e.g.:
  - NSVZ β functions for gauge sectors,
  - non-renormalisation theorems for the superpotential,
  - *a*-theorem,
  - *a*-maximisation technique,
  - Seiberg duality.
- Exact (non-perturbative) investigation of conformal windows of FPs and critical exponents is possible without approximation schemes.

Goals of current and future works

- Extend previous analysis in the large-N Veneziano limit to NNLO, obtaining indications of conformal windows for large ϵ. (this talk!)
- Obtain exact non-perturbative parametric conformal windows and critical exponents of all other FPs.
- Search for (non-perturbative) interacting FPs in R-parity violating MSSM extensions including new quarks or leptons.
- Investigate phenomenological signatures of MSSM extensions with new SU(N) or U(1)' dark gauge sectors.

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Field content

- Semi-simple ($\mathcal{N}=1$) Super–Yang–Mills theory with gauge group $\mathrm{SU}(N_1)\times\mathrm{SU}(N_2)$.
- Chiral superfields (ψ,Ψ,χ,Q) with flavor multiplicities $(N_F,1,N_F,N_Q)$, charged according to

Matter	ψ_L	ψ_R	Ψ_L	Ψ_R	χ_L	χ_R	Q_L	Q_R
$SU(N_1)$	$\overline{\square}$	$\square$	$\square$	$\overline{\square}$	1	1	1	1
$SU(N_2)$	1	1	$\square$	$\overline{\square}$	$\overline{\square}$	$\square$	$\overline{\square}$	$\square$
Flavour	N_F	N_F	1	1	N_F	N_F	N_Q	N_Q

Charges and multiplicities of chiral matter.
Table from [Bond and Litim, 2022].

- Interaction given by Yukawa superpotential $W[\psi,\Psi,\chi]$ such that $W=y\,\mathrm{Tr}[\psi_L\Psi_L\chi_L+\psi_R\Psi_R\chi_R]$.

FP structure of the theory

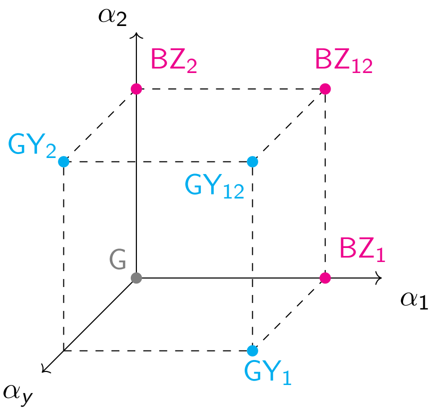

Fig. 1: Illustration of the 7 fixed points that may show up in such theories:

- the free Gaussian (G) FP, in gray,
- the 3 interacting Banks-Zaks (BZ) FPs, in magenta, and
- the 3 interacting gauge-Yukawa (GY) FPs, in cyan.

Theory parameters

- Theory characterized by field multiplicities (N_1, N_2, N_F, N_Q), with couplings

$$\alpha_{1,2} = N_{1,2} \left(\frac{g_{1,2}}{4\pi} \right)^2 \quad \text{and} \quad \alpha_y = N_1 \left(\frac{y}{4\pi} \right)^2 ,$$

  where $g_{1,2}$ are the usual gauge couplings.

- Physical quantities will be homogeneous functions of the multiplicities: parametrise with the three ratios

$$R := \frac{N_2}{N_1} , \qquad \epsilon := \frac{N_F + N_2}{N_1} - 3 = \frac{N_F}{N_1} + R - 3 , \quad \text{and}$$

$$P := \frac{N_1}{N_2} \frac{N_Q + N_1 + N_F - 3N_2}{N_F + N_2 - 3N_1} = \frac{1}{\epsilon R} \left(\frac{N_Q}{N_1} + \epsilon + 4 - 4R \right) .$$

Bounds on parameters

- Imposing $N_1, N_2, N_F, N_Q > 0$ result in the bounds

$$R > 0 \ , \quad R < 3 + \epsilon \ , \quad \text{and} \quad 4(R-1) > \epsilon(1 - RP). \tag{1}$$

- Parameters (ϵ, R, P) become continuous in the Veneziano large-N limit.

- In the $\epsilon \to 0^{\pm}$ regime, (1) reduce simply to

$$1 < R < 3 \quad \text{and} \quad P \text{ finite} \, . \tag{2}$$

Example of flow with GY fixed points

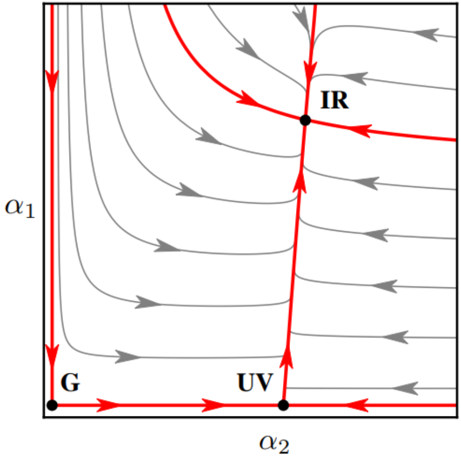

Fig. 2: Flow on the non-trivial Yukawa nullcline for $\epsilon, P < 0$. Figure from [Bond and Litim, 2017].

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Beta functions in NLO in perturbation theory

- NLO in perturbation theory $\Rightarrow$ leading order ($O(\epsilon^1)$) results in ϵ.
- NLO: 2-loop $\beta_{1,2}$ and 1-loop β_y.
- Let $\beta_i := d\alpha_i/d\log\mu$, with μ being the RG scale. Then:

$$\beta_1^{(1+2)} = \underbrace{2\alpha_1^2\epsilon}_{\text{1 loop}} + \underbrace{2\alpha_1^2\Big[(6+4\epsilon)\alpha_1 + 2R\alpha_2 - 4R(3+\epsilon-R)\alpha_y\Big]}_{\text{2 loop}},$$

$$\beta_2^{(1+2)} = \underbrace{2\alpha_2^2 P\epsilon}_{\text{1 loop}} + \underbrace{2\alpha_1^2\Big[(6+4P\epsilon)\alpha_2 + (2/R)\alpha_1 - (4/R)(3+\epsilon-R)\alpha_y\Big]}_{\text{2 loop}},$$

$$\beta_y^{(1)} = 2\alpha_y\Big[-2\alpha_1 - 2\alpha_2 + (4+\epsilon)\alpha_y\Big].$$

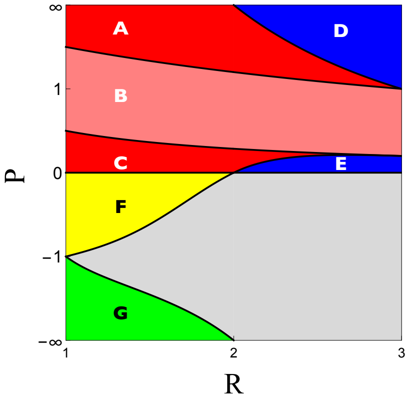
NLO fixed points and asymptotic safety

FP	BZ_1	BZ_2	GY_1	GY_2	BZ_{12}	GY_{12}
α_1^*	-$\frac{\epsilon}{6}$	0	$\frac{-\epsilon}{2(R^2-3R+3)}$	0	$\frac{RP-3}{16}\epsilon$	$\frac{R^2(R-2)P-4R+3}{(R-1)(3R^2-8R+9)}\frac{\epsilon}{2}$
α_2^*	0	-$\frac{\epsilon P}{6}$	0	$\frac{-\epsilon PR}{2(4R-3)}$	$\frac{1-3RP}{16R}\epsilon$	$\frac{R(R^2-3R+3)P-R+2}{(R-1)(3R^2-8R+9)}\frac{\epsilon}{2}$
α_y^*	0	0	$\frac{1}{2}\alpha_1^*$	$\frac{1}{2}\alpha_2^*$	0	$\frac{1}{2}(\alpha_1^*+\alpha_2^*)$

Table: NLO values of couplings for each fixed point.

- AS requires $P < 0$! Sign of ϵ determines free and safe sectors.
- In all cases, GY_{12} is a fully attractive IR sink!
- Possible interacting UVFPs: GY_1 and GY_2.

Regions of existence and relevancy of all FPs



case	region	eps	complete asymptotic freedom (eps < 0, P > 0)							UV	IR	
			G	BZ1	BZ2	BZ12	GY1	GY2	GY12		0	y
1	A	-	1---	2--+	3-++		5-++	6-++	7+++	1 AF	3	7
2	B	-	1---	2--+	3--+	4-++	5-++	6-++	7+++	1 AF	4	7
3	C	-	1---	2-++	3--+		5-++	6-++	7+++	1 AF	2	7
4	D	-	1---	2--+	3-++		5-++	6+++		1 AF	3	6
5	E	-	1---	2-++	3--+		5+++	6-++		1 AF	2	5

case	region	eps	asymptotic safety or effective theories (P < 0)							UV	IR	
			G	BZ1	BZ2	BZ12	GY1	GY2	GY12		0	y
6	F	-	1--+	2-++			5-++		7+++	5 AS	2	7
7	G	-	1--+	2-++			5+++			eff.	2	5
8	H	-	1--+	2-++			5+++			eff.	2	5
9	F	+	1--+		3-++			6+++		eff.	3	6
10	G	+	1--+		3-++			6-++	7+++	6 AS	3	7
11	H	+	1--+		3-++			6+++		eff.	3	6

Fig. 4: Parametric conformal windows and relevancy of all FPs in NLO.

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Effect of finite ϵ on conformal windows: GY_1

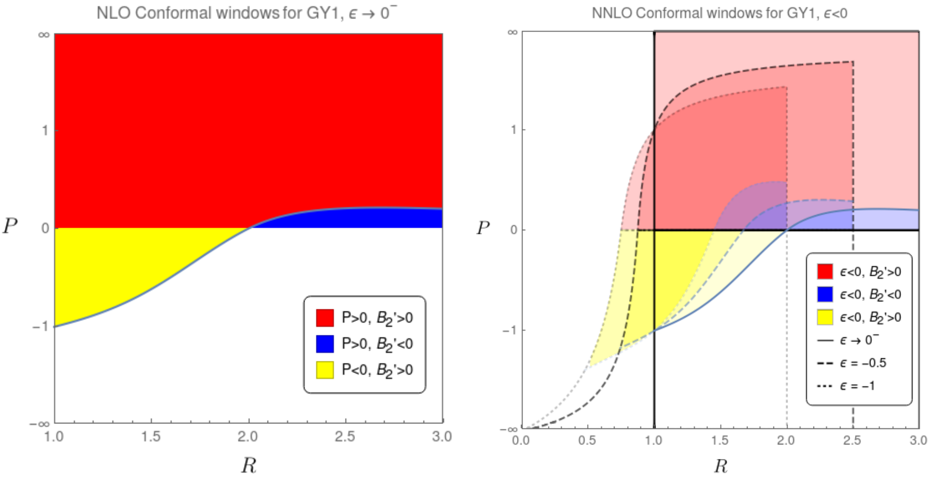

Fig. 5: Parametric conformal windows of GY_1 in NLO and NNLO.

Effect of finite ϵ on conformal windows: GY_2

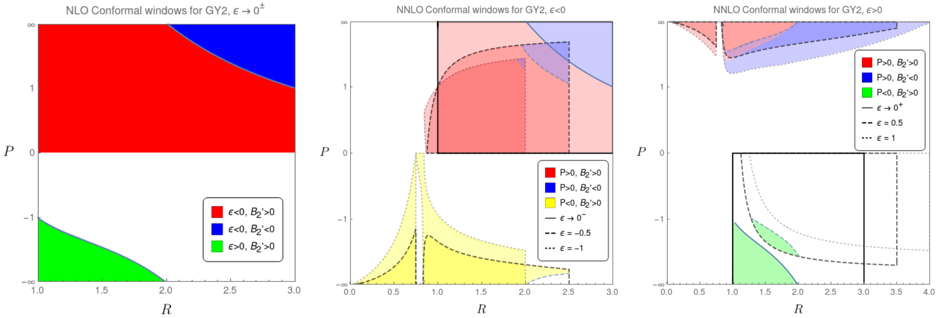

Fig. 6: Parametric conformal windows of GY_2 in NLO and NNLO.

Effect of finite ϵ on conformal windows: GY_{12}

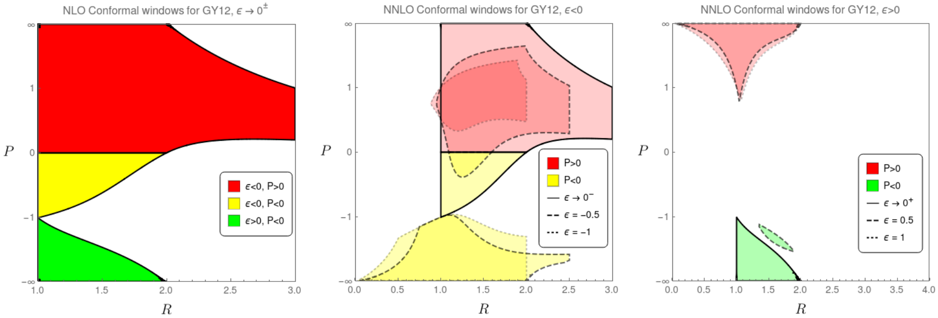

Fig. 7: Parametric conformal windows of GY_{12} in NLO and NNLO.

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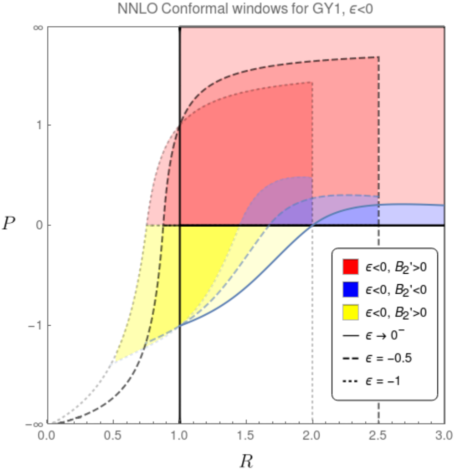
Non-perturbative superconformal window of GY_1



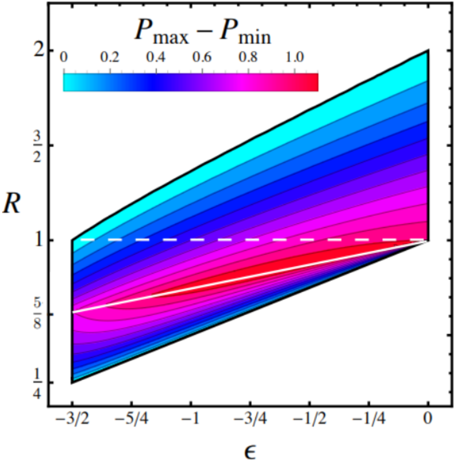


Fig. 8: NNLO and exact parametric conformal windows of GY_1.

Closed infinite-order β functions

- Gauge NSVZ β-functions are given by [Novikov et al., 1983]:

$$\beta_1 = \frac{2\alpha_1^2}{F(\alpha_1)}\left[N_F(1-2\gamma_\psi) + N_2(1-2\gamma_\Psi) - 3N_1\right] \ ,$$
$$\beta_2 = \frac{2\alpha_2^2}{F(\alpha_2)}\left[N_F(1-2\gamma_\chi) + N_1(1-2\gamma_\Psi) + N_Q(1-2\gamma_Q) - 3N_2\right] \ ,$$

where $F(\alpha) = 1 - 2C_2^G\alpha$ and C_2^G is the quadratic Casimir in the adjoint.

- Non-renormalization theorem for Yukawa superpotential: non-perturbative Yukawa β-function simply reads

$$\beta_y = 2\alpha_y\left[\gamma_\psi + \gamma_\Psi + \gamma_\chi\right] \ .$$

Future directions

- ~~Extend the analysis in the large-N Veneziano limit to NNLO, obtaining indications of conformal windows for large ϵ.~~
  – NNLO result is qualitatively compatible with exact analysis of GY_1!
- Obtain exact non-perturbative parametric conformal windows of FPs of 4D supersymmetric templates for particle physics.
- Search for (non-perturbative) interacting FPs in R-parity violating MSSM extensions including new quarks or leptons.
- Investigate phenomenological signatures of MSSM extensions with new SU(N) or U(1)' dark gauge sectors.

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Thanks for your attention!